Advanced Structural Analysis
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Module No. # 6.2
Lecture No. # 34
Matrix Analysis of Plane and Space Frames

Good morning. This is session number 34, module 6 – matrix analysis of plane and space frames.

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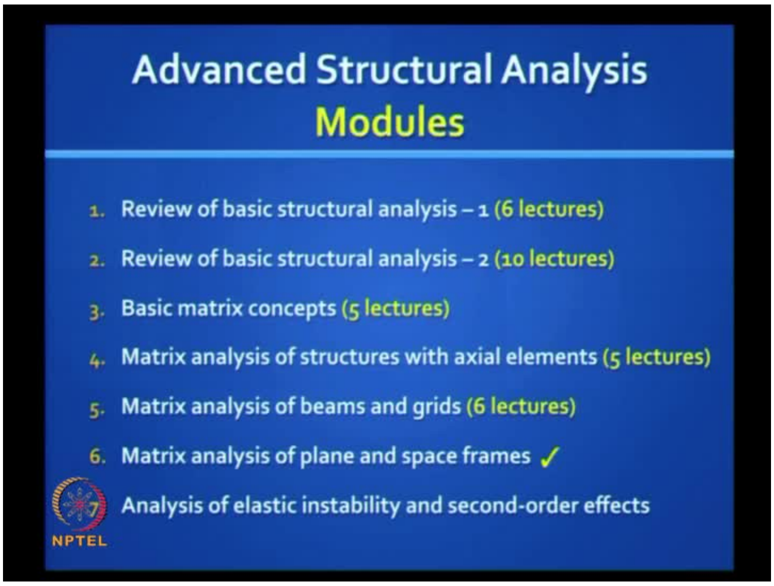

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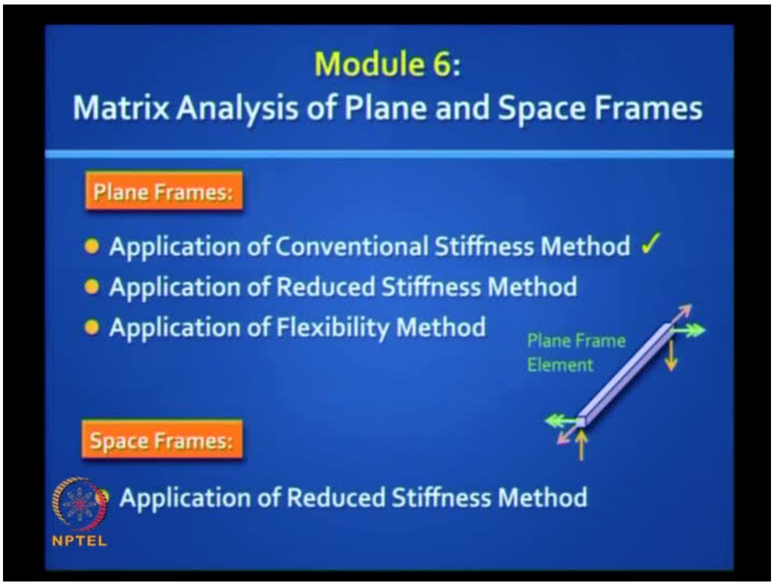

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If you recall, we had just started the application of the conventional stiffness method to plane frames. This is covered in the chapter on matrix analysis of plane and space frames in the book on Advanced Structural Analysis.

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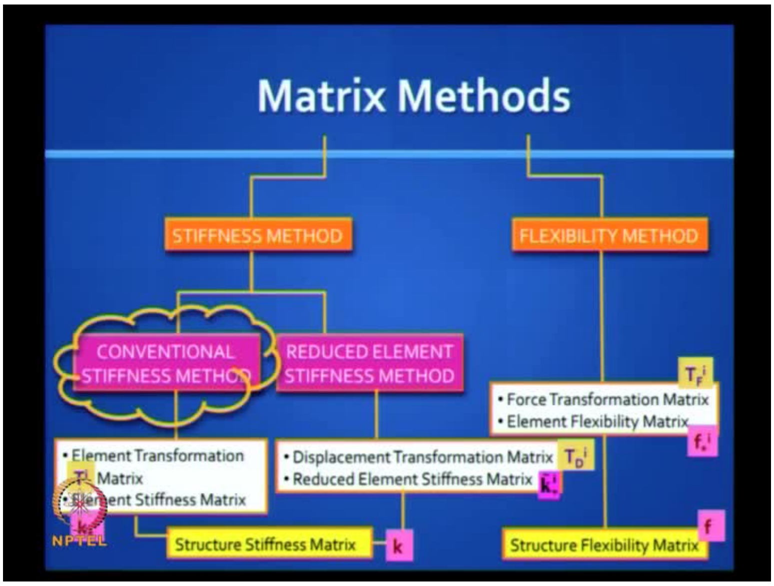

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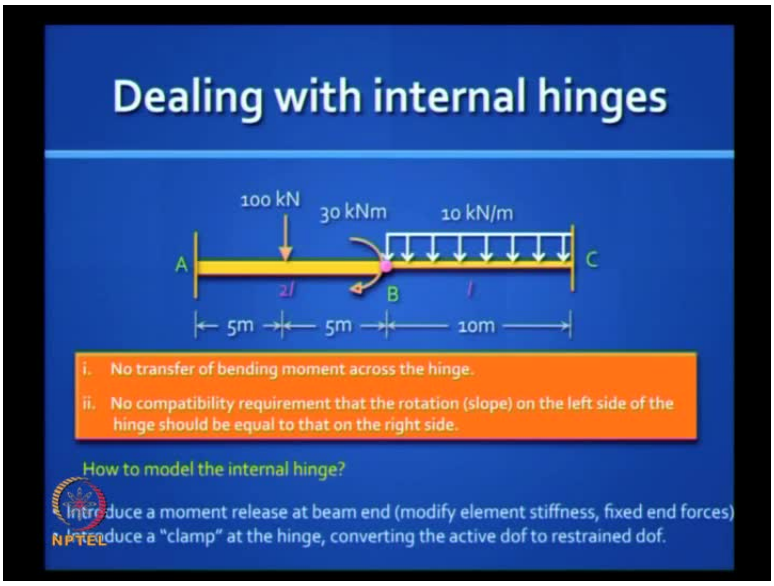

As you know, there are three broad methods we studied. We are looking at the first of them – Conventional Stiffness Method. We will now look at problems where you have internal hinges in frames.

We have already learnt how to deal with these in beams, so it is basically the same. I am just showing you some of the old slides on how to deal with an internal hinge. There is no transfer of bending moment, across the hinge, and no compatibility requirement regarding rotation. So we introduce a moment release and we introduce a clamp.

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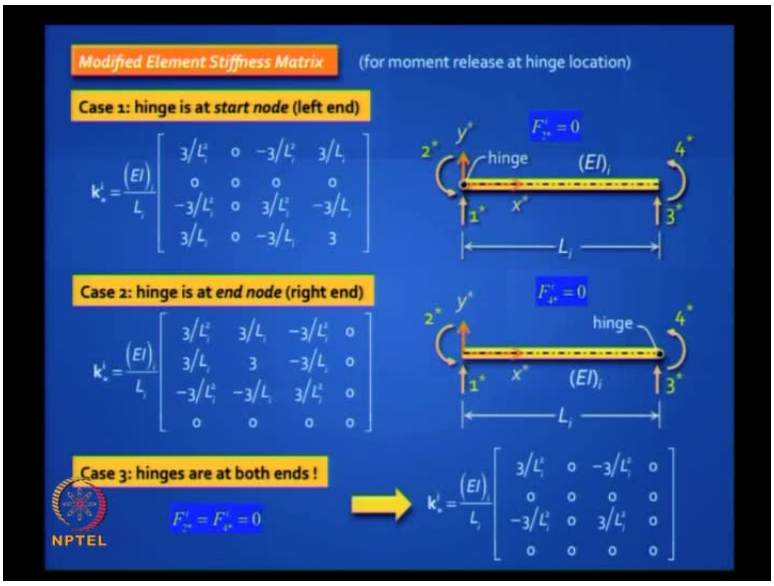

If you recall, we need to modify the element stiffness matrices. Essentially, one of those rows and columns will become full of 0s, corresponding to the release degree of freedom.

If you have a hinge at the start node, it is a second row and second column that gets released. Not only that, from 4 EI by L, it goes to 3 EI by L, the flexural stiffness, and 6 EI by L square becomes 3 EI by L square, etc,. We have done this earlier. If you have hinges at both the ends, the second and fourth rows and columns will become 0s. We are just refreshing something that we did earlier.

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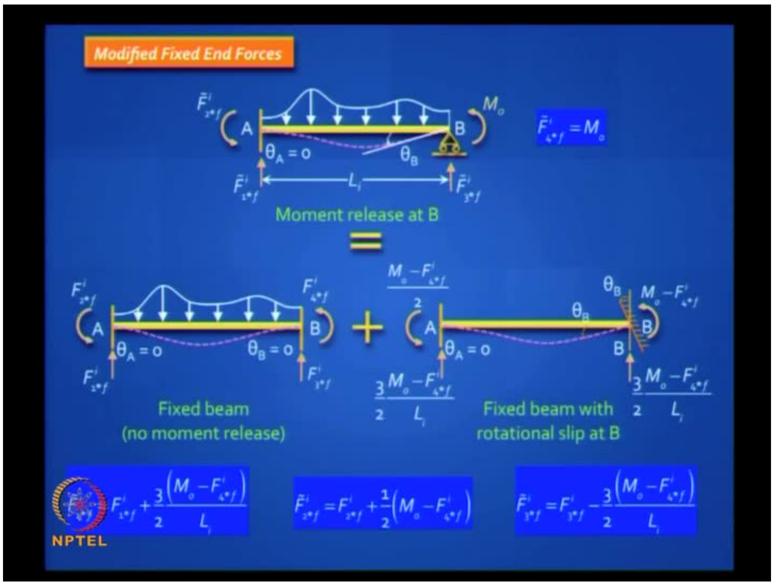

We also know that we need to modify the fixed-end forces, where you have a moment release. Essentially, the fixed beam will become like a propped cantilever and you need to modify the fixed-end moments.

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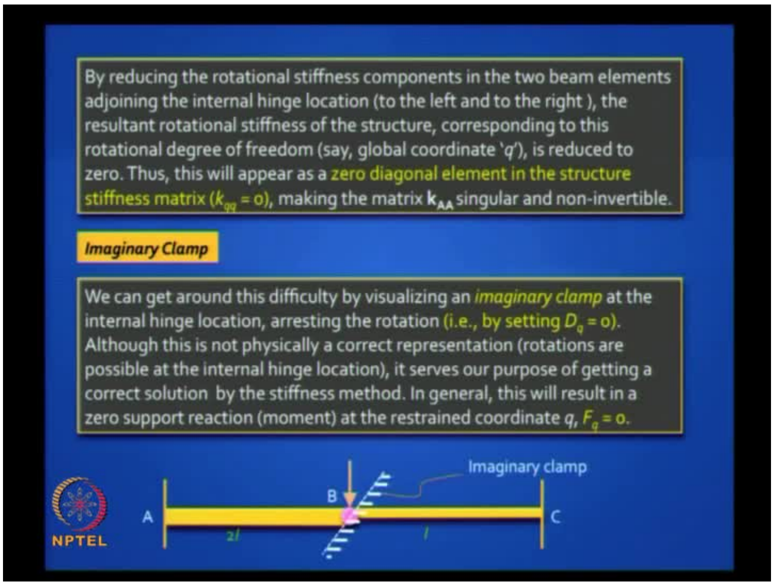

Finally, we said that we need to deal with the fact that we will end up with zero stiffness in corresponding to the degree of freedom, where we have the moment release. Because how we deal with the moment release is – we are going to shifts that global coordinates from active category to restrained category.

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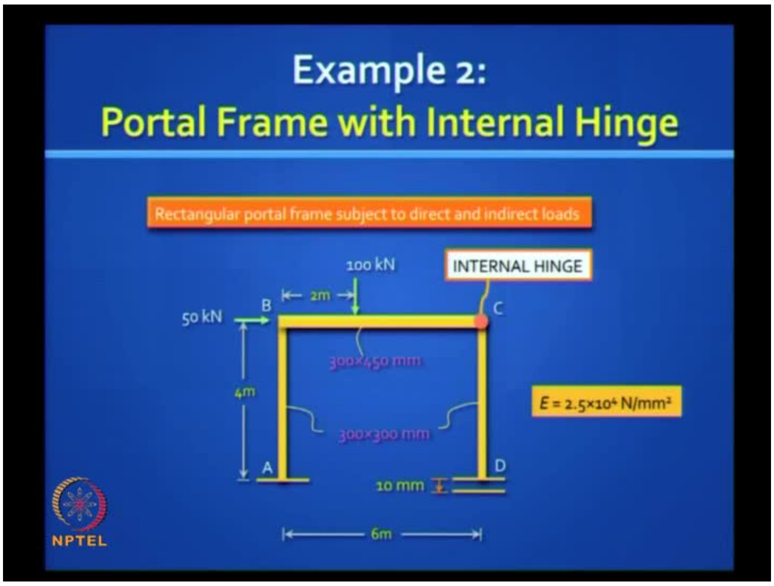

You do not need to inverse the k_{AA} matrix with the 0. Diagonal element in it moment you have a release. It will have a 0 diagonal element. So we conveniently pass it on to the restrained coordinate and it does not really matter. We do this by means of an imaginary clamp. We have done this for the beam. We are just repeating this idea for a frame.

If you remember, we did this problem in the last class – portal frame with a concentrated load. The only change we have now introduced is we bring an internal hinge at a beam column joint. In this case, joint C.

How do we deal with the same problem? Can we make use of some of the work we have done earlier. Can we use the same stiffness matrices? Partly yes. For A B

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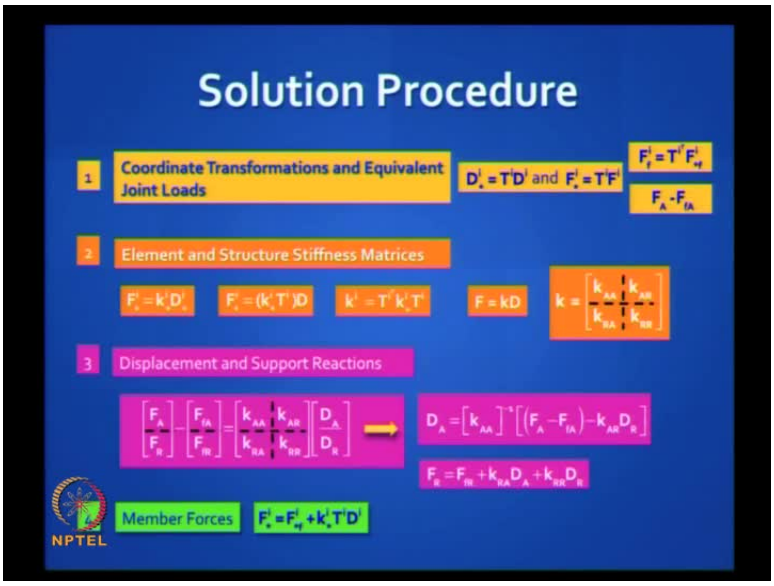

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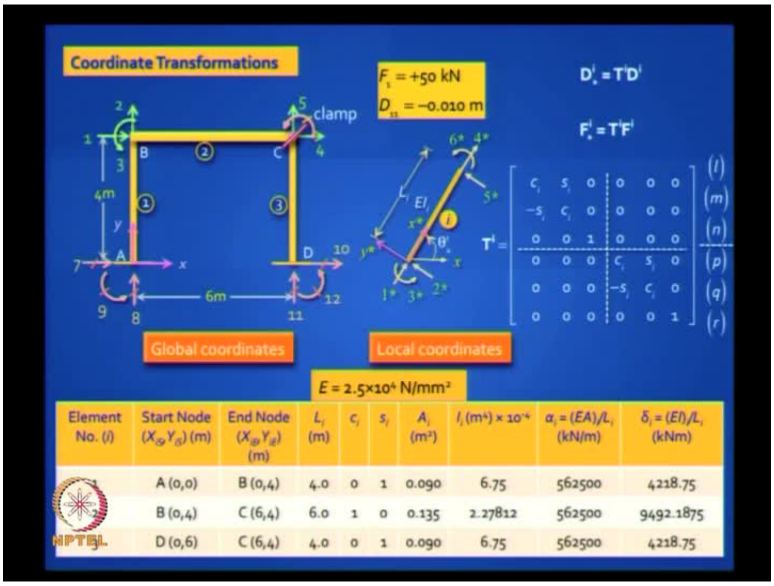

Let us go through this problem. The procedure is exactly the same as we did earlier, except that we need to make modifications for the internal hinge. If you recall in this slide, we have looked at the six global coordinates and the six restrained coordinates. Just reproducing a slide which we covered in the last lecture. But there are going to be changes now. Firstly, you have the internal hinge, at that hinge at C. We are going to convert one of those degrees of freedom. Which degree of freedom? What is the number? Six. We convert it from active to restrained, and we will introduce a clamp. We do all that. Is it clear? We do it so that and that degree of freedom goes to the restrained category. Nothing else changes.

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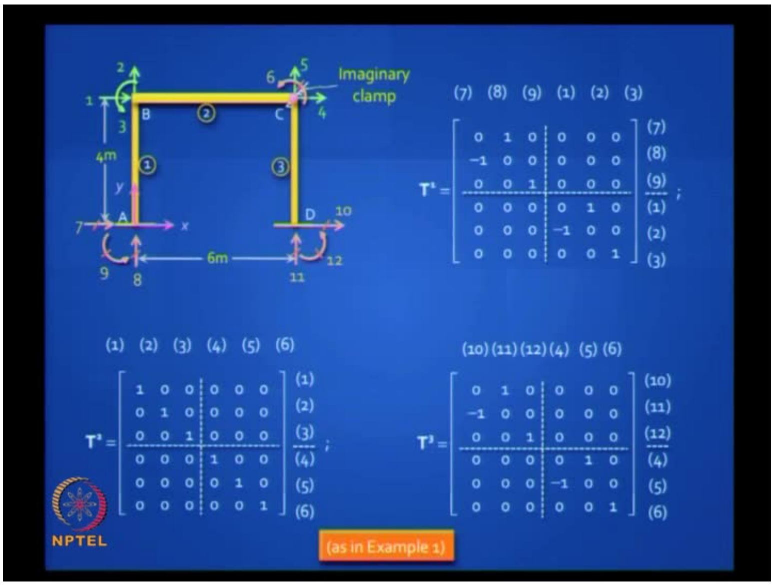

So, how do we deal with this problem, with the internal hinge? Do we need to make any changes for the transformation matrices? Absolutely no change, because it matters little whether the global coordinate is active or restrained.

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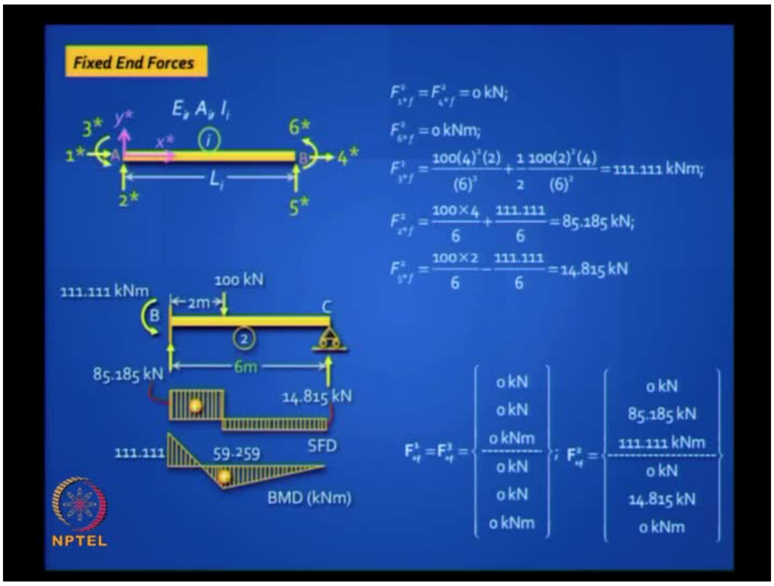

So we the T_1 T_2 T_3 are what we got earlier. There is no change. Do we need to change the fixed end forces? Yes, we do, because if you recall, at C, you have a hinge, so it is a prompt cantilever you have to do.

I think we know how to do this. How to calculate the fixed end moment for a propped cantilever? So we have to work this out. You are familiar with this. We have done this kind of problem earlier. Find out the fixed end forces and then work out the fixed end force vector. What do we do next after this? What do you do after you find the fixed end forces?

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You have to

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You have to convert to global coordinates. And after that?

Net load (( ))

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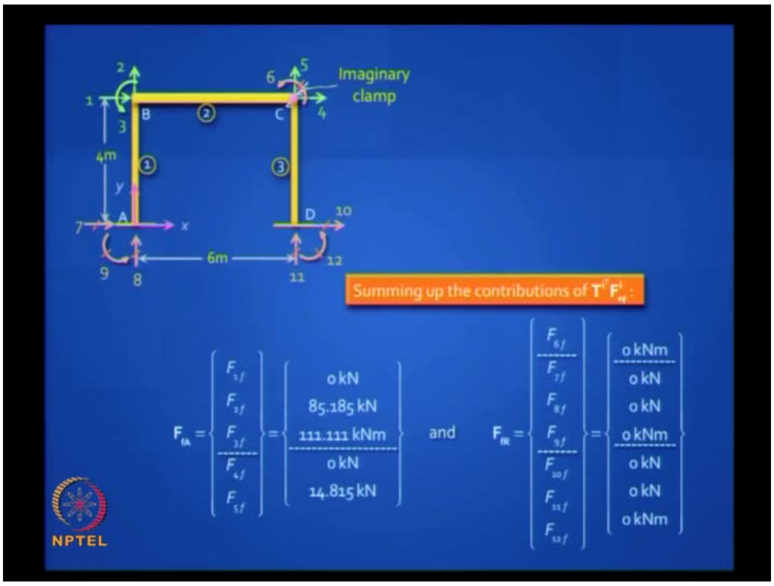

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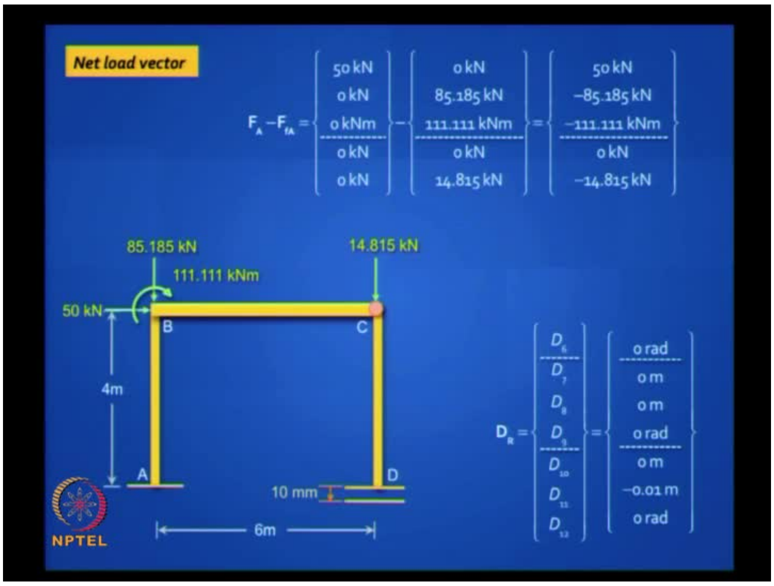

Get the net load vector or the resultant load vector. How do we do that? We do it by T_i, transpose F_*. We are just repeating what we did earlier, except that the force vector is changed slightly, and we get F_{fA} and F_{fR}, and we get the net load vector.

You can draw a sketch, showing that the loads that you get. This is identical to what we did earlier, except that now you have an internal hinge at C and values of those forces have changed.

Earlier, we also had a moment applied at C, now that is released. There is no moment applied at C. Do not forget that there is an indirect loading in the structure. You have D R coordinate, 11 D_{11} will be minus 0.01 meters because there is a support settlement of 10 mm at D. Exactly as we had earlier.

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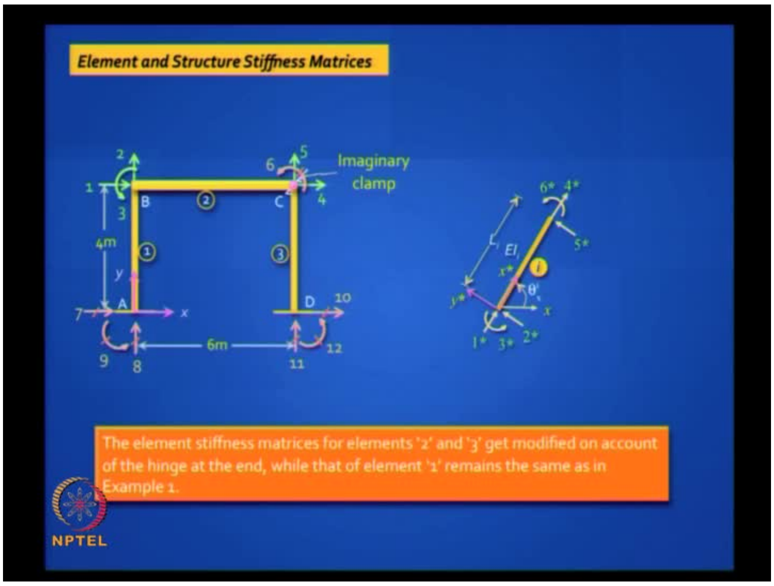

Then, we need to generate the element and structure stiffness matrices. What changes we need to do now? For element 1, any change? No, so you are right. We need to bring changes only for elements 2 and 3. Can you write them down? Can you write down the stiffness matrices for elements 2 and 3?

For element 2, the end node has a hinge, so it will broadly take this shape.

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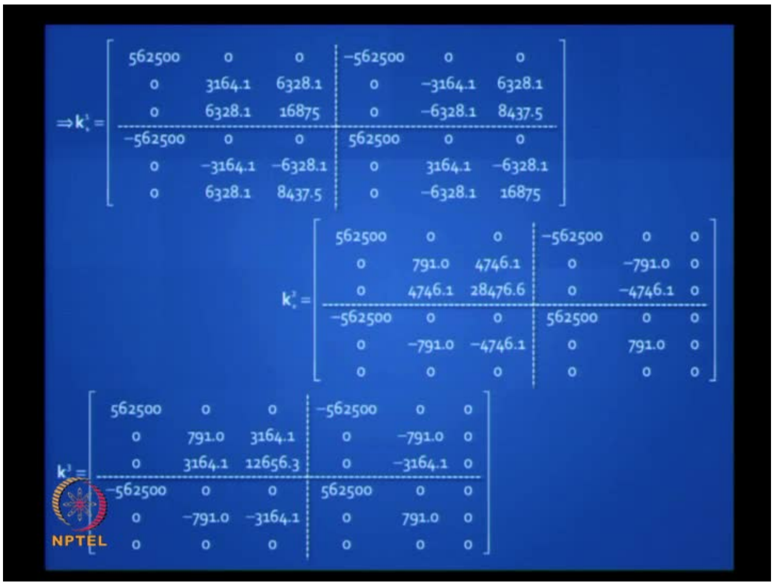

You have EA by L for your axial degree of freedom. And you will find that for the element 2, the 6th degree of freedom. That means, the sixth row and the sixth column will become made up of 0s. And you need to modify the 4 EI by L to 3 EI by L and so on.

Also because our start node is at D, for the third element, the last row in the last column will become 0. Can I proceed? This is you need to go through the book and see this example in greater detail. Just repeating because we cannot possibly solve these matrices in class. You need to solve them by yourself.

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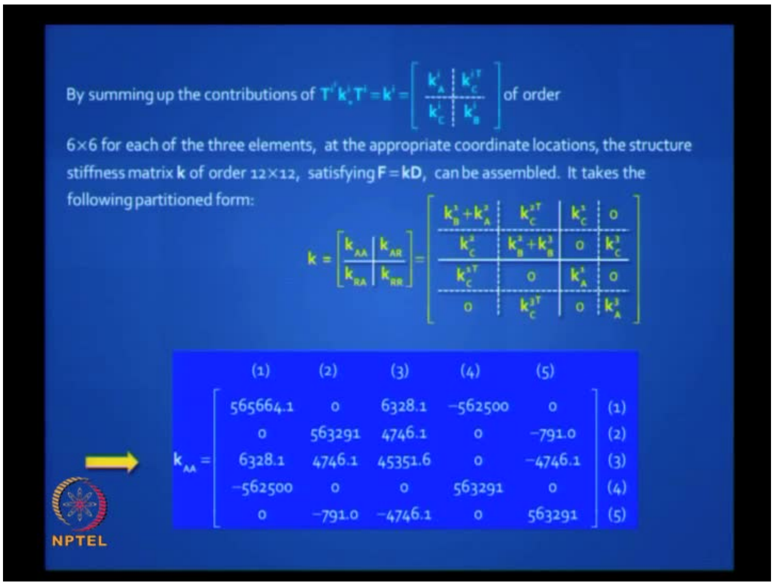

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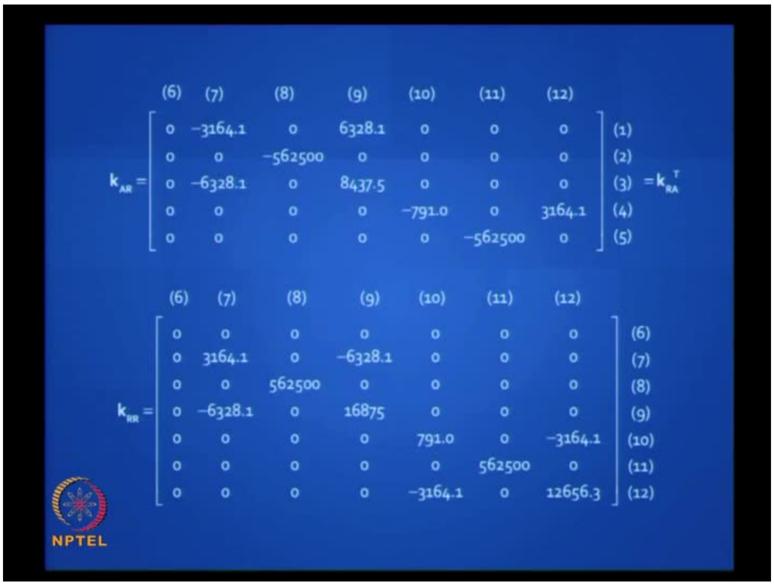

And we have to assemble the structure stiffness matrix exactly the way we did in the last class. I have explained this earlier. So you get k_{AA}. But note, k_{AA} has a size 5 by 5. Without the internal hinge, it was 6 by 6, and then k_{AR} and k_{RR} can be worked out. Please note k_{RR} will be now having a size 7 by 7. That is it.

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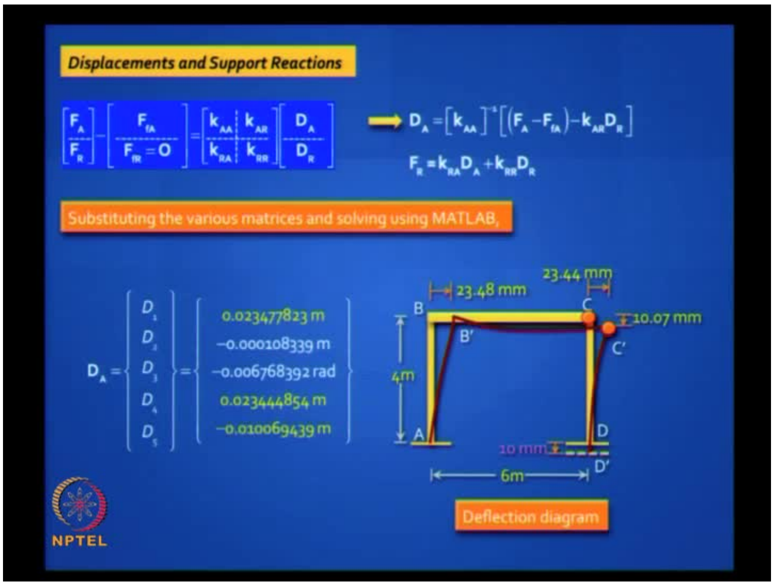

Then you solve for the unknown equations. The method is the same. You will notice that the deflections have increased substantially as compared to the previous problem. Now, you have a sway of 23.4 mm. That is how you interpret the DA vector. D_1 is the sway you get at B, which is shown in yellow, and D_4 is the sway you get at C.

Can you see something about the behavior of DC? What does it look like? It is a cantilever. So, maybe there is a shortcut way of getting that deflection. We will see that. It is good to understand behavior by interpreting the results that you get.

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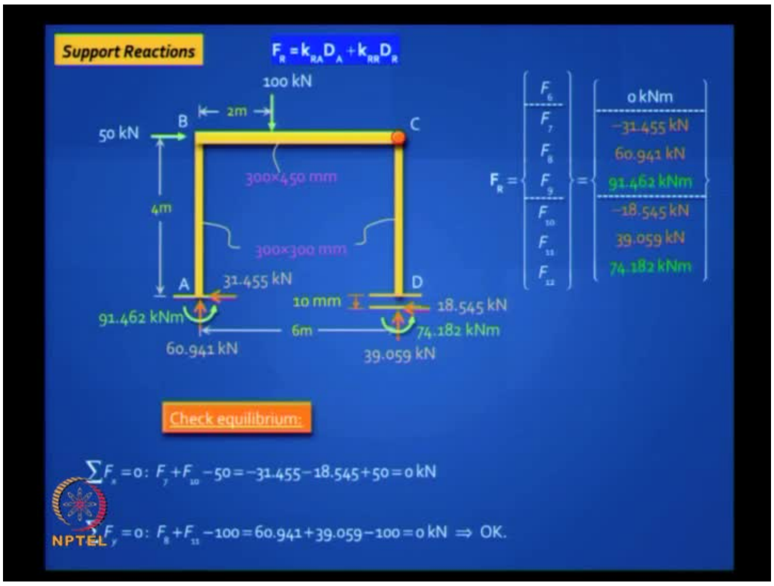

Then you find the support reactions in exactly the same way the second equation you solved. You can write down your support reactions. You should check your equilibrium. All these steps are identical to what we did earlier.

What is the last step?

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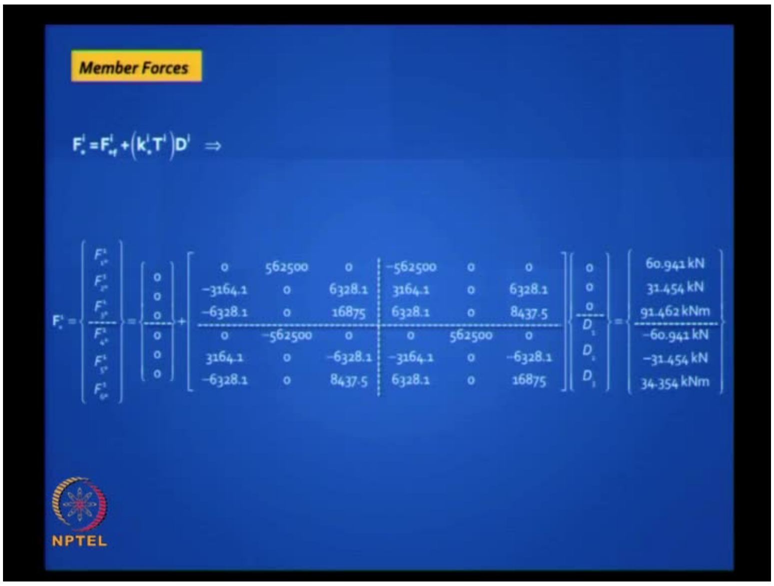

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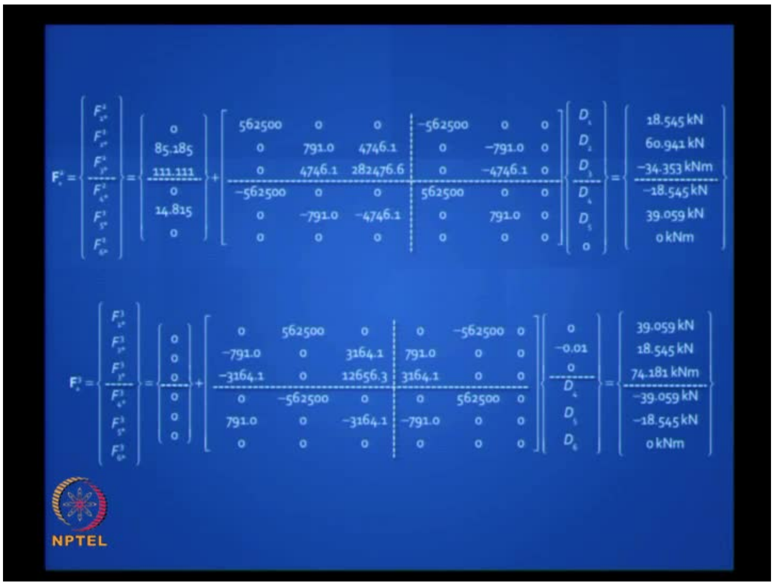

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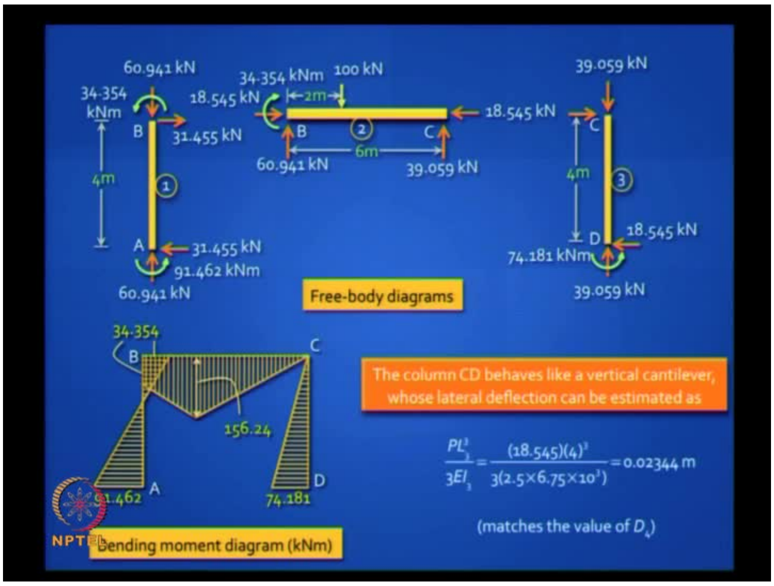

Member end forces again. The method is the same you find out for the three elements and you draw the free body diagram. What you do next? Well, the most important diagram for a frame or a beam is the bending moment diagram, but you can draw all the diagrams.

Let us just draw the bending moment diagram, and you will see very nicely that at C, there is 0 moment because that is exactly where the internal hinge is located. It is here that you can interpret. You can clearly see CD acts like a cantilever, as we have mentioned earlier.

There is only one load. If you isolate CD as a cantilever, you have a vertical load which does not goes bending and you have a lateral load. What is a lateral load? 18.545. You know the formula -- PL q by 3 EI, you plug it and you get the exactly the same answer that we got the hard way -- 23.4.

These are small checks that you need to do as an engineer. You should always be suspicious of the results that you get from a software. You need to check with your intuition and some thumb rule calculations to validate your solution. Is it clear? You will find that almost everybody knows, practicing engineers know, how to use software's. But interpretation of results is not everybody's cup of tea. That is the difference between a really good engineer and not so good engineer. You should be able to interpret, you should detect errors, you have to back to input and correct them.

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Dealing with hinged and guided fixed supports exactly similar to what we did for beams. You have to release some degrees of freedom.

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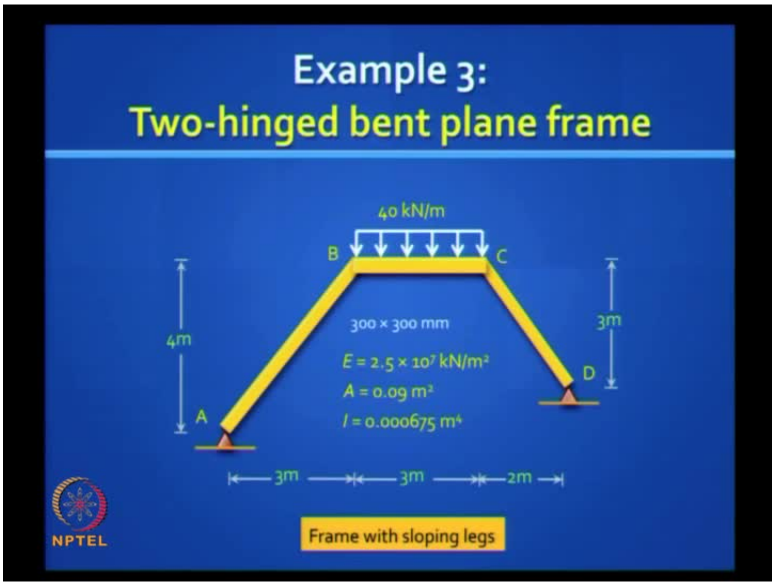

Let us take another frame problem. This is a bent frame. Remember we did this frame earlier by slope deflection moment distribution method. But here you will get slightly different answers. Why do you get different answers? Because here we are doing it more accurately. We are including axial deformations, which we did not have earlier in the slope deflection and moment distribution method.

How do we deal with this problem? What is a degree of kinematic indeterminacy?

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How many active degrees of freedom we have in the conventional stiffness method? Where we do not take any shortcuts? At A, how many active degrees of freedom do we have?

1.

At B?

3.

At C?

3.

At D?

1.

It adds up to?

8.

How many restraints?

4.

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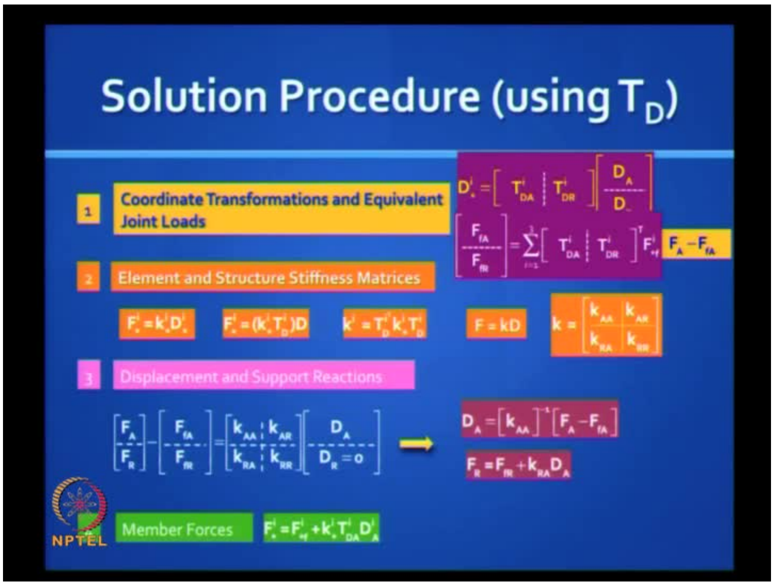

The method of analysis is identical to the previous problem, except here luckily we do not have any support settlements. Also, we are going to use a slightly different technique of generating the stiffness matrix and the fixed end force vector. We will use T_D, instead of the T_1.

I want you to recall the difference between using T_D and T_i. T_D is the displacement transformation matrix. It has the big advantage of you not worrying about the slotting. But it has a disadvantage, in terms of storage and programming.

What is the disadvantage? You will have to deal with larger matrices. But for problem like this, it does not make too much of a difference. Do you understand? Let us say, you had a multi-storied building and you had, say, 100 degrees of freedom. Then, your T_D matrix for even a plane frame element will be 6 by 100, that is very [unwilled/unwilling].

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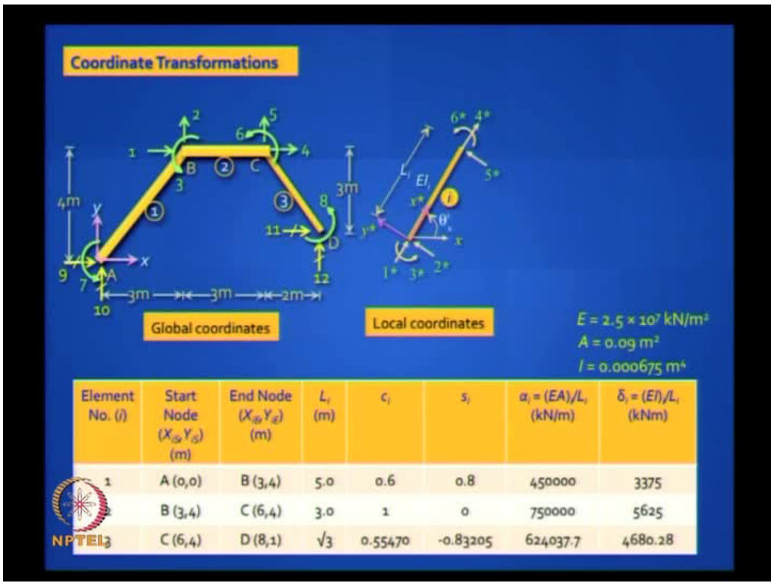

But if it is T_i matrix, the conventional, it will remain 6 by 6 for every element. But then, you have to worry about the slotting – the linking global coordinates. So, we did one problem of that variety. Let us do another problem using T_D, and otherwise, the method is the same. As you rightly pointed out, we have eight active degrees of freedom. I have marked them there in green color, and you have 4 restrained degrees of freedom.

You can identify your start nodes and end nodes for the three elements. You could draw the three elements separately or you could choose to draw a typical element and you can generate the matrix. The T_i matrix and the T_D matrix actually are similar, except that the size changes. So, we are familiar with the T_i matrix.

You have to still reopen the cos theta, sin theta, and 1. So, the method is the same. Only thing that you have to do it with your eyes is keep them open and fill up these matrices.

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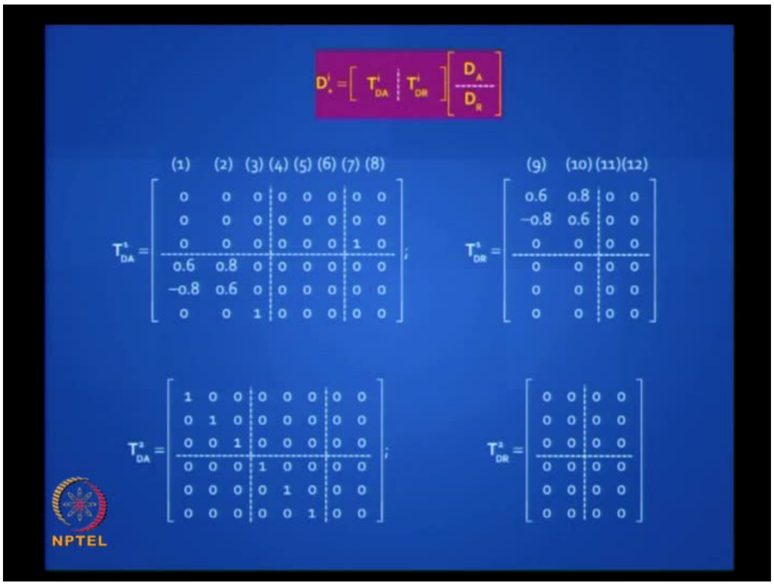

It is convenient to separate out the active degrees of freedom from the restrained. So actually the T_D matrix for each element will have a size 6 into 12, which you can partition into 6 into 8 for T_{DA} for active degrees of freedom, and 6 into 4 for restrained degrees of freedom.

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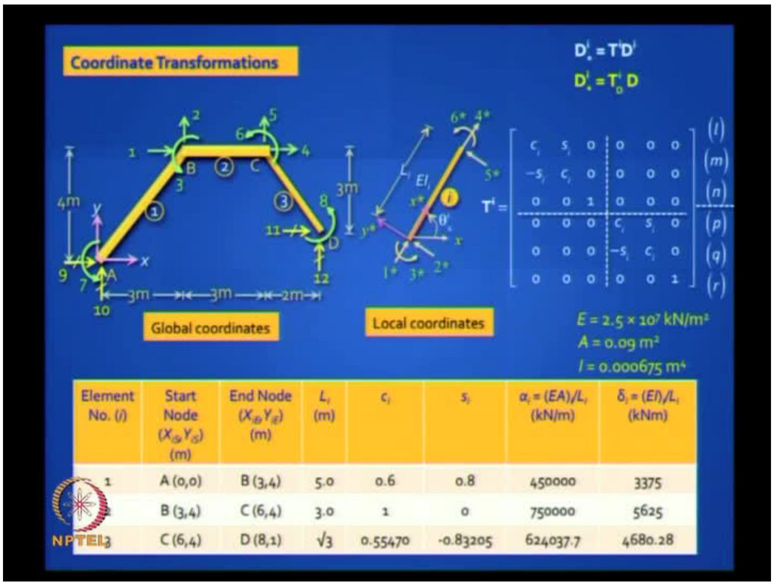

Most of these elements in the matrix will be 0s, except where you have the degree of freedom, which is active. Let us see, take the first column in these two matrices for T_{D1} and T_{D2}. The first column actually corresponds to D_1 equal to 1.

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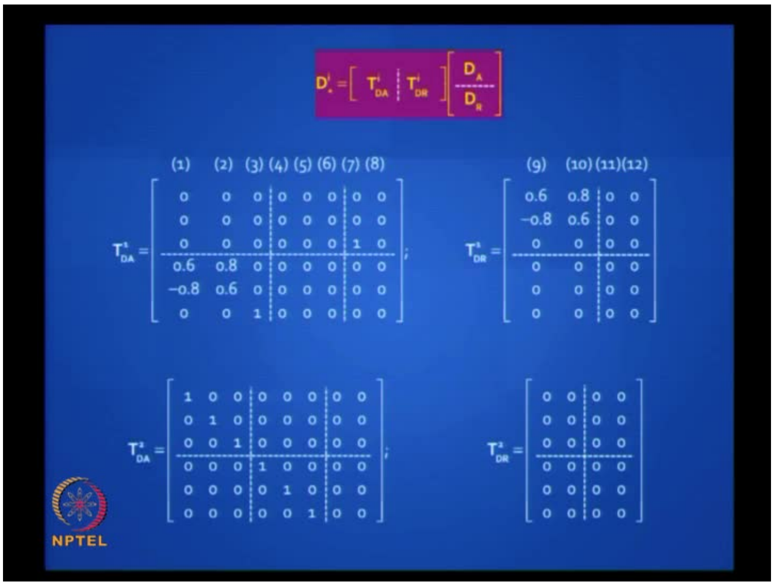

If you apply D_1 equal to 1, it is going to affect elements 1 and 2. It is at the tail end of element 1 and at the beginning of element 2. So basically, you have to use this transformation matrices and fill up at the appropriate column. You see the tail end gets filled in element 1 and the start end gets filled in the element 2. It is only 1. Is it clear? I hope you know how to fill up this.

This is actually similar to what you would get, if you did the T_i formulation, except that you have to keep track. Now on the top row, I have written 1 2 3 4, all the way to 12, but you need not do that in this case. It is obvious it has to be a sequential form from 1 to 12. Is it clear? I hope you know how to do this.

We have done it for a truss, you have done it for a beam, it is not a big deal to extend that to a plane frame.

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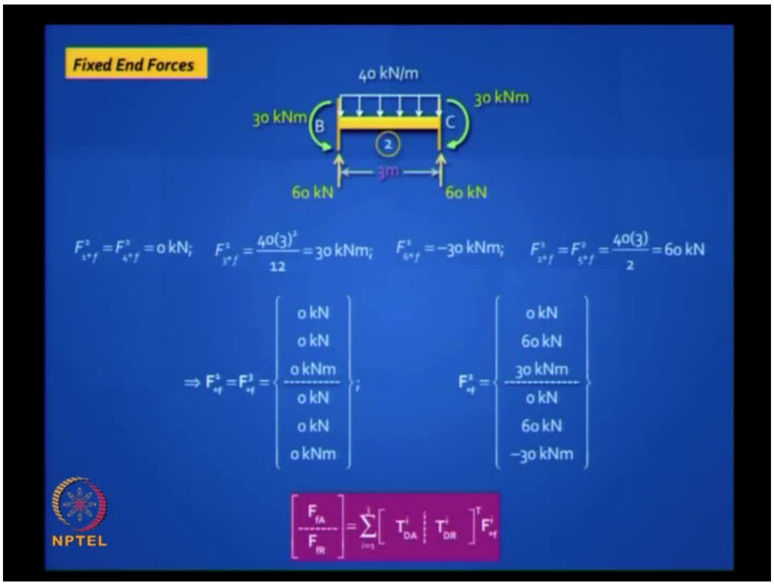

Similarly, for the third element, you can fill up now the fixed end forces. It is very easy to calculate in this case of the three elements. Only the second element is loaded. It has a UDL. so you can find the end moments and the vertical forces, write it in a vector form.

You have F_{2*f} having that non-zero vector, whereas F_{1*} and F_{3*} will be null vectors because there is no load in those two elements. Clear?

What do you do next? You convert it to...

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You want to get it for the whole structure. You want the F_{fA} and F_{fR} vectors.

How do you do that?

T_D transpose.

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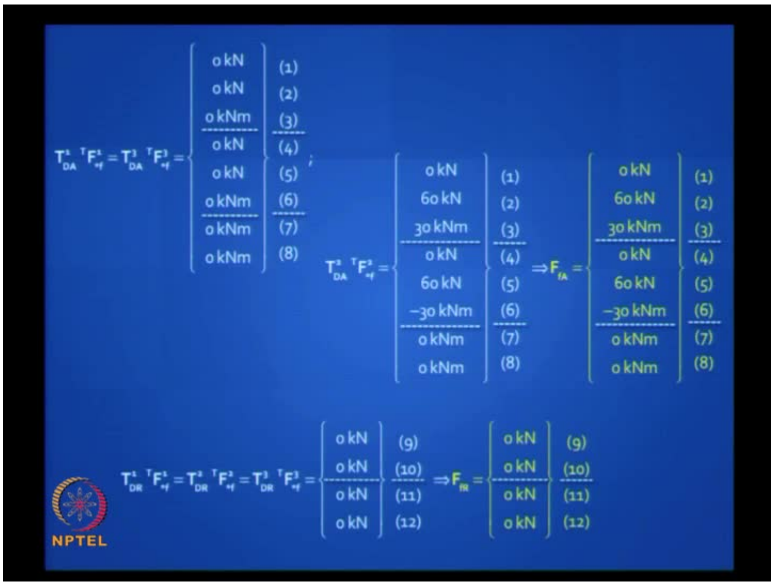

T_D transpose, so that contragradient principle works. And you can sum up for do one element at a time and sum it up for the three elements. This is something you can work out for all the elements -- you get F_{fA} and F_{fR}.

You need not write down those numbers – 1 2 3 4 – all the way to 12, because it is obvious. Those numbers you need to do when you do the T_i matrix. When you do T_D, it has to fall into this numbering scheme. Is it clear? So you get F_{fA} and F_{fR}. What do you do next? You have got the load vector. What do you do next?

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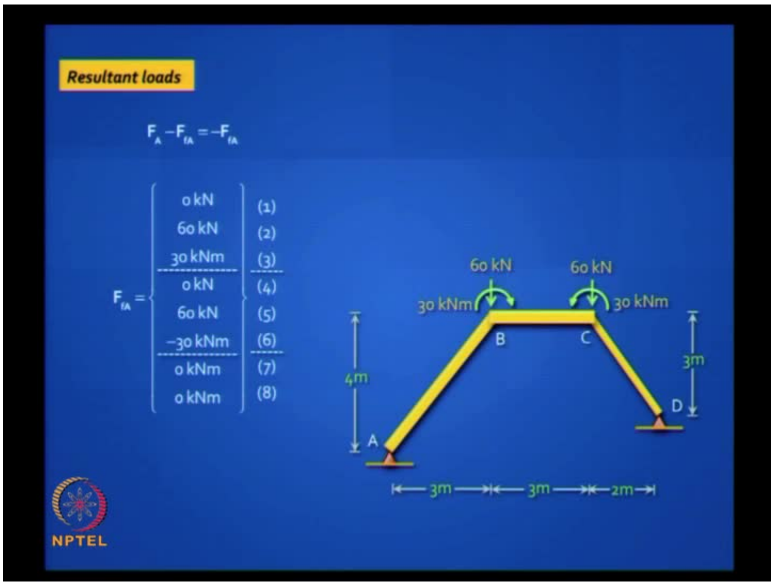

You can draw a sketch. Look at this. This is your resultant load vector F_A minus F_{fA} F_A is a null vector because there are no nodal loads in this particular problem. You had only a UDL and F_{fA} is given there. Put a negative sign to it and interpret those results. You will find that the original problem had a UDL. What you are having here will give you exactly the same active displacements.

The same D_A vector, the same deflections at B and C vertical and horizontal and the same rotations at B and C, and at A and D. You have to superimpose these results with the results that you get in the primary structure, when you have that UDL.

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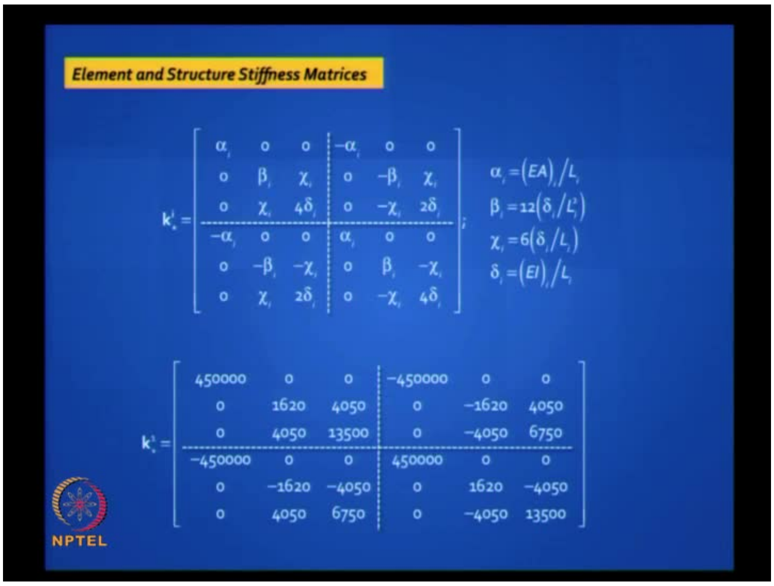

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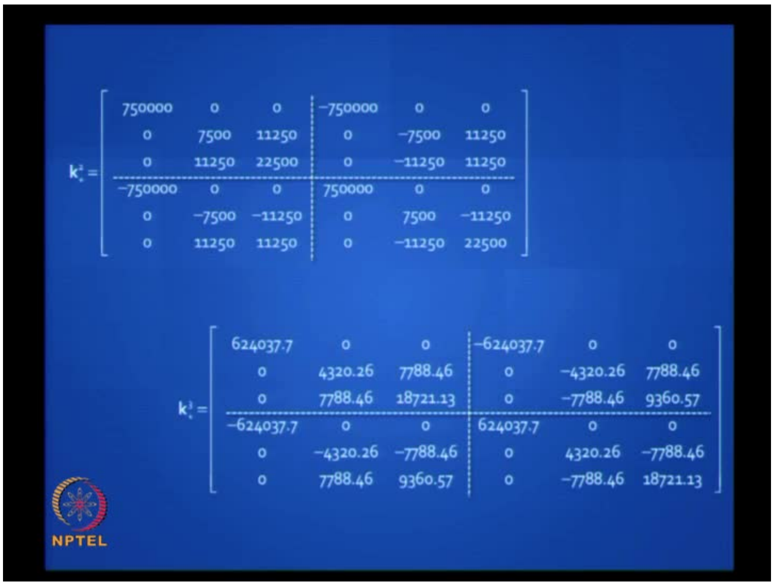

So, next you get the element and structure stiffness matrices, standard formulation for a plane frame element, you just have to plug in the values of E_A by L EI by L, and it all falls into place. Generate this for all the three elements.

What do you do next? you have to generate the structure stiffness matrix. how do you do that?

T_D transpose $k_* T_D$.

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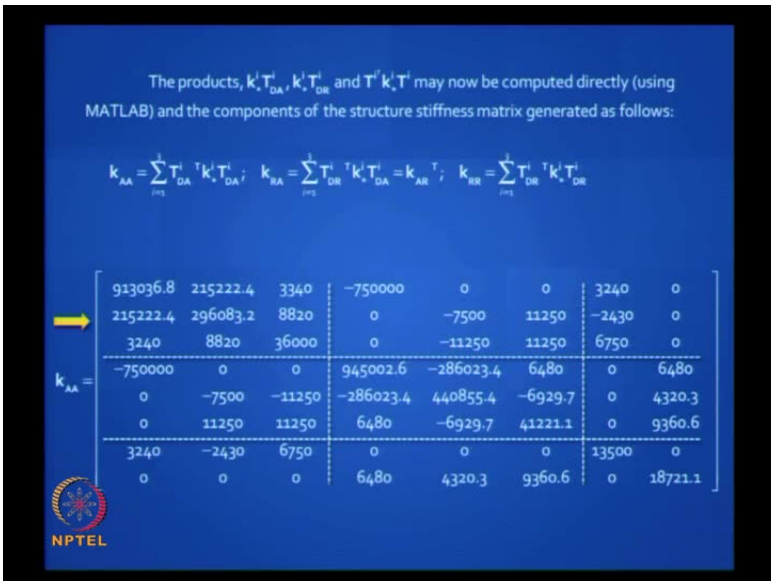

Which you can do it in two stages? You can first do $k_* T_D$ and then do the T_D transpose or vice versa. It is best to program all these and let MATLAB do it. Just check, after you get the results, that you have a symmetric matrix and that your diagonal elements are all positive. You will get some null some 0s somewhere. If you number it more intelligently, it will fall into a nice banded matrix with minimum half band width for our convenience in storage and solution.

I hope, up to this stage, you know how to do it. If you write the full k matrix, you cannot fit it into that slide because the size is 12 by 12. It makes sense to do it – k_{AA} k_{AR} k_{RA} k_{RR}. Even here, it helps to draw partitions as we have done it here, because 1 2 3 refers to the joint B. So, I have drawn a partition for that – 4 5 6 refers to the joint C, and the last two refer to the rotational coordinates at A and D. Is it clear?

You must have your own system of identifying for convenience because you need to interpret and you need to know why those zeroes are where there are. They should make some kind of sense.

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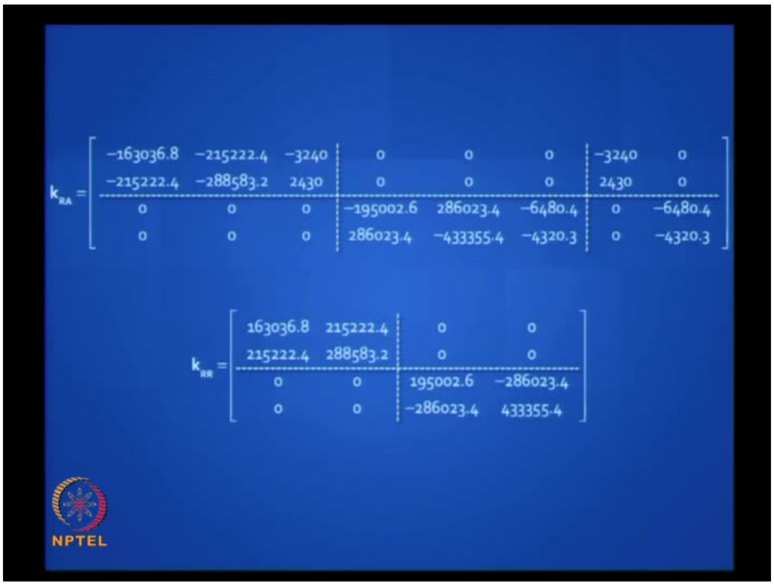

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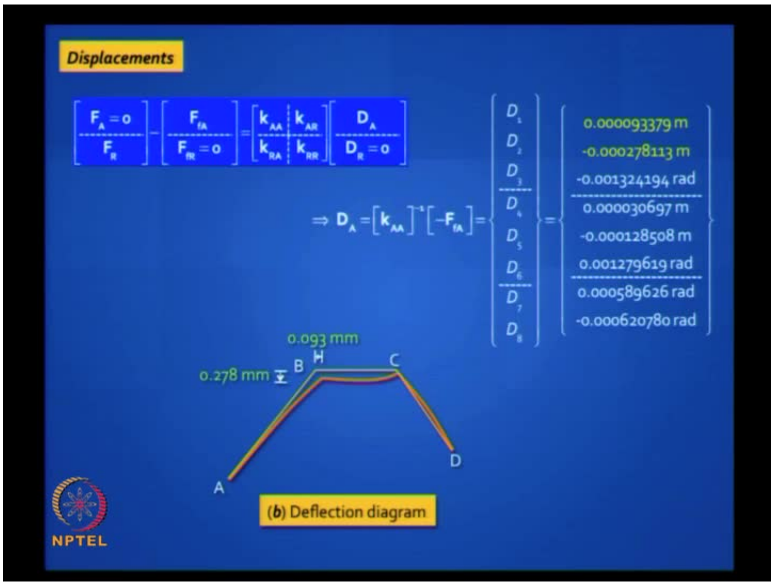

Next, you generate all your matrices – $k_{RR.}$ k_{RR} is 4 by 4, and k_{AA} is 8 by 8. Then, you solve this. Here there are no support settlements. Solve these two equations. First, you find the deflections. When you look at that kind of deflection vector really, you can make sense mostly only after translations.

The order of magnitude of the translations should make some sense in a real structure. So, I mark them in yellow color, the once that are significant; the once that are almost 0, I have not marked them. And you have to draw a sketch.

The sketch would look like this. Imagine, you have these two hinged frame, bent frame with a UDL on B C. This is how it is going to deflect. And the deflections are very small -- 0.093 mm to the right. That is the sway at B, and vertical deflection of 0.278 mm, which is extremely small. So you got this?

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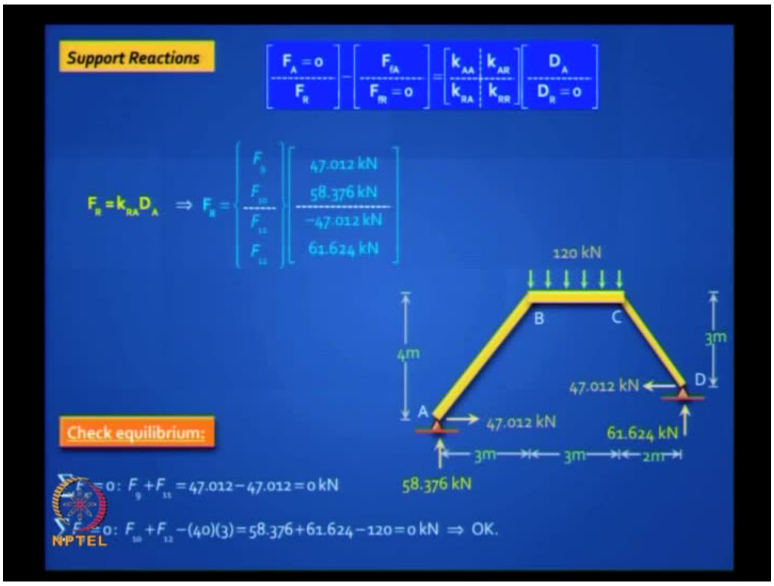

Then, you can find your support reactions solving the second equation, interpret those results, check equilibrium. The least you can do is check force equilibrium, so sigma F_y should add up to 0. Those two vertical reactions should add up to total load of 120 kilo Newton and the horizontal reactions must cancel out. It makes sense.

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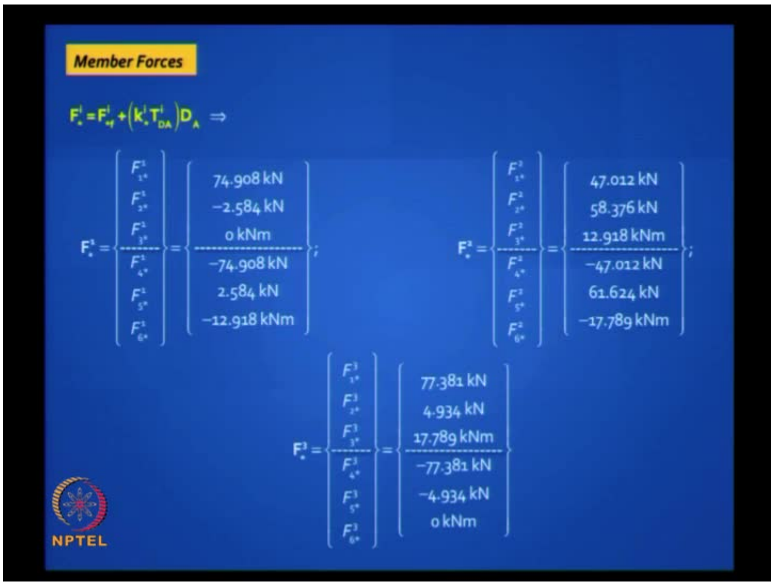

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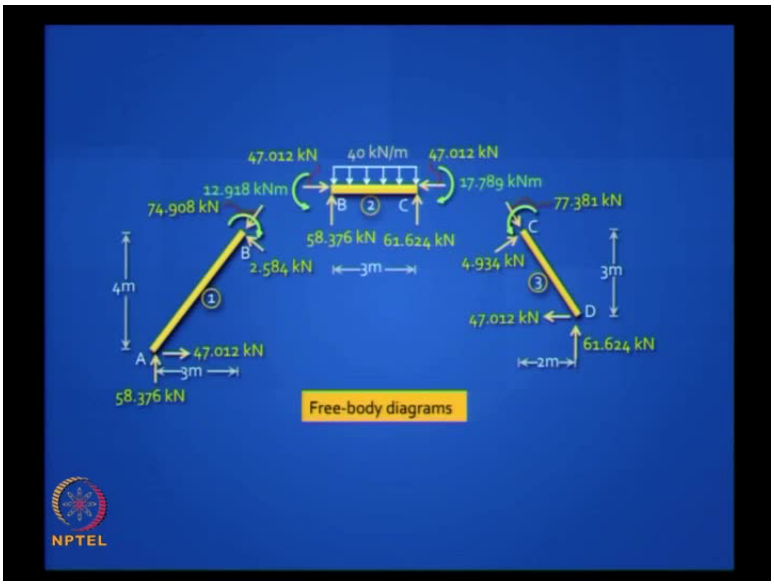

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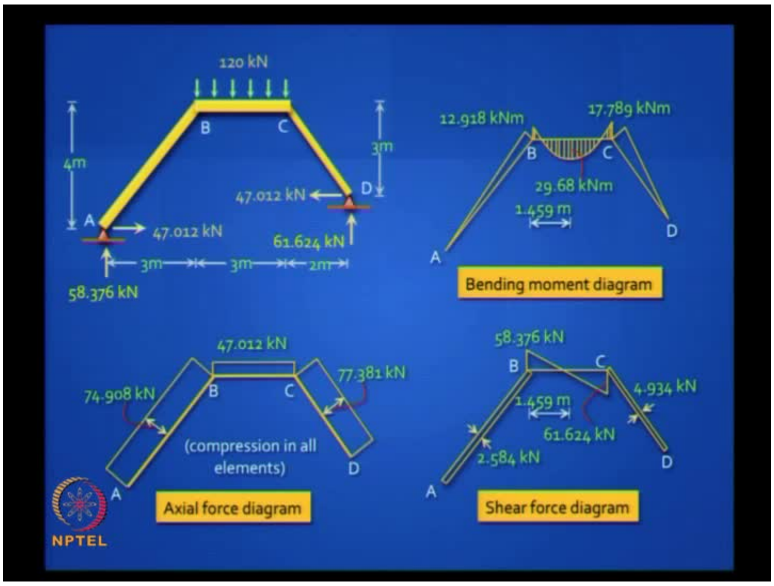

Last step is to find the member forces by using the T_D matrix and draw the free body diagrams, and draw the bending moment diagram, draw the axial force diagram, shear force diagram.

All this can be done. And compare these results with the results we got earlier by the slope deflection method, not moment distribution. Moment distribution is not good when you have sway. You will find that they are almost identical.

This is more accurate because you are accounting for axial deformation. We will do the same problem by the reduced element stiffness method, where we ignore axial deformations and compare the results.

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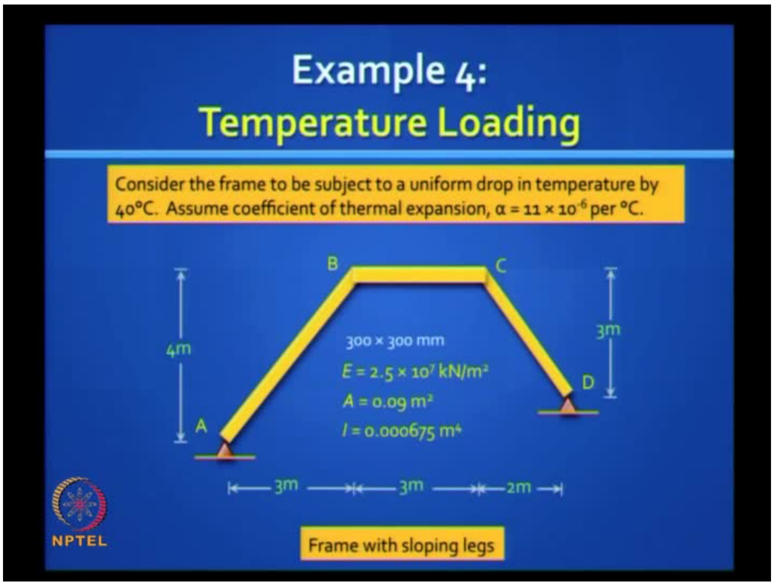

Here is one last problem. A really interesting problem. Remember at the end of the last class, we also said that let us look at temperature loading. This is more tricky because the legs are inclined. The same frame. Let us do this slowly so that you get it. Same frame with sloping legs. We are now talking of a fall in temperature by 40 degree Celsius. Coefficient of thermal expansion is given. How do you solve this problem? There are no loads -- no direct loads, indirect loading.

(( ))

Yes, tell me step by step, how to do this problem?

Find the elongations first (( ))

What are yes first

Find the elongations and then

Elongations you have to speak very clear

(( )) change in length

Find the free change in length.

Yes sir.

For each element, which is L, alpha delta T, then what?

And then (( )) fixed end forces for this (( )) fixed end forces

It is more right to say that this structure is kinematically indeterminate. So, the primary structure is one and which all the degrees of freedom are restrained. In the restrained structure, if you allow this temperature change to take place, you will end up with what kind of forces?

(( )) compression (( ))

Obviously tension, because if you take any element it wants to shrink. It is prevented from shrinking you will get tension.

Will you get bending moments and shear forces? Not in the primary structure, but in the final structure, you should get. You will be in for lot of interesting results. So let us look at it slowly.

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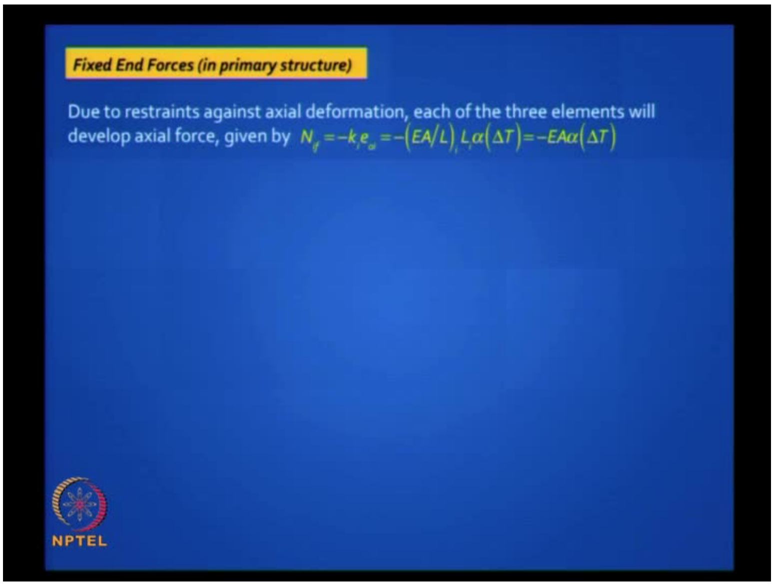

First, as you rightly said, we will find the fixed end forces in the primary structure and that is simply the axial stiffness k_i. N_{if} is k_i into E naught i. We put a minus sign in general because normally you have a rise in temperature and E naught is an extension. But in this case, E naught will turn negative, so when you substitute the results you should get a tension.

Now, it is interesting that k_i e is EA by L and E naught i is L alpha delta T. L cancels out. It is interesting. Am I right? So for all the three elements, you will get the same axial force. Yes or no? That is what logic says. You get the same axial force for all the three elements. Can you work out how much that force is it? Small force or a big force? Plug in those values.

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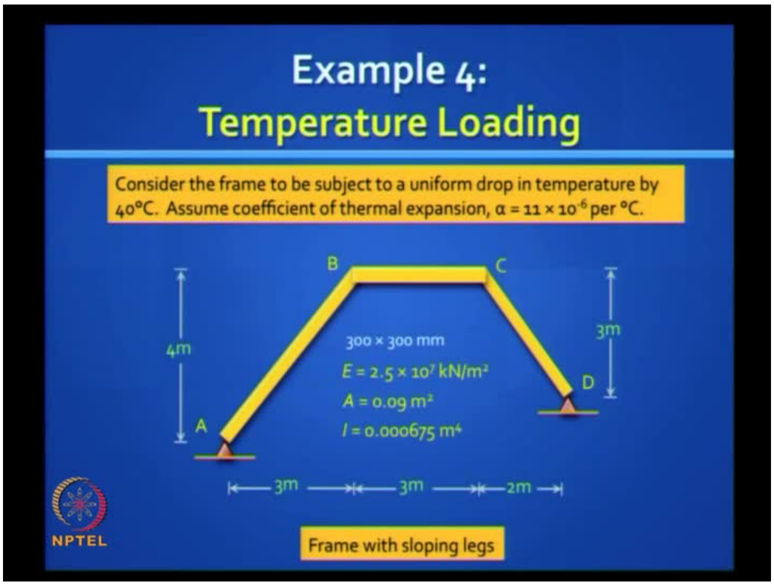

Let us go back. All the members have cross section 300 by 300, so the area is 0.09 meter squared, material has elastic modulus of 2.5 into 10 raise to 7 kilo Newton by meter square.

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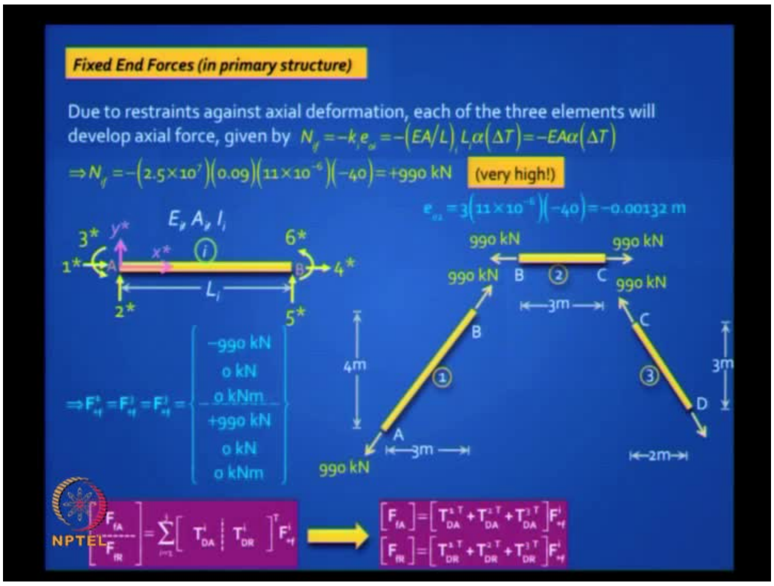

With E_A and L, can you work out what that force is? That is it. E_A alpha delta T, where delta T is minus 40 degree Celsius, you get a huge value.

990.

990 is not a small value. It is a huge axial force. Do you think the final force will be so large? How much you think?

(( ))

See this is what you would get. If you were to arrest those degrees of freedom, prevent that cooling from manifestly in the primary structure, you get a huge force of 990 kilo Newton.

But had this structure, for example, being simply support, what will be the force in all those members? 0. So it is actually not simply supported, its tweaked. It is hinged at both the ends, so there is some restrain.

Can you guess what you think in practice is 990 will reduce to? 100. Let us share some guesses.

10.

10 kilo Newton.

10 to 20 kilo Newton.

40.

100.

let us see what we get at the end. Good your guesses are, not bad. Let us see. But this is alarmingly high. It is good to also have a check on what is that length free length that would give it a 0 value. If you allow it to cool fully, then the extension will be L alpha T, where L is 3 meters, here alpha is 11 into 10 raise to minus 6, delta T is minus 40.

You just need 1.32 mm. That is a small moment. May be it will be allow do so. Let us see. You must get a physical understanding due to really understand this problem and its entirely.

These are the forces that you will get in those three elements, if you were to arrest the degrees of freedom. All 3 element will have a force of 990. How do you convert this to loads on the structure?

990.

How do you convert this to loads on the structure?

(( ))

How do you do…

By transformation matrix first transformation (( ))

(( ))

Well, to figure out what will be loads, it is not easy, so you need help. And what is the best thing you can do? Use the transformation matrix. What transformation?

(( ))

First, we will write the fixed end force vector. There are six degrees of freedom of which only the axial degree of freedom will have a non-zero value. There is no shear force and bending moment. So, we write it as minus 990. It is what you will get at F_{1*f} and plus 990 is what you will write at F_{4*f}. Is it clear? This is same for all the three elements. Now, what do you do?

(( ))

Right, you do that. You have to sum it up for all the three elements. For each element, you have to do the transpose of T_{DA} and T_{DR}. And if you expand it, you can write it like this because the s star f is a same for all the three elements.

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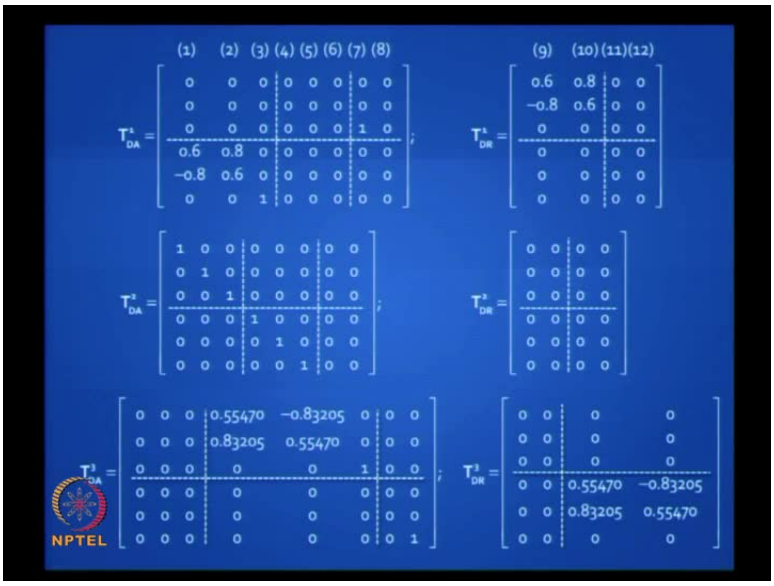

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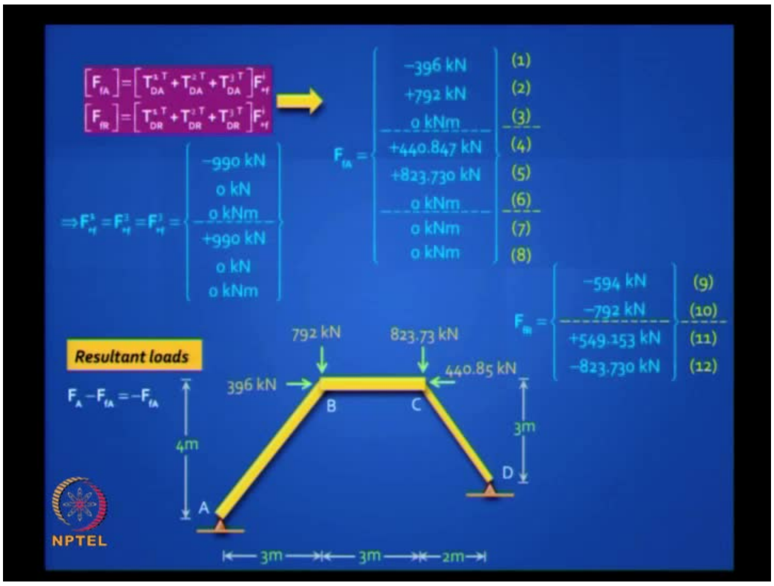

You just have to add the T_D matrices. You do that and this is something we have calculated in the last problem. So I am picking up those values from the previous problem. And if you plug it in, you will get the answer. You will get F_{fA} and F_{fR} and this is beautiful. Only matrix methods does it so efficiently. If you had to do it manually, figuring out what are those equivalent forces, it is going to be trickily.

So this is the power of the contra gradient principle. How effortlessly from the axial force in the restrain structure, you are getting equivalent joint loads. It is beautiful. Very powerful. So you can find the resultant loads, which is F_A minus F_{fA}. F_A is a null vector because this structure has no direct loads acting on it. So this is what you get.

It is as those someone is pressing down those corners – B and C. You can visually it make sense that is what going to happens, when it wants to cool. Those loads are not small, they are huge loads -- 792 396. Is it clear? How do we proceed from here?

(( ))

K matrix

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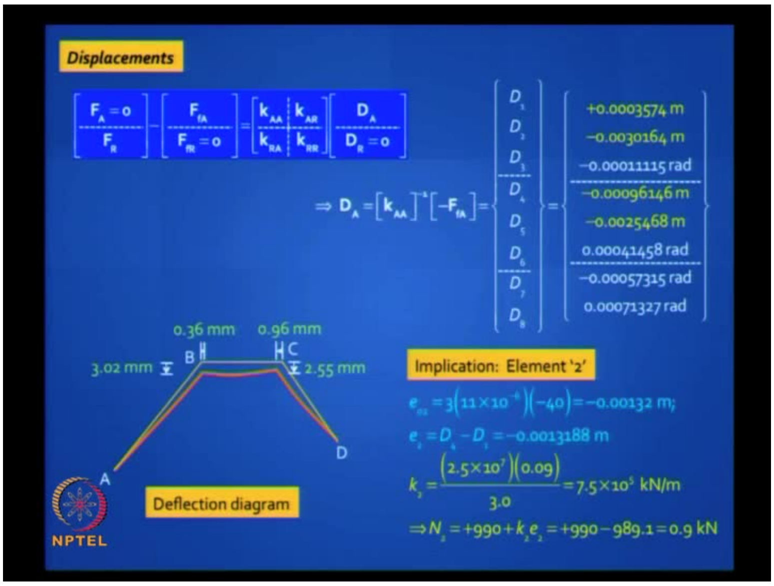

We use these equations. You solve the equations – the k_{AA} matrix. You have already got and you look at the answers.

If you plot them, it is going to look like this. B is going to go down by 3 mm, C is going to go down by 2.55 mm, and B C is going to shrink move inwards. B goes to the right by 0.36 mm, C goes to the left by 0.96 mm.

Now, if you are an intelligent engineer, what will you calculate first to figure out whether your residual force would be larger?

(( ))

Yeah, we calculated E naught 2, which was 1.32 mm or something.

Yes sir.

Subtract from it and you see the value.

Subtract from it what you think you will get?

Negligible.

Let us just look at the element 2 E naught 2, for example. You can do for E naught 1 and 3 as well, but let us just take the second element because it is nicely horizontal. So E naught 2 was 1.32 mm. And if you want to find the final elongation, which you can get from D 4 minus D 1 because D 4 is a horizontal deflection at C, D 1 is the horizontal deflection at B, that must be the elongation in second element BC. It turns out to be very close to E naught 1. So, close that if you work out, you can find the axial stiffness of that element E_A by L of the element, multiplied by that and add the fixed end force plus 990 minus 989.1.

Ladies and gentlemen, your force is less than 1 kilo Newton. This is really beautiful. Just 0.9 kilo Newton is the force that you get – not 20 kilo Newton, not 40 kilo Newton, not 100 kilo Newton, not 990 kilo Newton. At the end of the day, you will find that, thanks to this analysis, you can confidently say I do not need to worry about temperature changes for a frame like this. But you would not be able to say it with that degree of confidence unless you could analyze this structure and interpret the result. Is it clear?

So, it is not enough just to blindly use matrix analysis. As an engineer, it is more important to interpret the result. So, 0.9 kilo Newton is nothing for a structure of this magnitude. But 990 kilo newton is shockingly large.

Why is it so low, can you tell me?

(( )) because A B and C D are

Structure will adjust itself and there had D been a roller support, it would have nicely moved around a little bit, so that nothing happens. You get that 1.320000 exactly. So structure adjust itself, so nothing much to worry in a system like this.

A similar situation you will get in buildings, where you have differential movements in beams. Typically in a multi-storied building, let us say, you have columns and shear walls. The shear walls are quite stiff actually because the cross sectional areas are large, the columns are comparatively more actually flexible. So, when you have a huge beam sitting on top of them, where you have the shear wall, you would not get much movement. Where you have the columns, you will get a movement. And that movement gets enhanced with time because in concrete we have a phenomenon called creep. You have differential settlement in that beam. And if the beam is not long 6 EI by L squared is a kind of fixed end movement that you will get. That is a huge moment. If you work out, it is very large, but when you do the frame analysis, you will find you will end up with much smaller moment. But unless you can do that distribution through an analysis, you cannot prima facie say whether its negligible or not.

So, when you have more members coming into play or more flexibility in the structure, you really do not have to worry too much about in direct loading. It kind of takes care of itself.

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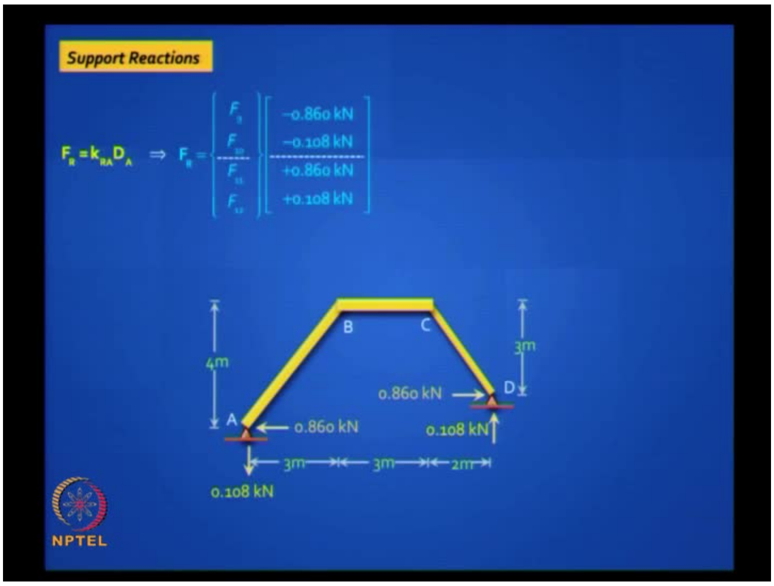

Let us finish this problem. The next step is -- here you find is negligibly small and you find out the support reactions. Support reactions also turned out to be very small, not surprisingly, and itself equilibrating. Look at that, there is no load on the structure. You cold the structure and you got support reactions. It is just self-equilibrating. This no external direct loading acting on the structure.

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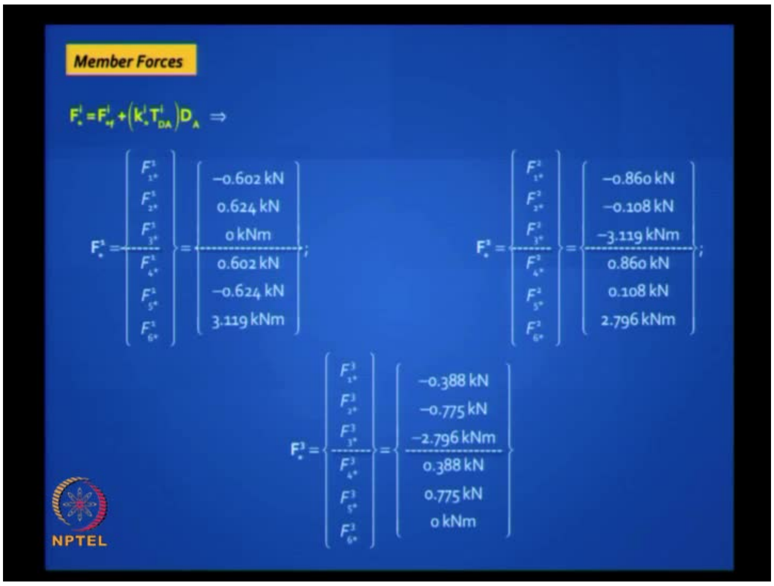

You can find the member end forces. Now you get shear forces and bending moments in addition to axial forces. But they are of small (( )). You see that value which we got 0.9 for the second element. The exact answer is 0.86, which is close to 0.9

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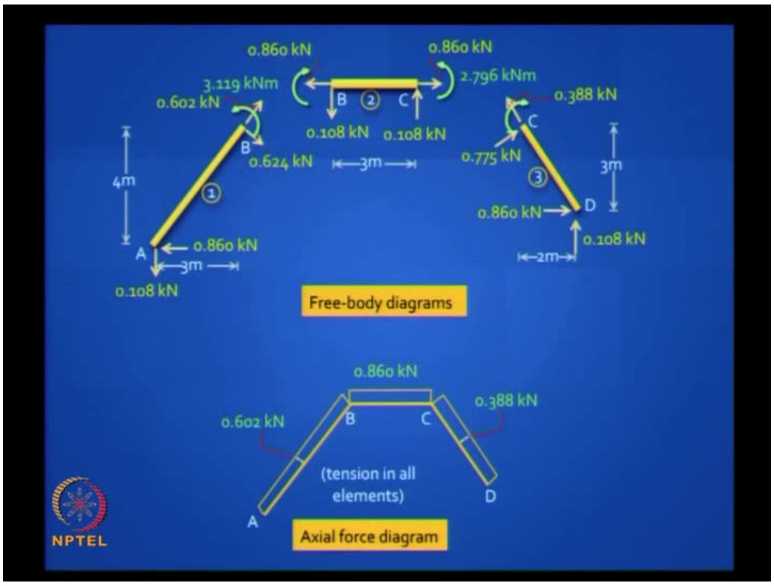

These are your final diagrams. You got axial force diagram, shear force diagram and bending moment diagram. The axial force diagram is more critical among these and you will find that they are very small.

This simple problem teaches us many things and shows us a power of matrix analysis. Now, with this, we have finished the conventional stiffness method.

Q: A and D are at the same level, then here the force at vertical force at A and vertical at D creates a moment no sir.

Yeah.

And it is balanced into horizontal force. If it is at the same level, which will balance it?

A: That is a good question. It is a very good question.

Suppose you had a symmetric frame, let us say, you had a symmetric frame and A and D were at the same level, what do you think will happen? You cannot have a vertical reaction. Why cannot? You have a vertical reaction because they have to be equal and opposite they will create a couple. So the moral of the story is you cannot have a vertical reaction but you still can have a horizontal reaction.

Sir, the member must be in tension right tension or compression now.

It still be in tension.

(( ))

Then you can you take a frame and apply a horizontal (( )) (( ))

So, there is always an answer to any question. It should make sense. Of course, one answer is a horizontal reaction is 0 but you know that is not going to happen because it is an over rigid system. It is statically indeterminate. They have to be internal forces, they have to be support reactions, but the reaction will cancel out. Is it clear?

Good question. So, are you now confident of dealing with conventional stiffness method? We have completed the conventional stiffness. We have done all kinds of problems with it. Next we go to reduce elements stiffness method, then we will do flexibility method, and finally we will do space frames.

With that, we actually cover most of structure analysis. We have a seventh module, which we will take a look at. Thank you.