Advanced Structural Analysis
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Module No. # 6.4
Lecture No. # 36

Matrix Analysis of Plane and Space Frames

Good Morning.

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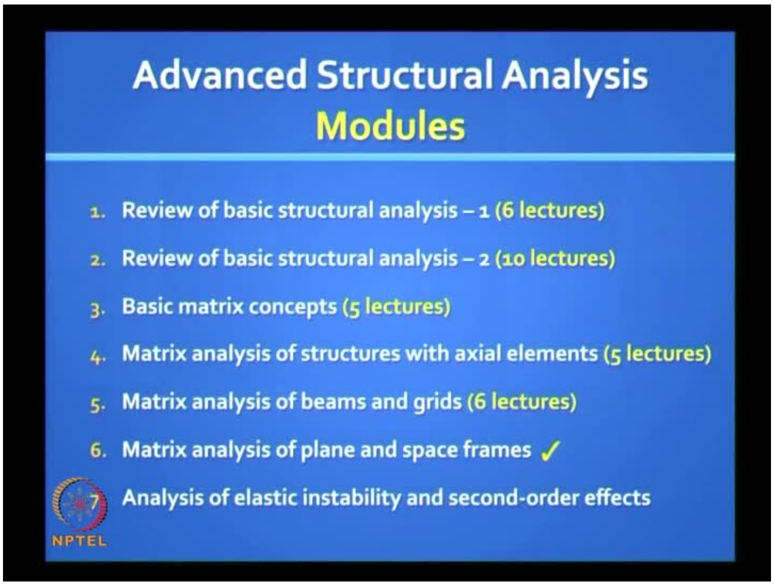

This is lecture number 36, module 6, Matrix Analysis of Plane and Space Frames. If you recall, in the last session, we covered the reduced stiffness method;

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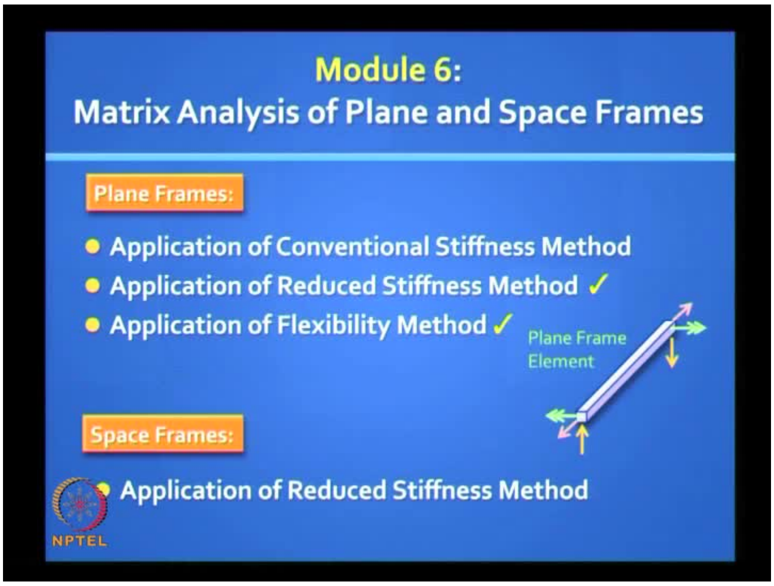

We will continue with that method, and also complete the flexibility method, as applied to plane frames.

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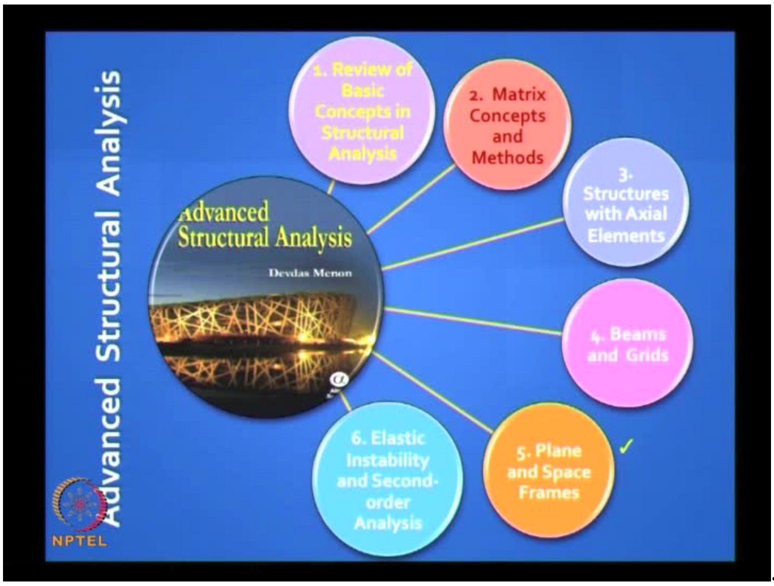

This is covered in the chapter on plane and space frames, in the book on Advanced Structural Analysis.

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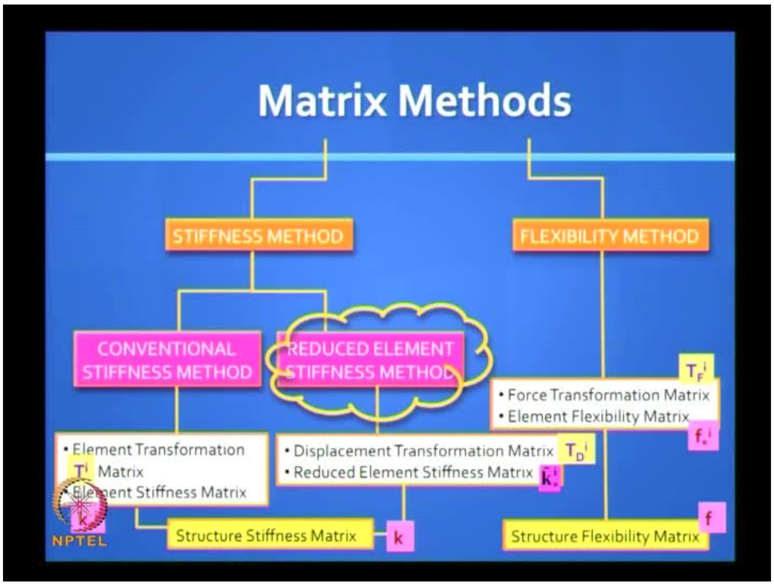

So, this is the reduced element stiffness method.

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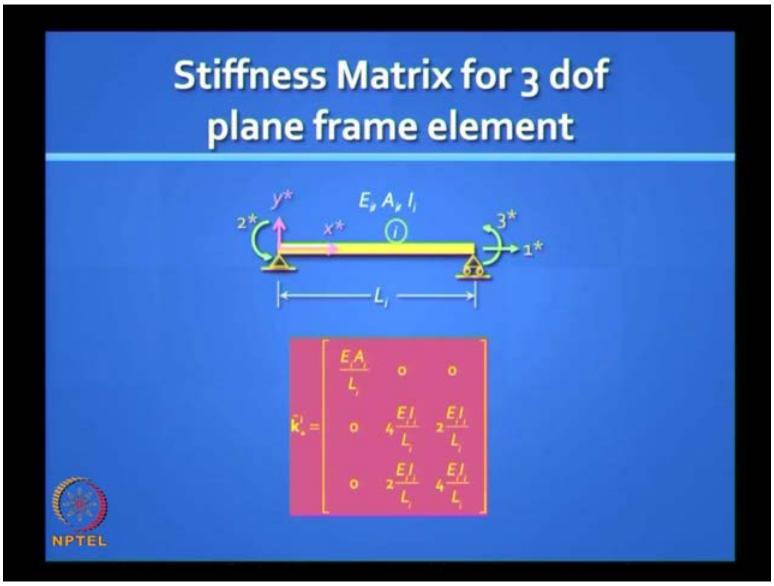

You know that the size of the plane frame element stiffness matrix is 3 by 3. We have done a few problems.

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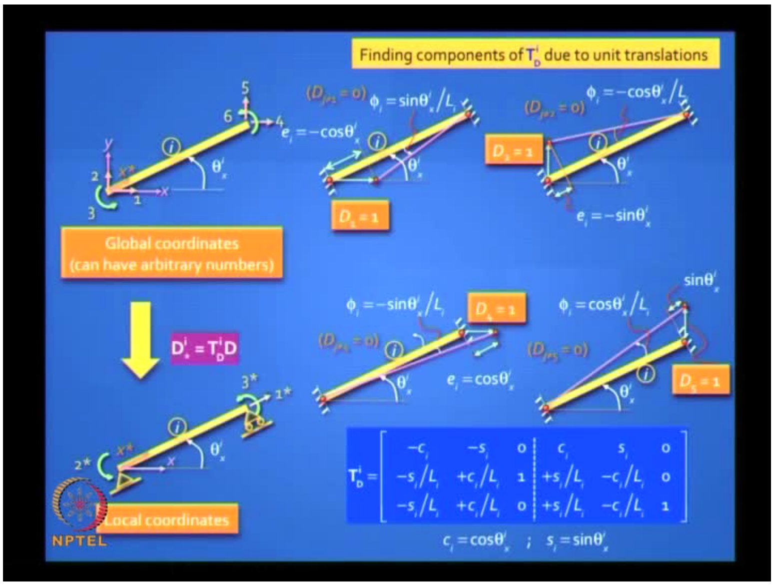

This is how you do the transformation the T D matrix in the reduce element stiffness method.

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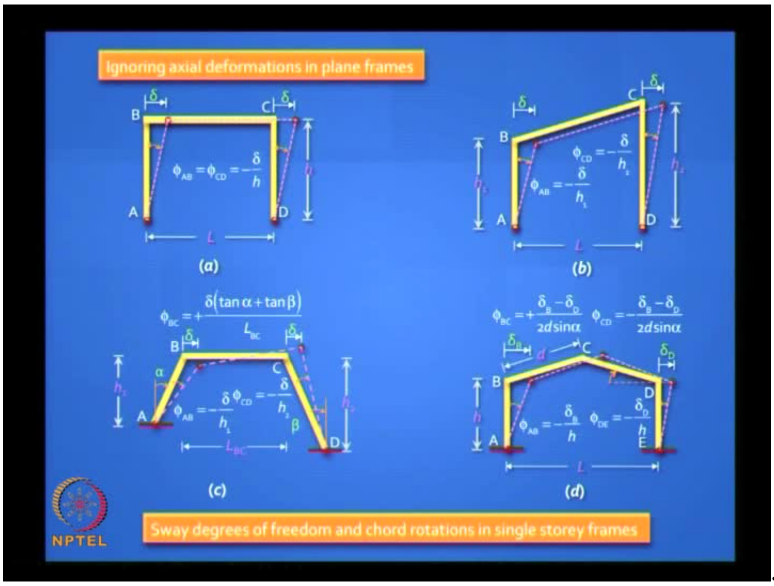

Now, let us look at a problem with sloping legs, which is actually a complicated type of problem, when you apply the reduced stiffness method. So, you have to intelligently identify the sway degrees of freedom; we have discussed this earlier. And you need to express the chord rotations in terms of the identified sway degree of freedom.

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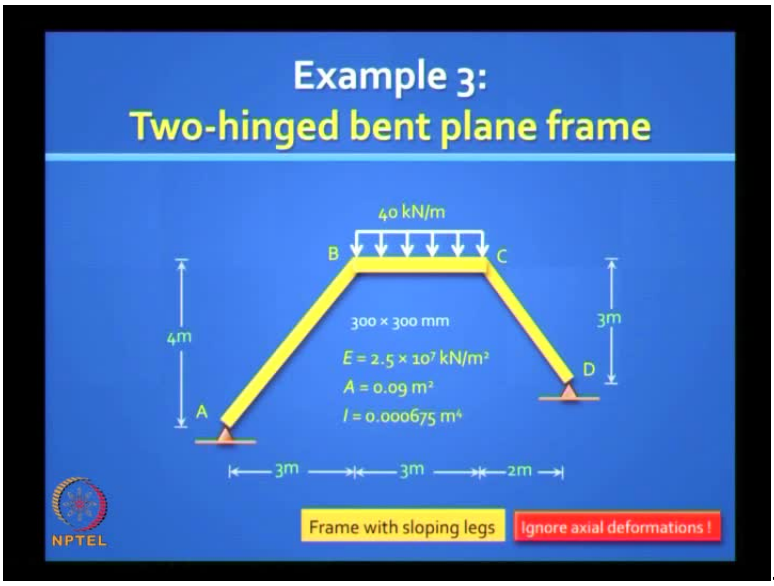

So, let us look at this problem which we have solved earlier by the more regress method. So, let us solve this by the reduced element stiffness method and let us a take advantage of the fact that we can ignore axial deformations. We have solved this problem earlier by the slope deflection method. What is the degree of indeterminacy, kinematic indeterminacy?

Shortest needed degree of kinematic indeterminacy refers to the active global coordinates. The absolute minimum when you have when you do not have axial deformation.

8, 5, 8 [Noise]

Ok.

Five

At the supports A and D, do you need to have anything?

No, No.

No. We do not even bother about finding reactions. We do the minimum work. with So, at B and C, how many do we have?

[Noise]

At each of them.

[Noise]

You have a rotation at B; you have a rotation at C.

Vertical reaction.

And you have just one sway degree of freedom; that is what we did in slope deflection method; remember - theta B, theta C, and delta BC. Correct, but we have done this problem by the conventional stiffness method. There the degree of kinematic indeterminacy was - at B and C, you had each 3 degrees of freedom; 3 plus 3; you also had a rotation at A and D; it was 8.

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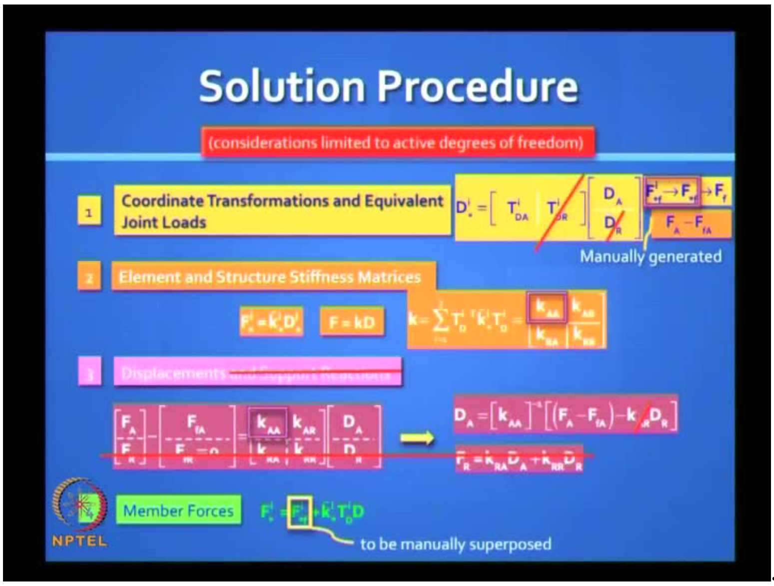

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So, we have, we are reducing 8 to 3, but we are ignoring axial deformation; that is a major reduction. The method of solving is, as we have discussed earlier, the steps are the same plus we take advantage of the fact that, we limit the considerations only to active degrees of freedom because we can always get the support reactions at the end from the free bodies.

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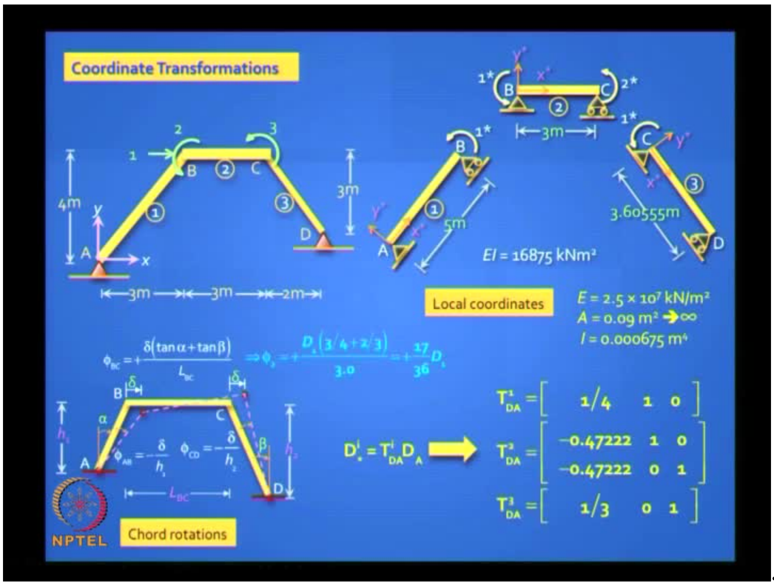

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So, let us do this problem. So, I have got 3 degrees of freedom; global coordinates 1, 2, 3; that is all I need. Local coordinates, again, I take advantage of the fact that the there is a hinge at A and D.

So, my 2 degrees of freedom B model will now become 1 degree of freedom model. What I have achieved is massive reduction in the quantum of work A, I am not going to use a plane frame element; I am using a beam element. A plane frame element has 3 degrees of freedom in the reduced formulation, but I am not worried about the axial degree of freedom. So, the 3 becomes 2.

The plane frame element becomes a beam element and I am using that formulation for element number 2 because there is a fixed end force coming in there. There is an intermediate load, and B and C are not hinged. But for elements 1 and 3, I take advantage of the fact that, I have a hinged support at the ends A and D where, the bending moments are 0. And so, you know, I can take advantage of a reduced stiffness. What is the element stiffness for that flexural stiffness? 3EI by L. So, I just need 1 degree of freedom which I marked as 1 star; is it clear?

So, with this, I can proceed. I first need to write down my T D matrix T D A matrix. Will you try that?

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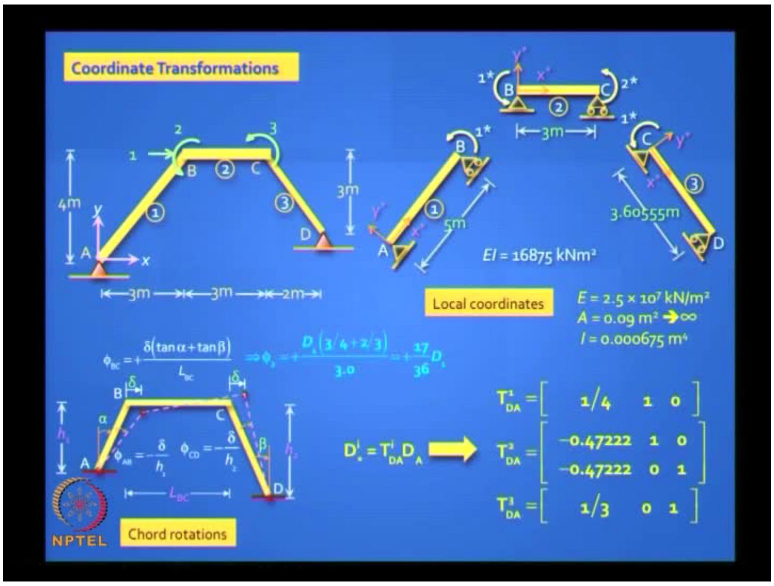

To do that, you have to be very careful. When you are dealing with sloping legs, you have to correctly express the chord rotations in terms of the sway. So, the displacement D 1 corresponds to the delta in this figure. right The figure at the bottom is a generic figure.

So, you have chord rotations of delta by the height for the elements 1 and 3. right And for the element 2, you have to work this out using trigonometry; it turns out to be as shown in that formula delta tan alpha plus tan beta by the span L. I am not going through this all over again because we did exactly this, when we did the slope deflection method. So, if you plug in the values of tan alpha and tan beta because this the dimensions are shown there, you will get the chord rotation for the element 2 as plus 27 by 36 times the sway D 1.

Just plug in the values of tan alpha tan beta; this, the span of that element BC is 3 meters. So, you will get this. Shall we proceed? We have done this earlier. It is just a repetition. We put a positive sign because you are getting an anticlockwise chord rotation for element 2, but please note - for elements 1 and 3, the chord rotation is clockwise; clockwise; chord rotation is clockwise. So, and it is given by delta by the respective heights.

Now, can we write down the T D matrix. We can So, the T D A matrix for the three elements are as shown there. If you apply D 1 equal to 1, then for the first element, the chord rotation is 1 by 4; it is a clockwise chord rotation. You get equivalent anticlockwise end rotations and it will be plus 1 by 4 because anticlockwise is assumed to be positive in the sign convention.

The same is true for the element 3. So, you get 1 by 3 because the height is 3 meters. As far as the element 2 is concerned, it is 27 by 36. right It is We just worked it out know 5 2; it is 27 by 36, if you expand it, sorry 17 by 36, it turns out to be 0.4722. But it is a clockwise rotation. So, your equivalent end rotation will be, sorry, it is an anticlockwise chord rotation. You are you get clockwise equivalent end rotations. So, it turns out to be minus. This is the only difficult part; that first column in your T D matrix has to be very very carefully written because it is totally dependent on your correct assessment of the chord rotations; both magnitude and direction.

The others are simple. this If you put D 2 equal to 1, we are now talking about the second column in the T D matrix. Then, you will find that you get it will effect only elements 1 and 2. The right end of element 1 and the left end of element 2. So, you get… and element 3 is [un effective] So, you get 1 1 0 0, and similarly, for D 3; is it clear? This is easy to generate. Can I proceed?

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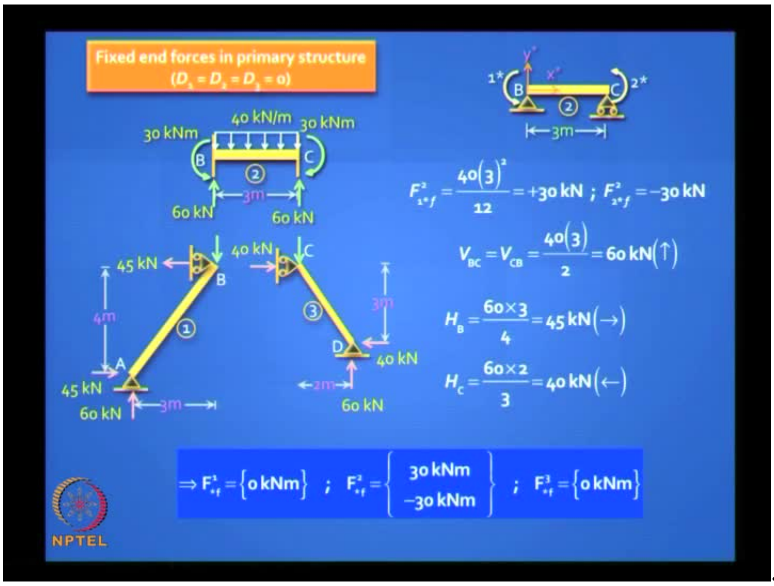

You got your T D matrix. Now, you have to carefully write down your fixed end forces. We did this earlier, and remember, we just calculated the fixed end moments. But here, you have to be careful because you have only 3 degrees of freedom D 1, D 2, D 3, and you do not have those vertical forces which we put on the last occasion. right There are no vertical degrees of freedom. So, you have to manage with rotational degrees of freedom and the sway degree of freedom. You have to handle the problem in that framework. So, how do you do that?

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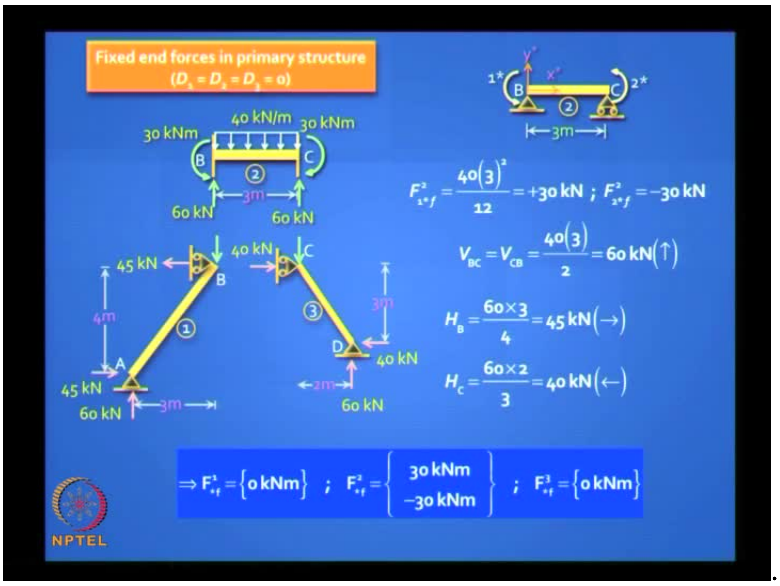

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Well, first of all, from this analysis, you can find out that there are going to be no fixed end forces for elements 1 and 3. Why? Because there is no intermediate load, and for element 2, the fixed end moments are 30 kilo newton meter minus 30 kilo newton meter and there is no axial force. So, this is what you get from this diagram, but how do you include the effect of axial force here? You have only 1, 2 and 3 in the global coordinates. How would you transfer this understanding to equivalent joint loads, the global coordinates?

What is In other words, what is F 1, F 2, and F 3? F 1 f, F 2 f, F 3 f. What is F 2 f? F 2 f is plus 30 because it comes from element 2. What is F 3 f?

[Noise]

Minus 30. What is F 1 f? What is f one f?

[Noise]

It is not 0. If it is 0, then you make a mistake. What is F one f?

40 [Noise]

Yeah. You look at what is happening at B and C. You have restrained those degrees of freedom at B; you get a horizontal reaction to the left of 45 kilo newton at C; you get a horizontal reaction 40 kilo newton. What is happening to the whole beam?

So, you will get a net force of 5 kilo newton acting to the…

[Noise]

Left or right.

[Noise]

Okay. To the left; you are right. But, when you find the net loads, you have to oppose everything; you put a minus sign. Have you got it? So, that is it.

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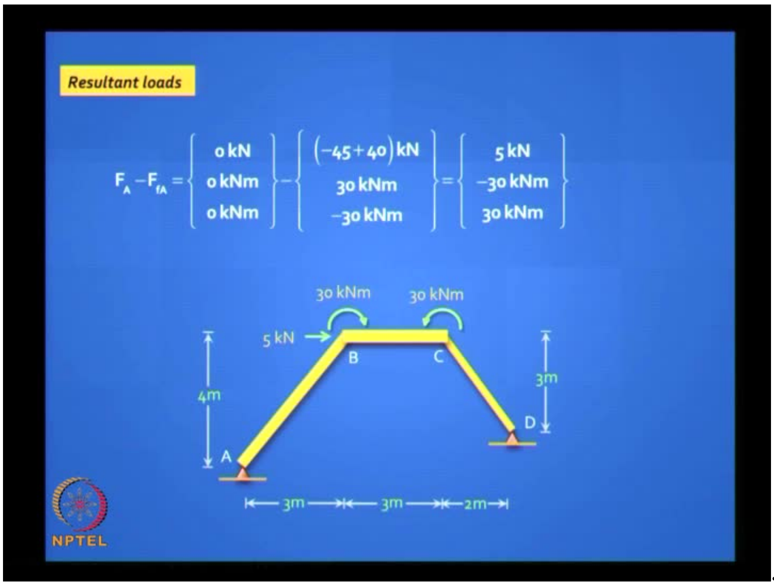

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Look at this. It is completely different from what we did earlier, and you are going to get the same results. So, there are so many different ways of solving these problems, but sloping legs problems are tricky. They are notorious; students generally make blunders;

practicing engineers do it. But luckily, for you, today you have nice software which is the black box to most engineers. We just feed in that input and you get the output, and you hope all is well.

But we are learning and we are learning to do these problems by different methods including flexibility method. And we know one thing for sure - the answers are have to match the answers have to match. We will expect some difference between ignoring axial deformation case and including axial deformation case, and that will be a little more pronounced when you have sloping legs. We will see that; apart from that, you should get the same answers we got when we did the slope deflection method.

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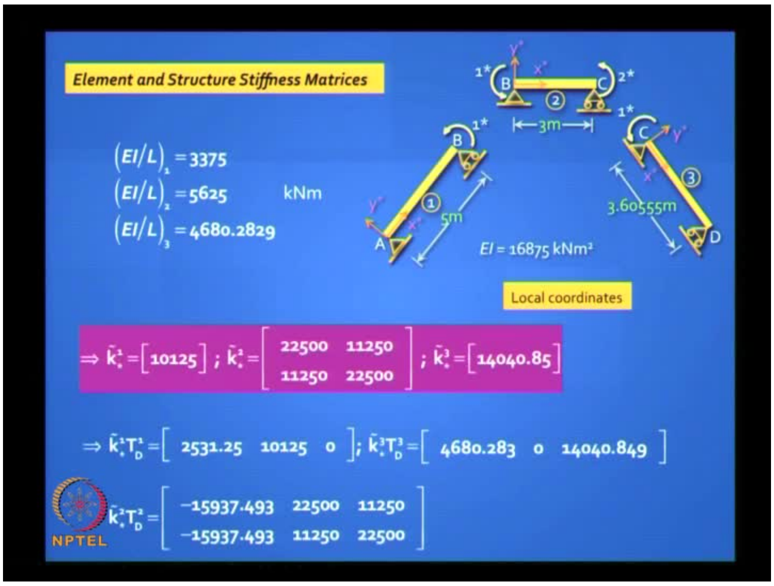

So, we proceed. We write down the element and structure stiffness matrices; very easy to write down; EI by L for all those values are known, and 3EI by L is what you will put for elements 1 and 3. And for elements 2, it is a standard 4EI by l, 2EI by l 2EI by L, 4EI by L; is it clear? We can generate this and then carry out these products, and do the final super position and assemble all the matrices. We will get the answers. This I am going fast because you know how to do it.

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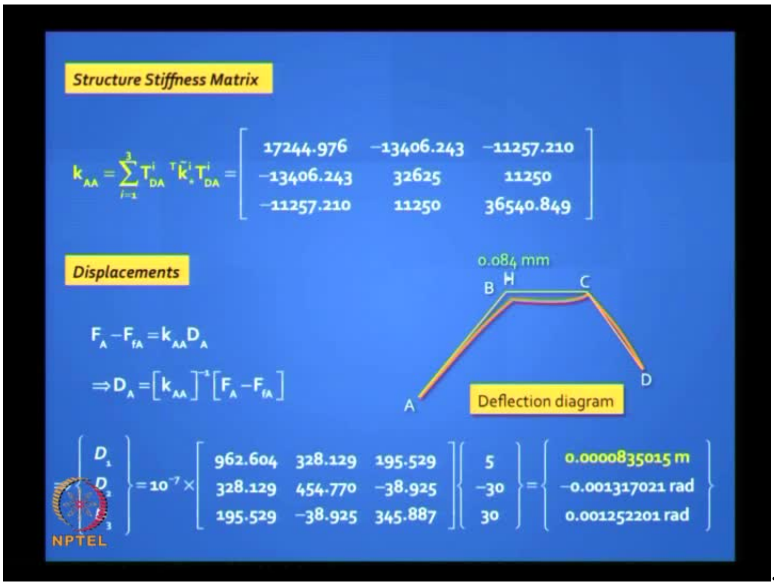

Now, you are ready to take any load. You give any D 1 D 2 D 3; you get any F; F 1 minus F 1 f; F 2 minus F 2 f; F 3 minus F 3 f; that is your net load vector. You will get the solution with the power of this matrix. So, we will do that. We will invert this matrix; plug in those values of 5 minus 30 and plus 30. You got those answers. And that is what your deflected shape looks like.

Now, you find that, that deflection, if you compare with the earlier result has the little discrepancy, but that is you have to live with that because that is the approximation you get from ignoring axial deformations, but this is exactly the answer you get when you do slope deflection methods. right

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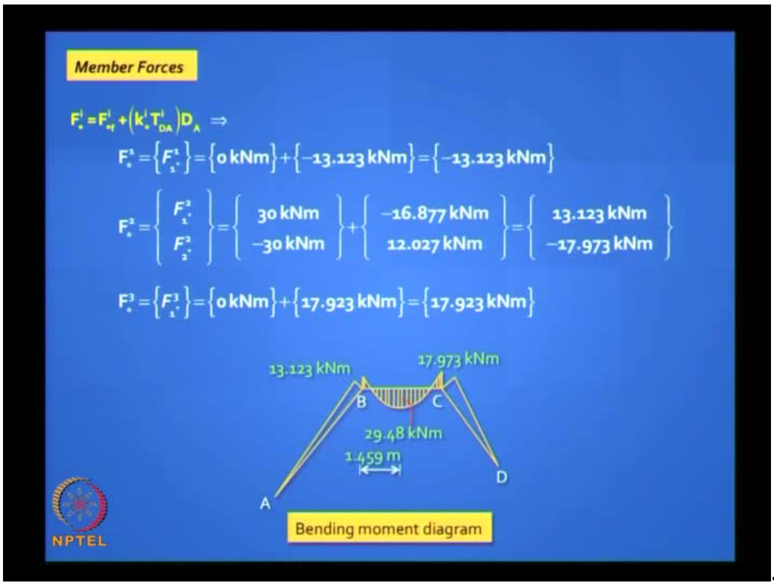

It is also exactly the answer you get from flexibility method, ignoring axial deformation. If you do a strain energy formulation, you will get this. Then, you find the member forces when you draw your final bending moment diagram. The bending moment diagrams do not change significantly; it is only if the deflection [that] ok

You know how to handle this problem. You got a similar one in your assignment. You got a hinge, support, you have a support settlement also, but luckily, for you, you do not have sloping legs. They are vertical. So, please do those two assignment problems and you will get a good understanding of stiffness method.

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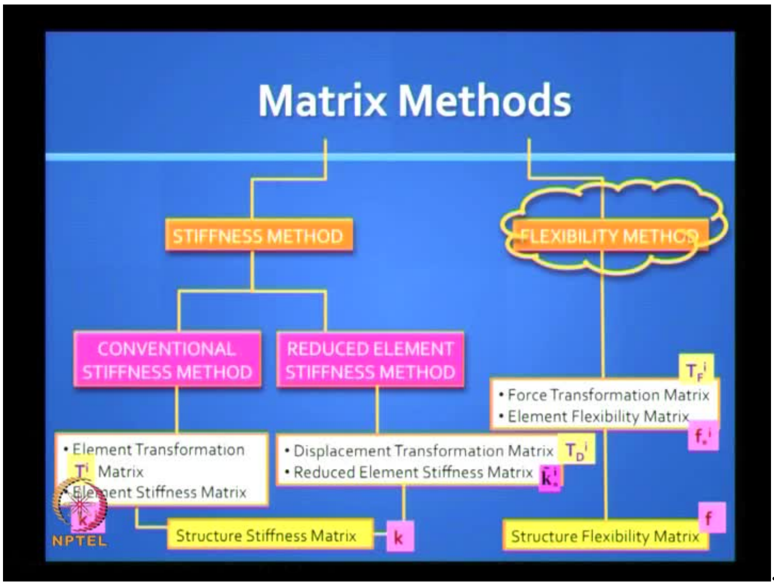

We now look at the last remaining topic of flexibility method, not applied commonly in practice for frame. We do it for beams; we do it for trusses; frames it is not common, but the logic is the same, and at least conceptually, let us learn it. You will find that in your assignment. I have not given a problem on flexibility method; in your final examination, it is unlikely. I will ask you a question for plane and space frame, but, let us just go through the theory, and see how it works.

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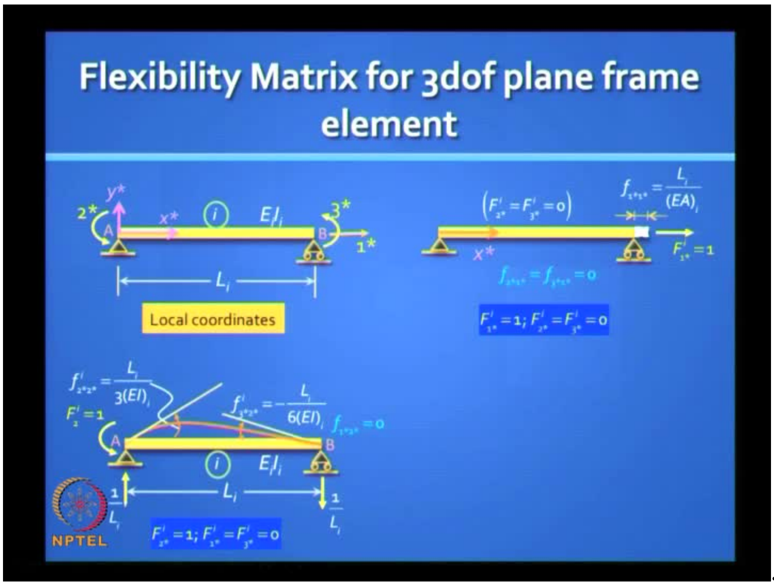

First, you need to identify the degrees of freedom. You know that, it is the same as you use for the reduced stiffness method; 3 degree of freedom system. You can generate the flexibility matrix from first principles. You apply 1 unit load at a time. Here, you apply loads. You can do one thing. You can take your stiffness matrix and invert it; you will get the flexibility matrix, but that is no fun.

Flexibility is for people who want fun because you want to understand exactly what is happening. So, here, when you pull it, you are not going to get any moments. It is just like an axial truss element. You know that the deflection, the lateral elongation you get is an extension in that element. It is the flexibility, axial flexibility l by E, Clear? And that is your f 1 star 1 star.

Next, you apply a rotation. Now, you are doing what you did for the beam. You get rotations of L by 3EI and L by 6EI. One will be anticlockwise; the other will be clockwise. You do not have any axial you do not have any axialchange in length. That is an assumption we are making because we are saying that the flexibilities do not interact. The axial degree of freedom and the flexural degrees of freedom are independent. That is an assumption we are making here. We will see through that assumption in the next module, and will bring in that interaction.

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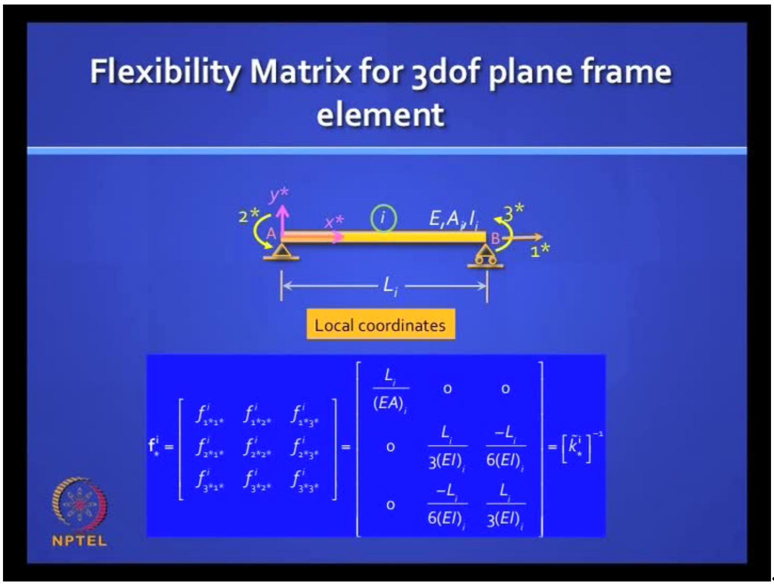

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Now, we accept this and you can do it at the other end, and you can generate your flexibility matrix. Very easy to remember because you know what you did for a beam element. you are know You know it is L by 3EI minus L by 6EI L by 3EI and minus L by 6EI. You just have to add the axial stiffness as you did in the stiffness matrix L by EI.

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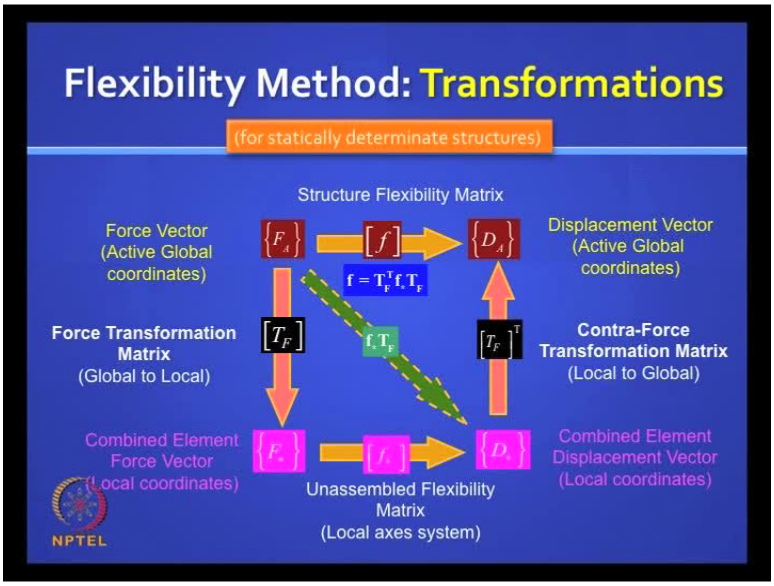

Then, you do the transformations and standard procedure. let us This is for the beam element.

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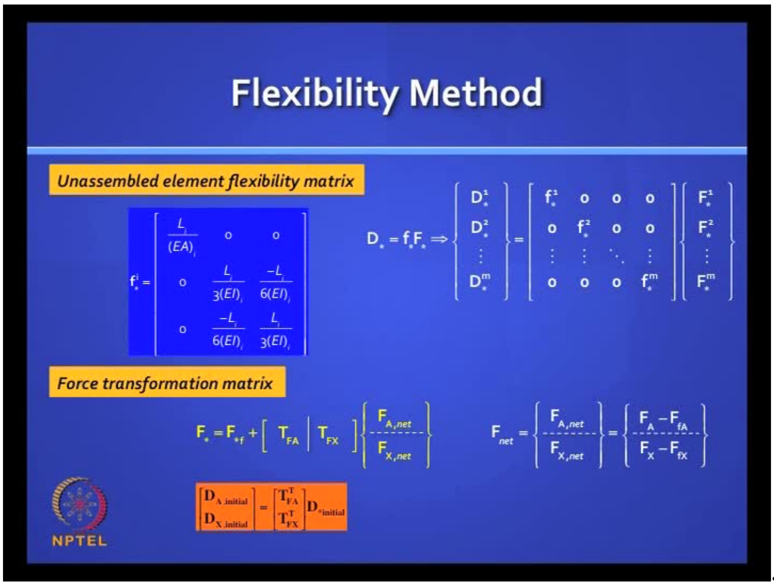

For the frame element, it will have 3 degrees of freedom; the rest of the procedure is the same.

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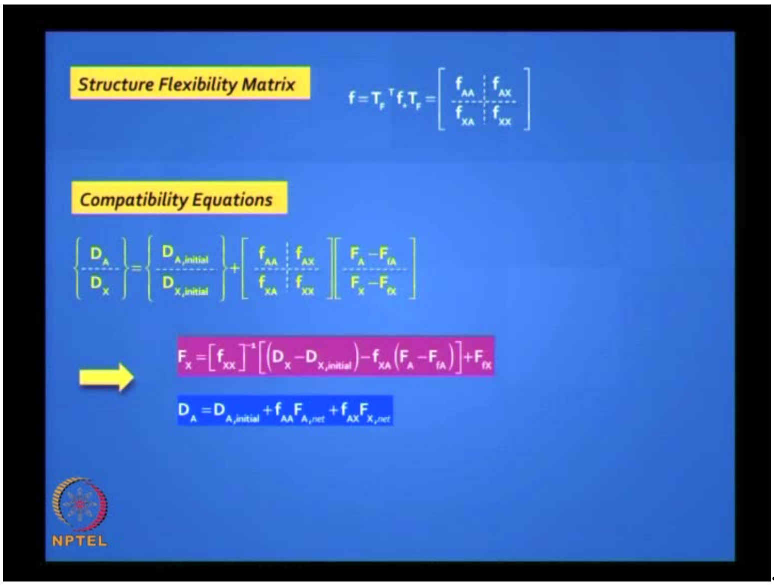

You generate the structure flexibility matrix. You write the compatibility equations. You solve and find the redundants; then, you can find the joint displacements if you want to. In the stiffness method, you had no choice. You have to find D A to proceed to get the member forces; here, you do not need to find. By solving that second equation, you get the redundants. If you can draw the free bodies and get your member end forces, you finish everything.

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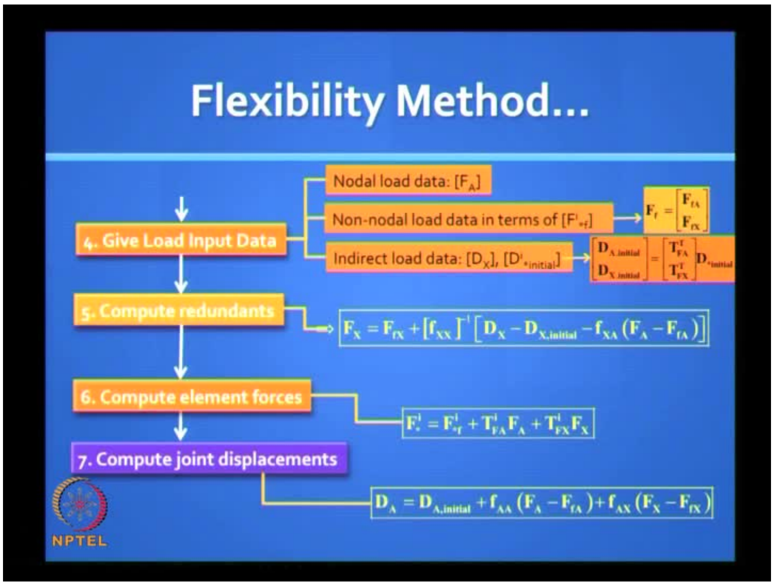

So, this is the procedure. The displacements come at the end.

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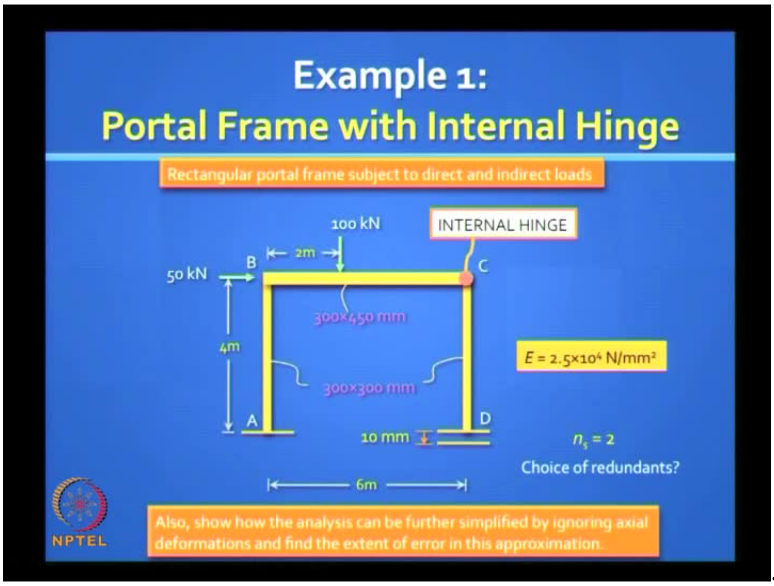

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Now, let us do just one example with and without axial deformations. Now, please note, when we did the method of consistent deformation, when we did the theorem of least work, we assumed axial deformations were negligible. So, we are now doing for the first time flexibility method with axial deformation.

So, let us see how to do that. What is the degree of kinematic indeterminacy? Here, we put in two complications, just to see how to apply this method; one is you have a support settlement; the other is you have an internal hinge. So, if you did not have that hinge, you know that the degree of static indeterminacy is 3. That hinge brings your moment release. So, it drops to 2. Now, you can choose your two redundants.

Which would you choose? You want the primary structure to be a cantilever.

[Noise]

Then, that third element will be dangling in the air. It will be unstable. Can you please think carefully? I mean you preferred keeping the fixed end, A as fixed end right; that is what you meant by cantilever. Mind you, you can still do that, but at D, you do not remove that support. What kind of roller?

[Noise]

You want the roller to roll vertically or horizontally.

Horizontally, vertically.

You have a choice; not only that, you can choose any two. So, let us do something interesting in the next slide.

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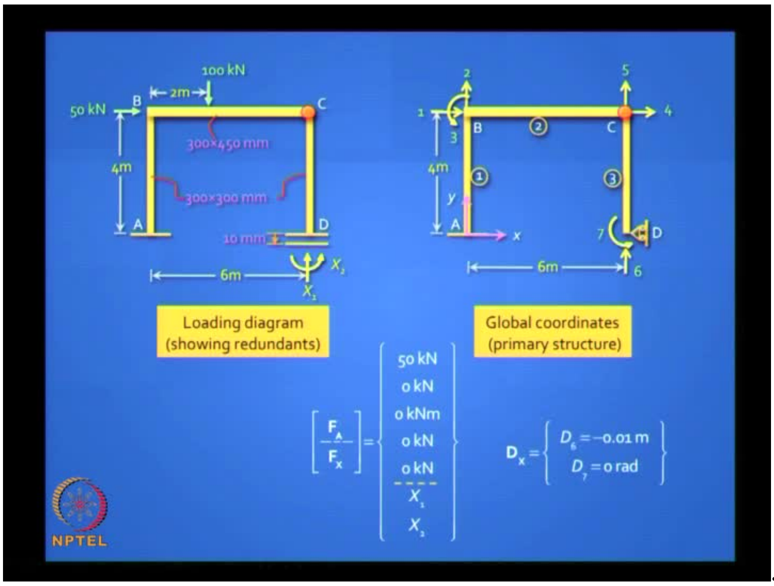

So, I am going to choose my redundants x 1 and x 2 as the vertical reaction, and the fix and the moment at the support D which was I have to in my primary structure. I have to I have to release those two corresponding restrains to displacements. So, what is the support I will put there?

[Noise]

Ah.

[Noise]

Put the roller in the vertical direction; put the roller in the vertical direction, and number your coordinates appropriately. So, we know, you know that we have always put the redundants at the end of the list. So, we have 1, 2, 3 degrees of freedom at B, 4 and 5 at C. There is no rotational degree of freedom. You cannot bring it there because you do not have a unique rotation there. Each of those two elements will rotate by different amount. So, you do not number it.

We did the same thing in the reduced element stiffness method, but here, we add 6 and 7 as redundant coordinate. So, this is how it will look, if you write the force vector. We separate F A and F x. F A is the load applied. It is 50 kilo newton because there is a horizontal load at B; rest of the quantity is the 0, and you have x 1 and x 2. x 1 and x 2 are unknown. Any question? ok

You also have to write down the compatibility equations to solve for the unknown redundants x 1 x 2. What are What is D x? What is D 6 and what is D 7?.

[Noise]

Right D 6. that is why you bring in the support settlement D 6 is minus 0.01 meters and D 7 is 0; yeah. You have a question?

[Noise]

I am not ignoring. Go back to the question. We said we will first include; then, we will ignore, and then we will see the difference. So, I had we will remove that in the next phase. Right now, we will include.

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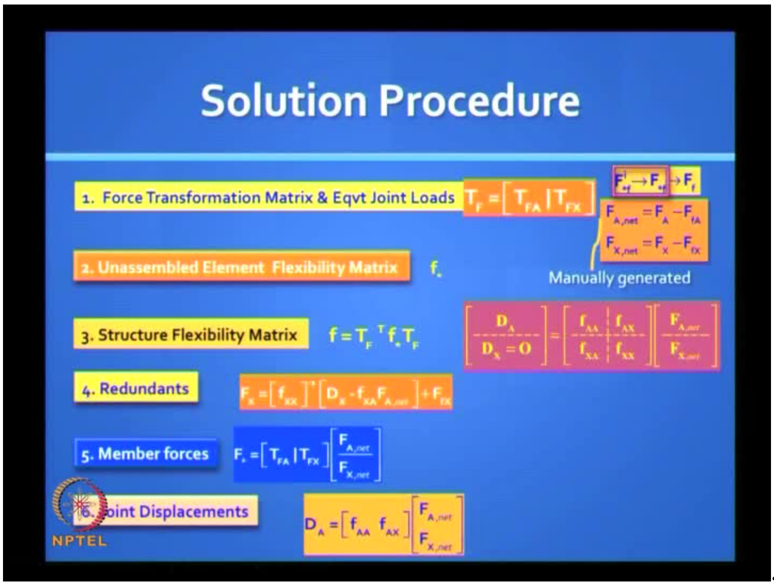

So, the procedure: First write down the force transformation matrix; find out the equivalent joint loads. You have to do the... This part is borrowed from stiffness method which is unfortunate because your flexibility method is not able to stand on its own. It has to borrow this concept because you just can handle intermediate loads in a matrix formulation.

Ah unassembled element flexibility stiffness matrix, structure flexibility matrix, T F transpose f star T F; write down the displacement equations, all the compatibility equations, get the redundants, get the member forces, and if you wish, get the joint displacements.

The procedure is very clear. The items 4 and 6 emerge from those set of equations. Then from the second equation, you get the redundant; from the first equation, you get the joint displacement. We have done this earlier; the procedure is the same.

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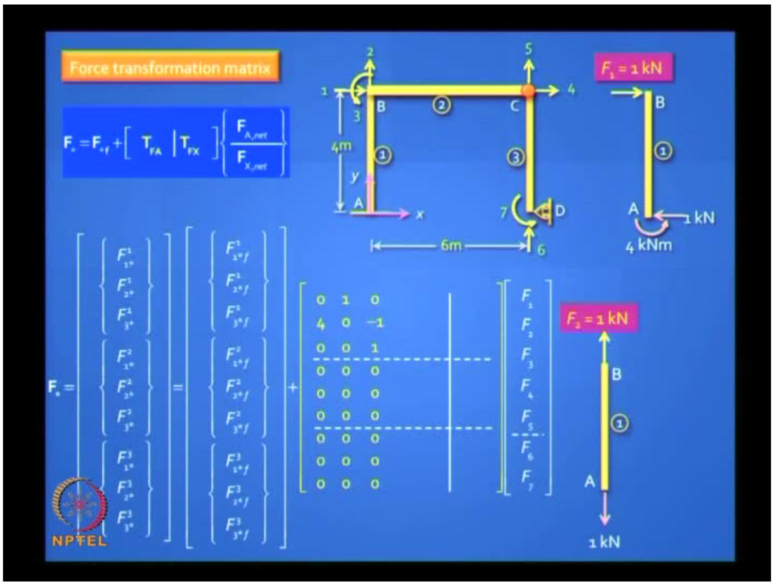

So, this is interesting. Let us generate the T F matrix. T F matrix has two parts; T F A and T F x; how do we do that? You have to apply unit load at a time. So, it will look like, if you apply unit load F 1 equal to 1, take that huge structure and apply. Do you think anything will go to the support D? No. So, it will be, only element 1 will behave like a cantilever. right

Am I right? If you apply F 1 equal to 1, and no other loads, then, you need not draw the whole frame; just pull out that cantilever as it is; you got those results. right

So, I am going to demonstrate here, how to generate the force transformation matrix. F star is f star f plus T F A times F A net plus T F x times F x net. Right. We will work out that F A net and F x net later, after we handle the fixed end forces, but right now, our task is to write down T F A and T F x.

So, I have written down the first three columns, corresponding to F 1 equal to 1, and I will show you F 2 and F 3 shortly. But you look at that first column 0 4 0. Is it right? Because only the first element will have non zero items; second element will not be affected; third element will not be affected because this is a cantilever action for element one. Agreed. So, that is why you got zeros here.

I am applying this load on this structure. Here, only this element gets effected. Elements 2 and 3 remain unaffected. So, the first column corresponding to F 1 equal to 1 will have 0 0 0 0 0 0 for elements 2 and 3. For element 1, these are the reactions I get if you go back, and I have not shown the element coordinates. Element coordinates would be… what would they be? They will be 3 by 3. right They will be 3 by 3.

So, you will find that the first item here, what does this correspond to? What does f one star correspond to? Axial force. Will there be any axial force in this element? No. That is why I put zero there.

Second element corresponds to what? Moment at this. Well, it depends on how you put your element, but it is here 4 here, right and the third one, moment at B 0; it depends on start node end node. I put start node at A. So, anticlockwise is clear; so, it is 4.

Now, let us take the second case: F 2 equal to 1. Can you try drawing the response for F 2 equal to 1? I am going to show it to you; F 2 equal to 1 means you are going to… what is going to happen if you apply F 2 equal to 1?

[Noise]

You will just get one axial force in.

First.

So, that is the simple thing. So, does it make sense? You get just one. You get just one here and the rest of it are all zeros. Clear? Very easy to fill up. Next one, can you try? F 3 equal to 1; F 3 equal to 1; F 3 equal to 1, will it affect element one only or …?

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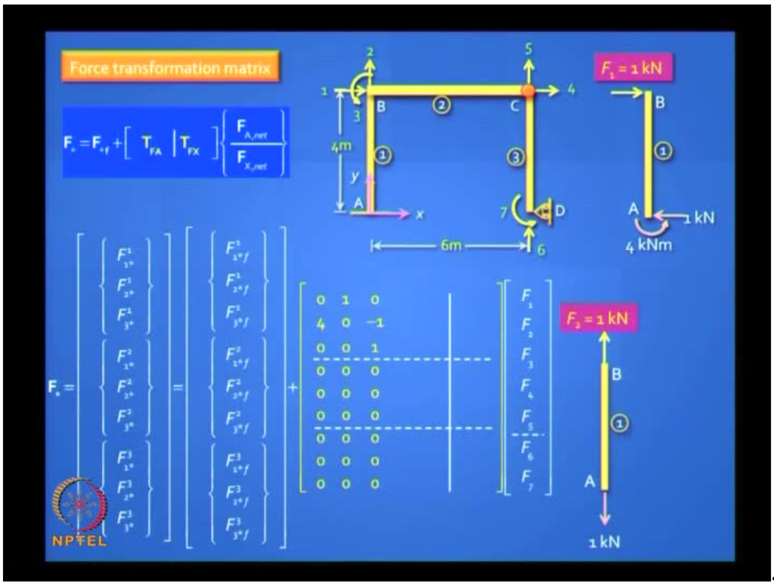

Only one [Noise].

It will affect only element one. Try drawing. Will you get any reactions at D when you apply that unit format? No. No. I will tell you one way to understand. Remember, when we talked of the internal hinge, usually you have a parent child relationship. Now, clearly, element 1 element 3 is a child. It badly needs elements 1 and 2 for it to hang on to for its own their life. right

So, the child cannot take loads when the parent is loaded. When the child is loaded, you will get something in the parent. Let that be very clear. right So, you can quietly remove the child from the picture and look at the parent. Parent is a cantilever. Now, if you apply a moment at joint B in that L shaped cantilever, do you think anything will go on to element 2 and 3? No.

So, this is flexibility method; really have to enjoy the physics of the problem and get the answer. So, you just have to draw this.

In (( )) coordinate.

Yeah.

Um, one - axial degrees of freedom, and two - moments.

Right.

In the element one.

Right.

But element 2 has one axial different or one moment.

No. They are all frame elements; all are identical; all have one star as the axial, and because there is going to be an axial force in that element…

[Noise]

Sorry.

Element 2 there is no…[Noise]

Ah We do not take advantage of it here. We do not take advantage of that hinge here because that we will do when we ignore axial degrees of freedom. You understand, this is here we are not taking advantage of the release. There, you can do it. I am not taking advantage in this case, but I will take, I will take your suggestion in the next case, where we ignore axial deformation.

So, your point is valid. I can actually reduce the degrees of freedom for element 2 and 3. Right now, I have not done that. It is I will still get zero moments there. So, your point is good. We will do it later. Is it clear? Now, I will go faster.

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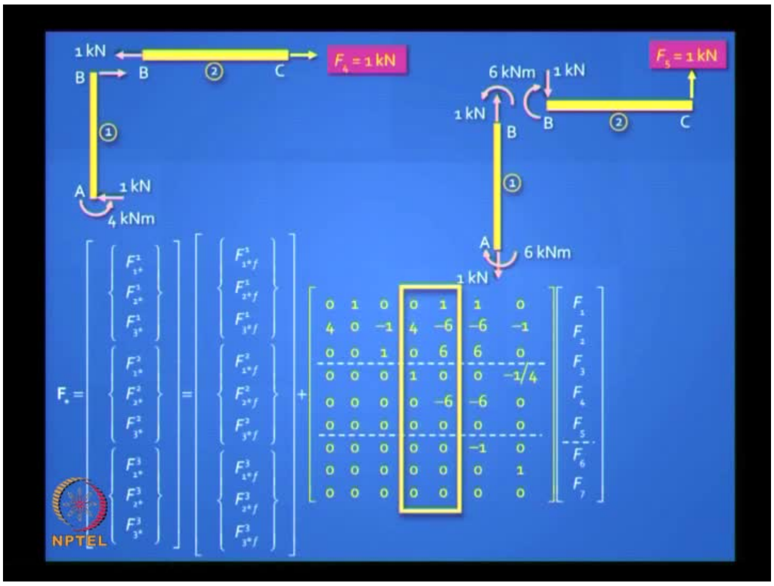

Ah If you apply F 4 equal to 1, again the child is not affected. So, I have not shown the child. Here, child is element 3. So, can you generate the middle part as I have shown here?

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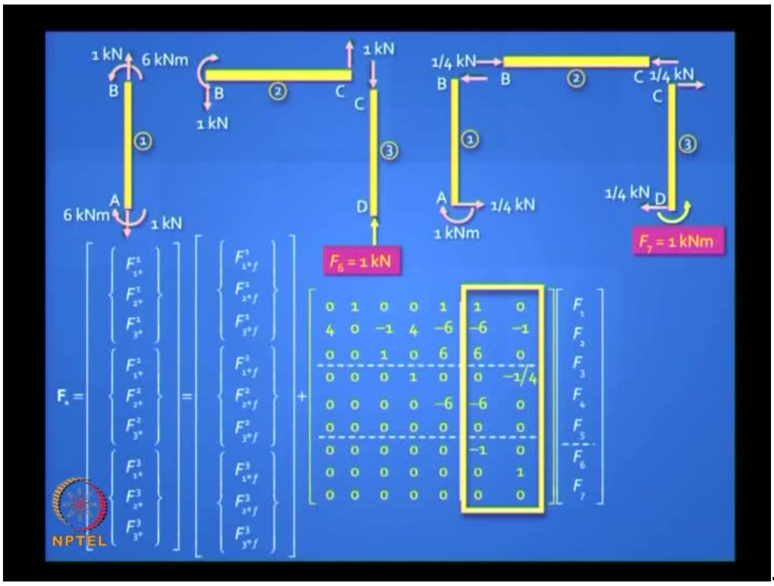

You can work it out. Next one, apply F 6 equal to 1 and F 7 equal to 1. You get the remaining parts. Here, the child is being pulled and the child is hanging on to the parent; parent also gets pulled. Clear?

So, you have got all three elements activated when you apply (( )). These are simple things. These are statically determinate system. You can write down those forces, but it is fun writing down the T F matrix. Now, just look at this complete matrix. What we are saying is – we have written a set of equilibrium equations; if someone gives us this vector which is a fixed end force vector, someone gives us this load vector including the redundants; we have drawn it for a primary structure; the redundant are treated as loads on the primary structure. F 6 and F 7 are loads on the primary structure. Is it clear?

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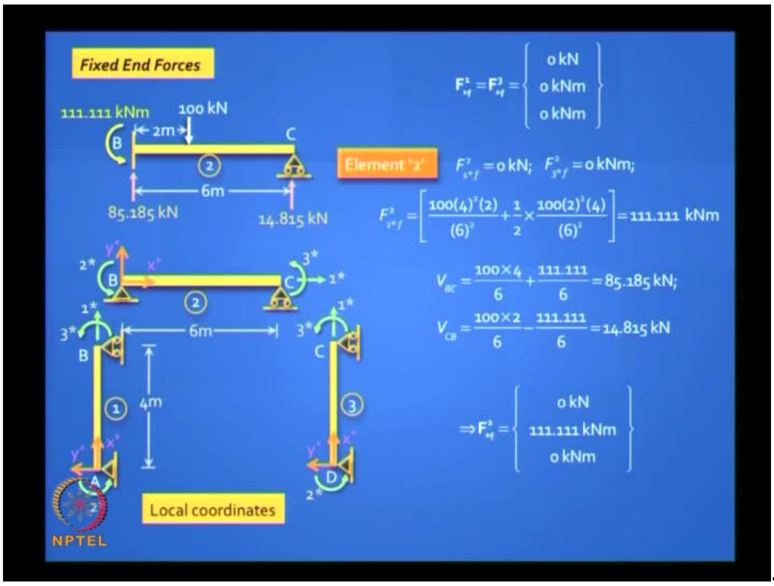

We get the bar force, get the member end forces, and we will do this in the last step. So, how do we proceed now? We have to write down the fixed end forces. Elements 1 and 3 have no fixed end forces. Now, these are the local coordinates by the way. So, this analysis, we have done earlier. We have done it for the reduced element stiffness method. Here, you do analysis where you do not get a moment at C which means you have to treat it like a prompt cantilever to get the equivalent joint forces. Is it clear?

You get the vertical reactions and write down the F 2 star f vector. You have only one moment there because here you are accounting for the (( )). At C, there is a hinge. So, you do not (( )) So, does not make sense. Now, I am taking take care of the hinge, but I am still keeping my full-fledged 3 degree of freedom matrix, but I will take the advantage in the next situation.

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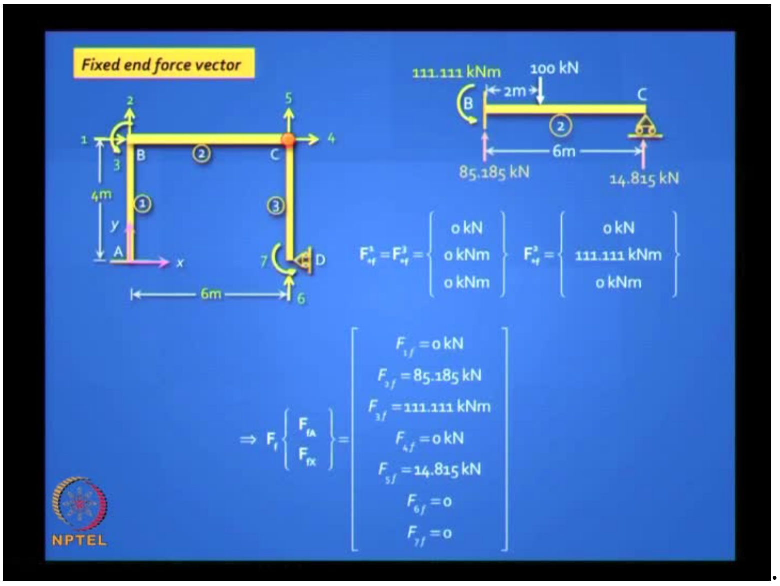

So, I have got my fixed end force vectors for the first and third. They are null vectors. For the second one, I just have one moment there. 111.11 and that is anticlockwise, and I can write the F f vector. Clear?

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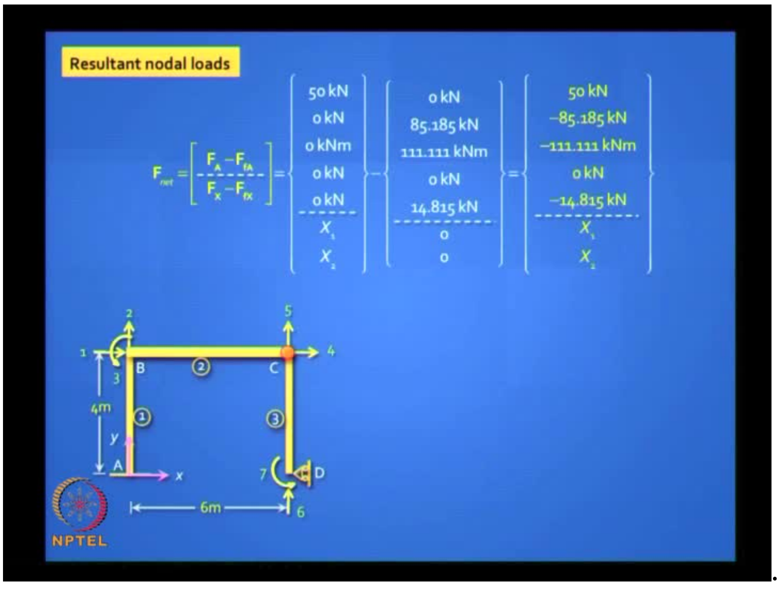

Next, I should get the net joint loads, resultant nodal loads. So, that is F f net is F A minus F f A and F x minus F F x. I just do this substitution. When I write the second one, I have to that is where I bring in those forces. You see, this is given to me. This is written in the beginning. I have 50 kilo newton load and I have x 1 and x 2. I do not know. This is my F A F x vector.

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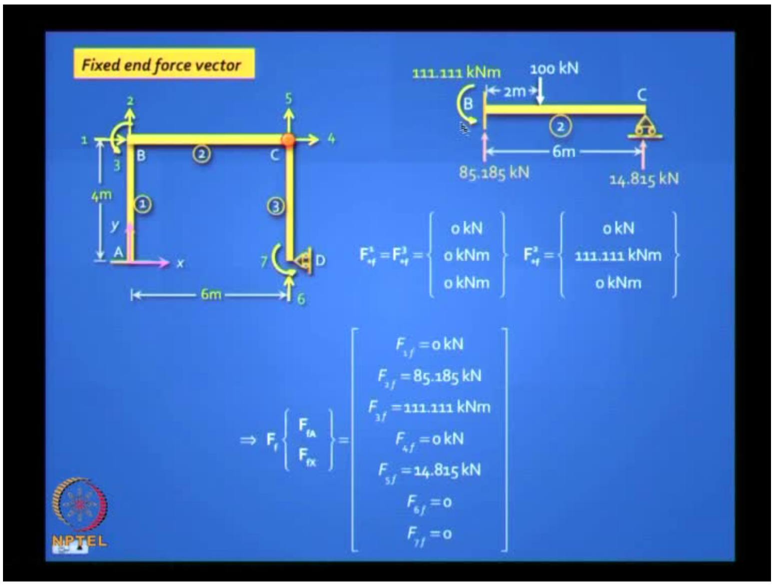

This is my fixed end force vector which I compute by inspection. by inspection If I go back here, I see that, not only this will contribute to F 3, and these two reactions will contribute to F 2 and F 5. Is it clear?

That is how I get these two quantities here. Does it make sense? They are pointing upwards with a positive, but I put a minus sign, and now I get the final picture. This is the structure I am going to analyze. Is it clear? It is completely taken care of the F x of the intermediate loads.

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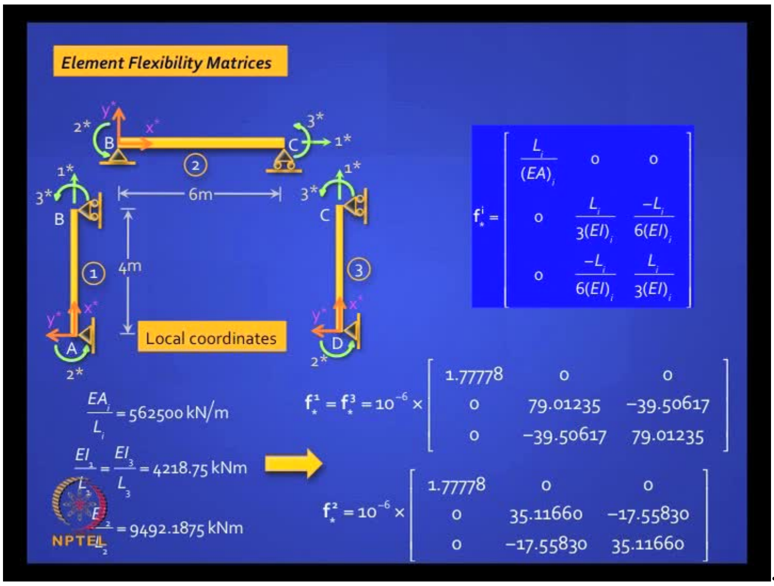

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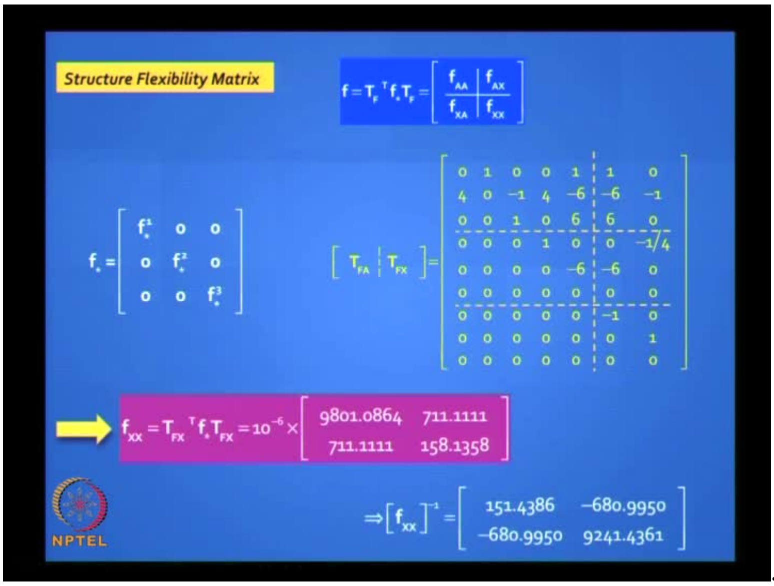

How do I analyze this? I first generate my element flexibility matrices. I have taken them all. To have this format, it should be L by EA and all these values [are known/unknown]. You can generate the flexibilities. What do you do next? Generate the structure flexibility matrix. You you have the unassembled flexibility matrix by putting them in a diagonal form. You have the T F A matrix. You do that product. You will get F x x. It is a 2 by 2 matrix which you can generate.

Please note, when you actually do it on matlab or on a computer, you have to be careful when you deal with flexibilities because stiffness values are going to be large when you write in kilo newton meter units but the reciprocal of it, the flexibilities are going to be very small.

So, if you work with the absolute figure, you will have massive rounding of errors. So, you have to work with significant figures. So, you have to cleverly take out ten raise to minus 6 outside. So, you get large numbers inside and you do not truncate those digits.

This is what you have to be careful when you deal with flexibility. Is it clear? In stiffness, you do not really have that problem, but in flexibilities, you have. You understand? If you have this quantity and you write it in decimal format with ten raise to minus 6, you might write it as 0. So, 9800 becomes zero because you are used to writing by hand, point naught. Now, you get tired of writing so many zeros, you say, may be it is meaningless and you put zero. Is it clear? So, be careful because you you need to work with significant figure.

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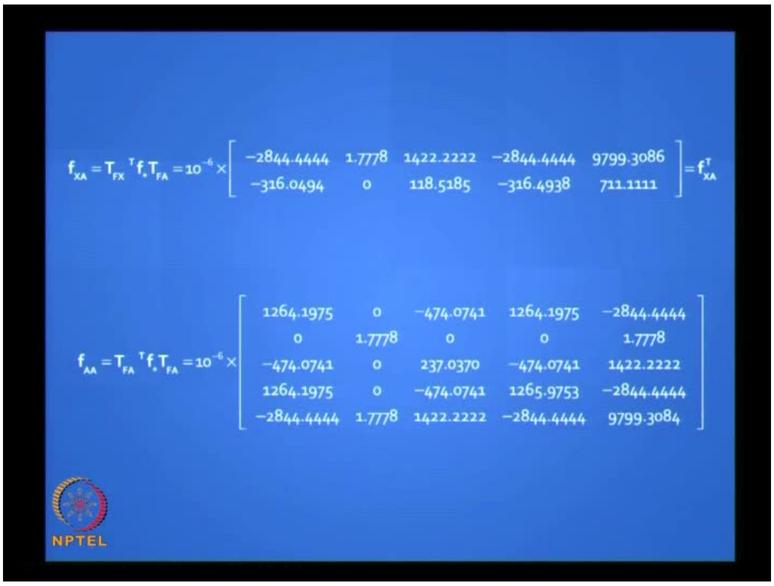

So, you do this, get the inverse of it, and you you are ready to do the final calculation. Similarly, you can generate f x a and f x a T. All these can be done through matrix operations.

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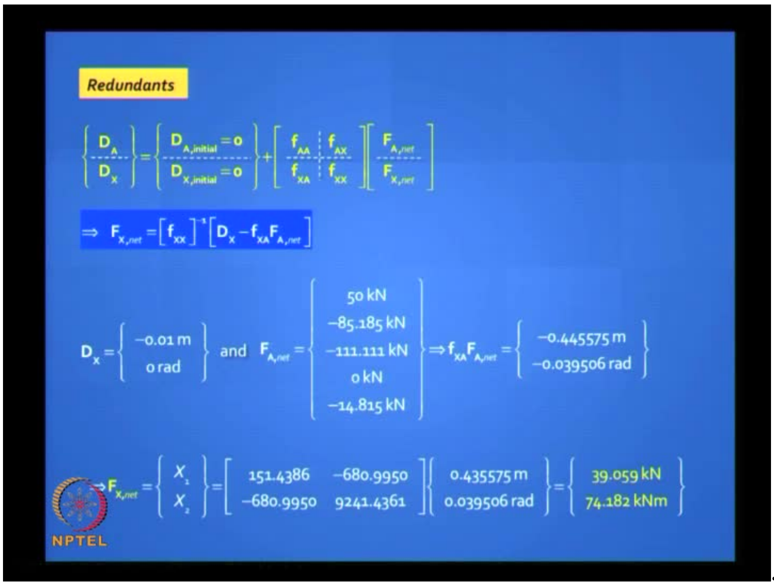

Now, you find the redundants by solving that second equation. You do not have any temperature effects or you know constructional errors. So, you do not have any initial forces here. You plug in those values and you finally get you get the F x net ok.

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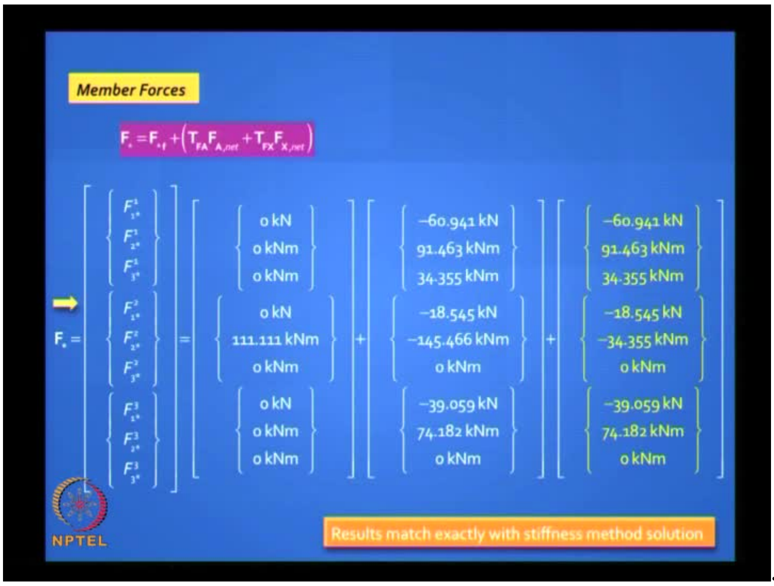

So, you get x 1 and x 2 in the last step. x 1 and x 2 is 39 kilonewton and 74.18 kilonewton. Find the member end forces. You remember those large equations we generated with the T F matrix. Now, you are in a position to put, and all those unknown values you get the final end forces exact.

When you draw your… if you compare with what we did earlier, earlier in the exact method, the conventional stiffness method or the reduced element stiffness method without axial deformation. You will get these answers absolutely; no error till the sixth or seventh decimal place; it is exact ok.

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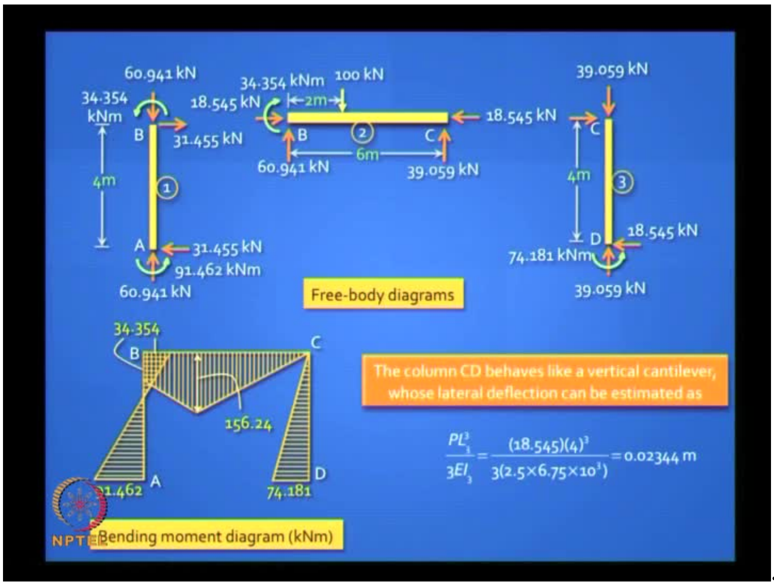

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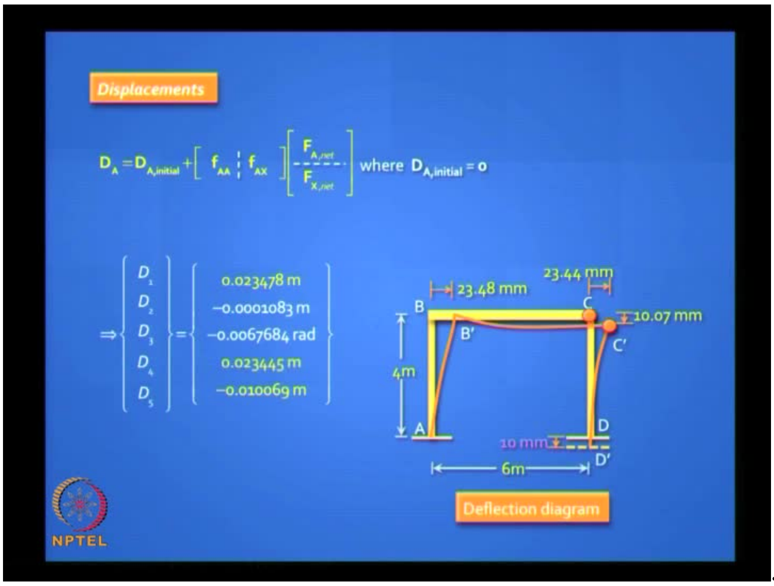

Then, you draw the free body diagrams draw the bending moment diagram and again remember we said that you can get the deflection straightaway you are really [interest in this way/interesting the sway] by recognizing that c d base like a cantilever we have done this earlier but, if you still want to work out displacements for the hard way you are welcome to do so do this calculation and interpret the results we will get exactly the same values we got earlier ok

So with this we complete the flexibility method including axial deformation which is something new you discovered you have not done it earlier you are not done it in method of consistent deformation you are not done it in theorem of this work or column analogy method that is another method.

Now let us see how to take advantage of of axial deflection one way to get the solution if you writing a program is put e a tending to infinity there large value of e a you get the result of solution without axial deformation because if axial rigidity is infinitely large axial deformation is is small that is what you get.

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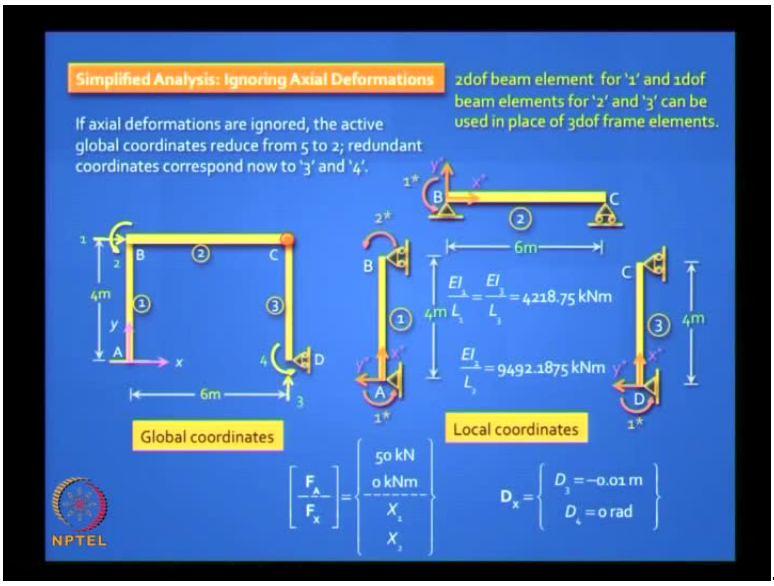

But let us do it this way now what is interesting is your degrees of freedom now reduce you had seven earlier now you have four for reasons which we explained earlier so you have only one two and three four corresponds to your restrained degrees your redundant degrees

Also for the beam element we will take the advantage that you had pointed out the two degree of freedom beam element for one anD 1 degree of beam elements for two and three can be used in place of frame elements.

So we we are going back to beams and we are taking advantage of the modified flexibility so we can do that for element one you still have two degrees of freedom but, because we want to take advantage of the internal hinge at c i put a roller support there so i have only one degree of freedom and my f a and f x vectors are easy to calculate it is smaller much easier to work out d x is the same only thing in earlier we wrote d six d seven now you write d three d four.

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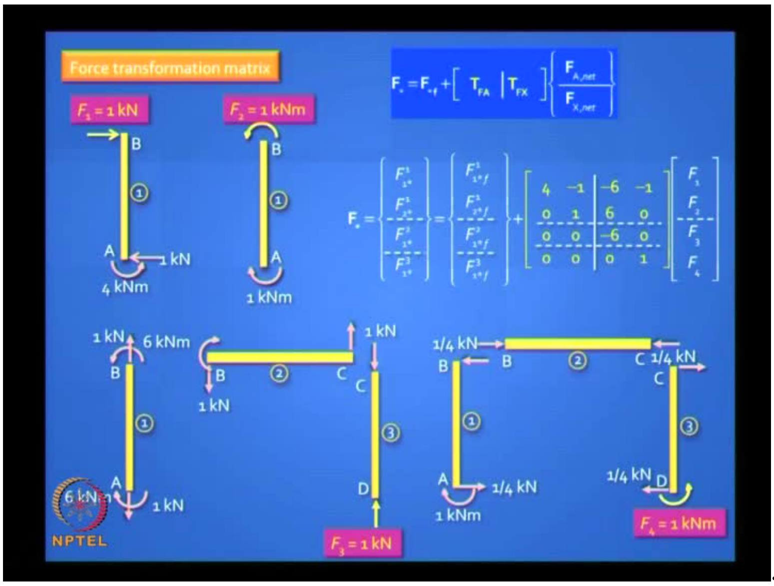

Force transformation matrix much smaller much easier f one equal to one f two equal to one same results these two do not change f three equal to one f four equal to one same only we have renumbered them f six became f three f seven became f four your size of your t f a matrix is shrunk to a nice compact size right

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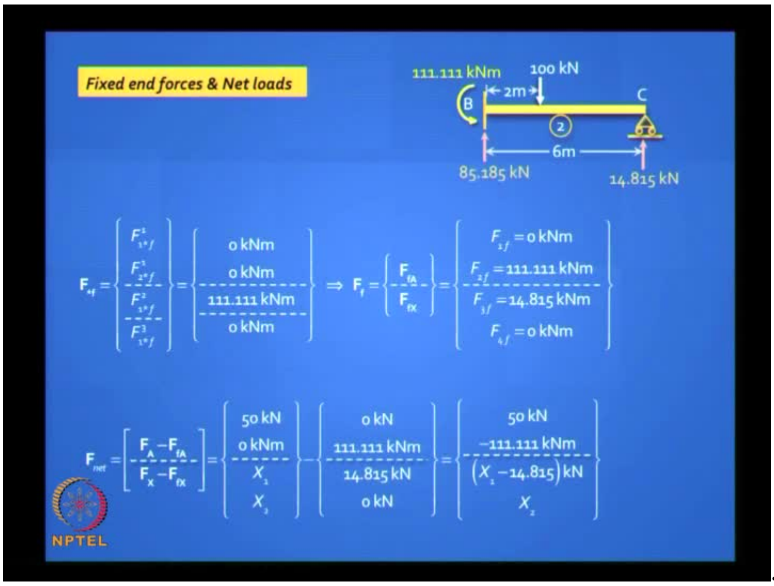

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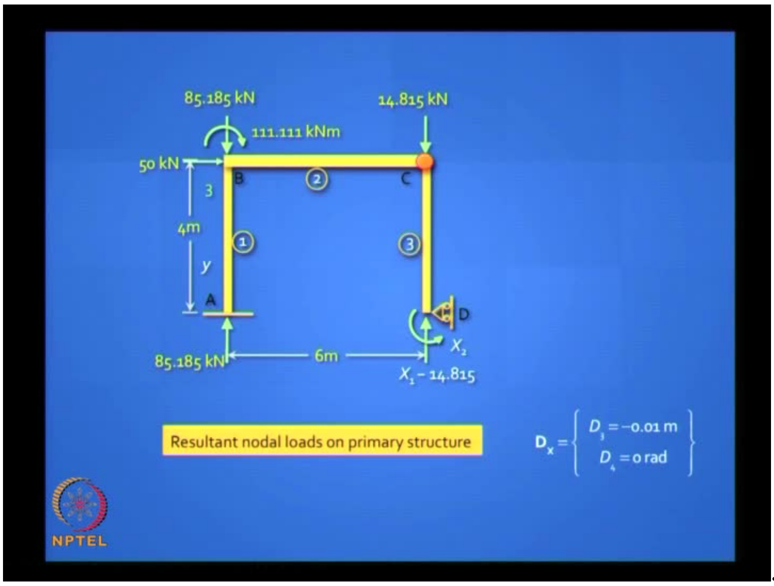

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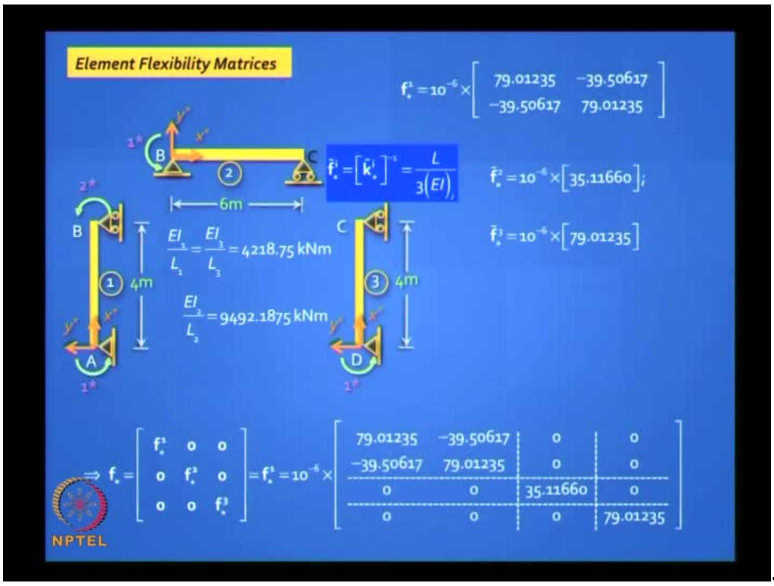

You can interpret the results very easily. You got your T F matrix. Find the fixed end forces exactly as we did earlier. There is no change in this. Only your size of your vectors are become smaller. You got rid of things that were not required. You are dealing with the same system element flexibility matrices f 1 star is the same as we got earlier. But for elements 2 and 3, you take advantage of L by 3 EI. This is a clever trick you can do. 3EI by L for stiffness, L by 3EI for flexibility. So, you get so I have for the first time, I have introduced this symbol f star tilde because it is like k star tilde. It is different. You know you are working with a reduced flexibility right. Generate your flexibility matrix. This is the unassembled flexibility matrix.

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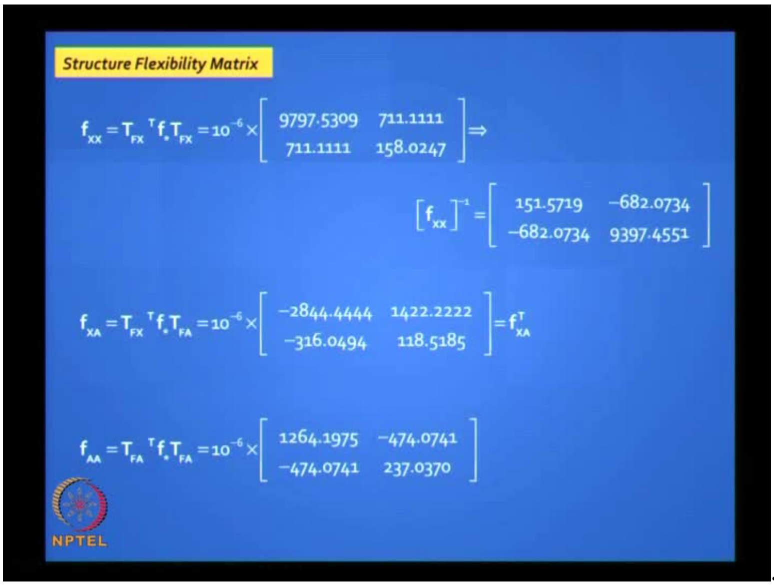

Then, you do the operation and generate your… You can do them separately; f x x is T F x transpose f star T F x, and f x A is T F x transpose f star T F A; f A A is T F transpose f star T F A. Is it clear? Or you can deal with it altogether and then partition it at the end, but this is much easier to handle.

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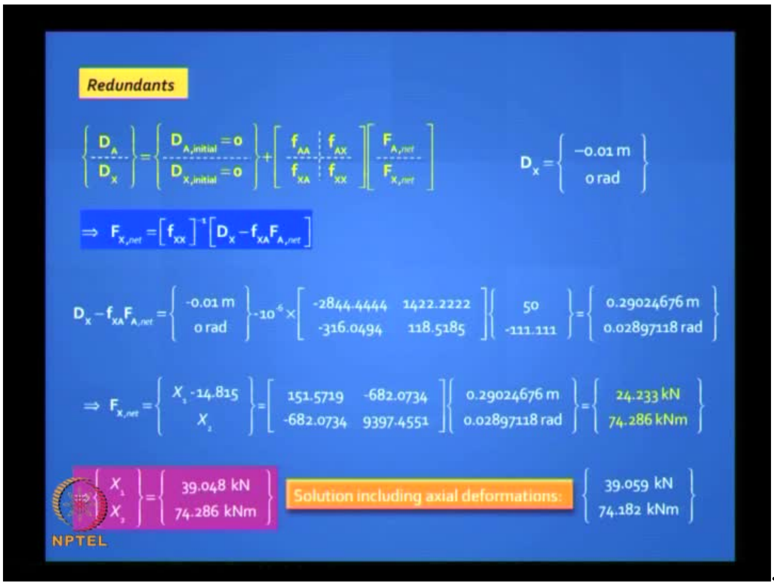

Find the inverse of your f x x matrix. Find your redundants; plug in those values; you get your solutions as 39.048 and 74.286 now; what did you get earlier? 39.059 and 74.182; big deal.

So, that is why we do not waste time dealing with the axial deformation; the errors you make are negligible. And in the context of what we discussed yesterday, this is certainly accepted; this kind of… because they are going to be much larger errors in reality. ok.

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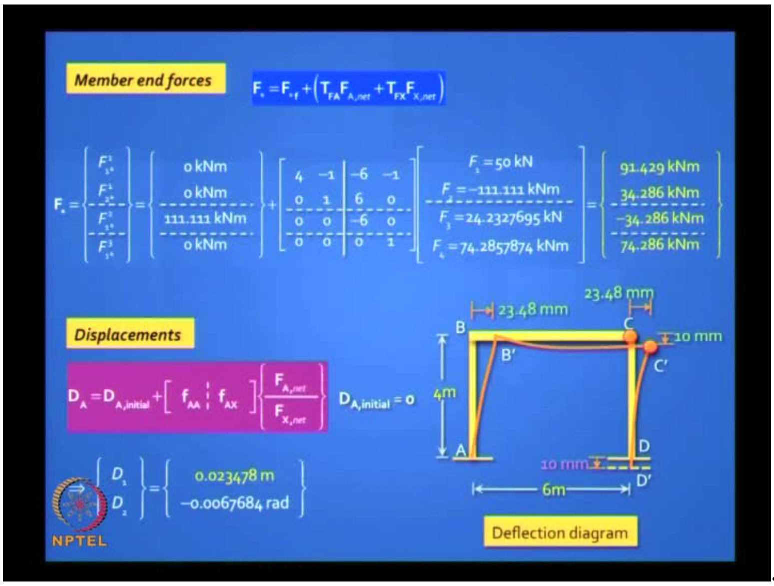

Find the member end forces; plug in those values as we did earlier. Find the displacements if you wish. Now, you get the same sway at both ends, but it is very close to what we got earlier. And the deflection at C earlier was 10.07 mm; now, it will go down only as much as the support D goes down; is it clear?

So, we have tackled this problem in many different ways. We are very clear on what is going on. Proof of the pudding is in the eating; which means, the answer should match. Completely different methods and same answers.

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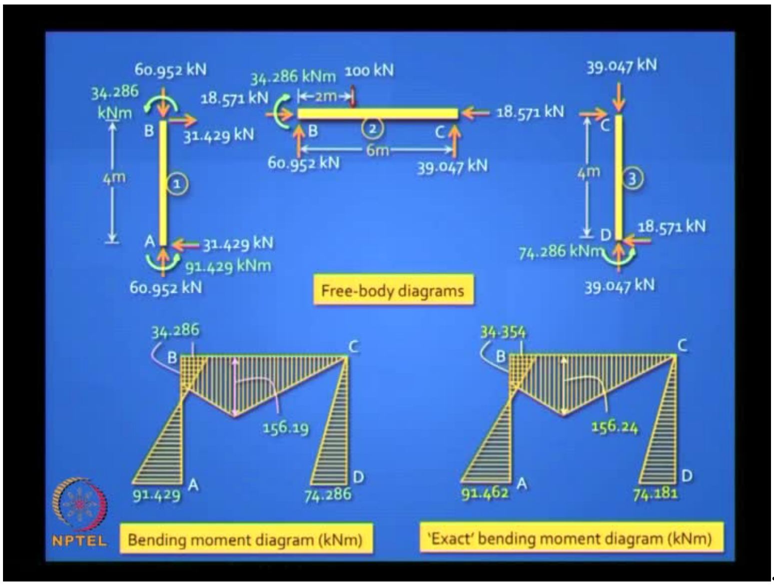

Finally, your free body diagram; bending moment diagram. Compare these two diagrams; exacts with axial deformations and without axial deformations.

It is all the same. May You are making errors in the decimal places; not worthwhile doing it. So, powerful methods when you do manually, do them the easy way out. When you are writing a program, do not do flexibility method; do the stiffness method. You have a choice - reduced elements stiffness method or conventional stiffness method.

With this, actually we have completed everything except space frame, which are really tough, but I am trying to make it simple. And again, you do not need to do anything in your assignment or your exams, but let us get a taste of it.It is like moving from one dimension to two dimension to three dimension. The concept is the same, but you have to really think; especially, if you want to do the reduced method this which you should learn to do manually. So, we will cover it. I am not expected do any example. We will do this in tomorrow's class.

After that, the seventh module again is an exposure I want you to have. I want you to be present; not so much to learn, to solve it because it is really advanced; it is second order analysis, but you should have a flavor of it. And those of you, who are interested can actually sit and solve those problems. I am planning to finish that in three sessions, and then, we will have one last concluding session. So, we are nearing the end of this course. Thank you.