Advanced Structural Analysis

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Module No. # 6.3

Lecture No. # 35

Matrix Analysis of Plane and Space Frames

Good morning, this is lecture number 35.

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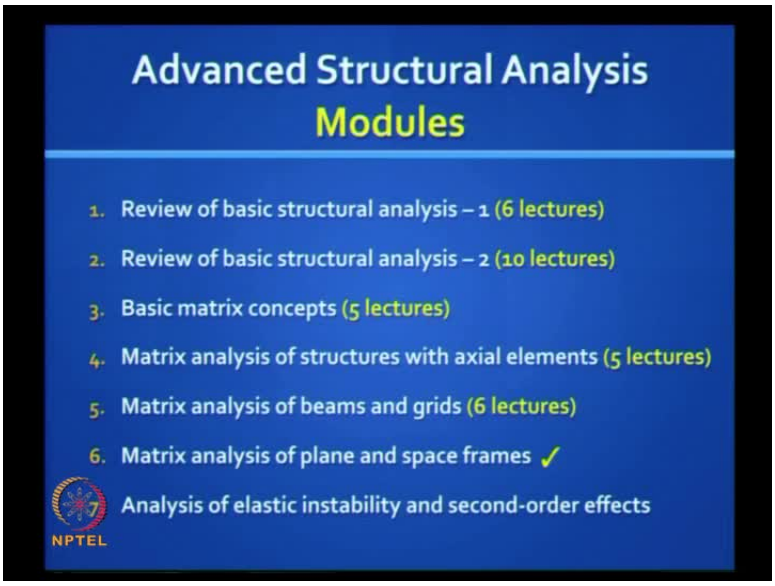

We are still with module 6 - Matrix Analysis of Plane and Space frames.

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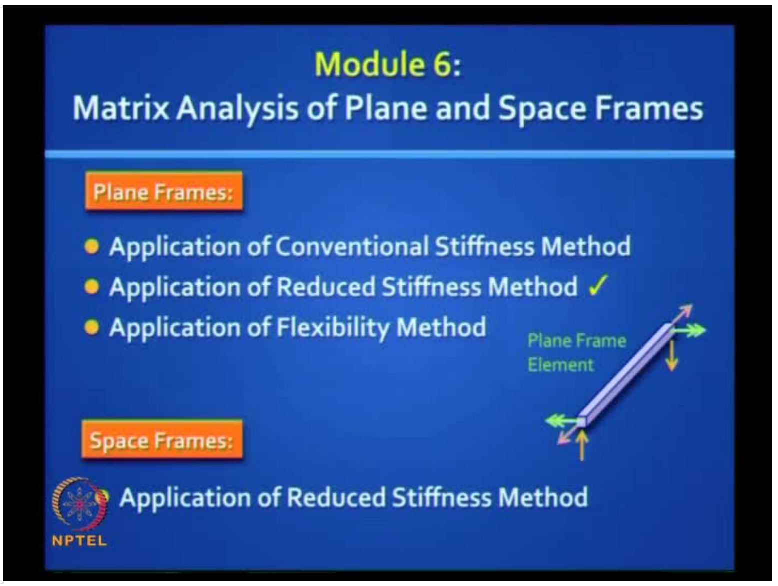

If you recall, in the last class we had covered the conventional stiffness method. So, in this session, we will look at the reduced stiffness method, as applied to plane frame elements.

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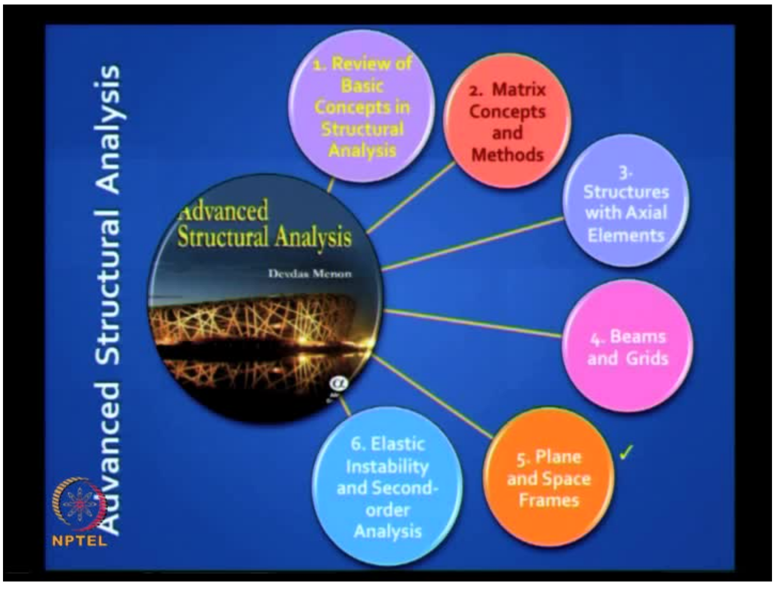

This is covered in the chapter on Plane and Space Frames in the book on Advanced Structural Analysis.

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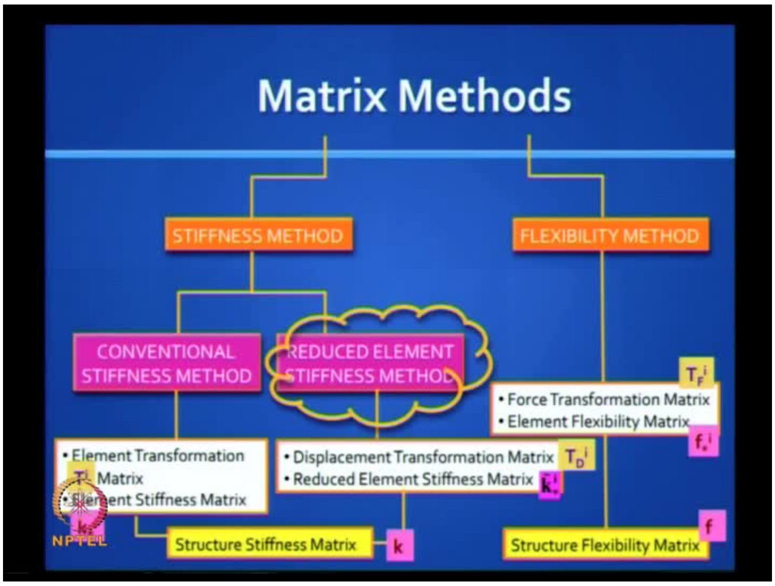

So, I keep showing you these maps, because I want you to see how the system of analysis is the same; the structure is changing, but the methodology is not changing. We saw how well it worked for the simplest type of axial element; then, we worked with plane trusses; then we worked with space trusses; then we worked with beams; then with grids, and now, with plane frames, and in the next class, or the class after that, with space frame.

So, we are covering all kinds of skeletal structures, and you can clearly see there are two broad methods: There is a stiffness method which is preferred for programming compared to the flexibility method. In the stiffness method itself, you have the conventional stiffness method; you have a simplified formulation called, the reduced element stiffness method, and that is a method that we are going to discuss in this session.

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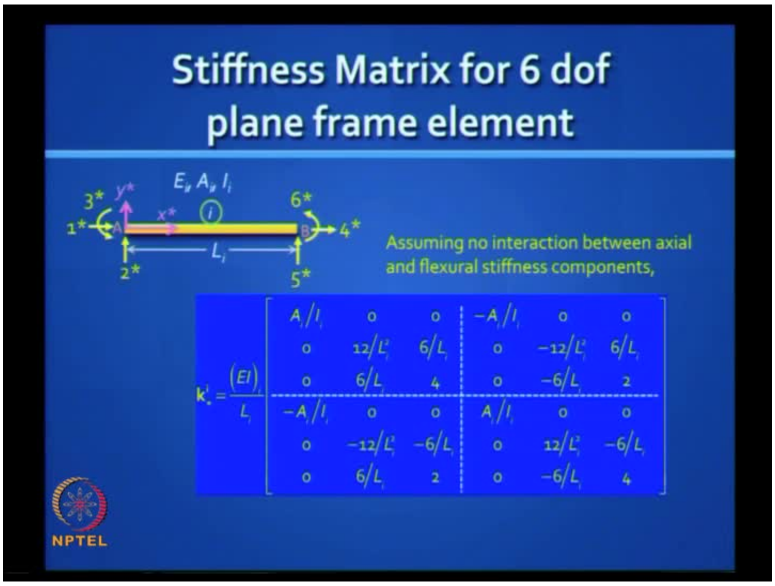

If you recall, this is the 6 degree of freedom plane frame element that we used for the conventional stiffness method. This is a large matrix and it is a singular matrix. What is the rank of this matrix? 3, and one way of understanding why the rank is 3, is because you have, mathematically, you have 3 dependent rows or columns, but physically, what does it mean? Physically For a stiffness matrix to be non-singular, what you have to make it is, you have to make the element stable.

Now, a singular stiffness matrix still works in a global scenario because your structure is stable. When you assemble the structure, stiffness matrix - the k a, a is non-singular, but here, you can begin with a non-singular element stiffness matrix by giving how many restrains? 3; then only you have stability, and you can choose your type of restraint. We have been assuming that the simply supported condition is convenient, and we will stick to that.

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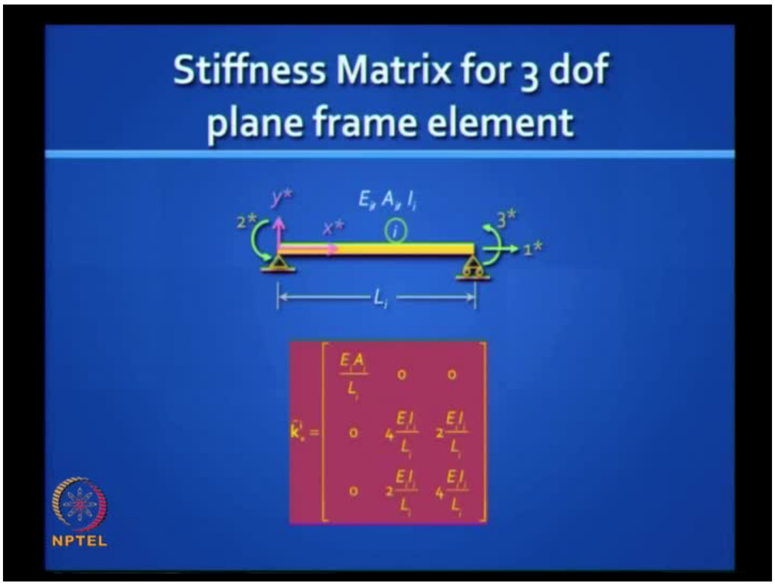

So, we are now going to use a 3 degree of freedom system, and it is very easy to write down; at this stage, you should find it very easy to write down the element stiffness matrix. You can do it from first principles.

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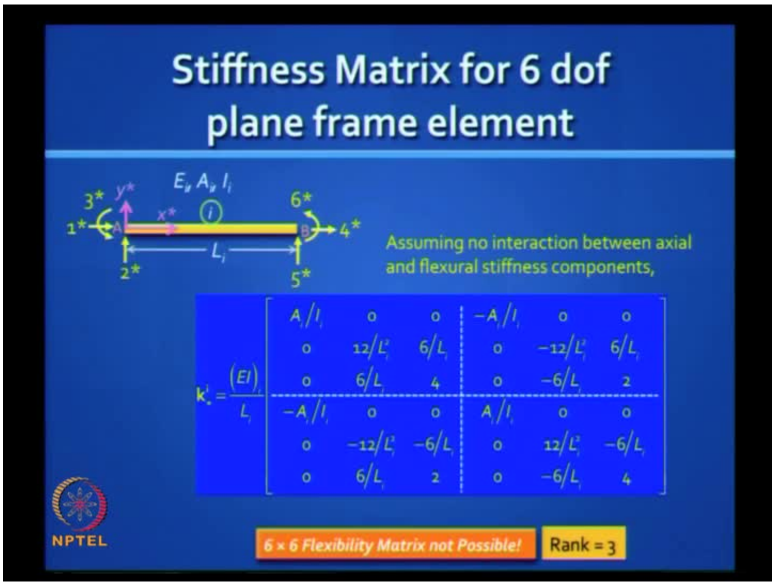

You can also do it from the 6 by 6 element stiffness matrix. how Just talk of the irrelevant rows and columns; there are 3 dependent rows and columns. If you make it simply supported, you will find that the first row is important; the second is not because we do not want the shear degrees; just delete the shear degrees of freedom, and you do not need 2 axial degrees of freedom. So, that is how, this reduces to this element. It is very easy to derive; very easy to remember. E A by L, axial stiffness 4 EI by L, 2 EI by L, 2 E I by L, 4 EI by L; is it clear? It is actually a combination of your axial element and your beam element.

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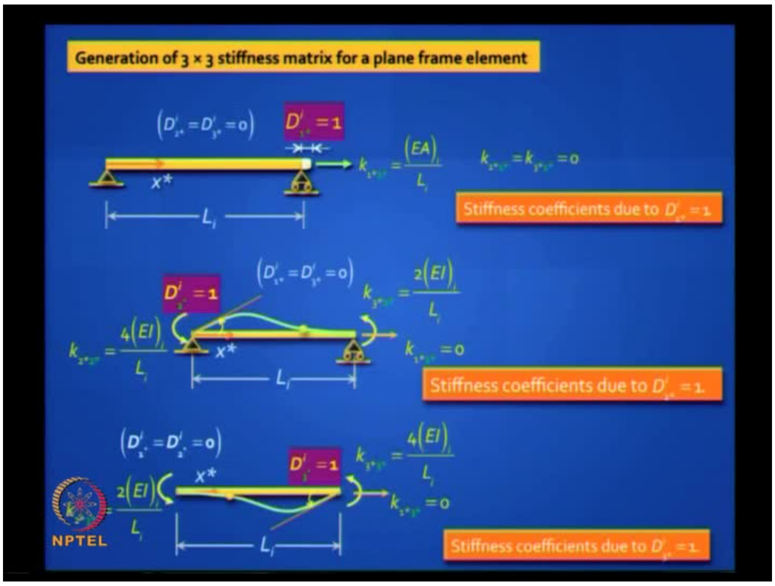

right You can also derive from first principles. You apply a unit displacement, one at a time, and you can generate these. This diagram will be very familiar to you now. It is not difficult; you can generate the element stiffness matrix.

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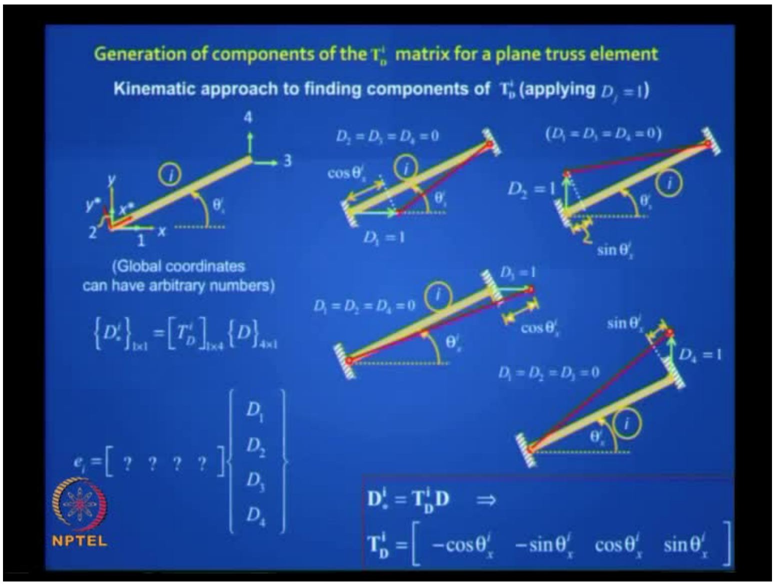

Now, we also need that displacement, the T D matrix that is the displacement transformation matrix. Well, we are familiar with this slide because we used this when we dealt with plane trusses. The plane frame is advancement on the plane truss because in a plane truss, you have 4 degrees of freedom in the conventional system. So, we are familiar with this transformation minus cos theta, minus sin theta, cos theta, sin theta. And you will recall there are two ways of deriving this. This is the kinematic way, but there is also an easy static way, where you get the T D transpose matrix. So, you are familiar with this.

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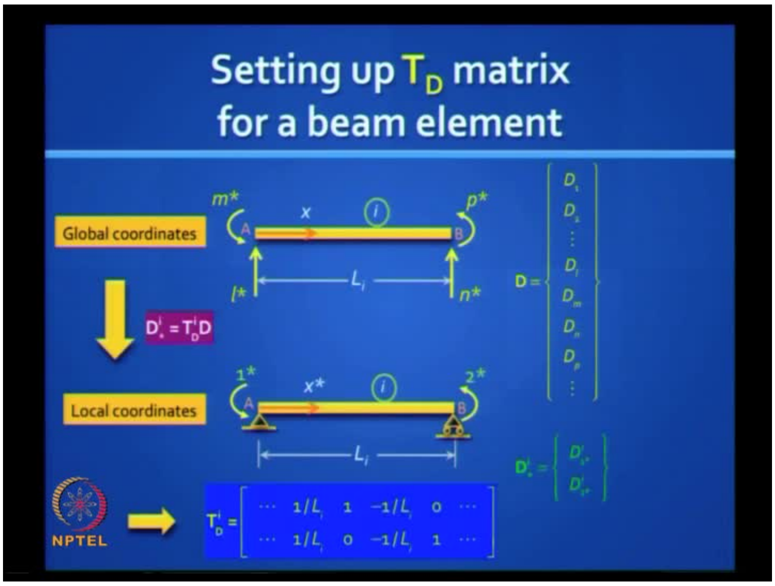

You are also familiar with what you do for a continuous beam. And if you have chord rotations, relative supports settlements, then you have to use this chord rotation which is given by 1 by L, and clockwise chord rotations are treated as negative, but the equivalent beam and flexural rotations which is 1 by L, will turn out to be positive. So, you are familiar with these two; if you put them together, then you get what you need to do for a plane frame element with 3 degrees of freedom.

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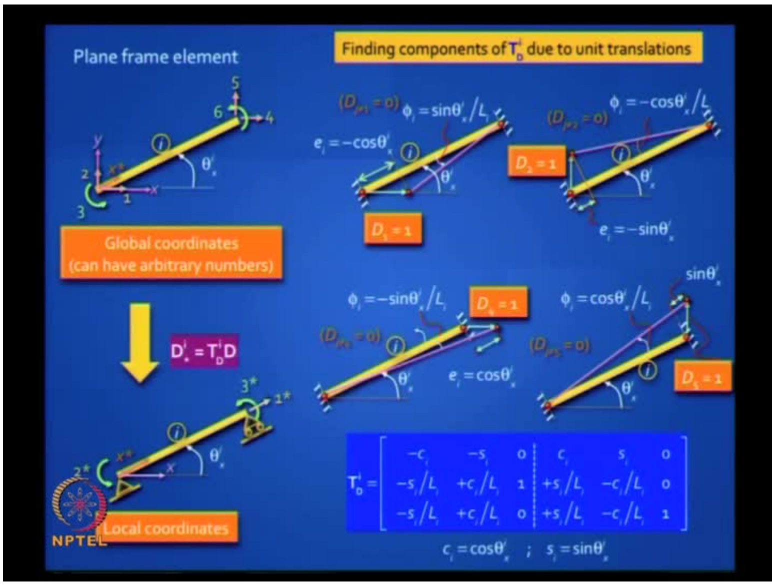

You can easily work this out from first principles. What you need to recognize is - if you have any of those translations, you have to find out what is the elongation you get in the element; right so, it is either cos theta or sin theta. If you get an extension, give a positive sign; if you get a contraction, give a negative sign; it is very easy. So, you can generate this by pushing one at a time. I have shown here, the translation effects. So, rotational effect is very straight forward because there is no transformation required when you have a rotation because it is same; you get 1.

So, let us just look at this. Let us look at the first one; this first column in your T D matrix corresponds to D 1 equal to 1 in your structure. So, if you have D 1 equal to 1, you need to look at this picture. If you apply D 1 equal to 1, that element undergoes a contraction of cos theta; so, it has got a minus sign; that is why, we wrote minus c I; c stands for cos theta.

At the same time, you get a chord rotation. Have you noticed? You get a chord rotation. The chord rotation is anticlockwise; the value of the chord rotation is 1 by… not 1; it is sin theta by l. right And so, you get equivalent flexural rotation which will be clockwise, and that is why, you get minus s i by l; minus s i by l (( )) is it clear? Any doubts on this?

From first principles, you can generate this; alternatively, you can use a force approach and generate this, and find the T D transpose. Did you understand what we did in the first column? Yes. This corresponds to the first degree, the axial degree. These two correspond to the rotational degree; so, if you have a chord rotation 1 sin theta by l clockwise, you will get equivalent flexural rotations minus s by l; is it clear? Like this, you can work out for the second degree of freedom. If you take the third degree of freedom, it is a rotation D 3 equal to 1; that does not need any transformation here. So, corresponding to D 2 star, you get 1. So, 0 1 0; does it make sense?

That is all. Once you have got the physics in this, you got it. This is the actually not difficult to do, once you realize that it is just a superposition of the plane truss element and the beam element. We did the same in the beam element. We had 1 by L because you did not have sin theta cos theta, but now, your plane frame element can be oriented in any direction, and not necessarily align with the global x axes; is it clear? ok

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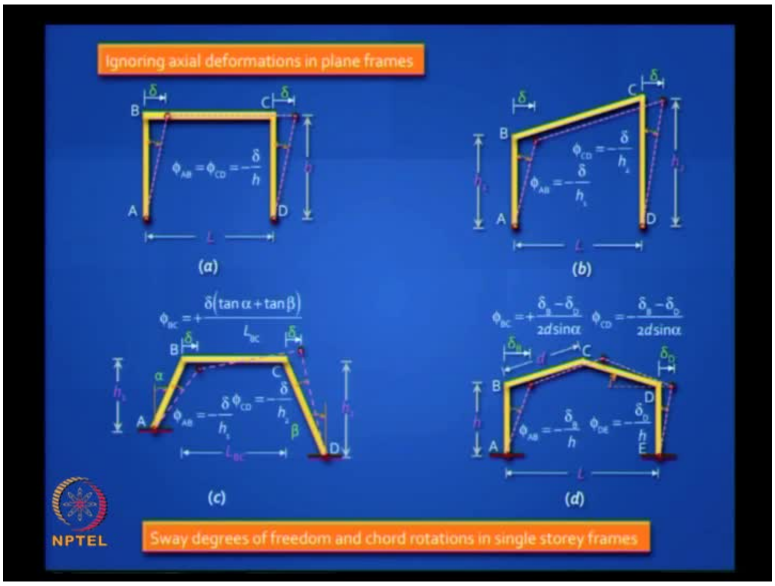

So, if you have got this, then we can go ahead. This slide should also be familiar to you. These are the shortcuts you will take when you want to avoid considering axial deformations. Remember, you can convert; you have to find out the sway degrees of freedom, the minimum sway degrees of freedom, and convert them to chord rotations. right We had done this when we did the slope deflection method, remember. So, we will invoke this concept when we want to do a simplified analysis.

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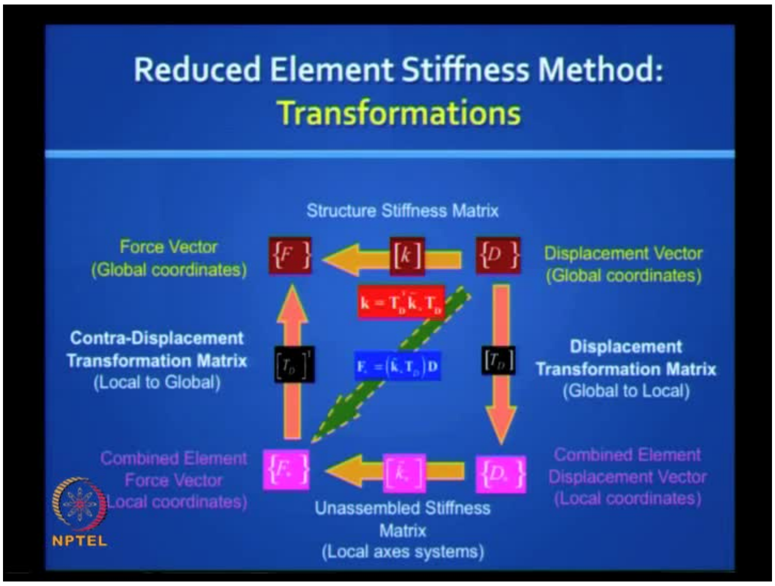

This slide is familiar to you. These standard transformations we do in the reduced element stiffness method.

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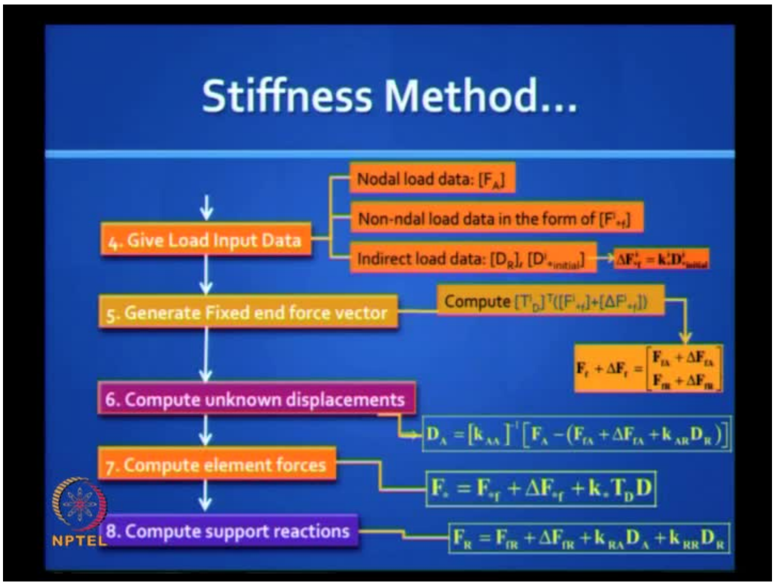

This step wise procedure for programming is also clear to you.

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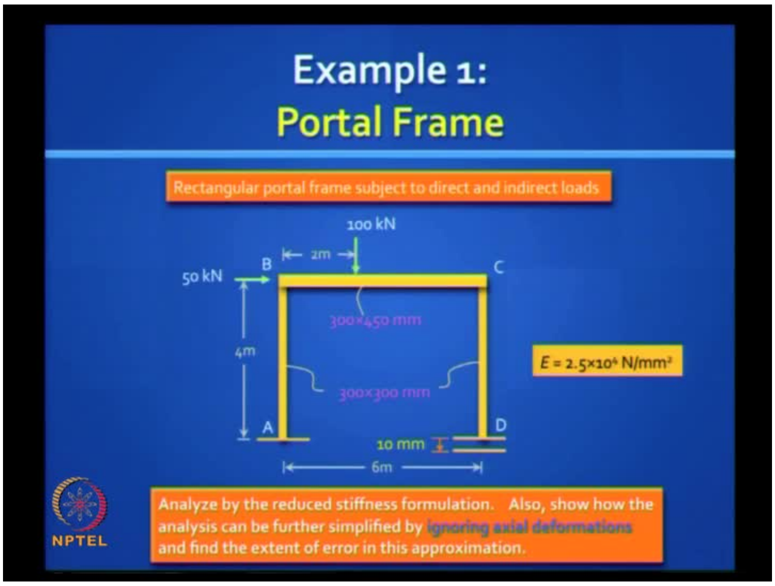

Now, we will take up the same problem we did by conventional stiffness method, and you can see, there is a tremendous reduction in effort when you do the same problem by the reduced element stiffness method. ok.

So, we will take this frame, and we will solve it in two ways: One - we will include axial deformations; that means we should get exactly the same answers we got by the conventional stiffness method. We will also ignore axial deformations, and that reduces the problem even further. It is good enough for most practical cases, in which case, the solution you will get will be what you would get if you were to solve by the slope deflection method or a moment distribution method, and let see, what is the order of error that you get, if you ignore axial deformations. So, you have that facility to either include or exclude, which you did not have in the slope deflection method; is it clear? Let us go ahead.

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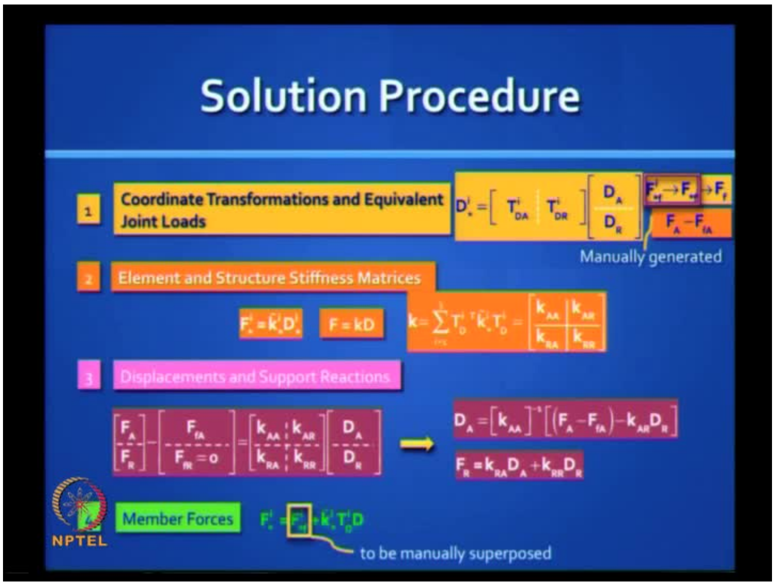

First, you need to do the coordinate transformations. We try to partition T DA and T DR. You have to find the equivalent joint loads, if you have a distributed load. In this case, you have consecrated load acting in between that beam; so, you have to do this manually for reasons we have discussed earlier. You cannot do the T D transpose and get this, which you could do in the conventional stiffness method because you had that many extra degrees of freedom.

Then, you generate the element and structure stiffness matrices. Mind you, the matrix you get here, the final k matrix is exactly that which you get in a conventional stiffness method, but you are working with smaller initial matrices. There is one more thing that I think is worth noting; you need not find reactions; if you are anyway, it is a manual method. So, you can avoid F R if you want to draw the free body and figure out the F R yourself from equilibrium; that option is there in this method.

So, if you want you can find the support reactions along with the unknown displacements, and you need to find the member end forces. This last equation is similar to your slope deflection equation; except that, you are now including the contribution, if any of axial deformations right ok.

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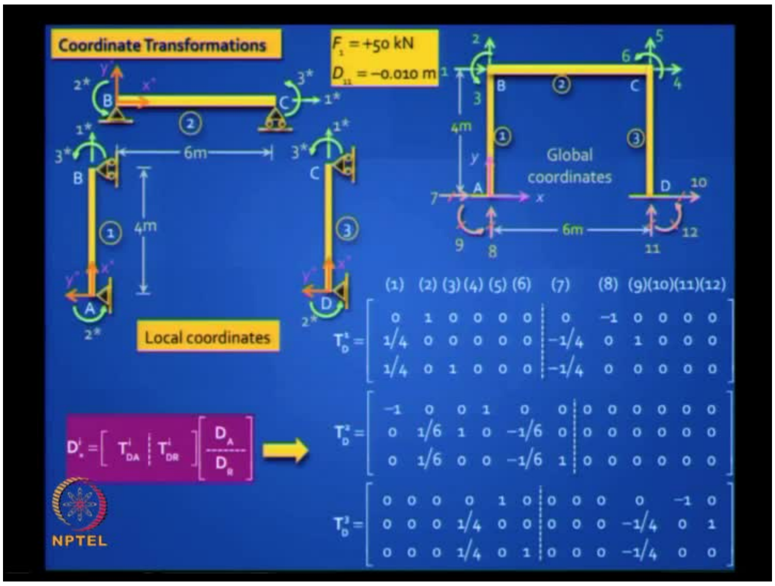

So, first, we will do the coordinate transformations. In this portal frame, we will use 6 degrees of freedom for the same 6 active degrees of freedom, which we did earlier, and 6 restrain degrees of freedom; the loading input is the same; F 1 is 50 kilo newton and D 11 is minus 0.10 kilo newton. Remember, we did the same frame with and without an internal hinge. We will also do the internal hinge in this case, and show you that you can solve the problem.

Now, you are dealing with three elements, and each of those elements has 3 degrees of freedom; 1 star corresponds to the axial degree; 2 star corresponds to rotation at the start node; 3 start rotation at the end node, and you can choose your start and end as you wish. In this case, you can see that, for elements 1 and 3, I have chosen the start node at the bottom.

You have to generate this matrix; give it a shot; what is a size of those matrices for each element? What is the size of the T D matrix? 3 by 12? 3 by 12. right Let us do it together.

We will demonstrate for 1 or 2 rows so that you get the hang of it. It is 3 by 12, and you can partition it where you separate the active degrees of freedom from the restrain degrees of freedom. Let us go through it; let us do a few of them.

Let us take the first one. If you apply D 1 equal to 1 in the structure, but do not allow any other rotation to take place, what do you think will happen? Well, clearly, this element 2 will undergo a contraction. So, you should put this as minus 1. right Nothing happens to element 3, and there is no bending in element 2.

So, straight away, you know that this is minus 1; this is 0; this is 0 0 0 0; got it? Now, what about element 1? Element 1 is going to undergo a clockwise chord rotation, but no change in elongation or extension. So, clockwise chord rotation, anticlockwise end rotations. So, it is, the rotation is 1 by 4; got it?

So, it is plus 1 by 4 plus 1; very easy to do because you are lucky. You have a reticulated frame where the angle is 90 degrees. If it is not, put cos theta sin theta the way (( )). By the way, in the in the book, examples are given with lot of inclined cases. You can go through it, but we will also do an inclined sloping legs problem.

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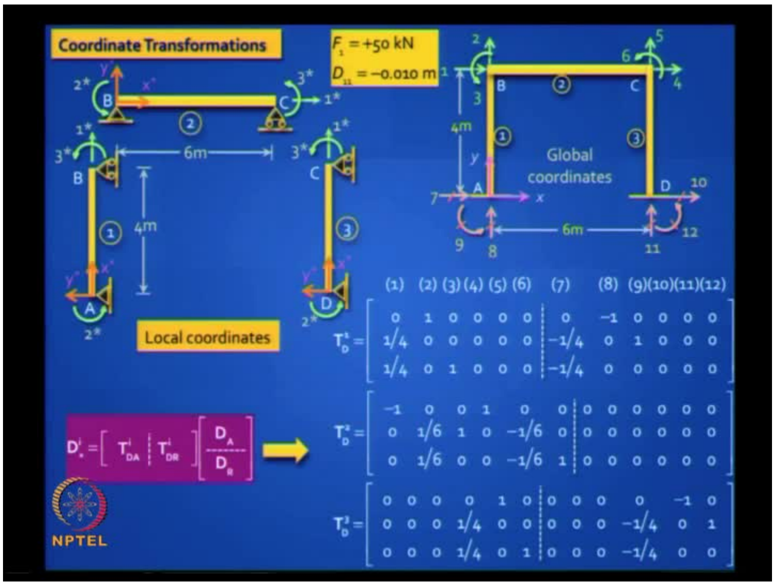

Take D 2 equal to 1; very easy if you lift up D 2 equal to 1; only element 1 will be affected (( )) get it.

So, can you work this out yourself, or you should be go through all the column; what do you say? You are IIT student; you are bright guys. Except that you have to go back and do your ground work, this is easy. We are only looking at concepts. Now, we are not going to sit and solve every second digit, but you got a method. It is very clear. You have got assignment problems where you really need to sit and solve.

So, my suggestion is - go through it carefully and do it; there is one more suggestion, I can give you. If you want to minimize your work manually, you can throw away half that matrix. You can throw away all the restrain coordinates because you can get those reactions from the member end forces anywhere. right Then, your size becomes much easier and it is full of 0s. So, you just have to fill, in fact, when you program it. you It is got null; it is a null matrix; then, you fill it only where you need to fill.

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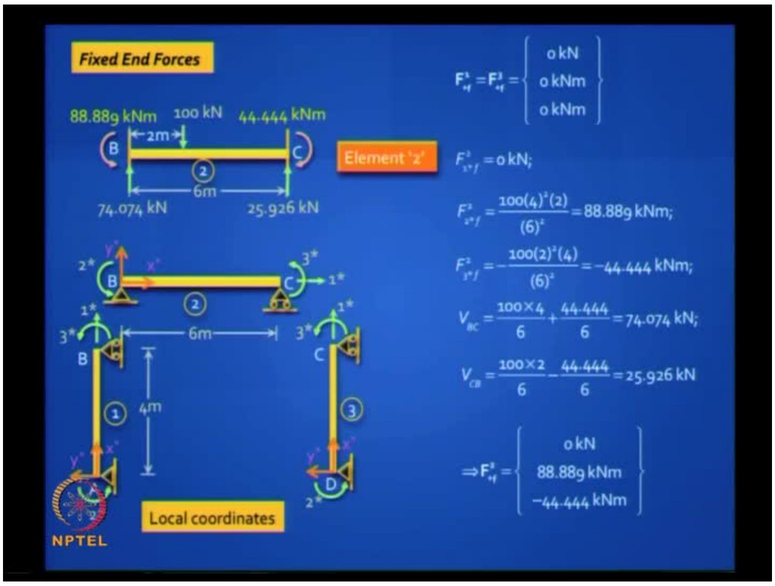

So, it is very easy to actually programming. So, you know how to get the T D matrix for a plane frame fixed end forces. We have done this before. So, these are your local coordinates; the elements 1 and 3 do not have any loads in between. So, it is 0 kilonewton; 0 kilonewton meter, 0 kilonewton meter. Remember, 1 star corresponds to an axial force; 2 star and 3 star correspond to the end moments; there is no fixed end force for the first and third element. Only for the second you have, and you have to be careful with them.

We have done this calculation in the conventional stiffness method. So, I would not repeat it, but it is important to work out those vertical reactions in the conventional stiffness method. Those vertical reactions went into your fixed end force vector because you had some coordinates. There you had 6 degrees of freedom; here you have only 3 degrees of freedom, but you still need to work out the vertical reactions. Why you need the vertical reactions? No. No, because each independent is treated separately till you put it all together in the structure. So, that answer is wrong.

You are finding fixed end forces for element, treating it independently. You do not pass on what happened in the second element to the first element, but you do it in the overall structure. So, you need that information to find the equivalent joint loads.

So, even though you are taking shortcuts in reducing the number of degrees of freedom element wise, you got to see the big picture, and put it all back in the right place. So, as far as your element load vector is concerned, you have only this; there is no axial force in element 2. So, 0 kilonewton. The left end fixed end moment is anticlockwise; 88.89 kilonewton meter. So, it is plus 88.89, and the one at the right end is clockwise plus clockwise 44.44, and therefore, minus 44.44; it clear? So, it needs clarity in understanding to assign this correctly; otherwise, you would not get the (( )). got it?

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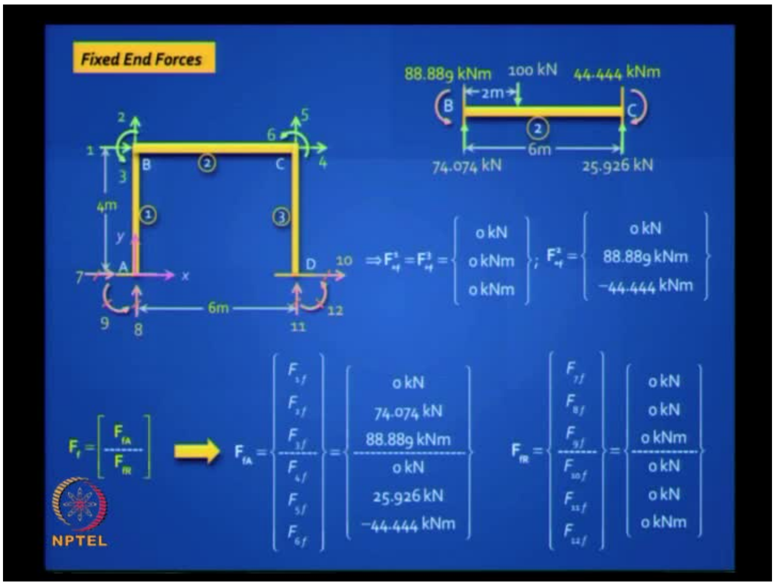

Now, what do we do? Now, we take this element alone as non-zero fixed end force vectors, and when you want to put it on to that structure, you have to do it by inspection; use a shortcut method; reduce element stiffness method; you cannot do T D transpose and get it.

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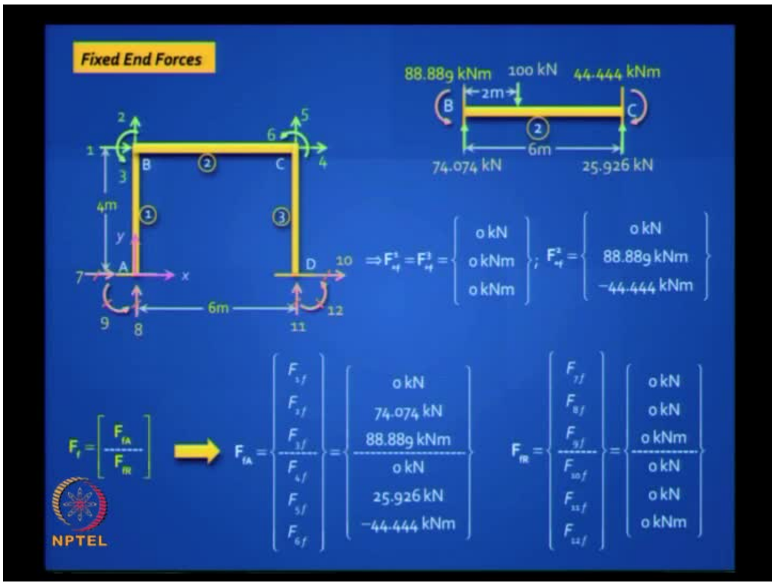

So, it does not make sense to write down your fixed end force vector like this because because what are the contributions from the 3 elements? The first and third elements have no contribution. right Now, this look at this; this 88.89 matches with this force right F 3 F and this 44.44 matches with 6. So, 3 and 6 get those numbers, but then, you have a degree of freedom here, 2 going up, and 5 going up. That is why you put 74.04; is it clear?

And it is positive; both are positive; is it clear? That is why you need to calculate those fixed end forces. Because once you look at the global coordinates, you are just looking. Do I get any contribution from the elements? If I do, put them all together; does it make sense? It makes absolute sense totally, rational logical method.

Only thing, this needs a little input from your side. It is not mechanical, the way the conventional stiffness method is. Conventional stiffness method, you can program it and just forget about it; do not even look at the physics of the problem, where you have to, but that is what makes it interesting. This is good, for human beings should do it; that is good for machines. ok

So, as far as the support reactions are concerned, there is nothing because elements 1 and 3 are where you have fixity. You have null vector that this is done by inspection. So, you got F F A, you got F F R, and you got it by looking at each element separately. Clear? Can we proceed? Is this clear to you? ok.

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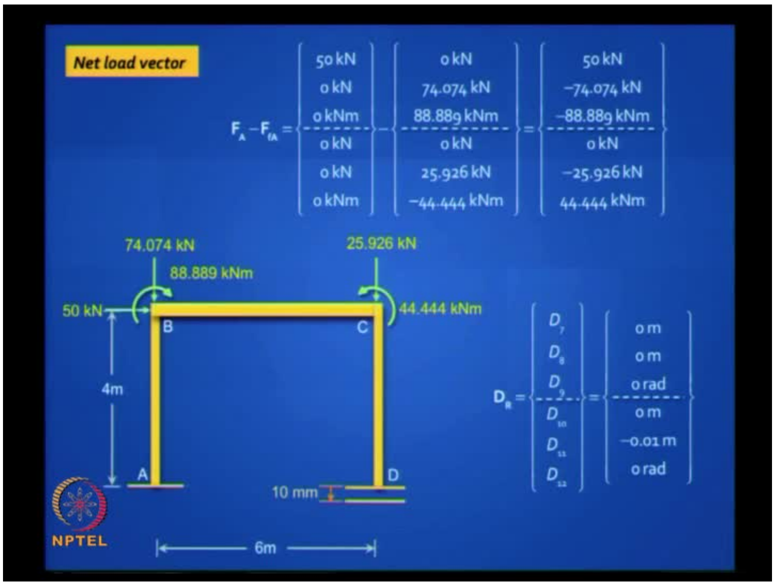

What is the next step? No. We still need resultant loads which is actually the same as this because you do not you have a 50 kilo newton acting at the third coordinate. So, you you do this and you draw, sketch, and the same. Now, this is a structure that I am going to analyze and superpose this result with the fixed end force values that I get. This structure is loaded with equivalent joint loads and this is identical to my original structure. I have got rid of the concentrated load acting in the middle of the element 2. I have replaced it with the equivalent end forces; both vertical forces end moment; got it? And what is guaranteed? What is guaranteed is the D A vector will be the same, and that is the beauty of the equivalents. right ok

And also, note in this problem, you have a 10 mm settlement. Now, I have given you an assignment. The last next assignment where you get two problems only to do; one is a simple problem for conventional stiffness method, but I have thrown in a bit of temperature because we discussed that in the last class.

Second is of a funny shape frame; a shape frame like that fixed here, hinged here. So, you should take advantage of that hinge and with the support settlement and U D l on that beam. So, this is kind of suspended from above, and this is resting on the ground below; it is an interesting problem; try it ok.

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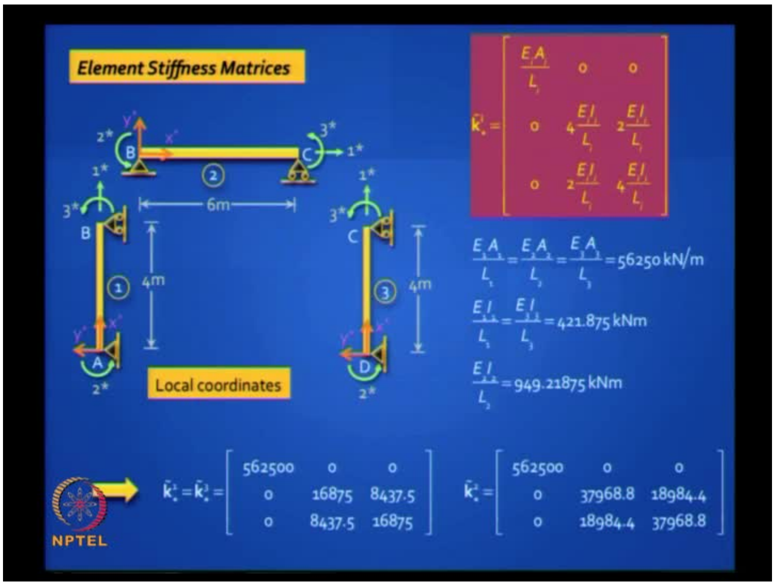

Element stiffness matrices, you know, the formula you know the EI by L. So, this is child’s play. You can do this EA by L is also known. So, for the first and third elements, it is going to be identical because they are identical columns. Only for the second element, it is easy to write always EA by L 0 0 and the rest is 4 EI by L 2, EI by L; very easy to write down; mechanically you can do this.

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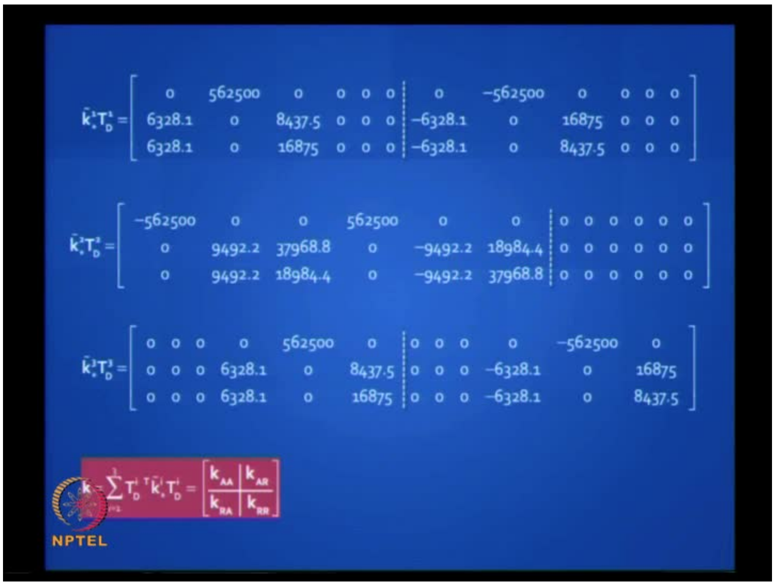

What do you do next? You generate the structure stiffness matrix. You do it in two phases; first you do this k i T D i for the three elements; all these can be done by matrix multiplication. If you are doing manually also, it is not difficult. It is only a 3 by 3, 3 by 6; then, you add up all the contributions. You do not go have worry about slotting here because you got the T D, and you can generate it.

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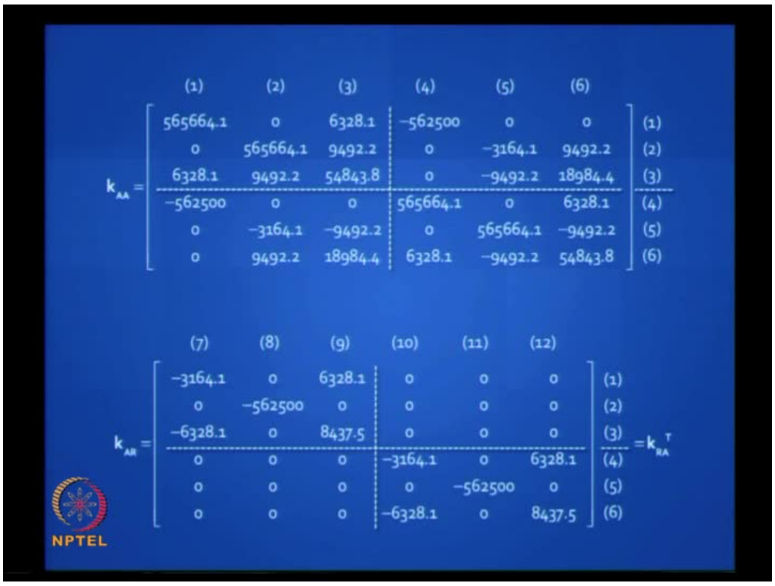

It is a big matrix you get; 12 by 2, and low end behold; this is identical to what we got by the conventional stiffness method, but the operations involved much less effort because you dealt with much smaller matrices. You did not have to worry about the slotting also.

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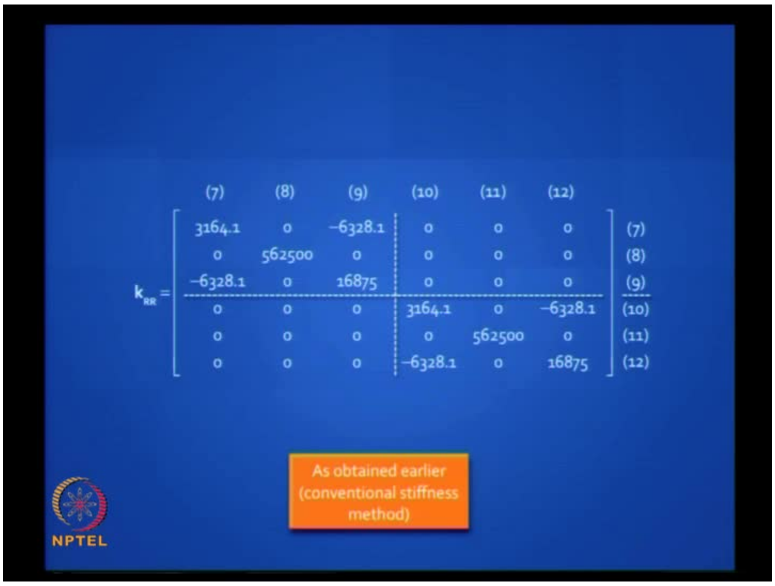

So, it is a much more powerful method, and it is including the effect of axial deformations, and you are getting exactly the same results.

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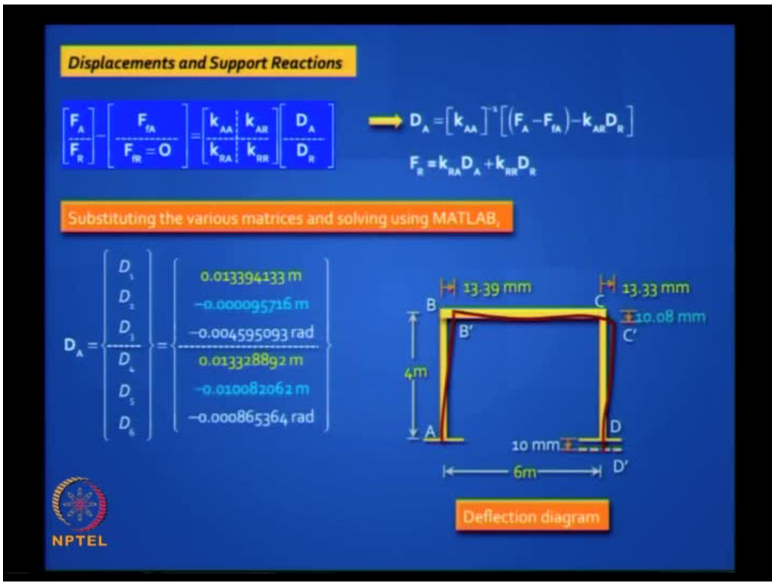

So, the next step should give you also the same result. You will get the same deflections and you will get the same rotations. So, you can do that, and we got the same solution as we got earlier.

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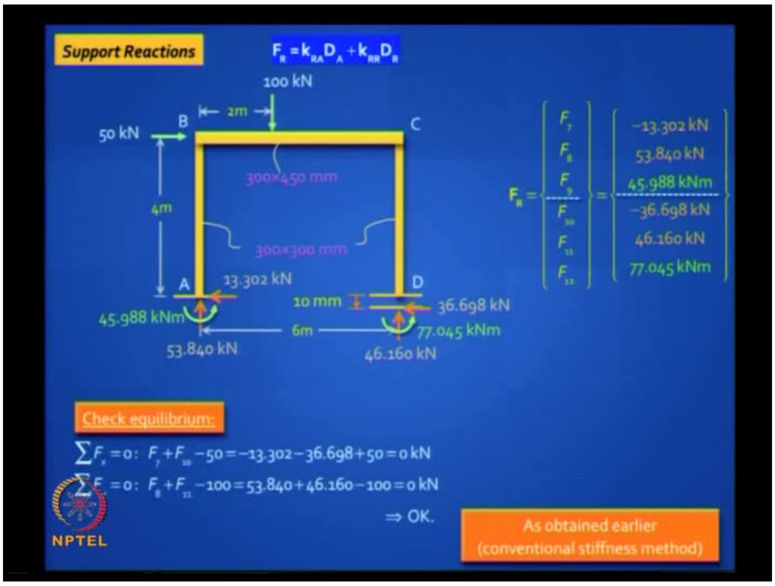

A support reaction also is identical. So, these two steps are common to both reduced element stiffness method and the conventional stiffness method. And so, you can check equilibrium, find out your reactions, make sure everything is okay; find out your member forces.

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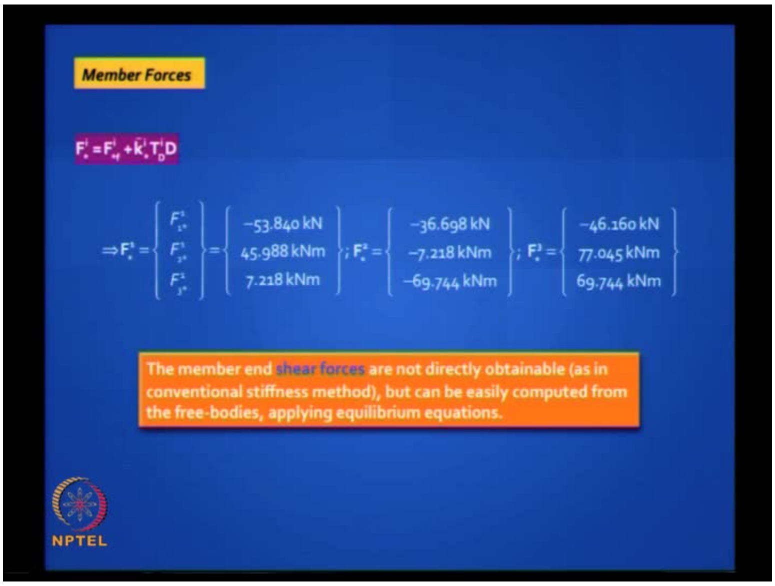

Now, this is where there is a slight difference from the previous method. What is the difference? You have only three degrees of freedom. In the conventional stiffness method, you got everything; you got the member end; moments member end; axial forces member end shear forces. Now, you got only three; the rest you got to figure out yourself, which is which is ok

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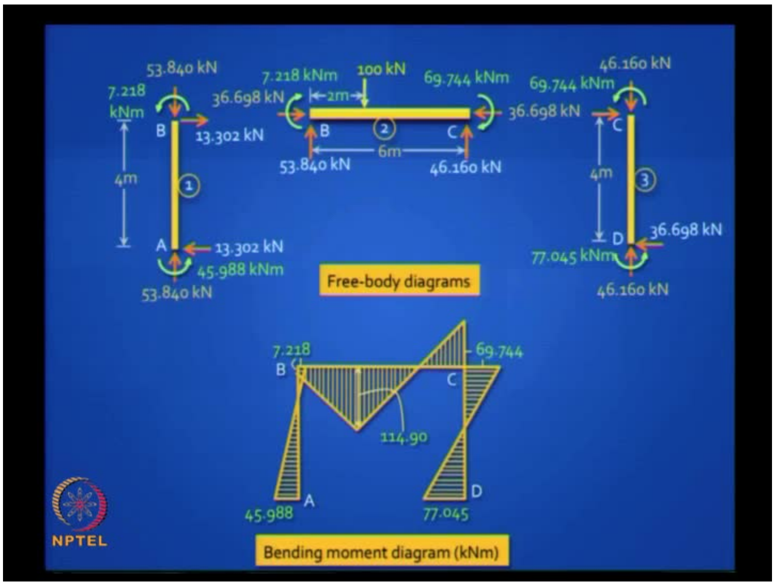

So the member end shear forces are not directly obtainable as in conventional stiffness method, but can be easily computed from the free bodies, applying equilibrium equations. So, let see how to do that. So, this is what you get from those vectors. You got the two end moments for each of the elements and you got the axial forces. If it is plus, it means extension; if is minus, it means it is compression.

Now, the rest of it, you can get from equilibrium; isn't it? You take the second element; you can get the vertical shear force; so, you do that. That is is one step away; that is it; you are ready; bending moment diagram is the same; exactly, what we got earlier.

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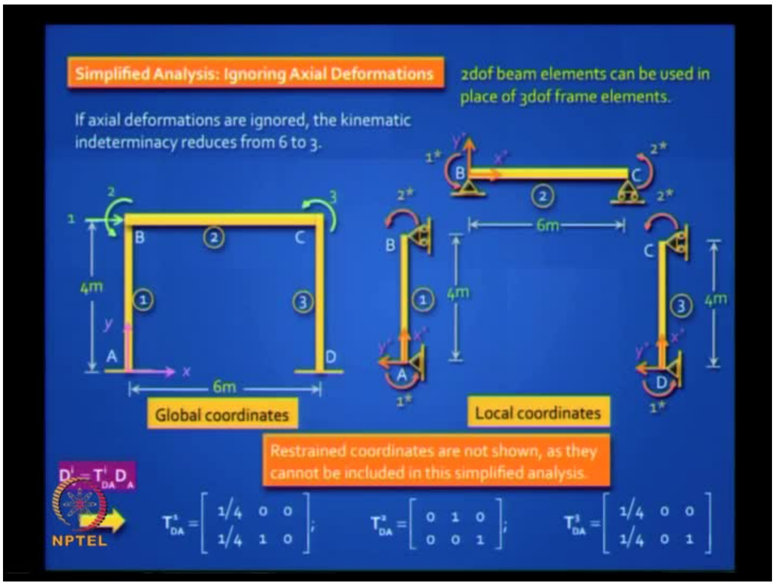

Now, is a interesting step; why should we work so hard with even that matrix? What is the advantage of ignoring axial deformations? The plane element reduces to a beam element. So, you can work with beam elements, throw away the axial deformation, and one way to check the answer is put EA tending to infinity in the problem because then it becomes actually rigid and you should get the same result, but your effort required is much less in this method.

So, if axial deformations are ignored, the kinematic indeterminacy itself reduces from 6 to 3; that is a big reduction. Why does it reduce from 6 to 3? because the columns A B and C D will not change in length. So, you got rid of two vertical degrees of freedom at B and C, and BC also will not change in length. So, you have only one sway degree of freedom. You can choose either the left end or the right end; the choice is yours. The deflection will be the same. This is what we did in slope deflection method and moment distribution method right.

So, it is a massive reduction of effort, and let us not waste time in calculating reactions through these techniques. So, we do not even put global coordinates for reactions. So, from a 12 degree of freedom model and now three degree of freedom model of the structure level, it is a tremendous saving in effort. So, the two degree of freedom beam elements can be used in place of three degree of freedom frame elements. So, we throw away the axial degree of freedom; that is it.

So, you have 1 star, 2 star, for each of the three elements, and the restrain coordinates are not shown as they cannot be included in the simplified analysis. The reason is - once you are going for chord rotation way of dealing with sway, then do not bring in do not bring in those restrain degrees of freedom; which means, now, how do you deal with the 10 mm support settlement at D? Do you understand?

Now, how did you do it in slope deflection method? Let us say, the portal frame D goes down by 10 mm; earlier you could handle it because in the D R vector, you could fit it in. Now, what do you do in the fixed end force? You have to handle it because if D goes down by 10 mm, C also will also go down exactly by 10 mm. If CD is not going to change in length, BC will undergo chord rotation, and that can you can get the fixed end moments. So, that is the clever way of doing it, and that is what we will do.

But first, can you write down the T D A matrix? Do not worry about T D R. There is no T D R here. Can you write down the T D A matrix for the three elements? What is the size of each of them? What is the size of each of them?

2 by 3.

2 by 3 right 2 by 3. So, write them down. So, this you should do; no excuse. This is simple. Write down the 2 by 3 T d A matrices for the three elements. Well, they are identical for elements 1 and 3, they have the same.

Yes sir.

They are going to behave identically; is it not?

Yes sir.

The start node is the same at the bottom.

So, reduces your effort considerably. Write them down and tell me the values. Let us take the second element; what is it going to look like? What is a T D A matrix for second element? Very easy; what is it going to look like?

0.

0.

There is no minus 1; anticlockwise is positive.

0, 1.

0 1 0 0 0 1 and for the first, and third element 1 by 4 0 0 1 by 4 1 0; is this clear to all of you?

Not identical.

They are not identical. Oh yeah, that 0 1 is shifted; yeah, you are right.

Only the chord rotation is identical; the rotational degree of freedom 3, the global level pushes that one to that corner; is it clear? Very easy to do; any doubts? Can we proceed?

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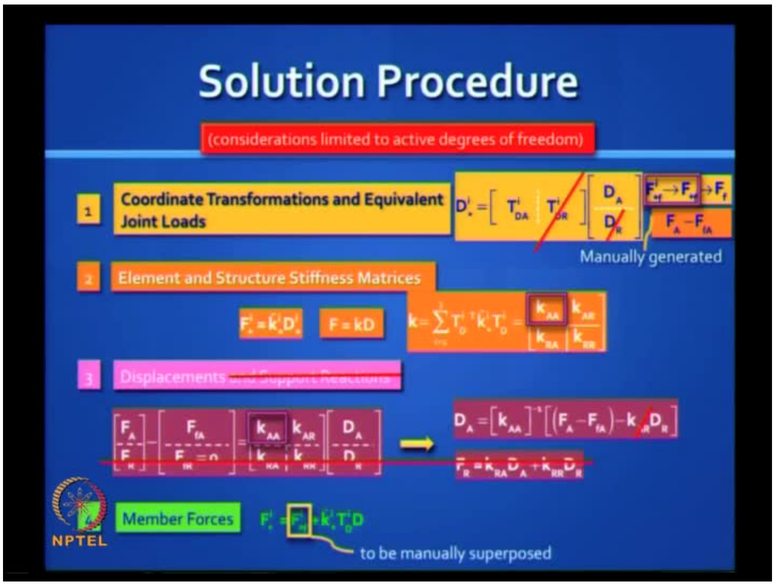

This is massive simplification; remember where we started? We had that 12 by 12, and then I have to explain each column and each (( )). Now, this is child's play; right the tremendous reduction in effort, and so, procedure is the same with much less effort, but consideration limited to active degrees of freedom. So, you can cutoff, chop off, some of these calculations. First thing, knock off the T D R. We do not need T D R then; just need k a a. What is a size k a a? 3 by 3; that is all. This is Nano technology working with small matrices and you are still getting good results for actual practice ok.

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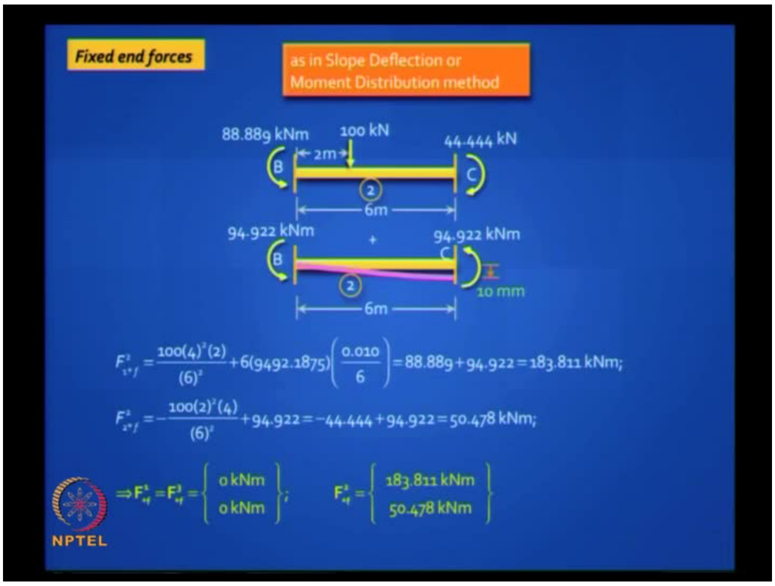

What about the support reactions? Who cares? Throw it off. So, you just need DA; that also DA minimal 3 degrees of freedom; plug it in and get it. So, this is what we are going to do. So, you have to take care of that 10 mm support settlement in the manner that we have did in slope deflection method. So, I hope you are familiar with this calculation. You have this additional fixed end moment caused by the support settlement; that element will just go down. There is no force in that element. So, you work this out. You can find the total fixed end moment. Shall I go ahead because we have done this problem earlier?

So, the first element and third element will have no fixed end moments because they are vertical columns. The second element will have fixed end moments caused by two loads: one is a direct loading 100 kilonewton; the other is an indirect loading; 10 mm settlement in the right; that means clockwise chord rotation of 10 divided by 6000. So, this can be done. You got the fixed end force vector; what do you do next?

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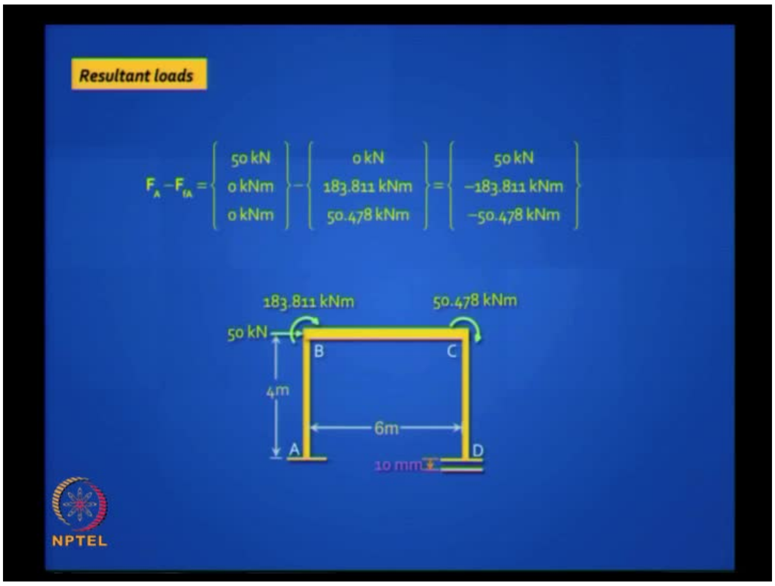

You get the resultant load vectors in the same manner, and here, you do not worry about those concentrated loads, as we did earlier, because they are not going to affect the displacements because they actually rigid. So, even if I put them, it is not going to make any deflection; it does not cause any axial deformation. Is it clear?

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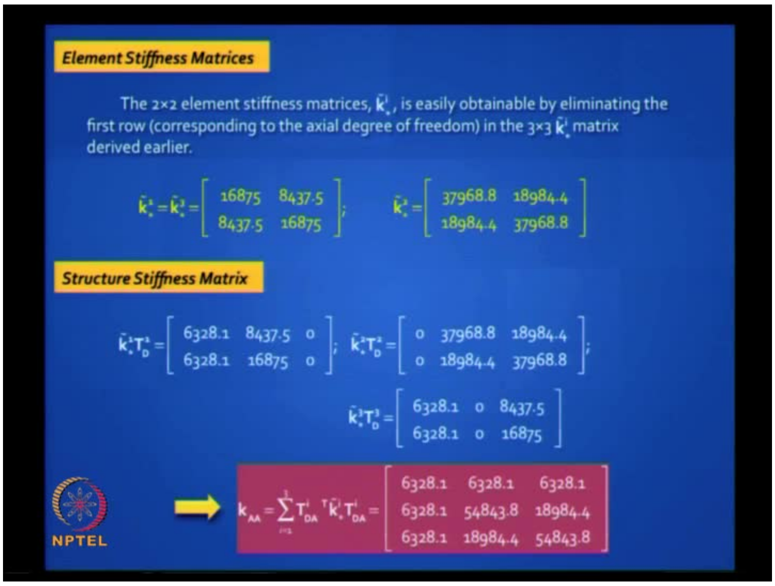

So, I do not even need to do that exercise, and so, this is identical to slope deflection method, but you made an assumption. Assumption is - axial deformations are negligible. How? Through That is, for this problem, we will know shortly. So, you have got these moments. So, your problem is now reduced to this problem 3 loads; no vertical loads. Element stiffness matrix is very easy. What is it? 4EI by L, 4EI by L, 2EI by L, 2EI by L.

So, that is easy. You can do it or you go back to your previous 3 by 3, and knock off that EI by L row and column; first row (( )) you will get this. So, no big deal; structure stiffness matrix same procedure you generate. Will this structure stiffness matrix be the same as what you got earlier? Will it be the same? Yeah. First of all, it would not be because first of all the size is different. Well, this is 3 by 3; that was 6 by 6. So, this is Nano. We are playing very small and you have got these values. They are, the units are same for all the components in the stiffness matrix.

No.

No. No. For your translation, your rotations, that is why I do not write any units outside. So, do not do that. There are no common units here. How do you find the displacements? Same method; you get some answers. right.

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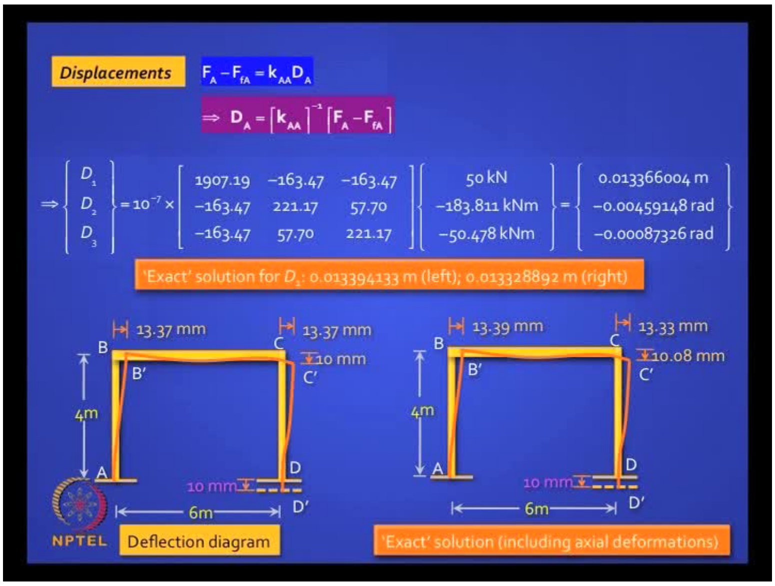

Now, it is interesting to check, whether these answers, these deflections are what you got when you considered axial deformations. This is what we are getting now, and it is pretty good. This is the exact solution – 13.37mm is what you get when you ignore axial deformations, and it is the same left end right end because we are assuming that beam will just move like a rigid body, but the exact answer is 13.39 mm and 13.33; is it okay as an engineer? Civil engineers make errors in the order of meters. So, compared to that, you know, .03 mm error is nothing; not only that, in practice, you must take all these solutions you get from the computer with the big pinch of solved. The reason is - so many unknown parameters are there. First of all, you have written a load of 50 kilonewton. Where did you get it from? From the wind; sorry, wind is very uncertain right.

So, there are so many uncertainties involved. You have to have a probabilistic look at the whole picture. You should also know what is a kind of material that you are using and what is the who is going to build the structure, but that is no excuse for an analyst; an analyst should say - you give me whatever you give me; I will give you to the level of accuracy that, and I should know what is that percentage of error involved. So, analysis is one thing; design is another. For design, you have to be very practical, but we are civil engineers, while we do structural analysis. So, we are quite happy with in fact, we know that the 13.37 could in reality be actually 15 mm; it is ok.

So, did I tell you that joke about 2 plus 2? No, then what is 2 plus 2? The answer depends on who you are. If you are If you are a mathematician, what do you think? 2 plus 2 is? What is the answer that mathematician will get? If you are a bad mathematician as you are, you will say 4, but if you are a good mathematician or professor of mathematics, you will never give anything that is our practical value to anybody. So, you will quote a theorem saying that, if there are two real numbers, if they are adding up, and you will give the lemma which says it can and you can plug in your numbers if you are interested because you are interested in the algebra. So, that is the mathematician.

If you ask a good a good civil engineer, what is 2 plus 2? What do you think good civil engineer…

(( )).

Ah.

4 plus or minus 2, 4 plus or minus 2.

Then you are a bad civil engineer, but at least you did not say 4, but you depends on what kind of errors you are willing to tolerate. So, I would say, most good engineers would say something; say between 3.5 and 4.5. That is not as bad as 2 and 6; 3.5 and 4.5 - why do you say that? because you know jolly well that you will never get 2 plus 2 to begin with the 2 itself will have (( )). But the joke is not on the civil engineer; the joke is on lawyers and charted accountants. If you ask this question to a lawyer or a charted accountant, what do you think will be the answer?

[Noise]

What do you wanted to be?

And if you say, you want it to the minus 320.6, they said.

So, be it my consultancy, if he will be this much.

And you will hire the mathematics professor and the civil engineer to to prove to you that 2 plus 2 is minus 325.

So, be a good civil engineer and appreciate the abnormity of the errors.

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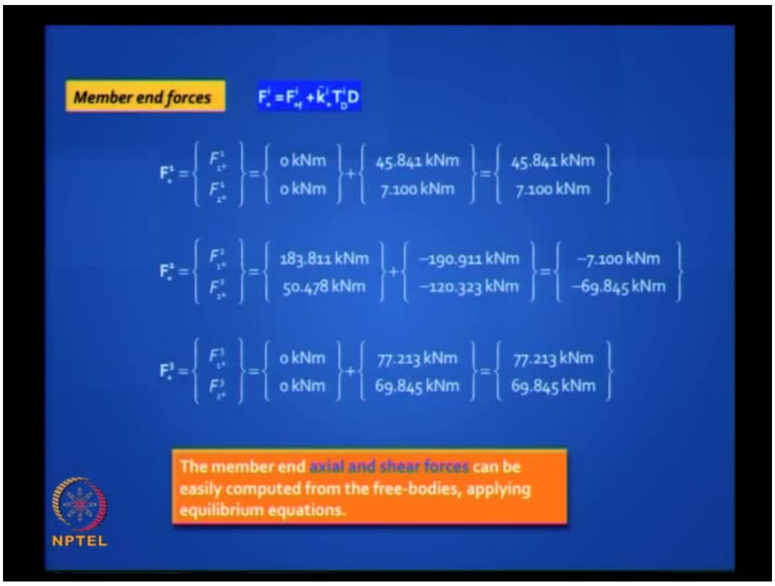

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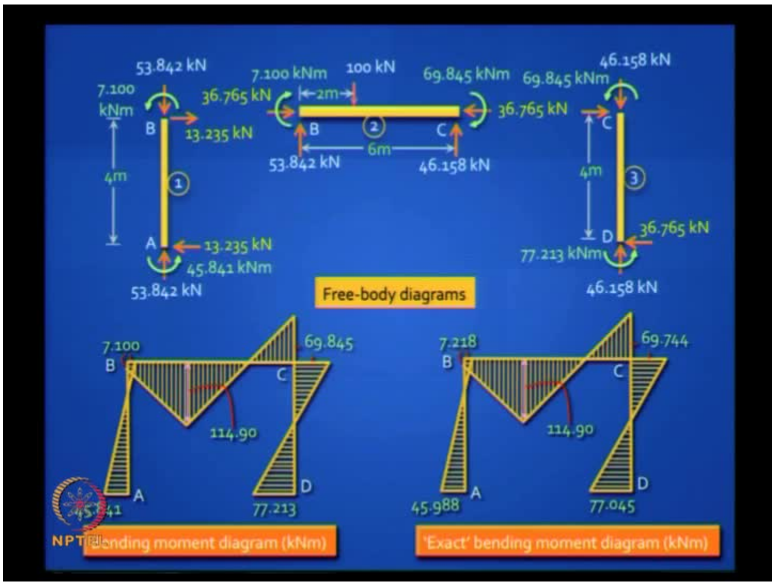

Member end forces, you get these, mind you, you are getting only the moments, and what do you do with those moments? You have to generate the axial and shear forces which you can from the free bodies. You will get some difference from what you got earlier surely you are right, but you are really and you can work work out the the shear forces and axial forces from equilibrium.

First, you get the shear forces. Then, you get the axial forces. Then you draw the bending moment diagram. Now, this is interesting; who cares about deflection? I want to design my structure for bending. How do these bending moments be compared? And you will find that this is pretty good. 69.845 is what you got without axial deformation. 69.744 as long as it is between 60 and 75, it is… well, you will design it for 70. That is what you do and design for 70, and on top of that, they will be factors set degrees.

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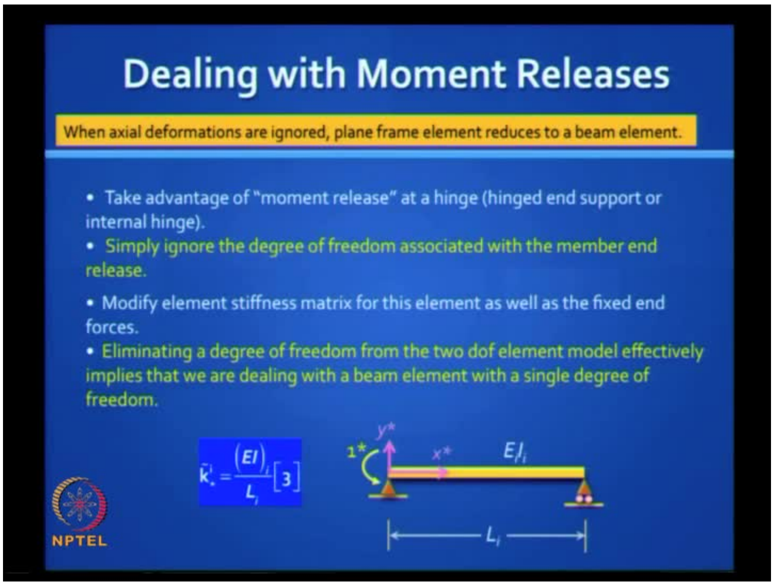

Next topic: How do you deal with moment releases? When axial deformations are ignored plane element, plane frame element reduces to a beam element. So, we can take advantages of moment release at a hinge hinged end or support, or internal hinge. We do what we did for the beam, and it is - you ignore the degree of freedom associated with it; modify the element stiffness; 4EI by L becomes 3EI by L, and you have only 1 degree of freedom. If the other end is hinge, if it is guided fixed, you know it is going to be EI by L.

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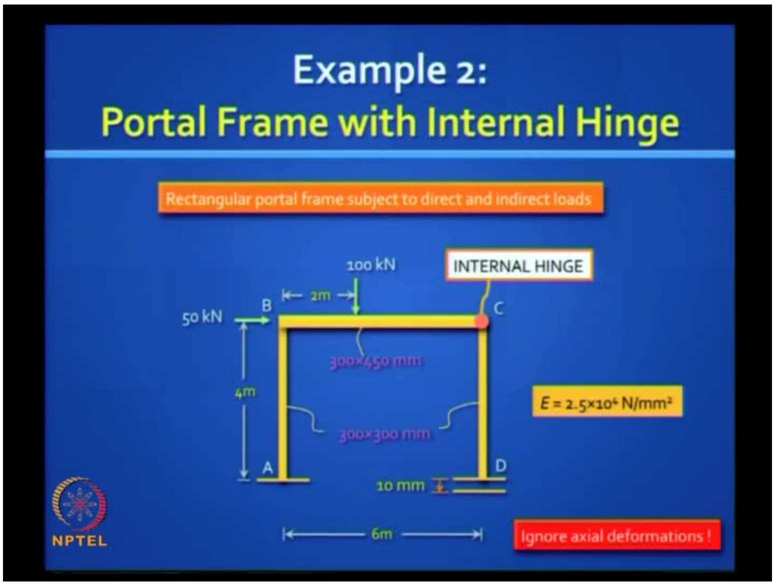

So, we will do this. This is what we have done earlier. Let us demonstrate. Let us demonstrate for that same portal frame, with an internal hinge, this is something we could not do in the slope deflection method. right We could not do such problem, but now, you can, in a matrix formulation. Is it clear? I have taken the same frame. We are going to ignore axial deformations; make the life simple, but we have got a hinge. There, is it going to make the life more complicated, or its going make it easier; that hinge, it should make it easier. So, let us make it easy solution. Procedure is same, coordinate transformations.

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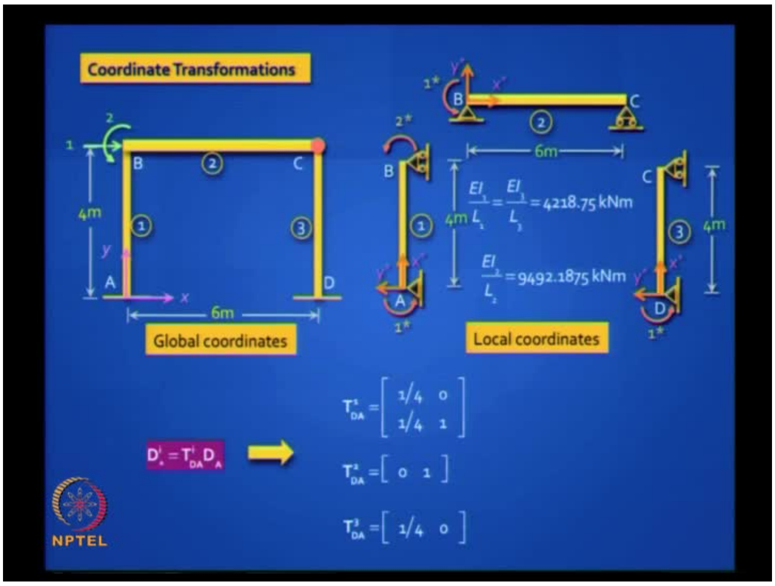

Now, look at the changes we are making now compared to the previous problem is in elements 2 and 3. We knock off the rotational degree of freedom at C. So, size of your stiffness matrix your T D matrix reduces. You have 2 degrees of freedom for element 1; no change there, and you have only 1 degree of freedom for elements 2 and 3, and you should release at the right place. Where there is a moment release, there is a hinge internal hinge at C, let us see. It matters a little; whether the hinge is internal or at the support. Clear?

How will this change? This is the change. Another thing, you are in your global coordinate 3 has gone now because you do not you have you cannot you cannot talk of D 3 in your structure because you have two different rotations in the connecting members. So, you remove it. You do not need to put a clamp here; just remove it silently. So, you have only 2 degrees of freedom; a degree of kinematic indeterminacy; only 2 in this problem.

Remember, when we did by conventional stiffness method, we had 5 with the with the hinge; without the hinge, it was 6 here; without the hinge, it was 3; with the hinge, it is 2. So, you have D 1 and D 2 in your structure as your unknown rotations; is this clear to you? The T D matrix; it just follows from the previous case. Just ignore that rotational degree of freedom at c.

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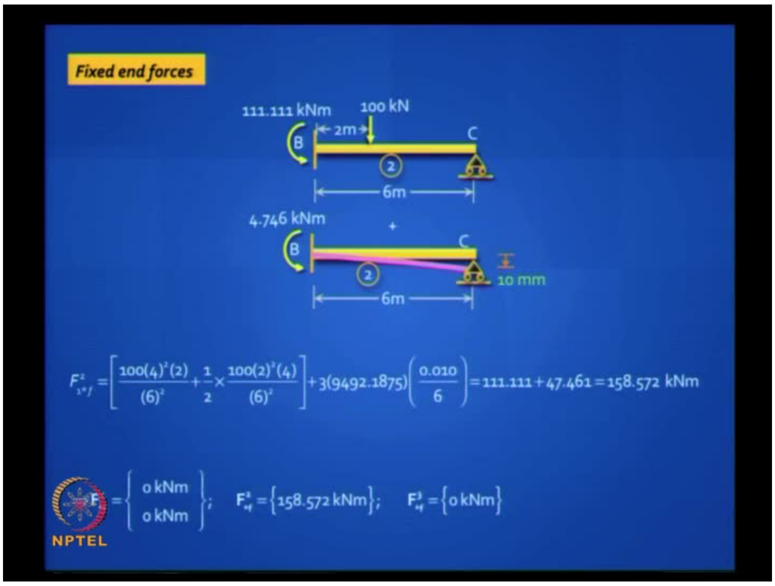

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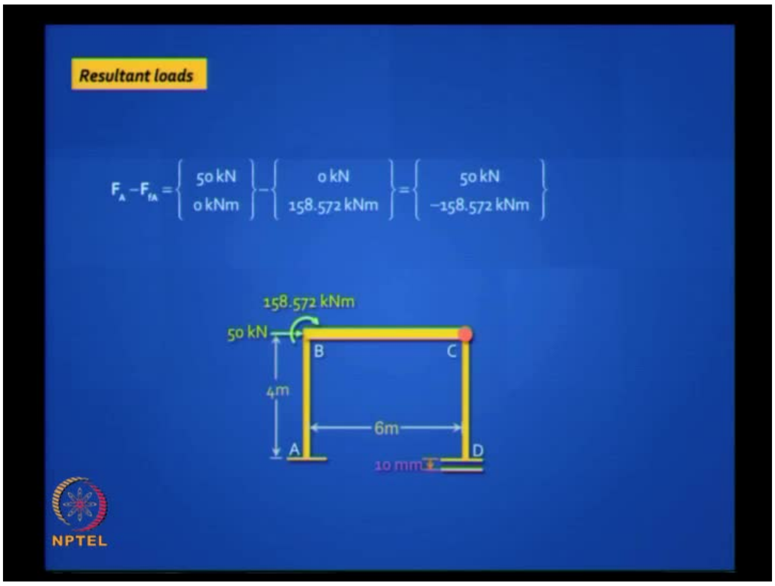

Then, fixed end forces, what is the change we need to do? Compared to the previous one, you have to release at C, the moments. So, you are dealing with the propped cantilever. You know how to find out. You know how to find out right for of for that consternated load. You know what to do you have to take. Half carry over half, and for the support settlement, it is not 6 EI by L squared; it is 3EI by L squared. Is it clear? You know that, for a cantilever, that is what you do, and you get the answers. You get the fixed end forces; size of a matrix has now come down from 3 to 2 vector, and that is that is what you get; your fixed end forces, your nodal forces.

Your equivalent joint loads are now, that 50 kilo newton was given to you as input. The only thing that came extra is that 158.572. So, you are going to get the same response with these, as you got earlier. Is it clear? You do not put any vertical forces; you do not put any moment at C, but here, you might have a bigger effective of axial deformations. The reason is - this frame is very flexible. So, how much it moves will be effected a bit by axial deformations. So, let us take a look at what you get. What about your stiffness matrices?

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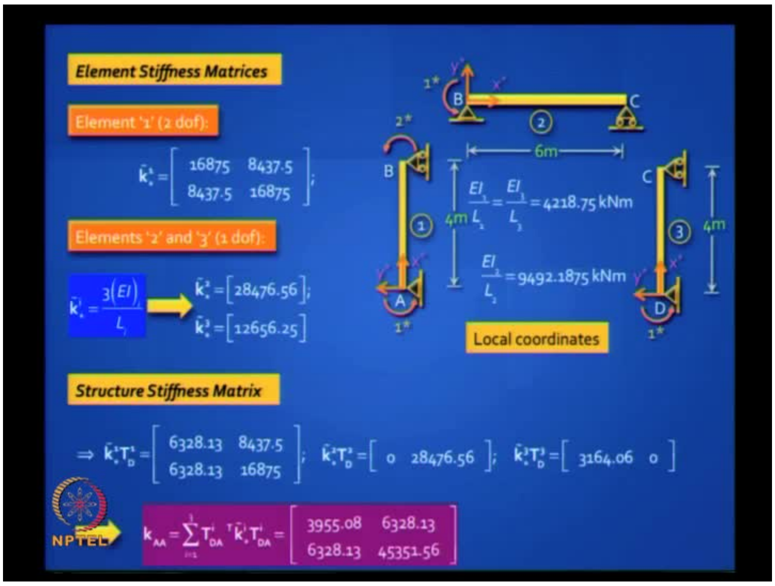

For element 1, no change. You have the same 2 degree of freedom. Elements 2 and 3; the change is 3EI by L. right So, you do that 3EI by L. Write down these values. It is a 1 by 1 matrix; very easy to calculate structure stiffness matrix you can generate; it is a 2 by 2 structure stiffness matrix right because you have only 2 degrees of freedom. Find the displacements.

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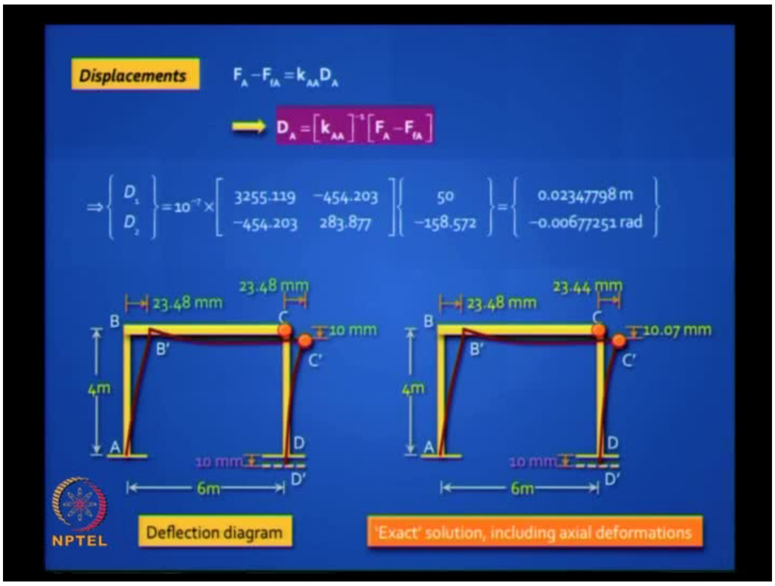

Now, your deflections are 23.48 and the exact solution is also 23.48. So, this is very good. This is excellent solution, excellent answer. I think, the error you get is when their legs are inclined in that, I show, there we get very good answers; also, the vertical displacement is 10.07 compared to 10 at the bottom. ok

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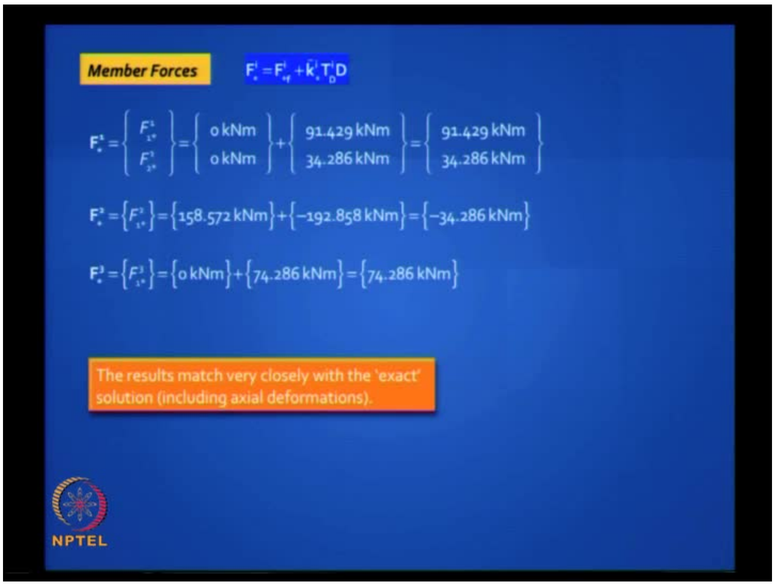

So, member end forces calculated; these results also match what you got earlier.

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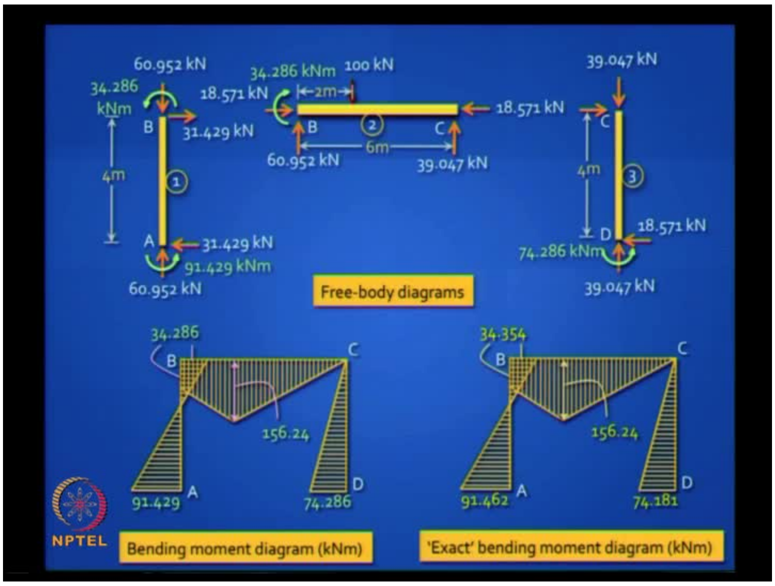

Draw the free body diagrams, bending moment diagram. Final comparison, some minor error, but certainly, No. No major error; is it clear? So, with this, we have covered reduced element stiffness method as applied to frames. Clear? Thank you.