Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of Technology, Madras

Module No. 2.1

Lecture No. 07

Review of Basic Structural Analysis-2

Good morning to you.

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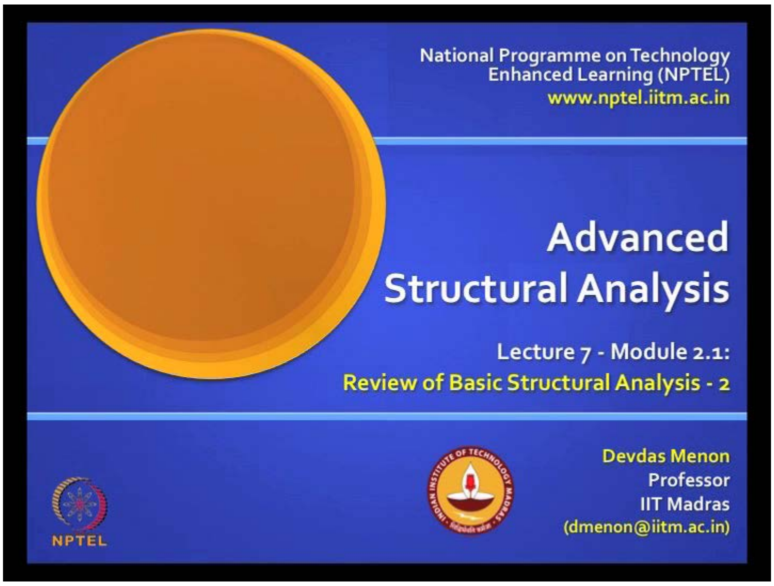

This is lecture 7 in the second module on Review of Basic Structural Analysis.

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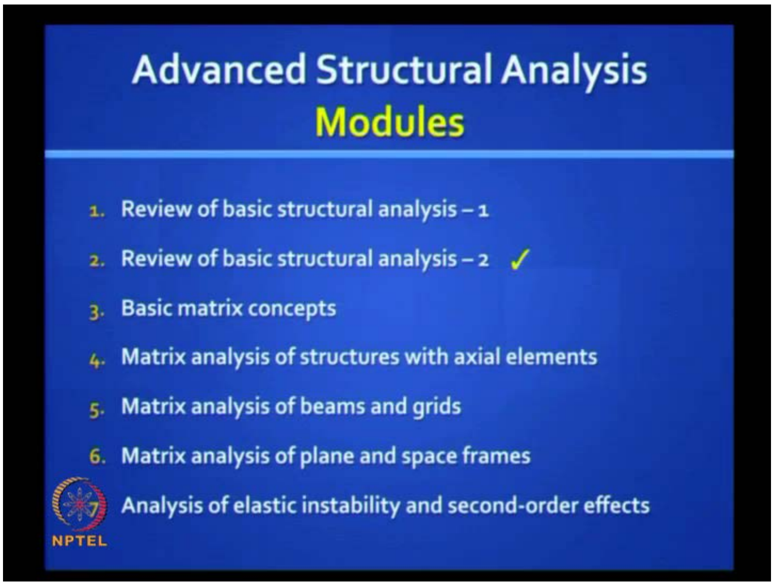

We finished the first review where we covered statically determinate structures. Now, we are going to look at statically indeterminate structures, as well as kinematically indeterminate structures.

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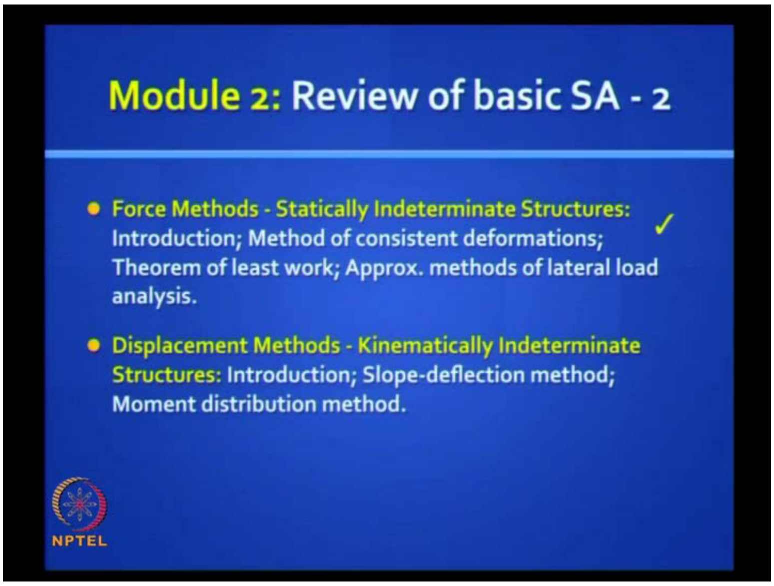

The first three lectures in this part will deal with force methods, as applied to statically indeterminate structures; you already had an exposure to this in your earlier course. So, I will go a little fast.

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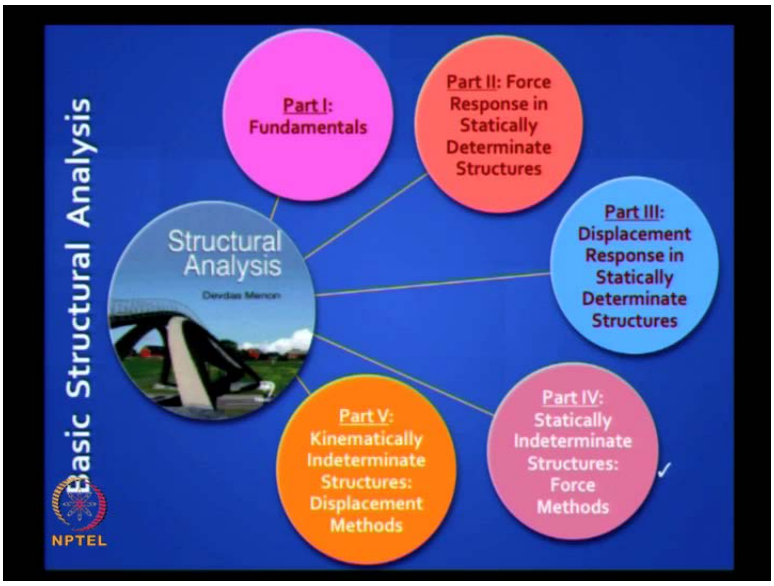

As mentioned earlier, we are going to refer to this book on structural analysis, part IV: Statically Indeterminate Structures - Force Methods.

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You really have not got much exposure to displacement methods, but you had some exposure to force methods. The most popular force methods are: The methods of consistent deformations and Theorem of least work, which you have been introduced to. These lead to a more generalized method called flexibility matrix method, which we will study in depth in this course.

There are other methods like the column analogy method, which are good for manual use, but we are not going to cover it in this course. We will look at displacement methods later, whereas, it is important to know the main differences between force methods and displacement methods; they are both aimed at analyzing indeterminate structures; completely different paths; both are very interesting.

Displacement methods are probably better suited for computer applications, but can you tell me, what are fundamentally the main differences between force methods and displacement methods? First, with regard to the type of indeterminacy - static indeterminacy, which means you are talking about unknown elements in the force field; that is part of the force methods. In displacement methods, we worry about kinematic indeterminacy. That is one major difference. What else? So, first is the type of indeterminacy, which you answered.

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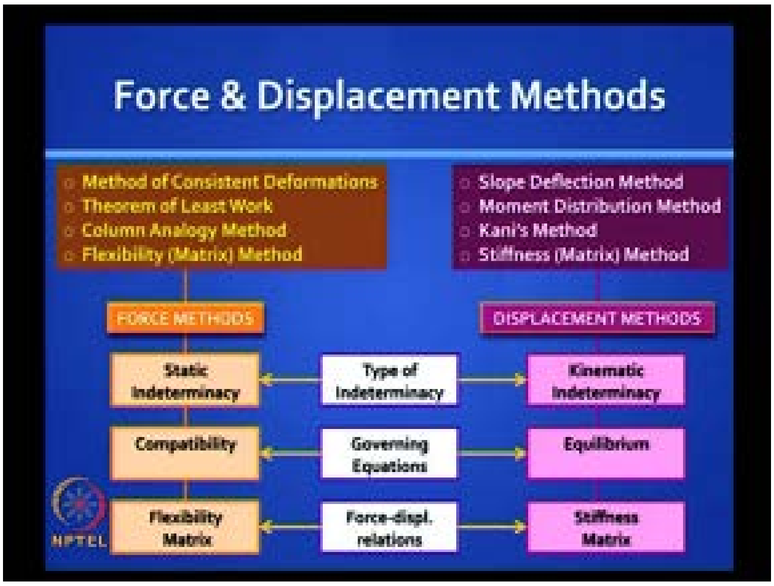

Secondly, to solve the indeterminacy, you need to apply some equations which are called governing equations. As you know very well, in force methods, they are of the nature of compatibility equations; in displacement methods, they will be of the nature of equilibrium equations. You need to express the displacements in terms of forces. You can do it in two ways: you can either use a flexibility approach where you deal with the flexibility matrix, or you deal with the stiffness matrix. So, these are broadly the fundamental differences. As I mentioned, in the first three lectures we will focus on force methods and in the next four lectures we will focus on displacement methods.

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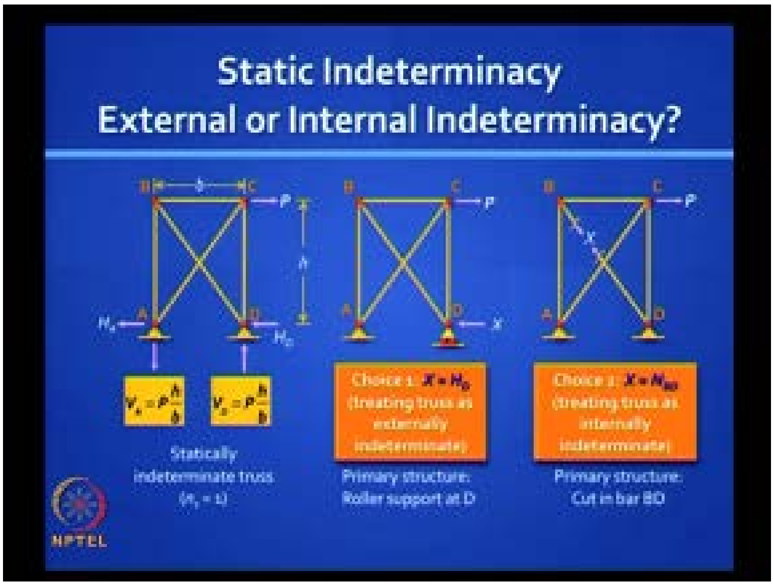

Let us begin with something you know very well. Look at this pin jointed frame or plain truss. What is the degree of indeterminacy - static indeterminacy?

Yes, it is 2. Would you describe this? Is it 2? Look carefully.

1, sir; it is one.

It is 1. It looks deceptive; it is 1. You can use the m plus r greater than 2 j equation. For example, what is M? How many members are there?

5.

There are five members. How many support reactions are there? It is 4.

5 plus 4 is 9. How many joints are there?

There are 4; 4 into 2 is 8. So, the number of equations available is 8. M plus r minus 2j will be 9 minus 8 equal to 1. This is a simple problem with one degree of static indeterminacy.

Can you tell me, whether this kind of indeterminacy is external or internal?

External.

You say it is external. You could view it as external? Which reaction here is indeterminate? Is it vertical or horizontal?

It is horizontal. So, the vertical reactions are determinate. As an off shoot of this, remember that, if such plain frame were to undergo support settlements, let us say that support B goes down relative to support A, would you get any forces in the structure? You do not, because the vertical reactions do not really change; they are known; so, they are determinate. What is indeterminate is the horizontal reaction? H_A and $H_{D.}$ So, you are right. You could treat horizontal reaction - one of those reactions, say, $H_{D,}$ as the redundant X. When you release that reaction, you get the primary structure which is statically indeterminate and it is slightly rigid; this is one way of approach.

Let me ask, if you can treat the same problem as internally indeterminate? Yes, that option is also there. You could remove which member or cut which member? Anyone of those members can be removed, not necessarily the diagonal; the diagonal is what we normally remove. Can you remove the member to get the primary structure or should you cut it? Some text books mention about removing members, which is fine in one sense, but inaccurate in the other sense. You will find that, that member - if you cut it, it will have no force; if you remove it also, it will have no force. But if you cut it and apply the unknown redundants - a pair of equal and opposite tensile forces, as shown there, then you are also invoking the stiffness or the flexibility of that member, which would be missing if you remove that member all together. So, the stiffness of that member BD has a role in governing the answer of X. So, do not remove members; you just cut them. When you cut them, you release the member. We will examine this more carefully, later. So, we have multiple choices of indeterminacy and it is left to you whether to treat this problem either externally or internally indeterminate.

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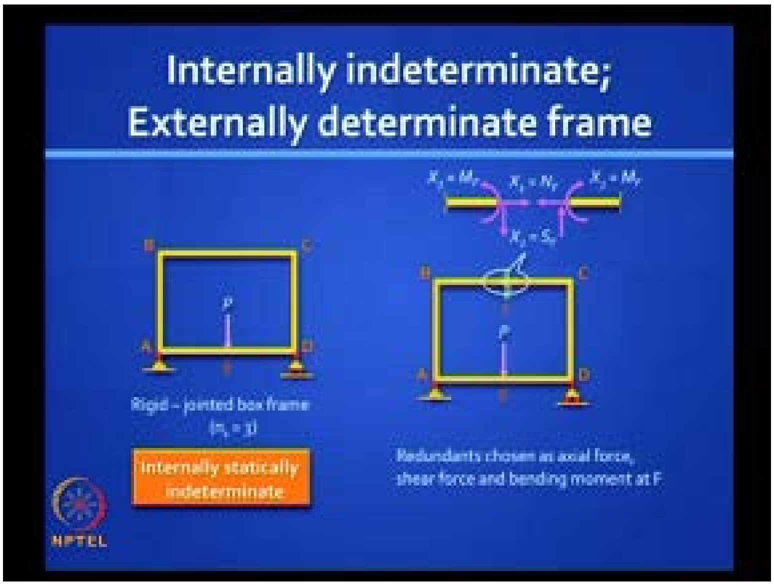

There are some other types of problems. For example, this - where the support reactions are very well known; it is externally statically determinate. Here, the indeterminacy is internal. If you put a cut anywhere, you will expose a shear force, an axial force and a bending moment; you have to apply them equal and opposite, as shown here.

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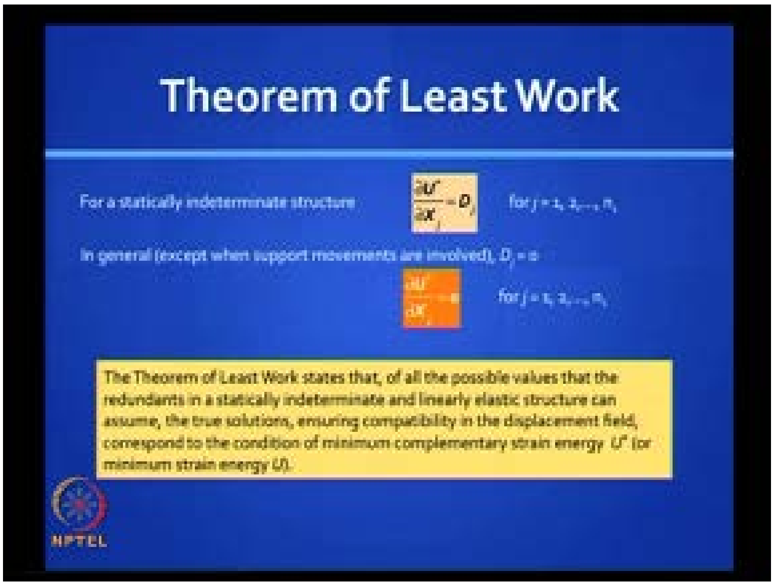

We covered the theorem of least work in the previous class. You are familiar with this. The first equation, dou U star by dou X_j equal to D_j, is caustically announced second theorem. Usually, D_j will turn out to be 0. If it is so, you can also interpret this as a minimization of the complementary strain energy in the structure. Now, the D_j that we refer here is a displacement conjugate with X_j. If you look at the previous problem where you took the support reaction as X_j - the horizontal reaction, then D_j would be the moment in that support. That moment is 0 in that example. Is it not? Say, moment in the original strength is not allowed; so, D_j is 0.

Similarly, if you have a situation as in the previous case where you have treated the bar as redundant, what would the D_j refer to? (Refer Slide Time: 09:01) I will go back to the previous problem. Here, what would the displacement D refer to at the redundant location? It will be the relative distance between the cut ends of the bar.

Here, we interpret it in that sense. You apply two equivalent and opposite forces X, to reflect the redundant. The conjugate displacement is basically the elongation in that bar which will be represented by the relative movement in that bar. Since the bar is not cut, compatibility demands that, the two ends should remain joined to each other; there is no relative displacement between the cut ends. Actually, the bar can elongate. D is not the elongation in the bar. It is the relative displacement between the cut ends and that is how it is reflected in Castiglione’s theorem. The interpretation of it is that, we are actually minimizing the strain energy.

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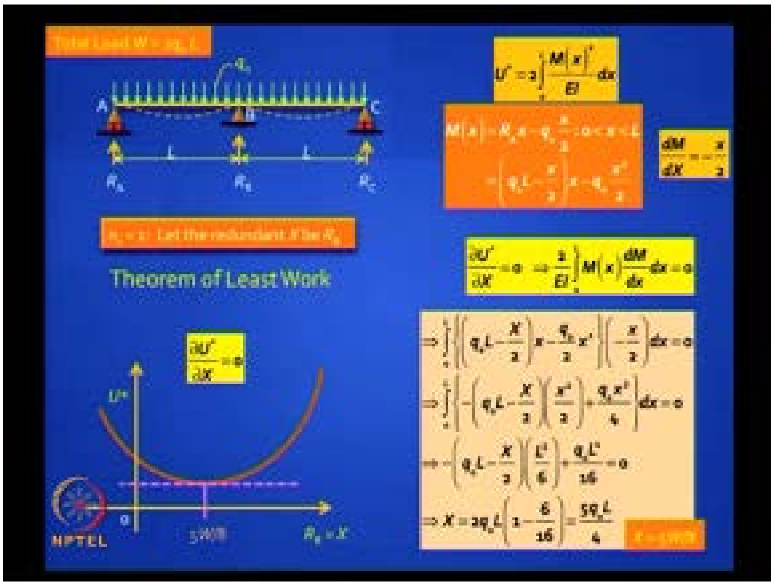

We demonstrated this with a simple problem of two span continuous beams, if you recall. You need to carry out this integration and you will get X equal to 5 W by 8.

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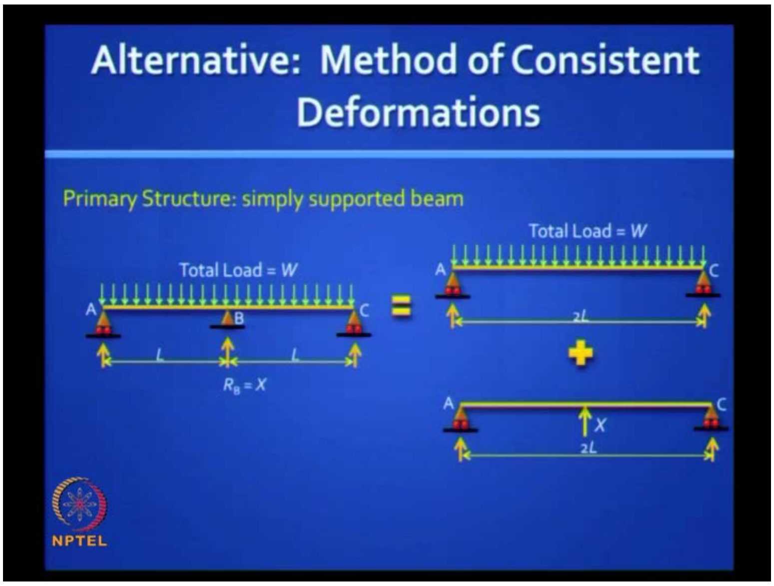

There is another way of doing the same problem. That is the method of consistent deformation. We choose the same redundant X as the support reaction in the middle support. Then, we release that reaction. How do we release it? It is released by removing that support all together. So, we do a superposition of that primary structure, which is a simply supported beam loaded with the external loading - in this case UDL, plus the unknown redundant, now shown as a separate load case. So, we treat X as a load, but an unknown load and our job is to find X. How do we do that? Equilibrium is not enough; you need to invoke compatibility. What would compatibility be here? What would be consistent? The deflection at B should be 0. So, that is what you do. You look at the deflected shapes for a better understanding. In the picture on the left, you can see very clearly that delta B is 0. Here, B - this deflection will be maximum; so, will it be here? (Refer Slide Time: 11:43). But they are in equal and opposite direction and you can calculate those values; you know those standard formulae.

What would be the deflection due to the UDL here in the middle? 5 by 384 into W into, not L, there are 2L coming here, 2 L whole cubed by E I, which can be derived as well. Similarly, due to concentrated load, it is X into 2 L whole cube by 48 E I. These two should be equal and that is compatibility. When you invoke that compatibility, easily you get the answer, X equal to 5 by 8 times W. Is it clear? This is a nice way of doing and you will understand the meaning of consistent deformations.

We are going to explore these two approaches. One is an energy approach where you minimize complementary strain energy. But mind you, the theorem of least work is strictly applicable, only when you do not have support settlements and only when you have D_j equal to 0. If you had support settlements, you could still invoke the energy method, but you are basically doing method of consistent deformations; you are just finding displacements using an energy formulation.

If you want to complete this problem, after you get the answer X equal to 5 by W 8, you can superimpose the free bodies and you can get these two bending moment diagrams.

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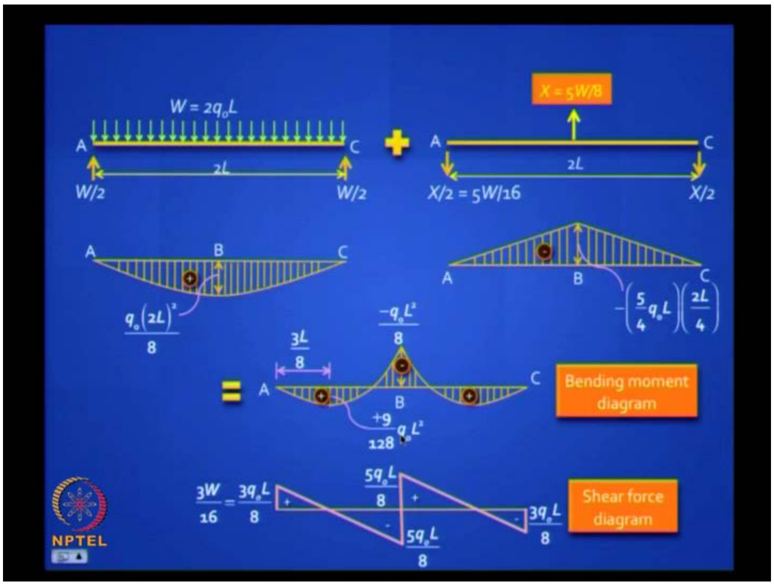

The first due to the load is causing sagging moment here and is causing a hogging moment here. When you join these two, you get a diagram like this. It is very straightforward. You can draw your shear force diagram and so on.

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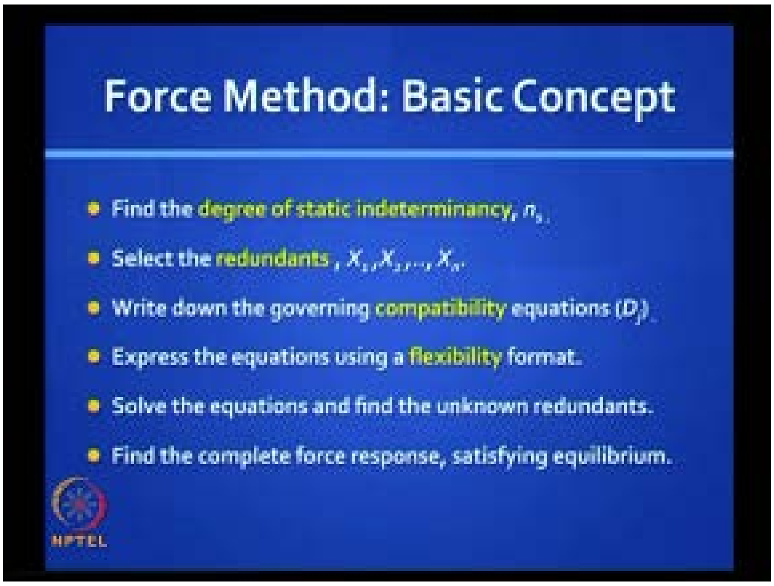

Now, let us summarize the basic concepts underlying force method. What is the first thing you need to do in the force method? You have to identify the redundants, which also depends on the degree of static indeterminacy. Find the degree of static indeterminacy and choose; you have choice here. If you have n - n subscript $_s$ stands for static indeterminacy, then, we have to choose as many redundants as the indeterminacy demands. What do you do next? Once you have chosen the redundants, you have also chosen the primary structure, where those values of X are 0, where you release such constraints.

Then, on those primary structures, what should you do? You first apply the?

External loading.

External loading and then you apply the redundants one at a time, if you wish and write down the governing compatibility equations. Wherever you have made those releases, write down those expressions. Usually, D_1, D_2, D_3, D_4, at those redundant locations will be 0, but there are some exceptions. Where they are 0, you have a choice of invoking the theorem of least work, but otherwise, we have to actually calculate.

Let us say, the support settles or moves horizontally. Then D_j is equal to that movement. Is it clear?

Express the equations using a flexibility format. Which equations? The compatibility equations; you have choice here. Then, you solve these simultaneous equations and find X_1 to X_n. Once you have found X_1 to $X_{n,}$ you draw the free body; you have got a statically determinate system. You can analyze it, draw the shear force diagram, draw the bending moment diagram or find the actual forces in the case of a truss. You could do it directly on that overall free body. Alternatively, you could find out what is happening under the action of each of those loading systems on the primary structure and superpose everything.

So, you find the complete force response. It satisfies the equilibrium, but more important is that, it also satisfies compatibility.

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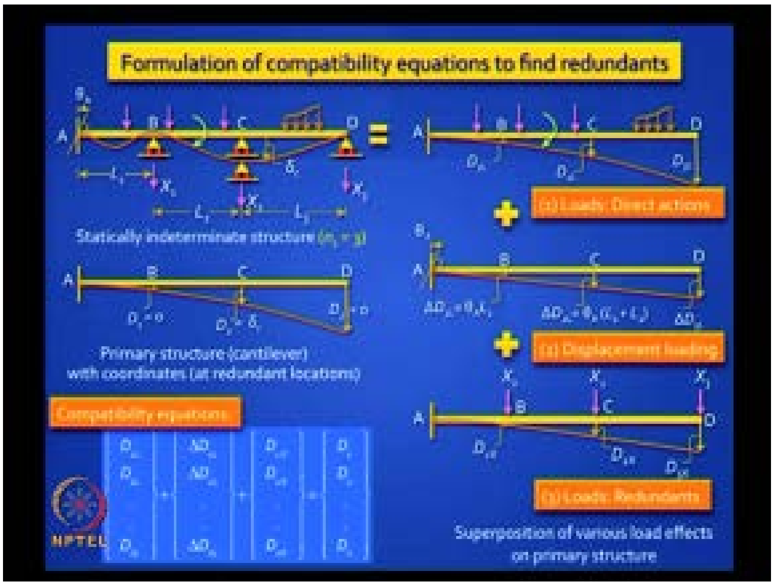

Let us quickly look at a broad generic way of describing this method. I have given you a piece of three span continuous beam subjected to some arbitrary loading and on top of that we have got some support movements. Support A, which is a fixed end is shown to rotate by a known amount, theta A; it is a rotational slip; Support C deflects by delta C. The degree of static indeterminacy of the structure is 3. So, you have many choices of the primary structure; which one would you prefer? Cantilever - It is the easiest to draw. So, you remove all those three supports and you can basically identify that the reaction there is X_1, X_2, X_3.

One would imagine that those reactions will be pointing upward, but there is a good reason to assume them to be pointing downward for calculation purpose. I will show you the reason in a while. Then, they turn out to be negative, which is totally acceptable.

Now, this is your primary structure (Refer Slide Time: 17:10). You have released those constraints at B, C and D; you have got a cantilever. This can deflect and the deflection that you get at B conjugate with the chosen coordinate 1 is $D_{1;}$ deflection at C downward is $D_{2;}$ deflection at D downward is D_3. You know that the answers to D_1, D_2, $D_{3.}$ They are not 0, not all of them are 0. What is D_1? D_1 is 0 in the original problem. What is D_2? It is plus delta C. Then, what is D_3? It is 0. So, these are your governing compatibility to equations. These are what the right side of those equations should look like, that is, 0, delta C, 0.

Now, you take the primary structure and do a series of superposition. Please note that you have two loads here. One is called direct loading; the external loads are directly caused by those arrow marks that you see there. Some are forces and some are moments. It is a concentrated moment shown there. You also have another kind of loading here.

What is that loading called?

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Yes, that is called indirect loading, in this case, by support settlements. These two, you should be able to put on your primary structure.

Loading - the direct forces are pretty easy to do. You can analyze this, can't you? You have to find the deflections D_{1L}, D_{2L}, D_{3L}, the L stands for the loading - external loading. Is it clear? At those locations 1,2,3, where the redundants operate, you must able to find D_{1L}, D_{2L}, D_{3L}. You know enough techniques of finding deflection; any method you can use; we will use the unit load method because it will help us generate the flexibility matrix very easily, in a generic fashion.

Then you take the same beam and now you apply support settlements. What will it look like?

Rigid body moment.

It is a rigid body moment. Of the two support settlements, support movements shown in the original figure, delta C is accounted for on the right hand side of the equation. Do you understand? Delta C is occurring at a redundant coordinate, but theta A is not at a redundant coordinate; it is at a non-redundant coordinate - that you apply as a load, as an external indirect loading on the primary structure. If you will allow that cantilever beam to rotate by theta A, it will undergo a rigid body rotation. We are assuming very small deformations; so, sin theta and tan theta are equal to theta. So, can we write down those additional moments at 1, 2 and 3 as shown here? (Refer Slide Time: 20:18) This is easy to understand; theta A into L_1 is moment here (Refer Slide Time: 20:21), and so on and so forth. Can we work them out? So, you have got all the moments caused by the external loading on the primary structure.

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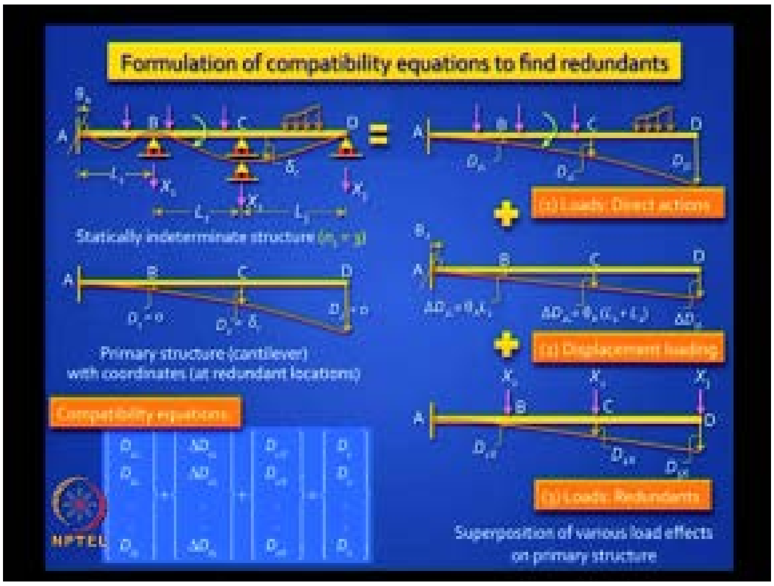

Now, you apply the redundants. Let us put them all together, in the beginning. You apply X_1, X_2, X_3 and let us see the deflections that you get at 1, 2, 3 are D_{1X}, D_{2X} and D_{3X}. Now, you are in a position to write the equations of compatibility. What would they look like? They will look like this.

In general, you may have n locations, n displacements, n degree of static indeterminacy. So, what have we written here? The first vector, first column on the left side, it is deflections that you get by the direct forces on the primary structure, which is what we showed here (Refer Slide Time: 21:17). Next are the additional deflections which you get in case you have indirect loading at non-redundant locations. If you have displacements at redundant locations, they get covered on the right hand side here, not here (Refer Slide Time: 21:35). Is it clear?

Then, we are putting together all the deflections caused by the redundants on the primary structure. You see how your learning has progressed. You first studied force response in statically determinate structures; then you studied displacement response in statically determinate structures and you need that understanding to write these expressions in the compatibility equations, because you are still dealing with a statically determinate primary structure.

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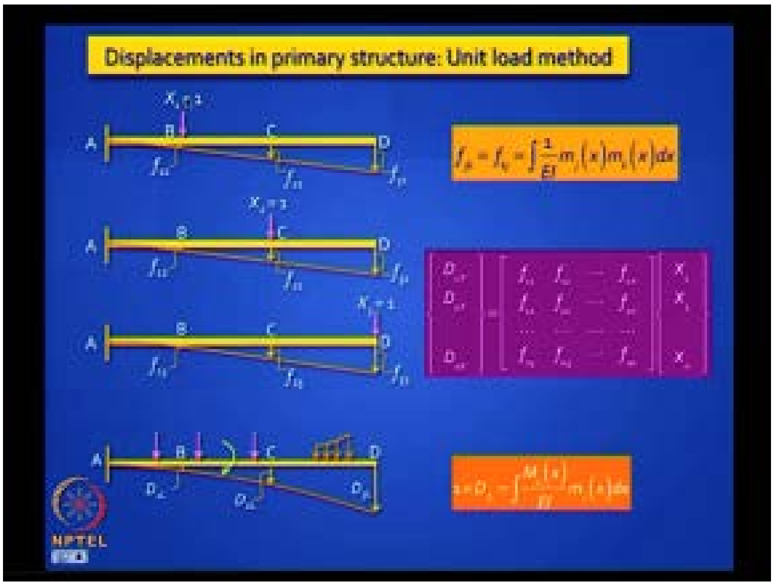

Now, you will find it convenient to apply one load at a time, according to the unit load method. You are familiar with the definition of flexibility. I have shown here three diagrams, three cantilever beams; they are identical beams, but I apply X_1 equal to 1, X_2 equal to 1, X_3 equal to 1; not all at the same time, but separately.

The definition of flexibility coefficient is that, the deflection here, caused by X_1 equal to 1 is called equal to f_{11}, this is f_{21} and this is f_{31}; these caused by X_2 equal to 1 is f_{12}, f_{22} and f_{32} and so on and so forth. Is it clear? So, this is a physical meaning and you will find that these flexibility coefficients are properties of the structure; they do not depend on any loading. How do you find these values?

Unit load method.

You can generate a nice formula, that is integral m_j m_k by EI into dx. This is unit load method. You can do that and write it in this format (Refer Slide Time: 23:20). So, all the deflections caused by all redundants put together can be written like this: D_{1X} is f_{11} into x_1 plus f_{12} into x_2, f_{1n} into x_n etcetera. Similarly, for D_{2X}, and so on. I can write that over all set of equations. Also, I get the deflections caused by the load using the unit load method. So, the beauty is - whatever unit bending moment diagram I drew for this and this and this (Refer Slide Time: 23:54), come in handy when I find these deflections; because if I want to find D_{1L}, I need to apply a unit load here (Refer Slide Time: 24:04). Do you understand? Bending moment diagram caused by this is called M_1 and so on and so forth. Those moment diagrams come in here.

So, you need how many diagrams?

You need one moment diagram, which I call M_L, caused by the external load on the primary structure and I need a separate diagram caused by X_j equal to 1; X_1 equal to 1, X_2 equal to 1, which I call m $_{1,\ 2,\ 3,\ 4}$, etcetera, with the help of these diagrams. If you want to do it manually, with area multiplication method I can get all the answers.

We will see later, in matrix methods you do not have do this manually; you write a program which will generate everything automatically.

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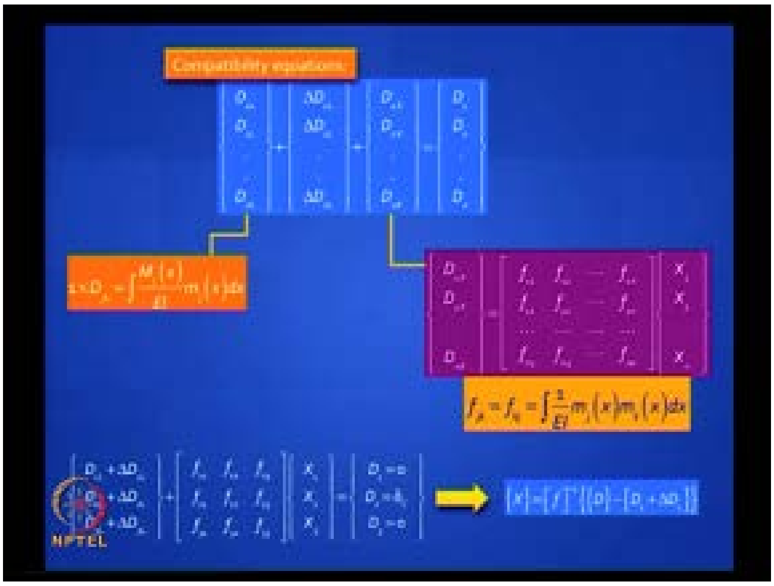

Now, I can write my compatibility equations like this. I am writing all the expressions. In this particular problem where I have only three unknowns, I can write that equation and then solve for $X_{1,}$ X_2, X_3 by inverting the flexibility matrix or using some elimination method and then solving it. Once I have the solution for X, I have got the correct force response and I can draw my bending moment and shear force diagrams on my overall free board. Is it clear?

This is the board method; we have done this.

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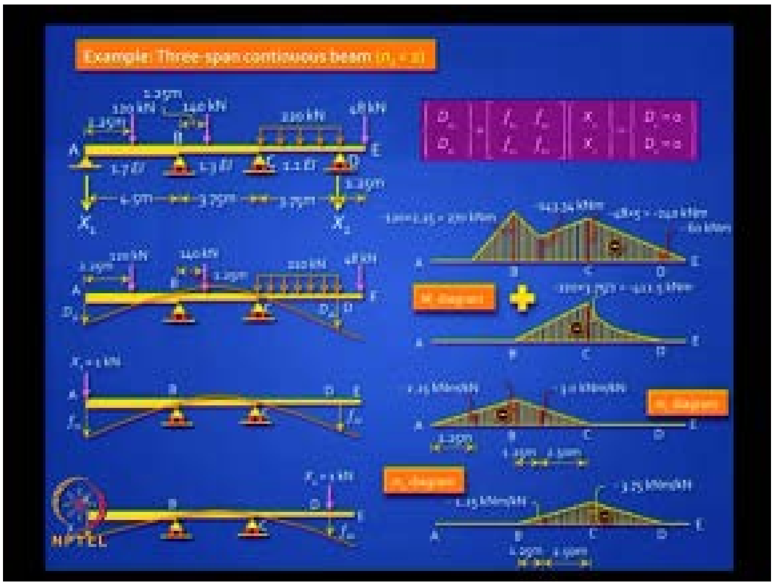

I will go quickly over an example which we have done earlier. This is a three span continuous beam with a cantilever over hanged. This is familiar to you; you have done this. Here, what would be a good choice of redundants? What is the degree of indeterminacy here?

Two.

It is two. Which two would you choose?

Last two.

You can choose any two reactions.

You would set B and C because you are most comfortable with the simply supported beam, where the supports are wide apart. Let us do something contrary to that. Let us choose the reaction at A and D as the supports. There is nothing wrong with that. Of course, you are right; this is not a good structure to build because you will have very large deflections at A and D, but mind you, you are not building this structure in real life. You are building it in your mind just for the purpose of solving this problem. You can straight away write the compatibility equations based on what we have discussed.

We have only two unknowns here. Do you agree these are the correct equations? (Refer Slide Time: 26:44) On the right hand side, D_1 and D_2 are both 0, because you do not have any support settlements in this problem. Your primary structure is 1, where you released the supports at A and D. These are the deflections caused by the external loads. I removed the supports at A and D and I let it deflect under the external loads - this deflection, in this direction, matching this assumed direction of X_1 is D_{1L} and this deflection is D_{2L} (Refer Slide Time: 27:09). Do you know how to calculate D_{1L} and D_{2L}? How would you do that? First, you have to get the bending moment diagram for this loading. Then, you have to apply a unit load f_1 equal to 1 at A and separately, f_2 equal to 1 at D. Then analyze those, which is what we will do.

In the first case, you apply X_1 equal to 1 and incidentally when you do that, this deflection that you get here is f_{11} and the one you get here is f_{21} (Refer Slide Time: 27:45). You can do the same with f_2 equal to 1. Now, you have to solve and get the equation. In other words, we now need to fill up this matrix, which is the coefficient matrix and we need to find these values and then we can find X_1 and X_2. That is it; the concept is straight forward.

Now, the problem with this beam is - you have bending moment diagram, which is made up of not only the straight lines which are easy to handle, but you also have a parabola coming into play. It may be convenient to split it into two for the purpose of calculation. Here, I have done that. I just took the UDL part and through the bending moment diagram, due to the UDL separately, which you can recognize and the rest of the load, three consequent loads would give me a bending moment diagram like this (Refer Slide Time: 28:42). You will notice that both the diagrams are hogging moment diagrams because the overhangs are really making the beam hog; the deflected shape also reflects the hogging. Now, the other one is easy to do.

When you apply X_1 equal to 1, the bending moment diagram, that is, small m_1 diagram will look like that. When you apply X_2 equal to 1, this is what the diagram will look like (Refer Slide Time: 29:09). Let us not worry too much about the calculations. You know how to do this, but conceptually, once you get these diagrams, you can multiply these diagrams, the way we did in the earlier classes and get the answer.

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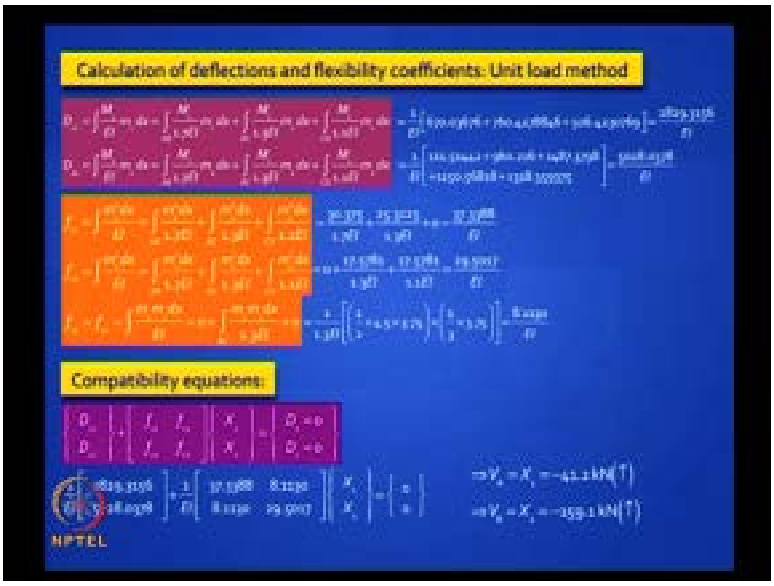

For example, if you want D_{1L} and D_{2L}, you need to do that integration. These are the answers, if you work them out. They are solved in the book. You can study it in detail. You have done this last year; so, you are familiar with it. You can find f_{11}, f_{21}, which is equal to get f_{12} and $f_{22;}$ they are pretty straight forward. You can get these numbers and plug them into the equations. These are very easy to solve in your calculator; you can invert the matrix or directly solve it. You get X_1 next. You notice that they come negative. They come negative and so you interpret it as the force is acting upward.

The reason why we assume them to act downward in the first place is - you will find that all the bending moment diagrams have the same sign, in general. So, you do not have to worry about the sign when you multiply; then negative into negative turns out to be positive. That is the main advantage.

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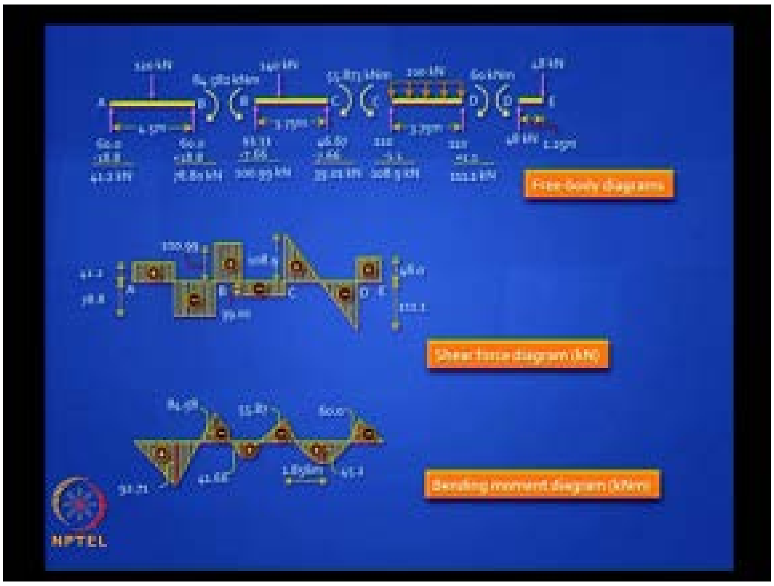

You can draw the free body diagrams. Once you know X_1 and X_2, you can do this. You can draw the shear force diagram and the bending moment diagram. This is the least expected from a student at this stage. That means, you have learnt how to do statically determinate structures; you have got a statically indeterminate structure; you have found the redundants; after you found the redundants, the structure is reduced to a statically determinate structure. You can handle any structure. We have shown a beam, but could very well apply this to a truss.

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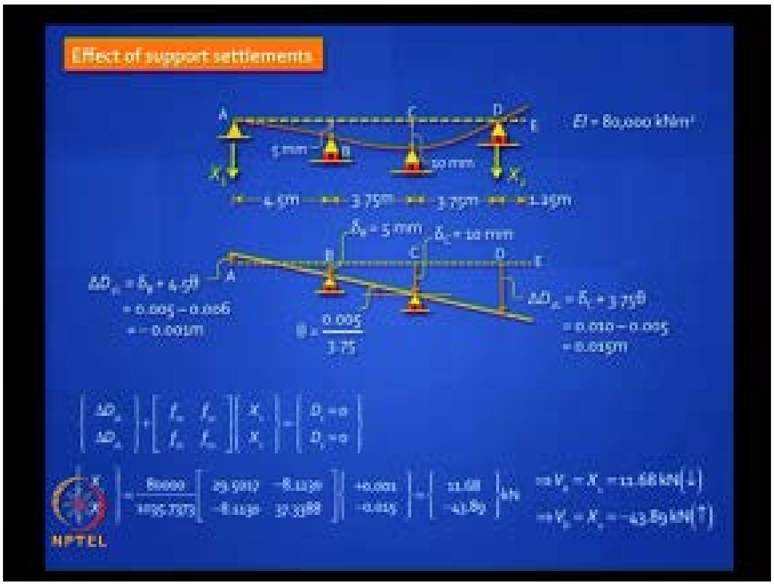

Usually, it makes sense to handle support settlements as a separate analysis problem rather than mix them along with the direct forces because you would like to know, out of curiosity, how much effect this has, independently. Here is the same continuous beam where you have given that the support B has settled by 5mm and the support C has settled by 10mm. Mind you, the same flexibility coefficient which you calculated earlier are applicable here. You do not have to do any extra work; you just have to write the equations correctly.

We will choose same redundants. You will find that 5mm and 10mm have occurred at the non-redundant locations. So, you have to treat it as a load. So, you take the primary structure and you allow these moments to take place, you will find you will get a nice straight line rigid body movement. From this moment, you can figure out what is delta D_{1L} and what is delta D_{2L}. Can you work it out? What is delta D_{1L}?

The whole beam rotates by an angle theta and the relative displacement at B and C is 5mm. So, you will find this angle is this difference (Refer Slide Time: 32:20), which is 10 minus 5 divided by the span 3.75; 10 minus 5 in millimeter has to be converted into meter. So, it is 0.005 by 3.75. That angle, you need to use to figure out those two movements. That is simple; you can easily do it using logic and get the two deflections correctly with the proper sign. You will find that delta D_{1L} turns out to be negative whereas, delta D_2L turns out to be positive. If you make a mistake here, you will get a wrong answer. So, you have to get this correct.

After that, we have same old equations. Only thing here is instead of D_{1L}, you write delta D_{1L} because this is the one caused by the loading. Suppose you have a problem in which you have both, the external loading and this, then you add those two quantities on the left hand side; that is it. f_1, f_2 is all known. Solve this equation; you get X_1 and X_2.

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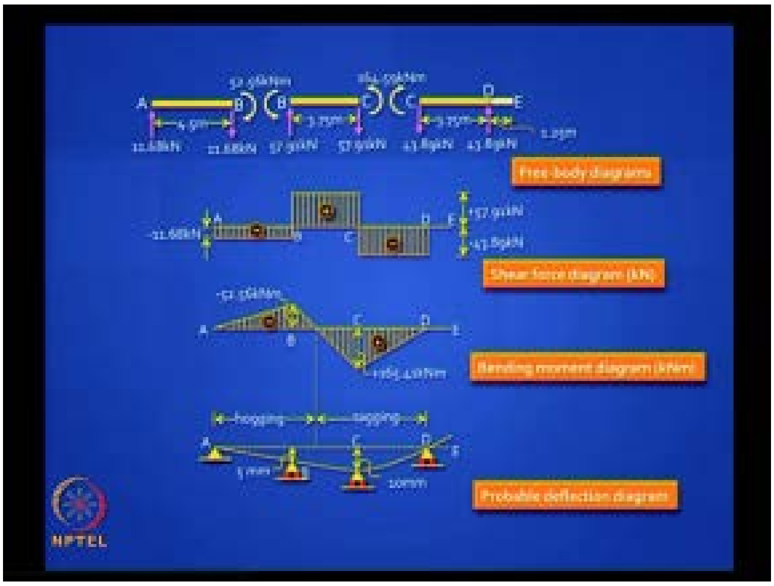

Then, you can draw free body, shear force diagram, bending moment diagram and deflected shape. Conceptually, it is very straightforward.

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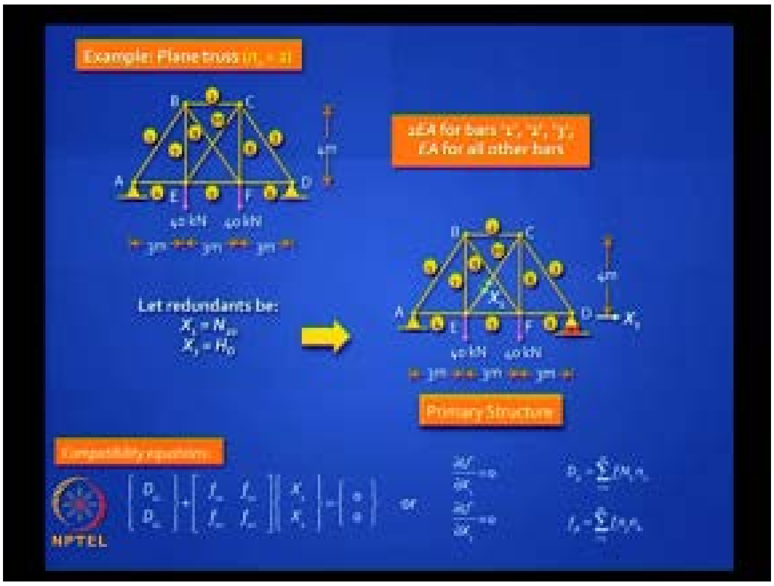

If you take the example of a truss, here, you have a truss with how many bars? Ten bars. Is it statically determinate or indeterminate? It is indeterminate. What is the degree of indeterminacy? One look at that structure should tell you. First, look at external indeterminacy; externally, you can view it as beam. It is not simply supported. So, you got one extra redundant. If you make it simply supported, that means release the horizontal reaction one external indeterminacy, and internally, you got an extra diagonal. Can you see that?

You can choose. That is what you can do.

You can choose one of those diagonal forces as one redundant and a horizontal reaction; may be a reaction at D as another redundant. So, your primary structure will look like this. You made a roller support at D and you cut the bar – 10 in this case. The forces, if you want to insert them, will be $X_{1,}$ which is the tension in that bar 10, which you do not know, and the horizontal reaction, we are calling it as $X_{2.}$ Is it clear?

Now, write down the compatibility equation. They take the same format; it is very easy to write them down. On the right hand side, D_1 and D_2 are 0. When you say D_1 is 0 you are actually making a statement that the relative displacement between the cut ends is 0; the bar should never have been cut in the first place. We are not saying that this point is not going to move; it will move when the bar elongates or contracts. But we are saying the relative moment is not there. So, those double arrows pointing towards each other refer to relative movement, when you interpret from a displacement perspective.

Physically, it is a little troublesome, because if they really approach each other, they will penetrate each other or overlap each other. That is something that you worry about in your mind. They can separate out or they can penetrate. Is it clear? That is the meaning of that.

You can also use the strain energy formulation and solve the same equations; they mean the same. How do you calculate the displacements? You can use a unit load method; I hope you are familiar with this. Apply X_1 equal to 1, get those forces; apply X_2 equal to 1, get those forces and so on.

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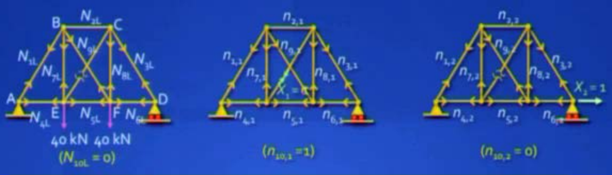

Bar no (i)	Length (m) (L_i)	Flexibility	Bar Force due to		
			(a) External loads (kN) (N_{iL})	(b) $X_1 = 1$ (n_{i1})	(c) $X_2 = 1$ (n_{i2})
1	5.0	2.5/(EA)	− 50.0	0	0
2	3.0	1.5/(EA)	− 30.0	− 3/5	0
3	5.0	2.5/(EA)	− 50.0	0	0
4	3.0	3.0/(EA)	+ 30.0	0	+1
5	3.0	3.0/(EA)	+ 30.0	− 3/5	+1
6	3.0	3.0/(EA)	+ 30.0	0	+1
7	4.0	4.0/(EA)	+ 40.0	− 4/5	0
8	4.0	4.0/(EA)	+ 40.0	− 4/5	0
9	5.0	5.0/(EA)	0	+1	0
10	5.0	5.0/(EA)	0	+1	0

If you do that, I have got three diagrams here. The first diagram is the structure in which I apply only the external loads, the primary structure.

The second one, I apply X_1 equal to 1 and I solve it by method of joints or by method of sections. In the second one, I apply X_2 equal to 1. When I apply X_2 equal to 1, X_1 is 0. So, this bar has zero force, this bar has zero force here as well (Refer Slide Time: 36:40). Here, this bar has a force of unit magnitude. We analyze this; we have done this last year, so I am going fast over it. It will be nice to write it in a tabular format. Write down the ten bars, note their lengths and write down the flexibility which is L by EA. So, the length of each bar is known; the EA values are given to you; analyze those three trusses and write them down.

If you take this particular case, it is so easy. There are only three bars. When you pull A horizontally, you will have an axial tension in these three bars and you do not have any force in any other bar. That is why this is just 1 1 1 and all the others are 0. Is it clear? These things should not take you too much time.

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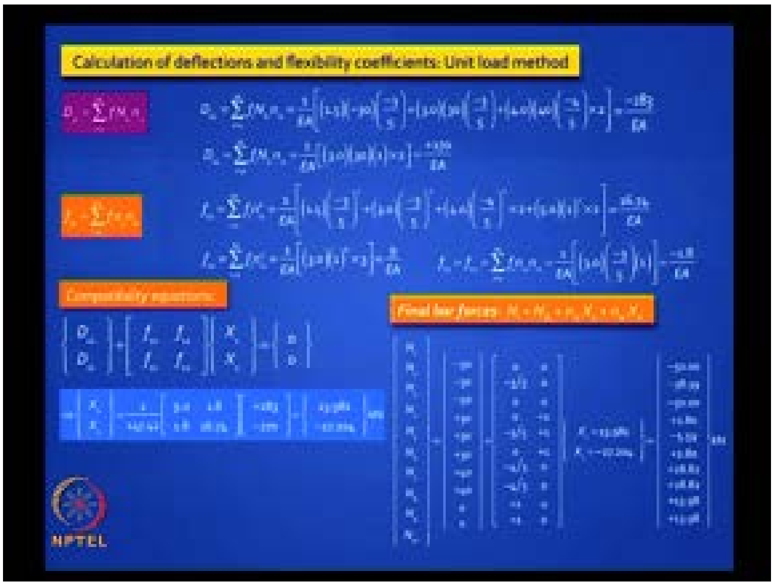

Once you have you done this exercise, invoke your unit load method equations. Are you clear about this? We have done this before. So, get those numbers in terms of VA and VA gets eliminated. Substitute in your compatibility equations and solve for X_1 and X_2. Final bar forces are given; you can do superposition. You have already solved. In that table, you have got forces; you have got force called by external loading. You have got force called by X_1 equal to 1. Take that column, multiply by X_1; take the next column, multiply by X_2 and add up everything to get the answers.

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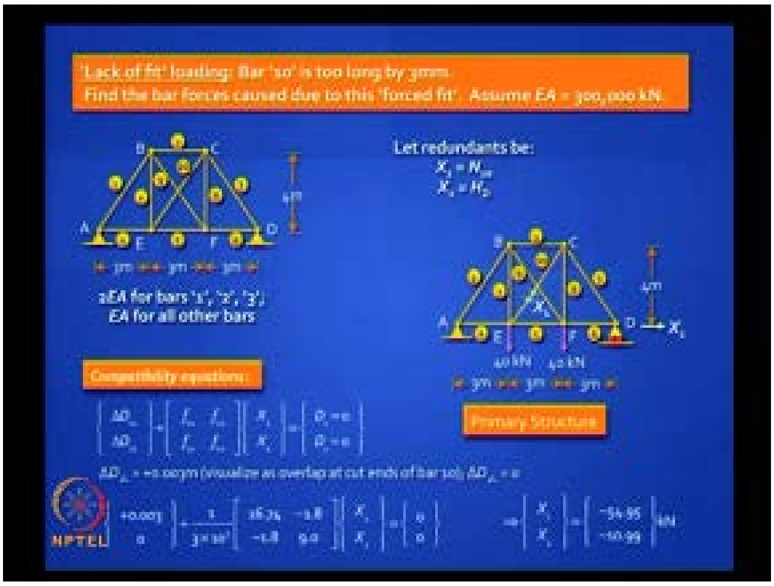

It is interesting to note that some of those bar forces will be positive and some will be negative; you must able to judge correctly. For example, in that particular loading, you would expect the bottom chord to be in tension and the top chords to be in compression, in the verticals and diagonals appropriately. You should figure it out.

Let us take a displacement loading problem. Let us say, you have a lack of fit problem, bar 10 is too long by 3 mm. When it came to the side, that bar was too long; so it had to be force fitted; that means, it had to be hammered into place. Because the structure is over rigid, all the other bars will resist this fixing. Had it been just rigid, they will move and accommodate this movement. You have a problem. You are forcing that bar to be fitted. First of all, can you tell me what will be the force in that bar? It will be compression, because it was too long and it will not be allowed to get that length of 3 mm; the final excess length may be 2 mm or 1 mm. It depends on the relative stiffness of all those bars. Because of this, the other bars will also be stressed. So, you have a self-equilibrating system.

How do you solve this problem? (Refer Slide Time: 39:49) Let us take the same redundants, as we have identified earlier. Now, the compatibility equation will look like this. Is it clear? This is because now you have a displacement loading.

How will you get delta D_{1L} and delta D_{2L}? This is not as easy as the previous problem where you had the rigid body moment in your beam.

How will you find this out? What is delta D_{1L} and delta D_2L? What does this physically mean? In which structure?

In the primary structure. If you take the primary structure where you have the roller support at D and the bar 10 is cut and when you allow this bar 10 to be too long by 3 mm, will you have any forces in the system? No, because it is just rigid and is statically determinate. You will have no forces.

Can you write down delta D_{1L} and delta D_{2L}? What will delta D_{1L} be? It will be 3 mm; it is free to take whatever length you want it to take.

Will it be plus 3 mm or minus 3 mm?

It will be plus 3 millimeter. It is too long; so, it will be plus 3 mm.

What is delta D_{2L}? Delta D_{2L} is 0. Now, this 3 mm, you should write in consistent units. So, it is 0.003 meters. If it is too long, you should visualize this as overlap of those two bars by 3 mm. Is it clear?

That is all you have to do. You have already got the flexibility matrix in the previous example. So, you just have to plug in these values.

Sir, why are we not taking D_1 as 3 mm?

This is a good question. What is D_1? D_1 is the final displacement in the original real structure, not the primary structure. When you identified X_1 as the axial force in that member and the primary structure was a structure in which you cut that member, D_1 is nothing but the relative displacement between the cut ends.

What is the relative displacement between the cut ends? It is 0. These are the typical mistakes that students make. I want absolute clarity in understanding here. D_1 is always 0, because it is the relative displacement between the cut ends. You had no business to cut that bar. 3 mm is the elongation in that bar in a statically determinate system, which is the relative distance between the cut ends in the primary structure. When we are talking about the primary structure, you not talking about the right hand side, you talking about the left hand side; because all these quantities on this side of the compatibility equations, that is, all these values relate to primary structure. Is it clear? This is a common mistake we done to make.

D_2 is always 0, so you do not have to worry. Is it not easy to solve? You already got the flexibility matrix, you invited it earlier and so it is very easy to solve and get X_1 and X_2.

Once you are familiar with the truss and its flexibility matrix, you can handle any loading; you put 100 loads, you get hundred answers just by solving this matrix.

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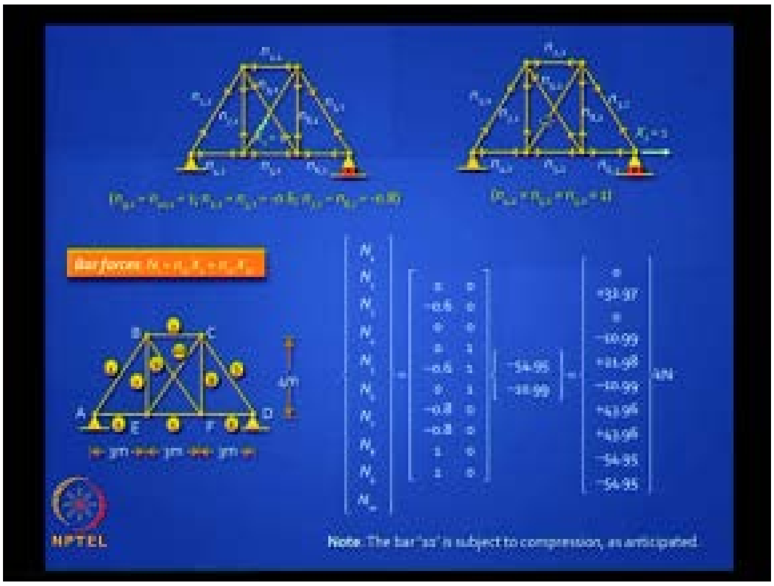

Once you have the solution, you can do superposition, you can get the forces and what is interesting is that you must check the answer. The bar 10 you suspected would be in compression and it does turn out to have a negative value here. So, it makes sense. If it goes into tension, then you made a big mistake. You must have these checks included.

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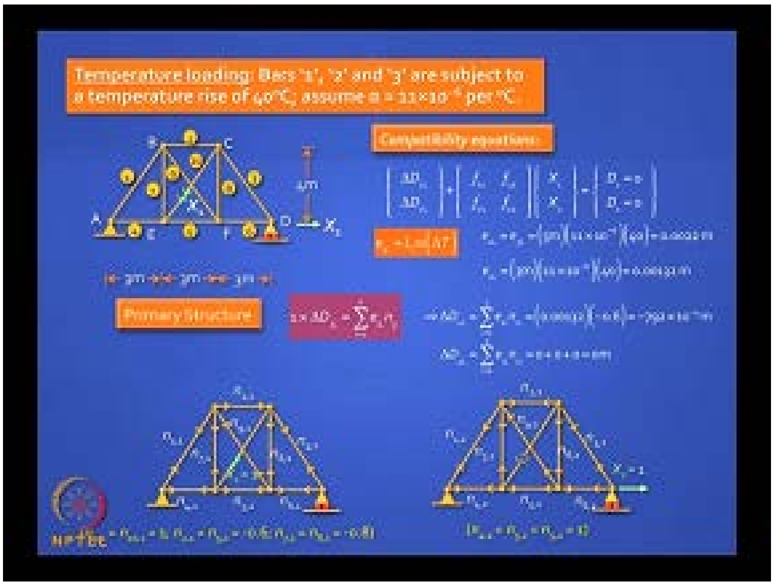

One last problem in continuation with previous one is as follows: Supposing you have temperature loading. I now take three arbitrary bars, 1,2 and 3 are subject to a temperature increase of 40 degree Celsius and the coefficient of thermal expansion given to you. How would you deal with the same problem? This is your problem; you can remove 40 kilonewton. How would you analyze this problem? You have to figure out in the primary structure what… you agree that your right hand side, D_1 and D_2 are 0. How will you get delta D_{1L} and delta D_{2L}?

[Noise] (Refer Slide Time: 45:10)

Why do you say Delta D_{2L} is zero?

[Noise]

You are right. It does not move. It does not move, for what reason?

How do you get delta D_{1L}? Let us say, we do not make any guess, we do it the hard way. How do you get delta D_{1L} then? The difference between this problem and the previous problem was that, in the previous problem, you had a moment in the redundant coordinate. In this one, you have moment in non-redundant location, like bars 1, 2 and 3 are not at the coordinates 1 and 2. So, in your primary structure, which is one with the roller support and with the bar 10 cut, if I heat bars 1, 2 and 3, can you tell me what those moments are? How do you find them out? You can go for the unit load method, Is it not? If you want to find delta D_{1L}, what should you do? You apply f_1 equals to 1.You do that, but first in your primary structure, if bars 1, 2 and 3 elongates, you had to find those elongations. Those are nothing but L alpha into delta T. That is a straight forward calculation. If your alpha is this and your delta T is 40 degrees and your L is this (Refer Slide Time: 46:42), you can figure out what are E_{1L}, E_{2L} and E_{3L}. That is straight forward.

Then, what do you do? You invoke the unit load method, which is in this equation, you apply f_1 equal to 1 or X_1 equal to 1 and you look at this displacement field, where you have these temperature changes. You have these known bar elongations; you have a pure geometry problem and you are applying a unit load invoking the virtual work method to solve for one unknown in that displacement field. We have done this in earlier class.

Can you find this out easily? You can do that; you have all the answers, you have the forces caused by X_1 equal to 1 and the forces caused by X_2 equal to 1. You will find that delta $D_{1L;}$ in this case, it is easy to work it out. Delta D_{1L} you can solve and you can sum up. You will find that only these three bar forces you need to worry about because all the other forces are 0. You have three bar forces; you can multiply them and get the answer. The answer would turn out to be equal to 0.6. This is easy to work out. You get delta D_{1L}. Delta D_{2L}, as you guessed correctly, will turn out to be 0, for the simple reason that when you pull here, only these three bars get a unit force and these bars do not have any force (Refer Slide Time: 43:25). So, you are multiplying a non-zero quantity, which is your bar elongation, with a zero quantity. We can guess this or calculate it. This is how you deal with non-redundant displacements.

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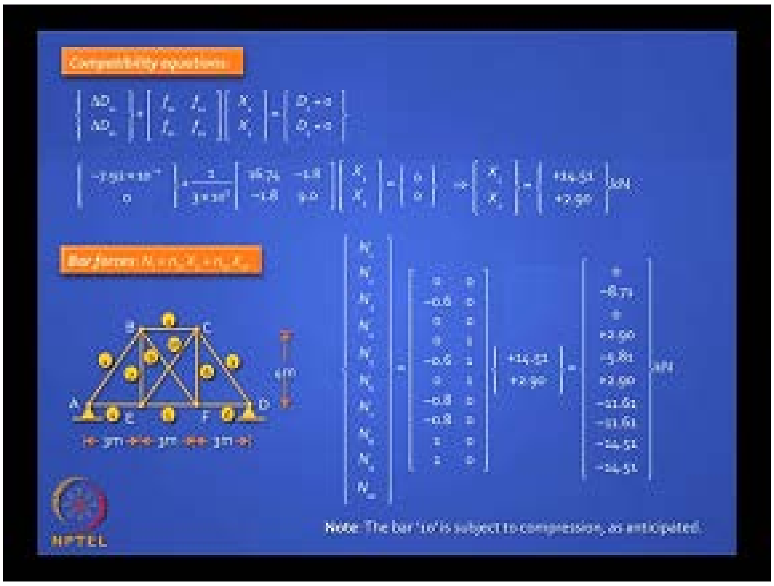

We are trying to tackle conceptually, different types of problems. You can substitute these values, get your unknown forces, do your superposition and here bar 10 turns out to be compression.

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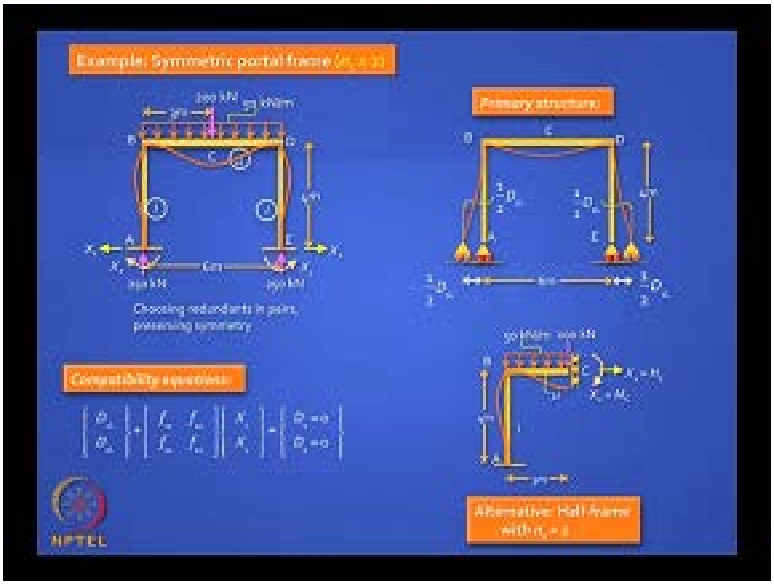

Let us look at one more example of a frame. We have a portal frame. You have done this earlier. What is the degree of indeterminacy? Normally it is three, but there is a symmetry in the frame. Can you reduce it? You are right. You can reduce by 1. For example, what do know for sure? Vertical reaction is statically determinate; you should be never been thinking of indeterminacy for the vertical reactions. This will be the deflected shape; there is symmetry in the deflected shape because the structure is symmetric, the loading is symmetric. The vertical reactions are total load, that is, 200 kilo newton plus 50 into 3, that is 350.

50 into 6. You are right. 500 divided by 2 is 250.

Vertical reactions are determinate, but the horizontal reactions are not; your moments are not, but they are equal and opposite. So, you have only two unknowns. You could do this. If you do this, you will have to release all those moments. That means, you must allow for a relative moment along X_1 and relative rotation along $X_{2,}$ which means you must replace those fixed ends with roller supports. Is it clear? Because it is all symmetric moment, the total horizontal moment, which is D_2 will be divided by 2. So, it is D_{2L} by 2, D_{2L} by 2 and this rotation here is D_{1L} by 2. Is it clear?

This is one way of doing it, but you can also cut it in the middle at C. The appropriate boundary condition would be a guided roller support. Why? Because it is free to deflect there, there is no shear transfer possible there. When you cut it into two, the constant load, 200 kilonewton should also get cut into two. 100 kilonewton comes on the left side frame and 100 kilonewton comes on the right hand side frame. Here, clearly there are only two unknowns. You can make it a cantilever. There is another way of doing it. Whichever way you do, these will be the compatibility equations.

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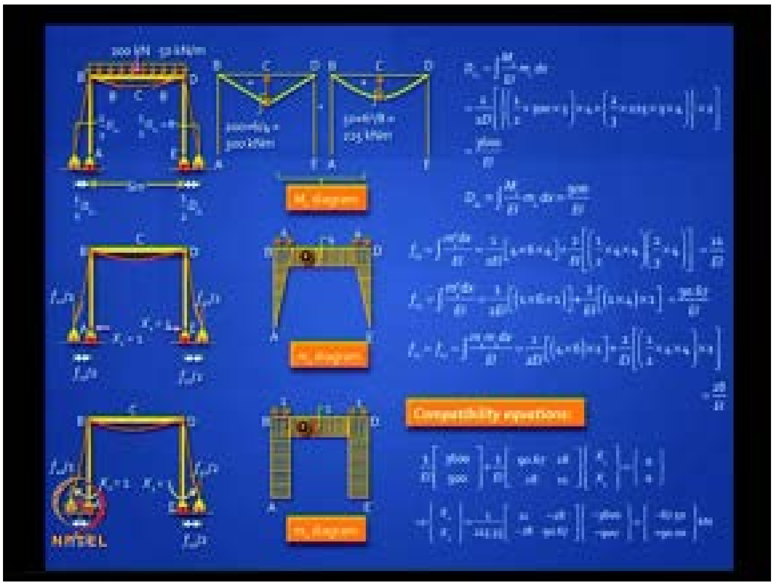

Let us take the first way. Can you find those moments? Well, draw the bending moment diagram, capital M_L diagram. The constant load gives you a triangular moment; the UDL gives you parabola; this is your small m_1 diagram, when you apply X_1 equal to 1; this is m_2 diagram, when you apply X_2 equal to 1. You do not even need to mark those deflections; they are just to help you understand; you can blindly go ahead and calculate D_{1L}, D_{2L}, $f_{11,}$ $f_{22,}$ $f_{12.}$ Once you have unit load, it is pretty easy to calculate. We have done this earlier. So, the method is straightforward.

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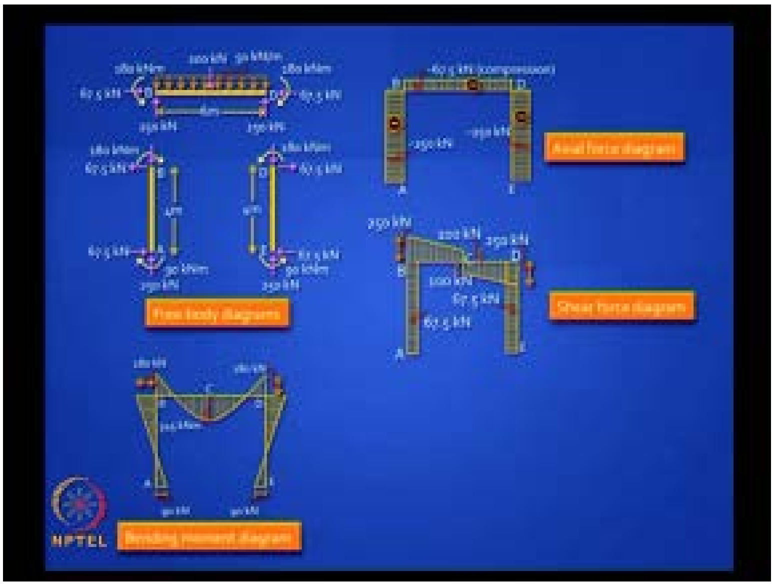

Write down the compatibility equations; solve for X_1 and X_2; draw your free bodies, draw the axial force diagram, the shear force diagram and bending moment diagram.

We will later do the same problem by displacement method. So, I will show you that it is much easier; you can actually do it in a couple of minutes and get the bending moment diagram directly; this is what the deflected shape looks like. So, we will stop here and take this up in the next class.

Thank you.