Advanced Structural Analysis
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Module No. # 5.5
Lecture No. # 31
Matrix Analysis of Beams and Grids

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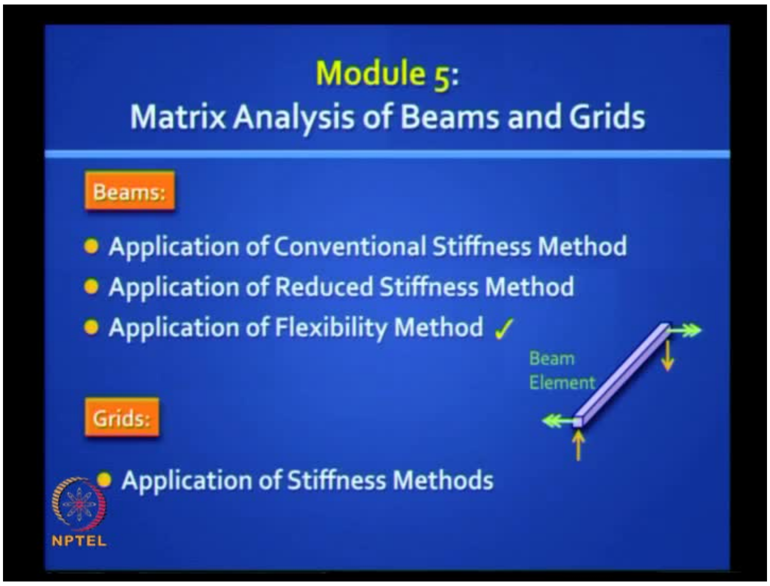

This is lecture 31, module 5, Matrix Analysis of Beams and Grids. If you recall, we had finished the reduced element stiffness method in the last session. Today, we will look at the application of flexibility method to solving beams.

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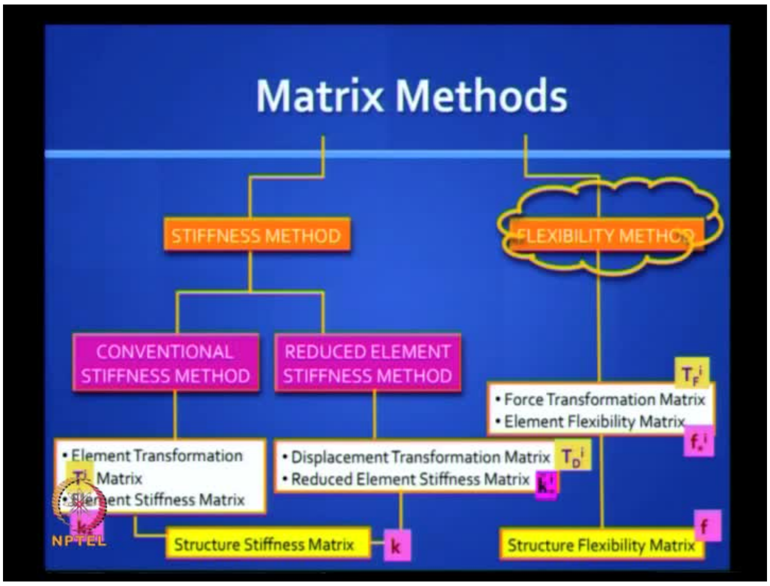

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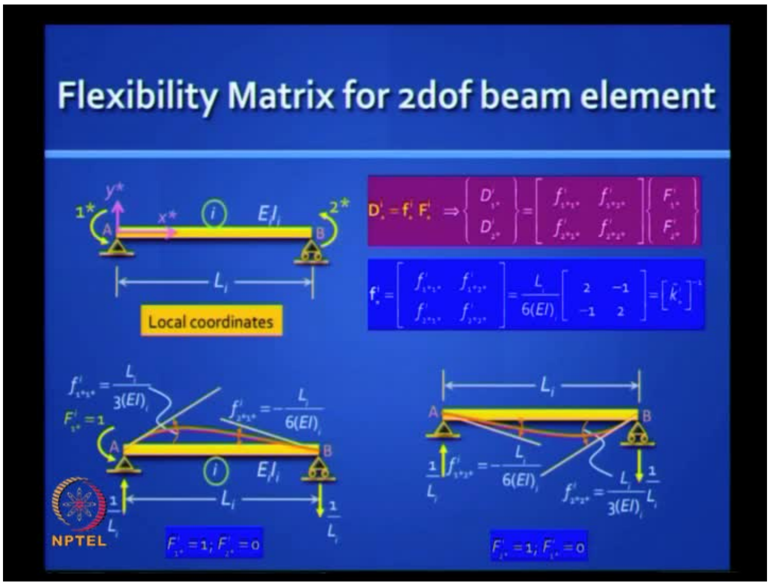

This is covered in the chapter on beams and grids in the book on Advanced Structural Analysis. Of the three methods, now we look at the last one - the flexibility method which you are already familiar with. The flexibility matrix for a prismatic beam element is something that you can easily derive from first principles; we have already done that.

But to refresh your memory, you have two degrees of freedom; you have a simply supported beam. You apply 1 unit force at a time - F 1 star equal to 1. You know the 2 end rotations, 1 will be L by 3 E I and at the other end it will be L by 6 E I. It is important to note that at the other end you will get a direction which is opposite to the direction you get at the first end.

So, you have to put a minus sign when you write this stiffness matrix. When you do F 2 star equal to 1, you get a picture like that and it is easy to write down the flexibility matrix. Remember this - 2 by 2 reduce element stiffness matrix is 4 E I by L, 4 E I by L, 2 E I by L, 2 E I by L.

It is going to be L by 3 E I L by 3 E I minus L by 6 E I minus L by 6 E I - there is a minus sign that comes. For convenience, you can take out L by 6 E I outside the brackets and so you will be left with 2 2 minus 1 minus 1. So, remember this - it is very easy to remember; it is for a prismatic beam element. If the beam element is non-prismatic, if it is step or if it is tapered, you can still derive it from first principles. How will you derive it? Which method will you use to get these rotations? You can use conjugate beam method; very easy but usually, we do it for the prismatic element.

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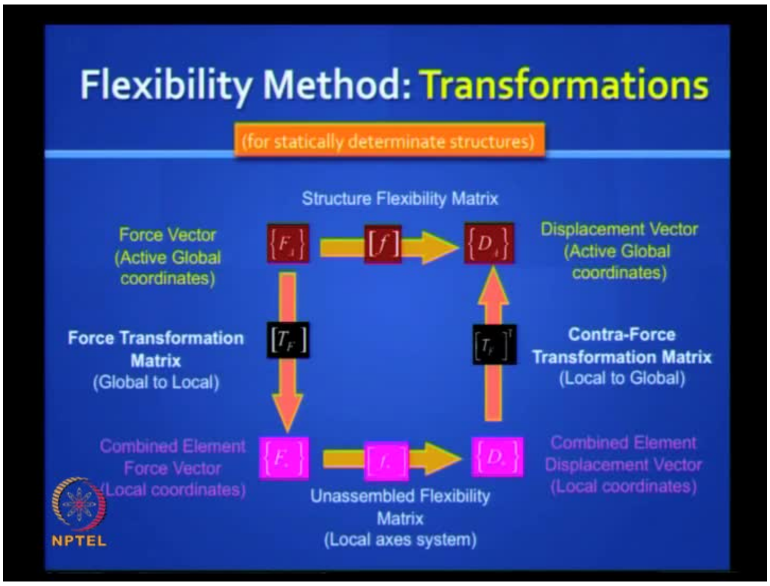

In the flexibility method, you are already familiar with the transformations and like this stiffness method. Now, we move in the anticlockwise direction in this chart. First, you have the element's flexibility matrix which helps you get the element level displacements if you have the element level enforces - let us see F star matrix.

Similarly, you have at the structure level; now I have written here F A and D A but for statically determinate structures you can just write F and D. Please note that, if we are trying to avoid including the reactions in the end forces, if you want to include them, it is possible; we demonstrated this in an earlier class by including rigid links and relating the support reactions to internal forces in those rigid links; but we will avoid that for convenience.

So, just like you had F A and F R, D A and D R in stiffness methods, you will have in place of that, you will have F A and F x and D A and D x. That is a big difference; you will get D x only if the structure is over rigid, statically indeterminate. If the structure is statically indeterminate, there are no redundant.

So, you have only F A and D A. So, that is shown here (Refer Slide Time 02:53), F A and D A and the basic force transformation is the T F matrix, which really tells you that if you give the loads on the structure, you can directly get the relevant internal forces at the element level. By doing that T F transformation, you will find that this is based entirely on statics, entirely on equilibrium; there is no compatibility in this relationship and it is a unique matrix.

It is generally a square matrix which you can derive by applying unit load one at a time; we will demonstrate all that. We have done that for trusses, we will do it also for beams and there is a contra gradient principle which links D star to D A. That is the T F transpose matrix. What is the use of this matrix? Can you tell me where does it help us? In trusses, how did it help us? Lack of fit.

If you have some initial displacements in a statically determinate just rigid structure, a truss, you can find out the joint displacements caused by these moments. There will be no stresses if it is statically determinate and you are actually solving a geometry problem using the help of statics.

This T F transpose matrix also establishes that while you ensure equilibrium in the structure - statically determinate structure, you are also, without your knowledge, simultaneously establishing compatibility. So, that is the other advantage. In beams, where do you, how, where do you anticipate using the T F transpose matrix?

Temperature loading of a segment - temperature loads we usually do not encounter in beams and they do not, they will come in frames because you need the actual degree of freedom; unless you are talking of a differential temperature, the temperature gradient across the depth of the beam which we will not consider for the time being.

So, you will encounter initial displacements when you have supports, moments at non redundant coordinates. You have 2 types of support moment - you have support moments at redundant coordinates which will directly going into the D X vector and you might have support moments at non-redundant location. We have done problems like that. Those you have to somehow convert to their effects in the primary structure and that is where you can use the T F transpose matrix. The other transformation you are familiar with. Once you have the element stiffness matrix, element flexibility matrix, you have the structure flexibility matrix by the standard transformation T F transpose F star T F.

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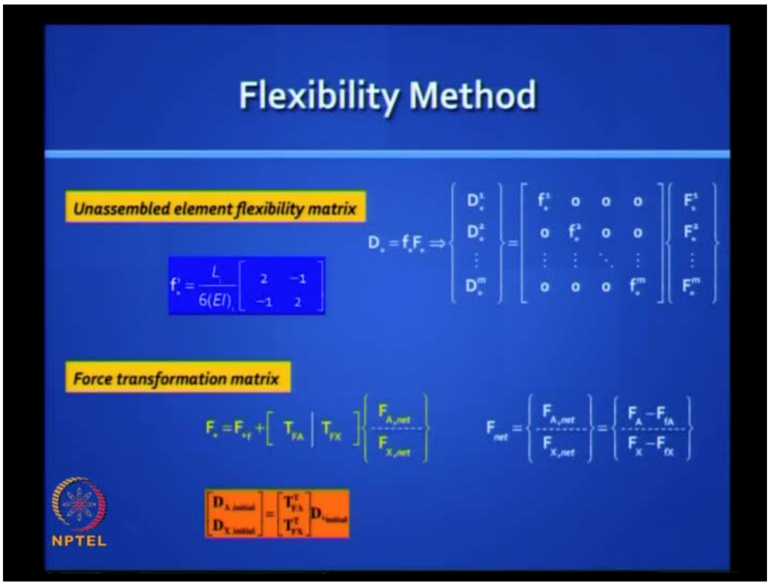

So, the F star matrix for the beam - the continuous beam will be a diagonal matrix because it is an unassembled matrix and each of those elements in the diagonal matrix will actually have a size of 2 by 2 because you have 2 degrees of freedom and F i star is given by L by 3 E I L by 3 E I, minus L by 2 6, minus L by 6 E I, minus L by 6 E I. This part is very clear.

Then you have the force transformation matrix, the T F matrix, you can separate out into T F A and T FX in case the structure is statically indeterminate and we can include the fixed end forces, which we will be required to do if you have intermediate loads on the structure.

So, this is a little aberration in the stiffness, in the flexibility method because we are forced to borrow concepts from the stiffness method to find the fixed end forces, and then import that concept here because there is no other way you can deal with intermediate loads.

So, we do that and do not put the T F transpose there. We had that problem in reduce element stiffness method; you had to manually get hold, assemble the F FA and F FX vectors. We will demonstrate that and so we can talk of net force vector as we did in the stiffness method - F A minus F FA and F X minus F FX. The only change we are making is, we did not have the X, we did not have redundancy in the stiffness method; instead we had reactions, we had r; that is the only difference. You can, as I explained earlier, in case you have initial displacements at non redundant locations, you can use the transpose.

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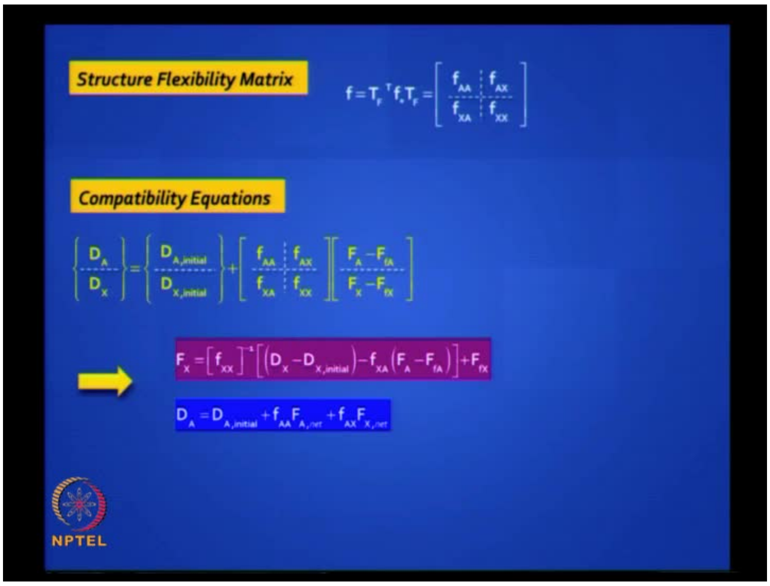

So, the structure flexibility matrix is given by this - the compatibility equations; you have to spell out and how do you solve for the unknown redundant?

Will you use the first equation or the second equation? The second; this is again another difference with the stiffness method. In the stiffness method, we use the first equation to get D A; now we solve the second equation to get F X and we plug that in to the first equation if required. Sometime we are not interested in knowing the displacement; we just want the bending moment and shear force diagram to get the D A matrix.

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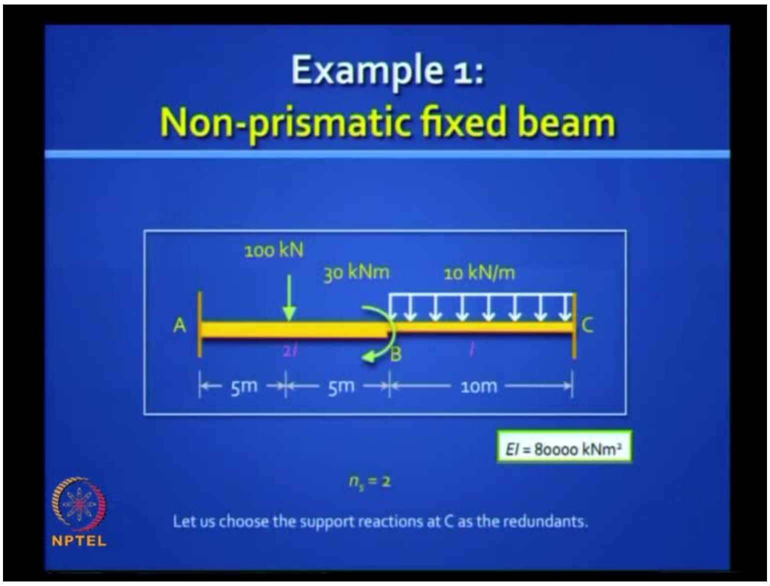

You can solve these equations. We have done this earlier and the method follows this format. Let us now demonstrate with a few examples. Let us take the same example which we did earlier; by the stiffness method, you have a non-prismatic fixed beam.

What is the degree of indeterminacy? Well strictly three, if you include the axial force but we are really not interested in the axial force. We want to draw the shear force and bending moment diagram and for that limited purpose I have explained this earlier, you can reduce it to 2 and what primary structure would you like?

Cantilever is the easiest; so, we normally have a preference, I do not know why, for fixing it at the left end but there is no need; you can fix it at the right end. We will do that.

So, we will release at c; then the structure becomes statically determinate and then we treat those 2 redundant as loads of unknown magnitude. That is a trick. F X is also now treated as a load on the primary structure but it is magnitude is not known; F A is known.

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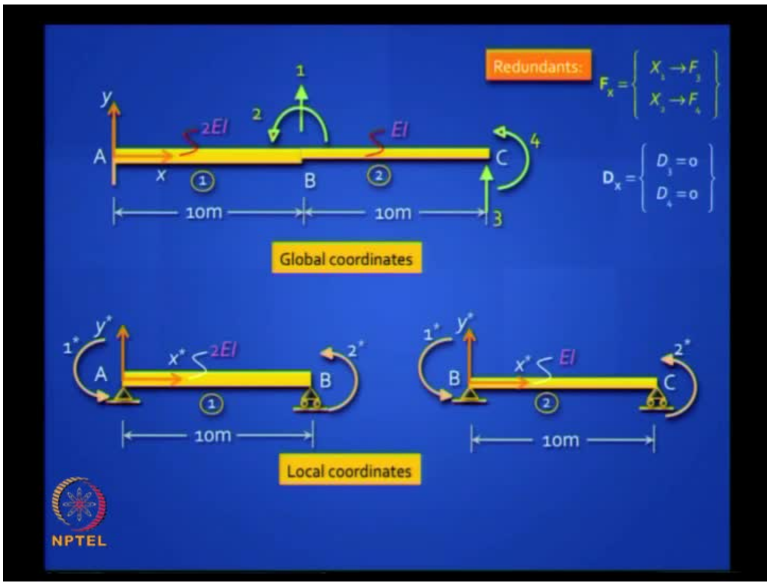

So, this becomes our primary structure - the global coordinates 1 and 2 at B. We will always number the redundant coordinates at the bottom of the list; so, 3 and 4 will go to the redundant coordinates and you should identify it. F X is X 1 and X 2. X 1 corresponds to F 3. We will see later when you have to do the net forces; it is not exactly equal to F 3.

But at least it is aligned in the same direction as F 3 and X 2 is F 4. Is it clear? The compatibility equations needed to solve this; in this case, you do not have any support settlement.

So, D 3 which corresponds to X 1 is 0, D 4 which corresponds to X 2 is 0. In case you have support settlements, in case support C settles by delta downwards, you will modify this as D 3 is equal to minus delta. In case you have a rotational slip theta at C, you will suitably modify D 4.

The question is supposing you have the support settlement at A, you cannot do it in D X; you have to do it as an initial displacement. We will see that little later.

You now need the element coordinates which are just like what we did in the reduced element stiffness method; they look the same, two degree of freedom system, simply supported beam.

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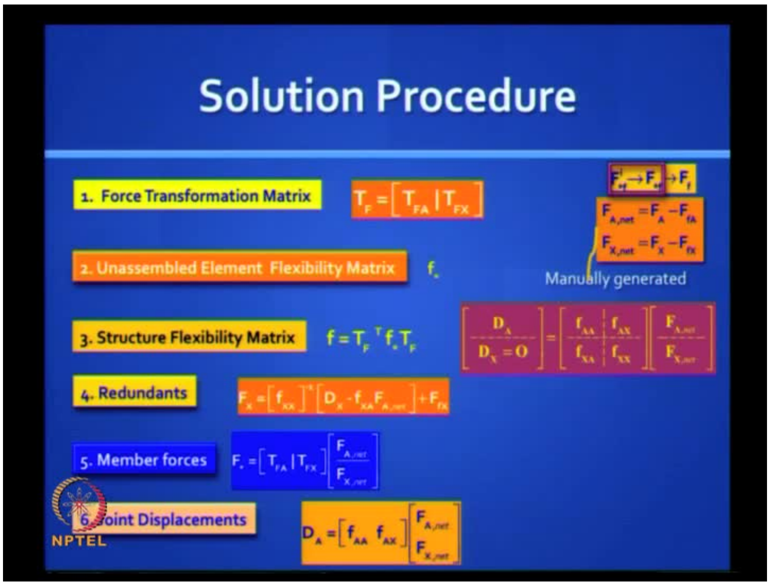

And you need to follow a certain procedure. Procedure is, find out the force transformation matrix, find out the fixed end forces - we have to generate this manually, find out the flexibility matrix at the element level; generate the stiffness flexibility matrix at the structure level.

Write down your compatibility equations appropriately, solve the second equation to find the unknown redundancy and if you wish, you can find the displacements also by solving the first equation. Otherwise, do not do that and in this case we are only interested in the shear force and bending moment diagram and find the member end forces directly by using the force transformation, where you have to put in the correct values of the net load vector. Joint displacements you can get also this way - D A directly.

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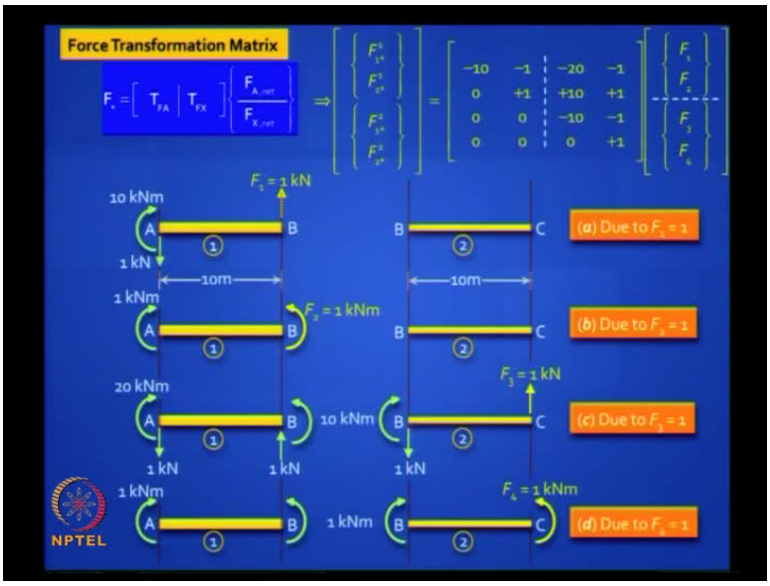

Now, let us do this together. Can you write down for this example, the force transformation matrix? You have to apply a unit load one at a time; let me help you. You have to fill in this matrix which I have left blank. There are only 2 elements.

So, F 1 1 star, F 1 2 star pertains to the n moments in the first element and F 2 1 star and F 2 2 star pertains to the n moments in the second element. You have four coordinates - global coordinates - F 1 F 2 F 3 F 4. F 3 and F 4 correspond to X 1 and X 2. So, I have drawn 4 sketches, I have drawn actually the free bodies, you can draw the whole cantilever beam. Apply a unit load at a time.

So, let us take the first case. Just draw the first one. We are now applying F 1 equal to 1. You can clearly see in a cantilever, if you apply F 1 equal to 1 at B, B C is not going to be affected because that is a free end; if you take a free body there, you would not have any forces there.

So, that is 0 0. How do you solve this one? What do you get? You will get, I will draw it; I will show you. These are your reactions. I am showing the reactions in a different color; you get a downward reaction at a of 1 kilo newton and you will get a moment of 1 into 10 - 10 kilo newton meter, acting clockwise.

So, this is the complete analysis of a simple cantilever beam subject to a unit load - F 1 equal to 1. Now, you have to fill in that first column in the T F matrix. How will you fill it? Just reallot those values - minus 10, next 0, next great that is it. It is just as easy as the displacement transformation matrix.

Now, will you fill in the next one; let us do the next one. Next one is pretty easy; you get pure bending, you just get one moment there. So, can you tell me how to fill up the next column? Minus 1 plus 1 0 0. Let us do the third one; third one, you are now applying a unit load at the free end of the cantilever.

So, what are the moments you will get? You can also draw the shears; they look like this (Refer Slide Time 14:38) - 1 into 10 is 10 kilo newton meter; that 10 kilo newton meter will act clockwise at B in B C and anticlockwise at B in B A, because every action has an equal and opposite reaction. But you can see clearly that it is a sagging moment for both and the moment you get at A will be 1 into 20 that is 20 kilo newton meter acting clockwise. It is easy.

Now, tell me how to fill the third column? Minus 20, plus 10, minus 10, 0. You will find that, let us say, I give you a three span beam, it will always follow this trend – minus, plus, minus, may be plus or 0; and the last one is very easy to get a uniform bending. How do you fill this up? Minus 1, plus 1, minus 1, plus 1; that is it. That is all.

So, you are comfortable with the T F matrix. We have partitioned it because F 1 and F 2 correspond to the active degrees of freedom, active coordinates and F 3 and F 4 correspond to the redundant coordinate.

So, we put a subscript A and X - A for active, X for redundant. Is it clear? So, you got this matrix and if someone were to give you F 1 F 2 F 3 F 4 or the net values, just plug it in and you will get all the n moments. Once you got the n moments, you can find the vertical reactions; you can draw the shear force diagram, bending moment diagram but remember - this can handle only nodal loads. You also have to add the contribution of fixed end forces for which we have to borrow ideas from stiffness method.

So, let us do that next. Fixed end forces are very easy to calculate; we have done it for stiffness method. So, we just have to copy those same diagrams for reduce element.

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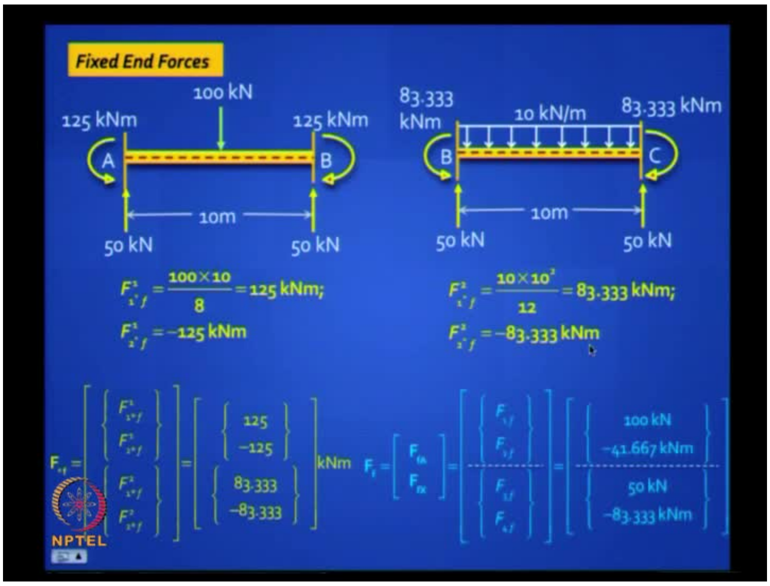

So, I would not explain this. You can find the fixed end forces; write them in a vector form like this - F star F is, in this case, plus 125, minus 125, plus 83.33, minus 83.33. You can put kilo newton meter outside. Is this clear? This is exactly what we did in reduced element stiffness method; nothing new.

Now, you have to generate F FA and F FX. Can you do that? You have to do it manually; you cannot use T F transpose.

So, you remember 1 and 2, 1 and 2 are at B, and 3 and 4 at C. So you have to get the vertical reactions. If you look at F 1, F, it is the sum of 50 and 50 is it not? So you have to see what adds up at 1.

So, you get 50 kilo newton here, you get 50 kilo newton here, they add up to the coordinate 1and that is how you get 100 kilo newton here. At the coordinate 2, you have minus 125 and plus 83.33; they add up to get minus 41.67. Is it clear? At the coordinate 3, you have plus 50 acting up and you have minus n. It is very easy; you just have to put together everything, all the contributions from all the elements algebraically, add up those numbers, you got the fixed end force vector at the structure level. Is it clear?

We actually did this when we did the axial element; did exactly the same. It does not matter, you have a beam element, you have an axial element, you have a frame element; the philosophy is the same. Can I proceed?

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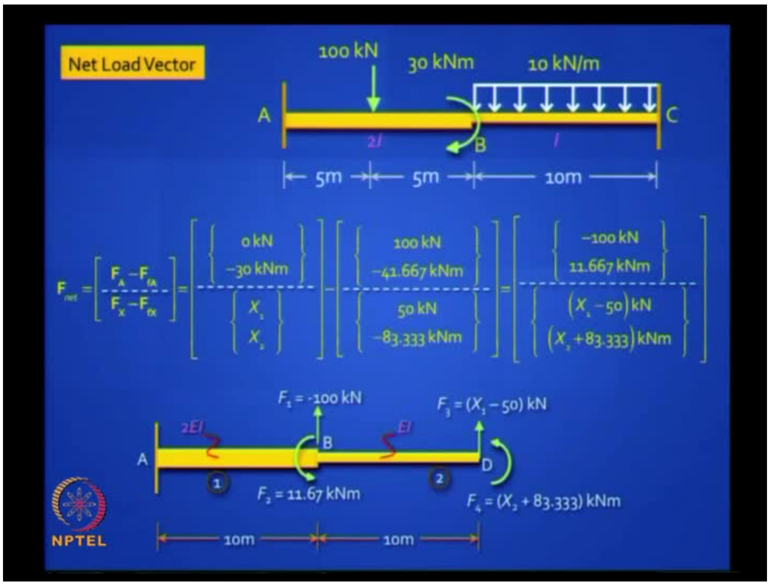

Now, you need to get the net load vector which is F A minus F FA, F X minus F FX. You had a nodal load; if you look at the original problem, there was a 30 kilo newton meter acting clockwise at B in the original problem. That is a nodal moment that is going at B is not going to any of the two elements unless you are told that it’s position is to the left or right. So, it is going to the center and it could be shared in any manner possible. So, it is a nodal moment and so your nodal force vector is 0 minus 30 and the x 1 and x 2 which you identified in the beginning. That is your nodal force vector.

Then you had intermediate loads which cause you to get fixed end force vectors, which we just computed. You have to add that with a negative sign because you need to eliminate them. Whatever you get, you get. So, you will get the unknowns X 1, along with the fixed end forces. Is it clear? Very simple, you have to write in this form; this is what we call the net load vector. Is it clear?

Now, what do we do next? Compatibility equations are very clear; that is D X is 0 in this case. Now, before we do all that, we need something. What do we need? We can draw a sketch to clarify what we are doing, what we wrote in that vector will actually be reflected in terms of these arrows marks. The net load vector I have just shown in arrow marks - there are four elements in that vector, four components in that vector - F 1 F 2 F 3 F 4. Is it clear?

So, what is the next step? You have gotten the transformation matrix, you have gotten the fixed end forces, you have gotten the net loads; you are ready to solve the compatibility equation but you need something.

Yes, you need first, before structure, you need the element and then you have to bend.

So, we go to flexibility matrix. The element flexibility matrix - what is the matrix for a prismatic element? Just so that your memory is okay, L by 6 E I at the diagonal, both diagonals and off diagonal will be minus no no L by 3 E I at the diagonal and minus L by 6 E I off diagonal. That is it.

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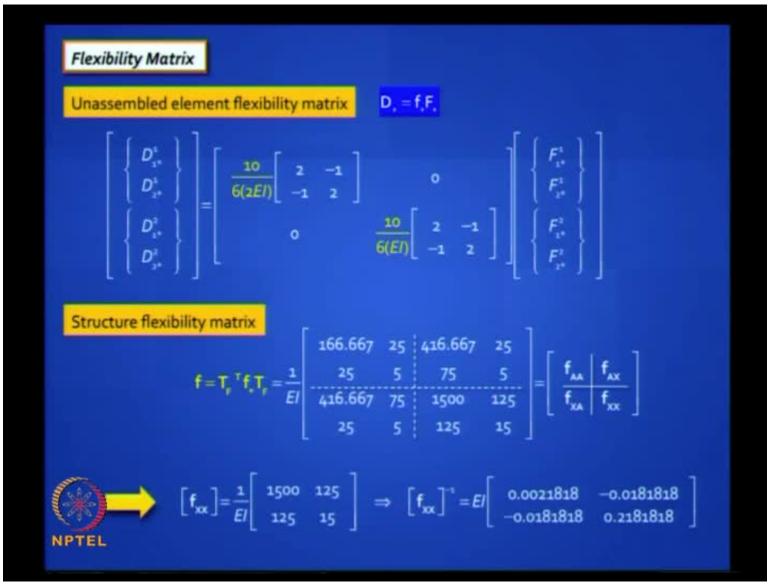

So, you do that; you can do it in one shot; you got the F star matrix. Is it clear? Just put the correct L; L is 10 meters for both the elements and the E I is 2 E I for the first element and just E I for the second element and the off diagonal elements are 0 or null. Is it clear? It is very easy to do.

So, F star is there, T F you got and you get the structure of flexibility; we would not waste time; we know how to multiply. So, let us say we get the answer, which you do first, I do not care; whether you do F star T F first, or T F transpose F star first, you do not need all those things, unlike the stiffness method.

So, here, finally you must get this answer and it must be right; it must be symmetric. Now, which is the one you need to invert here? In stiffness method, we always looked at the top left hand corner; that was K A A but in flexibility, because everything is upside down in flexibility, you have to look at the bottom right hand corner - that is F X X. Is it clear?

So, that is 2 by 2, because your degree of static indeterminacy was 2. Take it out, invert it also; be ready. Inverting a 2 by 2 matrix is very easy. You are now ready to handle any loads. In this problem, they gave you some loads; you got the net load vector; you use that.

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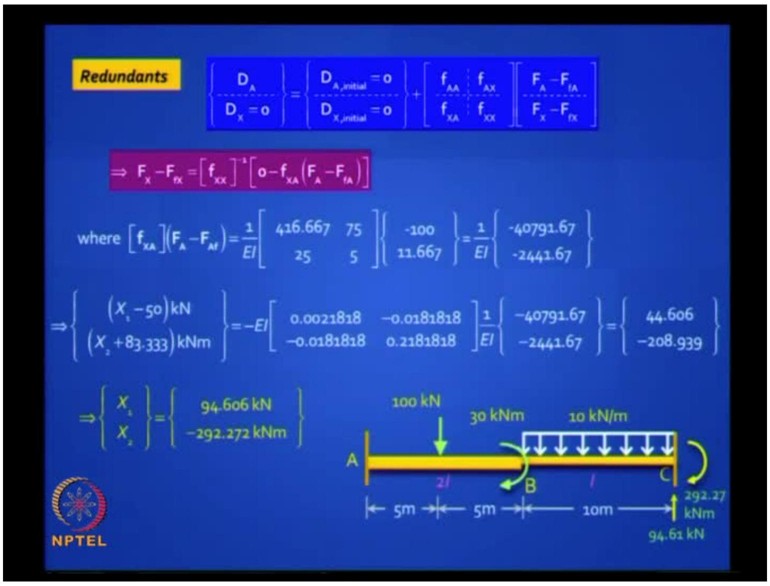

So, write down the equations clearly. You do not have any initial displacements; you do not have any support settlements. So, you write it clearly and solve for X. You just expand it and I leave it you to do it. You get some answers; you have done it systematically.

But do we need to explain these steps or it is quite clear? It is clear; you need to practice on your own. You get X 1 and X 2 and you can see what it means. It means you have an upward vertical reaction at C - 94.61; you have a clockwise moment at C, fixed end moment is 292.27. We got exactly the same answers in a completely different way when we did the conventional stiffness method, when we did the reduce element stiffness method. Is it clear? This is a completely different method but giving you exactly the same answer.

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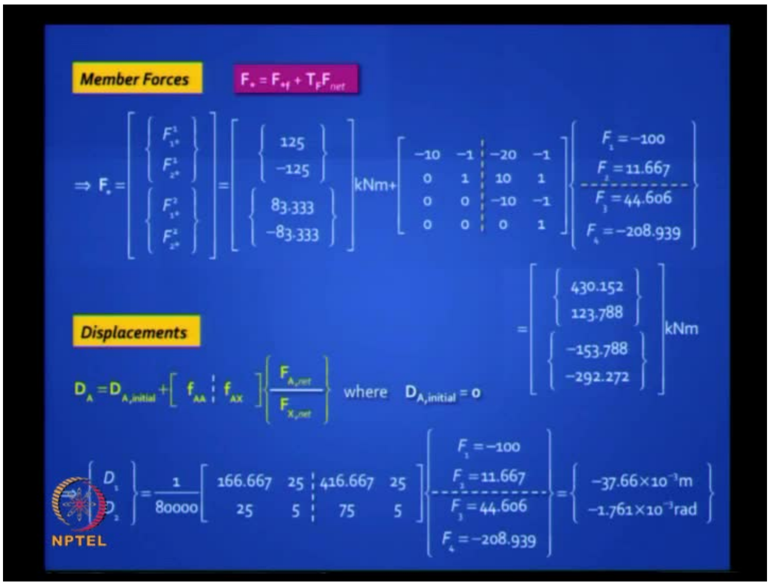

Now, last thing you need to get the element level forces - actually you do not need to do this (Refer Slide Time 26:28); you got the free body but since we are doing matrix method, we might as well do this the hard way; but you need not. You got the free body, you draw the bending moment diagram there itself.

But let us just play this game and if you take that F F star vector which you got the T FA T FX vector and you put in the net values of F 1 F 2 F 3 F 4, you will get those answers and you can also get the displacements if you want. You will get D 1 and D 2 perhaps D 1 is useful because you know how much it will deflect; it is going down and in actual practical problems, you need to control deflections. Usually in fix beams, deflection is not a problem but in simply supported cantilever beams, it can be a big problem.

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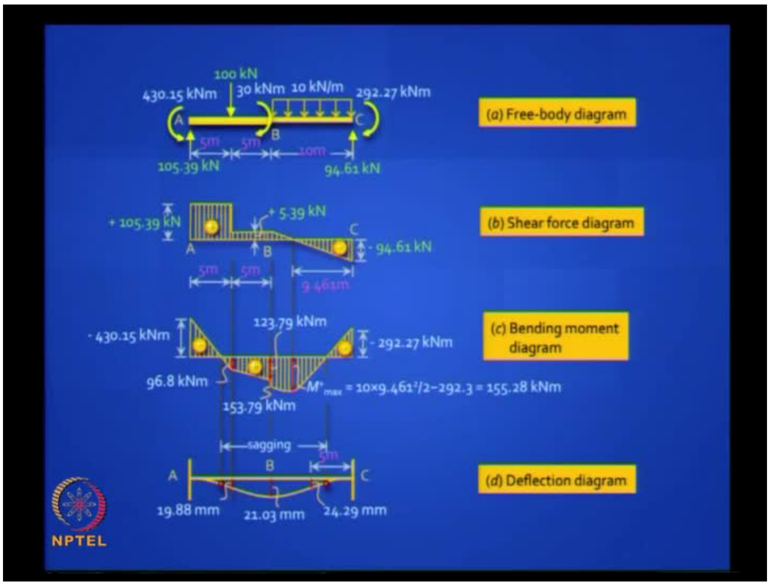

When you draw your diagrams, we have done this before, we would not waste time. Problem number 1, fix beam, non-prismatic we have solved. Clear? We will do another problem.

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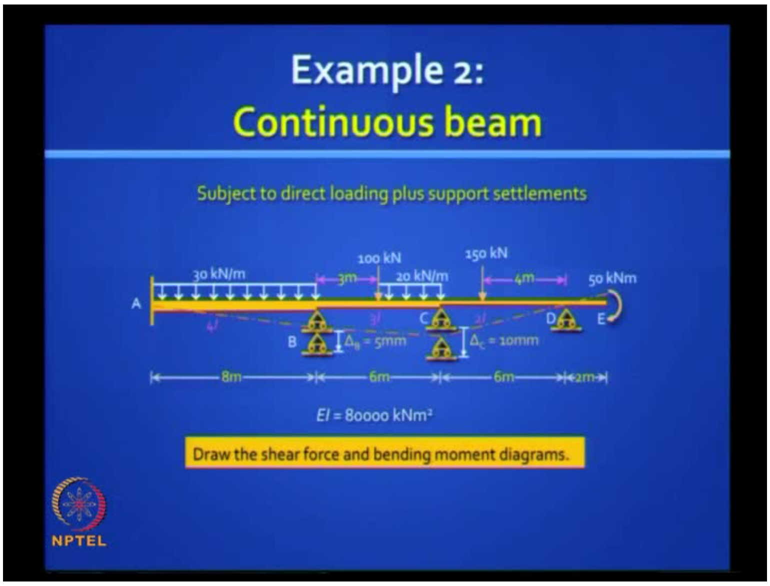

Continues beam - this is the toughest problem you can get in the sense that it has gotten all kinds of complication. What is the first thing we will do? First we will simplify work.

So, we will get rid of the overhang, because it will unnecessarily bring in more coordinates we do not want; we want to minimize our work.

So, you do that; but still, even after doing that, what is the degree of static indeterminacy? 3. Just think of the primary structure; what primary structure is most comfortable for you? Cantilever. So, cantilever means 3 props you have to throw away. That is the easiest way to understand; but why should it be cantilever? We can make it simply supported, you can do anything you want; but let us do it the way we are comfortable with - we will make it for cantilever. So, n s equals 3 and let us choose a support reaction at B C D, as a redundant, we will called them X 1, X 2 and X 3.

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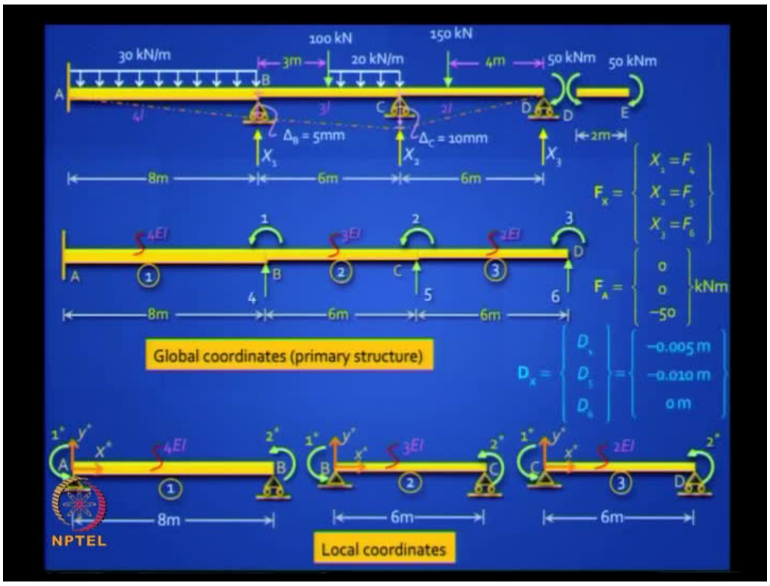

So, this is what, this is the structure we are going to look at. We will quietly remove the overhang and bring it back when we draw the final bending moment diagram; and that 50 kilo newton meter now goes as a concentrated moment at the end D. How do we proceed now? Well, first global coordinates - global coordinates, you have six of them here; that is the minimum you have, because you have 3 rotations on top - 1 2 3 and you have these three redundant are 4 5 6. Is it clear?

You can number them any way you want, but because we are following, normally we would have put the vertical translation as 1 and the rotation as 2; but here, this flexibility method, we are bias to forces not to displacements; and we have said that all the redundant we put at the end of the list. Is it clear?

So, the redundant are 4 5 6; they have to go to the end of the list and the active coordinates are 1 2 3. Is it clear? And straight away, write down the nodal forces; you have only one nodal force. W F 3 is this constant moment, which is minus 50 kilo newton meter; write it down and your F X vector is X 1 X 2 X 3 which is connected with F 4 F 5 F 6. Write down; make the framework ready to do analysis.

If two students choose different numbering systems, not a problem; you should still finally get the same shear force and bending moment diagram. The numbering, the naming can be different.

So, you also have support settlements here at the redundant coordinates; that is, you have D 4 and D 5, minus point naught naught 5 and minus point naught 2 0. I would like you to think of what you need to do; I am not going to solve it here, but that is something for you to worry about. Supposing I had a rotational slip, theta A equal to point naught naught 1 radian. How do you deal with it? So, please note down, that is the question you will get in your examination. If in addition to this, just formulate the solution. If in addition to this, for choosing the same redundant, if I had a clockwise rotational slip at A, of point naught naught 1 radian, how would you solve the problem in addition to all the loads that you have? So, think about it; we have done similar problems earlier; but I want you to figure it out yourself.

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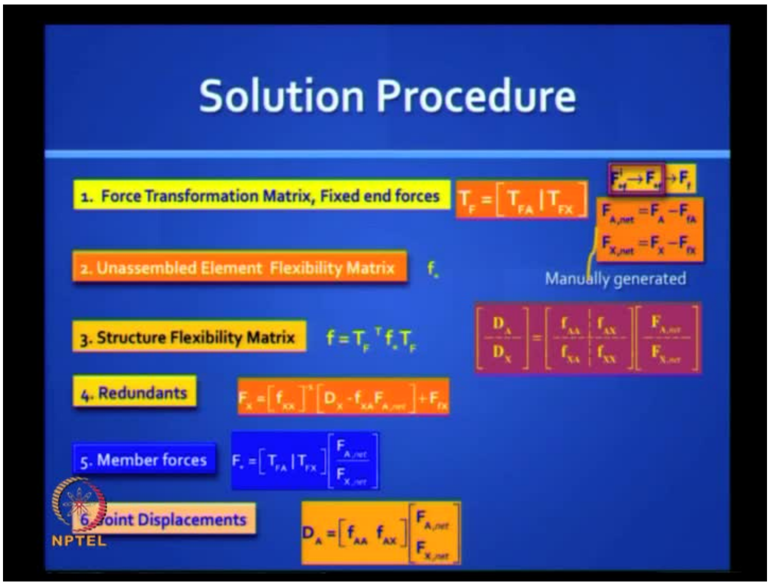

Now, you have three elements, 2 degrees of freedom per element; the spans are different - 8 meters, 6 meters, 6 meters and the E I values are different - 4 E I, 3 E I and 2 E I. What is the first thing you need to do? Force transformation matrix, fixed end forces; you can do which ever you want first. So, that is the first thing you need to do; either do this or do this.

Next, find out the flexibility matrix at the element level, find out the structure flexibility matrix, solve for the redundance; find the member and forces, find the joint displacements; standard procedure.

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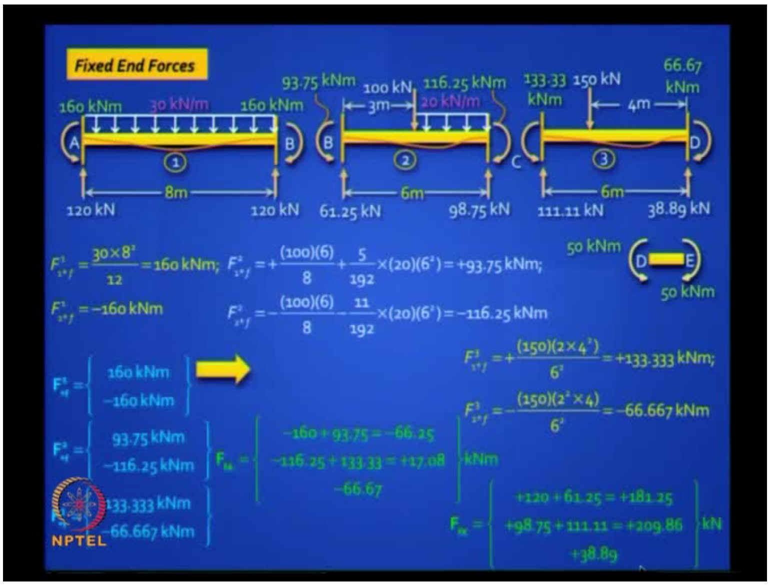

Fixed end forces - we did this, in fact, only yesterday. So, this is easy; this is also easy. This, be careful; yesterday, what was the difference? We did the same things yesterday, but there was a big difference between yesterday and today; what is it? No, that was the prop cantilever; element 3, we took advantage of a reduced element stiffness matrix, because of the hints and we had only a 1 by 1 matrix and it was 3 E I by L.

We do not play those games in flexibility method. Flexibility method is always 2 by 2 elements. Is it clear? So, you have to arrest the degree of freedom D. So, it is a simple case of fixity at C and D; and so your fixed end force is, as you calculated, this is what we did in the conventional stiffness method. We did not take any shortcut. Is it clear? So, that calculation is easy to do. 50 kilo newton meter you just keep aside; it does not come into your fixed end forces at all. You are handling it as a nodal load, all you put it as F 3 equal to minus 50; you handle that separately.

So, I hope you are familiar with these calculations. Then, what do you do next? You put them all together in a vector form; it will look like this. If you compare this problem to yesterday's problem, this vector is identical except that last F 3 star f vector because last time, it was having only one value; that value was different from these two because it dealt with the prop cantilever. Remember?

But we use these very values in the conventional stiffness method if you go back, but in the conventional stiffness method, we also had a fourth element that is overhang, which did not have any fixed end force vector. It is good to look at the same problem by all the methods; you can get really thorough and you can pick and choose which ever method you want. What do you do next? You have to assemble the F A and F FA and F FX vectors; will you do that? With the help of this diagram, write down the F FA and the F FX vectors. Just have to add up, please do it, so at least you know the steps and the procedure; fill up the F FA and the F FX vectors. Can you tell me the 3 values in the F FA vector? You have 3 moments I think, the first 3 of moments and the next 3.

So, are you getting these answers? In fact, the solution is shown there; what you have to add - you just have to algebraically add. Is this correct? Do you need explanation? Yes Praveen, what is your doubt? Last one, you are talking about F FA, F FA the last one is, you have a clockwise fixed end moment here - 66.67 and nothing else. This is just, you need not show here, they should be thrown away.

So, that is all you have; it is a minus because it is clockwise. Actually, I should not show this, but this is a carryover effect from the previous. It is already dealt with, that cantilever, that overhang is already dealt and I hope you are familiar with how to get F FX; it is also straight forward.

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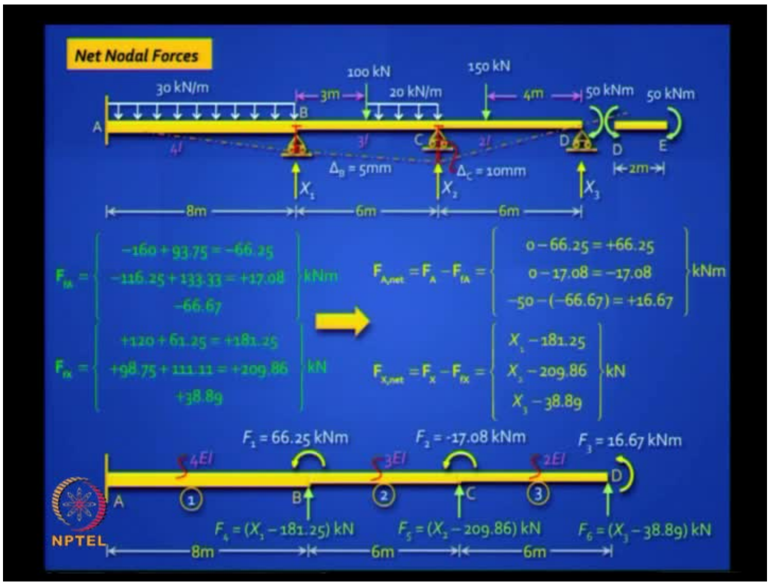

Now, you are ready; you have got this, I have just copied that diagram. What do you now need to do? You need to get the net load vector, which is F A minus F FA and you can also get F A minus F FX and keep it ready to solve your compatibility. Will you do that, net load vector?

When we say load vector on a primary structure, it also includes the unknown redundant; they also form a part of the loads, but those loads are unknown loads because the redundants are not known. So, it is a systematic method, fool proof method, you do it step by step; do not make any mistakes anywhere; you cannot go wrong. Of course, we have not yet found out the force transformation matrix; we will do that in a minute.

So, are you getting this? You got the F A vectors - 0 0 minus 50, you got the F X vector - X 1 X 2 X 3, you have got F FA and you have got F FX; it is just algebraic addition. No problem, then what is nice to do next? Take a look at those in the diagram; you see how beautifully you have converted all those intermediate loads into equivalent joint loads.

That is the power of matrix methods and you get the same displacements at B C D that is the basis of the equilibrium. But there are some unknowns hanging around - X 1 X 2 X 3 and to get those X 1 X 2 X 3, see this is a free loading diagram which does not suggest that B C and D cannot move; in fact, B C and D will move, but the correct answer X 1 X 2 X 3 correspond to the reality that you know, how much B C and D will move. It is not 0, because you have a known support settlement at B and at C. So, that is the value you have to forcibly impose and you will get the correct values of X 1 X 2 X 3.

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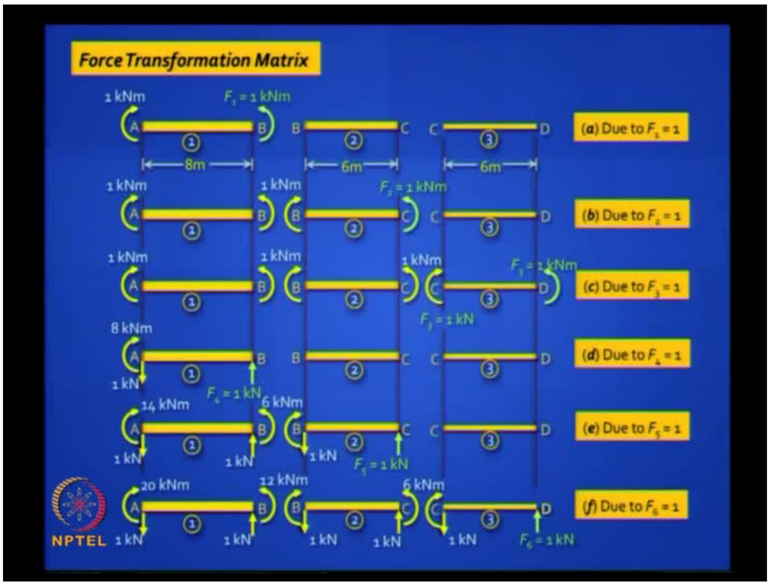

So, let us do this; now, you do it on your own. Please generate the force transformation matrix for this 3 element continues beam. Very easy the first one, I mean you have to repeat what we did last time, because all of them look alike. Will you draw, fill in the blanks there quickly? First one, second one, third one, they are all similar.

While you do it, let me ask you a question? When we did this solution by the consistent deformation method, we did some extra work. What was the extra work we did? We had to draw the unit load bending moment diagrams m 1 m 2 m 3 and so on, and we also had to draw the capital M L diagram due to the load. We are not doing any of that.

How are we avoiding finding the m 1 m 2 m 3 diagrams? Here, we are doing actually the same work because in the force transformation matrix, you are capturing the 2 n moments and you are assuming a linear variation between them. So, you are capturing the same essence in a matrix formulation.

So, it is easy to calculate, you will get all this. So, you will find the unit bending moment diagram, unit load bending moment diagram will either be linearly wearing or will be constant. If you are applying unit moment, you get a constant moment in element 1; if you are applying a unit load force, you will get a linearly wearing bending moment diagram.

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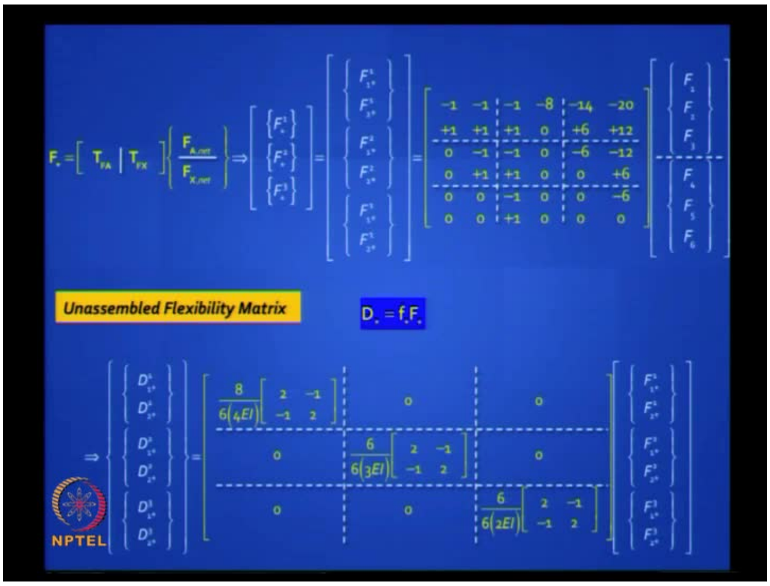

That is all automatically captured by F 1 star and F 2 star; so, you must connect what you are learning now with what you did earlier, but this is a blind, powerful method of doing it. Can I proceed? If you plug in all these numbers, you will get that matrix which will look like this.

It is similar to what we did earlier; earlier we had only 2 elements, now we have 3 elements. Not much difference. What do we do next? You got the T F matrix, T FA and T FX; what is the next step? The flexibility matrix.

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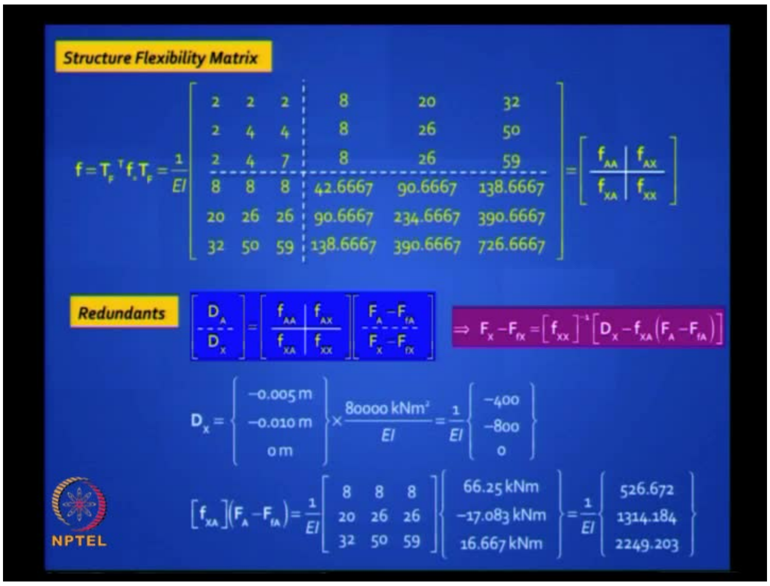

So, earlier we had a 2 by 2; now we have a 3 by 3; by the way, when I say 3 by 3, it is actually 6 by 6, because of each of those elements has a 2 by 2 matrix; all you need to know is, what is the length? What is the span? 8 meters in the first case, 6 meters in the second case, 6 meters in the third case. What is the E I value? 4 E I in the first case, 3 E I in the second case, 2 E I in the third case. That is all; just plug it in. So, you got the F star, you got the T F, what do you do next? Get F; do not even waste time, you get it.

You get F and just check that it is symmetric, because that is and the lower right hand corner holds a secret; that is you are and make it as precise possible; that is why I put 4 decimal places there. You know I put 6 significant figures, because that is a fellow you need to invert; if you round off heavily there, the inverted matrix will get you some answers but when you multiply it with this matrix, you would not get an identity matrix unless the accuracy is high; you are not doing it, the computer is doing it. So, you can do it, you can do double precision if you wish. You will get very good result. Please note - unlike the stiffness matrix, the flexibility matrix is not guaranteed to be well conditioned.

It has been proved that in some cases, depending on the choice of redundant, it is reasonably well conditioned, but sometimes, it can be ill conditioned. If it is ill conditioned, when you invert it, you would not get a good inverted matrix. When you check out the product of the inverted matrix and the original matrix, you will be in for big surprises; you will not get an identity matrix, you will get a full matrix with all kinds of numbers in it. So, your answers will be all wrong. This is one of the reasons why the flexibility method as a general method for large structures is not chosen.

But stiffness method is guaranteed to give you a good solution; it is a robust method; it does not have any choice unless you do reduced element stiffness method, which the software programs do not do; they do conventional stiffness method; a statically determinate truss which has degree of static indeterminacy equal to 0 will be analyzed as a highly kinematically indeterminate structure. The computer will solve 100 by 100 matrix inverted successfully and give you the solution which you could have solved manually in a minute.

But, you need to check, see a good engineer will use whatever software is available but as a tool, the engineer is the master and you must able to check because sometimes you get ridiculous solution.

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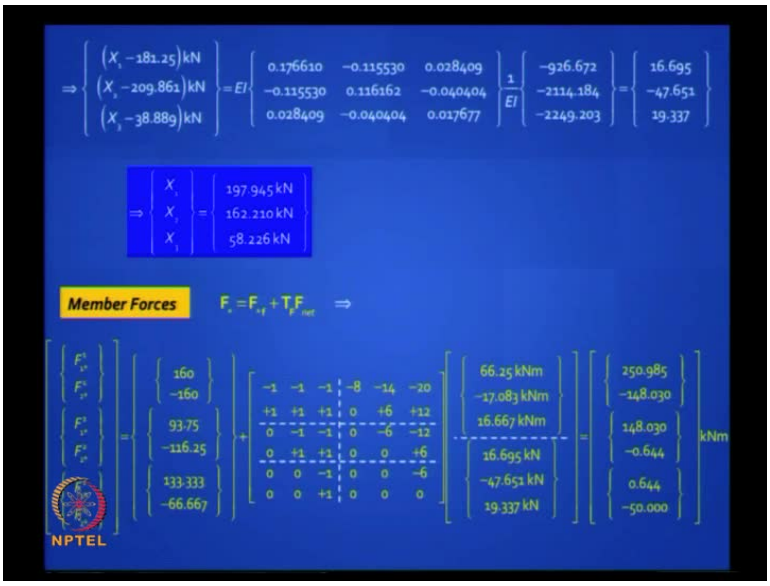

So, you can solve for this and get X, X 1, X 2, X 3 and if these numbers look familiar to you, it is because you have done this problem about two, three times and find the member end forces. Just invoke those equations; fixed end forces you have already got, T F matrix you have already got.

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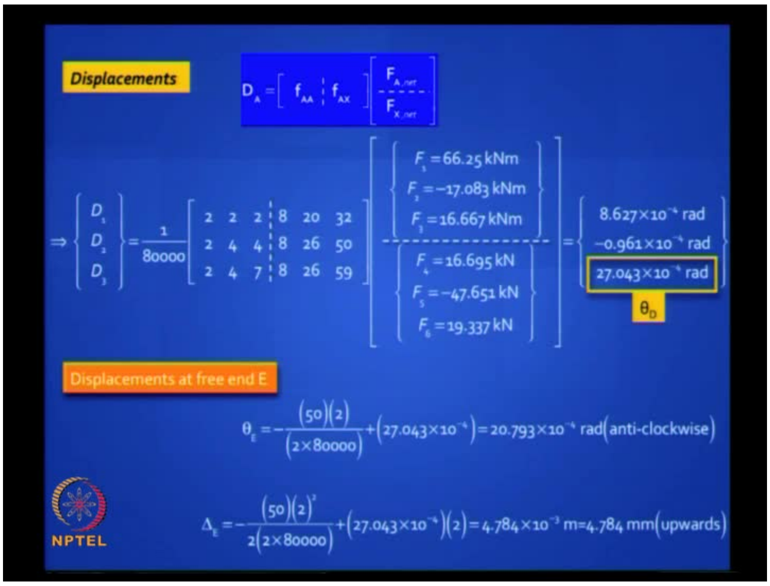

The displacement, the net load vector you have already got. Just multiply and add; you will get the final answers. What are those final answers? They are that end moments in, final end moments including the fixed end force vectors in all the 3 elements. Once you have that, if you draw the free body, you get the vertical reactions; you also put in the external loads and you can draw the free body. You can also find that displacements if you want. Now, in a problem like this, which displacement you think will be of interest? This is a 3 span beam with an overhang; which do you think will be interesting? The overhang part, you would like to know how much it went up; but you are not getting it here.

So, how do you deal with yes how do you find it out?

Get the rotation at

D 3

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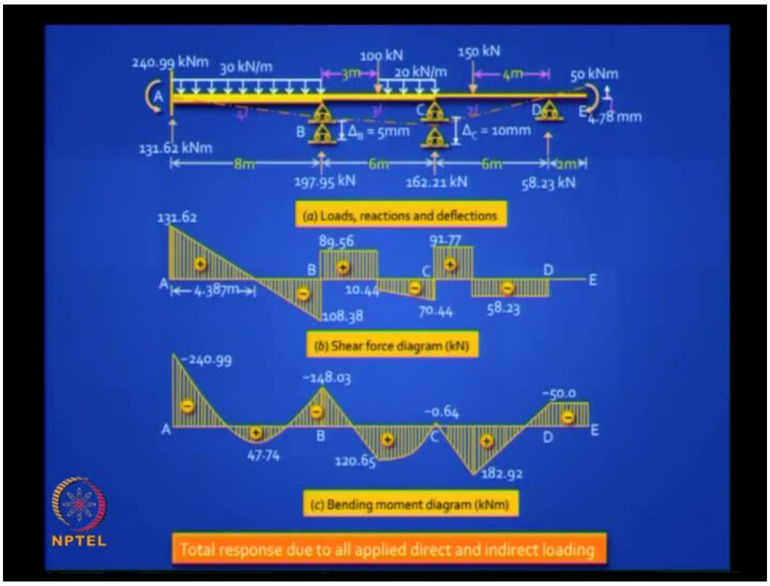

D and then multiply it by the, good you guys are smart. So, that is what you can do; you can find, knowing theta D, you should understand that the third displacement theta D, you can just multiply by the span; you can get delta E. You can also get theta E because it is a rigid body movement. Except for that 50 kilo newton effect for which you have a standard formula; so you have to add that. You can do that and this is your final answer. We have done before. So, is it clear?

So, we have really covered lot of ground; we finished. Please note that we have actually finished the stiffness methods and the flexibility method; we did only two problems in flexibility because it is not really used much in practice.

But we have studied all the methods exhaustively. Only one topic remains to be completed. What is that? Grids, which is very interesting and in your assignment I have given a grid problem. I will actually show you how to solve it, but you solve it on your own. Thank you.