Advanced Structural Analysis

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Module No: # 1.4

Lecture No: # 04

Review of Basic Structural Analysis-1

Good morning. We are now on to lecture 4 in the first module which deals with review of basic structural analysis.

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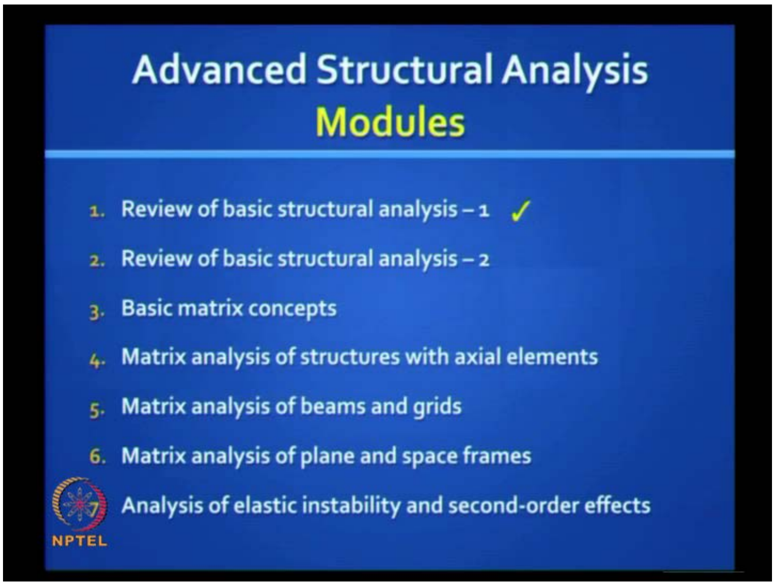

This is the first of 7 modules and in this module, we will be covering work and energy methods.

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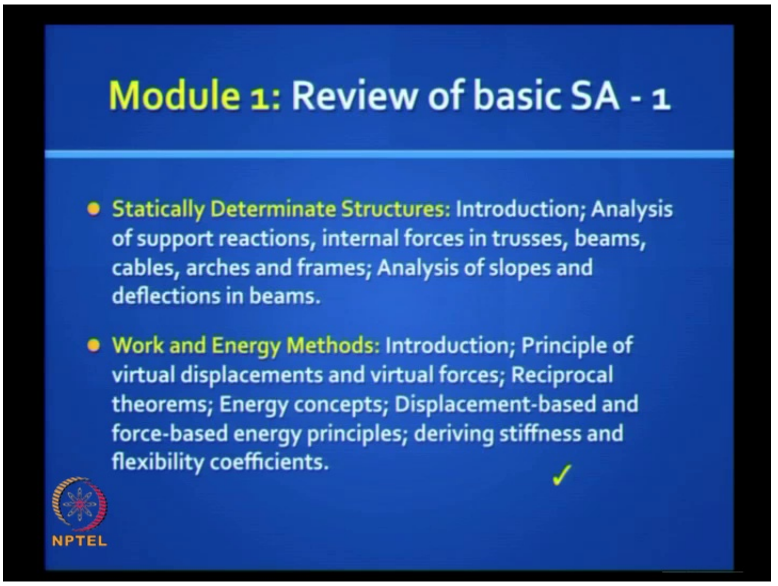

In the last class, we covered statistically determinate structures.

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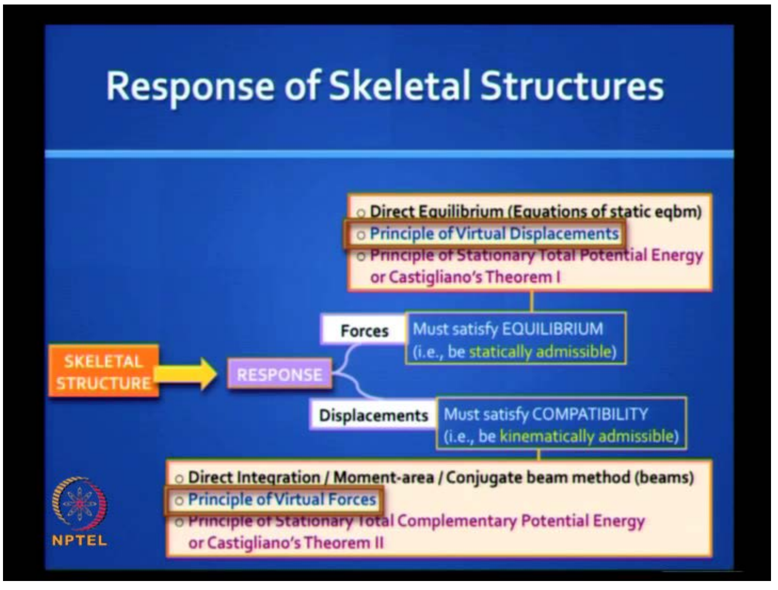

This slide shows the summary of what we do in structural analysis. We try to find the response of a given structure subjected to a given loading and we are interested in both the force response and displacement response. We will now look at virtual work formulation of finding these two responses. To find unknown forces in the force field which satisfy equilibrium, we can invoke the principle of virtual displacement; to find unknown displacements in the displacement field, we can invoke the principle of virtual forces.

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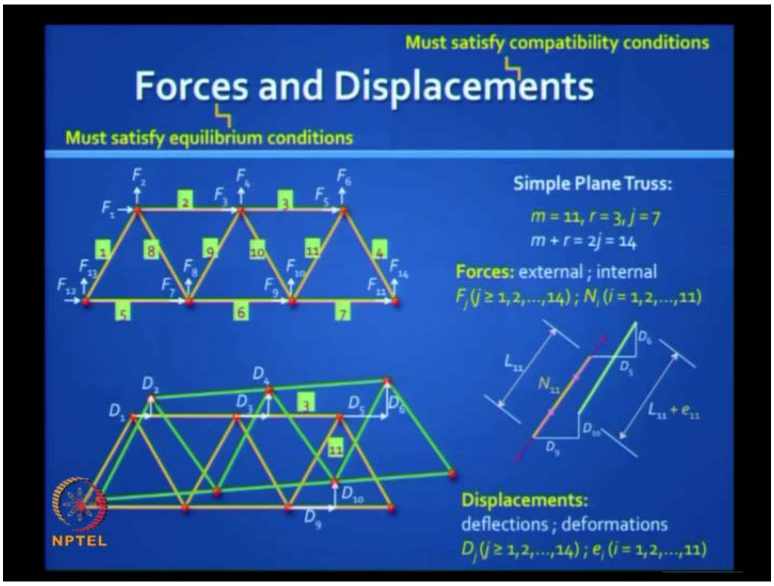

To begin, this slide shows what we mean by coordinates and how we use these coordinate indices to identify external joint forces and joint displacements. F_j and D_j are the respective joint forces and joint displacements in a truss at the pin joint. Some of these forces could be reactions and the corresponding displacements would be arrested. The rest of the joint forces would be direct actions; the direction of those loads and the locations are marked. From the internal side, you have bar forces N_i and corresponding bar elongations e_i.

The bar elongations are related to the joint displacements through relationships called compatibility and the internal forces in the various bars are related to the external joint forces through equilibrium. The force field must satisfy equilibrium, which means it must be statically admissible; the displacement field must satisfy compatibility and it must be kinematically admissible.

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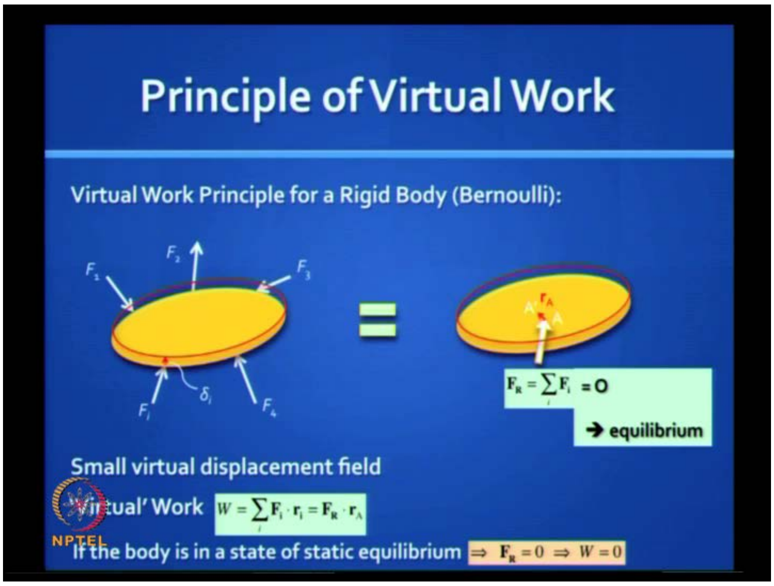

Here is a very useful principle called the principle of virtual work, originally discovered by Bernoulli. In its original form, it actually dealt with the behavior of a rigid body. You see here a rigid body subjected to various external forces. We could resolve all the external forces to a resultant force and if that resultant force is equal to 0, we know that the system, the structure, the body is in a state of static equilibrium.

What Bernoulli suggested was to give a very small displacement to the whole body. It is an imaginary displacement, it is a virtual displacement; it is a displacement to be given in such a manner as to not to disturb the force field. If you find out the delta components, you have F_1, F_2, F_3, F_4, that is, the change in position of the points of application of the various forces, and if you do a scalar product of all of them and add up algebraically, you get the total work done on this body. That total work done must… Actually, it is a dot product of the force and the displacement vectors; that must be equal to 0 if the body is in equilibrium; it easily follows from this.

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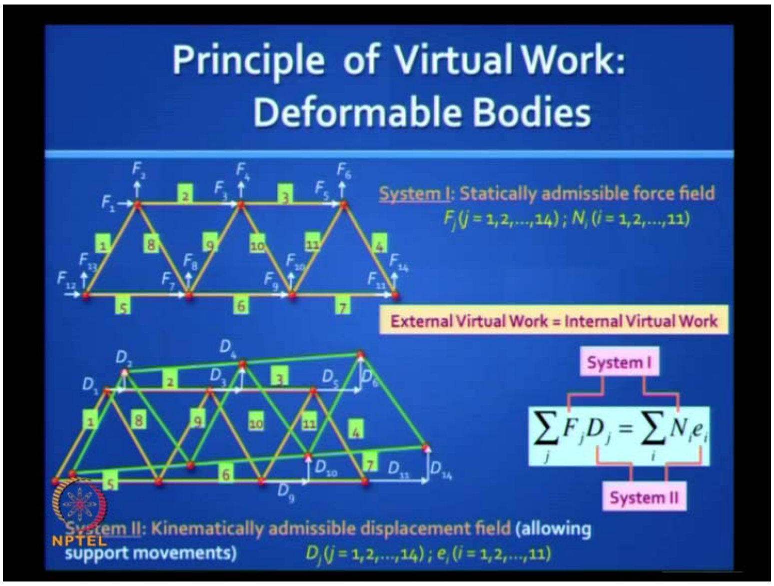

Although this was originally proposed for rigid bodies, this can also be applied for deformable bodies; all our structures are deformable. If you go back to the earlier problem of the truss, we can actually extend the principle of virtual work to a more expanded and diverse application where we can even visualize two identical structures; we look at the force field in one of them and the displacement field in the other. The advantage of looking at it this way is you remove the cause-effect dependence between the force field and the displacement field.

Let us call the first field where we have a statically admissible force field as system I and the other system where we look only at the displacements as system II, where we have a kinematically admissible displacement field. In this field, we look at the D_js. In this particular example, you have 14 values of D_j and you have 11 values of e_i because there are 11 members and they are all interrelated – bound together by compatibility. The compatibility ensures that the whole truss is held together. You do not have any separation of any joint from any member at any location.

In the first field, we have 14 values of the joint forces F_j and 11 values of the bar forces N_i. These two are also interrelated through equilibrium relationships. You could have a situation where there is a cause-effect relationship between system I and II, but that is not necessary for the proof of this principle. This is a powerful principle and we will accept it for the time being without any proof. It says that if you were to multiply F_j with

D_j and you do it so that you have a conjugate product of F_j and D_j, F_j and D_j are at the same location pointing in the same direction. If one points in the opposite direction, you have a negative value of that scalar product and if you do an algebraic sum over all the joints, in this case 14 joints, you get a product which has the unit of work, but it is appropriate to call this as virtual work and external virtual work.

It is virtual because it is not real – you are just multiplying two numbers. There is some commonality between those numbers, but there is no relation – no cause-effect relationship. The sequence of loading and all do not come into this, which would come in in an energy formulation; that is the advantage of a virtual work formulation. Now if you take the internal field, the internal force is N_i in system I, and corresponding e_i in system II and do product and an algebraic sum, then you have the internal virtual work.

These two numbers, they have units of joules and will be exactly equal. There is no requirement that the structure should behave elastically. You can use this for even non-linear behavior and for even plastic behavior. The only requirement is that the force field should satisfy equilibrium and the displacement field should satisfy compatibility. The joints can also move; you can see here that the entire system is moving; the supports can also move, but the internal integrity of the structure should be present. Is it clear?

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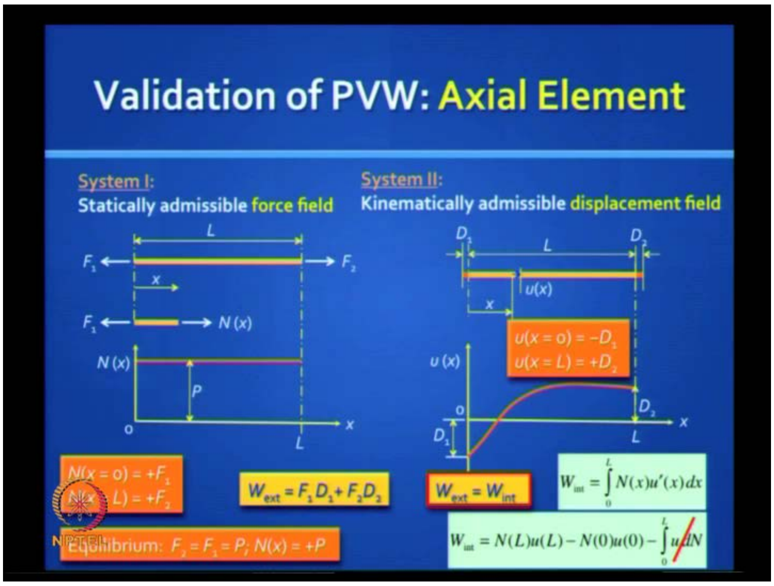

Now, we will show a simple validation of this theorem because it is a powerful theorem. Let us look at one bar. Let us look at it in system I where you have a statically admissible force field. You have forces F_1 and F_2 and you have an internal force at any location N. Theoretically, that force can change along the length of that bar N of x. Simple equilibrium demands that for the overall free body, F_1 and F_2 must be equal and opposite to each other and the internal force in that bar must be a constant and equal to this external force. Let us say that that force is P; so, N of x is constant and it is equal to P. This is the complete information as far as the force field is concerned.

If you want to express in terms of boundary conditions, the axial force – the internal force at x is equal to 0 must be equal to plus F_1 and the internal force at x is equal to L must be equal to plus F_2; F_1 and F_2 must be numerically equal to each other. Now, let us look at another field, a completely independent field in the same bar; we are looking at the displacement field here. Let us say that the left end of that bar moves to the left by an amount D_1 which is conjugate with F_1 but not related to F_1 by cause-effect. Similarly, the right end of that bar moves to the right by D_2, which is conjugate with F_2.

Let us say that any point in that bar located at x moves axially by a distance U of x. You know that we commonly use U, V, W in the Cartesian space system to relate to displacements in these three directions: along x, y, and z. Let us just take some random variation in U. Let us say you heat that bar and you have a different temperature at different locations; it is possible to get this kind of variation.

The compatibility requirement is that you must not have a break in the bar anywhere, which means that the curve should be continuous and there must be a compatibility relationship of U at x equal to 0 being equal to minus of D_1 and U at x equal to L being equal to plus of D_2. This is the complete displacement field. It is kinematically admissible and we will check out and see whether the principle of virtual work operates here or not.

This is the external virtual work product: F_1 D_1 plus F_2 D_2. How do you write the internal work? You cannot write it so simply because at every location x, the value of both U of x and N of x can change and so you have to do an integration. You have to take a small element D x and it will look like this. Now, work is given by the axial force N of x and the change or the local elongation at that point which comes when you integrate the strain.

The strain is U dash of x dU by dx and so this is the expression for the internal virtual work. If you invoke the expansion of that integral and you apply the limits 0 to L, I think you are familiar with this, it will take this form. You have N of L into u of L minus N at 0 into u at 0 minus integral u dN. Now, because there is no change in N along the length of this particular bar, that last quantity will vanish; it is equal to 0.

If you now apply the boundary conditions of N of L, u of L, N of 0, and u of 0, you will find that you get exactly the same expression as for external virtual work. This is a beautiful validation. You can clearly see there is no relationship between the axial force distribution and the axial deformation distribution. A similar proof is possible when you bring in bending and shear forces. The proof is available in the book Structural Analysis.

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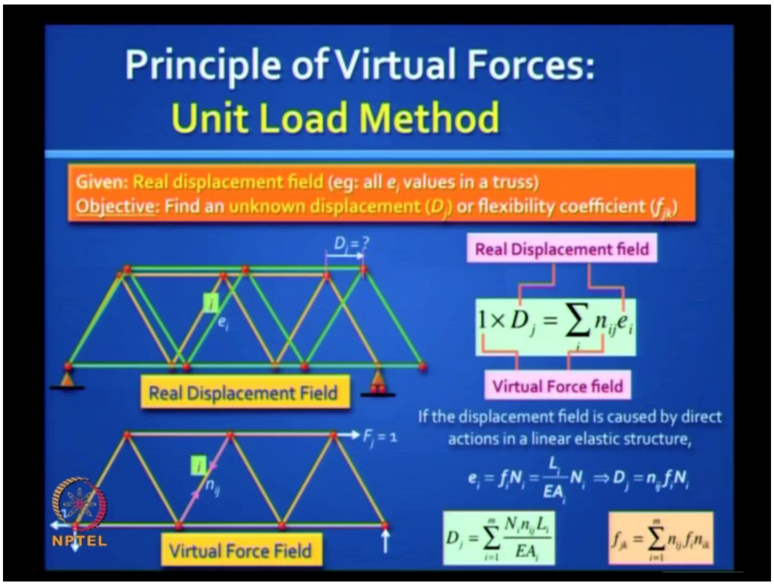

Now, let us apply this principle. You are familiar with the unit load method; let us use it first for finding an unknown displacement. What we traditionally do is we apply a unit force in the direction and at the location where we want to find the displacement. So, if you want to find D_j, you have to apply F_j and it is convenient to apply F_j equal to 1. You have a geometry problem. You have a real displacement field and you are creating an imaginary force field which you can analyze. In other words, you are using statics to help you solve a problem of kinematics. We are using the principle of virtual work because we are using work as a bridge that connects the force field with the displacement field.

Now, if you do this product, you will get 1 into D_j as the total external virtual work because there is no other joint, no other force acting which moves and the total internal work is n_{ij}. That small n_{ij} is defined here as the axial force in the element i caused by F_j equal to 1 – that is a notation we will use consistently – and e_i is the elongation in the i th bar. Is it clear? Those arrow marks should make it clear.

If you look at this, this is n_{ij} in the i th bar and this is e_i in the i th bar (Refer Slide Time: 15:26). We are just multiplying them for all the bars and summing over I; it does not matter what caused the displacement. If the displacements were caused by the application of loads on the truss, then you can get bar elongations from the axial forces in that real truss by multiplying n_i with f_i.

What is f_i? It is the axial flexibility in the truss; we have already seen this. This f_i is equal to L_i by EA_i (Refer Slide Time: 16:04). We assume that all the bars have the same material and so the e value should be common; if it is not, you can account for it. So, you have this formula D_j is equal to n_{ij} into f_i into N_i. The original formula is 1 into D_j. It is important to remember this because sometimes people remember only the formula. Then, you run into the problem of dimensional non-homogeneity because you will find that the right-hand product does not really match in terms of units with the left-hand product.

This is the big picture and you can apply this to find any unknown displacement or you could use it to find a flexibility coefficient – we will see this later. You can have two kinds of problems: problems in which you just have a geometry change caused by an environmental effect like temperature change or a lack of fit in a truss or you could have it for the more common application of finding an unknown displacement in a loaded structure.

Now, if you are finding the unknown displacement ((.)) loaded structure, here we are making an assumption of linear elastic behavior because the modulus of elasticity comes in here; so, you will get this form for a truss. You can now pull out the definition of flexibility coefficient from here. How would you define f_{jk}?

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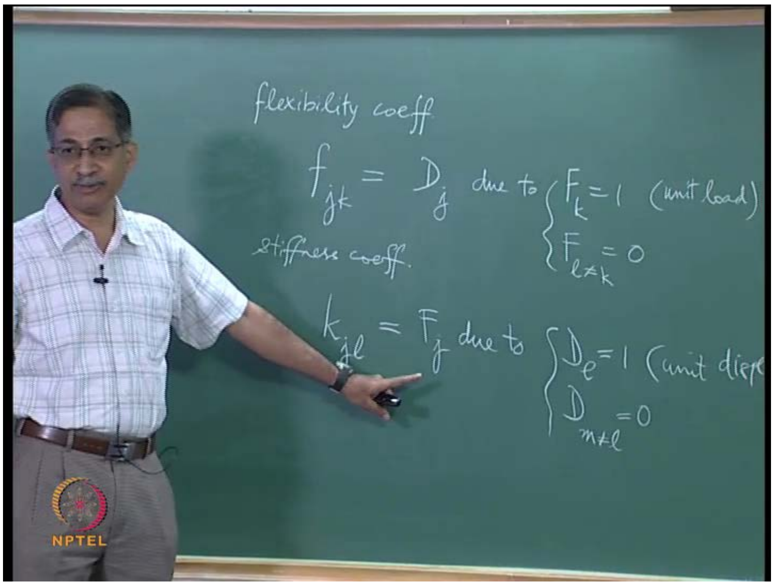

f_{jk} is defined as…. Is it a force or is it a displacement? This is a flexibility ((.)). Is flexibility coefficient a force or a displacement? It is a displacement. This is the displacement at the joint coordinate j due to the application of a unit load at the location k; at all other coordinates, let us say l not equal to k, there should be no load. There should be only one load at a time and while we are at it, we will also define the stiffness coefficient. We will be coming to this again and again and so we might as well look at it now.

We use a symbol k to refer to the stiffness coefficient and let me take j, but I will avoid k because it is a duplication. I will say j_l. What is this? This is the force at the location j due to a unit displacement. Here, we have a unit load or unit force, whereas here we have a unit displacement at this location; this is a unit displacement (Refer Slide Time: 19:39); that displacement for a truss is a translation, but in a beam you could have a rotation. It is complete only if you take other joints, let us say, m not equal to l; all other displacements should be arrested, which means you are dealing with a somewhat different structure.

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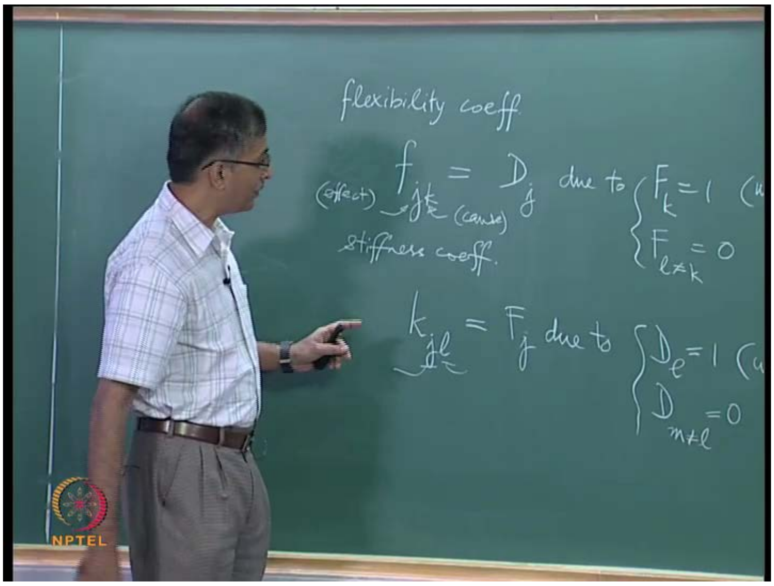

The important point to note in all these notations is this is the effect; this is related to the effect and this is related to the cause (Refer Slide Time: 20:13). You get this displacement because you applied a unit load. Similarly, here, this is the effect and this is the cause. Is it clear? Now, is there any relationship between these two?

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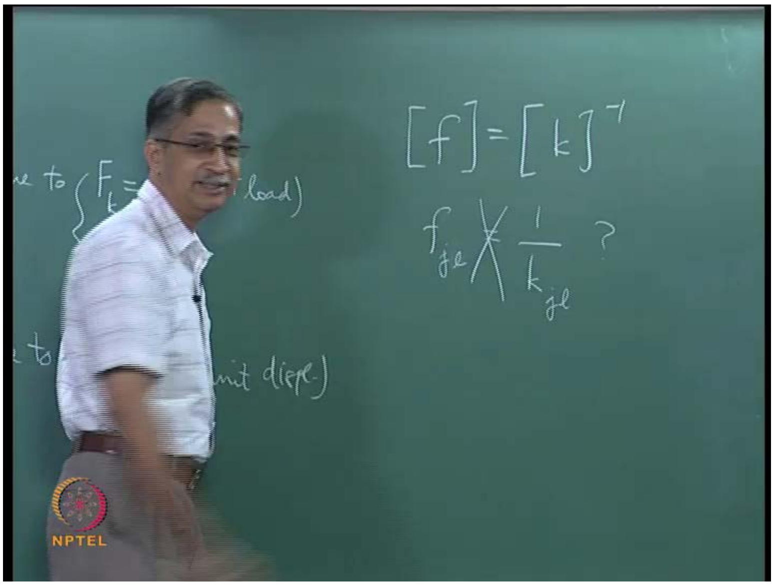

Is it right to say that the flexibility coefficient f_{jl} for a given structure is the reciprocal of k_{jl} (Refer Slide Time: 20:52)? Is this right? That is what one of you suggested. Is it right or wrong? After all, we have been told that one is the reciprocal of the other. Is it true?

This is not true. Why is it not true? This is wrong. What is true is that the flexibility matrix is the inverse of the stiffness matrix and that does not mean this. We will study this in more detail in the next module.

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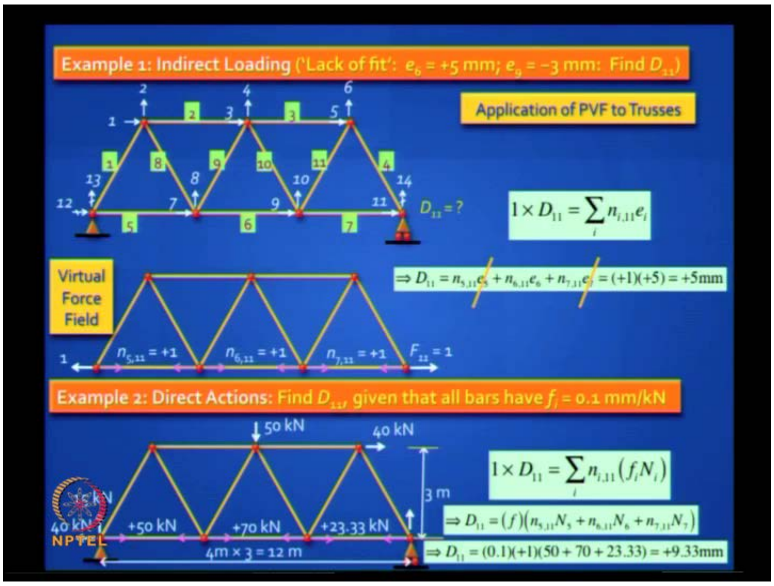

This can be applied to finding trusses. Here is an example of indirect loading. Let us say you have a lack of fit and two of the bars have a change in length. The bar number 6 has been manufactured and came with a length which is 5 mm more than its desired length and the bar number 9 is too short by 3 mm; so, you kind of fit things together. Will you get any internal forces when you do this? No, because you have just a rigid system. The bars will just move about a little bit and there will be no support reactions – no internal forces, but if the structure was over-rigid, it is possible that you have support reactions and you have a self-recuperating system.

Now, the question is: can you find the horizontal deflection D_{11}? How much will that joint move to the right? This is D_{11}. How do you find it out? Apply a unit load exactly there and analyze the system. When you analyze it, you quickly realize that if you put a unit load there, you will get a unit force in the bottom three bars and you will not get it elsewhere; it is very easy to analyze. You invoke the formula 1 into D_{11} is equal to the total internal work. The internal work relates to only the bottom three bars because all the other bars do not have any force; actually, it works out to be the sum of the elongations in those three bars. It makes sense; that is how the roller support will move; it is very easy to calculate.

If you have another example where the problem is that you are given a truss with some actual loads on the truss, say a 50 kilonewton load in the middle and 40 kilonewton horizontal load, you need to first analyze this truss, find the bar forces, and then for each bar, find the bar elongations by multiplying each bar force by its flexibility. That is easy to do. You do not need to go through the full exercise because only three bars really matter here.

Finally, you need to multiply with this, this, and this (Refer Slide Time: 24:03) because all the others do not have any force n_i. So, if you want do it fast, you just find the forces in the bottom cord and you can do that. Let us say these are the answers. Then, you can just multiply each of those by 1 and then the flexibility value and you will get the answer very easily. Is it clear? It is a simple demonstration. You can also apply this to beams and trusses. This is something we have finished.

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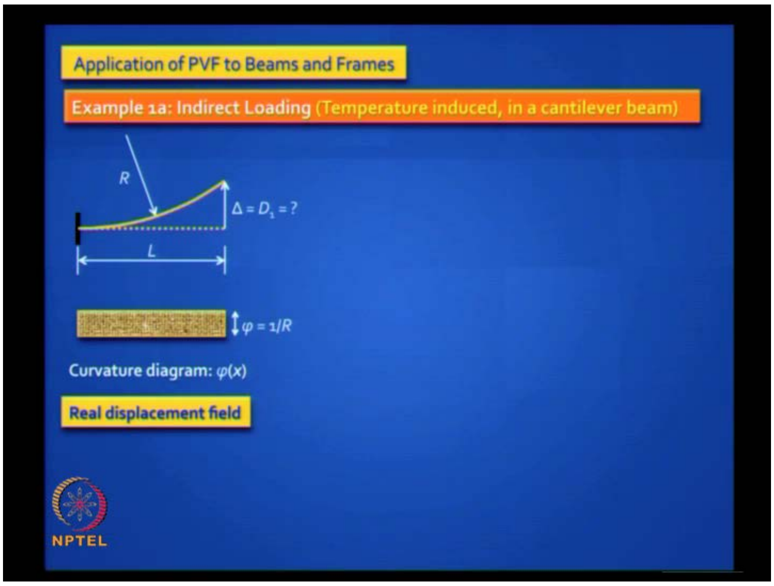

Let us apply this to beams and frames. Let us say you have a bar; you have a cantilever beam and that beam bends for some reason. Let us say it is bent because the temperature at the top is less than the temperature at the bottom and you have a linear gradient in the temperature. So, it will naturally take that curvature. The question is: can you find the vertical deflection?

Is there any method that you already know other than the virtual work method where you could solve this? Energy method we have not yet done. Conjugate beam method. Conjugate beam method because you remember on the conjugate beam, the fixed end become frees and the free end becomes fixed. What is the loading you put? There is no bending moment here. That is how students normally remember; they remember that you have to put an M by EI diagram. No. You have to put the curvature diagram. If the curvature is caused by a bending moment, you put the M by EI diagram; if it is caused without the bending moment due to environmental change, then you put the phi diagram, the curvature diagram, which is 1 by the radius of curvature. Is it clear?

If the radius of curvature is known, then that is what you put. So, you can do the conjugate method. Let invoke the principle of virtual work. Please note: in these applications, we use the word principle of virtual forces because your force field is virtual and you are imagining; you are constructing, you are cooking up, an imaginary force field to help you find an unknown displacement.

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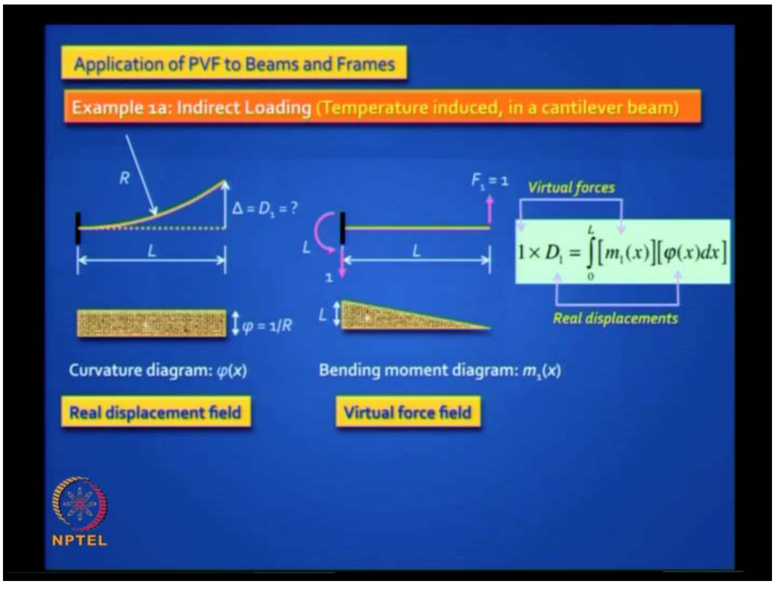

What do you do if you want to note the vertical deflection? Upward, you put a unit load exactly where you want the deflection and draw the bending moment diagram; we use the notation small m just as we used this notation small m earlier (Refer Slide Time: 26:47). Small m_1 means bending moment at the location x due to f_j equal to 1. This diagram is very easy to construct. Then, you invoke the principle of virtual work. The external virtual work is straightforward: 1 into D_j. The internal virtual work is actually a product of moment times rotation, but since the moment is changing from point to point, you need to integrate and you have to find the change in rotation.

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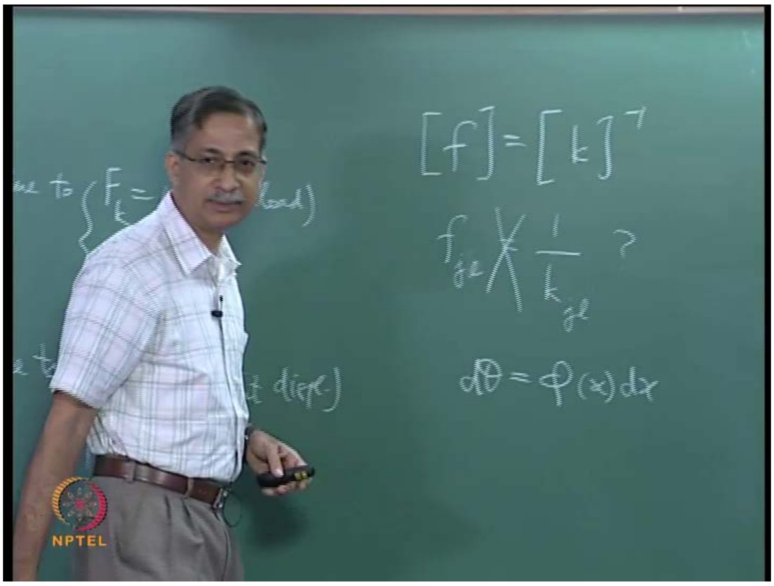

The change in rotation is given by d theta and d theta is given by curvature into dx; that is what we do; that is the integration we need to do; you can do the integration.

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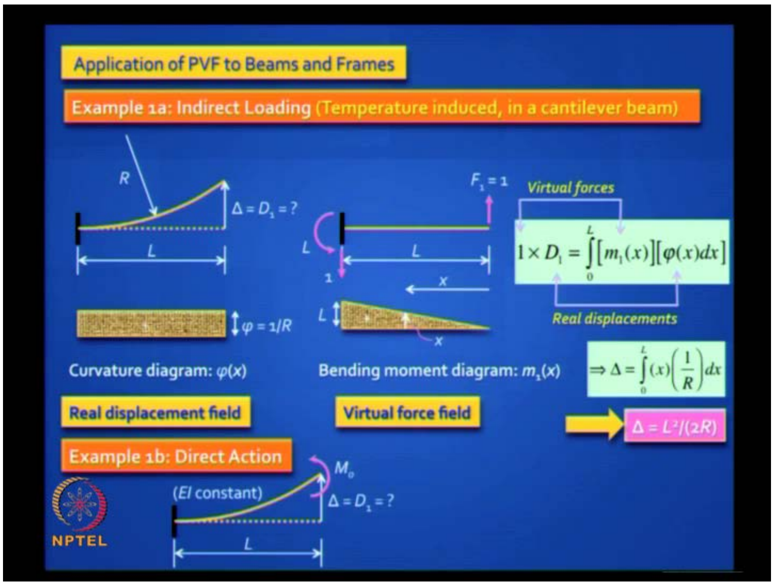

In this case, it is straightforward. If you take x from the right to left, it is convenient. The bending moment at any location is x; the curvature is 1 by R, it is a constant; so, you are actually multiplying the triangle with a rectangle. It is very straightforward to get the answer. Let us take the same problem and imagine that this has happened because you applied a concentrated moment at the free end. Agreed? So, it will lift up.

Now, the cause is not environmental change, but it is a deliberate action caused by a load. Then, the curvature 1 by R is given by what you said earlier – it is bending moment divided by EI. The bending moment is constant everywhere because it is a case of uniform bending – pure bending. You get exactly the same answer except that instead of 1 by R, you will write it as M_{naught} by EI. Is it clear? It is simple.

You have two kinds of applications: one, where it is a pure geometry problem and there are no forces involved; the second, you might tend to get confused, there are forces involved, but you are looking not at the force field – you are looking at the displacement field. It is still a geometry problem, an unsolved geometry problem, but you get some parts of that displacement field – for example, here, the rotations, the curvatures and there are some parts which you do not know, like the joint displacements. To find the unknown joint displacements, you are invoking the principle of virtual work.

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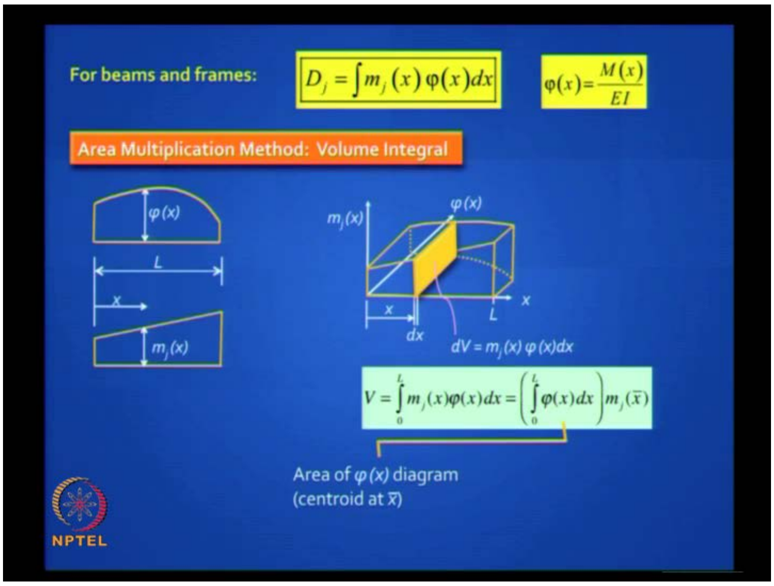

For beams and frames, this would be the formula that you would use. If you want to find D_j, the displacement or deflection at a coordinate j, then it is integral m_j into phi of x into dx. If the phi is caused by a bending moment, then it is M by EI; very simple and straightforward. If you look at it carefully, you are really multiplying two diagrams; you are multiplying a curvature diagram, which is real, with a moment diagram caused by a unit load, which is virtual.

If you look at that integral, you can see that it is really a volume and you can invoke what is called the area multiplication method by taking a slice of that volume. You will find that the local elemental volume is dV; it is m_j of x into phi of x into dx. It is very easy to understand this. Actually, you are integrating over the length of that element. Is it clear? You can do this. You can do it in a simpler way if you have laid down the curved portion.

The interesting thing is one of those lines will be straight. Which one will be straight? m_j will always be made up of straight lines because in a beam when you apply a unit load, you can get a straight line bending moment diagram. m_j is always straight; so, it is necessary to realize that your ordinates should be linear. Put that on top and the curved diagram, which comes from the curvature... The curvature could be straight or could be curved; in general, it is curved.

Put that on the ground, lay it down flat, take that area, and find out the location of the centroid of that area; the ordinate of m of j, m_j at that location, is the average height. So, you can reduce this problem to multiplying the area of the curvature diagram by the height at its centroidal location; this gives you quick solutions.

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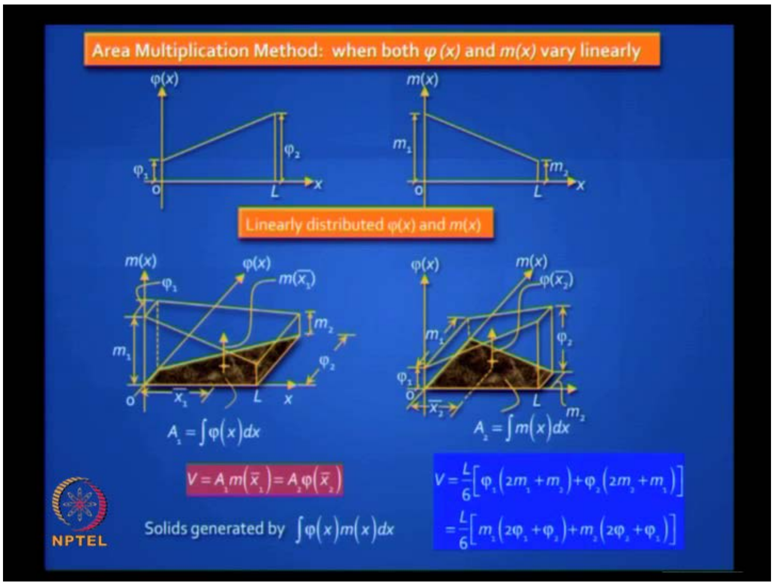

There are many examples; you have studied them last semester. One of the most common examples is when you have to multiply two trapeziums. Let us say the curvature diagram is also a linearly varying diagram. Then, you can either…. It does not matter which you put on the ground because you get an accurate answer either way and this is the solid that you get (Refer Slide Time: 32:00).

The formula you have to remember when you deal with a situation like this is V is equal to L by 6 into phi$_1$ into 2 m$_1$ plus m$_2$ plus phi$_2$ into 2 m$_2$ plus m$_1$, which is easy to remember. In case you forget and you flip it over, you will still not make a mistake. It is L by 6 into m$_1$ into 2 phi$_1$ plus phi$_2$ plus m$_2$ into 2 phi$_2$ plus phi$_1$. This is a very useful formula; it works even if some of those values are negative. As long as you have one equation for that trapezium, it will work. If one of them is a rectangle, you do not need to use this formula; if one of them is a triangle, you do not need it. You can have simpler methods.

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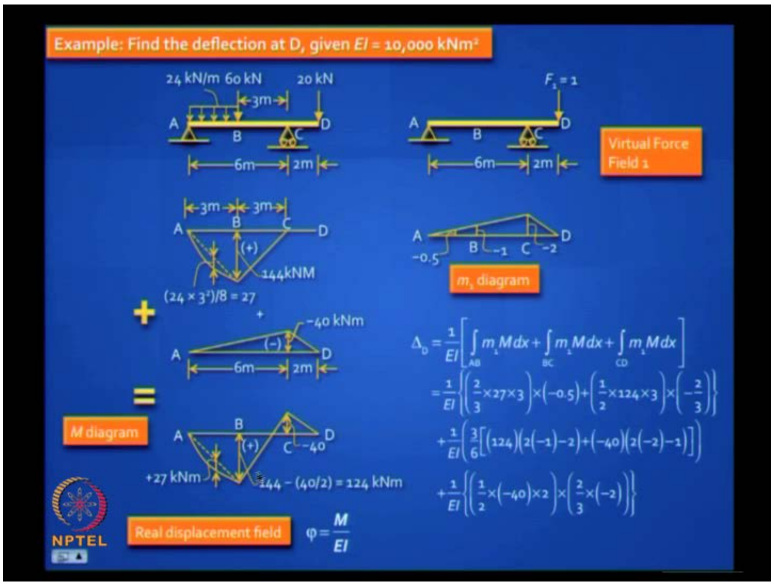

Let us take just one example and see how it can be applied. This is a little difficult; it is a simply supported beam with an overhang subjected to some arbitrary loading. The question is: can you find the deflection at the free end? You have to draw the bending moment diagram and it is good to separate out the curved part from the straight part. There are some shortcuts involved and I suggest you go through it carefully; you have done this last year.

The capital M diagram is a bending moment diagram in the real beam, but we are not interested in the capital M diagram; we are interested in the curvature diagram, which is the M by EI diagram. Once you have capital M, if you divide by EI, you have got the curvature. To find the unknown deflection at D, you need to apply a unit load F_1 equal to 1 to find D_1. When you do that, that diagram is made up of straight lines; that is your m_1 diagram.

This is what you need to do (Refer Slide Time: 33:53). You need to find delta by multiplying that m_1 diagram with the capital M diagram. It makes sense to separate out the curved part from the straight line part. If you work this out using the formulas we derived, you will get the correct answer. But just to go through this once again, this curved part has a maximum volume in the middle – 27 kilonewton meter (Refer Slide Time: 34:18). It is a parabola; so, the area of the parabola is two-thirds that of the equilateral rectangle.

This length is 3 meters (Refer Slide Time: 34:29). So, it is two-thirds into 27 into 3 and that is the area. Its centroid is at this location (Refer Slide Time: 34:38), where the ordinate here is minus 0.5; so, you multiply by minus 0.5. Then, this difference is 124; so, the area of the triangle is half into 124 into 3. The ordinate of this – the centroid of this – is at a distance of two-thirds from here; so, it is two-thirds into minus 2 by 3.

You can work it out and you will find that the value here is minus 2 by 3 and so on and so forth. You need to complete this product. Be careful; just look for equations. If you have a single equation for a certain length and you are multiplying with another equation for that entire length, you can do it together. But, if you have a break in the line, then you must restrict the integration to that region where you have the change in the slope. Work this out and you can get the answer. In this case, that is the value that you get.

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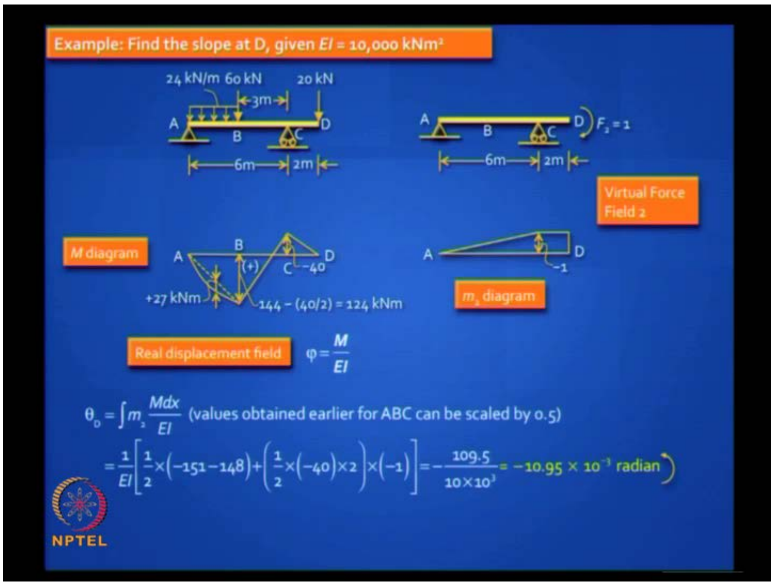

If you need to find the slope, you do the same thing, but now, you put a unit moment and you get this shape. Then, you can work out the answers; it is similar; you get something in radians.

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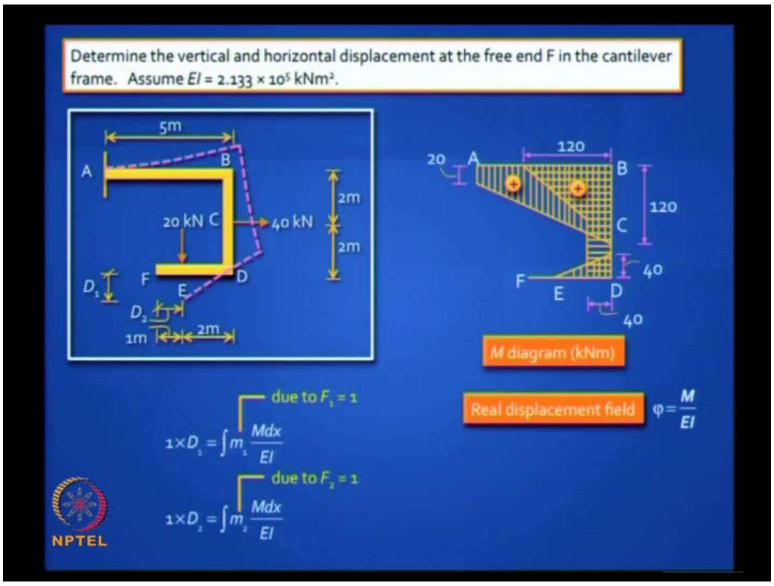

A more complicated problem is a frame; that is a cantilever frame. The question is: can you find D_1 and D_2? You have done this last semester; so, I will just go through the concept. What do we do? We first generate the bending moment diagram and doing this needs some skill, which you should have got by now; you must know how to analyze statically determinate structures. Draw the bending moment diagram on the tension side. We are not so much interested in the bending moment diagram as in the curvature diagram. So, if you divide this by EI, you have got the curvature diagram. The formulas to get D_1 and D_2 emerge from the unit load method.

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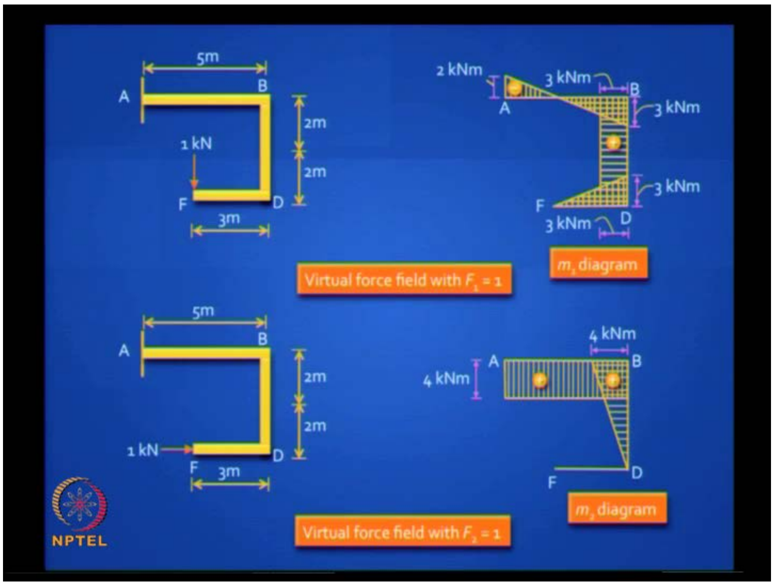

First, to find m_1, you apply F_1 equal to 1 and you need to draw correctly the small m_1 diagram, which is caused by the unit load; it is a virtual force field. To get m_2, you have to apply F_2 equal to 1 and you get another diagram. Once you have got these two diagrams, then you have got the capital M diagram. It is a question of integrating, but you will find it is much easier to do the area multiplication using the volume integral.

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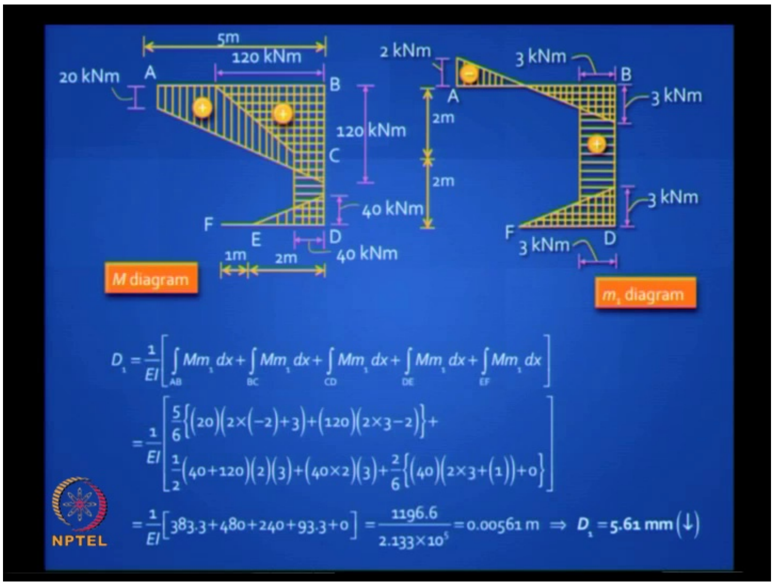

Let us take the first case. On the left side, you have the capital M diagram and on the right side, you have the small m_1 diagram; you need to multiply one with the other. In this case, both are made up of straight lines. It is very easy to do it and I leave it to you to work out the calculations. You can use that formula for the trapezium and you will get the answers very quickly. That is the first deflection.

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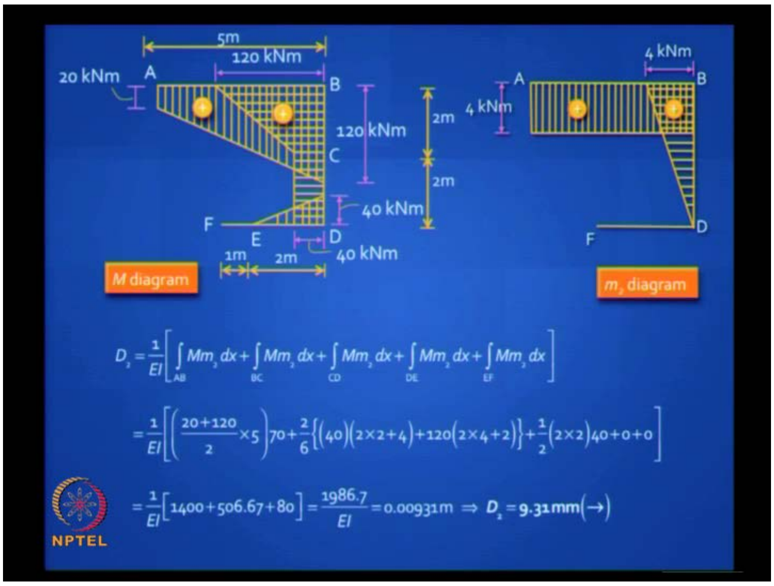

For the second one, m_2 will change. You repeat the same process to get the horizontal deflection. This needs some exercise. Once you practice a few problems, you will be very comfortable doing this. So much for the principle of virtual forces. What was the purpose of principle of virtual forces? Mainly to find some unknown displacement in the real displacement field. Now, we look at a less-used version of the principle of virtual work; Bernoulli's theorem originally dealt with this.

We are now looking at the force field which is statically admissible and you want to find out some unknown quantity in that force field. You might ask why we should struggle and do all this. Why do you need to use kinematics to find a static solution? Can we not do it directly? Yes, you can. If the system is statically determinate, you can directly invoke equilibrium but you will find there are some situations, for example, in a situation where you have multiple internal hinges, you will find it is much easier to invoke this principle. It is also called the dummy displacement method, but you could call it the unit displacement method like the unit load method when you put that dummy value equal to 1.

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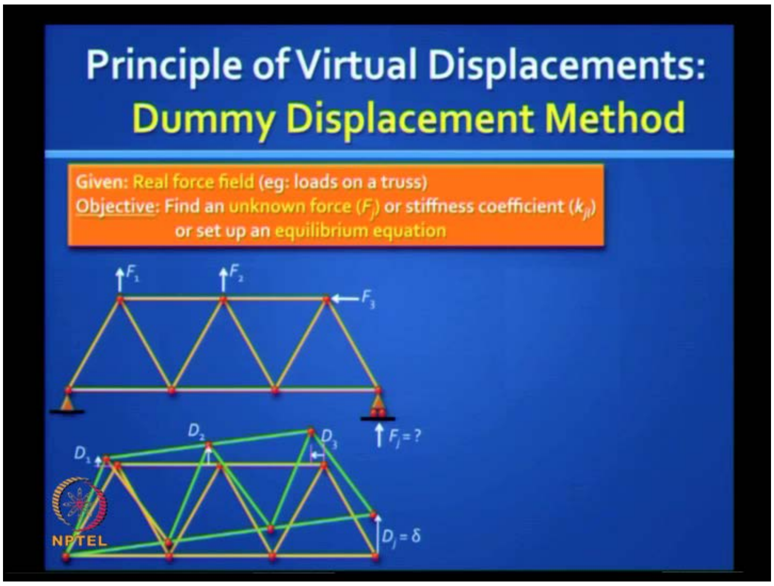

Here, let us say you want to find an unknown force or you want to find a stiffness coefficient. Let us say you want to find F_j. You want to find the support reaction in this truss when it is subjected to this kind of loading. What you do is you give it a dummy displacement D_j equal to delta. It will rotate about the left support; so, you have rigid body movements. It is possible for you to figure out how much those locations where 1, 2, and 3 are applied will move. Once you know those values D_1, D_2 and D_3, you can invoke this theorem. They are rigid body movements and so it is easy to calculate. The total internal work in such problems turns out to be 0. Why is it 0? Since you have rigid body movements, you do not have internal deformations in those bars. It is 0 because the internal displacement in the displacement field is 0 not because there are no internal forces in the real force field. Is it clear?

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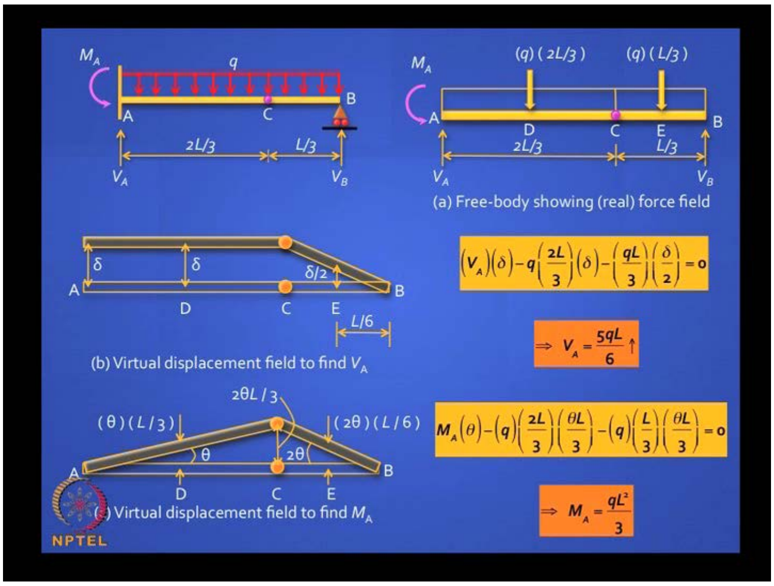

You invoke this and you can solve this problem easily. Let us demonstrate this with a problem where you have an internal hinge. At the left edge, it is not shown; at the left edge, you have a fixity. In this figure, please note there is a fixed end support at A and you want to find these reactions. What you see on the right side is the free body of the force field; the force field is real.

If you want to find the left reaction V_A, what you do is you lift it up by delta, but lift it up in such a way that the fixed end moment M_A does not do work; it must remain horizontal; you will find this is the only way you can draw the deflected shape. This is visualizing the deflected shape. Then, if you want to find the moment at A, M_A, you give it a rotation theta, but you should not allow any other movement. When you invoke the principle of virtual work, there should be only one unknown at a time, which means the other forces should not do any work; that has to be done cleverly. Then, if you invoke the theorem, it is very easy to compute the reactions.

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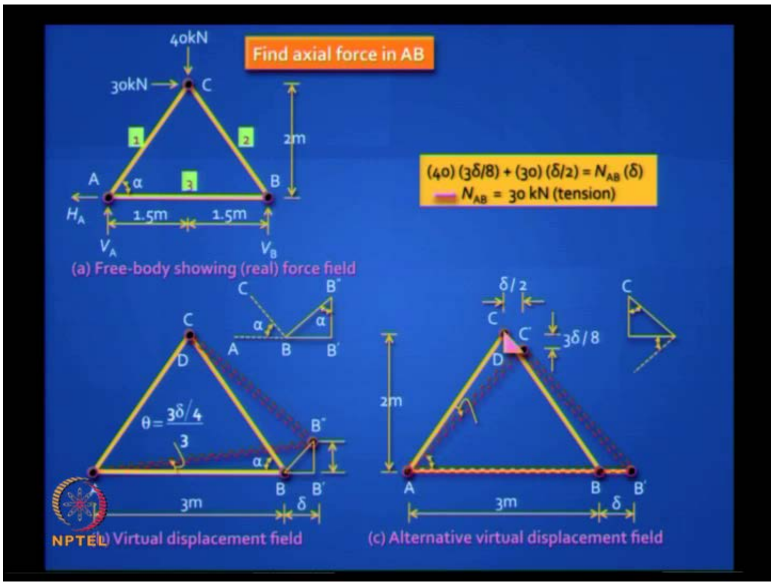

You can also do it in a truss. Let us say you want to find the reactions in a truss, this particular example. This is a tricky thing; not the reactions – reactions are easy to find; I want to find the actual force in AB. What should I do? Well, I should now have an elongation in that member AB. If I have any elongation in that member AB, this is the way to do it. Let it elongate by delta, but if I keep this B here (Refer Slide Time: 42:22), I have a problem because I will also be elongating the bar II, BC. I do not want that; I want BC to remain unchanged in length. It means I take C as center and draw an arc or a tangent; for small deformations, the arc can be replaced by a tangent; so, B moves to B dash.

In this configuration, the advantage is the only change in length is in AB and it is equal to delta. You have to work out the trigonometry part of this and find out how much the joint C has moved horizontally and vertically; this can be done. Once you have done this correctly, simply invoke the theorem and you will get the answer. It is a powerful technique. You might make mistakes if you are not careful with the trigonometry involved.

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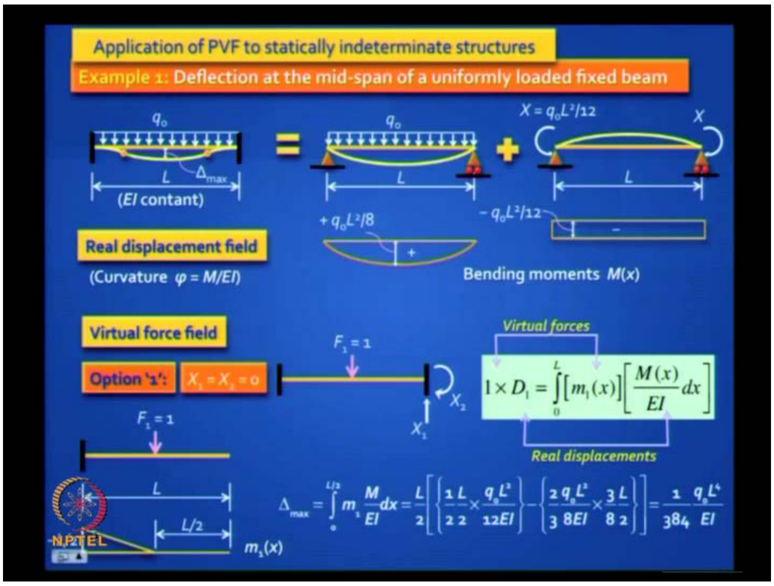

Last application of finding a deflection in… I want to show you that this can be also applied to a statically indeterminate problem – finding deflection in a statically indeterminate problem. Here, you have a fixed-fixed beam and the question is can you find the maximum deflection at the mid span in this beam? What do you do? First of all, you need to have the curvature in this system. You need to know the fixed end moment; you can use the conjugate beam method and find the fixed end moment.

What is the fixed moment? q_{naught} into l square by 12. I have separated out the distributed load defect and the fixed end moment effect? The m diagrams look like this. You have a parabolic sagging bending moment diagram q_{naught} into l square by 8 and superposed on that, you have a constant hogging moment, which is q_{naught} into l square by 12. Now, to find delta in the middle, what should we do? What should we do? Which theorem will we invoke? Which principle will we invoke? Principle of virtual work? You can do conjugate beam method, you are right, but we are now on the topic of virtual work. Virtual force or virtual displacement? Virtual forces. What will you do?

What we have here is a real displacement field. If you divide M by EI, you have got the curvature diagram. What you need to do is virtual force; this is what you should do; this what everybody does but there is a problem with this. The problem with this is you are again dealing with a statically indeterminate structure and you need to spend some time to figure out the small m diagram for this. Is there a way out which is easier? This really brings out the power of the virtual work. Is there an easier way? Yes and I want you to see this.

Let us say, the degree of static indeterminacy of this structure is 2; that means treat the cantilever as the primary structure and why can I not take any value of X_1 and X_2? I can, because if you recall the origin of the principle of virtual work, the only requirement is that the force field is statically admissible. Now in a statically indeterminate structure, how many force fields can you generate which are statically admissible? Infinite. Only one of them will be exactly correct for the boundary conditions that you have given. So, the unique solution is one which also satisfies kinematics, but we are not interested in finding the kinematically correct solution also, because this is just a device to help us find an unknown deflection.

All we need is any statically admissible solution. You would find that if I conveniently take X_1 and X_2 equal to 0, I have got a cantilever; I have reduced that fixed-fixed beam to a cantilever beam with a concentrated load in the middle, a unit load, for which the small m diagram is child's play; it is a small triangle; it is a small triangle and it is minus L by 2.

Now, will you try this out on your notebook? Can you get the deflection at the mid span? I will help you; just check out the solution. You are multiplying this triangle with these two diagrams. One is a parabola and one is a rectangle. Let us take this area. What is the area of the parabola? Let us do this rectangle first (Refer Slide Time: 47:44). You are multiplying this rectangle with this triangle. Do you agree that the area is half into L by 2 into the ordinate because you can take that full area of that triangle and any value of the ordinate here? Do you agree to this part? Now, we need to multiply this parabolic area. What is the area? Two-thirds of q_{naught} into L square by 8 into EI into L by 2; L by 2 is outside here. Its ordinate will be located at a distance of how much?

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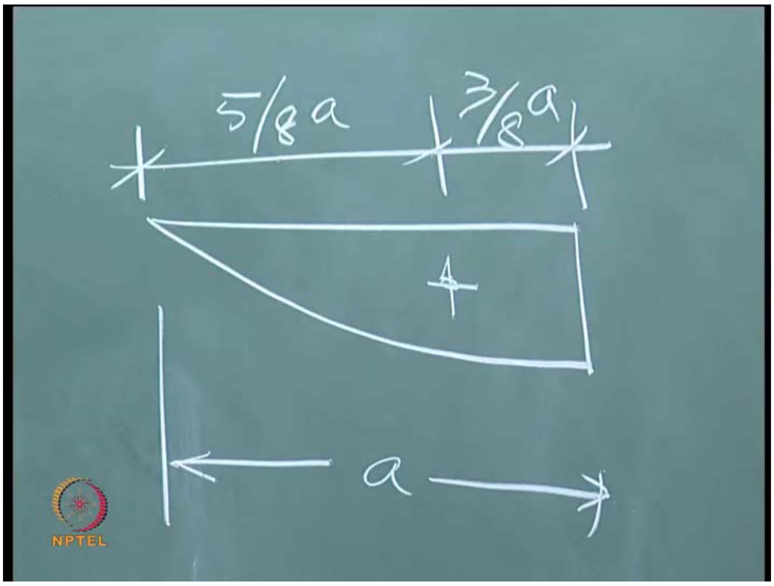

You know that if I have a parabola like this, the centroid will be located in such a way that these distances are…. If this length is l, or let us say a, how much will this be? 5 by 8 into a; this is 3 by 8 into a. That is how you get the ordinate in this triangle as 3 by 8 of L by 2. Agreed? 3 by 8 of L by 2. Now, you must be careful about the signs. This product will be negative because you are multiplying something positive with something negative (Refer Slide Time: 29:01), whereas this product will be positive because you are multiplying a negative with another negative. Clear?

Just check this out. The answer is 1 by 384 into q_{naught} into L raised to 4 by EI. Just pause for a while and see if the beam is simply supported, what is the deflection at the mid span? It is 5 by 384. So, making it fixed and giving it some hogging moment actually reduces your mid span deflection to 20 percent of the original value. In reality, many systems are partially fixed; so, the answer is between these two extremes. Now, what is extremely interesting is you could have chosen any statically admissible diagram.

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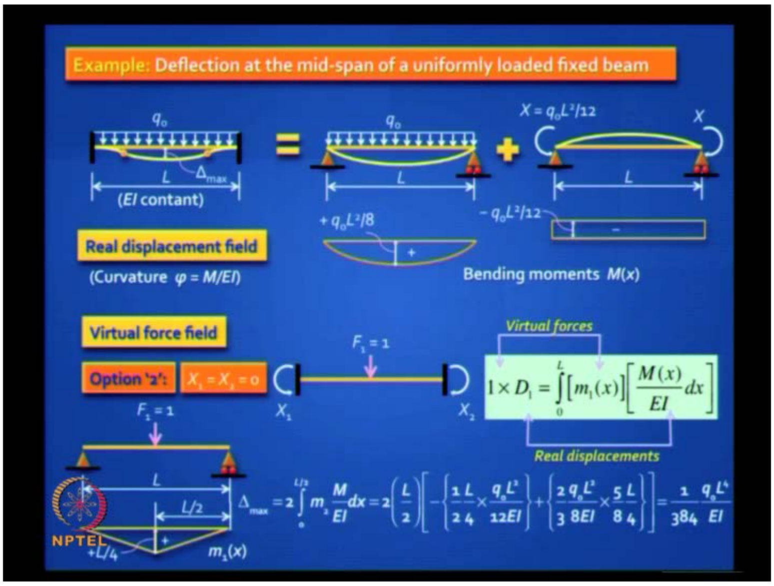

We will just see an alternative where we will treat the primary structure now as simply supported. That means here X_1 and X_2 are 0. Now, I am dealing with a simply supported beam and my small m diagram has changed completely. If I do the multiplication, you can check this out, I get exactly get the same answer. This is really a mind-blowing discovery; it shows the real power of the principle virtual work.

What are the implications of this? You take a complicated structure, a multistoried frame. If you have the bending moment diagram in any one beam for example, that is enough for you to help you get the deflection in that beam because you can strip it off from the rest of the structure and make it statically indeterminate and apply the unit load method. Similarly, we will see this later, you can find the drift in a tall building very quickly by invoking this.

We will stop here and we will continue in the next class. Can you just name the work theorems? There are three work theorems. Castigliano's theorem. Castigliano's theorem belongs to energy methods. Work theorems, three of them; they are applicable to linear elastic structures.

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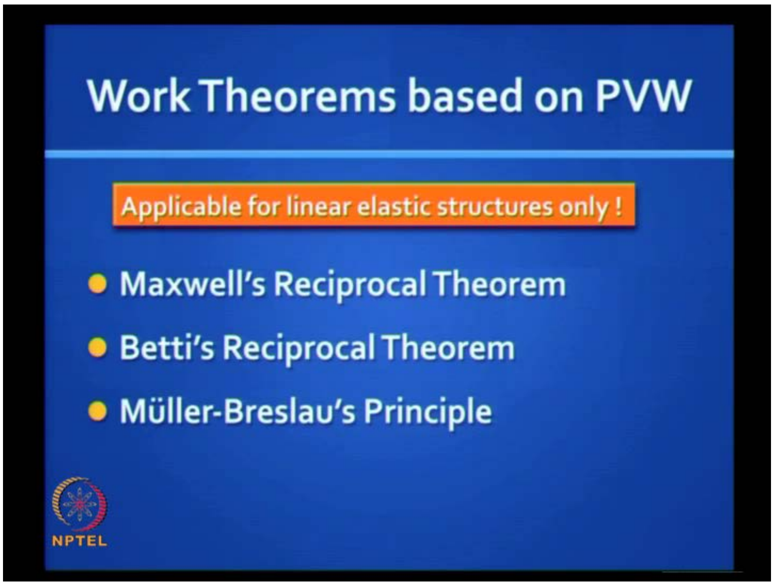

They are Maxwell's Reciprocal theorem, then Betti's theorem, and Müller–Breslau's principle. We will see this in the next class. Thank you.

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