Advanced Structural Analysis
Prof. Devdas Menon
Department of Civil Engineering
Indian Institute of Technology, Madras
Module No. # 5.6
Lecture No. # 32
Matrix Analysis of Beams and Grids

Good morning. This is lecture number 32, the last lecture in this module 5, on matrix analysis of beams and grids. With this, we will be completing six sessions on this module. The remaining topic is application of matrix analysis to grids, where we will be applying stiffness methods. As you know the grid element is different from the beam element, where you have an additional degree of freedom. You have the angle of twist and the corresponding forces, the twisting moment or torsion.

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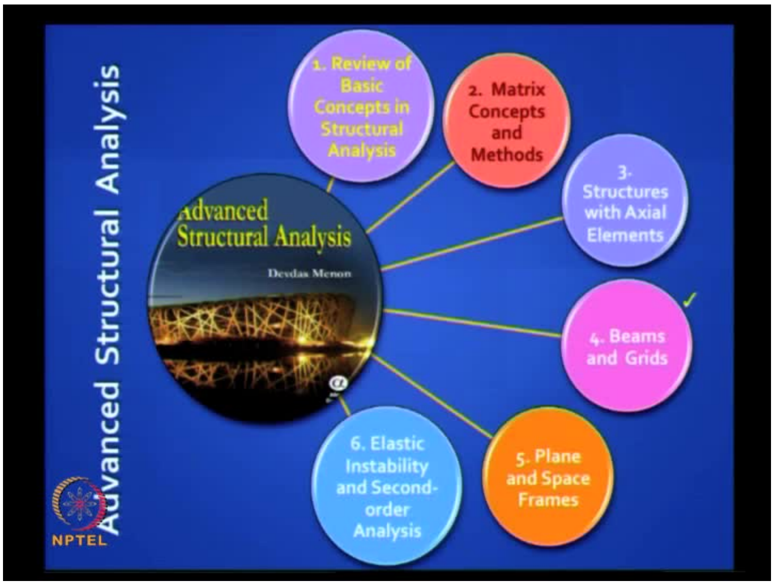

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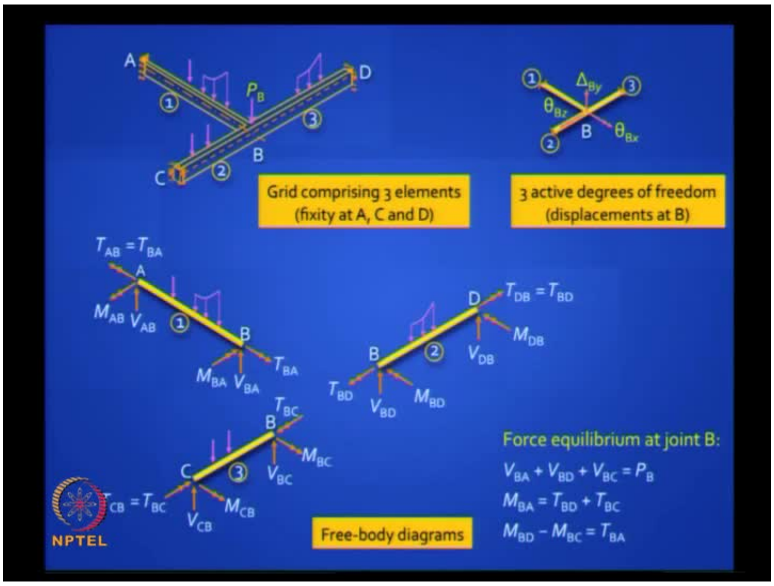

This is covered in the chapter on beams and grids in the book. So let us quickly review what a grid is? I have given you a segment of a grid. You have two interconnected beams, monolithically connected at the joint B. If you separate out the three free bodies, you will find that there will be twisting in all the three of them. If we say A, C and D are fully fixed against, both flexure and torsion, you will have only three active coordinates at b. You have a vertical translation, and you have rotation at about two axes, as shown in that figure.

If you take out the free bodies, you will find that, in addition to shear and bending, you have a twisting moment. The twisting moment at the two ends of each element will be equal and opposite as there are no intermediate twisting moments.

If you treat the beam CD as one beam then you would have to put a torque at B. But, if you separate out the two elements, they would look like this. Compatibility demands that the rotations at B should be the same for all. What would be a flexural rotation for one beam, would turn out be an angle of twist for the perpendicular beam. And equilibrium has to be satisfied at location B. The resultant force should add up to whatever force, acts at B. So in general, these are fundamental concepts.

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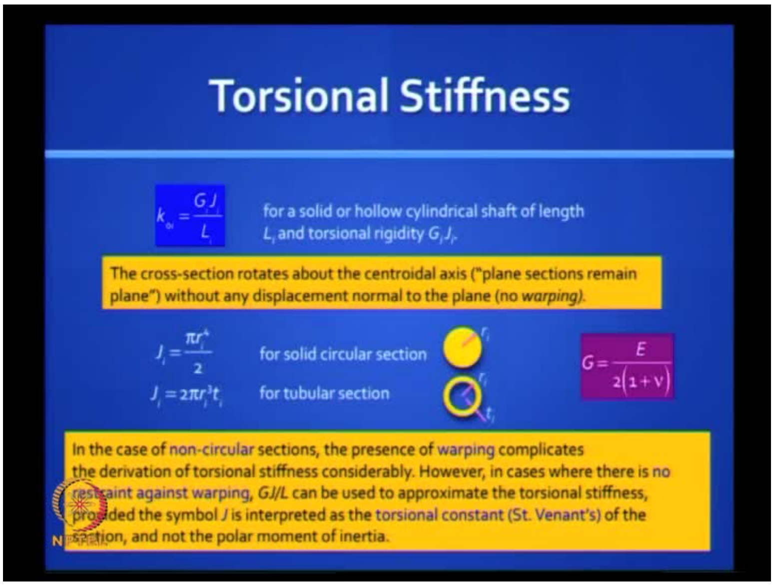

How do we address these issues in matrix analysis? If you recall, we had covered grids in the force method of analysis. We had actually applied the method of consistent deformation and had also used the theorem of lies worth. In that, we had separated out the beam elements and put some equivalent springs, and try to solve the problem in that manner. But that is not effective, if we have large grids. So, it is better to deal with the whole grid structure in one go, which is possible in a matrix framework. We will be looking only at the stiffness method of analysis.

There is something called a torsional stiffness for a grid element, which is what you get in a shaft. If a shaft, let us say, a circular shaft, solid circular shaft or cylindrical shaft, is fixed at one-end and you apply a constant torque at the other end, the angle of twist is given by GJ by L times the torque that you apply.

So, we can talk of torsional stiffness as GJ by L, where G is the shear modulus, J is the polar moment of inertia, and L is the length of the element.

In many ways similar to axial stiffness – EA by L. It also has vectorially the same direction, except that you will use a double arrow head. It is important to note that this concept of polar moment of inertia is strictly applicable only to circular sections, cylindrical sections. They could be solid or annular. The reason is – in non-circular sections, you are likely to encounter warping longitudinal moments, which needs a separate treatment.

Let us take a look at the two perfect circular sections. The polar moment of inertia formulas are very well known. The relationship between shear modulus and elastic modulus, in terms of Poisson's ratio, is also well known.

If you are dealing with the non-circular section then the presence of warping complicates the derivation of torsional stiffness. However, in cases where there is no restraint against warping, we can still use GJ by L. But J is not the polar moment of inertia; here, J is called an equivalent torsion constant, sometimes denoted with the symbol C. But let us stick to J. It is also called St Venant's constant because it is St Venant's torsion. If there is warping, you have to slightly modify these terms.

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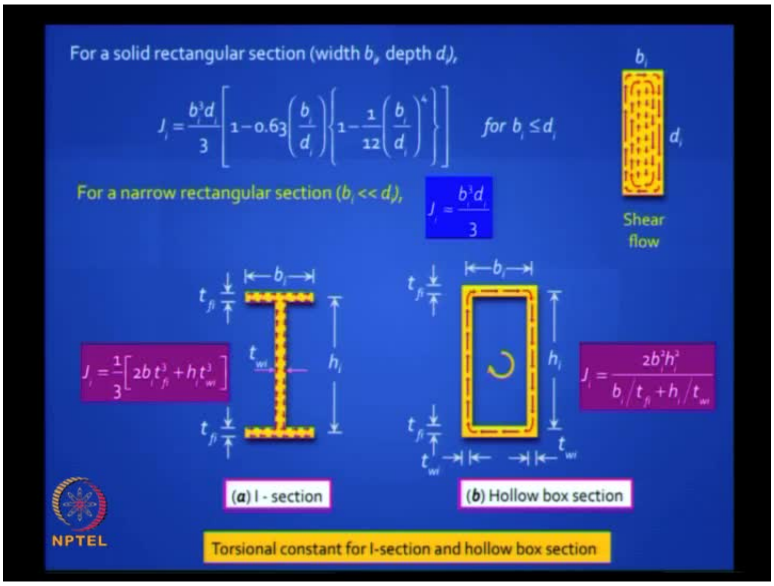

Let us take the simple case of no warping. If so, in a rectangular section, the value of J can be shown in that expression. It is a function of b and d, where b is the width of the section, and d is the depth of the section. b is assumed to be less than d, otherwise you have to interchange the terms. The shear flow is as shown in that picture. If it subjected to a clockwise torque, the shear stress is 0 at the 4 corners, though the distances are maximum from the centre because of complimentary shear requirements.

If you are dealing with a narrow rectangular section, then that formula reduces to a very simple formula – b cube d by 3. That is when it is infinitely long. This is an easy formula to remember. For thin walled open sections, we can just use this formula and add up different segments.

For example, I have shown here two thin-walled sections. One on the left, the I section. You can derive the formula using the formula for the narrow rectangular section because it is an open section. The one on the right, however, is different. It is like the annular, tubular section, except that it is a box section. It is commonly used in bridges. It is very good against torsion. The reason why this section is very effective against torsion is if you look at the nature of the shear flow, you will see that in the case of I section, the lever arm for the shear flow is very small, so you do not get much resultant torque. And you get a large twist as well. But in the case of a box, you have a constant shear stress across the thickness of the wall. It is like a circular section, so it is very effective against torsion.

That is because it is closed. If you cut that box anywhere, make a slit, then it is not effective – as effective. And you have a slit, you can go back and use the formula for a narrow rectangular section, and add up for the different segments. Anyway, these formulas are available, if you want to use them in practice.

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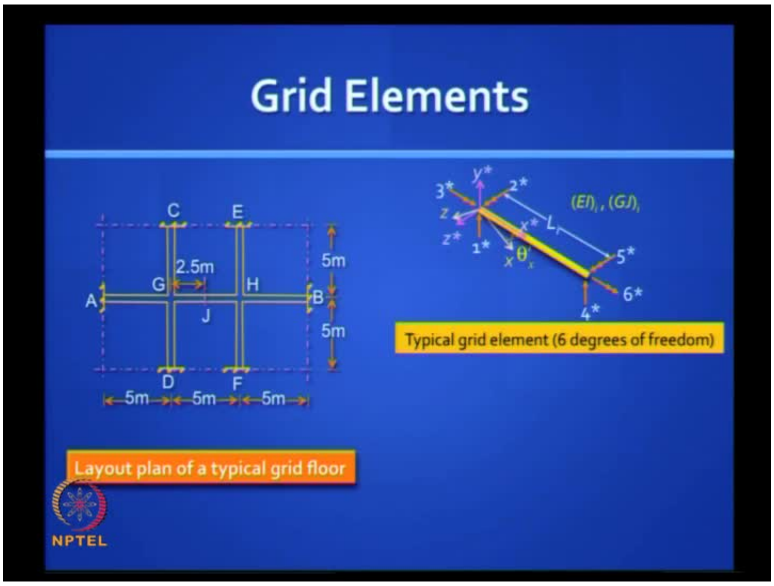

Let us look at simple problems like this. This is a grid problem, which we will try to solve. We have shown here, for convenience, 3 by 2 grid, but you could use it for any large grid as well.

The grid element in the conventional stiffness method will have 6 degrees of freedom. It is just like the beam element, except we have an additional – not axial – torsional degree of freedom at both the ends. So we have 1 star, 2 star, 3 star. The 3 star, we reserve for the torsional the angle of twist. Similarly 4 star, 5 star, 6 star. The element is assumed to be prismatic with the flexural rigidity EI, constant flexural rigidity EI and a constant torsional rigidity GJ.

Later, in the next module, we will look at the more complex space frame element where you have to add one more. A plane frame element where you have to bring in EA axial rigidity with space frame, and you have to bring out some more quantities, which makes it more complex..

But this is a spatial structure because the loads are acting perpendicular to the plane of the grid. And in general, the axis in the global Cartesian coordinates will not be the same as the one for the local coordinates. So you have theta. It is actually a rotation by the angle theta in x z horizontal plane, and we will see how to derive the displacement transformation matrix for such a situation.

Please note grids do not necessarily have to be orthogonal. We will see examples where they could have any orientation. And if you want to do the reduced stiffness method, that six degree of freedom model will simplify to how many degrees? Three. Because you will find that the rank of the stiffness matrix is only three and to make it stable, you have to give three supports. We will look at this later. Yes

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x and z are x y and z are global.

I have marked it there. x star, y star, z star are local as per our standard convention. x and x star need not be in the same direction like in a continuous beam. In this orthogonal grid problem, it will be either 0 degrees or 90 degrees. Have you got it?

But in general, it could be 45 degrees, 30 degrees, any angle.

So I am doing a generic formulation. But please note if x and x star are separated by an angle theta, then z and z star are also separated by exactly the same angle because it is just a rotation about the horizontal plane. And y and y star are identical because of vertical, the normal to the horizontal plane. This is same.

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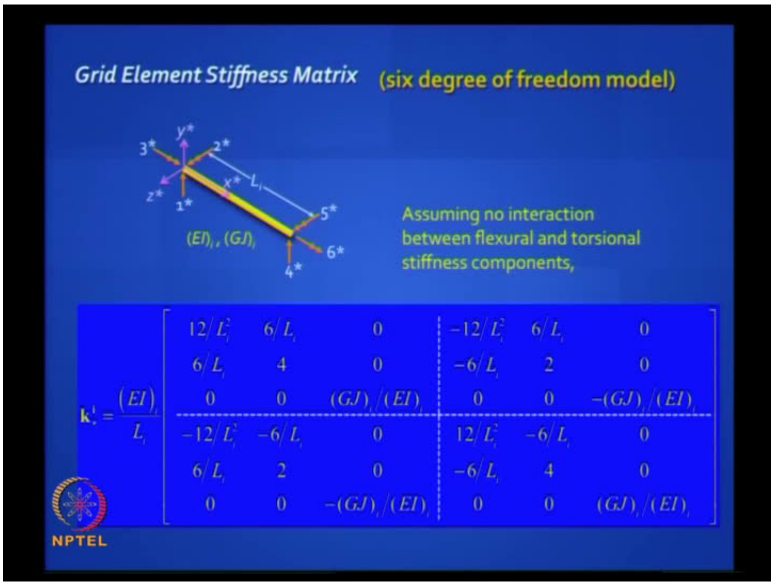

Let us look at this more carefully. We look first at the conventional stiffness method. This is the element we were talking about. The element stiffness matrix is simple if we make a basic assumption that the torsional stiffness component and the flexural stiffness components have no inter-dependents.

That is the convenient assumption that we are making here. You remember we looked at the plane frame element, we are going to look at it in detail in the next module. So you will have this GJ by EI coming here. You will have 0 0 here, 0 0 here, because this is your torsional constant. It should be actually GJ by L, but for convenience, if I take EI by L outside, I have to write here GJ by L. Please note that if this is GJ, then this will be minus GJ and so on, and so forth.

The diagonal elements will be always positive. The rest of the matrix is exactly what you had for that beam element. Remember, for a beam element, we had 12 EI by L squared, 6 EI by L, and so on, and so forth.

So we have to add it. Just two additional rows and columns at the right places because 3 star and 6 star come at the end of each of these coordinate numbers. It is not difficult to derive. It is just a super position of torsional stiffness to your beam element stiffness diagram.

There is no other change. We have added the third row, third column, and the sixth row and the sixth column; all of them are 0, except the diagonal terms and the corresponding terms at the other end. The diagonal terms will be positive, the other terms will be negative.

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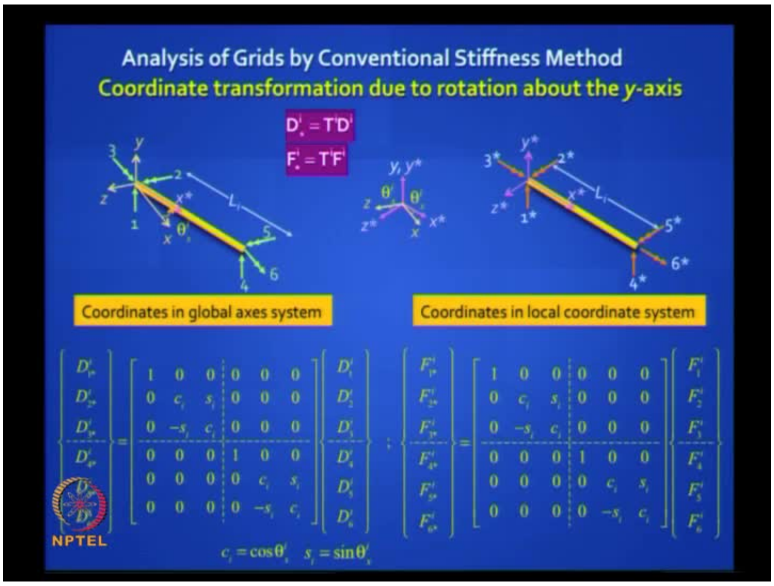

Let us see how to deal with the lack of alignment between the local coordinates and global coordinates? You have a difference between x star and x, and it is easy to write down because you remember, we have done this earlier, in the case of a truss.

It is similar kind of rotation we are doing, except that now we are doing it in the horizontal plane. D 1 does not change because that is a translation in the vertical plane. It is in y direction. D 1 does not change, so you have 1 there. And the rest of the transformation comes from your rotation and so it follows a familiar pattern – cos theta minus sine theta, sine theta cos theta. I hope this is not difficult for you, we have done this kind of transformation earlier.

We are doing it exactly the same way. It is very easy to remember and we need to apply it. But fortunately for you, we are not really going to solve any problems by the conventional stiffness method because it takes lot of time.

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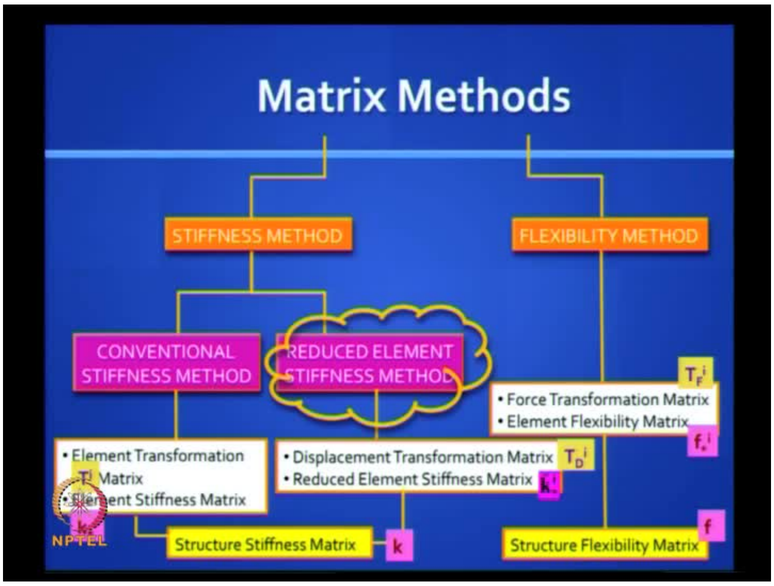

We will however try to solve the problems more conveniently by the reduced element stiffness method. Conventional stiffness method is the same in the book on advanced structural analysis, there are some problems solved but we would not attempt those here. This is conventional stiffness method, but we will go to the reduced element stiffness method.

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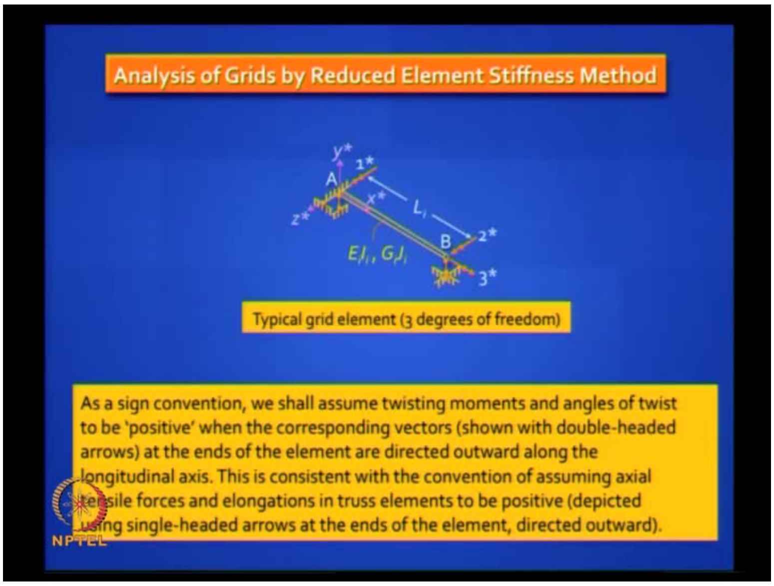

Here the transformations are much easier to do. Let us look at that. Now we are dealing with the three degrees of freedom system – 1 star, 2 star, and 3 star. The 3 star is the new addition that you have over the beam element.

The direction of it is pointing outward. It is a double arrow head and you need to give it a new interpretation. Earlier, remember when we had the axial element, we had a single arrow pointing outward.

The understanding was – if it is D_1 star, it refers to an elongation in the element. So if there is an elongation, it is positive; if there is a contraction, it is negative. We also had F_1 star for the axial element – if it is positive, it means that is axial tension; if it is negative, it means axial compression. You need to have something similar here for the torsional degree of freedom.

This is how we interpret a sign convention. We shall assume twisting moments and angles of twist to be positive when the corresponding vectors shown with the double headed arrow at the ends of the element, directed outward along the longitudinal axis.

This is consistent with the earlier sign convention for a single-headed arrow for the truss element.

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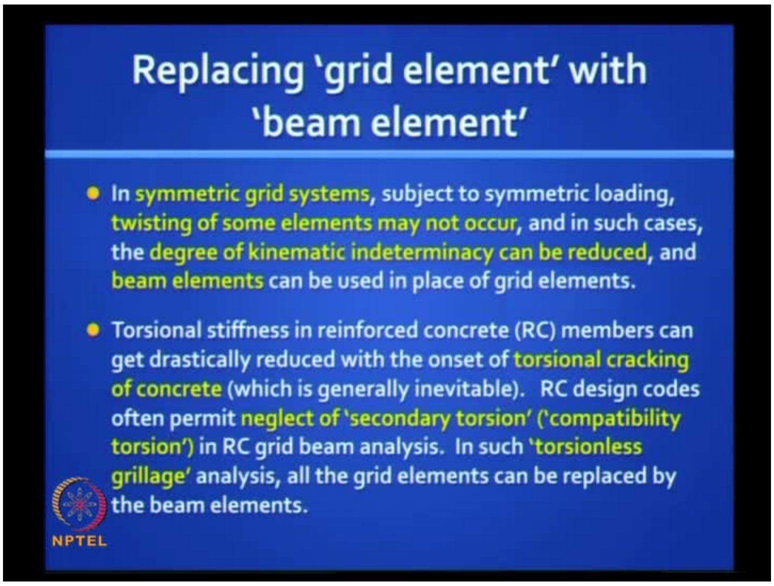

Just remember this. You will understand when we do it in an application. It is not always that you need to apply grid elements in a grid problem. You can replace some of the grid elements with the simpler beam element. Can you tell me where you have these opportunities?

When is there no torsion?

When we (( ))

When can you release torsion?

When torsion is (( ))

There are two instances. The first is when you have symmetry. If you have symmetry in the system, you might find that to satisfy symmetry in some of the beam elements you cannot have any angle of twist. So for that particular loading condition, which gives a symmetric response, you can avoid having that angle of twist.

Second is in reinforce concrete. In reinforce concrete, it is well known that concrete cracks easily under torsion. So, the torsional stiffness that you get from those formulas for homogenous elastic materials has to be factored. Usually, we assume a factor of around 15 percent or 0.15 because it drastically drops to 15 percent. It is not 0 but some design codes. In fact, all design codes allow you to simplify analysis by assuming it to be 0, which is called torsion-less grillage analysis. But it is not exact. If you want to do it exactly, take about 0.15 of the gross properties and do it.

The first instance is in symmetric grid system, as I mentioned. The advantage is that you can reduce the degree of kinematic indeterminacy. You can replace some of the grid elements with beam elements but you do all these tricks only when you are dealing with the reduced element stiffness method; otherwise, you pretended let it work out the values and you will get 0. Standard program will do it like that. In reinforced concrete, torsional stiffness can be drastically reduced with the onset of torsional cracking, which is generally inevitable.

RC design codes often permit the neglect of secondary torsion or compatibility torsion, which is different from primary torsion.

There is some torsion you cannot avoid. To satisfy equilibrium, you have to have that. But in grids, you do not have those instances in such torsion-less grillage analysis. We do this grillage analysis frequently in bridge decks, so there you can release these twisting movements. All the grid elements can be replaced by beam elements. But let us learn the right, royal proper way of dealing with torsion. Torsion will always be there. It may not be significant but you should appropriately assign the torsion stiffness values.

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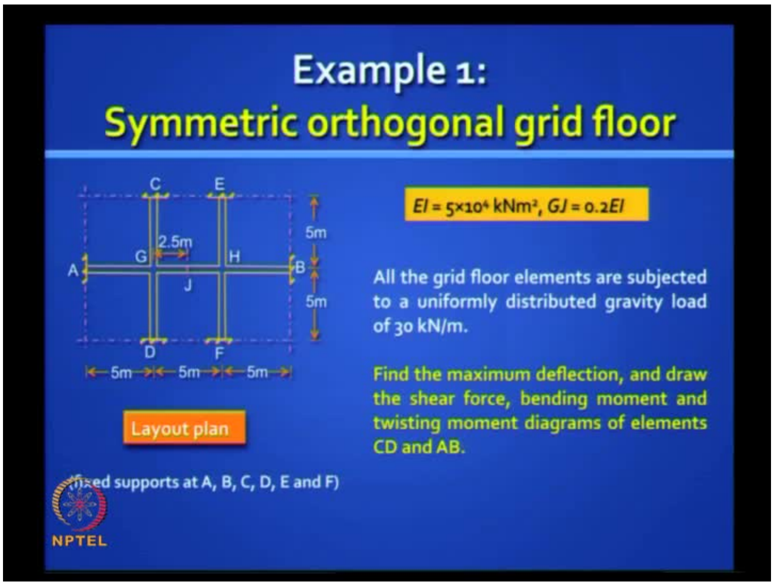

So, the method is the same as we have done earlier. Let us demonstrate this straightaway with one problem. This is the symmetric orthogonal grid floor. You have full fixity against flexure and torsion at the stream ends, which is A B C D E and F. It is assumed that all the grid elements have the same prismatic section – EI is given and GJ is shown as 0.2 times as EI. You have a gravity loading uniformly distributed at 30 kilo Newton per meter. So straightaway you can notice that there is some symmetry in this system.

You can see that the two shorter beams – CD and EF – will behave identically. You can actually cuts the system at the middle J, and take advance of symmetric. You can go one step further and you need not have grid elements for all of them, which can you replace with beam elements A B because you can see that D G and G C are going to be acting like mirror images. Hence, they cannot be an angle of twist along of the direction A B vectorially.

But C G D can twist. Why can it twist? Because if you take the beam A G J H B, it is going to deflect in an odd way. You have only showed that the slope will be 0 at J because of symmetry, and the slope will be 0 at A and D because the ends are fixed. But anywhere in-between it need not be 0, so it will have a rotation. It will hdefinitely have a rotation at G and H. And what is the flexural rotation for the longitudinal beam will be an angle of twist for the element C D. So, if you isolate the element C D, you will have to apply a torque at G, which come from this integral connection.

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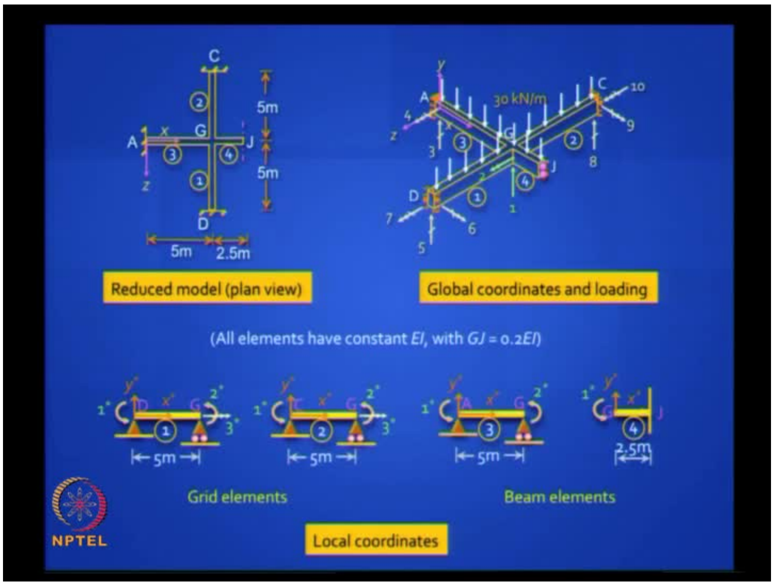

You must first visualize the physics of the problem. You can cut the beam at J. What is the boundary condition you will put there? You will put a guided roller because it can deflect but the slope is 0, so you can fix it against rotation and there is no twist there because it is a point of symmetry.

If you write down the degrees of freedom, you will find that it normally would take three active degrees of freedom at that common joint G. But you need to take only two because we have just established that there is not going to be any rotation at A J, along the x-axis because of symmetry. D. There is no rotation about that axis because that would mean twisting of that element at...

You have intelligently minimized your work by assigning coordinates carefully. So D 1 and D 2 are the only displacement unknowns. Then at the restrain coordinates, you can add all the remaining degrees of freedom. You have all together 10 degrees of freedom, two of which are active, and eight of which are restrain.

I am just demonstrating with one example. You identify the grid elements – they are the elements 1 and 2, and the beam elements are 3 and 4, of which 4 can be simplified with one degree of freedom. Why? Because you have a guided roller support. We have done that, so we can treat it like a cantilever.

We have made all the simplifications possible to reduce our work. Any questions? 1 and 2 are grid elements because there is going to be torsion in these two elements; 3 and 4 are beam elements. 3 will have 2 degrees of freedom, whereas 4 will have only one unknown degree of freedom because of the advantage that you have of a guided fix support. So we are really taking advantage of all the knowledge that we have to solve this problem.

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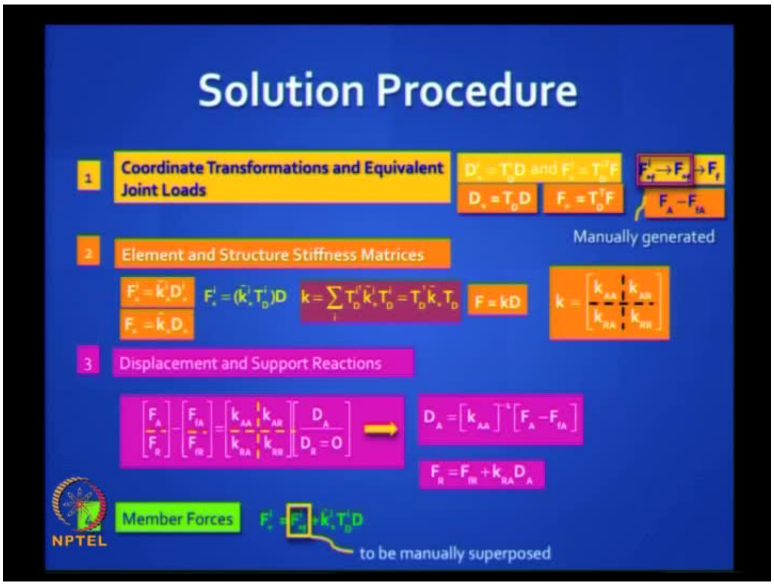

The solution procedure is exactly as we have done for all the problems till now. As you do not have any support settlements, so the steps are straightforward. We are using the reduced element stiffness method.

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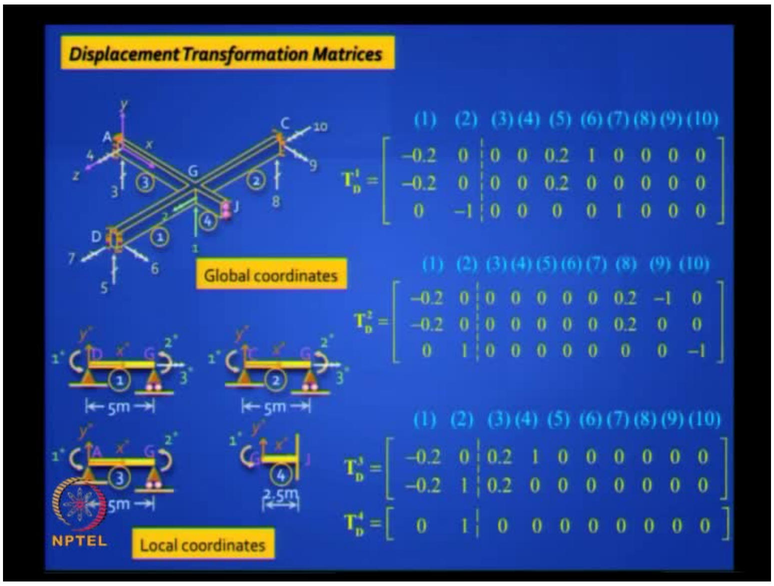

First, this is the simplified model of the grid that we are going to look at. You have to first generate the T_D matrix. Will you do that? You have global coordinates and you have local coordinates, can you write down the T_D matrix for these elements? And once you do that everything falls into play. To do the T_D matrix you have to apply D 1 equal to 1, and see what happens to those rotations; then D 2 equal to 1, etcetera. So I have done it here and let us go through it carefully. Listen carefully. D 1 equal to 1 means we are giving a vertical displacement here but not allowing any other moment. If this goes up by 1, what do you think will happen to all the different elements? It just goes up by one, you will have chord rotations. Let us take the first element. Will you have an anticlockwise chord rotation or clockwise?

Anticlockwise.

Anticlockwise, which means you will have clockwise equivalent end movements. So you get minus 1 divided by the span is, 5 so 1 by 5 is 0.2. So you get minus 0.2 minus 0.2. And you do not get anything in the third degree of freedom because that corresponds to an angle of twist. Is it clear? What we got here what about the next element? In the next element, it is clockwise if you look at in G to C, but we are looking at it from C to G, so the orientation is done this way. So you have to be very careful. We will get it identical. And what about element 3? Element 3 also has a span of 5 meters. There also you have an anticlockwise chord rotations. Does it fall in place? And for element 4, it does not matter because it is a cantilever you lift it up, the guided roller will also go up. So you do not get anything – 0. So the first column we filled up correctly.

The second column corresponds to 2. As far as the elements 1 and 2 are concerned, it is clearly an angle of twist, but it is going to be outward only for element 2 and inward for element 1 by the sign convention we took. So, we write here minus 1 and plus 2. And as far as the element 3 is concerned, it is a flexural rotation. So, it is 1 as far as element 4 is concerned. It is a flexural rotation, it is 1. Like that, can you fill up all the others?

Now you can do this exercise, go through it carefully. We have just demonstrated this. You have to be careful about the chord rotation and the sign conventions. It is quite straightforward. This is the displacement transformation matrix. You are doing it by looking by inspection and filling it up.

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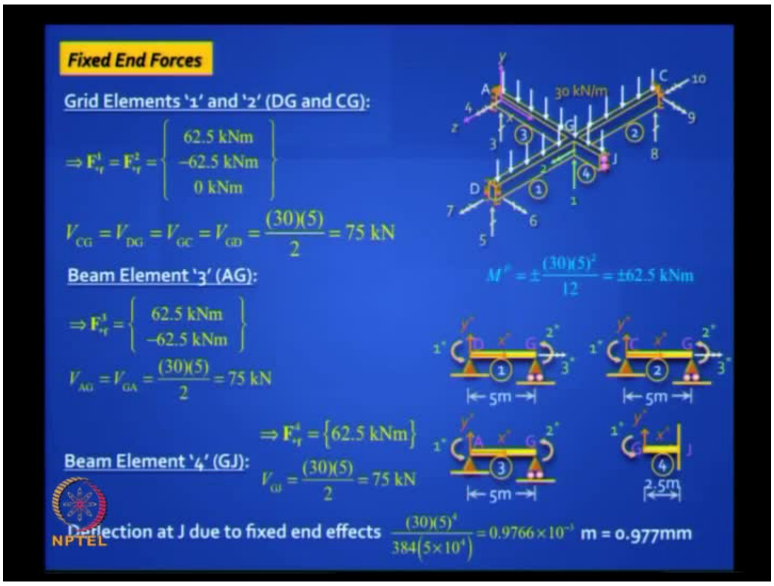

Next is fixed end forces. Luckily for you, all the beams are having a span of 5 meter and the loading is the same – 30 kilo Newton per meter. So the fixed end moment is plus or minus 13 into 5 squared by 12. It is turns out to be 62.5 kilo Newton meter.

If you take the first two elements – 1 and 2 – it is plus 62.5, minus 62.5 and 0. Anticlockwise positive, clockwise negative, and 0 because there is no torsion. Then you can also find the vertical reactions. Similarly, for the second element, it is exactly same. I mean the third element – it has the same span, the same loading and for the fourth element, it is a guided roller support. So you should take the full span, you will get the same formula 32.5. It is anticlockwise, it is positive and the vertical reaction is also the same.

We are dealing with the same number – 62.5 and 75, because all the spans are identical, the loads are identical. Not difficult. Then from this, you can also check out your deflection because that is the question asked. What is the maximum deflection in that entire grid is very important, when you design grids because deflection is one of the governing criteria for deciding the depth of the beam. The deflection should be within acceptable limits. So the fixed end deflection is very easy to work out. You know that for a simply supported beam, the deflection is 5 by 384, the new L rise to 4 by EI. If it is fix, its 1 by 384, so we have just used that formula and you get a very small deflection of 0.977 mm in this case.

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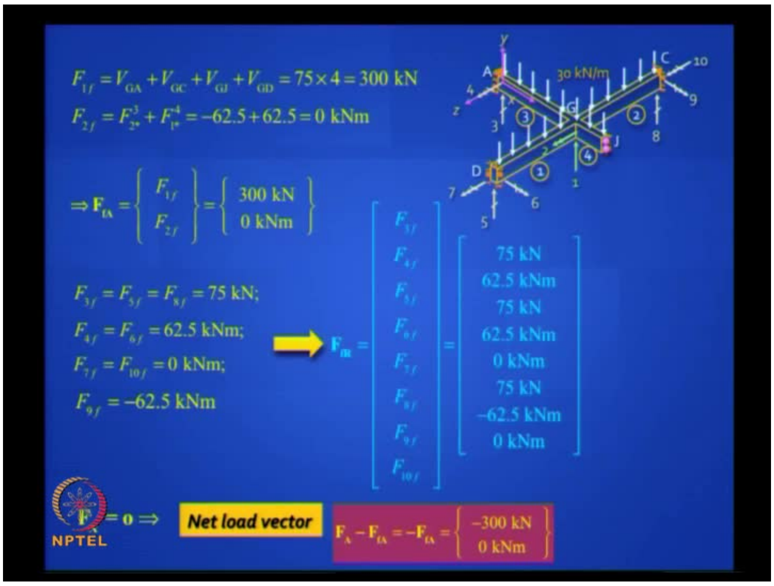

You can add up all the vertical reactions and find the equivalent force vector F A 75 into 4 is 300 kilo Newton. It is acting upward, so that is your fixed end force for F 1. F 2 gets cancelled out because minus 62.5 and plus 62.5 neutralize each other. Similarly, you can get the fixed in force vectors at the restrained coordinates. It is easy to compute.

You just pick up those values from your fixed end forces. So you are ready with this. You can find your resultant net load force vector. There is no concentrated load given in the original problem, it is only uniformly distributed load. So you got your net force vector exactly as we done in the earlier problems – F A minus F fA. F A is a null vector, F_{fA} we have already computed.

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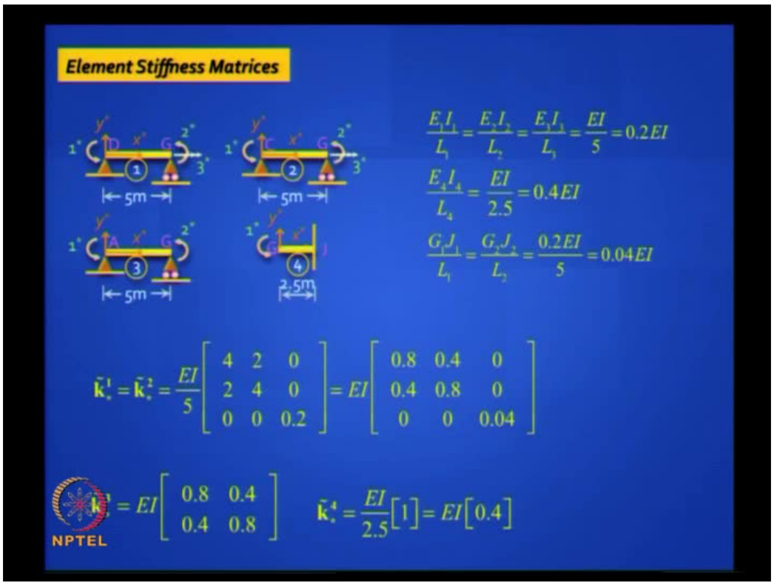

The element stiffness matrices are very easy to calculate. You know the formulas -- EI by L is known, GJ by L is known, the first two are grid elements. So, you can write $k1_*$ and $k2_*$, it is a 3 by 3 matrix.

The only new item here is a third row and third column. You just put GJ by EI value, which turns out to be 0.2. You can expand it. For the third element, it is a conventional beam element -- remember 4 EI by L, 4 EI by L, 2 EI by L, 2 EI by L. And for the last element, what will it be it is going to be? 1 by 1. It is cantilever.

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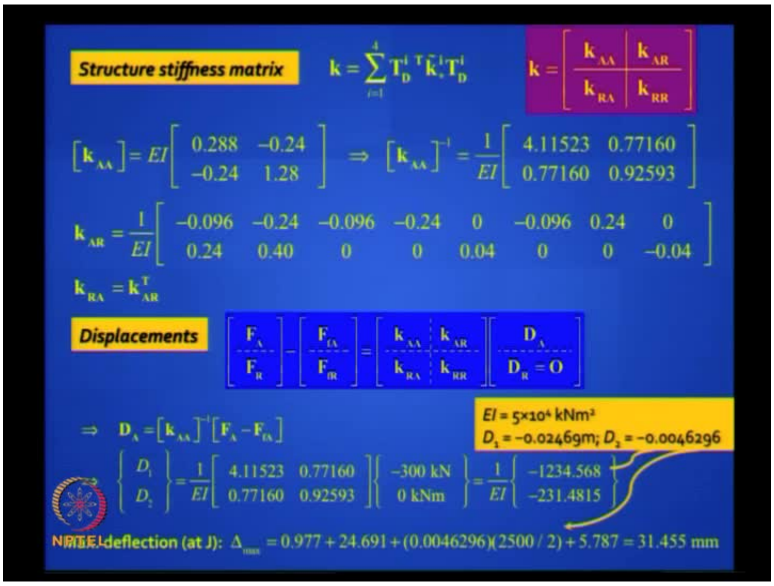

So what will be the formula EI by EI by L? That is it. And L for this is 2.5 meter. So you are ready with all the k matrices. You got the T D matrices. This exercise is best done with the help of may be MATLAB or something. You can get k_{AA}, and you can get k_{AR}. You can get k_{RA}, you can find the inverse of k_A matrix, and you use a standard equilibrium equation to find the displacement.

You have got D 1 and D 2. You got the net load vectors, so it is easy to find out. After you found D 1 and D 2, you can actually substitute the value of EI and get the values. Remember D 1 is a vertical deflection there, but we want the final deflection. So how do we get that?

This deflection D 1 has come from

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Yes. But the other part will give the same deflection. You have to add something to this. You have already calculated due to fixed end forces what the deflections. You have to add that. You have to add something else.

That rotation in a cantilever will give you some additional because you have simplified your problem here. It will give an additional value. Remember, for a cantilever at the guided roller support, we are finding the deflection at the guided roller support, not at D 1, please note.

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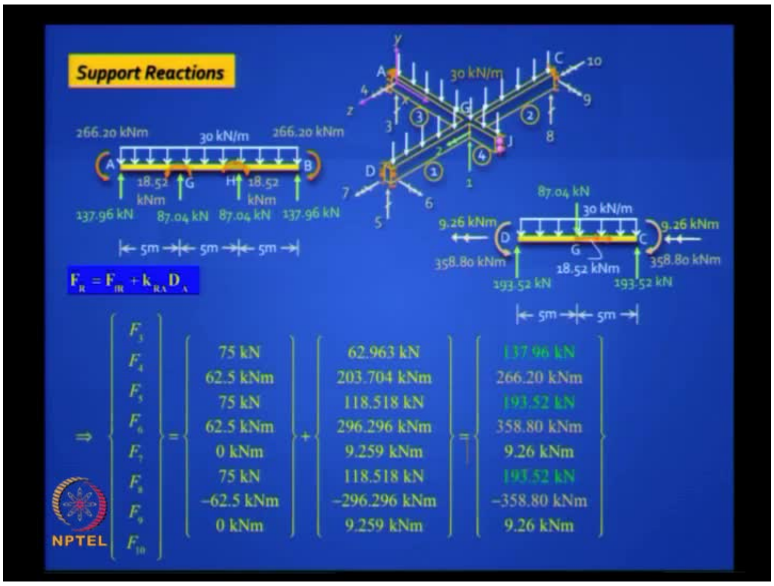

We are finding the deflection at J, so at J we have something more. Then what you get at G? What is the additional thing you get? It is a slope into span by 2. You can prove for a cantilever, so you have to do it very carefully and work out the deflections. Support reactions are easy to calculate, use that formula and take a look at the final free bodies. You will find that in that long beam, you will have some local torques coming because of the connections with the shorter beams.

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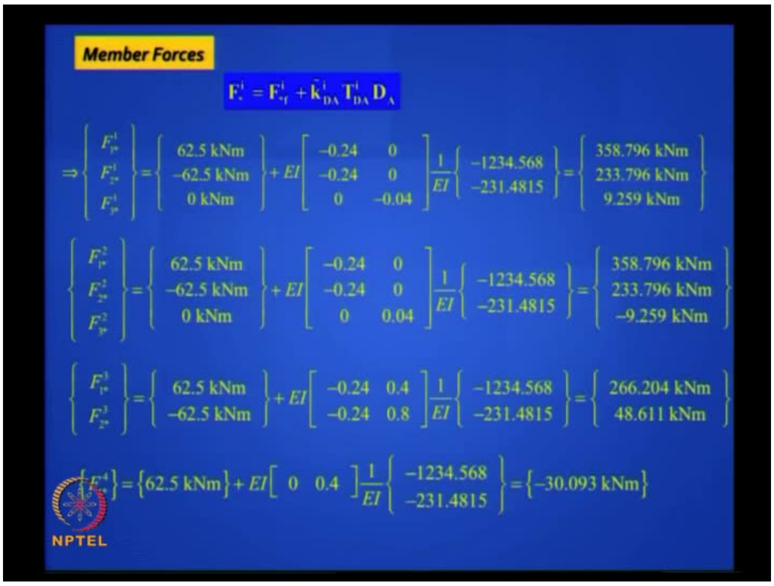

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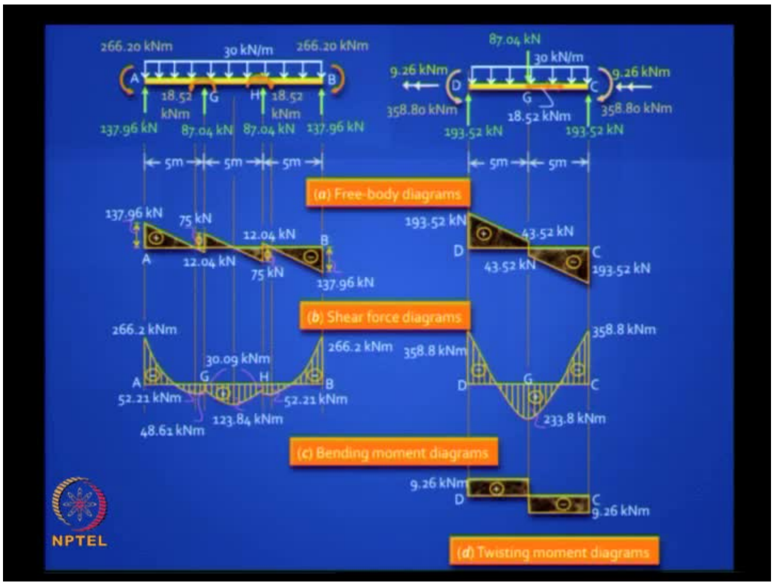

You do this – draw a free body, check your equilibrium, find the member end forces by this formula, and draw your shear force diagram carefully. We have done grids earlier, so I am going fast. We have done this kind of problem earlier. Draw your bending moment diagrams. We will have jumps in the bending moment diagram, where you have local concentrated moments acting, and you will also have twisting movements in the shorter beam. So this is all you need for a design. You have got the full beam, you have got all the internal forces, you have got the support reaction, you have also got the maximum deflection so very powerful. We are solving such problems.

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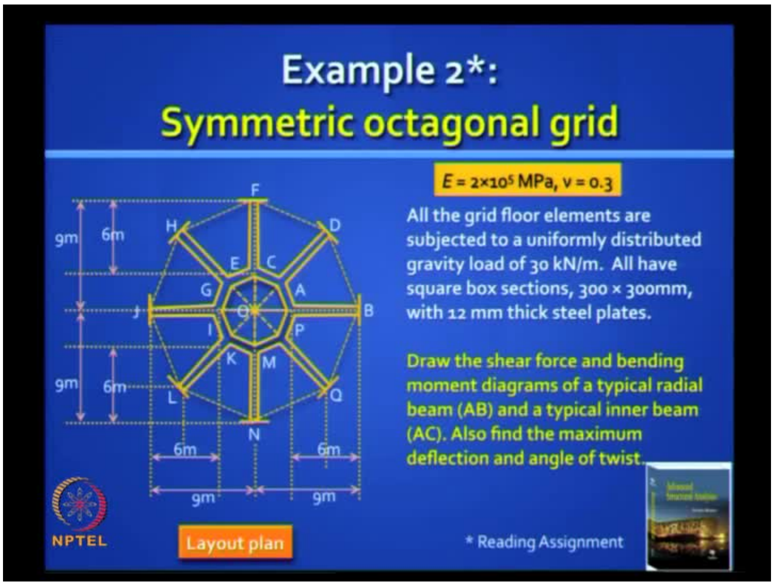

This is a problem that is very interesting but we do not have time to solve it. Please read it in the book. Do not worry, in the exam I am not going to ask you such a problem but architects like to give such a roof for example. You have an octagonal roof when you got these radial elements and circular elements can you design this from first principles. You can take advantage of symmetry and solve this problem. You just need to apply your brain, and you can actually use some shortcuts and crack this problem very easily. It is a reading assignment for those of you who are interested.

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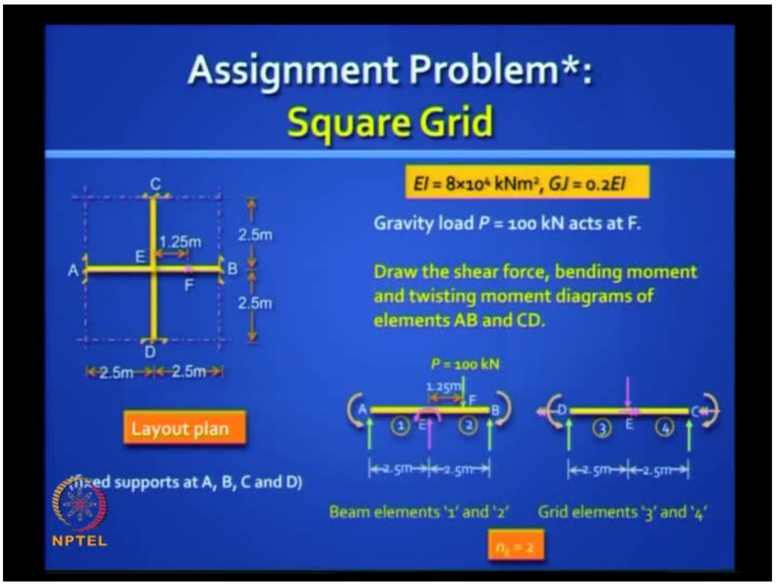

I have given you an assignment problem which is much simpler. Remember this problem? It is a square grid problem -- you have got two beams, they are identical forming a plus shape but you have eccentric load acting on one of them at F.

So you are actually asked to plot the rough shapes of the shear force bending moment and twisting moment diagram, which you can by inspection. Look at this carefully. Which of them will you treat as grid elements? If you draw the free bodies, its similar to the previous problem, and you will find that A B will be connected to C D through that joint at E, and for one of them – for A B – what do you have? Do you have beam elements or grid elements? Beam elements. And for C D, you have grid elements.

So, it is similar to the other problem. Please try to sketch and predict the shapes of the bending moment and shear force diagram, and then with these notation, you can go ahead and solve the problem.

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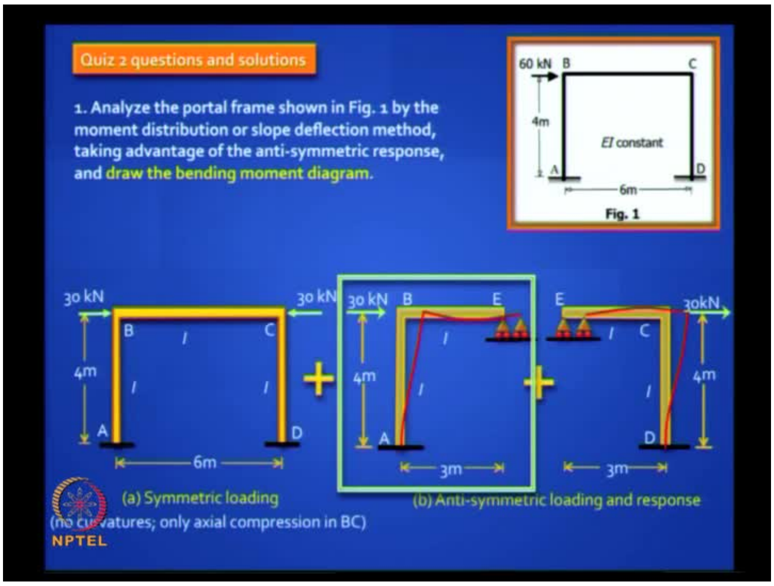

This is the last problem in your assignment which you should complete soon. And now it is time to look at your second quiz results. I have got the papers. Let’s take a look here -- the degree of kinematic determinacy. We are looking at your quiz questions, which you did. I am happy to say that some of you got full marks -- 15 out of 15; and I am sorry to say that we have a full spectrum. Some of you got very low marks.

Please figure out where you went wrong, if you went wrong and go through it carefully. It is very clear that some of you fully understand what is happening but some are not.

I have been liberal in my marking but take a look at this. Remember that such problems can be solved very easily if you take advantage of anti-symmetry. But some of you insisted on slogging the hard way – three unknown displacements. And you have got it right also but with so much of effort. You are not really taking advantage, it is actually very easy problem.

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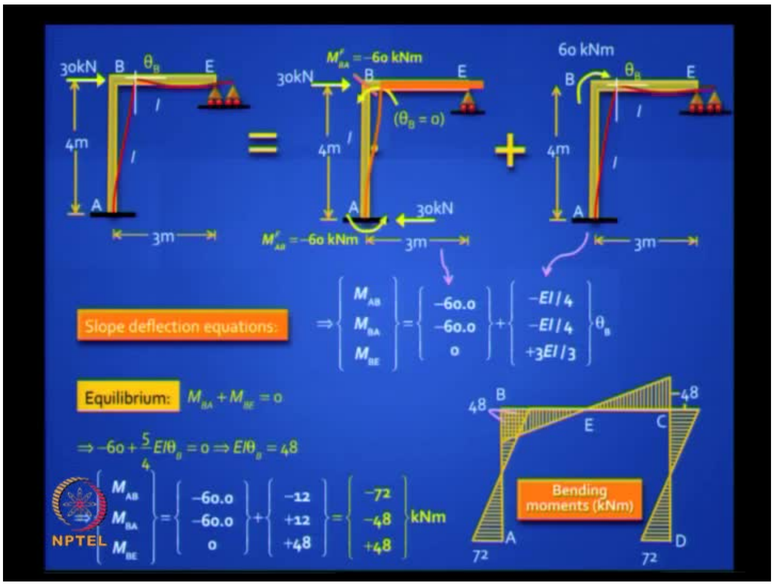

You remember, we have done this earlier. You can reduce the problem to superposition of symmetric and anti-symmetric components. Yes, we have done this. And you need to analyze only one part of it because the other part does not give you any curvatures. So that is all you need to analyze. And you have to find fixed end moments correctly and write your slope deflection equations correctly. The fixed end moment is 30 into 4 by 2, 60, anticlockwise.

This is conventional slope deflection method, so you put minus because it is anticlockwise. And it is EI by L because it is cantilever behavior for the element A B and is 3EI by L. L in this case is 3 meters. Some of you did it by memory based on the problem we had solved earlier. I do not know, you have phenomenal memory because you put value of 2EI for the beam element because in the problem we did in here, it was 2EI. It’s very the Indian memory is very good, but unfortunately, you are not seeing the problem here. EI is constant and so that is the change you make. And if you solve this, the equilibrium is very easy to solve. Your bending moment diagram should look like this. It is an easy problem.

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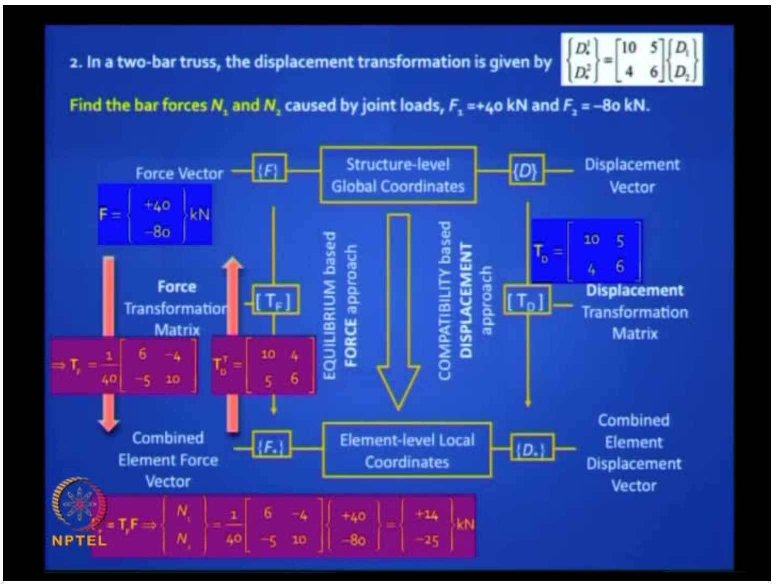

Next problem is actually an easy problem. Some of you try to figure out what truss shape it could be and did all kinds of funny things. But it is actually very simple. If you go back to basics – remember this picture. You have the structure-level coordinates, you have the element-level coordinates, you have T D flowing downward, from D to D∗, and you have T F flowing downward from F to F∗. The left one corresponds to the flexibility formulation, the right one corresponds to the displacement formulation. In this question what is given to you? You have got T D and you have got F. And what are you require to find -- F∗. So, you have to just use the right principles. You have T D, and it is flowing downward but you can bring in the contra gradient principle, which gives you T D transpose. But which gives you a connection between F∗ and F. So, you need to go to the other way, so what should you do? Invert it. And you get the answer.

Now I have got all kinds of permutations and combinations from you. Some of you have just multiplied the same T D matrix and hope that you get the forces, which is ridiculous. Some of you have took a transpose because you knew there was some contra gradient principle involved and you multiplied and you got again some ridiculous results. But many of you did it right. You have to invert it and do it and you do not need to figure out what truss is involved. It is a very simple problem.

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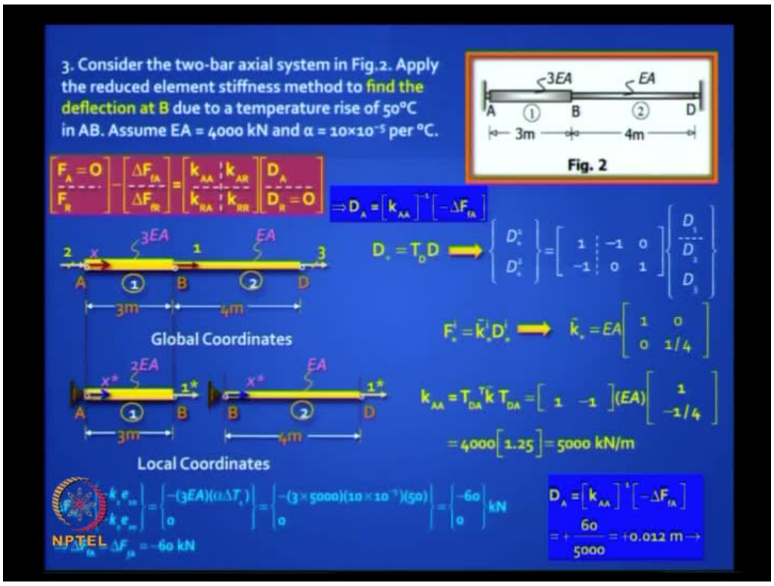

Third question we have done earlier, so you should have crack this. First of all try to minimize your work. Those you need to find only k_{AA} and delta F_{fA} because there is nothing else involved. So, what is the size of k_{AA} matrix – 1 by 1. You really do not need to even bother about the restrained coordinates. You could have managed with 1 but since all of you, in fact there is no one to took only one coordinates, so let us me also follow your example. This is the familiar picture, if you want the full T D matrix, you can write it down but all you need is only T D A and your k star matrix is very simple. It is a diagonal matrix.

Your k_{AA} is child's play, in one shot you can get it. Do not get full k but almost all of you got the full 3 by 3 $k_{A.}$ And then you picked up the top left hand corner and said that is k_A. It is very easy.

See, when you write in exam, you have to really be intelligent and do the correct thing in the least time involved. So what about your fixed end forces? Some of you forgot there only one bar is heated. You kind of heated both the bars and you got it wrong. So that is very easy to solve. The answer is 12 mm moving on the right side. Some of you moved it on the left side and never thought that it is foolish because if you heat that bar B, it is going to move to the right. How can it move to the left?

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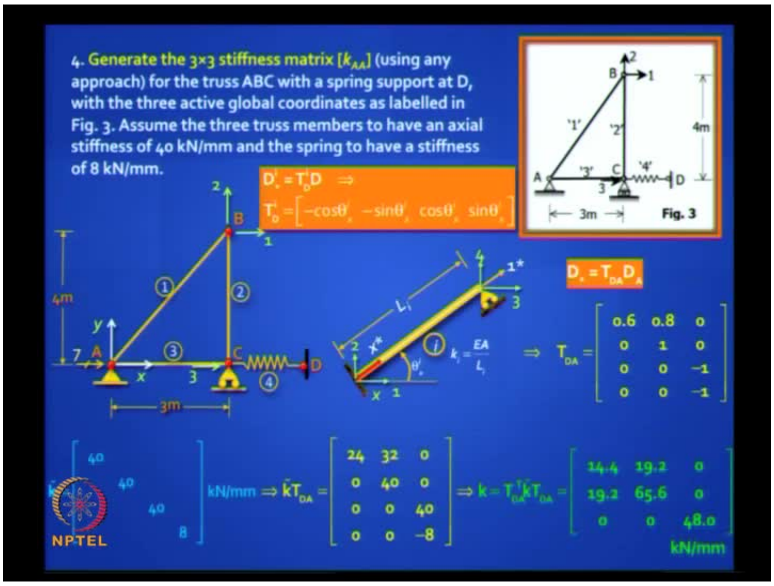

Last problem. Again simple. It is a truss. All you needed to do is to generate the stiffness matrix, nothing more, by just multiplying some matrices. This is the problem. You know how to write the T D matrix, it will look like that. You have four elements. Do not forget this. Spring is also an axial element, so it is very easy to write down. It is 4 by 3 matrix. Your k* matrix is child's play.

Remember everything is given to you. The axial stiffness is 40 kilo Newton per meter. But some of you thought it is EA and you divided it by the span. I do not know why. So you have got this – kilo Newton per millimeter, multiplied out, you get the final answer.

Thank you.