Advanced Structural Analysis
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Module No. # 6.5
Lecture No. # 37
Matrix Analysis of Plane and Space Frames

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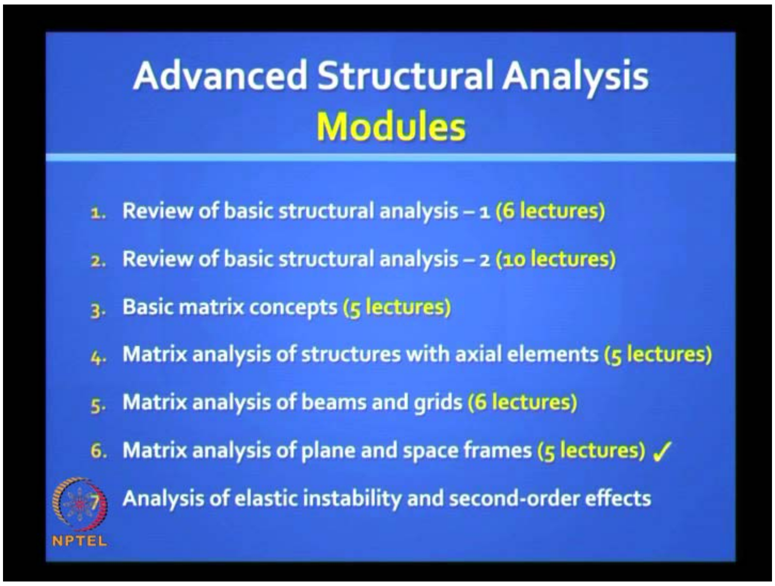

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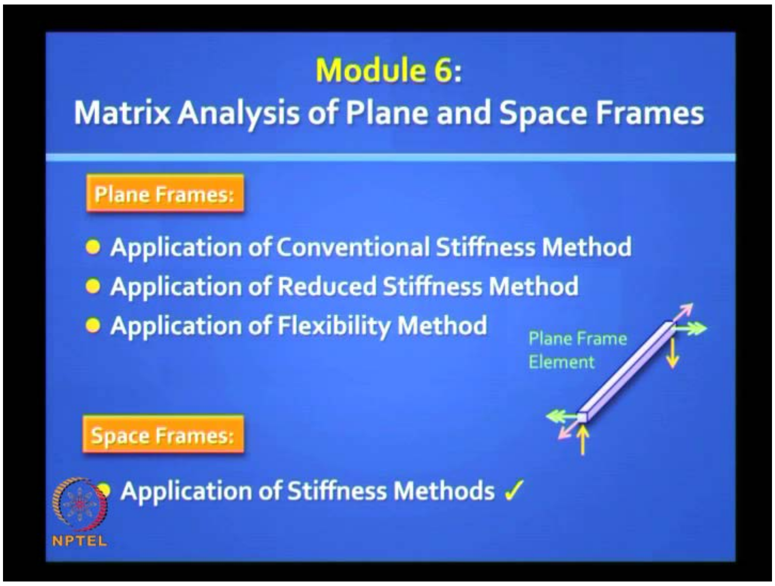

Good morning, this is lecture number thirty-seven - it is the last session we have on module six matrix analysis of plane and space frames, so this is the fifth lecture in this module. This is perhaps the most difficult topic we are covering because we are today going to deal with space frames.

The space frame element is an element, which has the largest number of degrees of freedom, you have six degrees of freedom at each of the two ends, so you have totally twelve degrees of freedom and it is difficult ok.

But, if you can really understand this then you will find that everything else you have done till now is just a special case of this. This is the ultimate you are dealing with - real life three dimensional structures - skeletal structures, and we are learning to analyze such structures when they are subject to any kind of loading direct loading or indirect loading.

So, we will look at the application of stiffness methods and really for such complex structures you need the help of a computer because you are dealing with very large sizes of matrices; you are dealing with many simultaneous equations that need to be solved. So, that is why we will use the stiffness methods and I hope to show you both the methods - conventional stiffness method and reduced stiffness method. You'll find the reduced stiffness method is also not going to be easy for a space frame because to visualize and to draw the write down the t d matrix is not easy.

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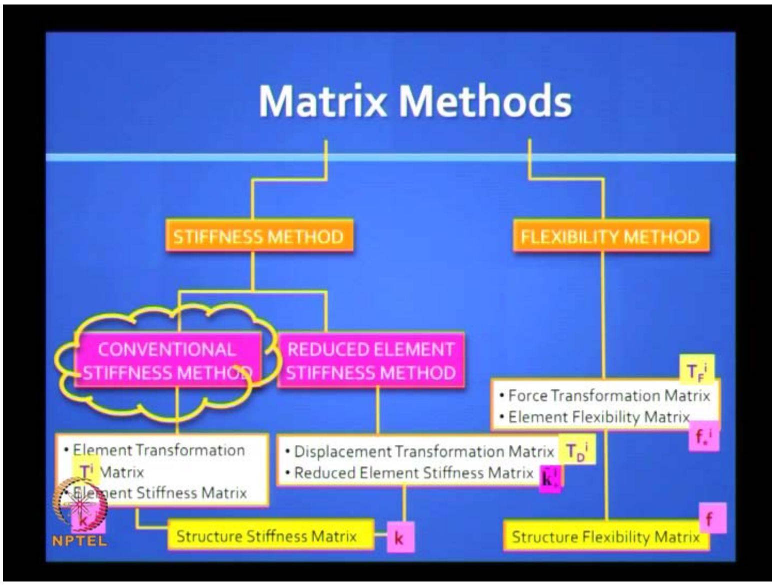

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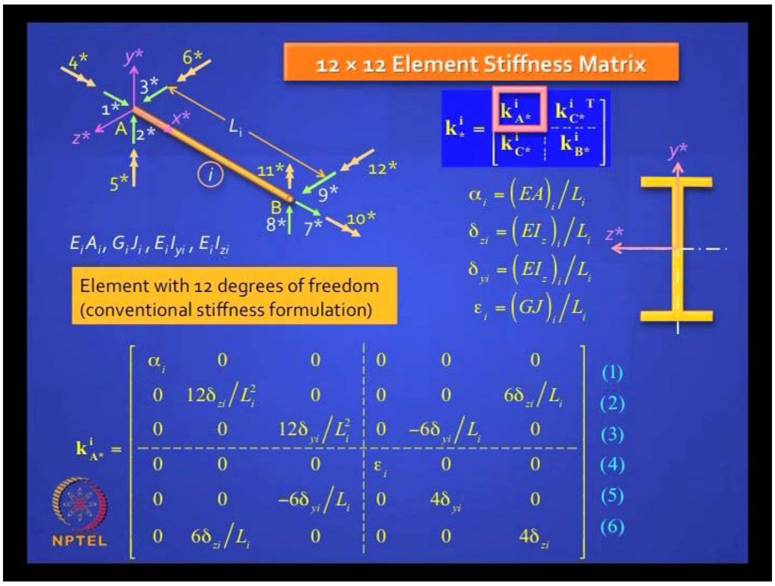

So perhaps in such an instance the conventional stiffness method is better because you can program everything so first we look at the conventional stiffness method so this is your element and this is the element with twelve degrees of freedom look carefully degrees one star and seven star refer to axial degrees of freedom that is what you had in your axial element in your truss element you had only that.

Then which corresponds to your beam the conventional beam can you identify two star is your conventional shear force so you you are familiar with that two star is a shear force which is a moment vector.

Six star

Six star perfect because that contributes to bending about the vertical plane the vertical plane is the x star y star plane so in your beam element you had two star three star as well as is it six star i thought it is eight star eight star and twelve star that is your conventional beam element right

Your conventional beam element had only two star six star two star is the shear force in the vertical plane six star is the bending moment in the vertical plane at the start node and at the end node you had eight star and twelve star but, you also had in the grid element you had the torsional degree of freedom so you also had four star and ten star so it is it is looking familiar now so looking familiar now

So we've we've looked at the truss element which is a special case of this space frame element we also looked at the beam element and the grid element if you bring in the plane frame element then it is just a combination of the beam element and the truss element.

But we have some additional degrees of freedom what do they correspond to what what are they let us look at them again five star is something new and three star is something new at the start node what do they corresponding to let us say shear they correspond to shear and bending about the horizontal.

X z plane.

In the horizontal plane that is in the x z x star plane you have situations where you can have simultaneously bending in the vertical plane and the horizontal plane ok

If you have unsymmetrical bending that'll happen in in any case that is if your section is not symmetric then the principle axis do not match with your global axes or with your centroidal axes so have you so at the end node the corresponding degrees of freedom are ten star and eleven star and nine star ok.

So you are familiar with all of them so you have twelve degrees of freedom six degrees in each node of the six degrees one corresponds to axial degree of freedom one corresponds to the torsional degree of freedom they are point along the centroidal axis the x star axis

Then you have two degrees of freedom to deal with vertical plane bending and two degrees of freedom dealing with horizontal plane bending the two would correspond to a shear and the moment is it clear that is it so you are you understood physically what they mean these are displacements and they also reflect the corresponding conjugate forces so you have translation and rotations and correspondingly you have forces and moments

Forces can be shear force or axial force moment can be bending moment or twisting moment torsional moment twisting moment these are words commonly used and the rotation corresponding to that is called an angle of twist the rotation corresponding to a bending moment is called simply rotation of flexural rotation or slope (( )) ok.

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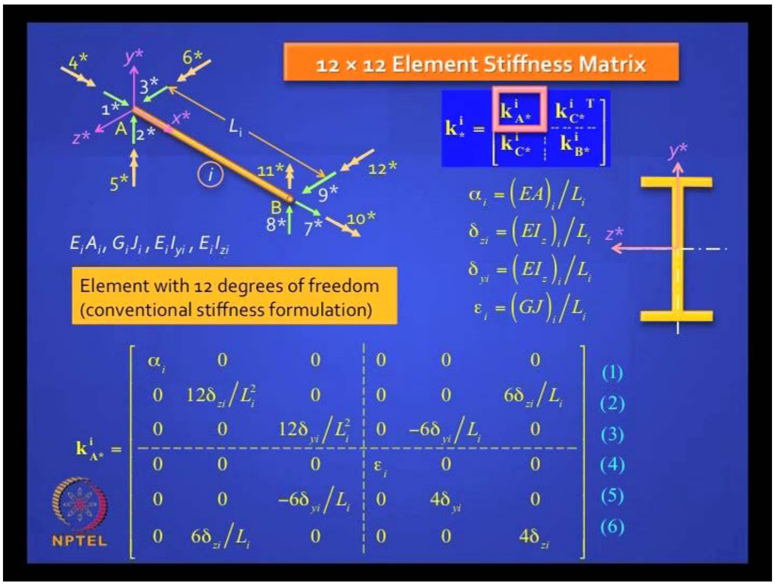

Now that matrix is too big for us to show in one one picture so we will break it up into parts you'll know that the element stiffness matrix which is twelve by twelve will be symmetric for sure and that is why we can break it up into three partition it into four six into six compartments of which you really have only k a star and k b star and k c star because the half diagonal quadrant will be the transpose of k c star is it clear

Now here is your exercise can you write down can you write down and this is a very least i expect you to do and i could ask you this question in the examination you do not need to solve any problem but, at least you should be able to generate from first principles with the knowledge that you already have can you write down for all the signs convention that we are depicting here where if you notice we define the x star y star z star axis and we aligned all the vectors in these three cartesian directions right

So what i required you to do is let us begin with this and let us try to understand the axis let us take an an i section it is a nice space frame element section to look at so you have if you look at it from the right side y star is pointing upward x star is along the longitudinal axis which goes to the center of the web out of the plane of the the that that plane and z star is pointing to the left if you are looking from the right is it clear

So with this as your reference can you write down and let us use these symbols alpha to represent axial stiffness e a by l delta you have two deltas you have bending about the major axis which is what we called e i z i by l i and bending about the weaker axis the horizon[tal]- you know horizontal bending that is e i y it should be y by i by l and then you have to multiply by four or two whichever is appropriate or six and then we have the torsional stiffness which is g j by l right.

J of course, has to be correctly assigned please note because this is a non circular section right can you write down in terms of alpha delta you have delta z delta y and epsilon can you write down atleast can you write down k a star given a short show me that six by six matrix based on all that you've learnt till now

What's the first row first column going to look like

(( )) alpha.

Alpha will one so it is going to be e a by l and then the rest of it will be zero so you got the first row in the first eleme[nt]- what about the fourth one

Zero and last will be epsilon.

The fourth will correspond to four star so can you read of can you tell me what that row will look like epsilon will come there you know c j by zero so read out that [re/row] row

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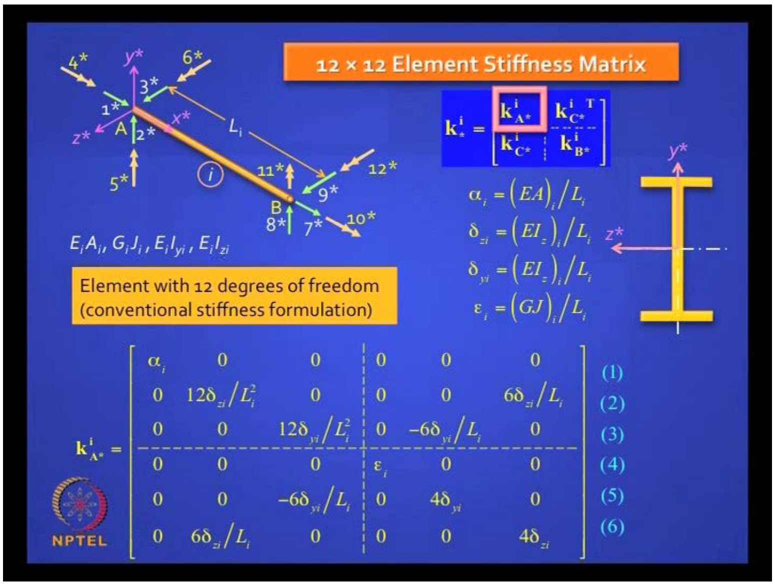

for me the fourth row zero zero zero epsilon epsilon zero zero so you got the first row fourth row and we get the columns also because the symmetric matrix now you have to worry about the second and third columns if you wish or rows

What's a second column going to look like zero you begin with zero because the first row is zero then what is the diagonal element k two star k two star look carefully what did you do for a beam element what is that shear force value.

Twelve e i.

Twelve e i.

By l square.

Twelve e i by.

L square.

L cubed twelve e i by l cubed right and so you got k two two star correctly which is the other non zero element in that.

Six star.

That's right k six star two star what would that look like that is a moment will it be positive or negative because if you lift up and a you are getting a clockwise rotation anticlockwise moment so be positive so let me help you where you are you we we wrote most of it it is not difficult you have to learnt to do this and no looking up books for this first principles is it clear.

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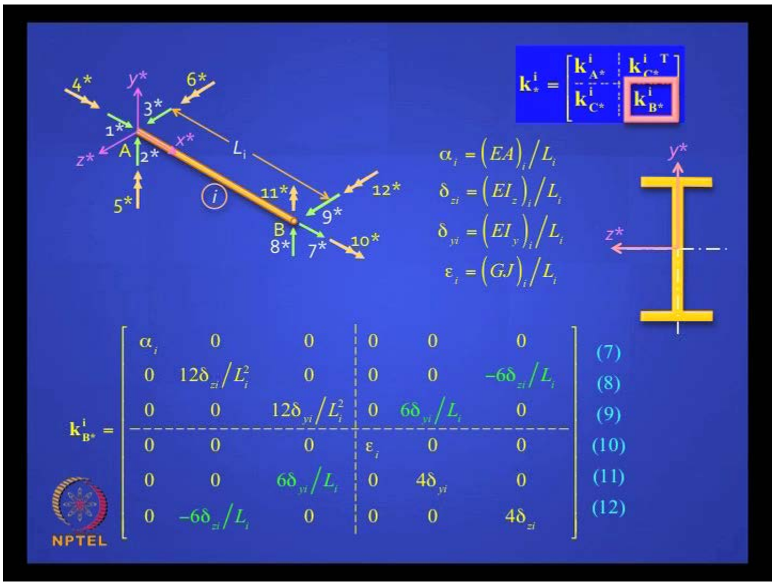

So we can write down k a star next let us move on to k b star will it look like k a star will there be any change between k a star and k b star look carefully and answer now you are shifting to the end node you are dealing with the coordinate seven eight nine ten eleven twelve which element so as far as your axial stiffness concerned no change torsional stiffness no change beam stiffness what is a change if any.

(( )) every bay.

Shear will get reversed moments will not get reversed perfect so i marked in green color the six e i by l squared value has an opposite sign to that which we assigned in k a star so i the diagonal elements will always be positive yes you had a doubt it is correct that is all you need to do ok.

Let’s move ahead what about the last one k c star this is now half half diagonal ok

K c star will it also look like k a star what is the different where will the he is right there will be a minus to the axial stiffness so e a by l will become minus e a by l and there will be also minus two torsional stiffness you get minus g j by l what else what else will change.

Shear force.

Ok now imagine the full matrix so full twelve by twelve matrix if you take any column you actually covering all the forces in the free body right so you have to satisfy equilibrium so the the shears must add up to zero then only you’ll have equilibrium got it so whatever shear forces you got in in the k a matrix will get reverse in the columns but, the moment do not get away because if you had chord rotation you’ll find the moments are going to be the same clockwise or anti clockwise so only those same

So here as you rightly said alpha and epsilon gets a negative sign i have marked in green color and here the shears of the diagonal elements shears are now the diagonal elements so you get minus twelve e i by l cubed that is the only the change and in fifth and sixth columns the shears will be six e i by l square they get reverse

So think about it and please come prepared finally, in examination you are likely to get this question atleast writing down correctly the twelve by twelve element stiffness matrix for a space frame element is it clear ok

Now you have other problems how do you do transformation in a plane frame the transformation was simple you had cos theta sin theta and one right but, it is not so simple in a space frame element because you have nine direction cosines you've nine direction cosines how do you deal with this situation.

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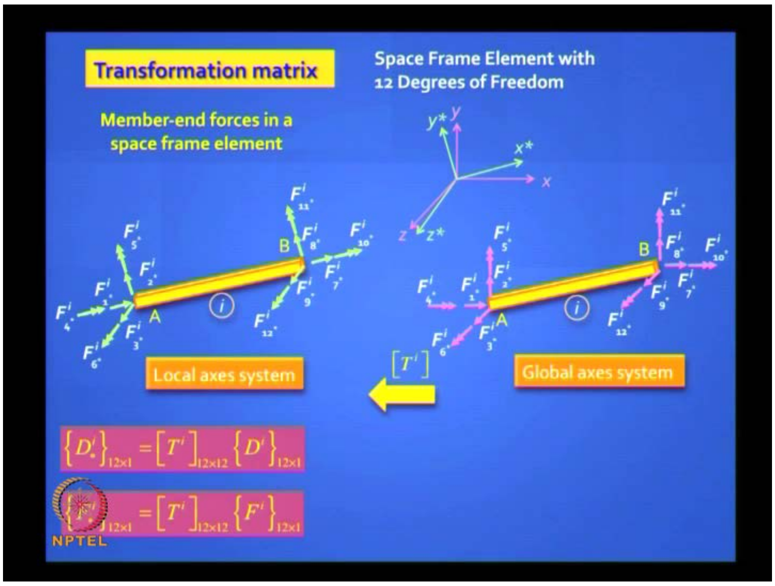

So if you go to the definition from the global axes system we want to switch to the local axes system the global axes and local axes are not going to necessarily be oriented in the same direction when you had a plane frame or a grid you had rotation about some plane which did not change

Here you can have a change in all the planes so if i have x y z i can rotate it arbitrarily in any direction and i get a new x star y star z star how do i do the transformation it is a well known transformation what is the matrix that is involved in that transformation

What’s that matrix called.

Angular transformation

In linear algebra it is well known it is a rotation matrix we actually used a rotation matrix in the earlier transforma[tion]- so this is what it is going to look like it is an orthogonal matrix the rotation matrix itself is an orthogonal matrix so you have to your transformation matrix for a twelve degree of freedom element will also be twelve by twelve it is going to have diagonal boxes and if you get one box correctly r i you got everything.

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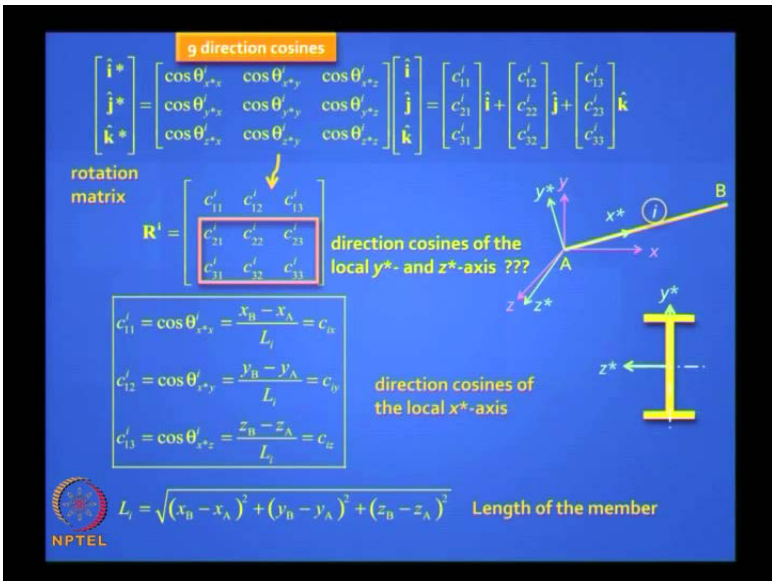

So it is quite simple the only thing is how do you get r i how do you get r i not so simple not so simple that is a first row of that r matrix what about the other elem[ents]- second row third row so let us go back to first principle take a look basically we are trying if you look at unit vectors let us say look at unit vector you can write down unit vectors corresponding to a global axes as x y z will correspond to i j k right

Then i write i star j star k star as unit vector corresponding to the local axes which are x star y star z star i've shown an element there a b the space frame element whose longitudinal axis matches with x star that is your only information you have and you need to get these nine direction cosine they make up your rotation matrix you can write them in a matrix form as c i one one c i one two etcetera etcetera

So as you rightly said the first row of that matrix is pretty easy because you have theta x theta x x theta x y and all that you have all you need is the coordinates of a and b in x y z in the global coordinates then you can write it down just say x b minus x a by l y b minus y a by l z b minus z a by l you get three direction cosines but, they fill feed you only the first row in that matrix

How do you get how do you get and you can get the length of the element automatically how do you get the rest of it that is the challenge and this has to be correctly assigned because and mistakes have been made because the i section must have its major bending axes in the direction you are going actually looked do in the construction site

Let's say i have an i section i can keep the major i can keep it this way or i can keep it this way it makes a big difference and i must be sensitive to it and in software's some software's allow you extrusion so you can actually see the final shape and you can say oh my god this this element i oriented the wrong way and could it turn it around but, you must have a full proof we've doing it so i think it is interesting from a vector algebra prospect how do you do this

I have got that line x star a b and i know how it is oriented with respect to x y z in the global axes how do i get the directions of y star and z star basically it boils down to that remember let me demonstrate i have this element and i have located it cartesian coordinates x y z so i have got x star i've defined this perfectly with respect to the first row direction cosine.

Now the y star of this is perpendicular there are many perpendiculars i can draw do you understand there are many perpendicular i can draw so how do i deal with it so i have to let us see this is an i section then it defines the perpendicular because it will be in the plane of the web right.

So a line alone does not define y star and z star i need a three dimensional cross section i need a real three d element how do you it so this problem has been yeah go ahead.

(( )) three elements (( )) in terms of the new coordinate i mean the local coordinates and we know the angle between the local coordinates and the global coordinates.

We do not know that is it we know only x star we do not know if you knew that then we could have gone to that rotation matrix can filled up where everything there itself we do not know that is the problem it is fascinating if you you know look at it from a coordinate geometric prospective so there are many ways of doing it i'll show you one simple way of doing it

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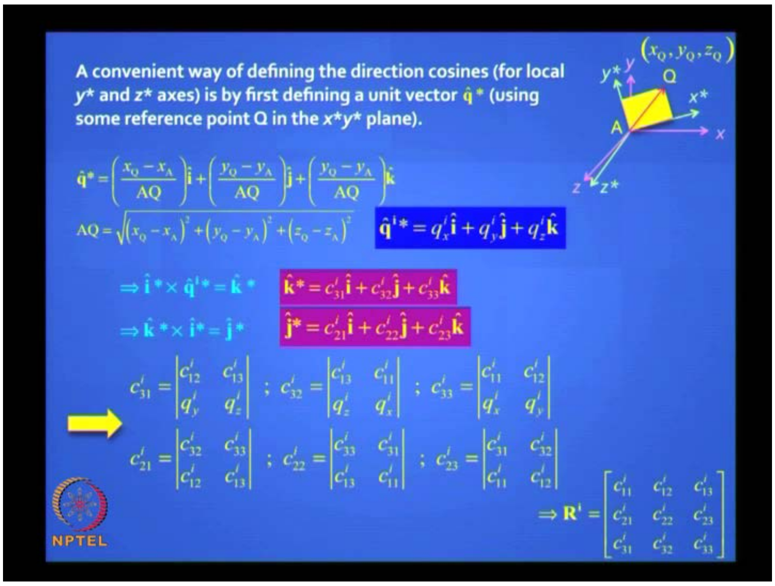

A convenient way of defining the direction cosines for local y star and z star axes is by first defining a unit vector q star using some reference point q in the x star y star plane so let us you have to define that plane if it is an i section you have to define the plane of the web to define a plane you need one more point and it need not be on that web it could be anywhere on that plane so you could pick up any other point probably a node in the structure itself and give those coordinates q star so it'll look like that and then if you can write down the the vector a q and normalize it you got a unit vector in that plane you can do that

Get some q get the coordinates of that you define those coordinates and then you can write write down a unit vector q star right can we do that you can do that and the length of the vector is known so you can get q x i q y i q z i as the direction cosines of this vector clear but, how does this help us get the unit vectors in the y star and z star direction what do we do next.

(( ))

There are tell me what property to invoke you are right so i have got the x star unit vector which is called i star i’ve got q star with the help of these two how do i get a perpendicular vector.

(( )) in terms of linear (( ))

How do i do it simple

(( ))

I will take the cross product then i get out to get the normal to that that is exactly what i do i take the cross product i know i star i now have q star i take a cross product and that’ll point in the z star direction perpendicular to that yellow plane there so i’ve got k star i’ve got k star now let us say then how do i get the j star

(( ))

Perfect that is what i do next k star i star j so simple vector mechanics a little bit of visualization then if you expand this equation and solve them you get these equation and with the help of that you got your rotation matrix so this atleast the theory of it you should know do not worry if you do not know most engineers have no clue about all this because they just know how to press buttons and then the design the software manual tells you which button to press.

Ah did i tell you that story about the lady who vacuum cleaner was it did i tell you that story it was not a vacuum cleaner it was a mixture so the house the maid puts it up together and the maid is illiterate and this ladies asked her how how did you manage to puts up together and this maid says madam if you do not know how to read or write you have to use your brain so please use your brain you cut this then you can do the transformations the rest of the procedure is simple.

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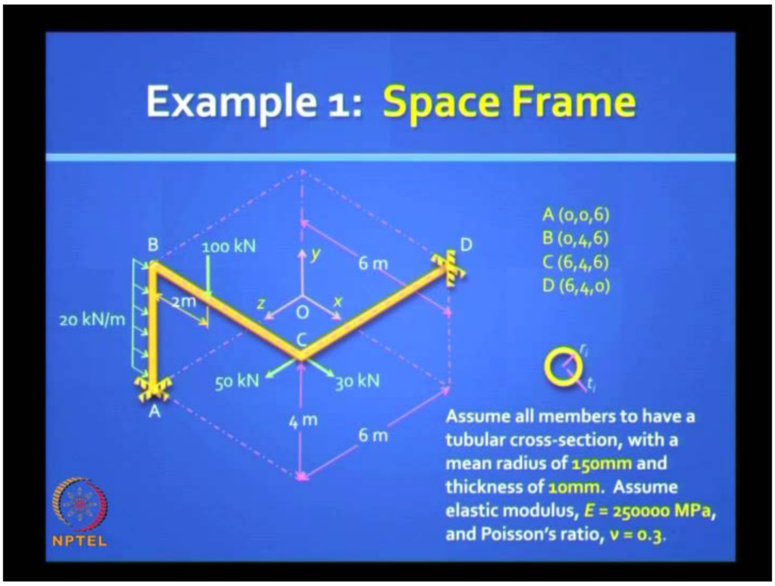

So let us straightaway get into a nice juicy problem if you can do this problem you can do any problem agreed but, this also an easy problem because i've conveniently put it all reticulated so all ninety degrees so i have got another problem in the book where that element c d does not nicely get itself aligned along the z axis you have a diagonal elements c o ok.

It solved in the book it solved by both conventional stiffness method and reduced element stiffness you believe me if you can do a problem like that with just three elements you can do a problem with two hundred elements you know exactly what to do you have a system of doing and that is powerful i mean you really have understood structural analysis anything else is a special case of this so let us do this problem.

So i have put all kinds of loading i did not throw in temperature loads and support settlement but, that is easy to do you know what to do so you got three elements oriented in three completely different directions and there are distributed loads and the first one and the second one you have been [inter/intermediate] intermediate load and you have nodal loads at c

So you can be pretty sure that all your elements will be subject to all things possible we will have axial forces we will have shear forces in vertical and horizontal plane you have bending in in vertical plane horizontal plane you also have a twisting moment to all of them this is really a great problem to solve i do not suggest you solve it it for your examination

But later in life when you have spare time when you want to look at these nice pictures or when you are like to do some programming some coding this is a good test case and some of your seniors have done it actually they validated the solution one way to validate the solution is use a stranded software package do the same thing you should get exactly the same solution

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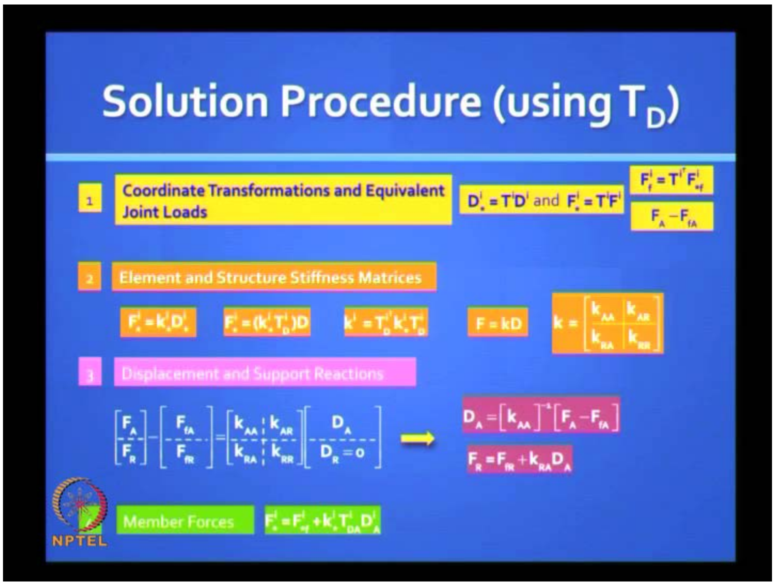

But we are going to do this from first principles the coordinates are i've shown there and you need some more information so conveniently let us assume that all the members are tubules nice tubular sections and you are given the mean radius is r i is one fifty m m and the mean thickness is ten m m t i is ten m m e value is given to you that steal and poisson's ratio nu is point three with this information you should be able to practice problem so let us do it.

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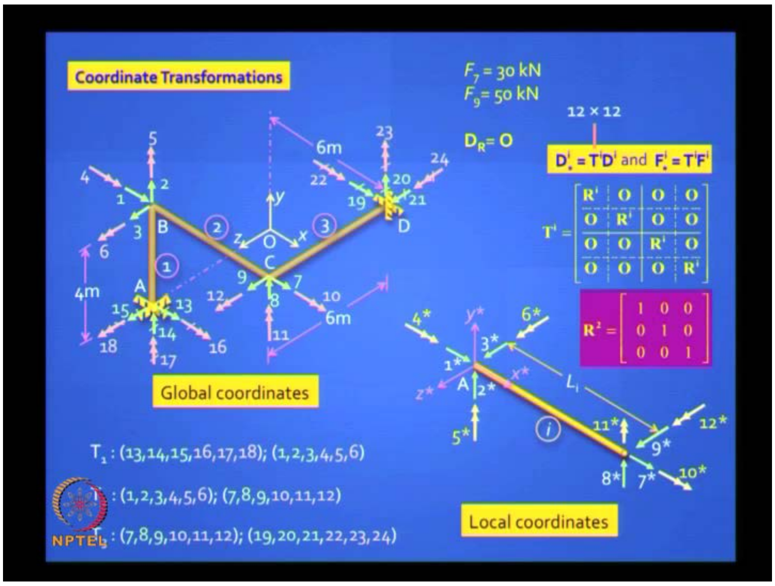

So the procedure is exactly as we did earlier we would not waste time with this luckily we do not have any support settlements we do not have any indirect loading so first we have to mark the coordinates how many active degrees of freedom do you think we have active degrees you will have active at b how many at b you'll have.

Six.

Let's take a look six and look look how nicely we've oriented them along with global x y z axis and the colors we have nicely put the green color for translations and that what color is that pinkish color for the rotations clear

You repeat this exercise at the joint c so how many active degrees of freedom do you have.

Twelve

twelve how many restrained degrees of freedom you have.

Twelve.

Again twelve so you have six here and six there

So what is a size of your overall stiffness matrix going to look like twenty four by twenty four luckily half of them are restrained so k a matrix will have a size of twelve by twelve there is no way manually you are going to invert that matrix you need the help of a computer right and it is a full full problem because you know you cannot ignore anything here ok.

So what is the loading that is given to you there is some nodal loads if you notice f seven is thirty kilonewton and f nine is fifty kilonewton and luckily there is no support settlements the restrained displacements are zero

Now let us write the local coordinates take a this is representative of all so i put i but, it it actually matches very nicely with two element two here we looked at this earlier so these all local coordinates what do you need to do next you need to you need to write the transformation matrix and you have three elements each of them will have a size of twelve by twelve and the transformation matrix will be made up of your rotation matrix ok

Can we write down atleast the rotation matrix for element two which is very easy to do what will it look like it is an identity matrix lucky you right it is an identity matrix those two x y z match with the global x y z but, not so for one and three so all you have to do you can do it by inspection because in this case it is quite easier do it or you can get the coordinate q and play that game and solve those equation it’ll be made up of ones and zeros but, you have to put the right one at the right place and it could be minus or plus so i leave that exercise to you.

You can generate it and you need the linking coordinates and they are pretty easy to remember the linking coordinates for t one will be remember the start node will begin with thirteen fourteen fifteen sixteen seventeen eighteen and the end node will be one to six similarly, t two the start node is one to six and the end node is seven to twelve t three start node is seven to twelve end node is nineteen to twenty.

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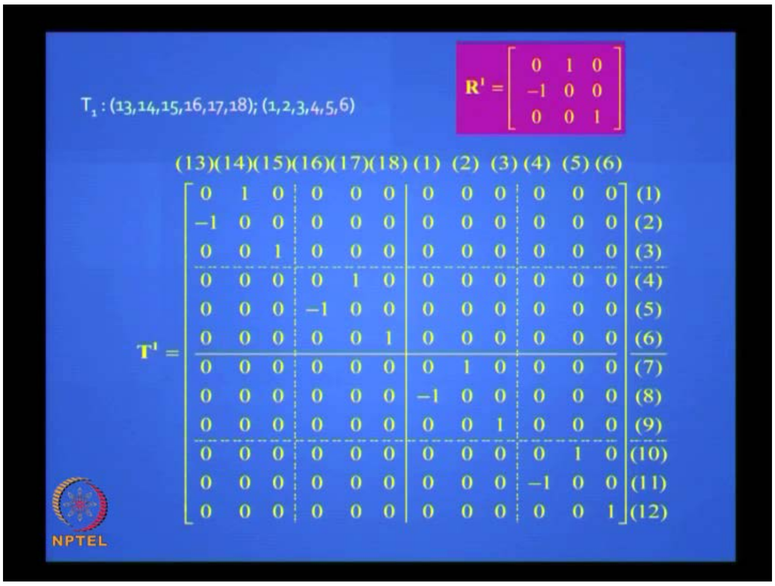

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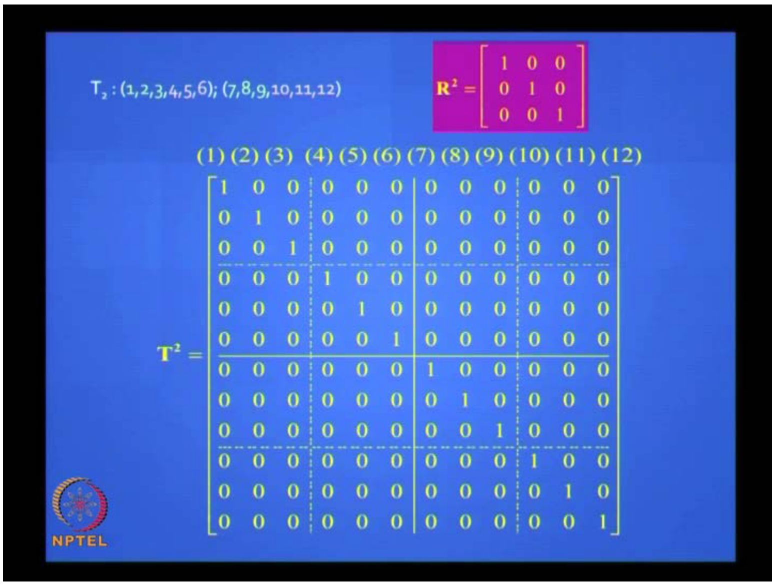

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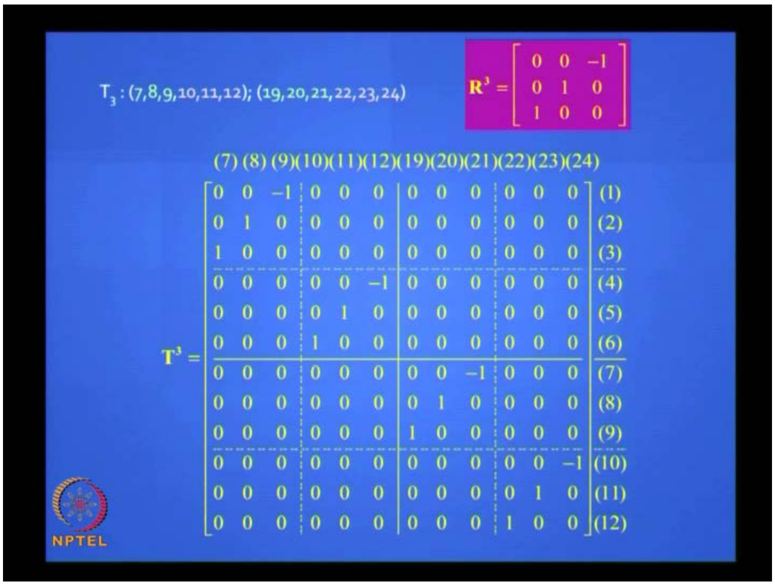

So you are ready then let us go fast t t one will look like this it is easy because you are just rotating that vector t two is you know identity matrix is is the easiest of the lot t three will look like this i leave it to you to figure out the rotation right because we just do not have time for that ok

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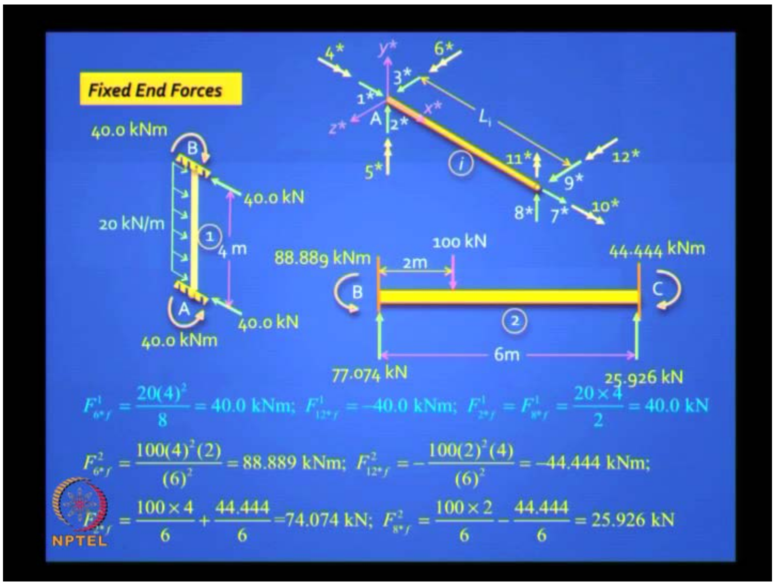

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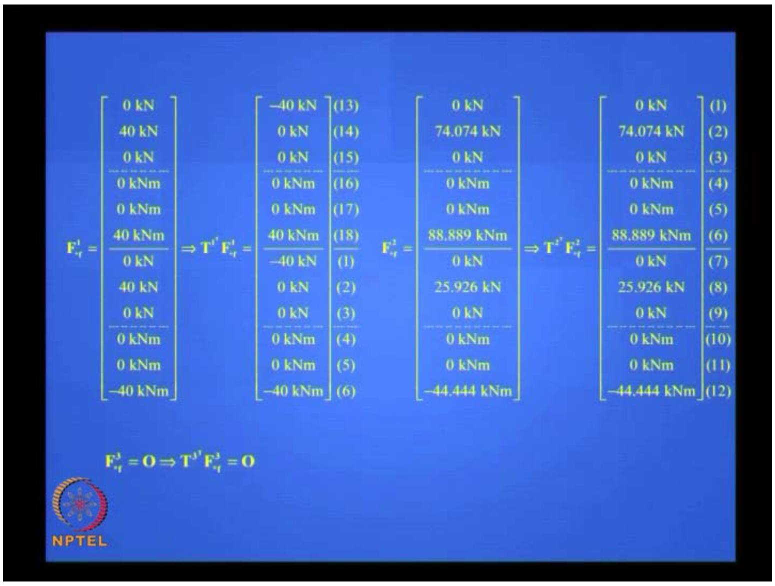

Next you have to find the fixed end forces you have a distribute load in element one that is child’s play for you by now you can find the fixed end moment and those forces for the second element also you can find out very easy to do you do that then what do you do next yeah you have to transform you have to do slotting game to do that you have to put that t i transpose you do that

See how easy to do it with a press of a button that is what the software does but, you have to do it correctly you got the t i you do not bother to write it down on paper let let it be there save paper.

Ah but, inspect it and make sure you have to make sense and then luckily for you there is no distribute load in element three so you do not have to do any transformation there.

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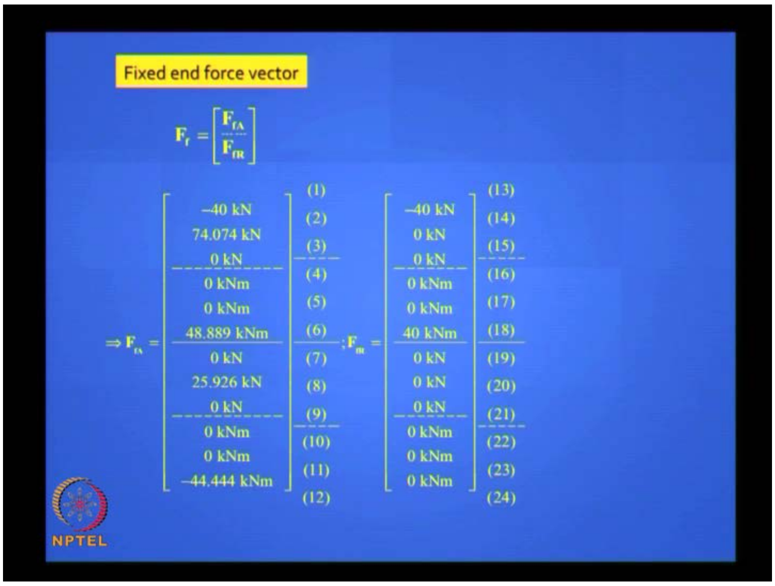

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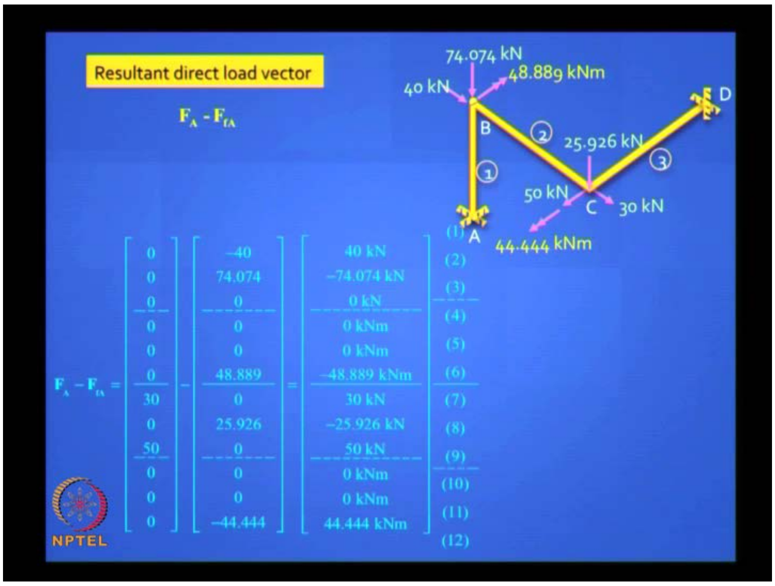

Leave the linking coordinates as we've done in earlier problems what you do next you have to add it all up at the right slots and then you get your resultant force vectors f f a and f f r then what do you do f a minus f f a is your net load vector and nice to draw a sketch this is what you get.

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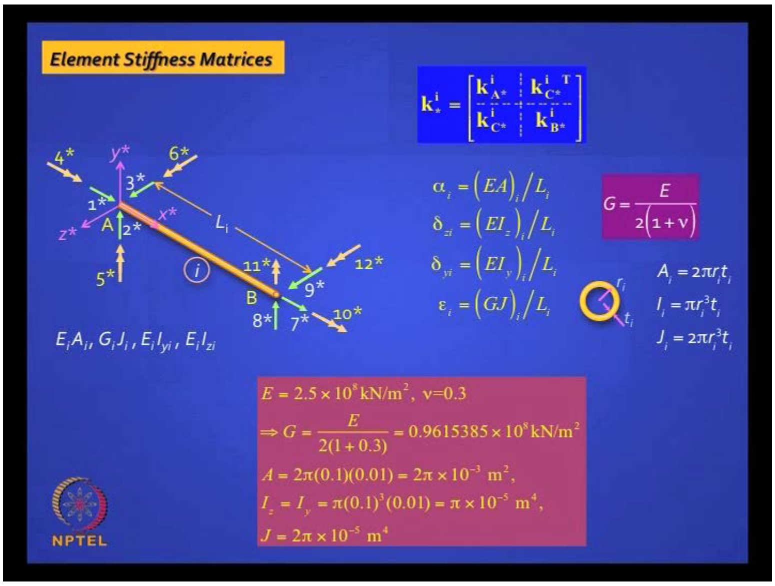

So your distributed loads have been conveniently convert to equivalent joint loads you are now dealing with this structure you are analyzing this structure and the displacements you get and this structure will be exactly equal at the joints to your original loading then you generate the element stiffness matrices so this story we know you have to go through process you have to first get the properties for a tubular section you can write the moments of the inertia and the cross sectional area properties you can you have given the poisson's ratio and g and e.

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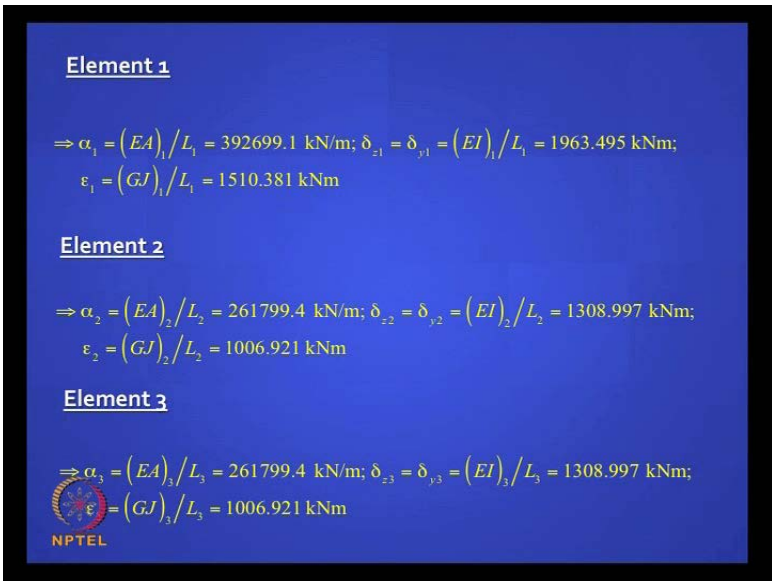

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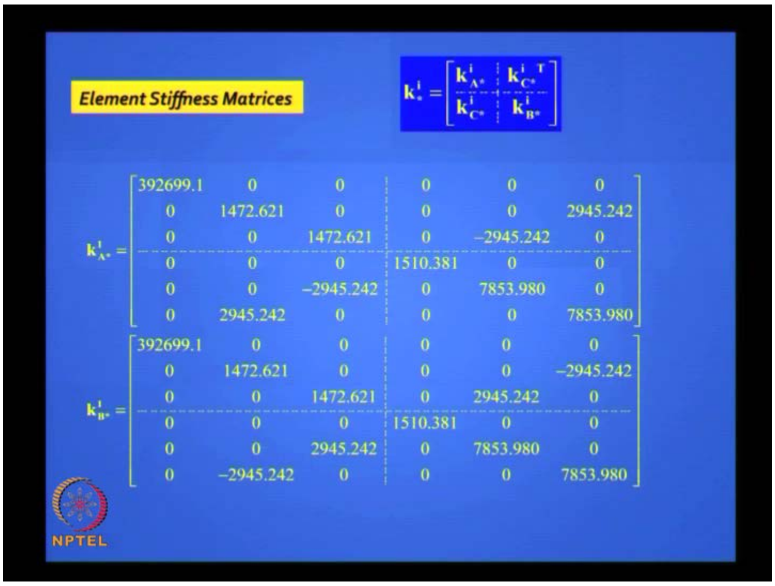

So you can write down all these values be consistent with your units and then you plug in those values into your three elements element one two three you get those numbers if possible write the units also and do it accurately and then then i am reproducing the you remember these three pictures i just discuss that is it you plug it all in you get the element stiffness matrices.

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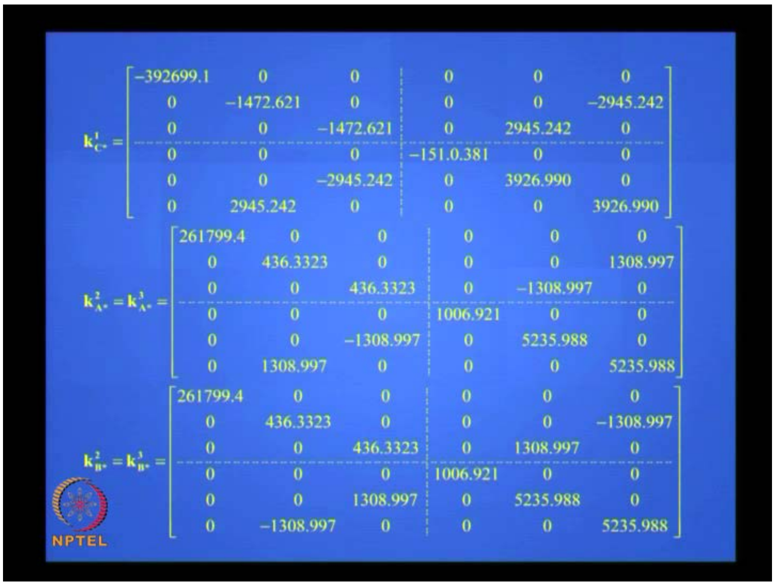

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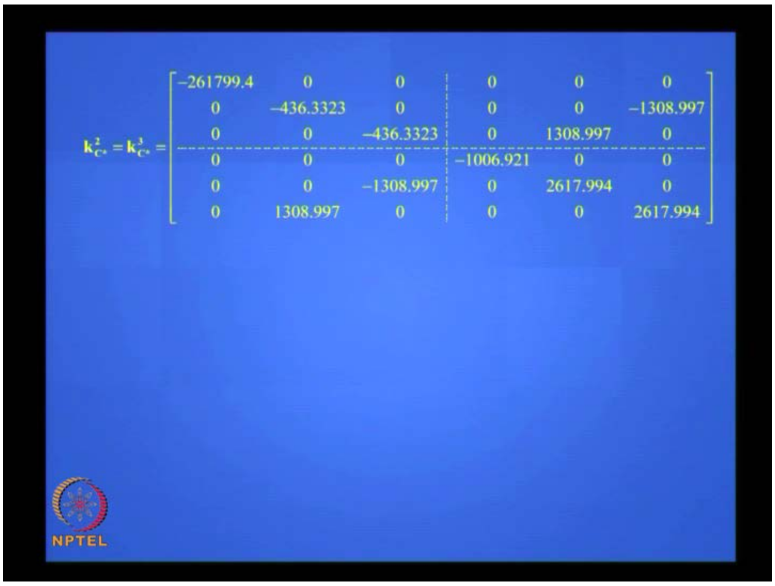

You have k one a k two k one b k one c k two a k two b k two c k three a k three b k three c some of them may be equal we have to because some have the same lengths so you work it all out that is it what do you do next what do you do next structure stiffness matrix so how do you do that here you have to intelligently do it we’ve done this earlier for the space truss or something

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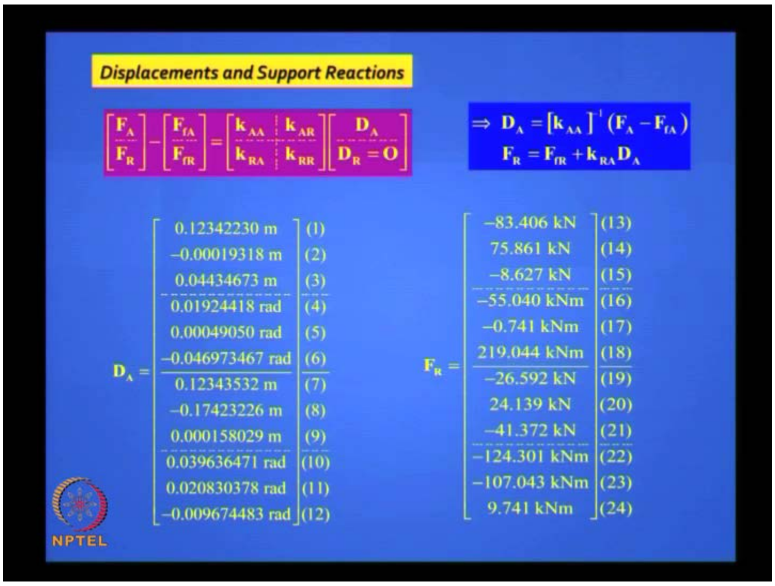

So you have to slot it correctly you should know which element will go away and once you do that you can generate the sub matrices first and then the full matrix personally it was lot of fun doing this for me i do not know for you so now you find the displacements and support reactions d a twelve you got the answer ok.

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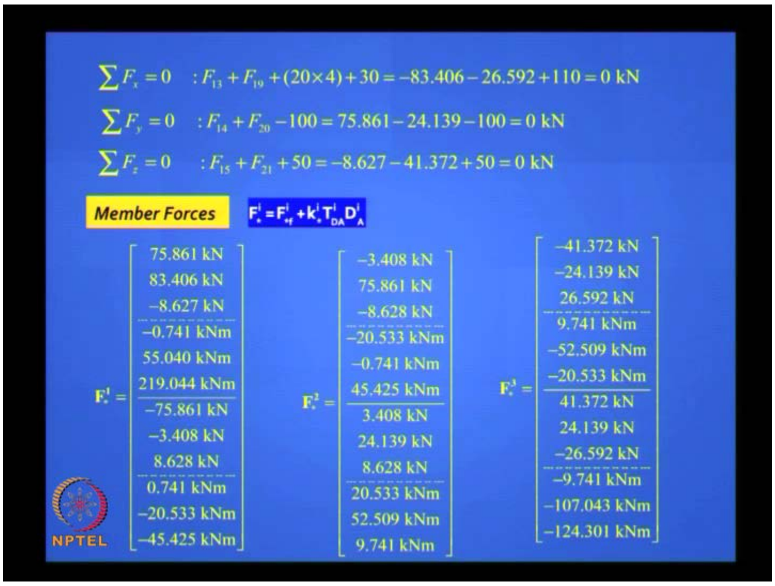

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Effortlessly the computer has done everything solved inverted a twelve by twelve matrix you got the support reactions you only wish they are all correct the first thing you should check is equilibrium you do that everything at least they all add up to zero right then what do you do next drawing is going to be very tough we will draw it later drawing is not easy member forces you do that like this these are the slope deflection equations now you draw draw your free body first free body

I should show you a some i think i've shown some of you i actually did this for an indoor stadium some twenty years back all done manually no computer i had four lined programmable calculator i entered everything i think those days it was all basic code was on basic and i have to do this in paper i used different colours for different vectors it was good fun.

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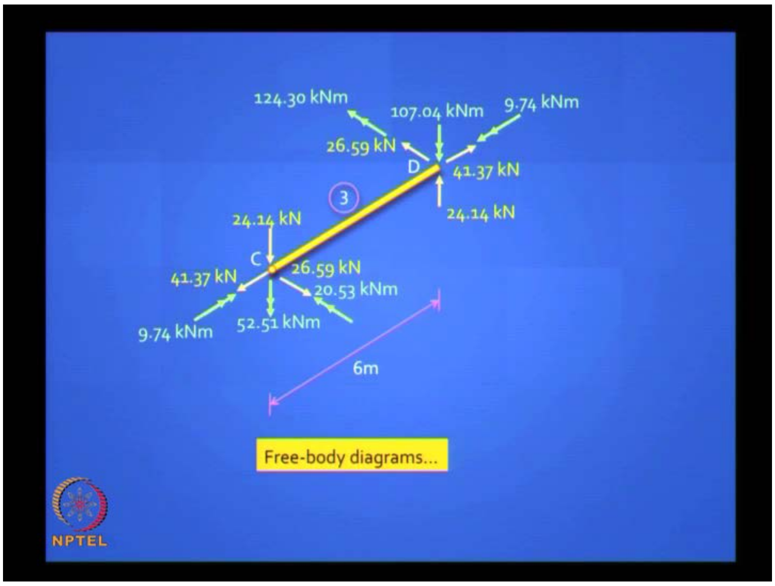

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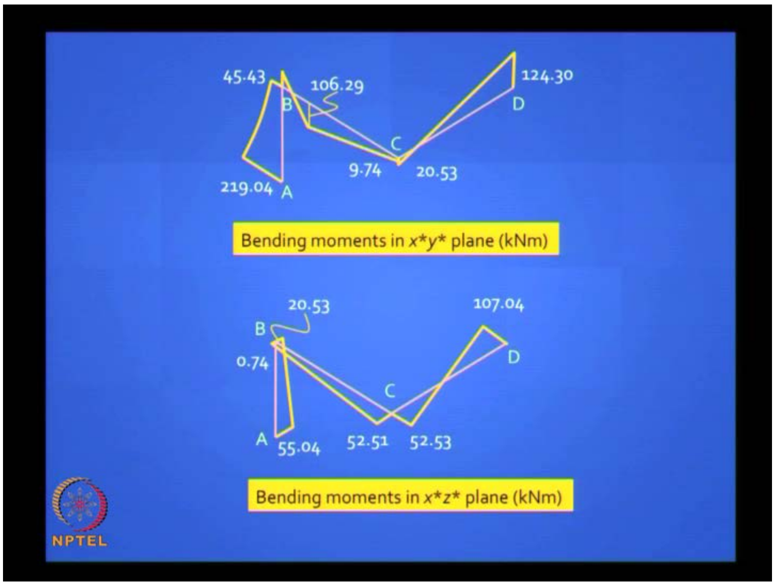

This is the first element this is the second element it is looks nice especially if you like colours and you should make sure that that everything matches equilibrium stiffness this is your third element look at that but, this is not what do you want what do you are really want to design so the axial force you do not want the bending mom[ent]- there shear the twisting moment you know the once at a worth drawing are the bending moment diagram.

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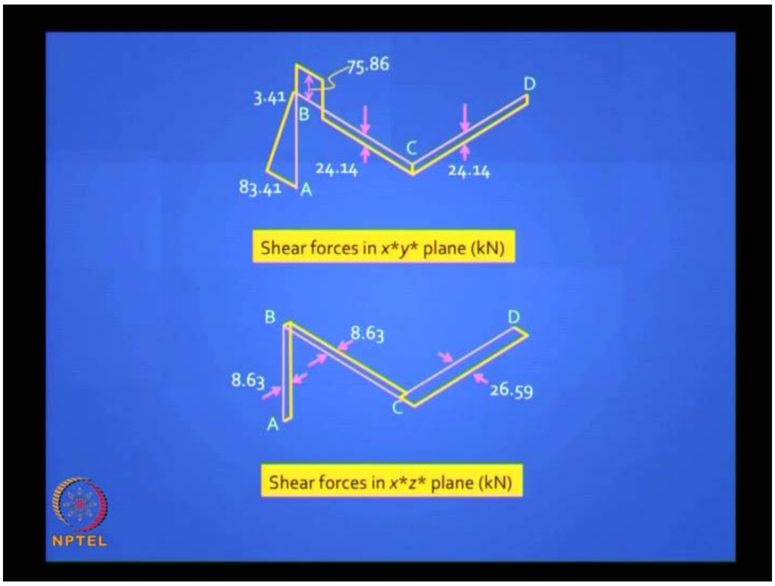

You have to draw two of them one in the x star y star plane the other in the x star z star plane look at that similarly, you need shear forces draw in the x star y star plane draw in the x star if you can do all this and do it accurately you've learnt matrix analysis of structures this is the ultimate test ok

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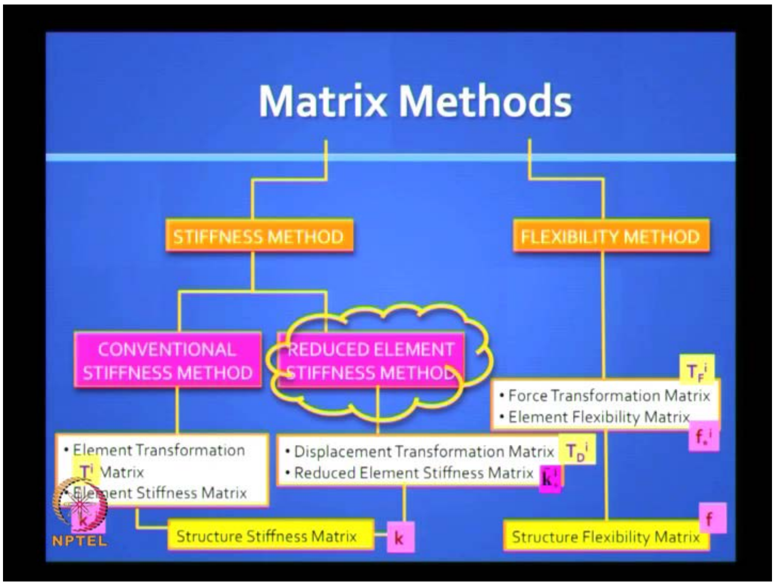

But let us do some simple problems let us quickly look at the reduced elements stiffness method not recommended for big problem because it needs much more thinking so i've done the diagonal element problem in reduced element stiffness method you can refer to the book do it

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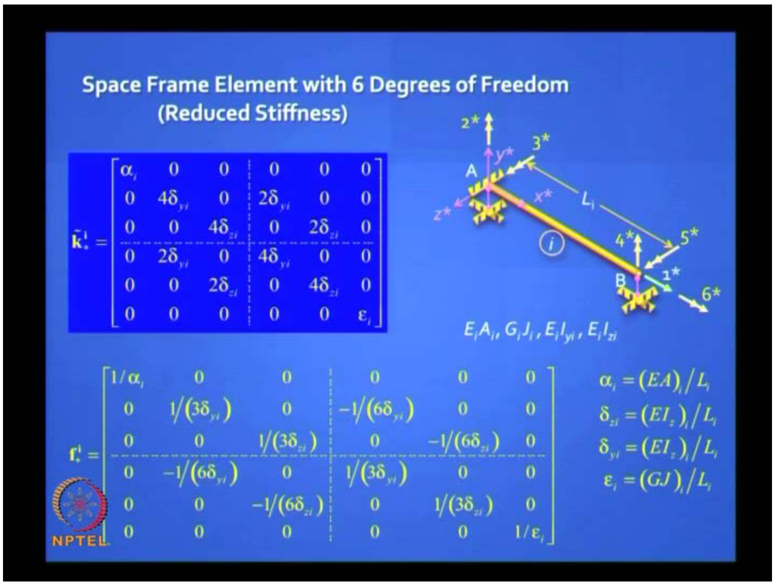

Ah let us take a very simple problem and do it so before that let us look at the matrix you have a six degree of freedom system so it is basically a combination of it is your grid element plus your plane frame element with bending about the horizontal axis ok

First you have to put six constrains to make it stable and that is how you get those vectors it is actually eliminating six rows and six columns from your conventional stiffness matrix you will end up with this

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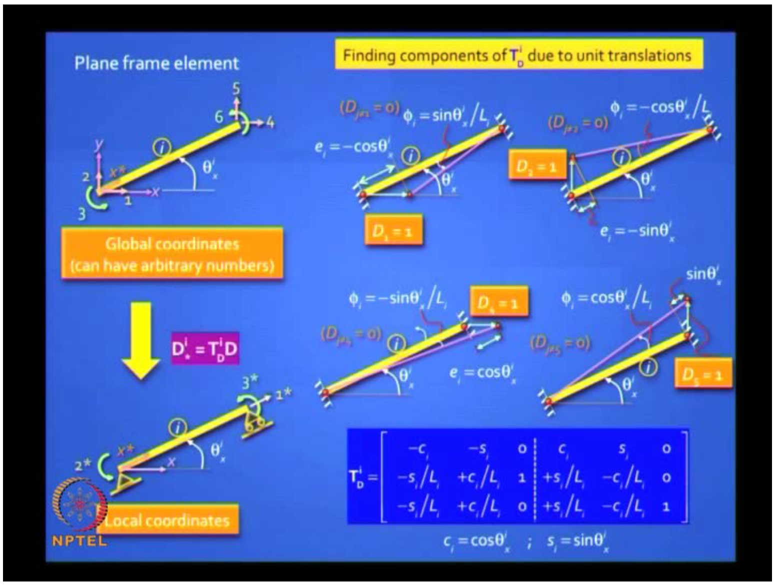

Ok incidentally you this matrix has an inverse and the inverse you get is the flexibility mat[rix]- so you have a axial stiffness there you have torsional stiffness there you have flexible stiffness components there then we this is a slide i borrowed from the plane frame you remember you here you have to worry about chord rotation and that is the tricky thing especially when inclined element

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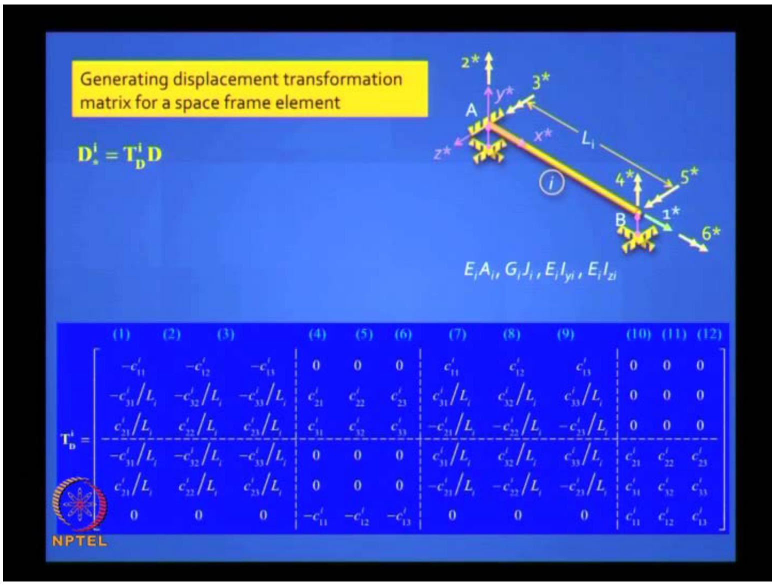

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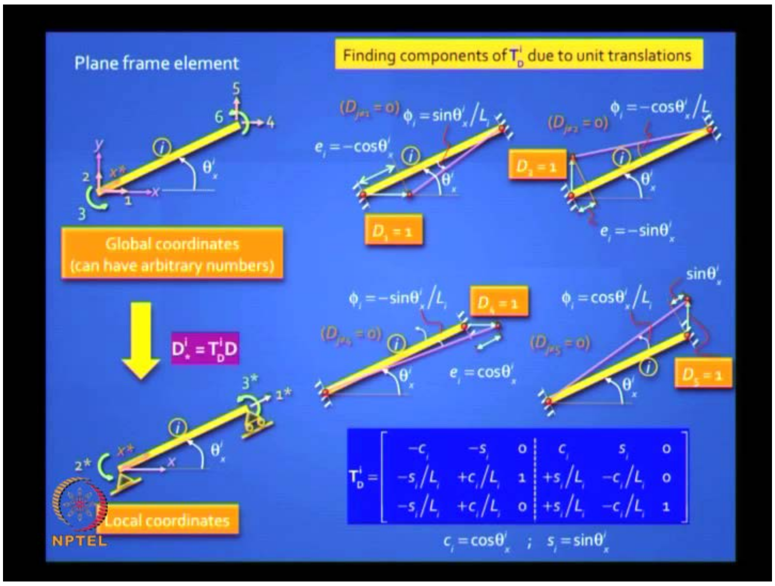

If you extend into a space frame that is what is going to look like that you figure it out yourself later because it takes a while to understand what get lifted up is it clockwise or anti clockwise but, it can be done can be done systematically you can do it but, you see it is not so nice as this one this is so easy so we did lot of problems with this in plane frames

We are not going to mesh up with space frame because if you make one mistake anywhere you will lost everything so do conventional stiffness method yeah.

(( ))

The local coordinates you need six local coordinates right because the rank of that conventional stiffness matrix is six so you have to arrest six degrees of freedom you have to cleverly choose those six degrees of freedom then only that structure is stable then only it can take any arbitrarily load in any direction anywhere

So here what we did was we picked up whatever we learnt from the plane frame you you are familiar with the plane frame element you are familiar with a grid element we just had to add some of constrains for the horizontal bending that is how it is worked out do not break your head too much over it reduced element stiffness method is not recommended for space frames do conventional stiffness method expect when you have easy problem.

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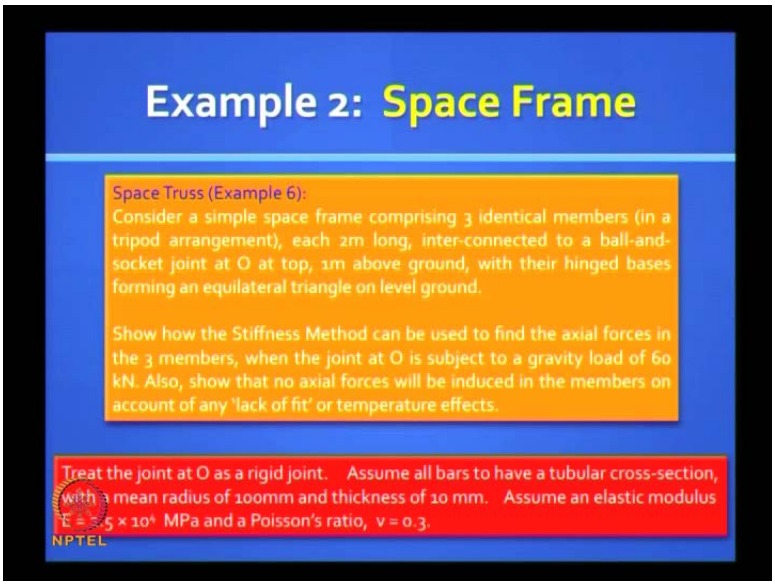

So let us look at one easy problem you can also if you find this difficult and i am show it is difficult there is a static approach where you can get the transpose of this matrix by the contra gradient principle you know that let us take a simple case remember we did a tripod problem beautiful problem which was a space truss now all we do is make that joint rigid become space frame simple problem

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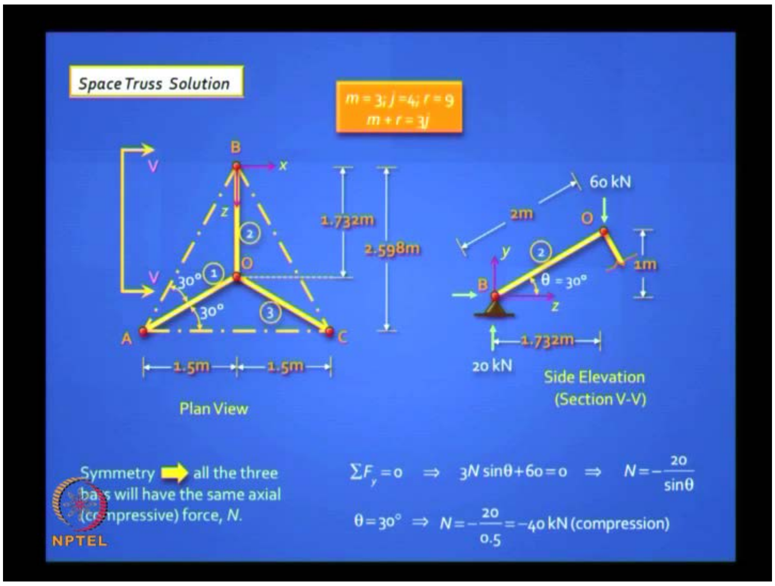

Just to end up on a happy note we will do one simple problem so it is a same old problem except that treat that joint at o as a rigid joint assume all the bars to have a tubular cross section earlier did not matter what cross section it had because it was statically determinate with a you know same hundred m m and thickness of ten m m tubes elastic modulus is given poisson's ratio is given its steel tube so this is what we did space truss and actually we had solved it using a flexibility approach because it is statically determinate.

Remember all the legs are identical and the force in each leg the vertical component is twenty kilonewton because you have sixty kilonewton hanging from the that ball and socket joint and then you can work out the force in the the axial force turns out to be forty kilonewton compression all the three legs have forty kilonewton

What do you think will be the answer if the joint o is rigid you'll get some bending moment you will get some shear force will you get some twisting moment no because of symmetry you would not get twisting mome[nt]- will you get bending moment in the perpendicular plane you will get bending in the vertical plane no doubt for each of them each element will be identical right

So you can do lot of tricks to simplify the problem but, my question to you that forty kilonewton answer that you got will you get a different answer once you have how much will it change.

It will not change

Some change must be there between rigid joint in that please note it will not change if you assume axial deformations are negligible

If you assume axial deformations are negligible you are dealing with a funicular structure there will be no bending no shear force it is not possible so funicular arrangement whether the joint is rigid or hinged makes little difference but, if you have a axial deformations this point can come down then you can have bending and then things can change but, in practice you will find that change is not going to be much.

That is the reason why it is it is that just a justification why even trusses truss members you you weld to one other they actual rigid we model conveniently as pin jointed because those secondary stresses you get because of the rigid in the connection i am not very significance so let us prove that.

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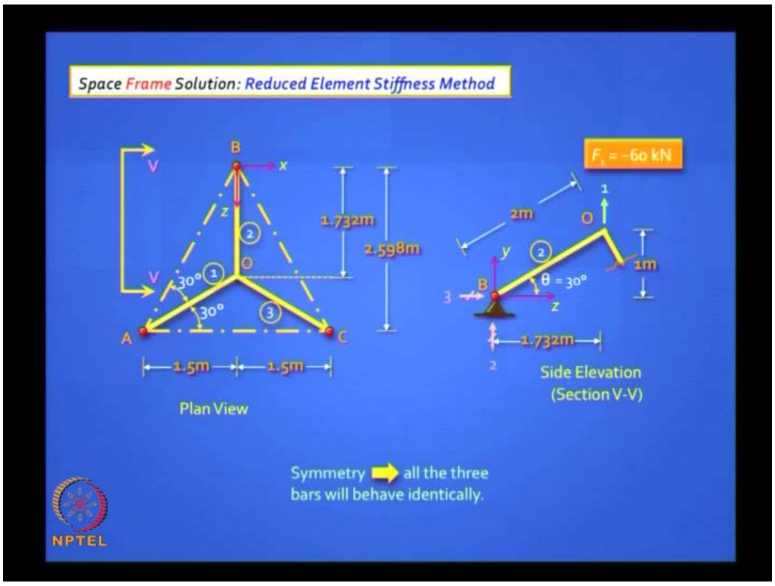

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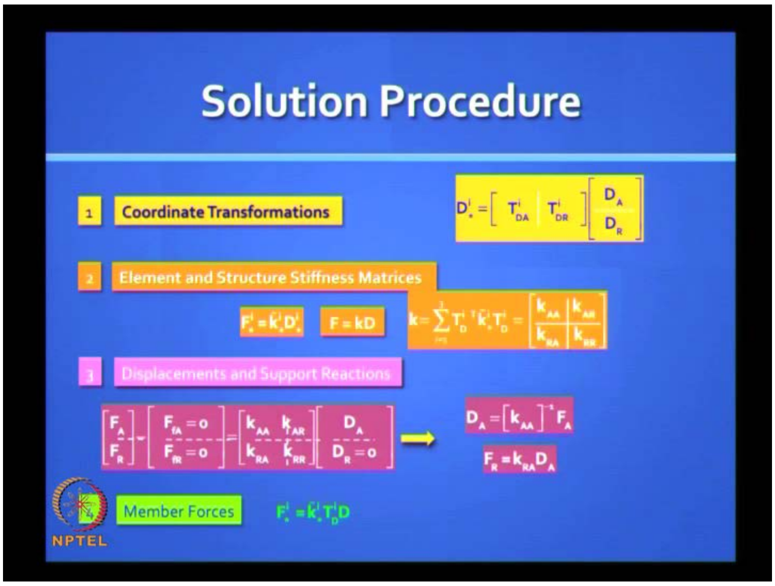

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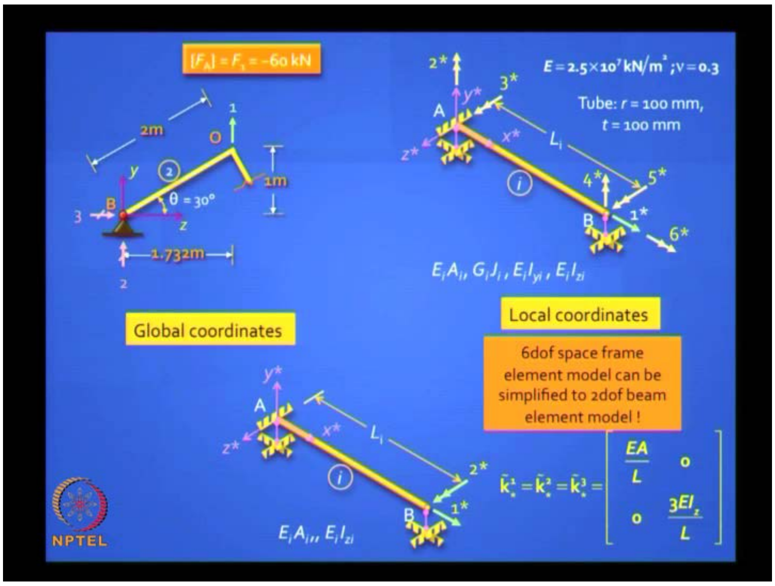

So we are now doing a space frame solution by the reduced element stiffness method that joint is now rigid there is symmetry and the solution procedure is the standard procedure no support settlements no indirect loading no fixed end forces so we will use this six degree of freedom element but, since we have only bending in one plane you can reduce that element to a to a beam element so that is a phenomenal simplification you could do.

So such problems do by reduced element stiffness method do not do by conventional because you can i mean but, you are using a in an elephant so for hammer to drive a nail so what do you do this becomes simple you have just e a by l and three e i by l because the bottom is hinged so you take advantage of a hinge also, it is a two by two matrix not twelve by twelve not six by six so it is easy.

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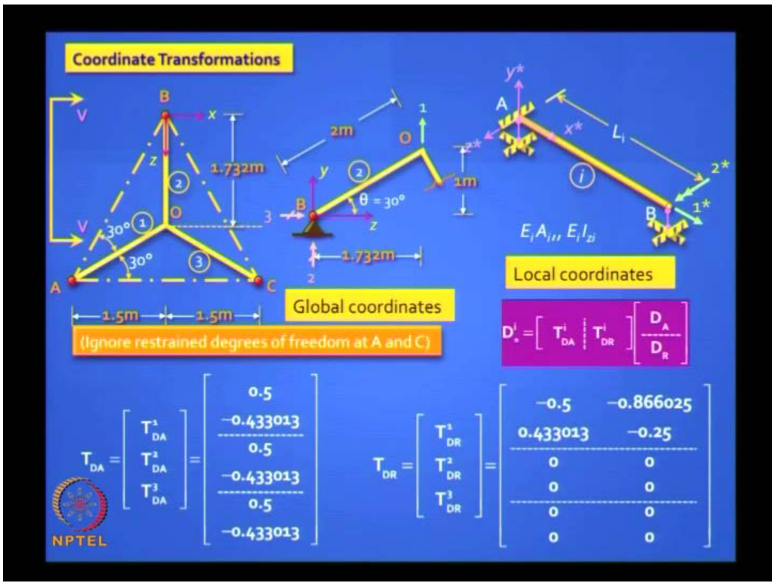

You generate those matrices from first principles you can do that and we are looking at one of those support reaction because we want them pointing in the y and z direction when on looking at the other two because all are identical

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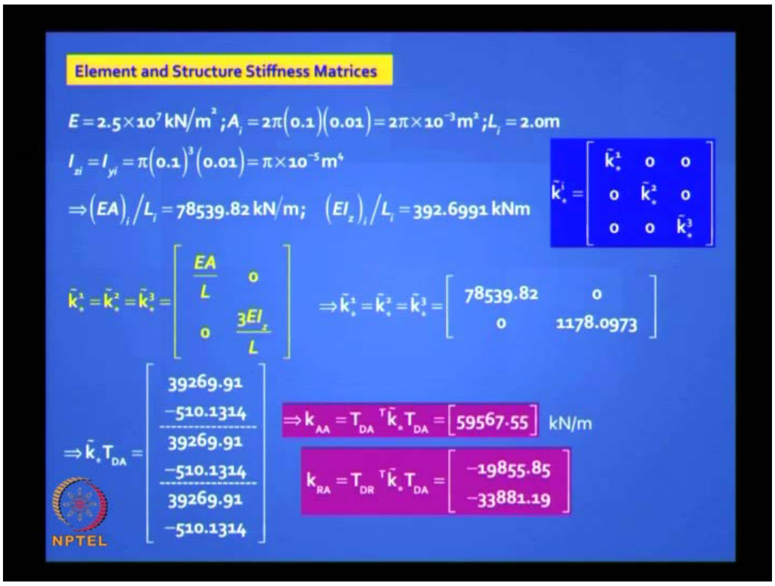

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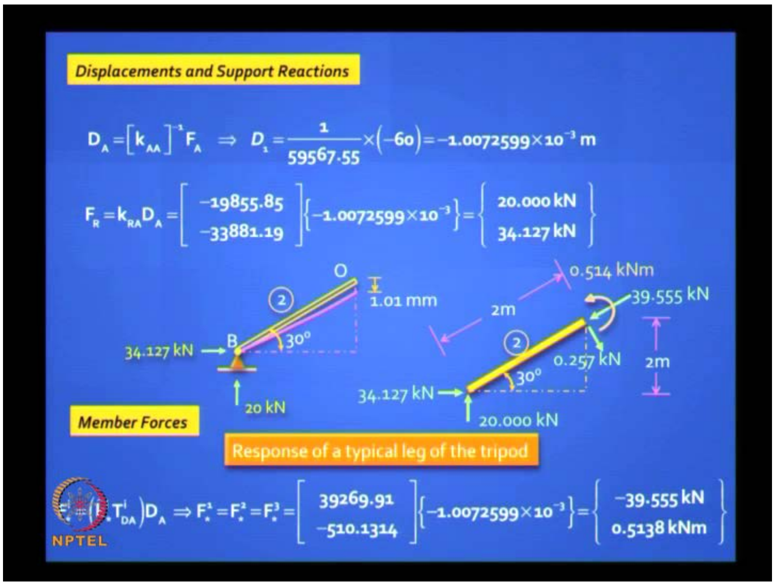

Write down the t d e matrix write down the element stiffness matrix generate your structure stiffness matrix these steps are and finally, solve and find your displacement look at the answer you are getting vertical reactions will always be twenty because three times twenty you must add up to sixty no question about that

but, your axial force need not be forty so you'll find that the that joint goes down by one mm one mm which is realistic and your internal forces you can calculate and what do you get instead of forty kilonewton you get thirty nine point five five five big deal so you still design for forty you are right we will end up and what are those moments like very small negligible.

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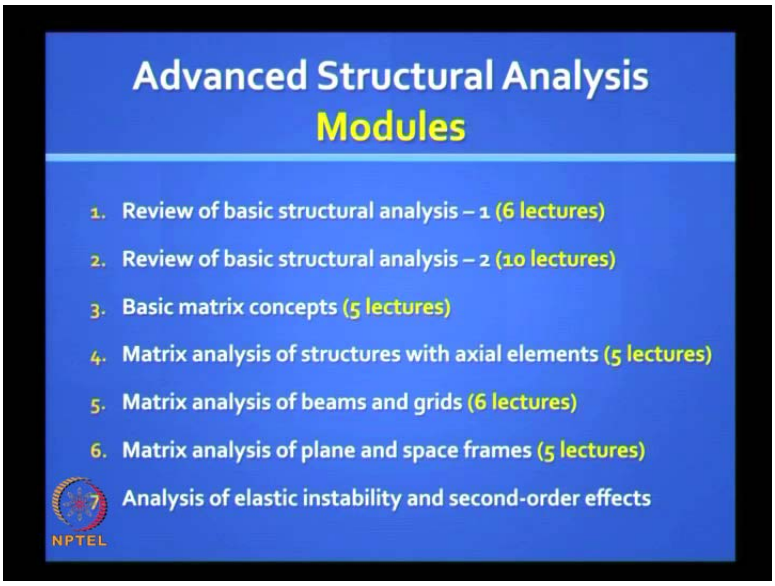

That is the reason why we design trusses as trusses even though the joints may be welded and frames so we have finished we come a long way we've finished six modules the toughest module we finished today and the next three sessions we will cover cover more from a conceptual point of view you do not have to study for your exams expect the concept on elastic stability and second order frames.

Thank you.