Advanced Structural Analysis

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Module No. # 2.3

Lecture No. # 09

Review of Basic Structural Analysis - 2

Good afternoon. This is our ninth session, second module - Review of Basic Structural Analysis. We are still doing the force methods, but we hope to complete it in this session. We will cover approximate methods of lateral load analysis and this is in part four of the book on Structural Analysis.

Now, I have given you an assignment and I would like to help you solve this assignment. So, take a look at this truss. What is a degree of indeterminacy in this truss? It is one, right. Is it internal or external?

Internal.

Clearly, internal. Now, the problem given to you is little more complicated. You have a cable; it is a cable suspended truss; now, what is a degree of indeterminacy? Three?

Two.

Two? See, you have an additional unknown force. What is that unknown force? Force in the cable. So, you have got a degree of indeterminacy 2. How do you solve this problem? Can you treat the cable as another truss element?

No.

Why not?

[Noise – not audible] (Refer Slide Time: 01:42)

Yeah, but in this case, it is obvious that it is going to be in tension. So, you can. So, in this case, you can treat the cable as another truss member. You can number it 7. How do you go about solving this problem?

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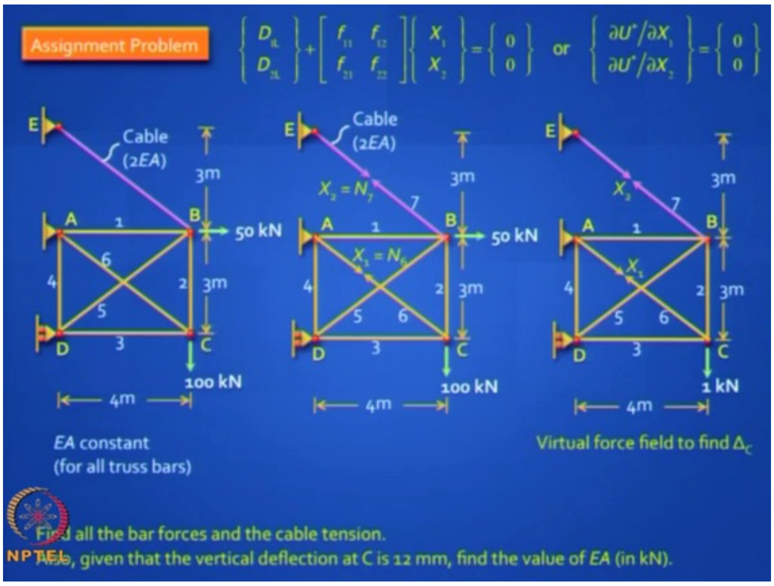

Now, the question asked is – analyze this truss, find the bar forces and the cable tension plus a deflection question. Can you find the vertical deflection at…(Refer Slide Time: 02:08)

Given that the vertical deflection at C is 12mm, find the value of EA - the actual rigidity, and for the cable, you are told the value of the actual rigidity is two times that of each bar in that truss.

Let us see how we can do this problem. You need to choose two redundants. One - clearly you can choose the cable itself and the other one could be any bar. Let us say bar number 6, the diagonal. You cut these two elements and you can number them. Let us say, X_1 is the unknown force in the bar number 6 and X_2 is the unknown force in the cable which is element number 7. Is it clear?

Assume tension positive. You know how to solve this. There are many ways of solving it. Basically, you have to invoke compatibility conditions. If you do the method of consistent deformations, you have to write it in the flexibility format or you could use the theorem of least work. You are familiar with this, you will do it. Now, this part is easy.

How will you find the vertical deflection at C? You give a unit displacement at C in the same system; unit force at C, unit load at C, but then that is statically indeterminate. Does it mean that you have to solve the statically indeterminate problem all over again for this condition or is there an easier way out?

This is what you would be doing. Solve again X_1 and X_2 for this problem, find the bar forces and so on, but that is a lot of work. Can we make it simpler? I have entered at this when we covered the principle of virtual work - the unit load method. There is a tremendous power in that method. What is a…

[Noise – not audible] (Refer Slide Time: 04:21)

You need to analyze this truss; this truss is statically indeterminate. You need to solve a 2 by 2 matrix to get X_1 and X_2 in this part. What is a simple way out?

[Noise – not audible] (Refer Slide Time: 04:39)

Anybody? [Noise] This really reduces the problem. It is mind boggling, yes? There is no zero for (()), those you have to do anyway, but it is indeterminate. Anybody?

[Noise – not audible] (Refer Slide Time: 05:00)

Just go back to the principle of virtual work.

What was the requirement of the force field? There it should be statically admissible. We are looking at that field. Is it required that system should also be kinematically admissible? No. So, can you take advantage of this? How many statically admissible force fields can you draw for this?

Infinite.

Infinite. So, choose the one which is easy. What is the easiest one?

[Noise – not audible] (Refer Slide Time: 05:37)

Assume X_1 and X_2 to be zero. All right. I mean, you can use, choose any numbers, any combination that you want, but it cannot be easier than this.

Now, you are left with finding five unknown bar forces, which is very easy to do. Then, you invoke the unit load method, that is it. This is something which most students do not know. It is a power inherent in this principle of virtual forces. Is it clear? Can you do this problem?

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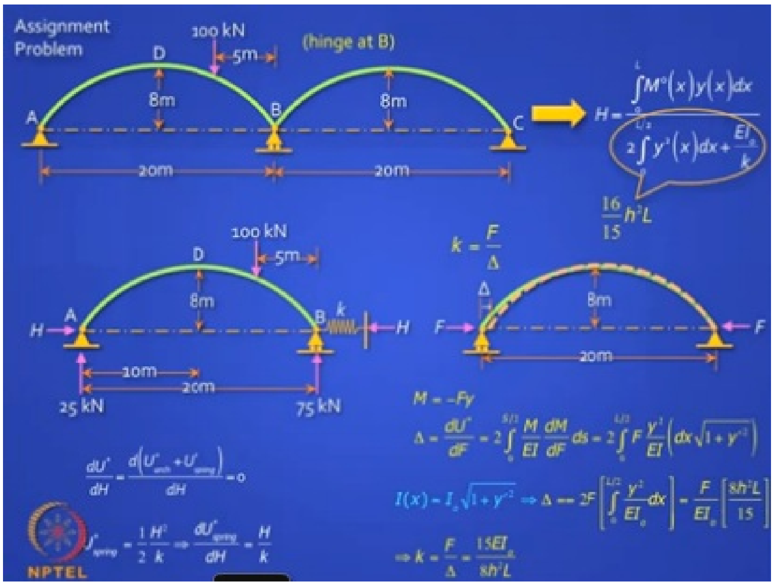

Now, let us look at another problem, which you have in your assignment. In this problem, you have a hinge. Please note there is a hinge at B. There is an internal hinge at... How do you solve this problem? The problem is you have to analyze, where you need to analyze both the arches; both are parabolic arches. You need to draw the bending moment diagram etcetera for the first arch ADB. You do not worry about the BC arch. So, how do you solve this problem? Can you simplify it?

[Not audible] (Refer Slide Time: 06:53)

No. You have two arches; you learnt how to solve one arch. You got two joints together like, what are they called? what twins are they? Siamese twins. Yes. So, what do you do?

[Not audible]

You separate them. When you separate them, how would you separate them?

[Noise – not audible] (Refer Slide time: 07:17)

You need to separate the two arches. You see the second arch is completely unloaded.

[Noise – not audible] same forces, same horizontal forces. We give the horizontal force from BC (Refer Slide Time: 07:33).

But how do you find the horizontal force? In the arch problem you studied till now, there were two hinged arches; that support B was a hinged support, not moving. Now, you have a roller there, but you have a restraint coming from the other arch.

So, can you separate out that first arch?

[Not audible]

No. Beam is still going to move. How do you… But it is not free to move because, the other arch does not want to be moved. So, how do you deal with this?

[Noise - not audible] (Refer Slide Time: 08:13)

Whatever method you do, it is not easy. Yes, we put a spring. That is right. You put a spring. Now, you can use energy method etcetera provided you know what that spring stiffness k is. So, how do you find that spring stiffness k? Yes?

Material property. [Noise]

Not displacement. You take the other arch and you push it horizontally at B, by some force say, F and see how much it moves, delta. By the way, look at this arch.

The vertical reactions are statically determinate; horizontal reaction H is not known. If that spring stiffness were to be infinity, you know how to solve the problem. If it is 0, H is going to be 0 because it is simply supported. But the value of k is going to determine the value of H. It will lie between these two bounds, 0 and the value you get when k is infinite. How do you get k? Well, take the second arch which is the same as the first arch and apply a force F. It is going to move. Can you see moving in that picture?

Now, how do you find this delta? How do you find? You can use any of the methods, but the easiest would be to use the energy method itself. So, do you agree k is F by delta? If you use energy method, you will find that the bending moment at any section in that arch is only due to F. So, it is F into y, it is a hogging moment minus Fy. Do you agree? If you bring in Castigliano’s theorem, delta is dU star by dF, which you can write in this form. Agree? You need to integrate only on one half because it is a symmetric arch.

Now, M is nothing but, minus Fy and dM by dF will therefore be minus y. So, want to take this format. Now, you have to use a trick, which we learnt in the last session. Yeah, you make that assumption I is equal to I_0 square root of one plus y dash square and that expression will delta will now simplify. I am actually, leaking out the solution to you; it will simplify to this quantity and this quantity is a constant for the arch. (Refer Slide Time: 11:00)

You remember we found this and that takes as value. Remember it was 4 by 8 or something. So, it is 8 h square L by 15. So, you have got an expression for k in terms of EI_0.

Now, what do you do? Now, you come back to the original arch. How do you still solve this?

[deflection we are at beam much be…[Noise - not audible] (Refer Slide Time: 11:26)]

Let us stick to the energy method which is what you use for solving the parabolic arch. So, you have to go back to the derivation of how you found capital H. How did you find it?

[Not audible] (Refer Slide Time: 11:38)

You began with complementary strain energy, but now you realize that the complementary strain energy has two components. One, that of the arch due to the bending of the arch and the second is due to the spring. Do you agree? That is the addition. This is how you should think because real life problem are like this. You want to simplify, this is how you deal with it. Otherwise, you have to work with both the arches.

Now, if you run that derivative through the second term you agree that the spring is half H square by k because the force in this spring is H. You agree? The derivative turns out to be H by k, which is nothing but, the deflection in this spring.

k you already know. So, you can... Now, use this concept and get an expression for H. So, go back to the original theory. You will find that when there was no spring, this term was not there. EI_0 by k was not there but, this k was infinity. So, you can…

I want you to derive it from first principle. You will find you can do this and you know the denominator; you can add up this value of k here with the value that we derived for the parabolic arch.

Now, it is easy to do. You need to first chalk out the road map given a problem and then proceed. So, it is an interesting problem. So, will you do this? I have given you two problems both relative force methods.

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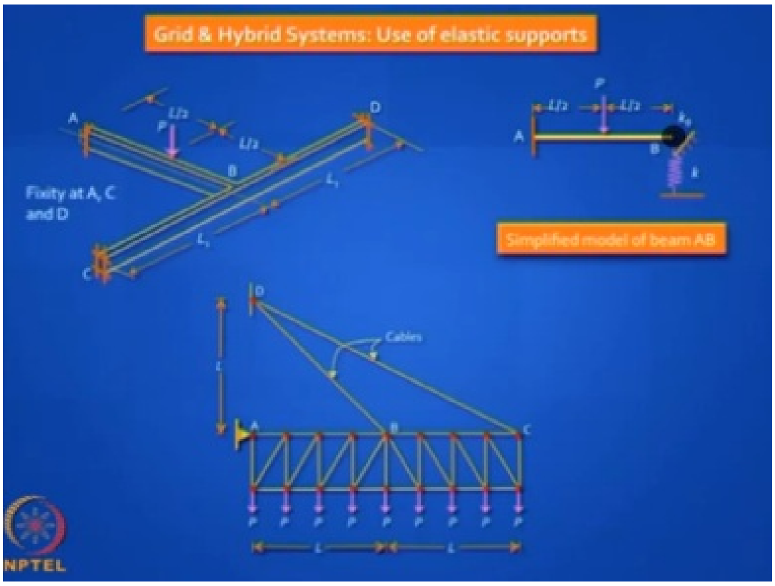

Now, in this session, before we take approximate lateral load analysis, let us complete some discussions on elastic supports. Now, take a look at this. It is a little complicated. We will study this in grids later but, you can always try to separate out elements but, you must put the right restrains when you separate them out. For example, if I want to separate out that cantilever AB, what should I put at B to simulate the effect of connecting beams CBD? One spring?

[Noise - not audible] (Refer Slide Time: 14:00).

You need to put two springs; rotational spring and translational spring and you need to figure out those stiffness values, which are simply cable B. You apply a force vertically downward at B and find out its deflection and the stiffness turns out to be k into… It is about 48 L cubed by EI and so on. The rotational spring you have to apply a unit torque at the point B and so on. I want you to get the concept.

You can handle many problems like this. Similarly, you can have a cable suspended truss and so on.

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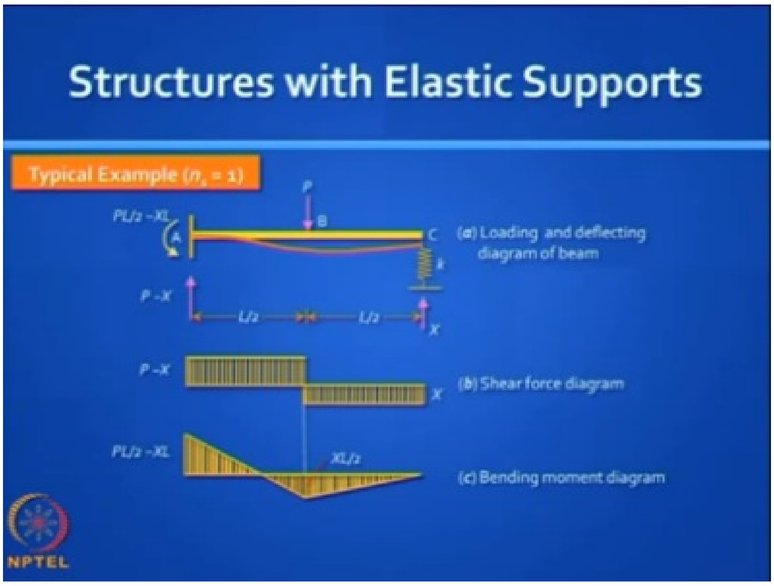

Let us take a very simple problem. You have a cantilever. It is not a prop cantilever because the prop is elastic. You have a spring there of known stiffness k. How will it deflect?

These are the… Let us say the force in that spring is X and so the other reaction will be P minus X and the bending moment. So, X is a single redundant. How will it deflect? It will deflect like that; C may go down a bit and so on. What will be the shear force in bending moment diagram? Something like that. (Refer Slide Time: 15:21). Clear? Easy to draw.

Now, it will be interesting to find out how X is related to the spring stiffness. Can you guess?

k equal to zero, X equal to zero.

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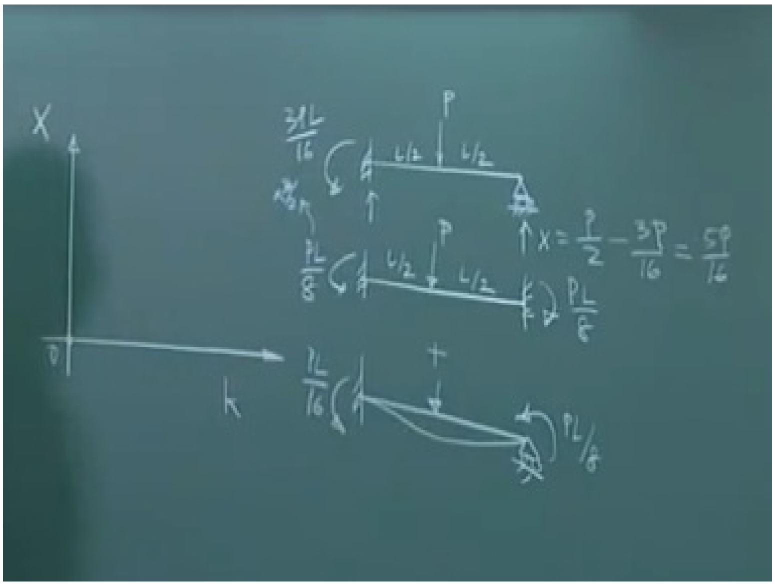

So, if you have to prop k with X, how would this figure look?

k is infinity…(Refer Slide Time: 15:56)

So, one point is here. If k is infinity, it is P by 2. Will it be P by 2? k is infinity means you are dealing with this situation. You know the answer for this. You know the fixed moment for a prop cantilever. We will need this again and again in future session but, let us go back to fundamental. Let us say it is a fixed beam. You remember the answer for this. This is P. This is L by 2, L by 2. What are these fixed moments? P L by 8. You can prove this by conjugate beam method. So, remember this is P L by 8 and this will be what?

[Noise – not audible] (Refer Slide Time: 17:01)

You need to take this beam. We will discuss this later. Reverse this moment. This is clockwise hogging. So, if I apply a loading like this, then when I add up these two, I get zero moment. When I do this, it will take a shape. How?

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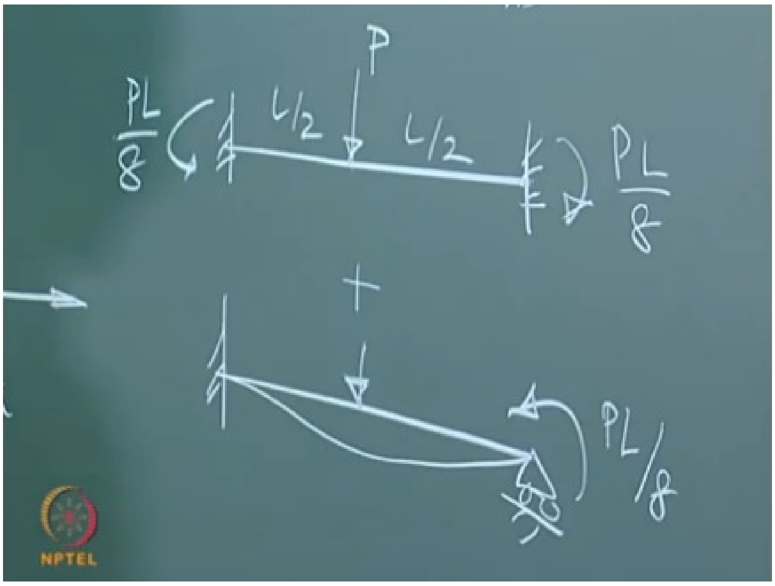

It will take a shape like that. What do you think this moment will be? This is going to be how much? You studied this and you will study it. This will be always half this value for a prismatic beam. So, take it right now, we will prove it in the next class in a day or two.

You will find that whenever you have a kind of symmetric loading this value will be always this value into one and half time its conventional fixed moment. So, how much will it be?

3PL by 16.

3PL by 16. Once you know this, you can find your reaction. How much will this be? When this moment was not there it will be P by 2, P by 2. So, this is X. So, it is P by 2 but, due to the moment, this will increase and this will reduce by how much? This divided by…(Refer Slide Time: 18:52). That is, 3 P by 16. How much do you get? 5 P by 16. So, as k tends to infinity, you cannot exceed this value. This value will be 5 P by 16.

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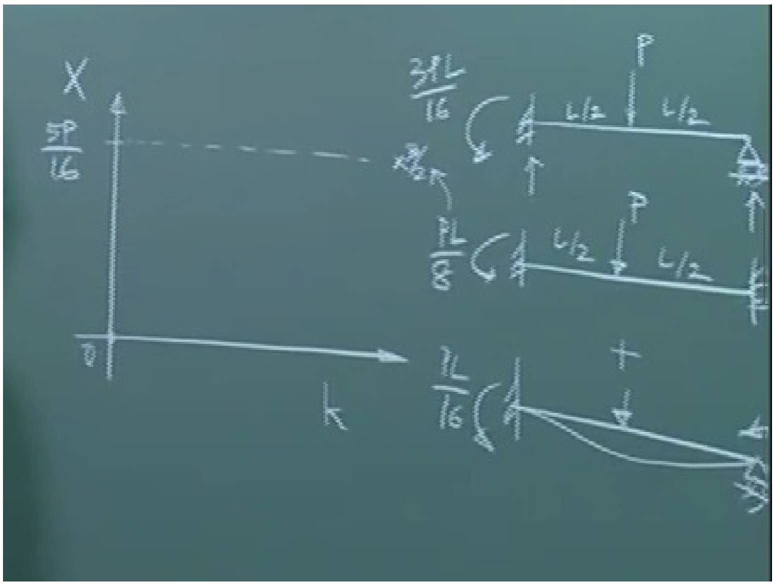

How do you think this moved from 0 to 5 P by 16?

Non linear way.

Some non linear way. Can you generate an equation? So, let us look at that. It is interesting to do these kind of problems. You can use the energy method. Remember there are two energies here.

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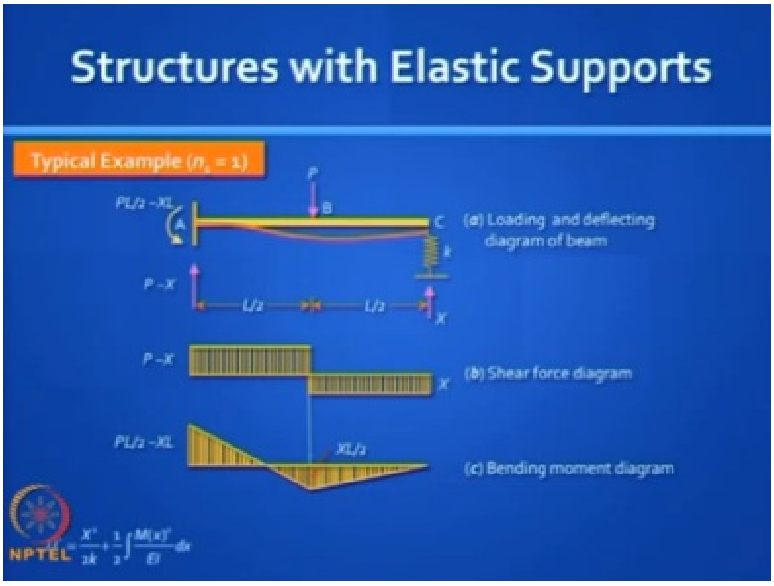

One from the beam and one from the spring; just like we did in the arch with a spring problem. Invoke the theorem of least work. Do you agree to these equations? It is a straightforward equation.

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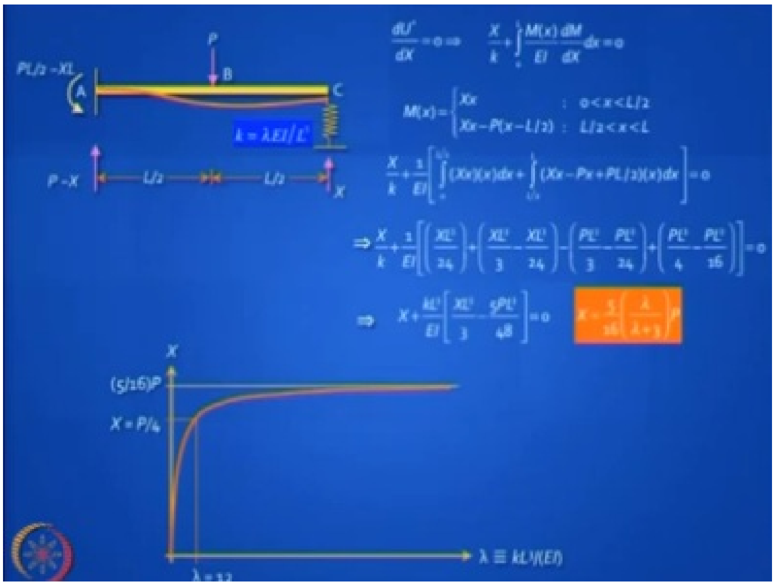

Let us assume you can do all these integrations. You see this part, you have to write the expressions for bending moment and you expand it and you can solve it. Finally, you will get an expression, which is a function of this k and EI.

Let us say k is lambda times EI by L cube. In other words, I am trying to relate the spring stiffness with the beam stiffness. The stiffness parameters in the beam are related to EI by L. So, let the spring stiffness be lambda times EI by L cube. I have chosen lambda in in a manner that it is non dimensional. So, it will be interesting to plug this relationship and you will find the relationship is non-linear as you rightly said. It will asymptotically hits the peak value of 5 P by 16. For example, if lambda is 12, X turns out to be P by 4. So, this is a complete way of understanding that spring type behavior.

So, it is somewhere…(Refer Slide Time: 20:55)

The actual beam is somewhere between a cantilever and a prop cantilever.

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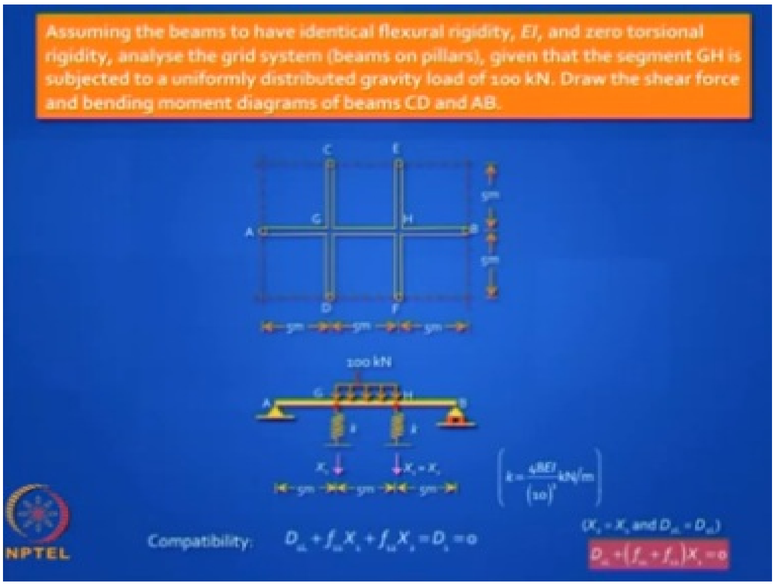

Let us take up more difficult problem and we will stop this section with that. Let us say you have a hall or room with a grid of beams like this. It is symmetric, a grid of beams. So, you have one long beam from A to B and two shorter beams and let us say they are simply supported. That means, they are resting on pillars; they are non monolithic supports.

Assuming the beams to have identical flexural rigidity EI, which comes from there having the same cross section, same width and same depth, and zero torsional rigidity, which comes from the fact that there are concrete cracks easily. Analyze the grid system given that the segment GH is subjected to a uniformly distributed load of hundred kilonewton. Draw the shear force and bending moment diagrams of beams CD and AB.

Now, here I am only going to give you the concept and I request you to go through the detailed calculations at your leisure. It is all given in the book. So, conceptually how will you deal with this problem? It looks complicated, separate out. Which one will you separate out?

CD and EF from AB.

Which do you think is supporting which?

AB is supporting…[Noise – not audible] (Refer Slide Time: 22:31)

AB is supporting or CD and EF are supporting?

All are interconnected. We cannot say.

You cannot say or can you say for sure?

Cannot. we cannot say.

See, GH is loaded. What is your hunch? What is your intuition?

AB is supporting CD and EF. CD and EF are supporting.

CD and EF are supporting because they are stiffer. See the stiffer fellows will support the… You know they are not so stiff beam. Cannot you feel it? Ultimately, the deflections are the same. So, it does not matter.

Let us say you made a mistake. It will show up with a negative sign in your reaction but, it is good to get it indubitably right straight. Do you think this is a good approach? That means, I have replaced the connecting beams; CD and EF with two identical springs. This is a loading. The GH is on AB. So, I have to take out AB separately. Do you agree? This is perfect. Now, I need a value of k. What is the value of k? k is the flexural stiffness of the beams CD and EF. So, how do I do that?

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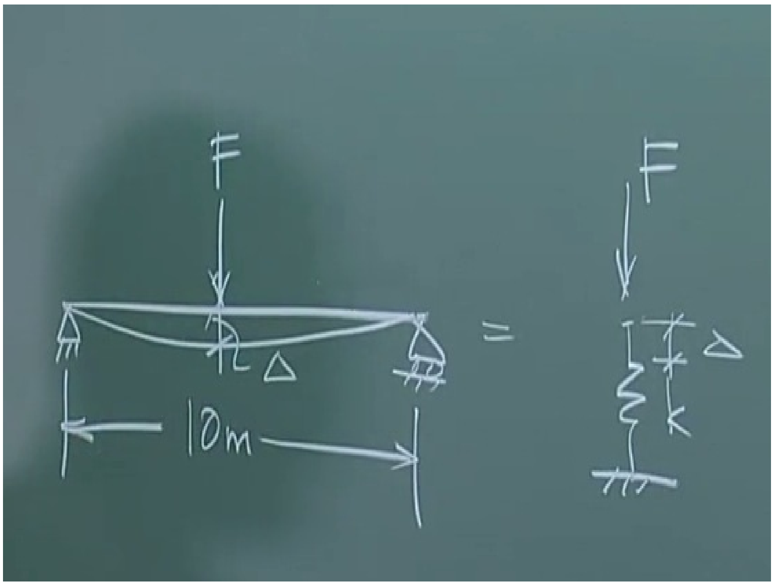

Both of them are simply supported beams. What is a span of the beam?

[Noise] 15 meters.

Total (()); I am talking of CD the one (()) we are going to replace. They are 10 meters. If I want to replace this with one spring k, I apply say, some force F; this is going to deflect. Let us say this is delta. Let us say this comes down by the same delta and that is how I replace the entire beam with a spring.

You see the power in this method. Is there a formula for delta that you can easily derive what is delta?

[Noise – not audible] (Refer Slide Time: 24:56)

You do not remember?

[Noise]

That is right. So, if the load is F you should say F. F into 10 cube by 48 EI. You are right. So, k turns out to be? FL, that is it. So, you can do that. It does not take you much time.

You got that and now, you can analyze the beam. So, the compatibility equation is, the deflection at either G or H. Your compatibility equation is, let us say X_1 and X_2 are the forces at the bottom of the spring, the reaction is - obviously, X_1 is going to be equal to X_2. So, let us cut the spring at the base. Let us see how much it moves. It should not move. There should be no separation between the cut end. Your compatibility condition is D_{1L}. That is, the deflection here caused by this loading in the cut springs plus F_{11} X_1 plus the F_{12} X_2; this must be equal to 0. That is your conventional compatibility equation. You can work out all those quantities and taking note of the fact that X_1 is equal to X_2, it reduces to that equation.

So, I will quickly show you how you can do that.

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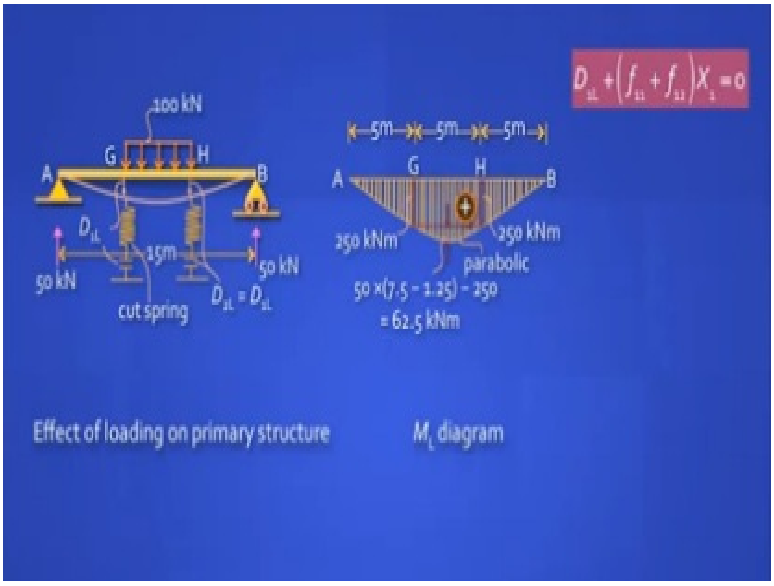

You need to now find out D_{1L}, $F_{11,}$ and F_{12}. First thing is you need the bending moment diagram when X_1 and X_2 are 0, in the primary structure. That is easy to do. You have a UDL, you have a parabola in the middle, and you have two straight lines. Those areas are easy to compute.

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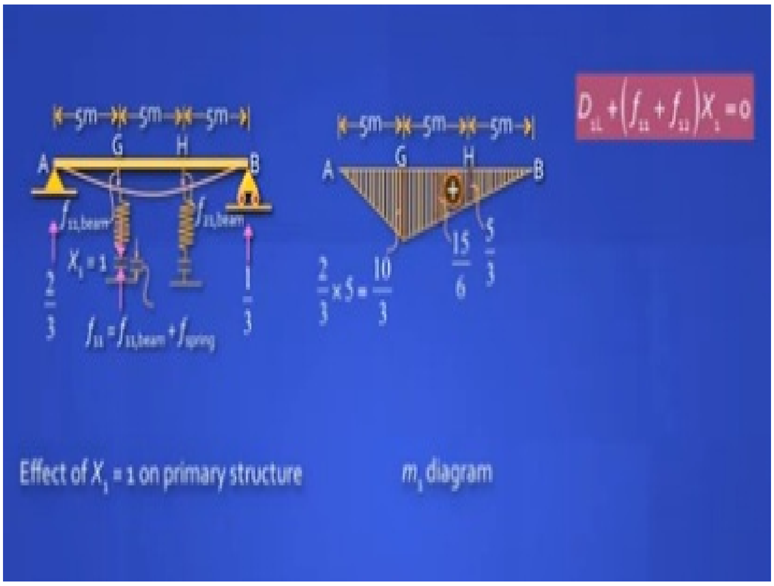

Then, you apply X_1 equal to 1. That means, you pull that spring down by X_1 equal to 1 and you have a force in the spring, but you also have a bending moment in the beam.

That bending moment diagram is a linear diagram like that. That is what we call m_1 diagram.

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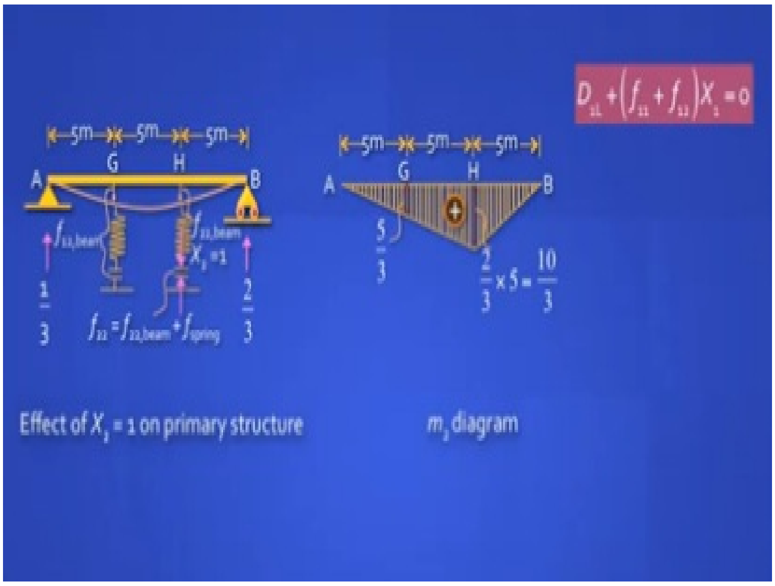

Then, you pull the other spring down with the unit load you have m_2 diagram, which is the same as the m_1, but laterally inverted. So, you have these diagrams.

What is D_{1L}?

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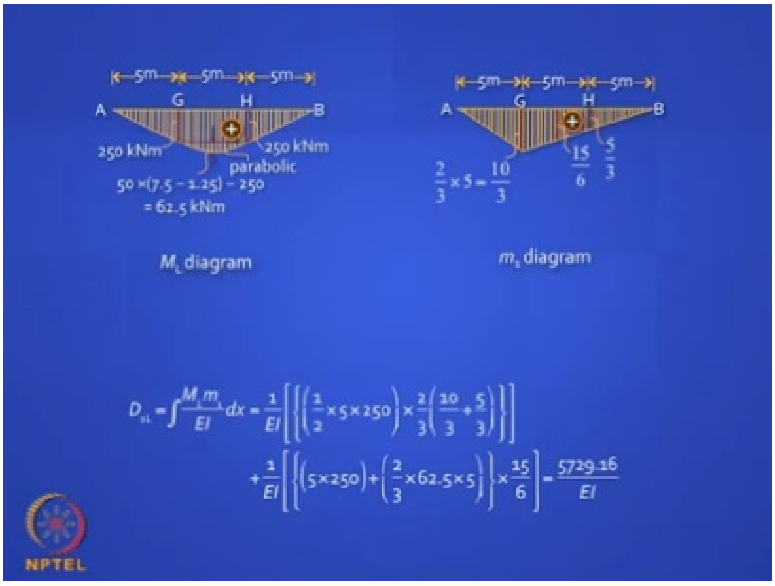

D_{1L} is a product of… comes from the product of this diagram and this diagram, which you can work out. It is not difficult.

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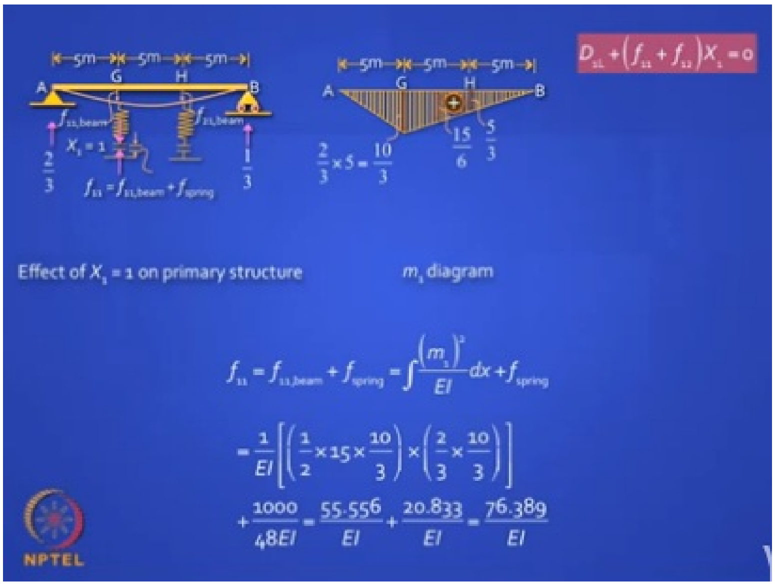

What is F_{11}? F_{11} is a product of this diagram with itself.

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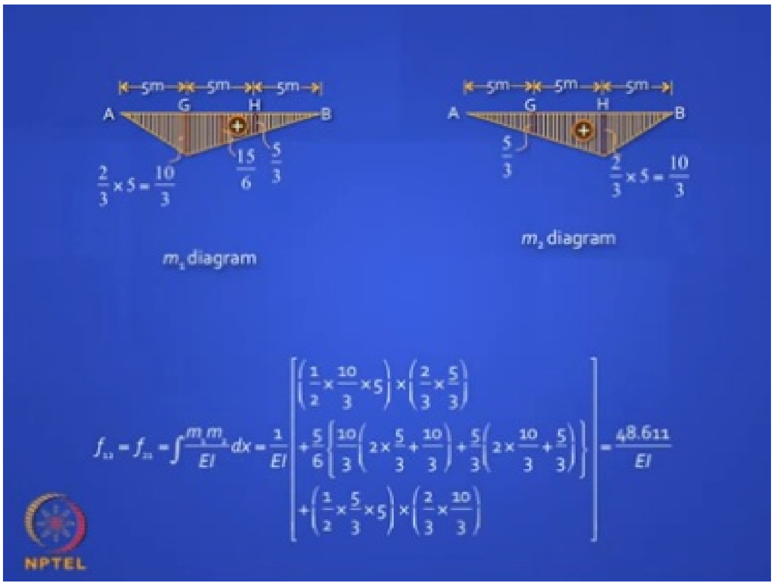

F_{12} is this with this.

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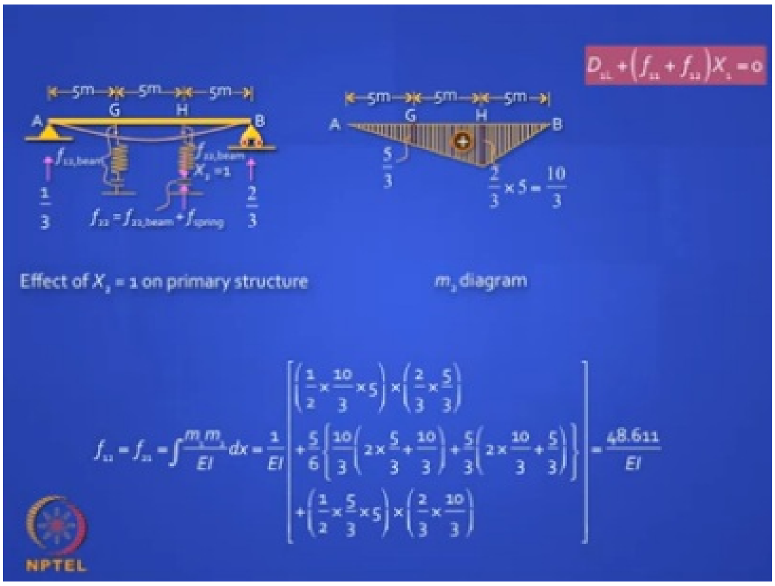

F_{22} is this with itself. So, these are simple things you have done before, but remember F_{11} has two contributions; one from the beam and another from the spring because the spring also elongates. The spring also moves. The spring movement is a value that you already know. So, you can work this out.

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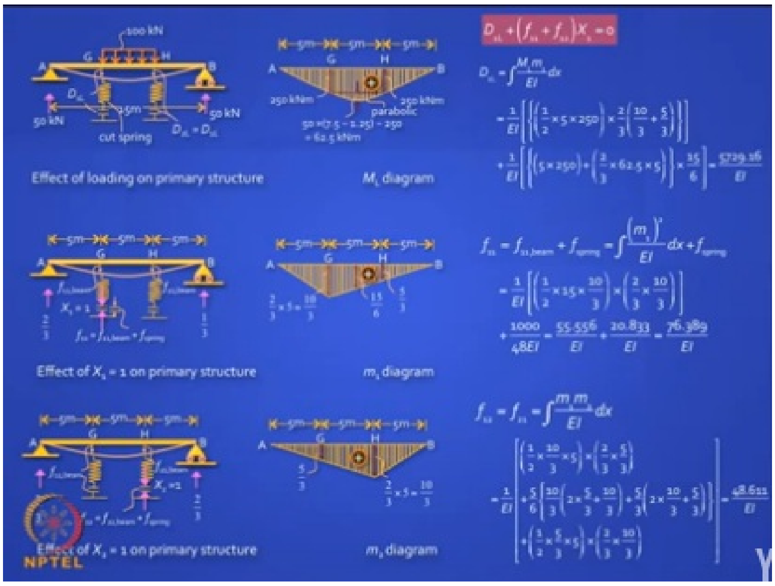

You can write F_{11}, F_{12} and so on.

Conceptually, is this clear? You might encounter such problems in real life. When you simplify, you have a powerful technique to solve them and you can plug in those numbers, solve for the X_1.

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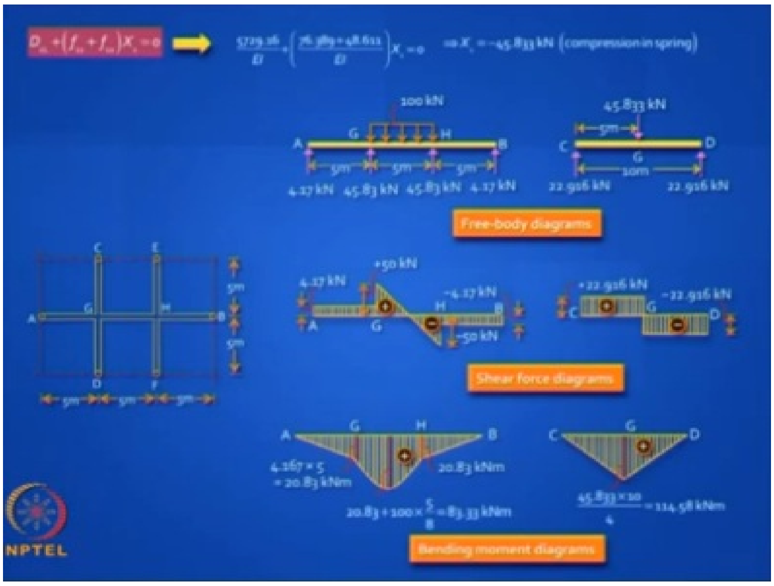

Now, once you know the value of X_1, you see how it comes. Let us say X_1 is 45.83 kilonewton; minus, which means it is compression. So, you have a force acting here and the same force acts downward on the supporting beam. Is it clear? These diagrams are easy to draw. You can draw the shear force and the bending moment diagram. This is complete. All you need to have is one unknown X_1 and X_2 is equal to X_1; you can solve this problem.

So, this how you deal with elastic supports.

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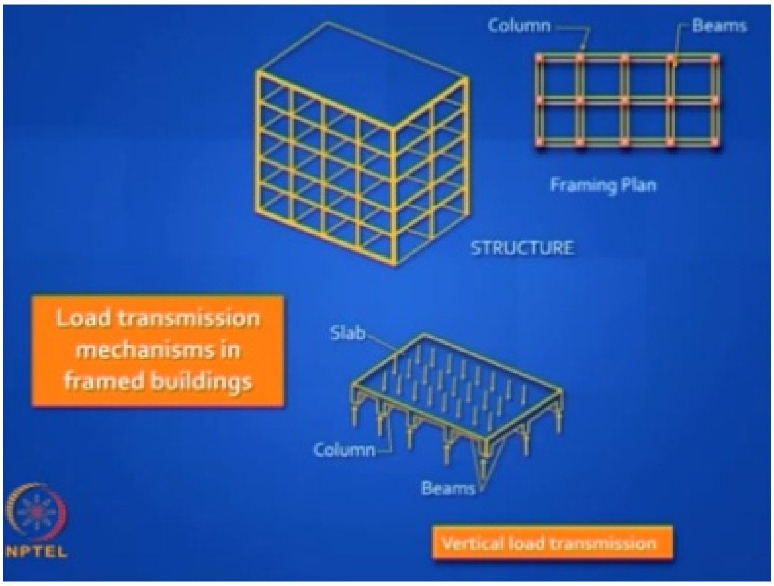

Now, straight away we will go to the last topic in this session. If you have tall buildings, you have a problem of both vertical load transfer and horizontal load transfer. So, let us take a typical frame building like this. You have columns and beams.

So can you see this? This is the framing plan, where you have columns and beams, they are integrally connected and the loads are transmitted from the slabs to the beams to the columns to the foundations, storey by storey. It is simple to understand. This is vertical load transmission.

What about horizontal load transmission? It goes to the foundation through frame action. So, you have to analyze this plane frame. If you separate out of a frame, you can do that. How do you do horizontal load transmission?

Let us say, wind is acting on the face of the building. You have cladding, which will come in the way of the wind velocity. There is wind pressure. You can calculate the wind force. Wind pressure will increase with the height of the building. Let us say you can work out those pressures.

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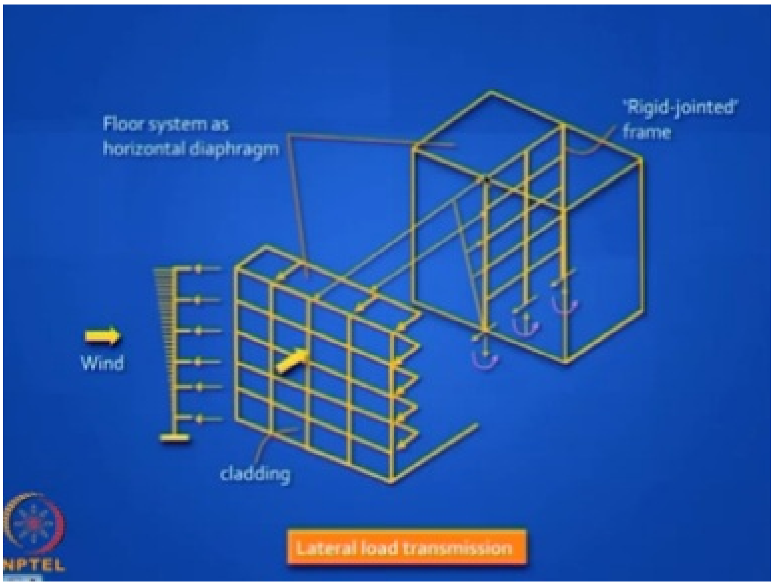

You can find out, let us say the force acting in an isolated frame at every floor level. How do you do that? Some people take just a wind pressure and multiply by the tributary area. That means half the width on one side and half…

That is one way of doing it. It works if your frames are all symmetric but, that would mean that your end frame; the last one would take half the load compared to an intermediate frame. Do you think that is the way it is going to behave? Why not?

[Noise – not audible (Refer Slide Time: 31:13)]

In reality, that whole building is going to move as a whole. It moves as a whole because of the slabs being integrally present there. So, the slabs serve a function. Not only transmitting the vertical loads but, also is acting as a diaphragm. You know what a diaphragm is? It holds a structure together. So, if you push one frame, the diaphragm as a whole make sure that load is shared by all the other frames as well. Is it clear?

Now, if the building is not very long; let us say it moves together. That means, all the frames are deflecting by the same amount at any particular floor level. Agree? So, if you replace them with springs - that means they all will take the same load because, all the frames are identical. Let us say if all the frames have the same size of beams and columns, they have the same lateral stiffnesses. So, they really take the same load. So, if you have total load, F at any floor level, you just have to divide it by as many frames as there are. Supposing the frames are non identical, then you will apportion the load in proportion to their relative stiffnesses. The stiffer frame will attract more load than the less stiff ones. This is some general knowledge.

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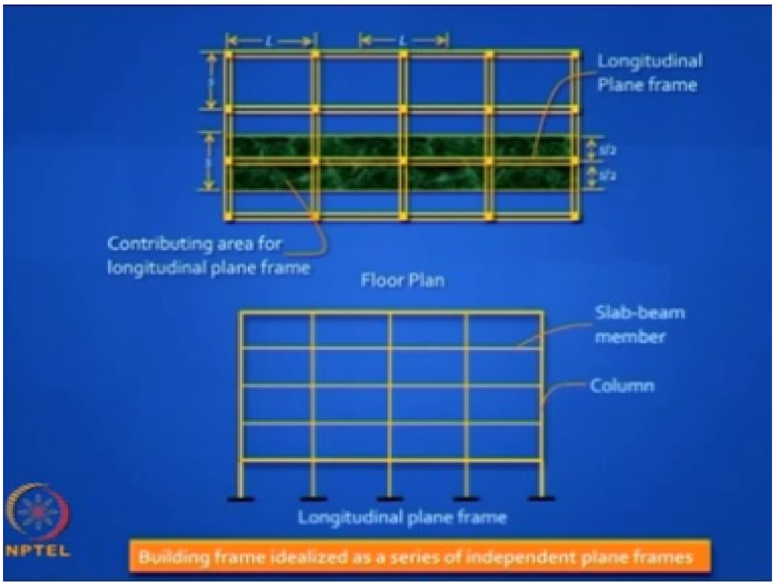

Now, if you take out a plane frame, will you have this longitudinal frame? You have to include the slab and the beam. Some people ignore the slab contribution, but the slab in compression actually helps the stiffness in the frame. So, you call it a slab beam member. So, this is a typical frame, but there are two frames; there are frames in the transverse direction as well. So, when the wind is acting normal to the front face of this building, it is a transverse frame, which will take the load. So, you have to work it out. Let us take a simple problem.

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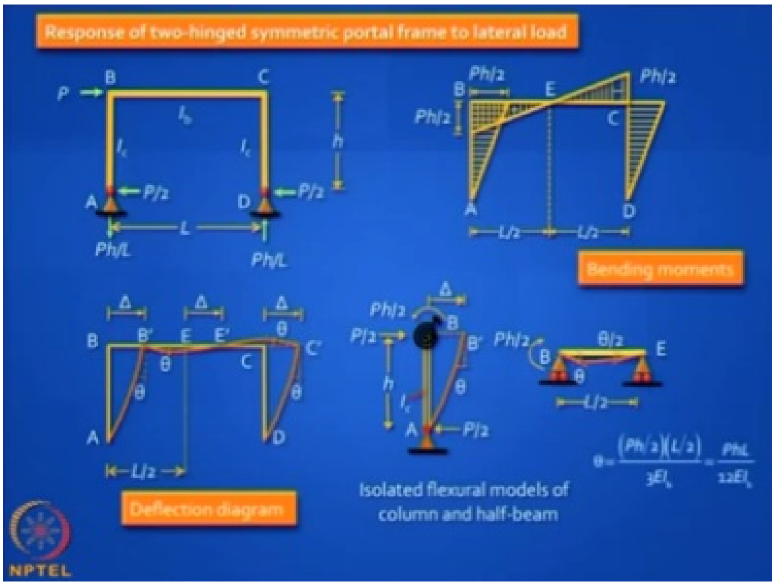

Can you analyze this frame? This is a portal frame. It is symmetric. There is a load P. We discussed this earlier that load will be shared equally, P by 2, P by 2 and that makes it statically determinant, if this frame is symmetric - remember, we proved this earlier. So, this has the bending moment diagram as shown; drawn on the tension side. It has deflection diagram as shown. The question I am going to ask you is - can you tell me what is that deflection in terms of P or What is the lateral stiffness of this frame? What does it depend on?

[Noise – not audible] (Refer Slide Time: 34:02)

It depends on I_b and I_c. Let us say I_b is infinity that means you have a rigid beam. How much will be that deflection?

[Noise]

Columns are flexible. Columns will sway but, the beam will move like a rigid body. How much will it move?

Let us say you isolate one of those columns. How will it look like?

[Noise – not audible] (Refer Slide Time: 34:38)

It is not a cantilever. At the bottom, you have a hinge. So, can you make it look like this? So, you see this picture.

I have taken out that column; on top, I have put a spring a rotational spring. If that spring stiffness is infinite, then when it sways it will sway with the slope $theta_B$ equal to zero but, it will still sway. Now, compatibility demands that the same theta is passed on to the beam. Now, I can cut the beam at E, at the middle because, that is where I have a point of contra flexure; there is no shear transfers; so, I can put a roller there. This is a clever way of simplifying the whole problem.

Now, the question is - how do I find delta? Let us take one extreme case where, k is infinite. Can you tell me what the answer will be?

[Noise – not audible] (Refer Slide Time: 35:43)

Let us say theta is 0. How much will it move?

[Noise – not audible (Refer Slide Time: 35:52)]

Now, it is a function of EI_C by H.

[Noise – not audible (Refer Slide Time: 35:58)]

How to find out?

[Noise]

It is same as cantilever.

That is right. It will be same as a cantilever. See, look at that picture;

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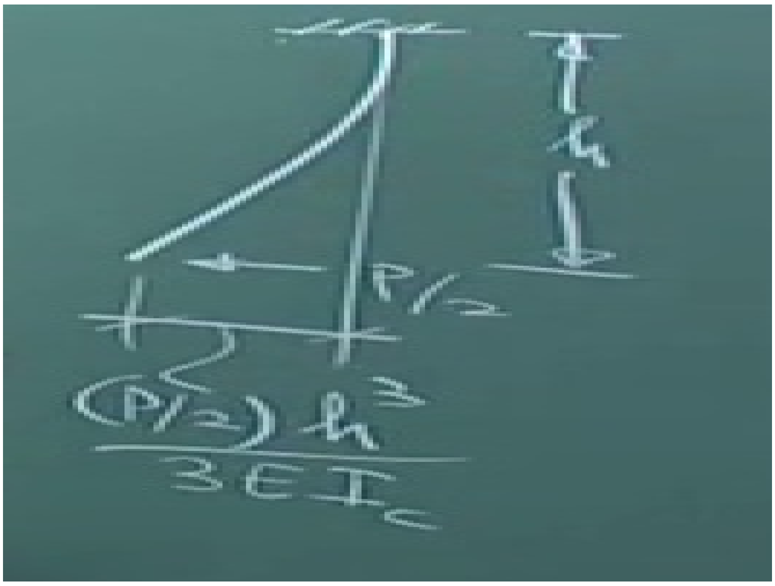

It has taken this shape; what is a force here?

P by 2

It is a situation like this. This is P by 2; this is H; what do you think this deflection is? Is it so difficult? P by 2 into h cube by…

3 EI_c

3 EI_c - it is not difficult. You must be alert. Whenever you see a deflection shape like this, it should remind you; it is like a hung cantilever. Is it clear? The formulas are there to help you. So, it is not that tough. You must develop the ability to write this down.

The other point to note is - you also know an expression for theta. If you have a simply supported beam of span L by 2, do you agree that the relationship between the moment and that beam…

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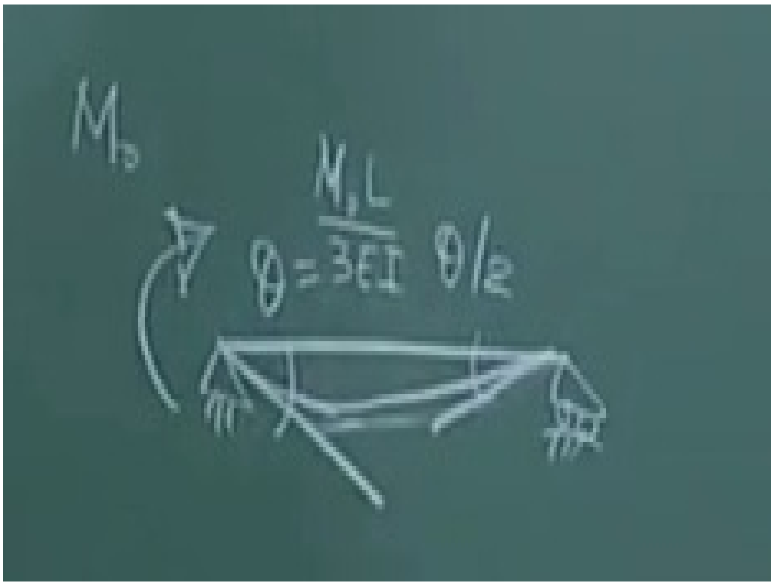

Let us say I have a beam, elastic beam and I apply some moment here, say M_0. This is going to rotate like this. What is the relationship between these two angles? Do you know?

Twice

This is twice that. What is this relationship? Theta will be what?

[Not audible]

M_0L by 3 EI. So, that is what we have done here. So, you have these relationships, you have an expression for theta and if you take...

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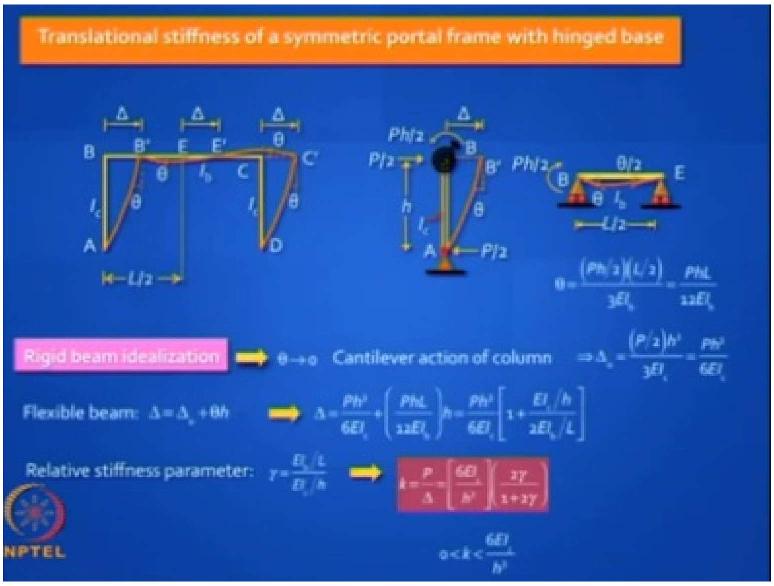

Take the rigid beam idealization. Do you agree to this? Cantilever action of the moment of the column. You get delta_0 equal to this quantity, which we just derived.

Now, let us say theta is not equal to zero. In reality, it would not be equal to zero because the beam is not infinitely rigid. Beam has a finite stiffness EI_b by L. Then, what is the additional deflection that you have? So, delta will be delta_0 plus what? Rigid body rotation. So, how much will that be? Theta into…

Theta into h.

You must think like that. If theta is not there, you have delta_0. If that spring is finite, you have a rotation here. It is a rigid body rotation. So, you have it also this...(Refer Slide Time: 39:35). Rotate it by an angle theta and so you have an additional moment here, which is theta into h. So, that is a clever way of simplifying and breaking down and understanding and you can do that. So, you have a nice expression. If you simplify it; you can write an expression for delta; you have a value of theta. If you bring in a stiffness parameter, which we have introduced earlier, EI_b by L to EI_c by h, then you can write an expression for k.

This is not very important. I just want you to get a general sense of how we can simplify things but, I want you to remember the limits. So, you can write k in terms of gamma and you will find that the maximum value will be 6 EI_c by h cube, which is when the beam is infinitely rigid. If the beam is infinitely flexible, we have a problem. The system becomes unstable. So, you have two internal hinges there and you have very large deflections. It will not survive. So, is this clear? This is one typical extreme problem.

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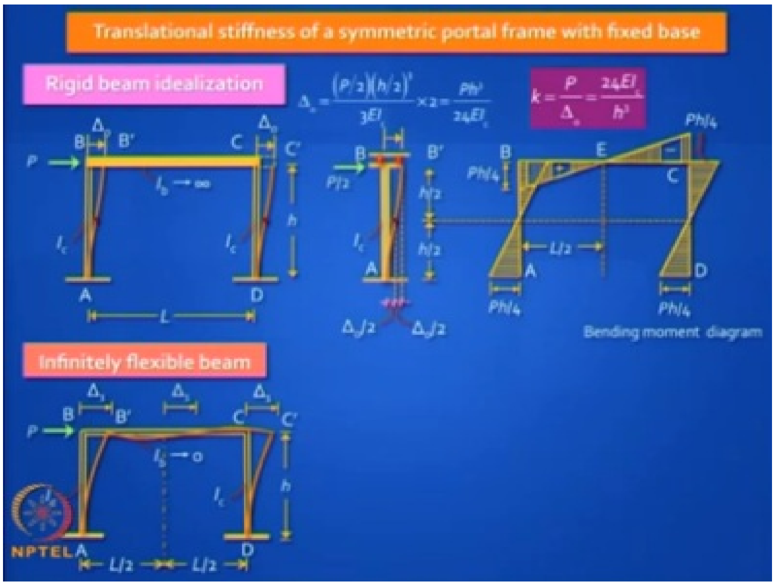

Let us take the same portal frame and fix it at the bottom. This is not statically indeterminate. Earlier it was hinged at the base. Now, let us begin with the rigid beam idealization. This building is sometimes referred to as a shear building. So, it is a shear mode of moment.

Now, you see how it is going to behave. If I_b is infinite, it will just slide horizontally and this is fixed on top, fixed here, it is kind of guided roller here; guided fix. So, the slope here is 0, the slope is 0. Do you agree that this point of contra flexure will be in the middle? Do you also agree at the speed that these two columns will move identically? They will have the same bending moments and same shear force? Do you also agree that this load P will be shared equally by the two columns? Now, can you give me an expression for $delta_0$ with that knowledge?

Look carefully. Can you give me an expression for $delta_0$ in terms of P, E and I_c, and h. Please work it out. See, this is how you check. Let us say someone in your structural engineering design office comes out with some value, you should at least make sure the value makes sense. It should lie within appropriate bounds. So, this is one extreme. Your actual deflection will be more than this. This is the least value of the deflection you can get when the beam is infinitely rigid. What is that formula for $delta_0$? Where is that hinge located? What is the moment?

Let us look at the isolated model. It will move by $delta_0$ by 2. In the middle, you see here if this overall moment is delta, do you agree that this is $delta_0$ by 2? Can you write an expression for $delta_0$ by 2? That is easy. That is a cantilever and it is two times that. So, is it okay to say that $delta_0$ is this into 2, where this is P by 2 into h by 2 the whole cube by 3 EI_c. It is the same formula except, I am taking half the height now. It is because of parity because, even if you flip it upside down, it does not change. It is like the settlement of supports problem.

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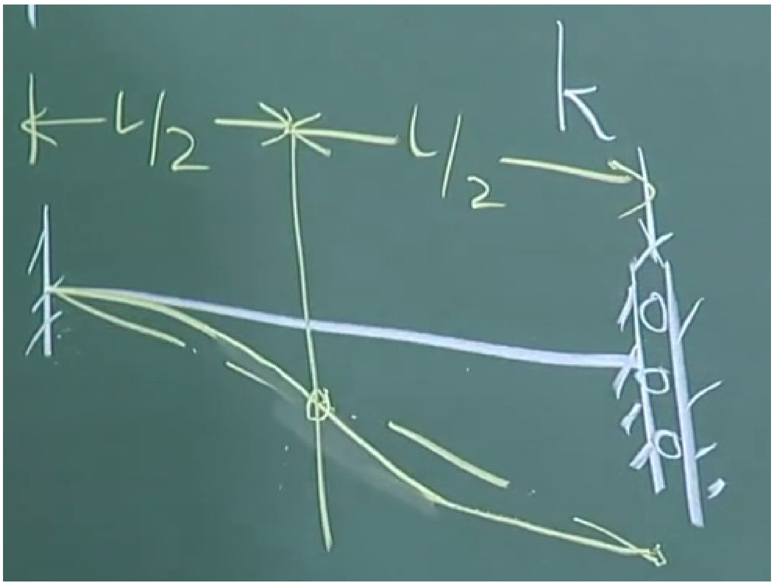

Let us say I had a beam like this, fixed here. Let us say I push this down. So, what is the deflected shape? Let us say this is a chord rotation. Deflected shape will be zero slope there and goes down like that. Does not matter whether this goes down or goes up. If it is a prismatic beam, cannot you feel that the point of contra flexure will be bang in the middle? Imagine this goes down. So, these are all reversible. So, this point of contra flexure will always be in the middle of the beam. You keep moving it up and down. This chord rotation is going to be like that and if this is delta, this has to be delta by 2. You are not convinced?

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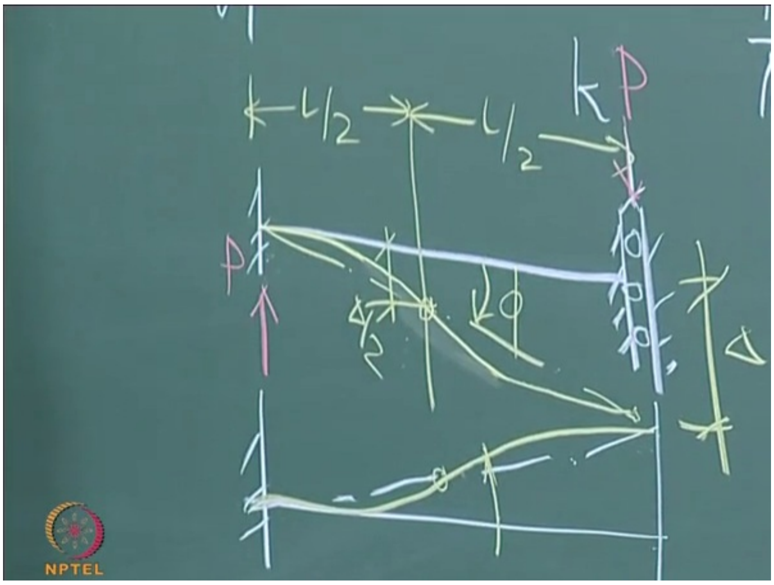

Let us say you move it up. What will be the deflected shape? The bending moment diagram for this is going to be like this. Now, when I added these two, what should I get? I should get 0. The beam will be flat. Agree? I can flip it over. So, it does not matter. So, we talk of chord rotation. It is not whether the right support goes down or the left support goes up. You have a clockwise chord rotation here, which we call phi here. You have anticlockwise chord rotation here. The points of contra flexure will always be in the middle. Let us finish it because we need this knowledge later. Let us work it out.

So, let us say this force here is P. Equilibrium demands at this also is p. What do you think this deflection is? Delta by 2 will be?

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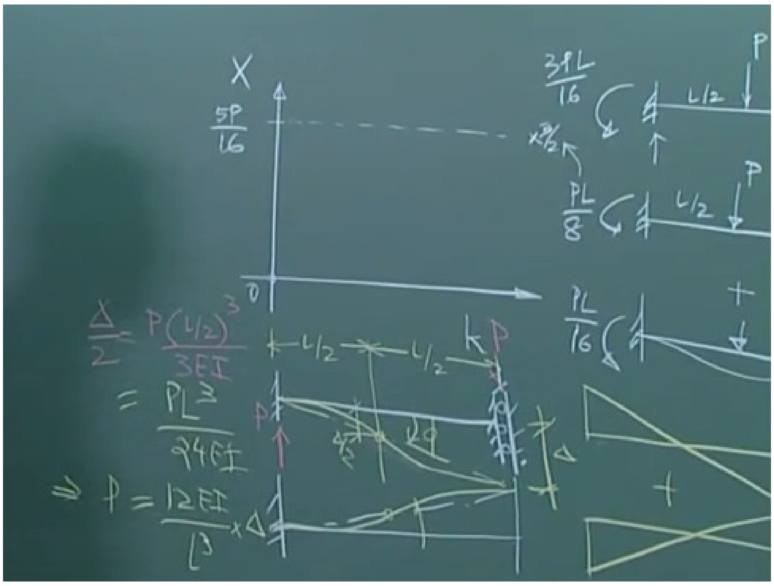

P into L by 2 the whole cube by? It is a small cantilever and I can cut it there by 3 EI. It is a clever way of solving the problem. So, this turns out to be… Remember this. This is PL cube by 24 EI, or you can say P is equal to 12 EI by L cubed into delta. So, you have a relationship between this delta and this P. It is very important to remember this relationship. Now, what is your moment? You get a moment here. (Refer Slide Time: 47:50)

It is acting down. You have shears acting down and up. You have a moment here. You also have a moment here. What are these moments going to be equal to? P into L by 2. How much does it turn out to be? P into L by 2 is how much, if P is this? So, it turns out to be? Moment is what?

6 EI by

6 EI by L square into delta. These are very important formula 6 EI by L square into delta. So, summary of what we have done here is - if you have a clock wise chord rotation, you can also write this as 6 EI by L into chord rotation phi because, phi is delta by L, assuming delta is very small compared to L. Is it clear?

If I have a clockwise chord rotation, I get a constant shear force like that, whose value is 12 EI by L square into phi. I get equal moments on both sides in the same direction. (Refer Slide Time: 49:11)

In this case, it is anticlockwise of 6 EI by L square into delta. It is 6 EI by L into phi. Is it clear? Even if you forget these, you can derive this. They are not very difficult. So, I hope you understand and you come back to this of how we got an expression for delta$_0$. Is it clear? Now, this is the stiffness that you get. It is called the translational stiffness of a shear building when you push it horizontally. You get this much reflection.

Now, in reality, the beam is going to bend and it is a little difficult to work out that formula but, let us take the other extreme. Let us say the beam is infinitely flexible. Can you draw the deflected shape for that?

Now, it is not going to be unstable if you put internal hinges there. So, how much will it move if it is infinitely flexible? This is the bending moment diagram. Can you see for this particular case, this is P, this is P by 2 into h by 2 is Ph by 4. So, you get typically bending moments diagrams like that. k has a value of 24 EI$_c$ by h cubed.

Let us put hinges. How does it behave? If you have an infinitely flexible beam, how will the columns behave? Simple cantilevers and so that is easy to calculate.

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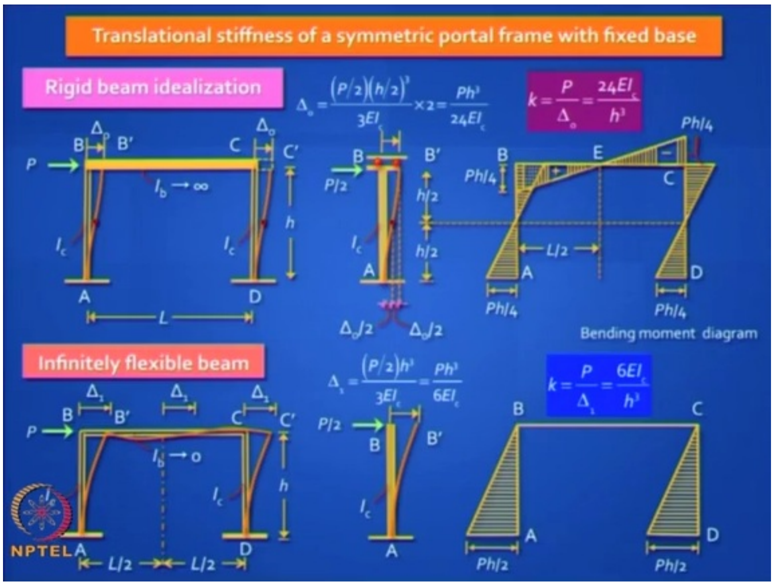

If you do that calculation, you get Ph cube by 6 EI. Your bending moment diagram will look like that because, your beam will have known bending moments. This is because of that hinges there. So, it is very interesting.

I looked at all the extreme cases. The summary of this - so, here it turns out to be 6 EI_c by h cube. So, take a portal frame, symmetric portal frame; take the base as hinged. Your horizontal stiffness is going to vary between 0 and 6 EI_c by h cube. That is what we saw in the previous slide. Fix the bottom and it is going to... The minimum value will be 6 EI_c by h cube. The maximum value will be the 24 EI_c by h cube. So, when your beam is infinitely rigid compared to the infinitely flexible case, the stiffness of the whole frame gets magnified by factor of 4. Some instances you can write in terms of lambda.

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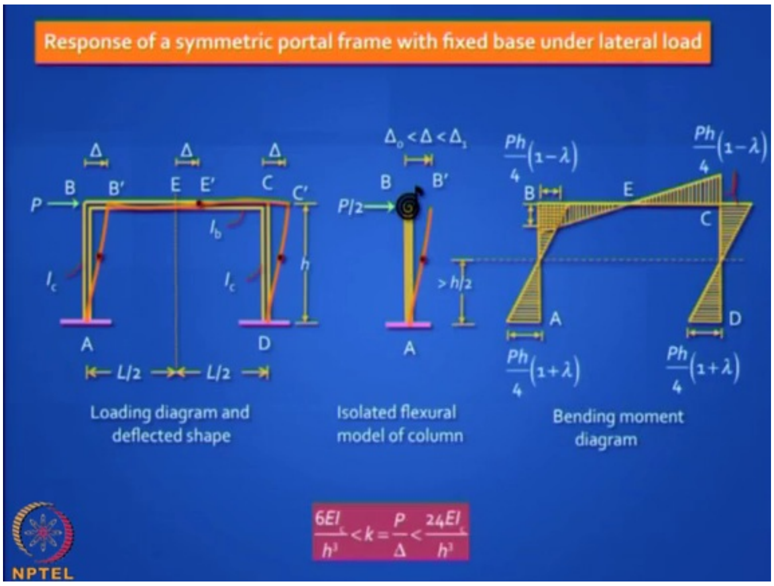

Finally, it is between 6 and 24.

Now, the whole idea of doing this is to tell you that when this first started, building tall buildings and tall frames - they found it very difficult to manually analyze this highly indetermined structures. So, they took shortcuts. For gravity loads, they used a concept, which we will see later called substitute frame method, where they just took one floor and they assumed that the columns are fixed at top and bottom. So, it was much easier to analyze. The argument was whatever happens far away from the story is not really going to affect the floor. It is a hunge, which works out well, where you can prove with Muller Brazil principle.

When you to came to lateral load, it took some time for them to work out a simple way of cracking the problem and they came up with a brilliant idea. They said, let us make some assumptions and make that whole frame statically determinant. Some methods involved over centuries and they are still popular today. How do you convert? I want you to give me the idea of how you convert an indeterminate frame to statically indeterminate, for the sake of approximate analysis.

Now, take a look at this.

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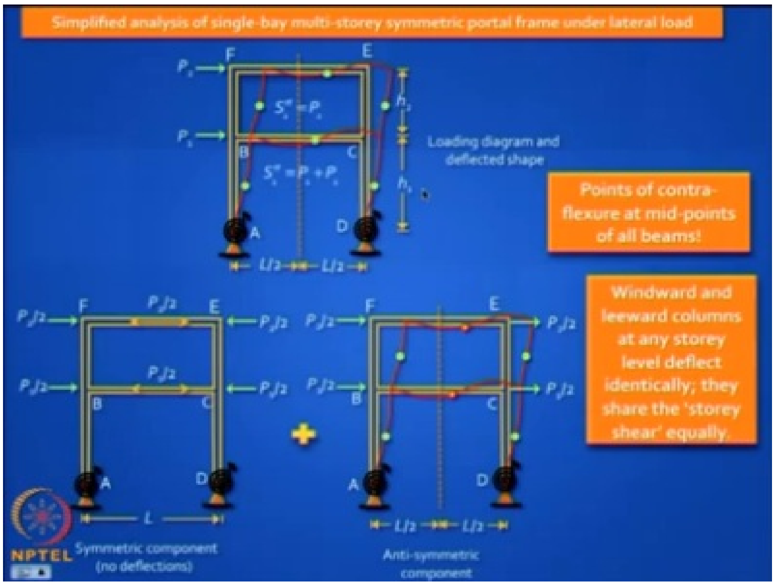

At this point, I want to convey, at the bottom, engineers often have this worry about should I model it as a hinged support or a fixed support. When do you model foundations as fixed? When do you model them as fixed and when do you model them as hinged? The reality is somewhere in between of it. When is it hinged?

[Noise – not audible (Refer Slide Time: 53:48)]

When your foundation can rotate? When can it rotate?

[Noise]

When you have a shallow foundation; when you have isolated footings and even sandy soil; when your safe bearing capacity is not very high - because, even a small moment there will release whatever moments you calculated. So, that is one extreme.

When do you model it as being fully fixed?

[Noise – not audible (Refer Slide Time: 54:23)

Let us say you even have an isolated footing, but it is resting on very hard (()). There is no way the rock is going to rotate. Or, you are on piles and the piles have a pile cap, which is heavily interconnected. So, in such cases, you can go for fixity, but the spring shows that you can handle all conditions in this situation. (Refer Slide Time: 54:54)

One thing you can notice. I am not taking one type of frame only, in this session. Let us say the frame is single bay. Single bay means just one two columns in one storey, but multi storey. Single bay multi storey frames. Let us say it is symmetric, which is what you normally... That means, all the columns on one side are the same as the columns on the other side. If the wind is acting from left to right, you call these columns as windward columns and these columns are referred to as leeward columns.

Now, because the frame is symmetric, you can assume this to be made up of two loadings: one is symmetric, the other is anti symmetric. Divide P_1 and P_2 by 2 and put them equal and opposite here, and then put them here on the same side. So, this is called an anti symmetric loading. We call symmetric loading because, when you add the two you get back the original loads.

There is a great advantage in doing this. The advantage is, this part - the symmetric part is very easy to calculate. You end up getting actual compression only of beams. Will there be any deformations?

[Not audible] (Refer Slide Time: 56:12)

No, you are assuming actual deformations to be negligible in frames.

The first part you can practically throw away. All the bending moments, all the curvatures, all the deflections are coming from the anti symmetric components. It is very interesting. Not only that, you will find that there are points of contra flexure always in the middle of the beams. They have to be in the middle of the beams because of this kind of behavior. Whether you push it to the left or right, your behavior will be the same. All the columns are moving in the same way. So, you will have points of contra flexure always in the middle of beams in such frame, in symmetric frame. But, in the columns, they could be anywhere. So, that is the other issue to know. Windward and leeward columns at any storey level deflect identically. They share the storey sheared equally.

Now, let us understand the meaning of storey sheared. We are seeing this hinge. I mean it is not a deliberate hinge. It is just the fact that the bending moment there is changing sign. It is going to be at the middle of the beam, but this hinge could be somewhere in the middle of the column. We have not yet proved that it is exactly in the middle. Similarly, this hinge in the lower storey is somewhere. Suppose in the bottom, is hinge, then this point of contra flexure will actually shift to the bottom. Is it clear?

We do not know where this is, but one thing we know for sure. We know that this hinge and this hinge (Refer Slide Time: 57:46) will be located at the same height. You agree? This hinge and this hinge will be located at the same height. Now, let us say, I cut a section here along this; wherever the hinge locations are in the columns, I take out the top part, then it is like a cantilever; it is taking some lateral load. That is referred to as a storey shear. Look at the ward - storey shear is the lateral load taken by all the columns in one storey. So, if your load here is P_2, the storey shear in the second storey is P_2. If I now cut a section at the lower storey, then the upper storey is part of that section. What is my storey shear? If I cut a section here, it will be P_1 plus P_2.

You will find that the ground floor columns take more shear than the upper storey column. So, they will be loaded more heavily and can you realize that? So, the point made is, this storey shear will be shared equally by this and this. This storey shear will be shared equally by this and this (Refer Slide Time: 58:55). With that assumption, we can solve this problem. We will stop here and we will complete this. This is very interesting simplified analysis.

Thank you.