

Advanced Structural Analysis
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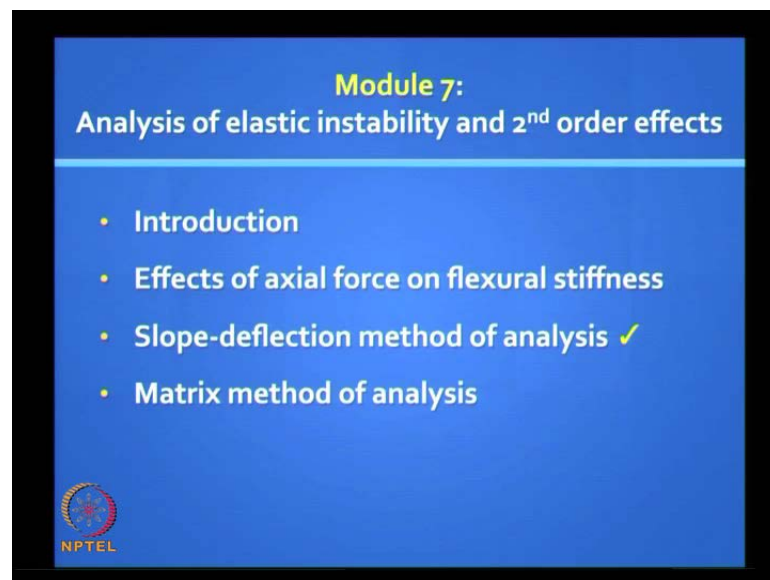
Module No. # 7.2

Lecture No. # 39

Analysis of Elastic Instability and Second-Order Effects

Good morning. This is lecture number 39, module 7. The topic is analysis of elastic instability and second order effects.

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We started this module in the previous class and we had covered basic background; the introduction and the effects of axial force on flexural stiffness.

There are two types of problems that we will try to solve. The first is we would like to know the critical buckling load in any given structure when some elements are subject to axial loading because you need to have that information to do second order analysis.

Why do you need that information?

[Not audible] (Refer Slide Time: 01:04)

No. That is to judge whether you need to do second order analysis, but to actually do it, you need P critical. Why do you need P critical?

[Noise – not audible] (Refer Slide Time: 01:14)

We will see that. You need to know the critical buckling load in order to do second order analysis.

P by P E .

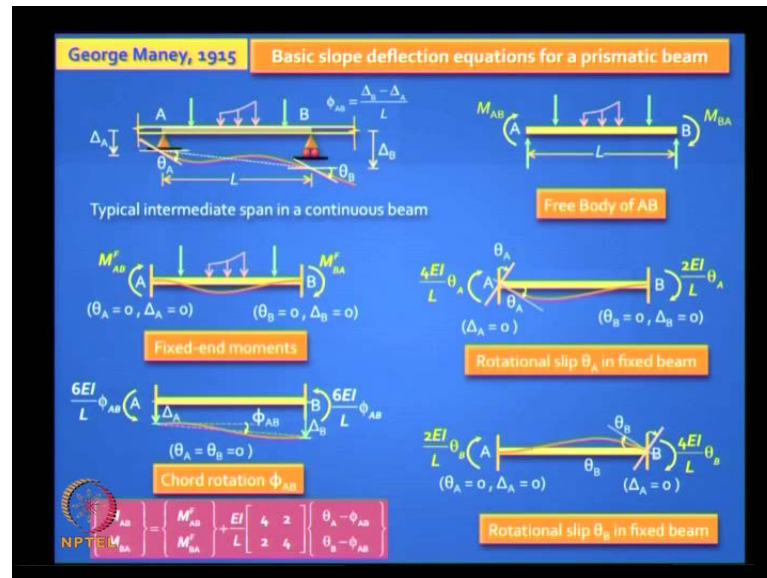
Yes because P by P E comes into play. We will take a look at that. Now, we are attempting a kind of manual method of analysis, which is the slope deflection method. However, in the next session, tomorrow, we will look at more sophisticated computer oriented matrix method of analysis.

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This is covered in the last chapter in the book on Advanced Structural Analysis.

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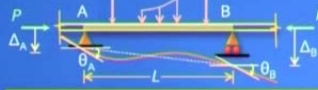


Let us go back to the old slope deflection method, which you are familiar with; goes back to 1915, George Maney.

Typically, we look at any intermediate beam in a continuous beam system. We are trying to write down slope deflection equation. That is, we are trying to write the end moment equations M_{AB} , M_{BA} , assumed clockwise positive here in terms of slopes and deflections. The slopes are θ_A and θ_B for that element and the deflections are δ_A and δ_B . However, we know that what is of relevance is not the absolute values of δ_A and δ_B , but the chord rotation which is ϕ equal to δ_B minus δ_A divided by L .

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Slope-deflection method



$$\begin{Bmatrix} M_{AB} \\ M_{BA} \end{Bmatrix} = \begin{Bmatrix} M_{AB}^f \\ M_{BA}^f \end{Bmatrix} + S \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix} \begin{Bmatrix} \theta_A - \phi_{AB} \\ \theta_B - \phi_{AB} \end{Bmatrix}$$

Typical intermediate span in a continuous beam

We can easily extend the classical slope-deflection method (George Maney, 1915) to systems involving beam-columns, by modifying the expressions for the various stiffness measures to include the effect of the axial force in the deflected configuration. The fixed-end moments also need to be suitably modified to include the effect of the axial force.

For convenience in the analysis, beam-column behaviour in some elements of the structure (such as beams in portal frames with low axial forces) may be ignored.

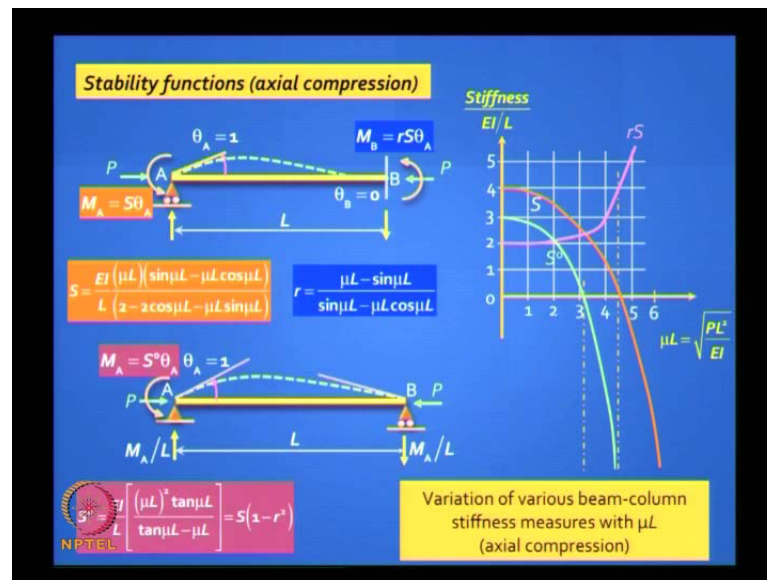
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What we are interested in is to understand how these slope deflection equations get affected when you have an axial force. We did this in the last class. Now, we want to extend the same slope deflection method from beams to beam columns by modifying the expressions for stiffness. We also need to modify the fixed-end force effects. This is the basic element in slope deflection method.

Now, we are going to apply an axial compression P and we are going to see how it affects the stiffnesses. What are the conventional stiffnesses? $4EI/L$, $2EI/l$ and so on. So, this is how it gets affected. It is very simple. That parameter capital S is the basic stiffness measure. When you do not have an axial force P , what is that value equal to? $4EI/L$ for a prismatic beam element. What is the value of the r ? r is the carryover factor; 0.5. So, you will find that the general form of that equation does not change. It is only now we have generalized it so that we can include the effect of axial force P .

For convenience in the analysis, the beam column behavior in some elements of the structure, let us say you have a multi-storied frame. You need not take the beam column effect in all of them. You need to worry only in those elements, where the P is significant. So, P by P/E of that element should be large and we suggested more than 15 percent. For the others, you can ignore this value.

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If you recall in the last class, this is how those parameters S and r vary. You can see that S starts with the value of $4EI/L$. It can drop down to 0 when you have a high value of μL . μL is the measure of the load P normalized with respect to L^2 by EI and EI/L^2 . So, you have those equations for r and S .

If you have any given value of μL , just plug that value into those expressions. You get readymade exact solution for r and S . These are known as stability functions. The default values of r and S are: for S , the default value is $4EI/L$ and for r , it will be half. It remains... So, r into S is the carryover moment. So, we have these expressions. We also have a tabular format if you wish to use, but the equation is very easy to put on a spreadsheet and you can get the solution accurately.

We also studied the modifications that needs to be done if you have the far end has hinged. Then, you know that $4EI/L$ reduces to $3EI/L$. The notation we use is S_{naught} and interestingly S_{naught} is related to S and r . S_{naught} is capital S into 1 minus r squared. You can prove it and interestingly, you can apply it for the default values also. If you take S as $4EI/L$ and r as half, then within the brackets, 1 minus r squared will become 0.75. So, 0.75 of four EI/L is $3EI/L$. So, we have those expressions as well. That is all we need if you have axial compression.

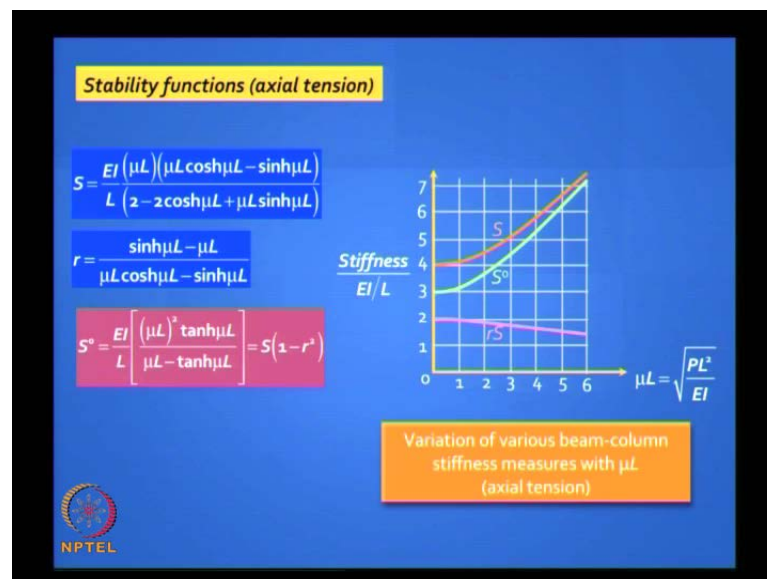
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Values of beam-column stiffness measures with μL (under axial compression)										
$\mu L = \sqrt{\frac{PL^2}{EI}}$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.493
$\frac{S^o}{EI/L}$	3	2.950	2.794	2.518	2.088	1.438	0.4082	-1.468	-6.518	$-\infty$
$\frac{S}{EI/L}$	4	3.967	3.865	3.691	3.436	3.088	2.624	2.008	1.173	0
r	0.5	0.5063	0.5264	0.5637	0.6263	0.7310	0.9189	1.3157	2.5605	$+\infty$

$\mu L = \sqrt{\frac{PL^2}{EI}}$	4.5	5	5.5	6	2π
$\frac{S}{EI/L}$	-0.0191	-1.9807	-5.6726	-20.634	$-\infty$
r	13.357	-2.5067	-1.3481	-1.0396	0

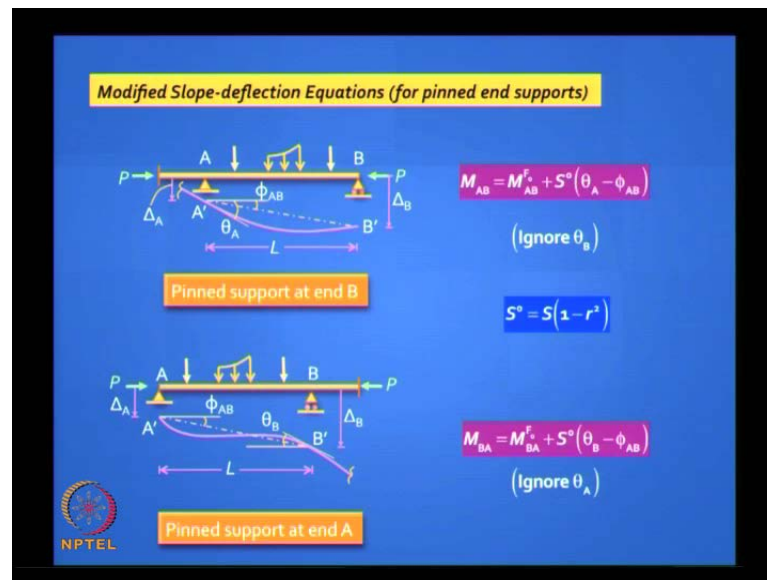
We have a tabular format if you want to use it.

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If you have axial tension, you have similar equations, but they involve hyperbolic functions. Interestingly, you will find that S and S naught will increase, but r into S , the carryover moment decreases. This is the outcome of having axial tension.

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All this we know. Now, when we did the slope deflection method, we took advantage of situations, where the far end is hinged or pinned. Is it not? What is the advantage of doing that?

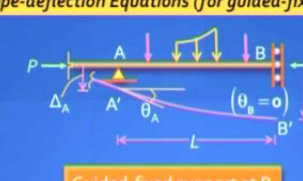
Equations to find the number of variables [Not audible] (Refer Slide Time: 07:31)

Not variable. What is the right terminology? The degree of kinematic indeterminacy reduces. So, we can take the same advantage here. Here for example, you can ignore theta B. Remember? **Instead of** 4 EI by L, we use 3 EI by L.

Now, instead of 3 EI by L, we use S naught. We replace capital S in the previous slope deflection equations with S naught. The chord rotation behavior is similar. If you have the hinge at the left end A, then you have a similar set of equations. Remember: The fixed-end moments will also be little different when you have axial force coming into play and we need equations to help us solve those.

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Modified Slope-deflection Equations (for guided-fixed end supports)

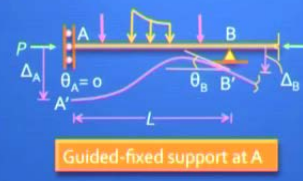


Guided-fixed support at B

$M_{AB} = M_{AB}^f + \bar{S}\theta_A$
 $M_{BA} = M_{BA}^f + \bar{r}\bar{S}\theta_A$
 (Ignore Δ_A, Δ_B)

$\bar{S} = \frac{EI}{L} \frac{\mu L}{\tanh \mu L}$
 $\bar{r} = 1 / \cosh \mu L$

$\bar{S}(\mu L) = \frac{1}{2} S(2\mu L) [1 - r(2\mu L)]$
 $\bar{r} = -\sec \mu L$



Guided-fixed support at A

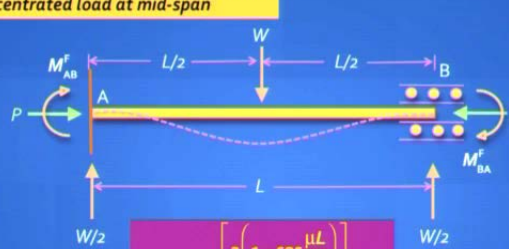
$M_{AB} = M_{AB}^f + \bar{r}\bar{S}\theta_B$
 $M_{BA} = M_{BA}^f + \bar{S}\theta_B$
 (Ignore Δ_A, Δ_B)

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You can also take advantage if you wish of guided fixed supports. You need to have some fresh expressions. Remember: The normal stiffness values EI by L ; the default value. However, you can have a **S tilde and r tilde**. I suggest you do not do this because it really needs a very good understanding. I am just showing it for you in case you want to use it, but you can stick to your basic slope deflection equations, where you have r and S and take advantage of S naught when you have the far end hinged. This you need not.

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Fixed-end Moments in Beam-Columns: Concentrated load at mid-span



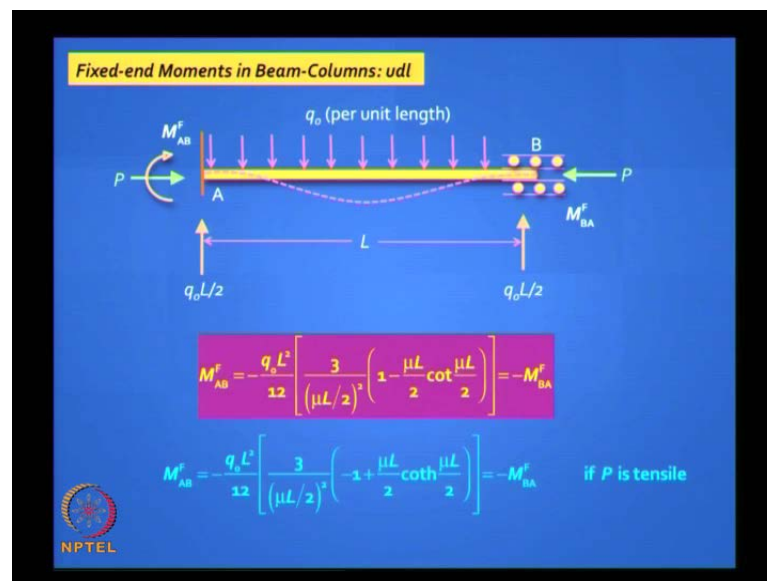
$M_{AB}^f = -\frac{WL}{8} \left[\frac{2 \left(1 - \cos \frac{\mu L}{2} \right)}{\frac{\mu L}{2} \sin \frac{\mu L}{2}} \right] = M_{BA}^f$

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Now, we need formulas for fixed-end moments. For a prismatic beam, look at the boundary conditions shown. To apply a force P, actually you must facilitate it. So, instead of showing fixity, you should show an ability for it to move horizontally. However, you can assume it is actually rigid, but this is a correct symbol. Can you see? This is a correct symbol instead of showing it fixed. However, some books show it as fixed; it is really not very important.

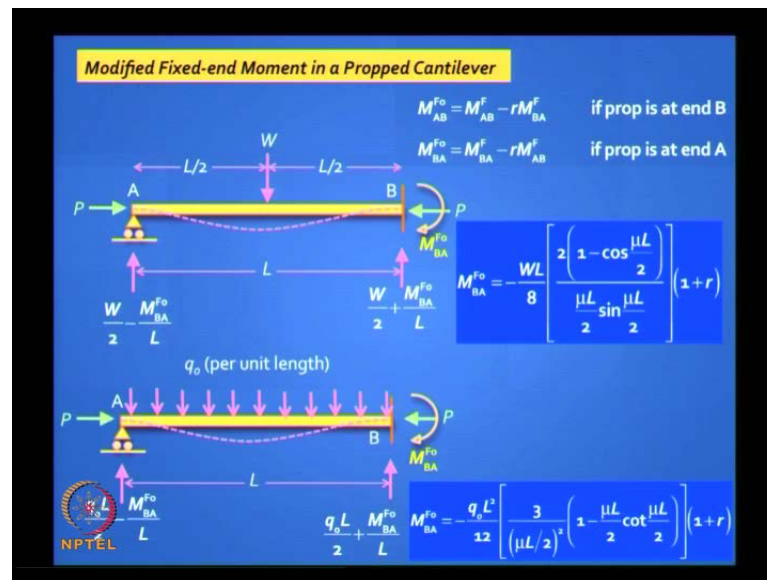
Now, if you have a force P acting, that fixed-end moment gets affected. The default value is WL by 8. You know that default value is WL by 8. Left side will be anticlockwise; right side will be clockwise. That gets multiplied by a factor, which you can see in those box brackets, which is the function of mu L. mu L is P L squared by EI; square root of that. So, we have a readymade formula. There is a derivation for this (Refer Slide Time: 10:16). It is all described in the book, but let us just use the formula when we need to do it. If P is tensile, there is a slight modification; the hyperbolic functions come into play.

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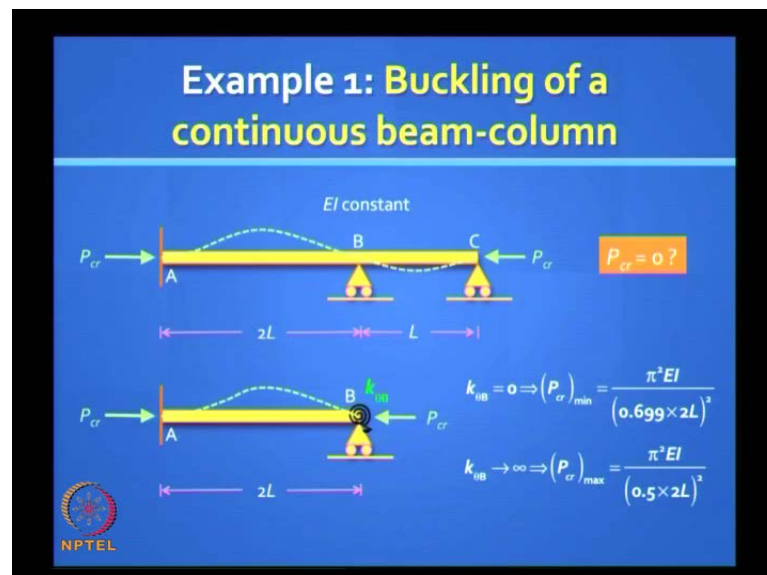
We look at one more case. The standard case is uniformly distributed load. You have similar sets of equations. Is it clear? That is all that we need to know for the time being, but you can derive for any arbitrary loading. Just go back here, the default value is q naught L squared by 12 multiplied by a certain factor.

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If you have a propped cantilever, you have to do the correction as we did earlier. In the conventional procedure, we had to take half and pass it on to the other end. Here, we do not take half, we take r because r is the carryover factor. So, you have to do something similar and you can work out those expressions in case you need to invoke these.

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Now, let us do some problems. I want to quickly show you how the slope deflection method can be used to find the elastic instability problem to solve the problem. That means let us take a problem like this; a two span continuous beam column. First, I want

to know if I were to apply an axial compressive load, what is a maximum load it can take? It is going to buckle. Can you give me some guesses? At least, can you tell me what that value will lie in between? I want to find P critical. Mind you: Both the elements are going to simultaneously buckle because of the continuity requirement at the joint B.

Yes, tell me, how do you arrive logically at a good guess?

[Not audible] (Refer Slide Time: 12:13)

You look at them independently. You have got two beam columns A B and B C. Which do you think is stronger of the two?

B C is stronger.

B C is stronger and shorter in length. So, you better look at A B. For A B, what are the 2 limiting; what is the lower bound and upper bound for P critical? What is the lower bound condition? What is the actual condition? If I isolate A B, what should I put at B? If I take out A B, A is fixed against translation rotation.

Roller and spring.

Roller? Yes, roller and spring; that is the picture I should draw. Now, it is easier for you to understand. The only variable here is $k_{\theta B}$. That is the rotational stiffness of that spring, which represents B C. What are its extreme values?

2 (())

No, of the stiffness of this spring? (Refer Slide Time: 13:14)

0 and infinity.

Fixed (())

So, if it is zero, what is P critical?

0.7

What is the effective length?

0.7 or more accurately 0.699.

If k is infinite, 0.5; that is it. You are good. So, 0.699 and the length here is $2L$. The other value is 0.5; that is all you have to remember; very easy. Got it? So, you have got a good guess. Your final effective length for that element must lie between 0.5 and 0.699. If you get something outside, something is wrong. So, you have got a good **hunch**. Incidentally, if you look at the element B C, it will give you only values, which are higher; you can work it out. So, you have made the right judgment by selecting A B to work out the bounds.

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The slide shows a beam element ABC with a horizontal span of $2L$ from A to B and L from B to C. A horizontal axial load P_{cr} is applied at both ends A and C. The beam has a constant flexural rigidity EI . The diagram shows the beam in its undeformed state (solid line) and deformed state (dashed line). Below the diagram, the following equations are derived:

$$\sum M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0 \Rightarrow (S_{BA} + S_{BC}^o)\theta_B = 0$$

$$S_{BA} + S_{BC}^o = 0$$

$$\Rightarrow \frac{S_{BA}}{EI/2L} \left(\frac{1}{2} \right) + \frac{S_{BC}^o}{EI/L} = 0$$

$$\Rightarrow g(P_{cr}) = \frac{1}{2} \left(\frac{S}{EI/L} \right)_{BA} + \left(\frac{S^o}{EI/L} \right)_{BC} = 0$$

Two boxed formulas for the stiffness terms are provided:

$$S = \frac{EI(\mu L)}{L} \left(\frac{\sin \mu L - \mu L \cos \mu L}{2 - 2 \cos \mu L - \mu L \sin \mu L} \right)$$

$$S^o = \frac{EI}{L} \left(\frac{(\mu L)^2 \tan \mu L}{\tan \mu L - \mu L} \right) = S(1 - r^2)$$

A yellow box indicates the solution method: "Solution by bisection method".

Initial bounds of $\mu_{cr} L = \sqrt{\frac{P_{cr} L^2}{EI}} = \frac{\pi}{k_*}$ are given as:

$$0.699 < (\mu_{cr} L)_{BA} < \frac{\pi}{0.5} \Rightarrow (\mu_{cr} L)_{BC} = \frac{1}{2} (\mu_{cr} L)_{BA}$$

Now, let us solve this problem. You can use slope deflection method to solve this problem. How do you do that? Theta B is your single unknown. **Write down the slope deflection equations.** Write down the equilibrium equation directly; M_{BA} plus M_{BC} must be 0.

What is M_{BA} ? Normally, you would have written $4EI$ by L . Now, you will write as S into EI by L for that element **A B S**. For the element B C, you will write S naught instead of writing $3EI$ by L . No r comes here; r goes to the joint A. So, is this **OK**, the equations that I have written? M_{BA} is S_{BA} into theta B. There is no loading on that beam except for axial load P . M_{BC} is S naught B C into theta B. The total moment must add up to 0; that is equilibrium; that is all you have to do.

Now, you have to write S_{BA} and S_{BC} in terms of $P_{critical}$ or μL . You have to do it in a clever way (Refer Slide Time: 15:26). First, somehow you have to bring in μ into the picture. If you divide throughout by EI by L , you can get a transcendental equation, g in terms of $P_{critical}$. Does that make sense to you? I divide that equation by EI by L . Can I get this equation? I need to solve this equation. How do I solve this equation? I have those trigonometric functions, stability functions for S and S_{naught} as the function of μ into L . So, I have to bring μL into the picture. So, I do that. So, these are my equations (Refer Slide Time: 16:18). I have to bring μL into the picture and I can use the bisection method.

The initial bounds as you rightly said are μL is $P_{critical} L^2$ by EI , which I can write as $\pi^2 EI$ by L^2 because $P_{critical}$ can be written in that fashion. What is Euler buckling load? $\pi^2 EI$ by L^2 ; if you plug that in, it will look like that. Is it not? L^2 squared for any boundary conditions. So, it is very convenient. You identified 0.699 and 0.5 for element $B A$. For element $B C$, can we still use $B A$? Yes, we can because the two spans are related. So, for the second element, it is half the value as that of the first element. Does it make sense? It is all very logical.

I can choose any value of μL and plug it into that equation. It should give me a value of 0 if I have got the correct solution. Now, how do you solve this equation? You can do trial and error, but you have to do a lot of trial and error. Is there a systematic way of solving that equation? Have we studied? These are numerical methods.

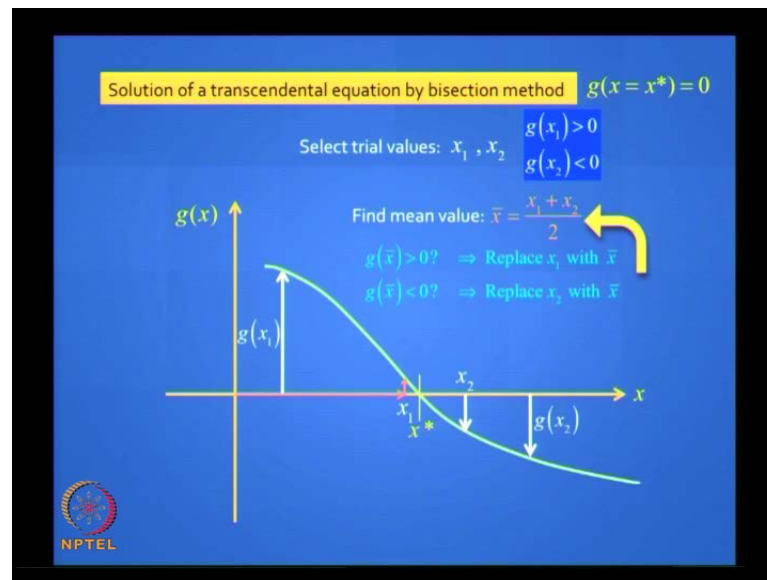
[Not audible] (Refer Slide Time: 17:54)

Newton-Raphson method will involve differentiation; it is going to be quite a mess.

Fourth order Runge-Kutta.

Runge-Kutta; some simple method, which works pretty well for any transcendental equation. Have you heard of the bisection method? No?

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Let us look at the bisection method. Instead of drawing on the board, I have got a slide; I worked out this morning to help you out. Let us say I have to solve any equation, g of x equal to zero; any equation. So, we are looking at the mathematical technique. Let us say where it is 0, x is equal to x star. So, let us draw a shape; let us say it looks like this. You are in trouble if there are many solutions. So, you must trap that region where you have got a single solution.

Now, take two values of x star, which will trap that solution. So, you need a lower value and a higher value. We have already got the lower bound and upper bound. So, let us choose those two values. If you plug in one of the values, you should get a positive value of the function. If you plug in the second value, you should get a negative value of the function. Let us say that the value, which gives the positive value is x_1 and the other one is x_2 . So, does this make sense? This is x_1 and this is x_2 .

Now, you tell me what is a clever way of getting x star?

[Noise – not audible] (Refer Slide Time: 19:24)

Yes, you bisect that interval between x_1 and...

Sir, this is Newton Raphson (())

Take the average of it.

This is Newton Raphson.

This is not Newton Raphson (())

In Newton Raphson method, you have to look at the slope.

It will be drawn tangentially (())

This is a crude method, but it works very well. So, you bisect that interval; that is, find the average value. So, that is what you do. Then, what should you do? You plug in that value and figure out whether it is positive or negative. If it is positive, you replace x_1 with this new value; if it is negative, replace... Then, what do you do next?

Then, again bisect.

Then, put it on a do loop. Then, you keep doing it till?

[Noise – not audible] (Refer Slide Time: 20:11)

Till you convert. You have to decide when it converts. So, you have to apply tolerance and decide. That is all; very simple.

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Trial	$(\mu_{cr} L)_{BA}$	$\left(\frac{S}{EI/L}\right)_{BA}$	$\left(\frac{S^o}{EI/L}\right)_{BC}$	$g(P_{cr}) = \frac{1}{2} \left(\frac{S}{EI/L}\right)_{BA} + \left(\frac{S^o}{EI/L}\right)_{BC}$
	4.49	0.010	1.804	+1.809
	6.28	-1971.6	0.005	-985.8
1	5.385	-4.486	1.101	+0.680
2	4.938	-1.613	1.487	-6.984
3	5.162	-2.808	1.303	-0.101
4	5.050	-2.165	1.397	+0.315
5	5.106	-2.474	1.351	+0.114
6	5.134	-2.637	1.327	+0.009
7	5.137	-2.655	1.325	-0.003
8	5.136	-2.649	1.326	+0.001

$$\Rightarrow (\mu_{cr} L)_{BA} = 5.136 = \frac{\pi}{0.6117} \Rightarrow P_{cr} = \frac{\pi^2 EI}{(0.6117 \times 2L)^2} = \frac{6.595 EI}{L^2}$$

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$$\text{Also, } (\mu_{cr} L)_{BC} = \frac{5.136}{2} = \frac{\pi}{1.223} \Rightarrow P_{cr} = \frac{\pi^2 EI}{(1.223L)^2} = \frac{6.595 EI}{L^2}$$

We do that here. We have the first two values of $\mu_{cr} L$ BA. Remember: We said π by 0.5. π by 0.5 is 2π ; 6.29; that is the second value. The other one was?

Pi by 0.666

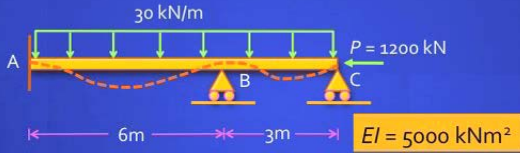
That gives you 4.49. So, you can do this exercise. Can you see **that** we have got a **pre-decent** convergence, where that function is nearly 0; on the right-hand side is the function. So, you can stop here. If you are doing manually, you can do it here, but if you want a higher tolerance, you can go further; that is it. Clear? That is a fastest; coolest way of solving of this problem.

If you want to do Newton-Raphson method, go ahead, but it is not going to be easy. You have to find the derivative of that function; that is a big problem. You may not be able to it also.

Now, you plug in those values and pull out P critical. You can use it either for element A B or for element B C, you will get the same answer. The answer is 6.595 EI by L squared, which gives you an effective length k e of 0.6117, which lies between 0.5 and 0.699. Got it? That is it. So, we use slope deflection method to solve a buckling problem. Clear?

(Refer Slide Time: 22:14)

Example 2: Second-order analysis of a continuous beam-column



30 kN/m

A B C

6m 3m

$P = 1200 \text{ kN}$

$EI = 5000 \text{ kNm}^2$

Analyse the structure and draw the bending moment diagram (including second-order effects) for the following two cases:
(a) P is compressive and (b) P is tensile.

Using the result of Example 1, $P_{cr} = \frac{6.595EI}{L^2} = \frac{6.595(5000)}{(3.0)^2} = 3663.9 \text{ kN}$

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Next, we will take the same problem, put some load on it. Now, it is a conventional two span continuous beam problem, but it is also got an axial load. We want to do second order analysis; you will get a good feel of it. You can do the conventional slope

deflection method, which we studied, but that cannot account for the reduction in flexural stiffness due to the axial compression.

Now, we will take two cases. One, axial compression of a fairly high magnitude. Obviously, that axial compression should not exceed the buckling load. So, first, you calculate the buckling load. From the previous example, P critical, plug in the value of EI that is given in this problem and plug in the value of the length. It turns out to be 3664 kilonewton. The actually applied load is only 1000. So, you can apply that load, but it is definitely going to affect the flexural stiffness; $4EI$ by L , $3EI$ by L will go for a toss. It will affect differently depending on whether the force applied is compressive or tensile. Does it make sense to you? Here, you cannot do the old slope deflection method, the old matrix method, **but** you have to look at this problem as a beam column problem. Clear?

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The slide illustrates a beam-column problem. A horizontal beam of total length 9m is supported by a pin at A and a roller at C. A uniformly distributed load of 30 kN/m acts downwards over the entire length. A point load $P = 1200$ kN acts to the right at the free end B. The distance between A and B is 6m, and between B and C is 3m. The beam is subjected to an axial compressive load $P = 1200$ kN. The degree of indeterminacy is 1, with the unknown displacement being θ_B .

Slope-deflection Equations:

$$M_{AB} = M_{AB}^F + (rS)_{AB} \theta_B$$

$$M_{BA} = M_{BA}^F + S_{AB} \theta_B$$

$$M_{BC} = M_{BC}^F + S_{BC} \theta_B$$

Equilibrium Equation:

$$M_{BA} + M_{BC} = M_B = 0$$

Stiffness Measures for Beam-Column Elements:

NPTEL $\frac{P}{EI} = \frac{1200}{5000} = 0.4472 \text{ m}^{-1} \Rightarrow \begin{aligned} \left(\frac{uL}{EI}\right)_{AB} &= (0.4472)(6) = 2.6833; \\ \left(\frac{uL}{EI}\right)_{BC} &= (0.4472)(3) = 1.3416 \end{aligned}$

Let us proceed. Take that problem. Degree of indeterminacy is still the same kinematic indeterminacy θ_B . Write down the slope deflection equations. Those are very easy to write. M_{AB} is M_{AB}^F . Normally, you would have written $2EI$ by L θ_B , but now, you will write r into S for the element $A B$. For $B A$, M_{BA}^F plus $S_{AB} \theta_B$; you can write S_{AB} or S_{BA} ; it does not matter, it is for the same element; into θ_B .

However, for $B C$, be careful because the far end is hinged and you want to take advantage of it. So, you will write it as M_{BC}^F for a propped cantilever; plus S

naught B C into theta B. So, the naught is to remind you that there is a hinge at the other end. That is why you put a naught on top. Clear? That is the equation.

What is the equilibrium equation? The same old equilibrium equation, M BA plus M BC is equal to 0. Plug it all in. Now, the value of mu is a known value because P is known. So, P is given as 1200. In this problem, I have taken as 1200 kilonewton; not 1000. P by EI is known. Plug in that value. You have got two values: mu L for A B and mu L for B C. So, these are known values. You do not have to do bisection method for these. Now, just plug in this value. This is the effect that is going to happen on the flexural stiffness.

(Refer Slide Time: 25:23)



(a) *P* is compressive:

$$S_{AB} = \frac{EI}{L} \left[\frac{\mu L (\sin \mu L - \mu L \cos \mu L)}{2 - 2 \cos \mu L - \mu L \sin \mu L} \right] = \frac{EI}{6} [2.93279] = 0.4888EI$$

$$r_{AB} = \left[\frac{\mu L - \sin \mu L}{\sin \mu L - \mu L \cos \mu L} \right] = 0.78660 \Rightarrow (rS)_{AB} = 0.3845EI$$

$$S_{BC}^o = \frac{EI}{L} \left[\frac{(\mu L)^2 \tan \mu L}{\tan \mu L - \mu L} \right] = \frac{EI}{3} [2.61989] = 0.8733EI$$

(b) *P* is tensile:

$$S_{AB} = \frac{EI}{L} \left[\frac{\mu L (-\sinh \mu L + \mu L \cosh \mu L)}{2 - 2 \cosh \mu L + \mu L \sinh \mu L} \right] = \frac{EI}{6} [4.88132] = 0.8136EI$$

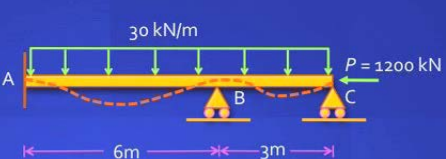
$$r_{AB} = \left[\frac{-\mu L + \sinh \mu L}{-\sinh \mu L + \mu L \cosh \mu L} \right] = 0.36965 \Rightarrow (rS)_{AB} = 0.3007EI$$

$$S_{BC}^o = \frac{EI}{L} \left[\frac{(\mu L)^2 \tanh \mu L}{-\tanh \mu L + \mu L} \right] = \frac{EI}{3} [3.34283] = 1.1143EI$$

If you plug in those values, you have those equations. You do not need to interpolate because we have nice calculators with us; or, you can put a spreadsheet. You will get those answers very quickly. If *P* is compressive, you can get those stiffness measures. If *P* is tensile, you will get those stiffness measures; just plugging into those trigonometric functions. You can have your own algorithm in matlab, where all these functions are readily available. So, you just have to press a button and you will get that number very easily.

(Refer Slide Time: 26:04)

Fixed End Moments
(with $\theta_B = 0$):



Case (a) : P is compressive

$$M_{AB}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(1 - \frac{\mu L}{2} \cot \frac{\mu L}{2} \right) \right] = -\frac{(30)(6^2)}{12} [1.14510] = -103.06 \text{ kNm}$$

$$M_{BA}^F = +103.06 \text{ kNm}$$

$$M_{BC}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(1 - \frac{\mu L}{2} \cot \frac{\mu L}{2} \right) \right] \left(1 + \frac{\mu L - \sin \mu L}{\sin \mu L - \mu L \cos \mu L} \right)$$

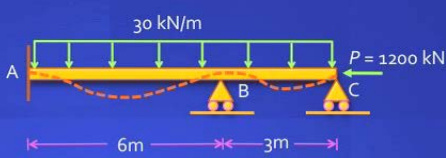
$$= -\frac{(30)(3^2)}{12} [1.03134] [1.54967] = -35.96 \text{ kNm}$$

NPTEL

However, take a look. It should look close to 4 EI by L, 2 EI by L and so on. Fixed-end moments, again plug in the formulas that we just looked at. Be careful and compare them with the normal fixed-end moment; they should not be too far away. So, you have to do it for the propped cantilever using a different formula; you know what to do.

(Refer Slide Time: 26:27)

Fixed End Moments
(with $\theta_B = 0$):



Case (b) : P is tensile

$$M_{AB}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(-1 + \frac{\mu L}{2} \coth \frac{\mu L}{2} \right) \right] = -\frac{(30)(6^2)}{12} [0.89744] = -80.77 \text{ kNm}$$

$$M_{BA}^F = +80.77 \text{ kNm}$$

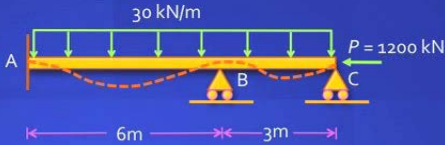
$$M_{BC}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(-1 + \frac{\mu L}{2} \coth \frac{\mu L}{2} \right) \right] \left(1 + \frac{\sinh \mu L - \mu L}{\mu L \cosh \mu L - \sinh \mu L} \right)$$

$$= -\frac{(30)(3^2)}{12} [0.97123] [1.45889] = -31.88 \text{ kNm}$$

NPTEL

We are not here to actually solve many problems. Next, when P is tensile, you will get another set of equations. Just look at the fixed-end moments.

(Refer Slide Time: 26:32)



Fixed End Moments
(with $\theta_B = 0$):

Case (a) : P is compressive

$$M_{AB}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(1 - \frac{\mu L}{2} \cot \frac{\mu L}{2} \right) \right] = -\frac{(30)(6^2)}{12} [1.14510] = -103.06 \text{ kNm}$$

$$M_{BA}^F = +103.06 \text{ kNm}$$

$$M_{BC}^F = -\frac{q_0 L^2}{12} \left[\frac{3}{(\mu L/2)^2} \left(1 - \frac{\mu L}{2} \cot \frac{\mu L}{2} \right) \right] \left(1 + \frac{\mu L - \sin \mu L}{\sin \mu L - \mu L \cos \mu L} \right)$$

$$= -\frac{(30)(3^2)}{12} [1.03134] [1.54967] = -35.96 \text{ kNm}$$

NPTEL

When it is compressive, the first one is 103.06. When it is tensile, it is 80.77 (Refer Slide Time: 26:38). So, there is a big difference whether it is compressive or tensile. Also, the magnitude of the force. If it is without any force, actual force is going to be? Let us work it out; $q_0 L^2$ by 12. Can you do that?

30 into 6 squared by 12 works out to?

90

90. So, you see that the default value is in between 80.77 and 103.06. So, that is the kind of variation you can get in fixed-end moment. Got it?

Let us proceed.

(Refer Slide Time: 27:24)

Slope-deflection Equations:

Beam AB ($L = 6\text{m}$; $\theta_A = 0$; $\theta_B = ?$; $\phi_{AB} = 0$)

$$M_{AB} = M_{AB}^F + (rS)_{AB} \theta_B = \begin{cases} (-103.06 + 0.3845EI\theta_B) \text{ kNm (Case'a')} \\ (-80.77 + 0.3007EI\theta_B) \text{ kNm (Case'b')} \end{cases}$$

$$M_{BA} = M_{BA}^F + S_{AB} \theta_B = \begin{cases} (+103.06 + 0.4888EI\theta_B) \text{ kNm (Case'a')} \\ (+80.77 + 0.8136EI\theta_B) \text{ kNm (Case'b')} \end{cases}$$

Beam BC ($L = 3\text{m}$; $\theta_B = ?$; θ_C not reqd ; $\phi_{BC} = 0$)

$$M_{BC} = M_{BC}^F + S_{BC}^* \theta_B = \begin{cases} (-35.96 + 0.8733EI\theta_B) \text{ kNm (Case'a')} \\ (-31.88 + 1.1143EI\theta_B) \text{ kNm (Case'b')} \end{cases}$$


Write down the slope deflection equations. You can do it in one go. Case A, case B will handle together because the equations do not change; it is only those values, which change.

(Refer Slide Time: 27:41)

Equilibrium Equation: $M_{BA} + M_{BC} = M_B = 0$

Case (a) : P is compressive $67.10 + 1.3621EI\theta_B = 0 \Rightarrow EI\theta_B = -49.2622 \text{ kNm}$

Case (b) : P is tensile $48.89 + 1.9279EI\theta_B = 0 \Rightarrow EI\theta_B = -25.3592 \text{ kNm}$

Final End Moments:

Case (a) : P is compressive

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -103.06 \\ +103.06 \\ -35.96 \end{Bmatrix} + \begin{Bmatrix} 0.3845 \\ 0.4888 \\ 0.8733 \end{Bmatrix} \{EI\theta_B = -49.2622\} = \begin{Bmatrix} -122.00 \\ +78.98 \\ -78.98 \end{Bmatrix} \text{ kNm}$$

Case (b) : P is tensile

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -80.77 \\ +80.77 \\ -31.88 \end{Bmatrix} + \begin{Bmatrix} 0.3007 \\ 0.8136 \\ 1.1143 \end{Bmatrix} \{EI\theta_B = -25.3592\} = \begin{Bmatrix} -88.33 \\ +60.14 \\ -60.14 \end{Bmatrix} \text{ kNm}$$

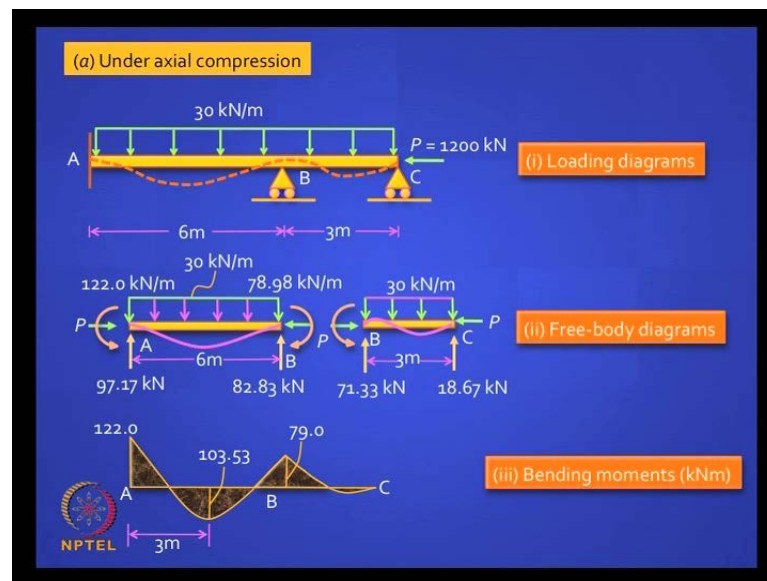

Then, write down the equilibrium equation, which is the same. Solve for the two cases: compressive and tensile. You see that there is a big difference in the solution. Theta B is almost half in one case. In which case will you get less theta?

When it is tensile.

When it is tensile. You can feel it, it is going to deflect less. Everything will be less. You will have a much stiffer structure when you have a axial tension, but if it is compressive it makes things worse. If you did not have tension or compression, you will probably get a value in between; exactly midway. Got it? Does it make sense?

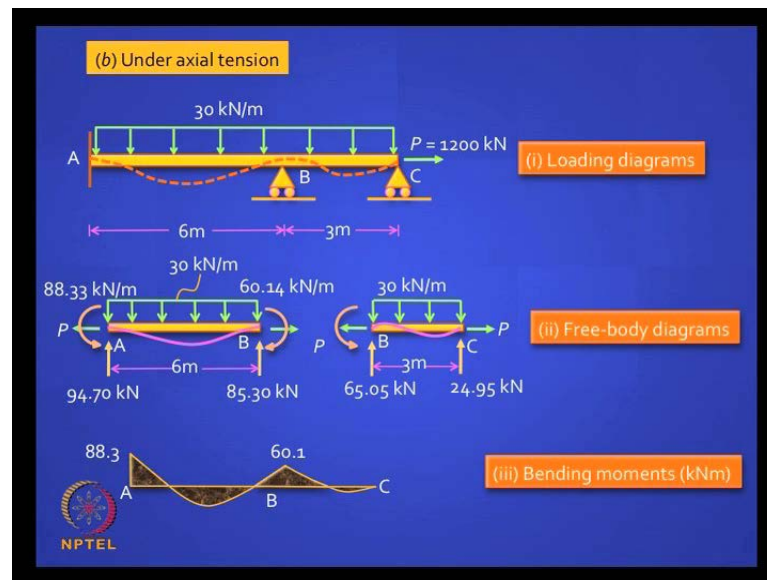
Then, you plug in the values of $EI\theta$. This is convention slope deflection method. Get the final moments; that is it.

(Refer Slide Time: 28:33)



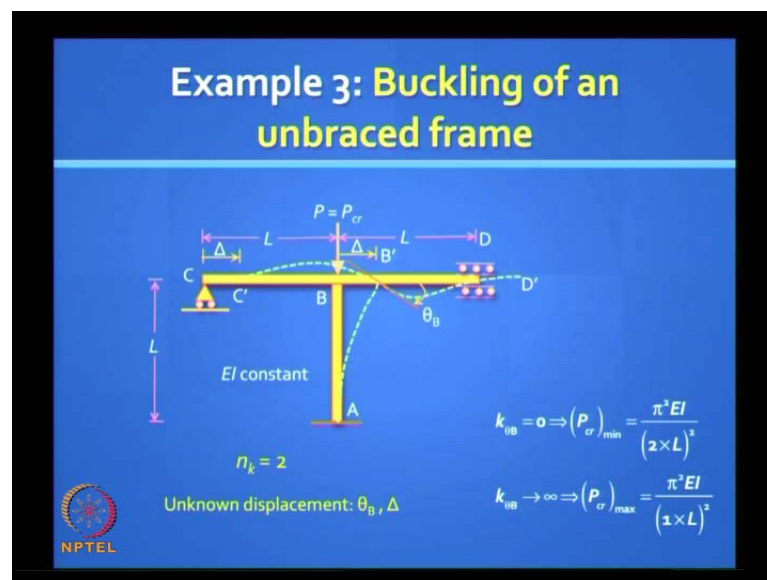
Then, draw the free bodies, bending moment diagram. In the bending moment diagram, you have to be little careful because for the mid-span moment, if you really want to do second order analysis, you need the deflection at mid-span. So, there is a formula you need for that. It is there in the book. If you invoke that formula, only then you can get the $P\delta$ value at the mid-span. However, you get the support moments here pretty accurately.

(Refer Slide Time: 29:03)



Similarly, you can do the axial tension. You will see that the moments are much less when you have axial tension as you would expect.

(Refer Slide Time: 29:14)



Let us take another problem. Let us take buckling of a frame now. We are switching from continuous beam to frame. So far so good; you are getting some idea of what we are talking about. Now, let us guess the solution. Now, you have a t-shaped frame, which is fixed at the base A, but mind you that top beam can roll. So, it is unbraced; it can

move sideways; it has got rollers. Can you guess the value of P critical, the low bound and the upper bound?

Here also, you can put a spring. Here, you have two degrees of freedom (Refer Slide Time: 30:05). You have two degrees of freedom: θ and δ . If you want to see how it deflects, it is going to deflect probably something like that.

[Not audible] (Refer Slide Time: 30:17) 0.7 and 2

Sir, 2.

0.7 is wrong.

It must be 2, sir.

Relax. You isolate that column. The column is a cantilever. The top can roll. So, it is a cantilever, but you can have some rotational fixity. So, let us take the case where the rotational fixity is 0. The rotational spring there is 0. Then, it is a simple cantilever. So, what is your k effective? 2.

Cantilever is 2. Let us see that it is fully fixed against rotation.

And guided.

And guided. How much will it be?

1

That is it. 1 and 2. No 0.7 here. Got it? You have to just use your brains. First of all, you should know that in an unbraced situation, you will never get a k_e value less than 1. So, 0.7 is ruled out; it has to be greater than or equal to 1. Clear? So, that is a good guess.

(Refer Slide Time: 31:30)

Slope-deflection equations:

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{BD} \end{Bmatrix} = \begin{bmatrix} (rS)_{AB} & -S_{AB}(1+r_{AB}) \\ S_{AB} & -S_{AB}(1+r_{AB}) \\ 3EI/L & 0 \\ 4EI/L & 0 \end{bmatrix} \begin{Bmatrix} \theta_B \\ \Delta/L \end{Bmatrix}$$

Equilibrium equations:

$$\sum M_B = 0 \Rightarrow M_{BA} + M_{BC} + M_{BD} = 0$$

$$\sum F_H = 0 \Rightarrow \frac{M_{BA} + M_{AB} + P\Delta}{L} = 0 \Rightarrow M_{BA} + M_{AB} + \frac{EI(\mu L)_{AB}^2}{L} \frac{\Delta}{L} = 0$$

Let us solve the problem. I want to ask you even before I go with the solution. You can write the slope deflection equations the same old way. You can write the equilibrium equations: you have a **moment** equilibrium and shear force equilibrium. Now, you have two equations. How do you find the critical buckling load? What kind of bisection method can you use now? Take a more generic case. I have degree of indeterminacy of 10: theta A, theta B, theta C, delta 1, delta 2, whatever.

I have got 10 equilibrium equations. How do I find the critical buckling load because it is the same buckling load, which is running through all the elements? I mean the mode shape will ensure that every beam and every column is going to buckle. This is global buckling; not local buckling. So, how do I get it? Any ideas? I will give you a clue. You can write the equations always in a matrix form. How do you get the solution?

Even in the previous case, did you realize that you got the solution in terms of theta B? You had something into theta B equal to 0. So, either theta B is 0 is one solution, which is called a trivial solution or the other quantity, which is g of P critical. So, it has to be an Eigen value problem. You are dealing with the ideal columns. Critical buckling load has to be an Eigen value problem. Now, how do you solve this problem?

(Refer Slide Time: 33:38)

$$\begin{bmatrix} S_{AB} + \frac{7EI}{L} & -S_{AB}(1+r_{AB}) \\ -S_{AB}(1+r_{AB}) & 2S_{AB}(1+r_{AB}) - \frac{EI(\mu L)^2}{L} \end{bmatrix} \begin{Bmatrix} \theta_B \\ \Delta/L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \left(\frac{EI}{L}\right) \begin{bmatrix} \left(\frac{S}{EI/L}\right)_{AB} + 7 & -\left(\frac{S}{EI/L}\right)_{AB}(1+r_{AB}) \\ -\left(\frac{S}{EI/L}\right)_{AB}(1+r_{AB}) & 2\left(\frac{S}{EI/L}\right)_{AB}(1+r_{AB}) - (\mu L)^2 \end{bmatrix} \begin{Bmatrix} \theta_B \\ \Delta/L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For a non-trivial solution, the determinant of the stiffness matrix must vanish (implying instability in the system). This condition yields the *characteristic equation*, whose roots provide the eigenvalues (critical buckling load), of which the lowest value (P_{cr}) is of practical interest.

$$g(P_{cr}) = \left[\left(\frac{S}{EI/L}\right)_{AB} + 7 \right] \left[2\left(\frac{S}{EI/L}\right)_{AB}(1+r_{AB}) - (\mu_{cr} L)^2 \right] - \left[\left(\frac{S}{EI/L}\right)_{AB}(1+r_{AB}) \right]^2 = 0$$

Initial bounds of $\mu_{cr} L = \sqrt{\frac{P_{cr} L^2}{EI}} = \frac{\pi}{k_e} : \frac{\pi}{2} < (\mu_{cr} L)_{BA} < \frac{\pi}{1}$ Solution by bisection method

Let me take you to one more step. This is what the equation is going to look like. You have theta B; you have delta by L. How do I get the solution for P critical? How do I get the characteristic equations?

[Noise] (Refer Slide Time: 33:59)

Minus lambda i and then (())

Do not bring lambda and all into the picture. This is very simple. I want the lowest. I do not want all the Eigen values. I do not want all the critical; I am a practical person, I realize that the lowest load will make the damn thing, but...

(())

What is an easy way of doing it? It is a property of this matrix.

Determinant

I can pullout the determinant and the determinant should be 0. That is the characteristic of an Eigen value problem. That lambda has already come into P critical. So, that is what you should do. So, anyway, you can write it in terms of EI by L; by dividing everything EI by L. So, for a non trivial solution, the determinant of the stiffness matrix must vanish. Remember: That is when there is no flexural stiffness left in that structure. It is

going to just **go on yielding**; go on buckling. So, this gives you instability. This condition gives you the characteristic equation and the solution of that.

The lowest Eigen value of that is what we look for. So, write down the determinant, which is easy to write down. This is now the (Refer Slide Time: 35:34) equation. You need a lower bound and upper bound; you have already identified. 2 and 1 was k_e ; solved by bisection method.

(Refer Slide Time: 35:45)

Trial	$(\mu_{cr} L)_{AB}$	$\left(\frac{S}{EI/L}\right)_{AB}$	$\left(\frac{r}{EI/L}\right)_{AB}$	$g(P_{cr})$
	1.57	3.6601	0.5707	+63.24
	3.14	2.4692	0.9990	-24.25
1	2.355	3.1995	0.6946	+24.64
2	2.747	2.8750	0.8090	+1.152
3	2.551	3.0463	0.7453	+13.18
4	2.845	2.7821	0.8472	-5.044
5	2.698	2.9196	0.7916	+4.298
6	2.771	2.8527	0.8179	-0.355
7	2.766	2.8574	0.8160	-0.040
8	2.765	2.8584	0.8157	+0.023

$$\Rightarrow (\mu_{cr} L)_{AB} = 2.765 = \frac{\pi}{1.1362}$$

$$\Rightarrow (P_{AB})_{cr} = \frac{\pi^2 EI}{(1.1362 \times L)^2} = P_{cr} \Rightarrow P_{cr} = 7.645 \frac{EI}{L^2}$$

You now know the bisection method. You get the answer; I would not waste time. You have got the answer. Check. Did you get a value of k_e between 1 and 2? Yes, you did. The fact that it is close to one suggests that the beam is pretty flexible. The highest it can go is to 2.

(Refer Slide Time: 36:06)

Example 4*: Second-order analysis of an unbraced frame

$P = 4EI/L^2$ $L/2$ $L/2$ $P = 4EI/L^2$

L L L

EI constant

* Reading Assignment

You can do one more problem. We would not do it. Second order effects for the same structure. Once you have got the P critical, ... It is done in the book.

(Refer Slide Time: 36:22)

Example 5: Buckling of a rigid-jointed portal frame

P P

$(EI)_b$ $(EI)_c$ $(EI)_c$

L L L

$\gamma = \frac{(EI)_b}{(EI)_c} = 0.5, 1, 5, 10$

Calculate the critical buckling load P_{cr} for typical values of γ (equal to 0.5, 1, 5, and 10), considering the frame to be (a) braced and (b) unbraced against sidesway. Hence, plot the variations of P_{cr} (normalised with respect to P_{e0}) with γ .

In fact, many problems are done in the book, but I am going to end with just one last problem, which is a good problem to look at.

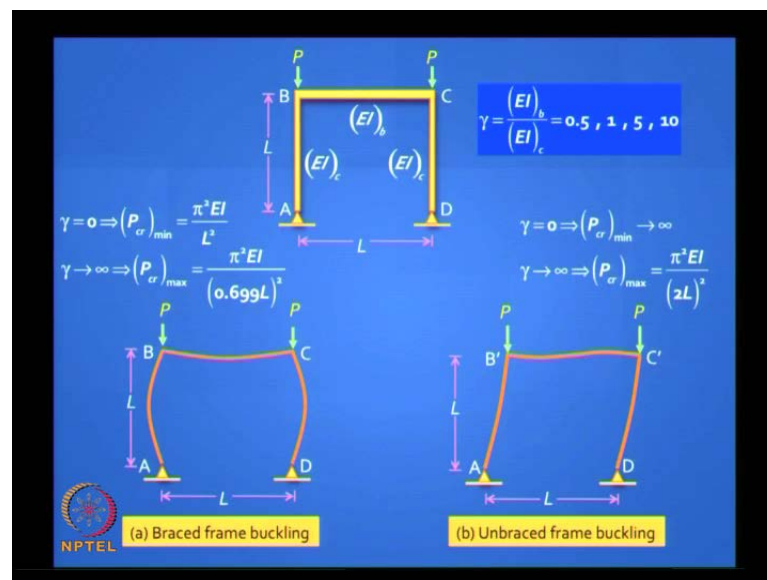
Now, I have a portal frame; rigid jointed portal frame. I want to look at two conditions: the frame is braced by other elements in the building and the frame is unbraced. I get two different solutions. Also, the relative stiffness of beam to column is crucial. So, I bring in

a parameter gamma, which is EI of the beam to EI of the column. Let us look at different values of gamma: 0.5, 1, 5, 10. Let us say EI b by EI c gamma is equal to 0. What does it mean?

Completely rigid body.

That means the beam is very flexible. So, those two columns are going to... Cantilever is not going to be stable. It is like having hinges there at B and C; it is going to collapse. That is one extreme, if it is unbraced.

(Refer Slide Time: 37:38)



If it is braced... Let us look at that. Let us look at the 2 modes of failure. If it is braced, how do you think it will buckle? Braced means no sway, how do you think it will buckle?

[Noise – not audible] (Refer Slide Time: 37:49)

It will buckle like that. You are doing like that.

Tell me what is P critical going to be bounded between? Two extreme values of P critical? What is k effect?

K effect is 1.

One is 1; one limit. The other limit?

0.7

0.7. Good. what about this case? (Refer Slide Time: 38:24). This is sway. These are conventional sway picture. What are the two limits here?

[Noise – not audible] (Refer Slide Time: 38:29)

Guided is 1 and...

What is the lower bound?

1

Lower bound is 1 and the upper bound?

2

Why is the lower bound 1?

[Not audible]

No. It is a cantilever because (())

It is going to behave like a cantilever.

Guided roller; it is guided and it is pinned here.

Guided roller.

It is a cantilever.

It is effectively a cantilever. We have looked at these things. know, if it is hinged here and can move there, the deflected shape is that of a cantilever. You have to think. If you can think, let us hope you can. So, these are the values which you correctly said. If it is **hinged**, it is going to be infinite. It is an unstable structure. So, it is going to be 2 and 0.

(Refer Slide Time: 39:40)

Case (a) : Frame braced against sidesway

Symmetric frame half-frame model

There is only one degree of freedom involved here, viz., the rotation θ_B

$$\sum M_B = 0 \Rightarrow M_{BA} + M_{BE} = 0 \Rightarrow \left(S_{BA}^0 + \frac{(EI)_b}{L/2} \right) \theta_B = 0 \Rightarrow S_{BA}^0 + \frac{2(EI)_b}{L} = 0$$

neglecting the marginal effect of axial force in the beam BE

NPTEL

$\gamma = \frac{(EI)_b}{(EI)_c}$
 $\gamma = 0 \Rightarrow (P_{cr})_{min} = \frac{\pi^2 EI}{L^2} = P_{E0}$
 $\gamma \rightarrow \infty \Rightarrow (P_{cr})_{max} = \frac{\pi^2 EI}{(0.699L)^2} = 2.047 P_{E0}$

Let us plug in those values. Let us look at the frame brace **against** side sway. How do you solve it? You can take half the frame, why should you take the full frame? Work with half the frame. What is the degree of indeterminacy?

[Not audible] (Refer Slide Time: 39:54)

theta B. **One**; Write down the equation. Now, we make an assumption here. We are saying that we worry about axial compression only in element A B, we do not worry about in B C. **This is** because if you worry about it in B C, then it is a more complicated problem. Why do we not worry about it in B C? Because it is going to be very small. So, these are the intelligent shortcuts that you can take. Only where you have high axial compression, you worry about treating that element as a beam column element. Is it clear?

With that assumption, do you agree that the equation will be as shown there? (Refer Slide Time: 40:46) M_{BA} is S_{BA} theta B and M_{BE} is $\frac{EI_b}{L}$ theta B. In this case, L is L by 2. It would have not be in $\frac{EI_b}{L}$ by 2 if you had axial compression; will have that **S tilde**. So, that is tricky.

(Refer Slide Time: 41:14)

Characteristic equation: $S_{BA}^2 + \frac{2(EI)_B}{L} = 0$

Dividing throughout by $(EI)_C$ and substituting $\gamma = \frac{(EI)_B}{(EI)_C}$

$$g(P_{cr}) = \left(\frac{S}{EI/L} \right)_{BA} + 2\gamma = 0$$

Initial bounds of $\mu_{cr} L = \sqrt{\frac{P_{cr} L^3}{EI}} = \frac{\pi}{k_e} : \frac{\pi}{1} < (\mu_{cr} L)_{BA} < \frac{\pi}{0.699}$

Solution by bisection method

$\gamma = 0.5$
 $\gamma = 1$
 $\gamma = 5$
 $\gamma = 10$



So, we will do it this way. It is very easy to solve. It is a simple equation. Solve. You have two bounds, which we discussed. The maximum value you can get is 2.047. We are asked to solve this for different values of gamma. This is the characteristic equation. Divide throughout by EI. See to get it in that form. You have the two limits: between 1 and 0.699. Use the bisection method. You have to do it for four different conditions.

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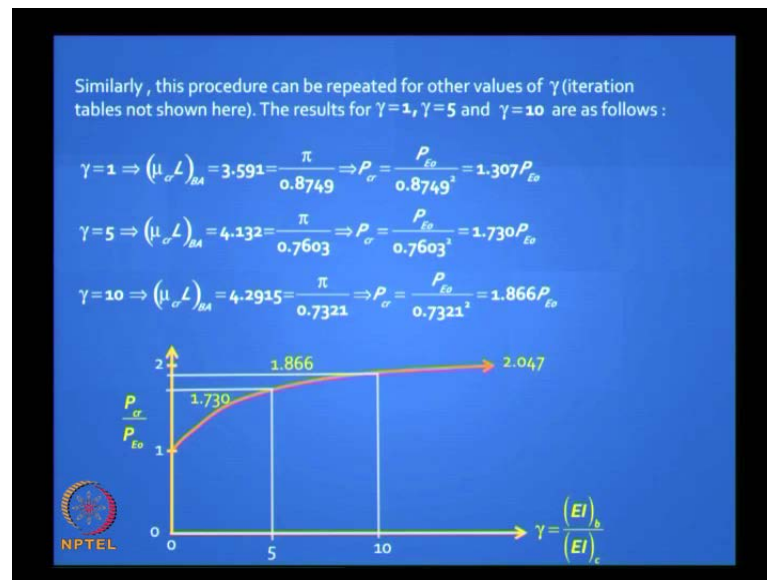
$\gamma = 0.5$

Trial	$(\mu_{cr} L)_{BA}$	$\left(\frac{S}{EI/L} \right)_{BA}$	$g(P_{cr}) = \left(\frac{S}{EI/L} \right)_{BA} + 1$
	3.14	0.0050	+1.005
	4.49	-1316	-1315
1	3.815	-3.8485	-2.848
2	3.477	-1.3471	-0.347
3	3.308	-0.5853	+0.415
4	3.392	-0.9383	+0.062
5	3.434	-1.133	-0.133
6	3.413	-1.034	-0.034
7	3.402	-0.9835	+0.016
8	3.406	-1.002	-0.002

$\gamma = 0.5 \Rightarrow (\mu_{cr} L)_{BA} = 3.406 = \frac{\pi}{0.9224} \Rightarrow (P_{BA})_{cr} = P_{cr} = \frac{\pi^2 EI}{(0.9224L)^2} = \frac{P_{E0}}{(0.9224)^2} = 1.175 P_{E0}$



(Refer Slide Time: 41:48)



Let us do it for one. Do it for 0.5. You have got the answer with fairly good tolerance. You plug in the value, you will get some value of the critical buckling load. You will repeat this exercise for the other gamma values. Finally, you draw a plot and the plot is going to look like this. If on the x axis, I increase the beam stiffness, beam rigidity relative to the column rigidity, the starting value is $\pi^2 EI$ by L^2 . So, it starts with 1 and the maximum value is when k effective is 0.699. So, you get 2.047 for a propped cantilever. Is it clear? You should be able to guess this kind of shape. You can get the answer for any value of gamma. How did we generate this? Slope deflection method.

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Case (b) : Frame unbraced against sidesway

full frame half-frame model

Unknown displacement: θ_B, Δ

Slope-deflection equations:

$$M_{BA} = S_{BA}^0 \left(\theta_B - \frac{\Delta}{L} \right)$$

$$M_{BE} = \frac{3(EI)_b}{L/2} \theta_B$$

Equilibrium equations:

$$M_{BA} + M_{BE} = 0$$

$$H_A = \frac{M_{BA} + P\Delta}{L} = 0$$

$\gamma = \frac{(EI)_b}{(EI)_c}$
 $\gamma = 0 \Rightarrow (P_{cr})_{min} \rightarrow \infty$
 $\gamma \rightarrow \infty \Rightarrow (P_{cr})_{max} = \frac{\pi^2 EI}{(2L)^2} = 0.25 P_{E0}$
 $n_k = 2$

NPTEL

It is going to be a little more difficult if you look at the case of **which** sway. Why is it going to be more difficult? 2 degrees of freedom, but you can correctly guess the bounds: lower bound and upper bound. So, 2 degrees of freedom. You have to write the equations carefully including sway. We have learnt how to do this. If you substitute and if you write down the equilibrium equations, you have to have a sway degree of freedom. We have done this problem earlier in slope deflection method. So, H_A should be 0.

(Refer Slide Time: 43:10)

$$\begin{bmatrix} S_{BA}^0 + \frac{6EI_b}{L} & S_{BA}^0 \\ S_{BA}^0 & S_{BA}^0 - PL \end{bmatrix} \begin{Bmatrix} \theta_B \\ \Delta/L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{S^0}{EI/L} \right)_{AB} + 6\gamma & \left(\frac{S^0}{EI/L} \right)_{AB} \\ \left(\frac{S^0}{EI/L} \right)_{AB} & \left[\left(\frac{S^0}{EI/L} \right)_{AB} - (\mu L)_{AB}^2 \right] \end{bmatrix} \begin{Bmatrix} \theta_B \\ \Delta/L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

For a non-trivial solution, the determinant of the stiffness matrix must vanish (implying instability in the system). This condition yields the **characteristic equation**, whose roots provide the eigenvalues (critical buckling load), of which the lowest value (P_{cr}) is of practical interest.

$$g(P_{cr}) = \left(\frac{S^0}{EI/L} \right)_{BA} \left[6\gamma - (\mu_{cr} L)_{BA}^2 \right] - 6\gamma (\mu_{cr} L)_{BA}^2 = 0$$

Initial bounds of $\mu_{cr} L = \sqrt{\frac{P_{cr} L^2}{EI}} = \frac{\pi}{k_*}$: $0 < (\mu_{cr} L)_{BA} < \frac{\pi}{2}$

Solution by bisection method

NPTEL

You plug it all in, you get the equation like that. So, what should we do? We should write down the determinant as equal to 0 and you will get a function in terms of P critical. Again, the 2 bounds are infinity and 2, but 1 divided by infinity is 0. So, the values are 0 and π by 2.

(Refer Slide Time: 43:41)

$\gamma = 0.5$			
Trial	$(\mu_{\sigma} L)_{BA}$	$\left(\frac{S^o}{EI/L}\right)_{BA}$	$g(P_{\sigma}) = \left(\frac{S^o}{EI/L}\right)_{BA} + 1$
	0	3	+9
	1.57	+2.4680	-6.074
1	0.785	+2.8745	+5.003
2	1.178	+2.7107	+0.208
3	1.374	+2.6002	-2.772
4	1.276	+2.6850	-1.238
5	1.202	+2.6983	-0.138
6	1.190	+2.7045	+0.035
7	1.196	+2.7014	-0.051
8	1.192	+2.7035	+0.007



$$\Rightarrow (\mu_{\sigma} L)_{BA} = 1.192 = \frac{\pi}{2.6356} \Rightarrow (P_{BA})_{\sigma} = P_{\sigma} = \frac{\pi^2 EI}{(2.6356 L)^2} = \frac{P_{E0}}{(2.6356)^2} = 0.144 P_{E0}$$

Solved by the bisection method, you get another set of solutions. You get a very low value. Naturally, under side sway, you do not expect high, you get a low value.

(Refer Slide Time: 43:49)

Similarly, this procedure can be repeated for other values of γ (iteration tables not shown here). The results for $\gamma=1$, $\gamma=5$ and $\gamma=10$ are as follows :

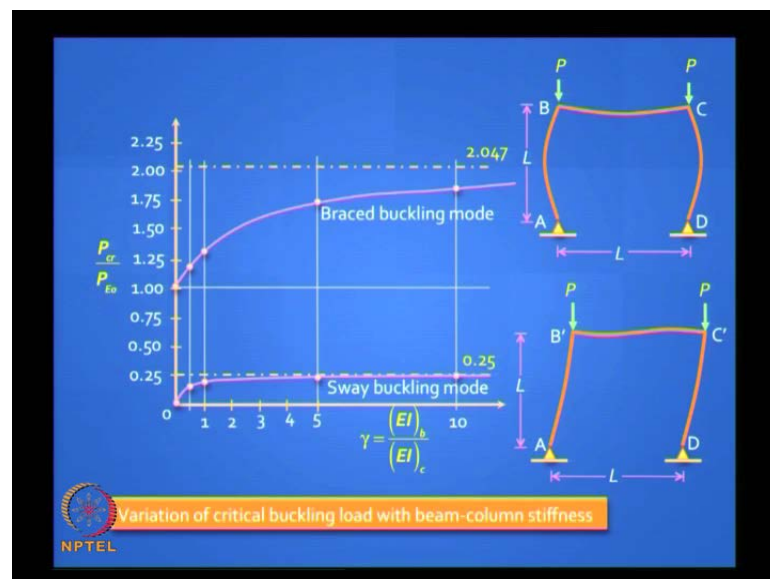
$$\gamma=1 \Rightarrow (\mu_{cr} L)_{ba} = 1.350 = \frac{\pi}{2.3271} \Rightarrow P_{cr} = \frac{P_{Eo}}{2.3271^2} = 0.185 P_{Eo}$$

$$\gamma=5 \Rightarrow (\mu_{cr} L)_{ba} = 1.520 = \frac{\pi}{2.0668} \Rightarrow P_{cr} = \frac{P_{Eo}}{2.0668^2} = 0.234 P_{Eo}$$

$$\gamma=10 \Rightarrow (\mu_{cr} L)_{ba} = 1.545 = \frac{\pi}{2.0333} \Rightarrow P_{cr} = \frac{P_{Eo}}{2.0333^2} = 0.242 P_{Eo}$$


Similarly, you do for the other gamma cases.

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Finally, draw the picture. This is the summary of the results. You will find that asymptotically as gamma increases peak, P critical will touch the upper bound that we talked about. The upper bound is $2.047 \pi^2 EI / L^2$ when the frame is braced and it is only 0.25. Why is it only 0.25?

[Not audible] (Refer Slide Time: 44:25)

Yes, we have seen how that upper bound works out. So, that is a dramatic. So, deciding whether a building is braced or unbraced has massive significance in terms of determining what is the critical buckling P_{cr}

Have you got some idea about how to use slope deflection method to solve problems where you need to solve it? If you do not have a problem of buckling instability do not do all this. This is a second order analysis, sophisticated analysis, advanced analysis. This is not there in your examination portion. It is just there for you to understand how this can be applied.

Now, we will do the same thing tomorrow using matrix formulation, a very beautiful formulation. That can handle any degree of indeterminacy. That is for tomorrow.

Thank you.