Advanced Structural Analysis
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Module No. # 7.3
Lecture No. # 40

Analysis of elastic instability and second-order effects

Good morning. This is lecture number 40, the concluding lecture in module 7, which deals with the Analysis of elastic instability and second order effects.

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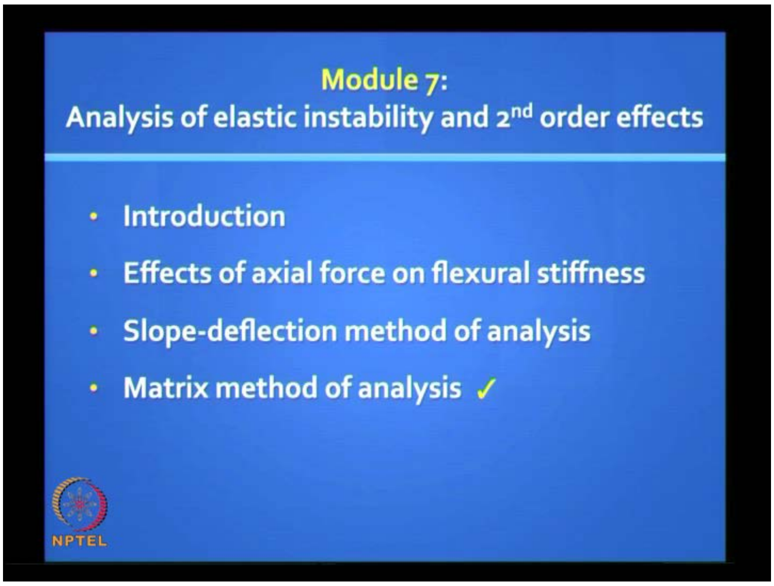

If you recall, in the last session, we looked at the slope deflection method of analysis which is the manual method. Now, we will look at more systematic way of solving the same problem using the matrix method of analysis. We will be using the conventional stiffness method.

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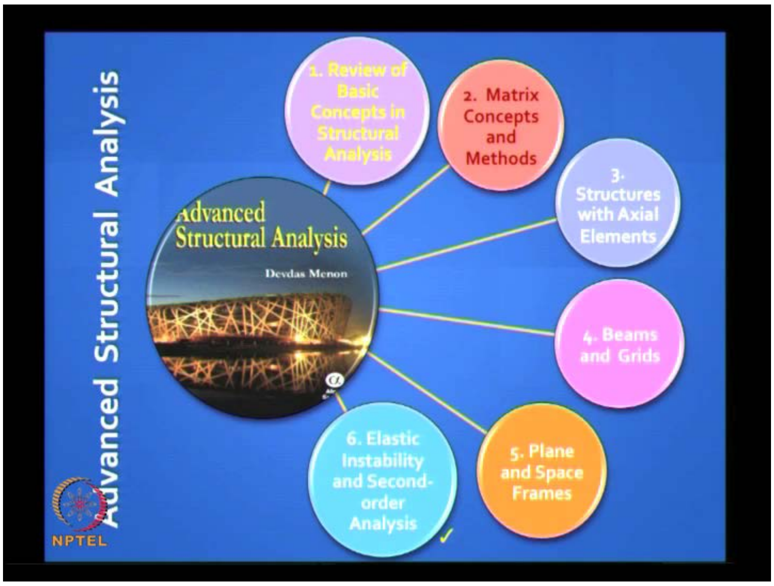

So, this is covered in the last chapter, in the book on Advanced Structural Analysis.

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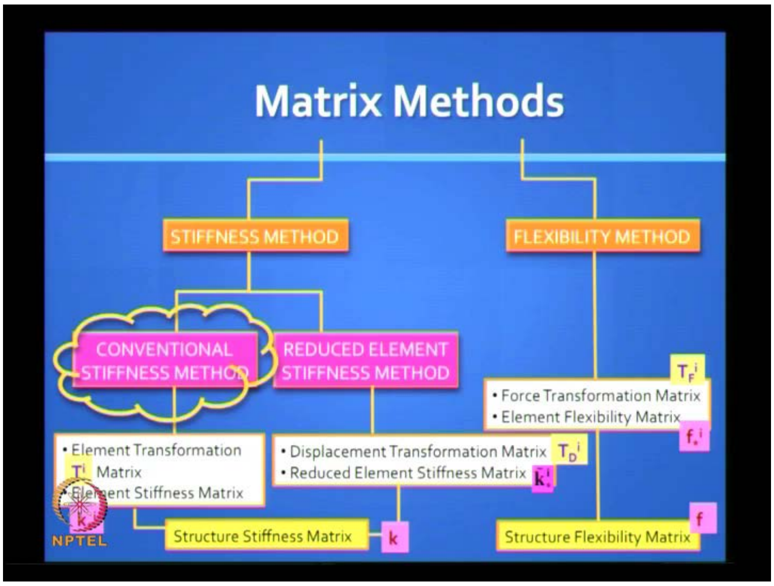

You are very familiar with all these methods. So, it is just to remind you that we will be using only the conventional stiffness method here.

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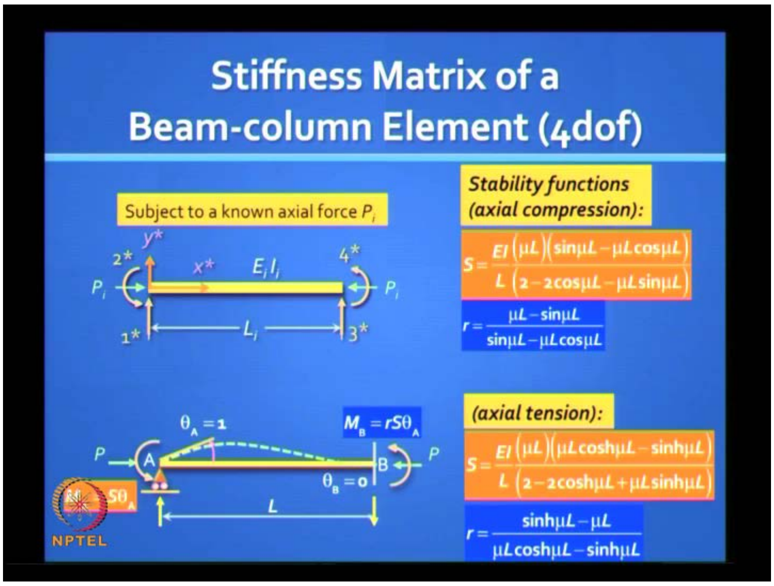

So, if you are using a beam element, which will now be called a beam column element because we are going to consider the interaction between the axial force and the bending and shear forces, and their effects. So, it is more appropriately called a beam column element, and so, we are switching over from the beam element to the beam column element with the difference that we now have an axial force P i whose value is known; P i is known in advance.

So, if you write down the measures of flexural stiffness, they will take this form; instead of 4 Ei by L, we have capital S; instead of 2 Ei by L as a carryover moment, we have r into S, and these are called stability functions; they are trigonometric functions. And if you have a case of axial tension instead of compression, then you have them in a hyperbolic form.

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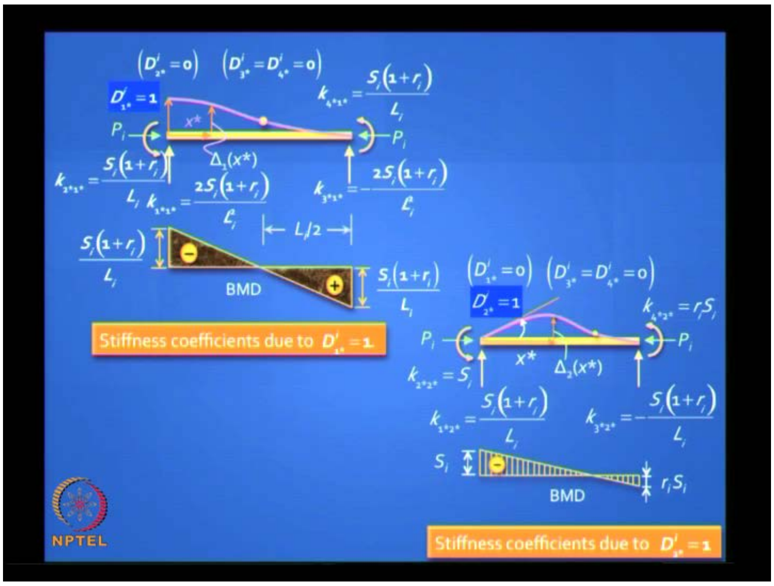

You are familiar with this and so we can easily generate the stiffness coefficients corresponding to the 4 displacements, applying them one at a time; the easiest to apply is when you have a unit rotation. At one end, instead of 4 Ei by L, we have S i; carry over moment will be r i S i, and then the shear forces come easily, as the some of those two moments divided by the span.

If you have translation with both the ends fixed, then you have an equivalent chord rotation given by 1 by L, and the formulation is similar; similar to what we did for a conventional beam element without any axial force acting.

If you recall, if you had a clockwise chord rotation, you have anticlockwise moments at the two ends in the conventional beam element; they were equal and their value was 6 EI by L square. Instead of 6 EI by L square, we now write it as s into 1 plus r divided by L. So, if you plug in those earlier values of S equal to 4 EI by L and r equal to half, you will get the same result.

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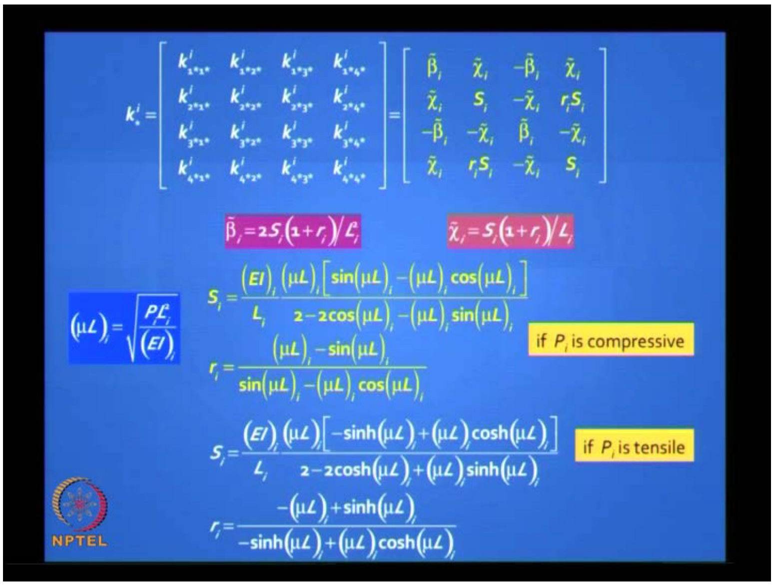

So, we have this familiar form of the 4 by 4 element stiffness matrix and we can easily generate all the parameters. If the axial force is compressive, you have it in this form; if the axial force is tensile, you have it in this form. Now, all this we are familiar with. We have actually used this when we did the slope deflection method, except that in the slope deflection method, we use what is equivalent to the reduced element stiffness formulation. Now, these trigonometric functions are not very convenient to use. So, luckily, for us, there is a major simplification possible, and let us look at that.

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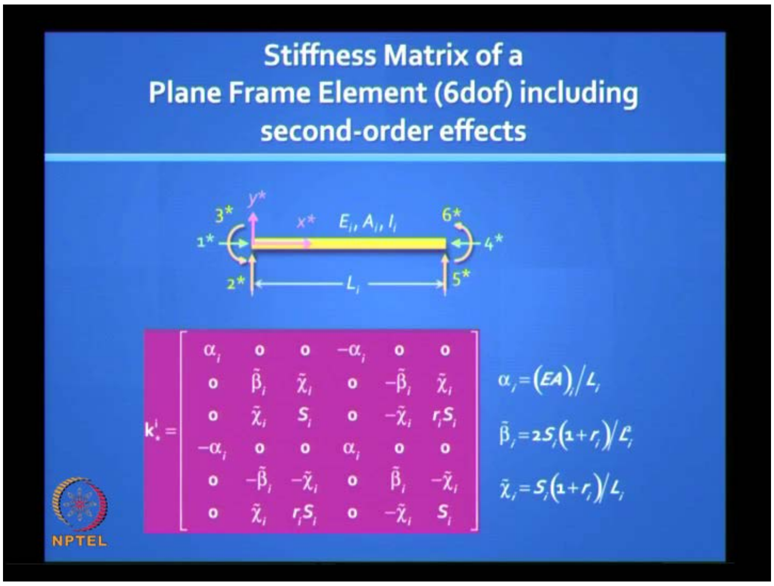

So, if you have a 6 degree of freedom element, if you had a plane frame element instead of a beam element, here also you have an axial force, but the big difference is you do not know the value of the axial force. So, we leave it as f 1 star and f 4 star, which must be in equilibrium with each other. And also here, we are allowing axial deformations which were not accounted for in the previous case. So, you can use this for plane frame problems and you have the standard 6 by 6 elements stiffness matrix, where the flexural components are suitably modified.

So, what is happening here is - the axial stiffness is still not changing; it is still A by L, but it is indirectly affecting the flexural stiffness components.

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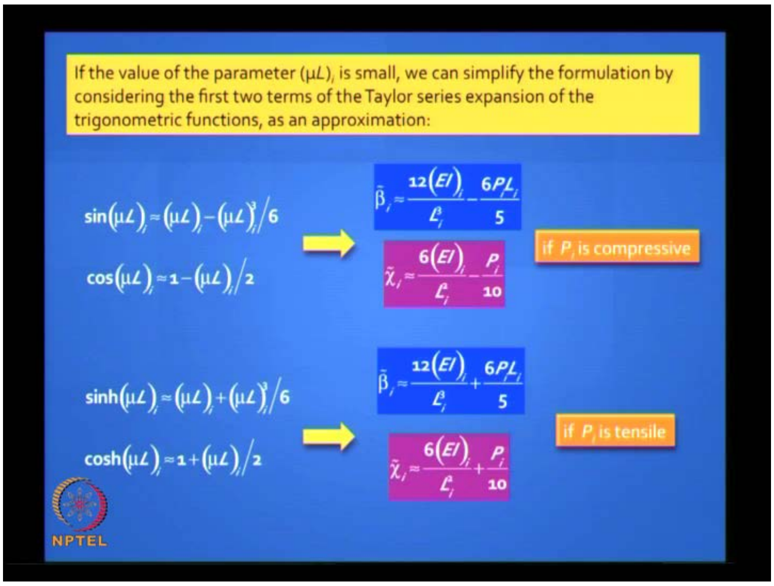

So, it takes this form, and if the value of the parameter mu L… what is mu L? Square root of PL square by EI. Then, we can simplify the formulation by expanding these trigonometric functions in a Taylor series form; so, it takes this simple form. If you take the first 2 terms for a sine function and a cosine function, and you plug in these values in the stability function, you get a very beautiful result.

You have only 2 terms and you have a kind of a constant, a linear relationship with P i. So, that makes the whole thing very simple. You do not have any more sine and cos, and if you have that hyperbolic function, then it takes this form.

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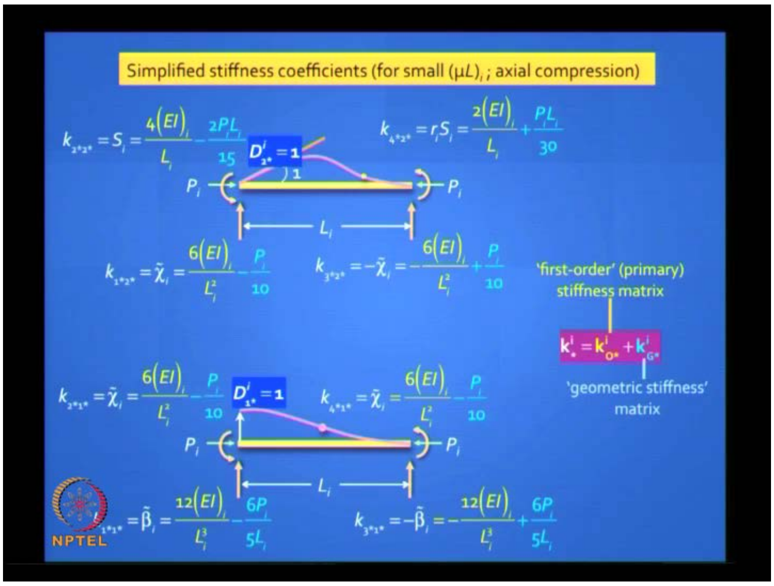

Now, one way to understand this, it is a simpler way rather than remembering these expressions, is to interpret them physically.

So, I have shown here in this figure, the standard force stiffness values for a beam element. The stiff standard coefficients are shown in yellow color. So, you have 4 EI by L, 2 EI by L, 6 EI by L square, as shown there; you are familiar with that.

You just need to add or subtract an additional term which involves the axial force P. So, it is a tremendous simplification, and what you see is naturally the flexural stiffness terms; the primary term 4 EI by L gets reduced when you have an axial compression, and if the compression is equal to the critical buckling load, it will actually degenerate to 0.

So, that term is minus 2 P i L i by 15, and the carryover moment on the other side which is normally 2 EI by L gets actually increased by a small quantity P L by 30; the assumption is P is constant and you know it is value. So, you can actually modify the stiffness if you know P. You have an absolute measure of stiffness and the vertical and vertical reactions are easily obtainable from these 2 moment terms. So, you will find that the 6 EI by L squared gets adjusted by a factor P by 20; it is a force unit. So, you reduce on the left side and you increase on the right side for this.

So, try to remember these formulations. Of course, you do not really need to for your examination because this is this portion I am excluding from your examination; this is a little advanced, but you can see that, it is quite easy to operate. And likewise, you have these correction terms when you apply a translation like this.

So, if you look at these two terms, the yellow term is your conventional stiffness coefficient. So, we will refer to those terms as first order or primary stiffness coefficients, and they buildup the primary of first order which I show with the subscript o; o meaning first order; k i o star is, in the local coordinate system, the element stiffness matrix without any axial force influence coming into play. So, that is the first order primary stiffness, and now, to that you have to add or subtract.

We will say, we add with the appropriate sign, whether positive or negative, and that term is called the geometric stiffness matrix. Why is it called geometric? Because it is kind of reflecting the geometric nonlinearity in this structure; it actually is looking at the effect of the behavior in the deformed configuration. It is only in the deformed configuration. The P delta effects come into play and those effects are second order effects. You can also call it as second order stiffness matrix and we can write an expression for it just by looking at these pictures.

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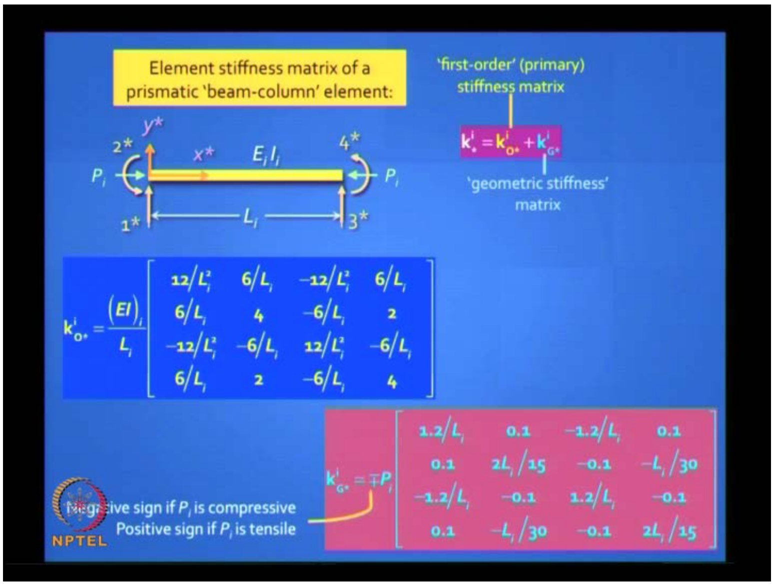

You can it take this form. So this is your primary - the yellow color, and this is your second order; is it clear? We put minus plus P I; the default is minus if the P i is compressive, but it is positive if P i is tensile because you know that the primary stiffness gets actually enhanced when you have axial tension.

So, we have actually simplified the whole problem so beautifully. It is very easy to now write down the stiffness matrix. We are including a correction for the axial force.

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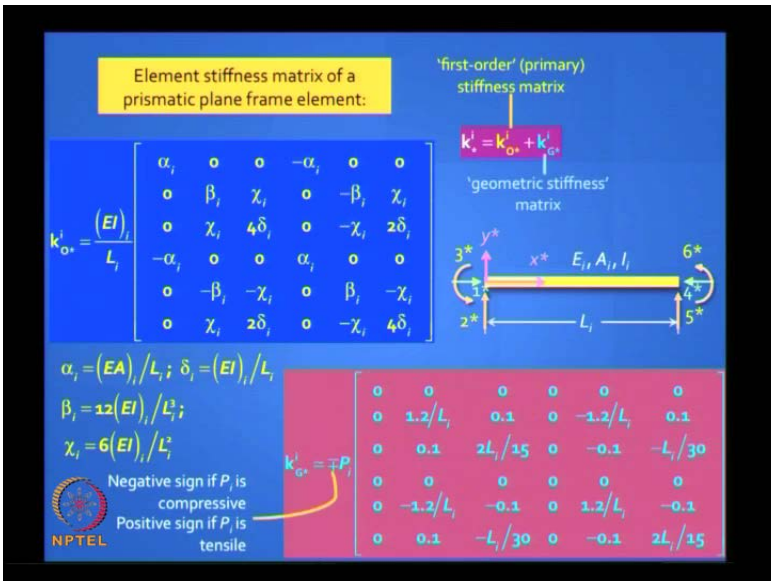

If you have a plane frame element, you can do exactly the same thing; only thing, now instead of 4 by 4 matrix, you have a 6 by 6; so, it looks like this; this is your primary matrix, where you have the terms as we did earlier.

Now, you have this additional term for the geometric stiffness matrix. You will find that the first row, and the third and the fourth row, and the first column and the fourth column, in the geometric stiffness matrix are filled with 0s. The reason is this effect is only for the flexural stiffness terms. The geometric stiffness does not get effect does not affect the overall stiffness matrix for the axial terms; axial term is still A by L; no change in that.

So, that is it. We are now ready to solve any problem. If someone were to give us a value of P, you just have to modify the stiffness and do the same, all there, but this also gives us the way of finding out the critical buckling load.

How do you find the critical buckling load? What? How do you find the P critical? Well, not in an element, in a structure, once you assemble the structure.

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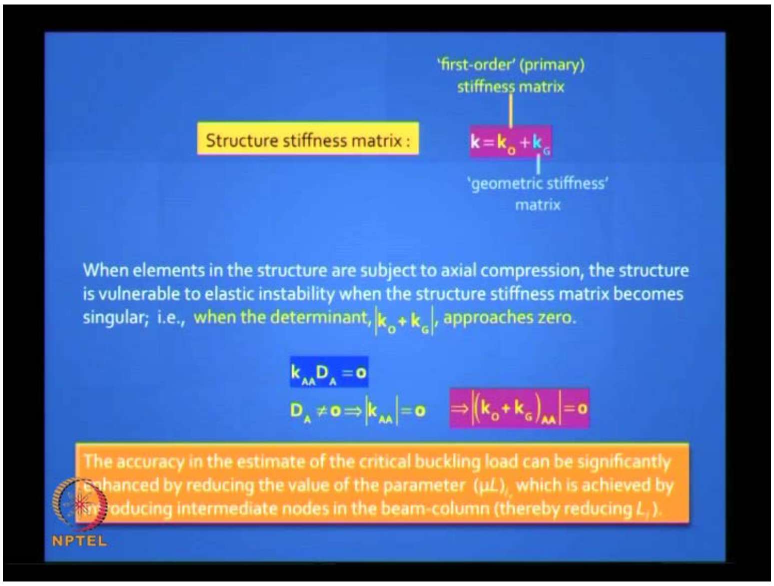

So, the structure stiffness matrix will look like this. After you assembled it, there is no I coming in here; you have k equal to k O which is primary, which is what we got earlier, plus k G, which is the geometric stiffness matrix for the whole structure. And let us say, there are loads acting on the structure, and very conveniently let us apply. They are equivalent joint loads and let us apply the loads at the nodes, and let there be no moments.

If you have moments, or if you have a distributed load, you realize that that it really does not affect the buckling load capacity of this structure. You remember, when you had primary eccentricity, you started of the lateral deflection at some initial point; then, you traced asymptotically, the critical buckling load.

So, how do you define a critical buckling load for a column? The stiffness does not become 0; well, if you had one element you could use that language, but for a structure you have many components in your stiffness matrix.

So, how would you say the same thing? If you gradually increase a load P, if it behaves in a linear elastic manner, you would find the stiffness matrix is also linearly changing because in this simplified form, in this approximate form, the stiffness coefficients are a linear function of P.

If I add, if I you know keep P constant and I increase the load, I still have a linear variation, but as I keep doing that, at some point something happens, and the structure buckles. That is called elastic instability. How do I locate that value of P, that value of P for which buckling takes place?

[Noise]

It does not become 0; that is a wrong word use; it does not change; it may change (( )) P

It becomes singular.

Fill in the blanks. It becomes in we have done this earlier in slope deflection method; how did you find the critical buckling load? It becomes singular; becomes singular; then only, you get an Eigen value formulation; it becomes singular.

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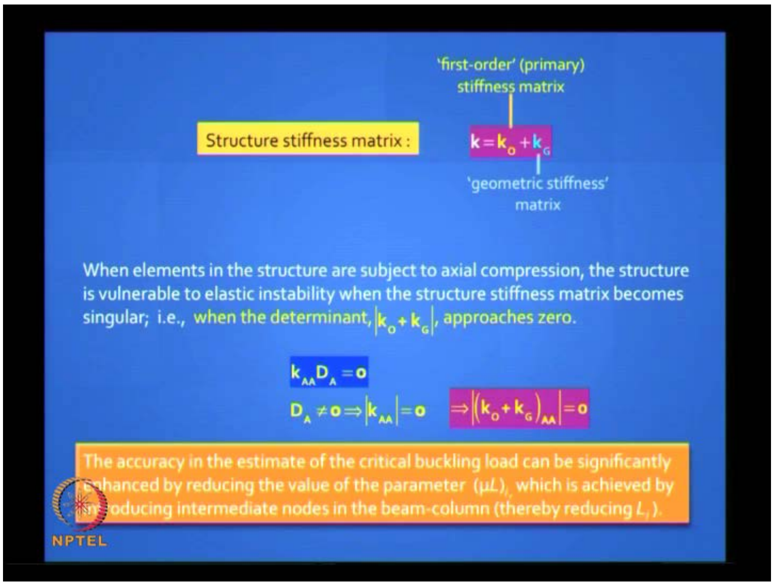

So, that is how you you just have to take the determinant of the overall stiffness matrix and then put the determinant equal to 0, and you have to take only for your active degrees of freedom; obviously because your equation is k AA D A equal to 0, you solve that. You have got an Eigen value problem here; you get Eigen vectors, not Eigen functions because you are dealing with matrices. That is how you solve this problem; is it clear?

So, supposing you had distributed loads, do not worry about them; convert them to equivalent concentrated loads; let they be only axial forces in this structure, initially. So, if you had what is known as pre buckling moments, it is okay; you keep them aside for the time being because they will they do not really affect your critical buckling capacity because you have studied earlier that, asymptotically you will hit that value. right Of course, probably, your structure will collapse well before that because you have other non-linearities coming into play.

So, the accuracy in the estimate of the critical buckling load can be enhanced by reducing the value of the parameter mu L. Please note, those approximations are valid; I mean you can reduce it just considering two terms in the Taylor series expansion of the sine and cosine function, only if that parameter mu L is small.

Now, mu L is square root of P L squared by EI; you have no control over P; you have no control over EI, but you have control over L. So, if you make your elements smaller, you make mu L smaller, and actually, you will get more nodes and you will get a better picture of the mode; shape also will look much better.

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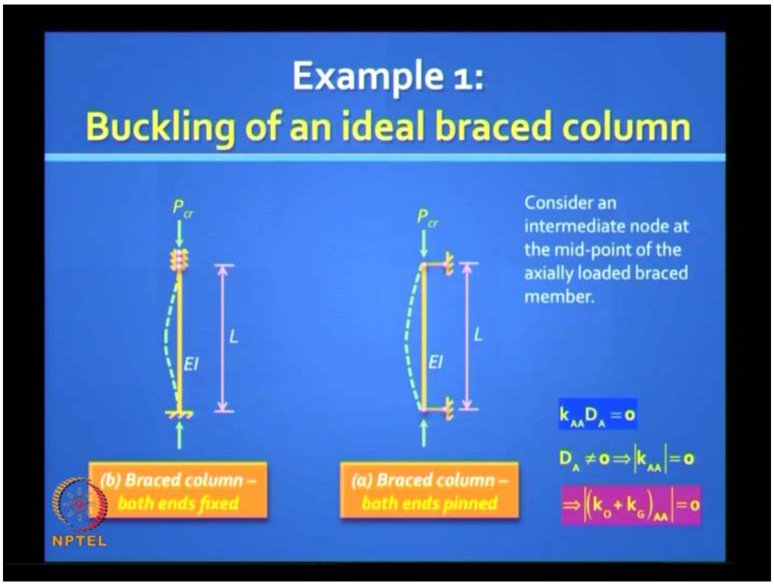

Let us apply it to very easy problems to test out this method. We know the solutions for these. We know the Euler buckling load for these two cases. They are ideal braced columns; case a - both the ends are fully fixed against translation and rotation; case b - they are fixed against translation, but they are free to rotate; it is a pinned condition.

What is the value of P critical for this? pi squared EI by…No, No, the second case pi squared EI by L square k - effective is 1, and the first case 0.5, which means the load will be 4 pi squared 4 pi squared EI by L squared; we know that; we know the answer.

Let us see, we can get this answer by matrix method. So, obviously, you need to create some nodes. How many elements would you recommend? This is just one element; you would not be able to get the result with one element; you need a, because the two ends have to, there has to be displacement lateral displacement. know

If you need a node, how many minimum well you just need? We will try this out; we will just put 1 node in the middle; you can do with 3 elements; you will get a better result with 4 elements; even you know much better result.

But let us check out with just 2 elements whether we get a reasonably good result or not; that will validate our method. So, we will do that and we will write the stiffness matrix, and we will put the determinant equal to 0 for the active degrees of freedom.

Now, if I divide this into two elements, how many global degrees of freedom will I have? Global coordinates in case a, active global coordinates in case a. If I put a node in the middle – active; well at that node, you have a translation and you have a rotation, and at the two ends if you are doing a beam element, you do not have axial deformations. So, you do not need to introduce that degree of it. So, you just have two active degrees of freedom, and for the second case, you have those 2 plus you have 2 rotations; so, that is it.

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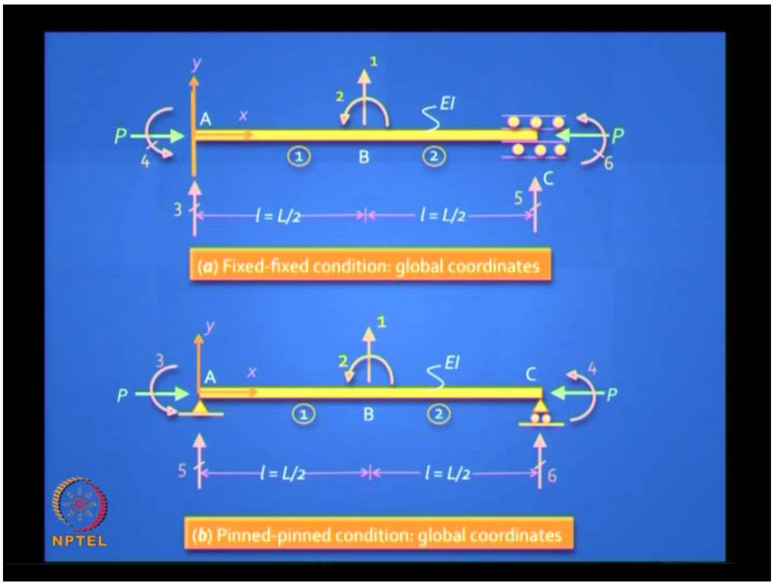

So, this is the first case. I have put it horizontally; does not matter. The column can be made horizontal. I have put a node in the middle. So, 1 and 2 active, and then, of course, I have restrained coordinates 3, 4, 5, and 6; does this is this clear?

In the second case, I have 1 and 2as before, but I have also have two end rotations 3 and 4, and I have only 2 restraint coordinates 5 and 6. Is this clear? It is very simple. What about local coordinates?

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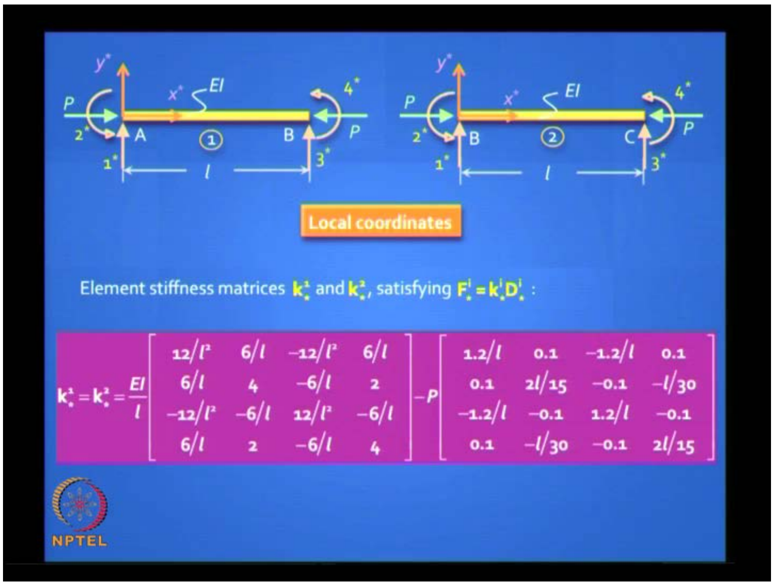

Well, that is a standard beam element. Except that, we are having a constant axial force P; it is the same P in both of them; it is the same P because it is a single column, and we write down the stiffness matrix which has the first order component which is the standard stiffness matrix, and we put a minus P to bring in the geometric stiffness part. Is it clear? So, this we know. These two matrices we have derived.

So, this is at the element level; at the element level at the element level, the stiffness matrices are always singular; do not worry. It is only when you build the structure stiffness matrix and you take the k A A part; that would not be singular normally, unless you have a instability caused by buckling; you follow? So, there must be some value of P which will reduce the flexural stiffness to an unstable situation.

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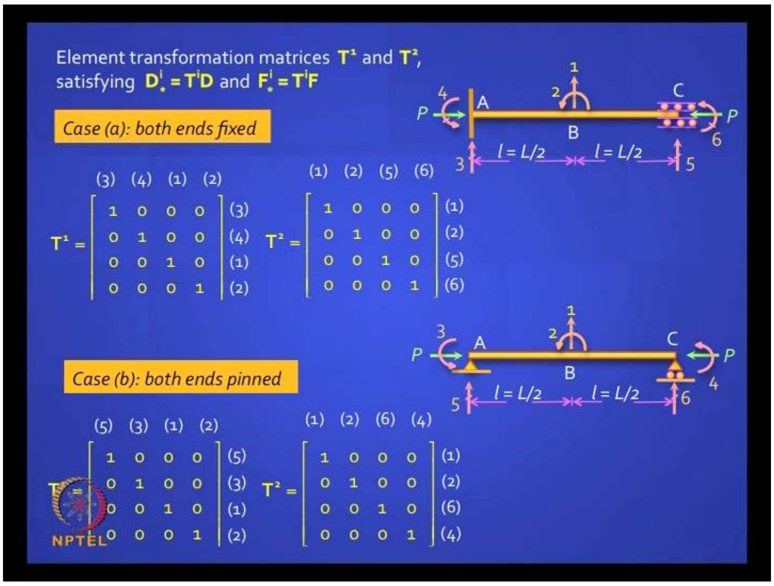

So, how do we generate the stiffness matrix for the structure? Well, first, you need the transformation matrices and they are simple. We have done this before. They are identity matrices. You have to correctly put the linking coordinates very clearly; it is 3 4 1 2 for the first element, and 1 2 5 6 for the second element, in case a. In case b, what will happen? The same T 1 T 2, but the linking coordinates will be… you have to be careful. It should be a written as 5 3 1 2.

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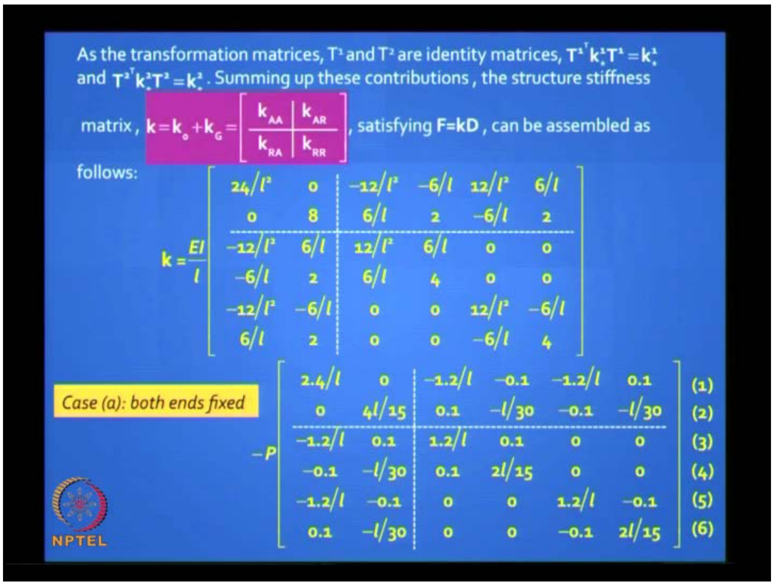

So, that, we need to do. Then, you generate the structure stiffness matrix; case a both ends fixed. Please note the minus P is stands out in the geometric part.

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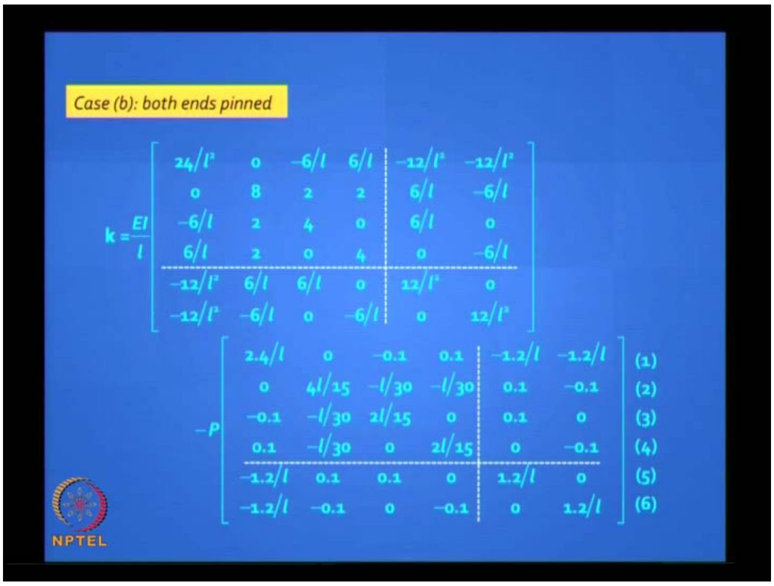

And second case, both ends pinned; you can easily generate this.

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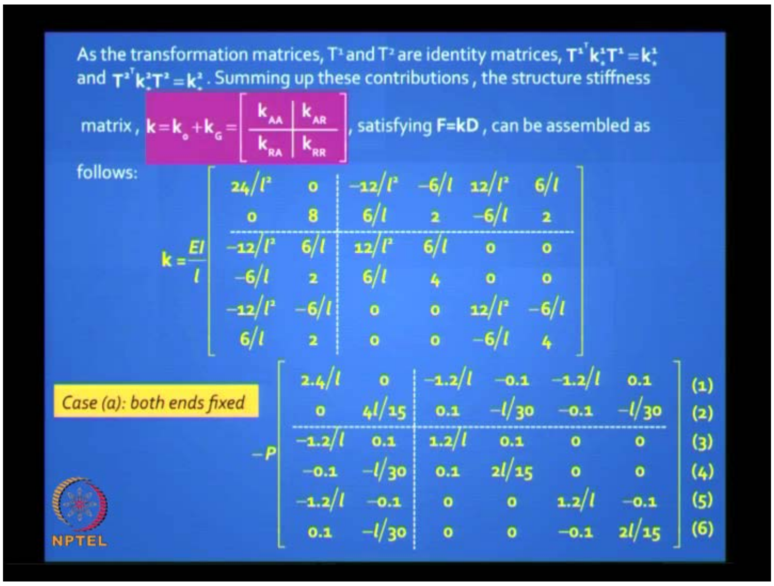

What do you do next? You got the k A for both of them. Let us take the first case. First what do you do? You write down the determinant; that is it. So, this is your… and I will you can see very clearly, why the determinant should be 0. If you go back to case a, this is look at the top left hand corner; k A is that portion on the left, and for the secondary, it is minus P into that portion.

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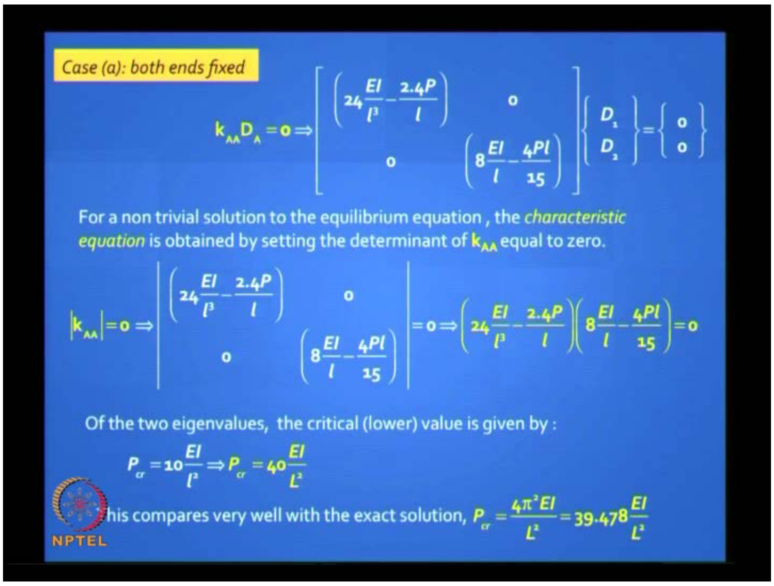

So, it is going to look like this and if you expand it, that is the equation you get when you write down the equilibrium equation because there is no external load acting on it. P is the load which is implicitly there, but there is no f 1 and there is no f 2, but it is just a you know actually loaded column; there is no f 1 f 2. So, this can have a trivial solution. What is a trivial solution? D 1 equal 0, D 2 equal to 0, which means the column remains straight; no buckling.

So, if you want a non-trivial solution, that determinant has to be 0; is it clear? That is how we say, the determinant. So, you made equal to 0; you will get an equation; that equation is called that is how characteristic equation; that is the characteristic equation. How many solutions will you get for that?

It is a 2 by 2 matrix. So, you have 2 solutions. What are those solutions called? Eigen values. They are called Eigen values. They are called Eigen values and we are really interested in the lower of the 2 because buckling will take place at the lower load. So, you can solve that equation and pick up the lower value, and you have got an answer, and the exact answer is 4 pi squared EI by L square which turns out to be 39.478. You got 40 is at good enough for an engineer; fantastic.

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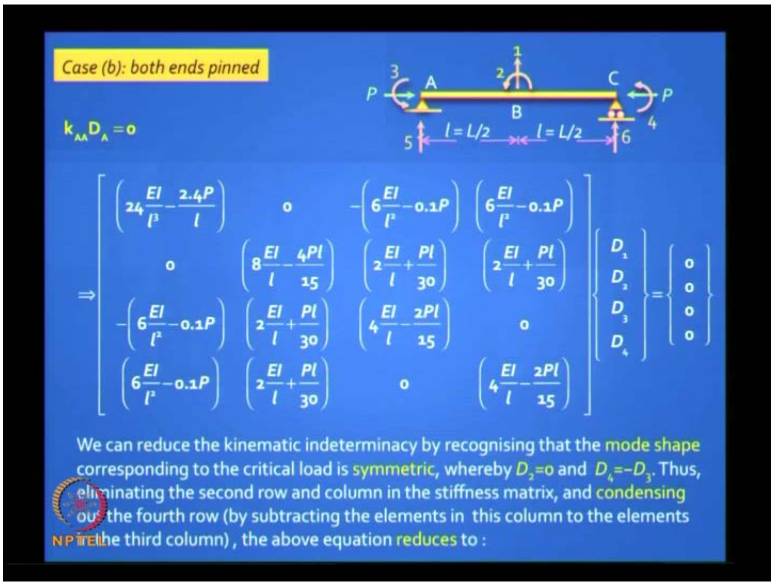

But you will get a better solution if you had 3 or 4 elements. right Similarly, you do it for both ends pinned. You have now 4 by 4; you get 4 Eigen values, but can we simplify this you can if you make some assumptions.

You have a rough idea what that first mode shape is going to be. So, is there any relationship between D 1, D 2, D 3, D 4 that you can intelligently utilize? What can you say, looking at the mode shape of a pinned?

D 3 is equal to (( ))

D 3 is equal to D 4, minus D 4 because it is equal and opposite. Yes because of symmetry, what else can you see? because of symmetry (( )) D 2 is 0.

D 2 is 0.

So, can we reduce this to a 2 by 2 form? Yes. You can. You know you can plug in wherever you see D 3 and D 4. You take one of them; you write D 4 is equal to minus D 3; so, you got rid of D 4. Wherever you have D 2, you put it equal to 0, and you rewrite, expand, and rewrite those matrices. This is called condensation. It is called static condensation in matrix methods. This is one technique of reducing the size of your matrix when you have inter relationships in your displacement vector.

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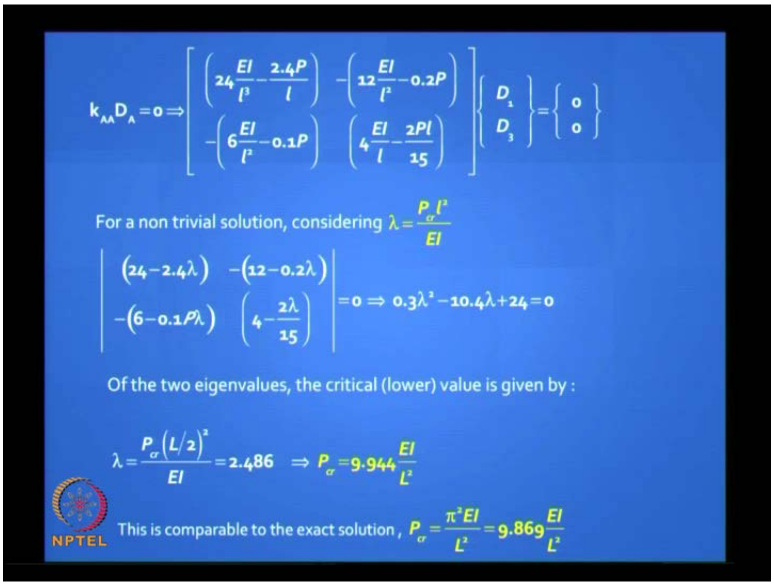

So, you do that exactly what you said; do that and then if you expand it out, it will reduce to this form. You have only D 1 and D 3; you got rid of D 4 and you got rid of D 2.

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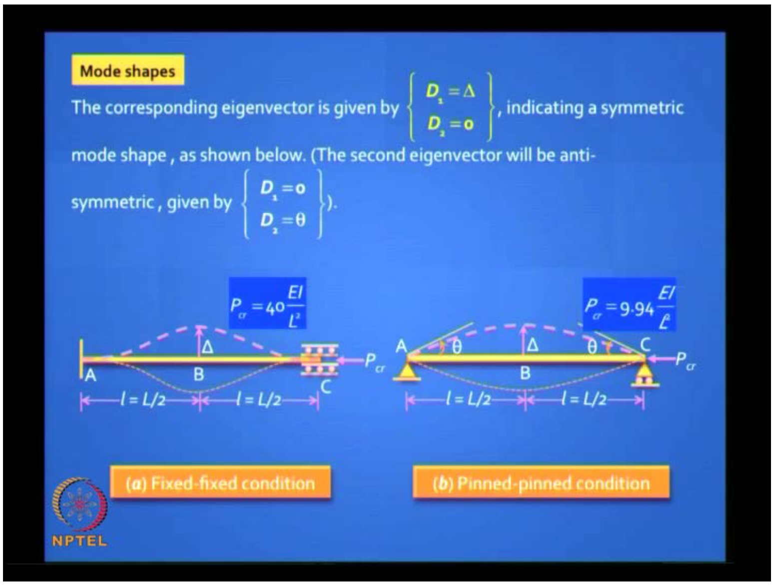

Now, you take the determinant of this. Take its Eigen value, the lower value; you get 9.944. What is the exact answer? pi squared EI. It is 9.869. Some error is there and if you want the mode shapes, you get them also, except that when you have 2 elements, you just get one delta. It does not really help you, but still, you can try to plot it. You do not know the curvature of those lines.

So, we are guessing that it is going to look like this because we know, the slopes have to be 0 at the two ends in case a. But if you really want to generate that mode shape more correctly, in a case you did not know what would you need to do, here, you have only delta and 0 0.

So, you could join with the straight line if you wish, but that is wrong. So, if you want it, let us say, I want value at quarters span point in relation delta; what should I do? I just need to create a node there, which means, if I have 4 elements, I get the quarter span displacements in relation to the mid span. You understand, that is how I do it, if I really want to know the mode shape; is it clear?

But in the exact solution, Eigen solution, you get the mode shape nicely as a sinusoidal function. So, that is more exact. That gives you the mode shape value at all the points on the beam.

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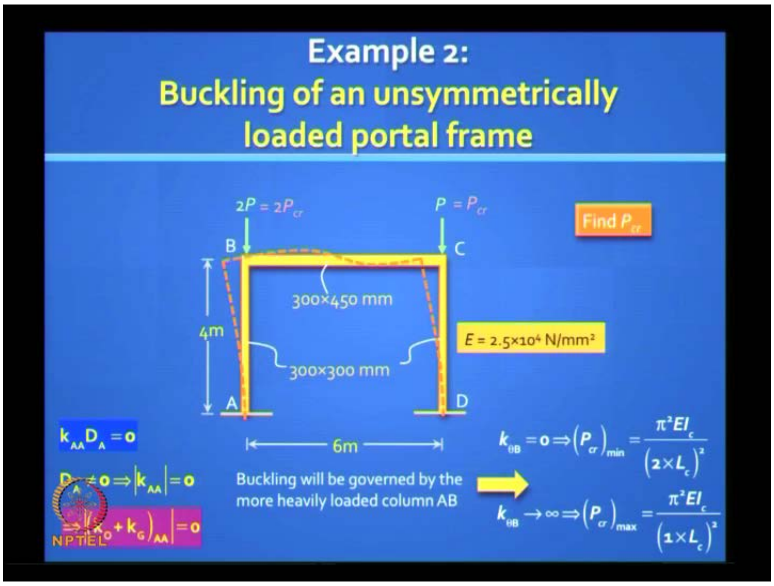

Let us demonstrate with the portal frame. It is an unsymmetrically loaded portal frame. Typically, how you find the buckling load is - you have to keep incrementing the loads. Let us say you start with 0, but you keep that proportionality; you know if one at C you have applying P at B, it should be 2 times whatever you are applying, and if you what to do an experiment you keep loading it, maintaining that ratio. That is called a load factor, by the way. So, if you had loads all over the place, you multiply all by the same factor; keep building it up, and at some points, what is going to happen? It is going to buckle, and most likely this is a shape it is going to take.

Now, your job is to find that critical buckling load, what is P critical? So, how do we do this? Well, procedure is exactly the same and we can have some guesses. Can you guess some initial values in that frame? Which of the 2 columns will govern the buckling load, the left one or the right one?

The left one.

Left; why because it more heavily loaded, and the length is the same and the cross sections are the same. So, the left one. So, what are the limiting values that you expect for that elements?

(( ))

It depends. You can remove the left one and put a rotational spring at B to take care of the remaining portion B C D, and the stiffness can have two extreme values to get you the lower and upper bounds; one is 0, when it is a cantilever. For a cantilever, what will be the k e cantilever?

Y 4.

Cantilever k e will be 2 L.

2; effective length ratio is 2 times. You know that we have done this problem earlier and if it is fully fixed. Come on; it is an unbraced frame. Just look at the deflected shape; there is going to a chord rotation; it will be 1.

So, those are your limits. You should able to guess them correctly. k e will be either 2 or 1; most likely it will be somewhere in between; you have to find out the value; that is so, you must always, as an engineer, have a guess have an initial guess - what the likely load is.

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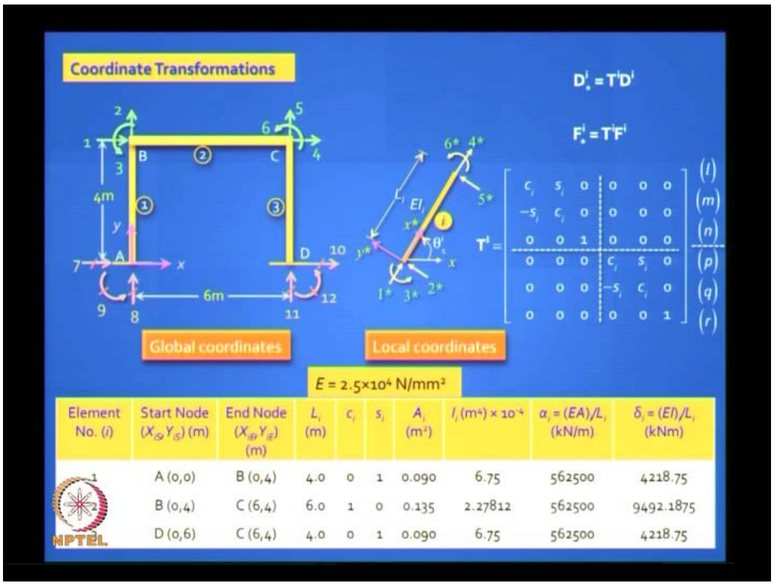

Element No. (i)	Start Node (X_{iS}, Y_{iS}) (m)	End Node (X_{iE}, Y_{iE}) (m)	L_i (m)	c_i	s_i	A_i (m²)	I_i (m⁴) × 10⁻⁴	$\alpha_i = (EA)/L_i$ (kN/m)	$\delta_i = (EI)/L_i$ (kNm)
1	A (0,0)	B (0,4)	4.0	0	1	0.090	6.75	562500	4218.75
2	B (0,4)	C (6,4)	6.0	1	0	0.135	2.27812	562500	9492.1875
3	D (0,6)	C (6,4)	4.0	0	1	0.090	6.75	562500	4218.75

So, you write down. This is a slide; I am picking it from what we did when we did portal frames without any beam column effect, without any P delta effect. So, this is straight. From that, you have 6 degrees of freedom which are active and 6 which are restraint. You can generate the T i matrix for all of them.

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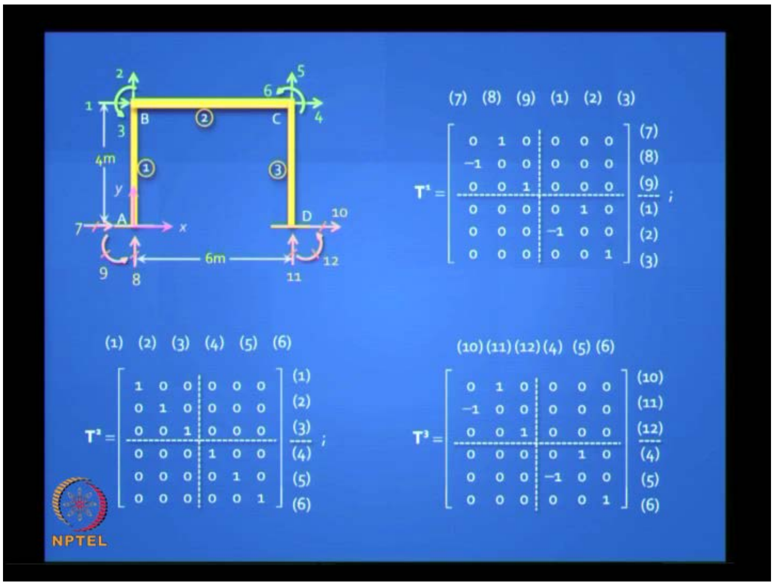

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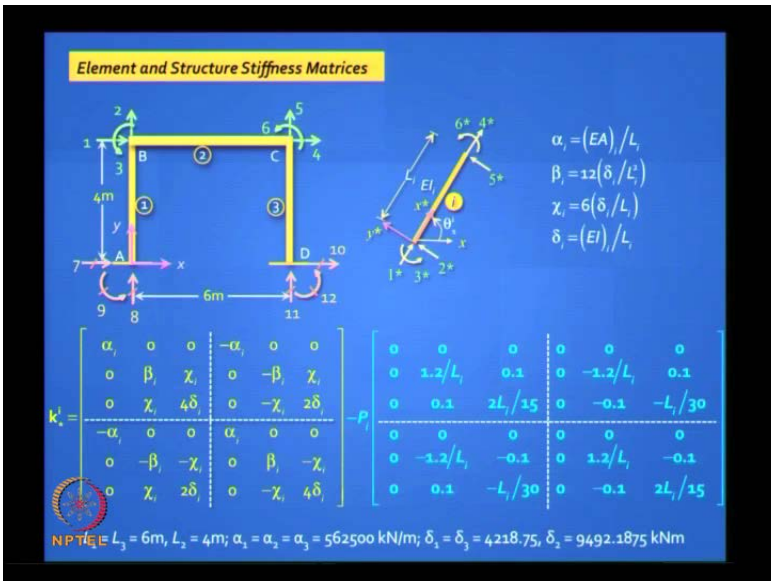

We have done this before - 3 elements, but now, when you write down the stiffness matrix, you have to make a small correction for P, but do you know P? Yes. We know one is P; there is a P for element 1 which is 2 P, and for element 3, it is P. What about element 2? Will it have any force at the point of buckling? In the deflected shape, you may get some force.

This is the This is what you need to do for all the elements, but you have to put in the correct value of P.

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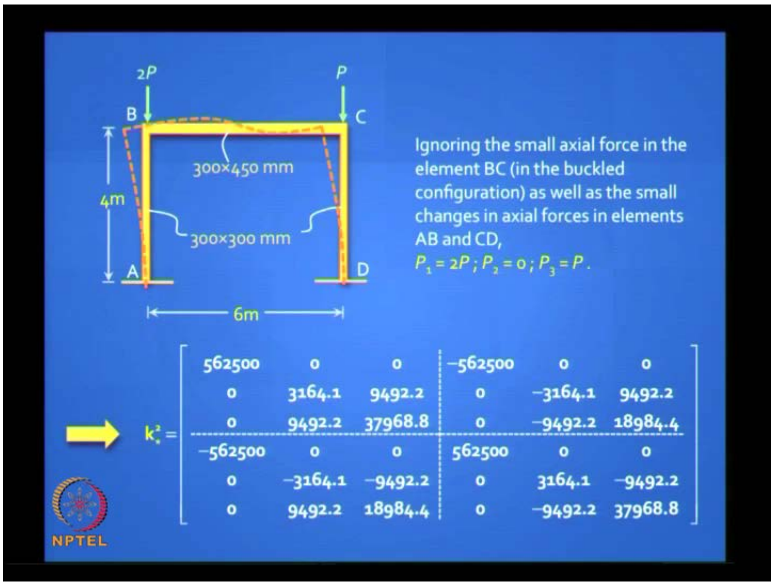

Now, you may get an axial force in B C, but it is very minimal. So, this is one convenient assumption we always do in a beam column. You worry about axial, the effect of axial load, only when the axial load is significant, if it is a very small. So, in beams, we usually ignore it; in columns you have to…

Now, for example, here, that 2 P can come from the floors. Let us say, this is a 20 storied building, and I am trying to simulate the lower storey, lower most storey; then, that is the weight of all the floors coming from above; is it clear?

So, these are the kind of physical meanings you can have. So, we are saying P 1 is 2 P, P 2 is 0, and P 3 is P.

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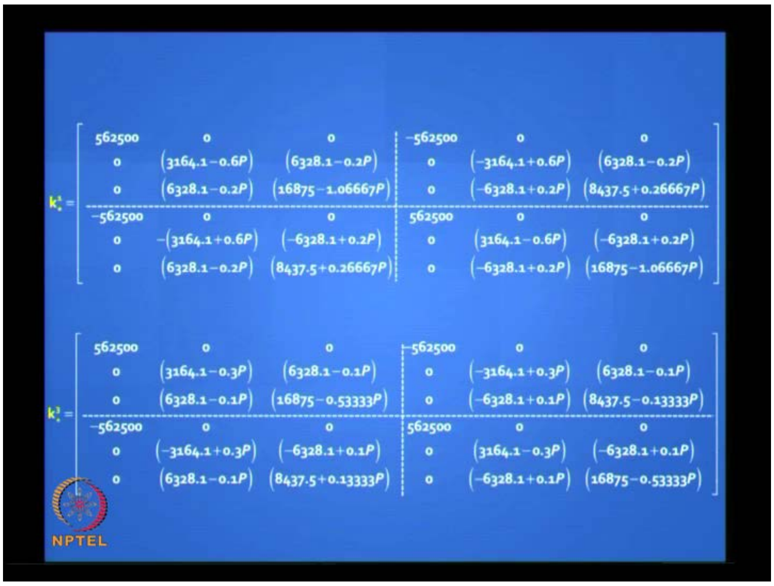

Once you know this and you know the properties of elements 1, 2, and 3, which we already derived, you can derive the 3 element stiffness matrices. For element 2, it is exactly what we had earlier; no change. For element 1 and 3 is going to change, and P comes into play. You have to just write down that k's, i's, o star plus k i g star. So, when you add it all together, it takes this form; is it clear for elements 1 and 3?

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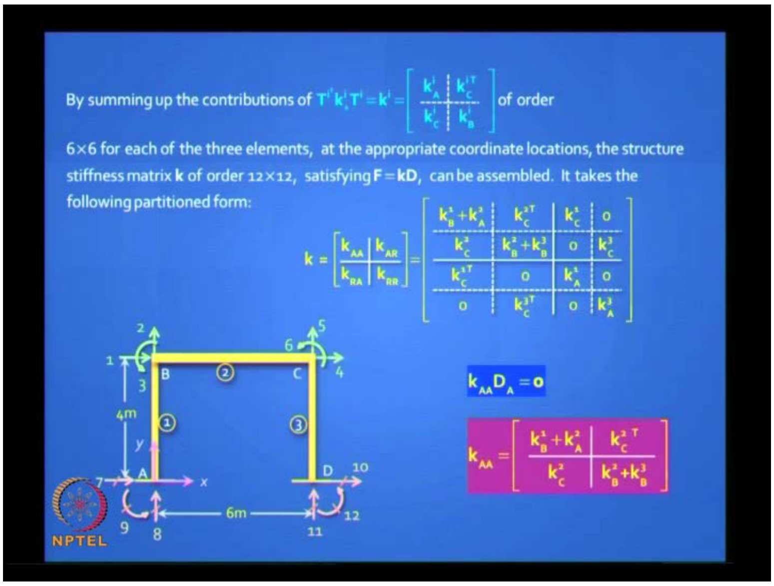

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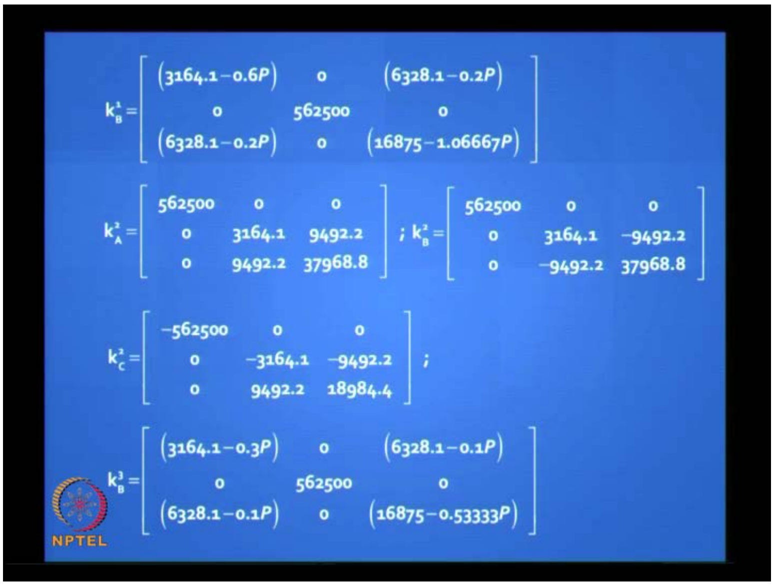

Then you assemble the structure stiffness matrix exactly the same way we did earlier, and you will get. You can write it in this form and it is exactly the same procedure; only thing, an unknown P comes into the picture.

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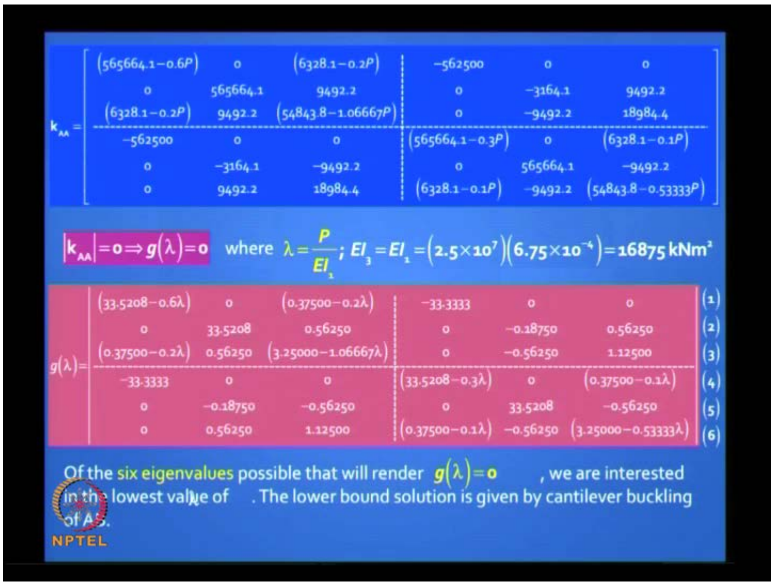

Finally, you get k AA in this form; that is all you need; you do not need k AR and k RR, and if you want to get the correct answer, then you must find the determinant of this matrix.

Now, you are going to deal with very large numbers. You can see there because 556, 565, etcetera you know. You got 6 digits out there. So, it is good to work with another alternative measure of the critical buckling loads.

So, let us bring in lambda which we can call P by EI; EI of any of those columns EI 1 or EI 3 - they are both the same, and when you do that, your equation simplifies to this format. If you divide throughout, if you normalize, it is the same equation. See, whether the determinant of the first matrix vanishes or an equivalent matrix vanishes, it is the same; it will give you the same result, but you are working with the better value. We have brought in lambda into the picture.

We just divided P by EI which means divide all the value by EI. You will automatically get lambda in the picture, and now what do we do? What method do you use to get lambda? You get 6 Eigen values in this.

We are not interested in all 6. We are happy with the first one; if it is a dynamics problem, when the dynamics is similar to instability in this respect, then you may be interested in higher Eigenvalues also because they get excited; higher natural frequencies get excited by arbitrary loading, but in buckling, you do not have that problem. It will fail. you know Some people made the suggestion: look, if I have a column, let us say, pinned; pinned when I apply a load here, it is going to buckle like that.

So, maybe I will hold it in the middle, and once I hold it in the middle and prevent it from latterly moving in the middle, the you know the it goes to the second mode before it buckles.

So, if I go to a slightly higher value than P, let us say, I put 2 times P; then, I remove my hands; may be it will stand. Well, theoretically, it will stand, but experimentally, it has been proved that it does not work. There is no way, it is going to go back and fail in the first mode.

So, this the lowest mode will always dictate the capacity of the structure. So, here, what you can do? You need You can do something like the bisection method, but you need some trial values. You have no idea how to pick them up. You do not know whether you can start somewhere; start with the lowest buckling load.

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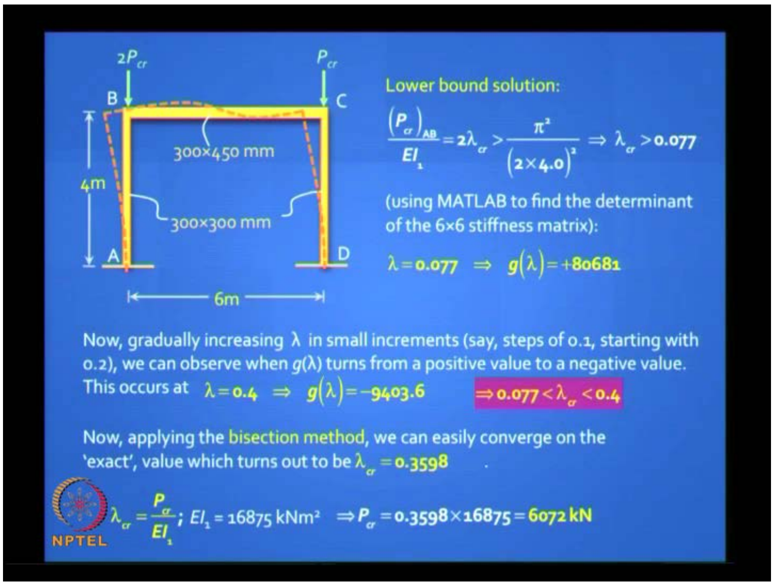

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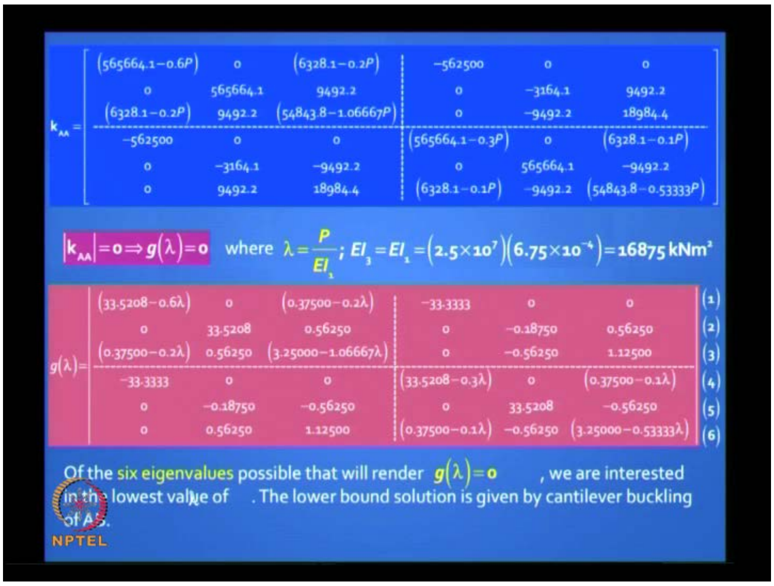

So, we want to find out what gives you the lowest value. So, you remember, the lowest is when A B behaves like a cantilever. So, you can easily find out for that, when it is a cantilever, what is the value of lambda? Now, if you put this value of lambda into that expression, for the determinant g of lambda, will it be positive or negative in the bisection method? You need to figure that also.

So, you do not know until you put it in that value. So, put it in and you can find the determinant easily by mat lab. It turns out to be positive. You get some value plus something. Now, you need to trap another value which is negative. Now, you need not go to the other extreme.

So, how do you get the next value? What is the clever way of doing it? You do not need to go to the upper bound. You need to think. So, what should you do? Cleverly do trial and error; few trials are enough. What is the next trial you will take? Double; you I mean work with round numbers; trial lambda equal to 2; trial lambda equal to 3; trial lambda equal to 4 because we do not want it exactly.

We are doing bisection method. We just want the nearest positive value. So, you do that little exercise incremented in steps of one. You will find when you put lambda equal to in this case 0.1, because 0.077 is very low; at 0.4, you are going to hit a negative value; at 0.3, it will still be positive; you can check it out.

So, now, you can do the bisection method. You have got the 2 values. You know the lambda critical is between 0.077 and 0.4, and you do the bisection method. You get the correct value of lambda critical, and you get the solution as 6072 kilonewton; that is it.

It is a little approximate because we are using that function. We are using the concept of geometric stiffness. If you want more accuracy, put in some more nodes, one more node, and check it out, but this is reasonably good.

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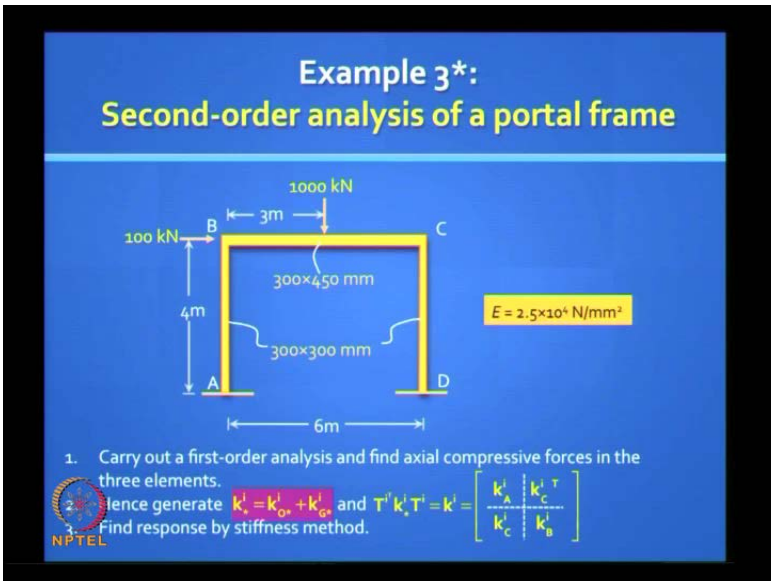

So, this is how you can do instability analysis for any structure. We do not have time to do a second order analysis, but this is explained at length in the book, what do you need to do? Let us take a problem like this. I am not going to solve it, but at least let us figure out how to solve it.

So, you have got the load at the mid span - 1000 kilonewton. You also got a lateral load 100 kilonewton; the frame is the same frame which we analyzed earlier. The first thing you should do is carry out a conventional first order analysis and find out the axial forces in the 3 elements. You can you will get it automatically from the program that you develop for analyzing plane frames.

So, we will get it. Then, you take those axial forces which you got and modify the geometric stiffness matrix and run it again; you get a solution; another solution; will that be correct? Why not, It is a reasonably correct solution, but if you want more accuracy, you should check out the axial forces you now get. So, typically, non-linear analysis is like this. You do not know the actual axial forces, but you get a first order of understanding when you do one run, then you pick up the value, modify the stiffness with those values and do a second run.

You can do a third run usually with 2 or 3 runs, and anyway, the computer is doing it; you are not doing it; the effort needed is just to program it; you have a tolerance level and that gives you your complete solution.

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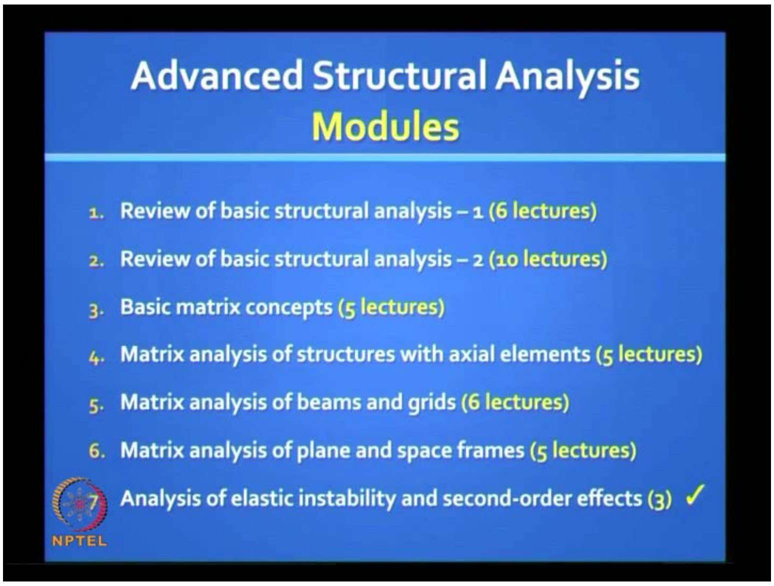

So, this is demonstrated in the book. You can do any number of problems of this type. So, my friends, we have come to the end. This is the last technical lecture we are having.

If you look carefully, what did we do in advanced structural analysis, first 2 modules - we actually reviewed basic structural analysis. In the first module, we actually looked at an introduction to what we do in structural analysis; specifically, we looked at statically determinate structures, how to find the force response in statically determinate structures.

We looked at typical beams, trusses, arches, funicular arches, cable systems, plane frames. You also briefly looked at space frames and space trusses. Then, we looked at ways of finding displacements, deflections in trusses, deflections in beams, rotations, curvatures in beams and frames. And we did this by many methods, but we found that the most powerful and general method is that of by the principle of virtual work. And then, we looked at other energy theorems which could handle both statics problems and kinematics problems, and they were they had their equivalent work theorems. You had the principle of virtual displacements; principle of virtual forces.

Then, in the second review, we spent lot of time on indeterminate structures. You first looked at statically indeterminate structures and we looked at complex problems including elastic supports and grids, and so on.

Then we did and that is a new thing. I think it is a IT topic which I believe is not very well learnt in many university's displacement methods.

We did not study them independently; we looked at them together; we did an introduction to displacement methods; then we looked at slope deflection method and moment distribution method. And the real thing we learnt is how we can dramatically bring down the degree of kinematic indeterminacy by having a feel for the structural response, taking advantage of releases, guided fixed supports, and hinge supports.

We also looked at how you can deal with frames which had the symmetry; subject to lateral loading, how you can simplify the analysis. We also looked at approximate methods of lateral loading analysis.

So, we did all that. Then, we got in to matrix methods. We first did a mathematical review of basic concepts.

Now, you see all those concepts we have been using, including Eigen values; all the way to the end, it is basically linear algebra. Then, we when you first saw that, you were not very comfortable because you know trying to solve structural analysis problems with matrices look daunting.

We looked at powerful transformations, we looked at the contra gradient principles, and the general theory that is in the third module, and we specifically went into every type of structure. We went into trusses axial elements; we went into space trusses in module 4; we looked at beams and grids in module 5; we looked at plane and space frames in module 6.

And in this module, we took a kind of overview. We did not go the…, but we have a clear idea on what to do, when you have to worry about elastic instability, and second order effect. So, 40 lectures.

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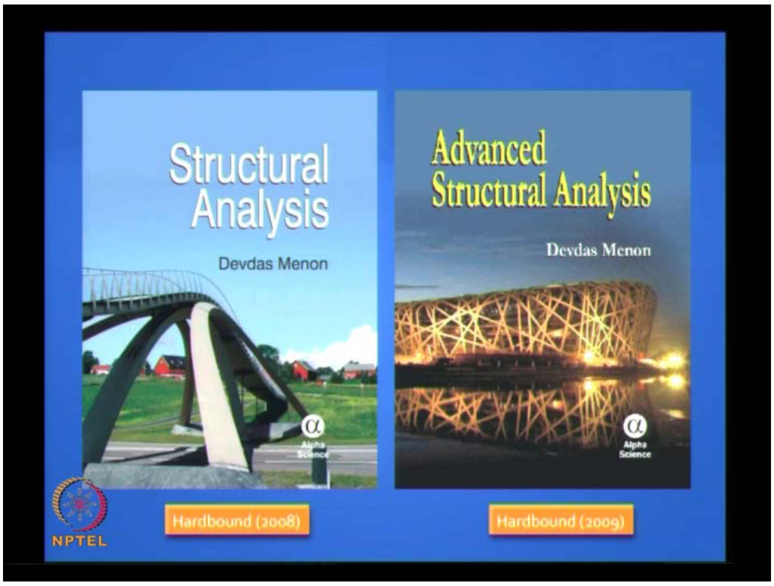

One more lecture to go; that is a general lecture, but to remind you, I have written two books and we have used the material in this book, these two books for the study. You need to look at those books to really understand what is going on.

And with that, yeah this is just to tell you that these, the paperback edition is available in India published by Narosa in 2008 and 2009, and the international edition hard bound published by Alpha Science from Oxford, UK; that is it.

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Thank you very much.

Before I end, I also want to place on record, the tremendous help I got from Nazeeb Sharif; Sharif just stand up; give him a hand.

He is being he is my M S student; he is being the TA who helped us in this. I am grateful to the staff of NPTEL without whose support we wouldnot have been able to do this. In particular, I wish to place on record, the support given by Manikandan who is not here, who is the one who made many of the slides that you have access to.

Thank you very much, bye.