Advanced Structural Analysis
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Module No. # 5.4
Lecture No. # 30
Matrix Analysis of Beams and Grids

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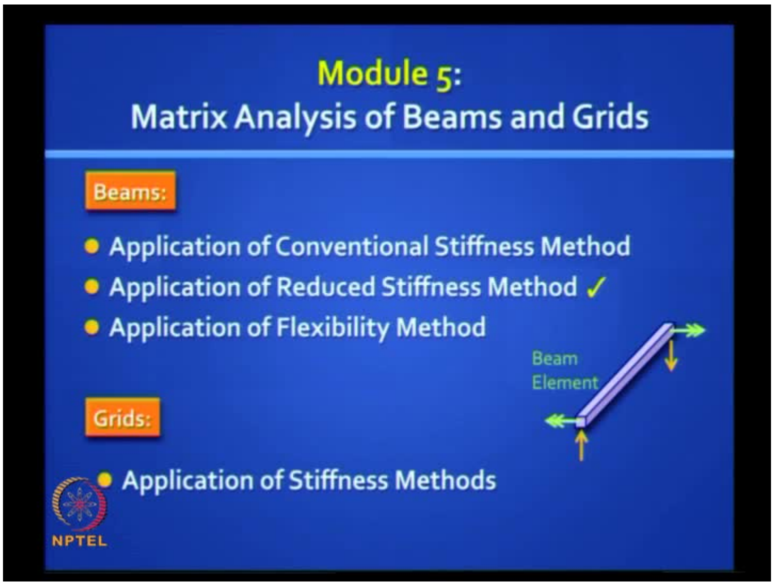

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Good morning. This is lecture 30, module 5 - we continuing with matrix analysis of beams and grid. So, we will complete the reduce stiffness method, which we started in the last class. For the beam element, and in the next class, we will take up flexibility method for the beam element, and after that, we will do again reduced stiffness method for the grid element and the conventional stiffness method. This is covered in the chapter on beams and grids, in the book on advanced structural analysis.

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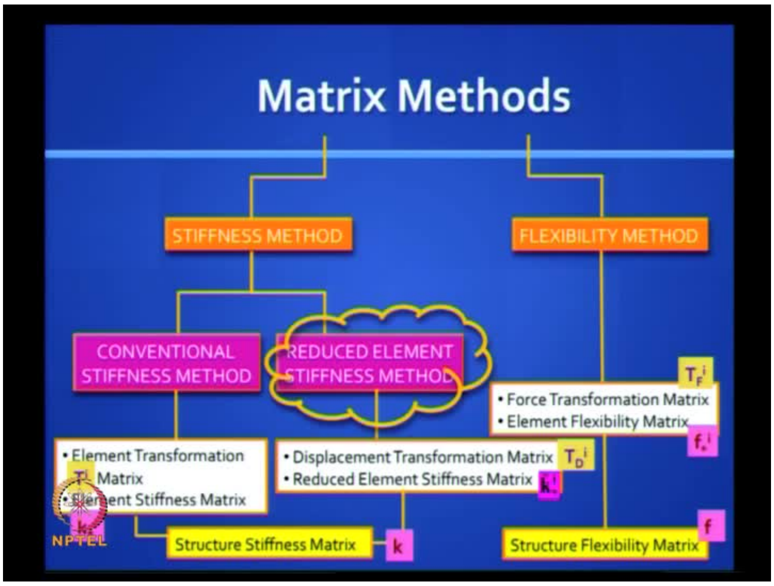

Of the three methods, we are now looking a reduced element stiffness method.

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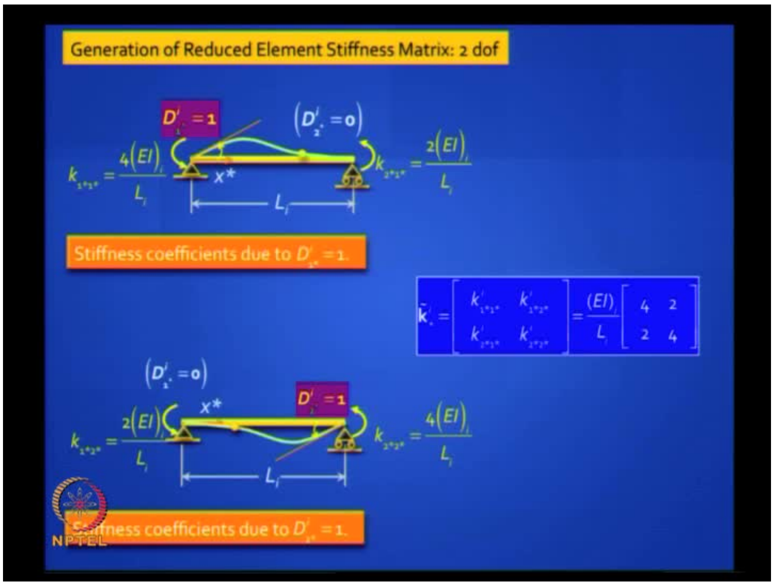

If you recall we learnt how to derive the stiffness matrix using various techniques, the easiest is the physical approach; that matrix is very easy to remember, 4 EI by L 2 EI by L 4 EI by L 2 EI by L. The diagonal elements are 4 EI by L; the half diagonal elements are 2 EI by L; they are all positive and you physically know the meaning of each coefficient in that matrix.

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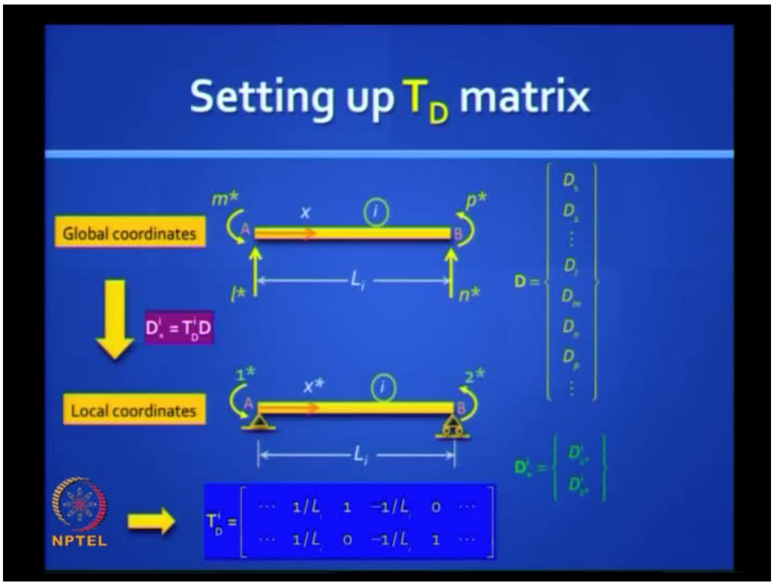

We also learnt how to setup the T D matrix. And the real problem is, when you have to convert deflections in the main structure in the beam, to a reduced element stiffness coordinate system at the local level. And we said, this can be done by converting the chord rotations to equivalent flexural end rotations.

So, if you have a chord rotation of 1 by L corresponding a unit displacement, you can treat that as an equivalent end rotation, but remember you have to reverse the sign; that means, if you have a clockwise chord rotation, you should have an equivalent anti-clockwise chord rotation. This is how you convert from typical global coordinates like m l m n p to the two coordinates: 1 star, 2 star; we have done this in the last class. But if you found this little difficult, there is an alternative way of arriving at the same derivation.

How do you do that?

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No, not the, you have to bring in the transpose, you have to invoke the contra gradient principle and some of you might find it easier to do it this way.

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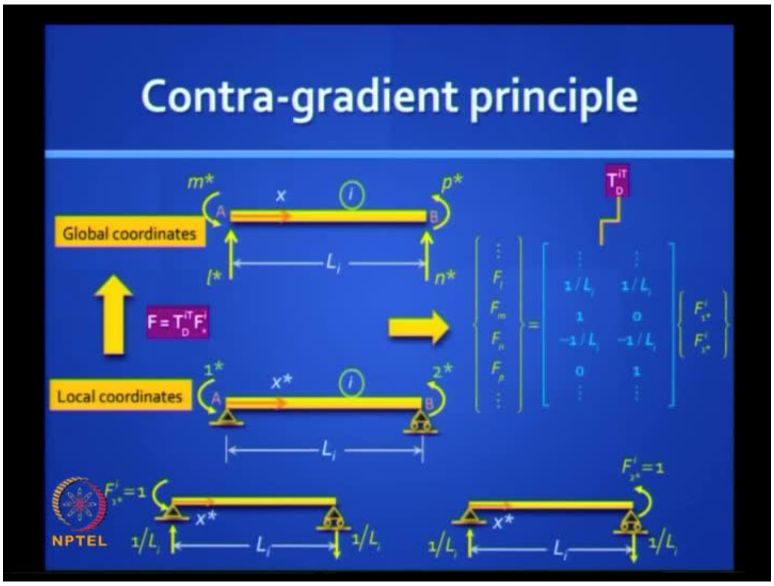

So, here, we do the reverse - we move from the local coordinates to the global coordinates; and we invoke the T D i transpose matrix, which means you apply a unit, moment - end moment - two of them, one at a time like this. You apply F 1 star equal to 1 satisfy equilibrium and figure out what are the corresponding values that you get in the global coordinate systems; next, you apply F 2 star equal to 1. And you can see that, after two end moments, only one is non-zero and it has value of plus 1, and you get two reactions, you can see very clearly - it is 1 by L.

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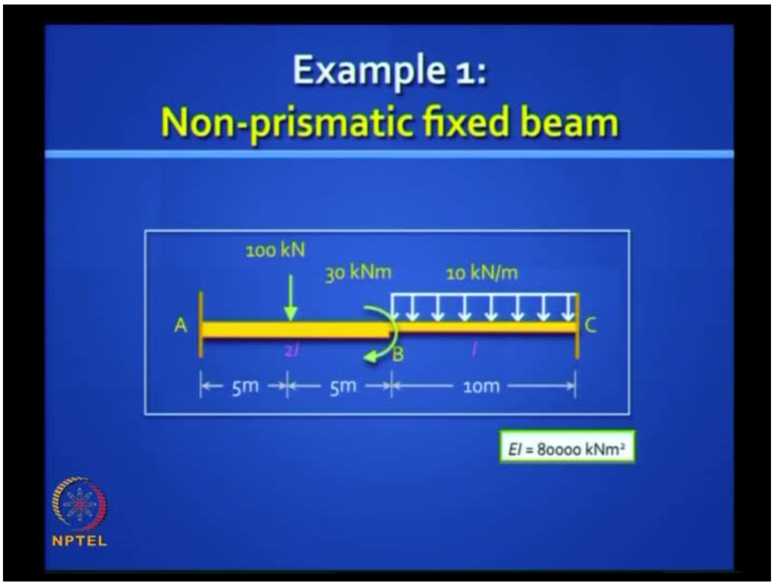

If it is pointing upward, it is positive; if it is pointing downward, it is negative; and you can very easily generate this matrix, you get it. So, this is another powerful way of generating the same transformation matrix, but we are getting it from the transpose. But my suggestion is, do it in the classical way - the T D approach using the displacement transformation. Now, you recall we did this problem in the last class. And we will now look at more problem, we will try to do three quick problems in this session; with that, will close the reduced element stiffness method.

We need to worry about how to deal with moment releases. There are two kinds of moment releases that you can get: one is when the far end is a hinged or roller support; the second is you have an internal hinge. Either way the bending moment in the beam at that hinge location is known, usually it is 0, unless you have an externally applied known value of a constant moment.

So, we have done this, we have taken advantage in terms of reduced kinematic indeterminacy, when we applied the slope deflection method and moment distribution method, remember. So, we can do the same thing; that means, here we just ignore that global degree of freedom all together; we do not even give it a number. If you have a hinge, whether an external hinge or an internal hinge, do not assign a global coordinate active or restrain at all, just keep it; we will see how to do that.

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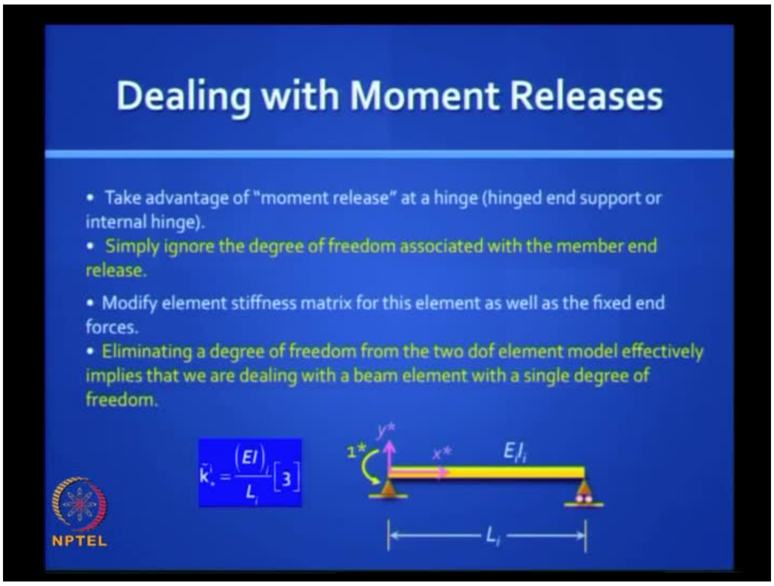

And modify the element stiffness matrix, you know that, 4 EI by L will now become… 3 EI by L, when the far end is hinge; so, that advantage we will take care of. And you also need to modify the fixed end forces, because normally you have the formulas for a cantilever beam which is fix. If one of the ends has a moment release, it becomes a prop cantilever and you know what to do, we have studied that.

And the big change that you get is, in the conventional stiffness method, how many degrees of freedom did you have? 4, you dealt with the 4 by 4 element stiffness matrix; that reduced to 2, when we did the reduced element stiffness method, where we did not take any shortcuts, that will now further reduce to 1. Remember, we had 4 EI by L and 2 EI by L, and then we had this 2 by 2 matrix.

Now, we do not worry about rotations at the other end - at the hinged end - and so you have only 1 by 1 matrix, just 1 coordinate; the other end is hinged. So, that is a major advantage we are trying to take in this method. You reduced a degree of kinematic indeterminacy and you know that the element stiffness matrix is 1 by 1, and the coefficient is 3 EI by L, is it clear? We have done exactly this in slope deflection and moment distribution method, when we took this simplification. So, we will see how to do this.

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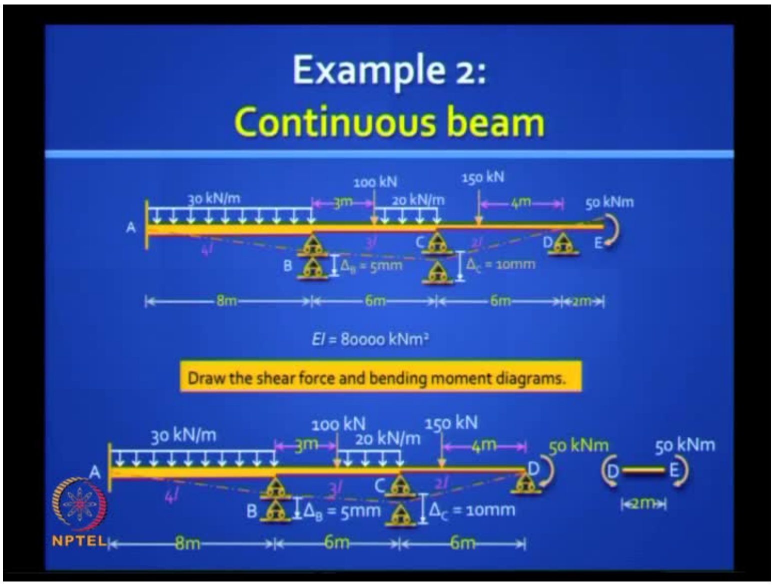

Let us take this problem, we have done this problem earlier? Yes, I think so, in matrix methods or in moment distributions slope deflections, one of them, anyway. So, how do you or did we do it in the… I think we did the conventional stiffness method, remember. What was the degree of indeterminacy? We did this, yes.

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Not 3, it was 5, because theta B was D 1, theta C was D 2, theta D was D 3, then delta E - we did not take any shortcuts - was D 4, and theta E was D 5, remember. They were five active degrees of freedom and they were also five restrain degrees of freedom, we had a 10 by 10 structure stiffness matrix; the k A was 5 by 5, remember that?

Now, we are going to have a massive reduction in kinematic indeterminacy. So, what should we do first? Because this is aimed at manual analysis, so we do not want to solve 5 by 5 matrices. The first thing you can note is, you have an overhang, which you can just get rid of it, because that reason is statically determinate.

So, that is the first thing you do; you separate it out and you bring it back, at the end when you need to draw the bending moment and shear force diagrams. So, when you remove it, you know that, if you have a clockwise moment of 50 kilo Newton meter at the free end E, you have an anti-clockwise moment at D. And Newton’s third law says, the same moment will act in the opposite direction in the segment A B C D at D.

So, you are now applying this as a nodal moment, except that it is not really going to a global coordinate, we will see, we will explain that shortly. Now, tell me we have got a reduced problem now - just a 3 span continuous beam. What is the degree of kinematic indeterminacy in this? Is just 2, if you want to take advantage of the hinge at D, right.

So, from 5, it is a massive plunge to 2; so, that is a clever thing and we should do it correctly without making any mistakes; so, please pay attention. By the way, we did this in slope deflection method, we did this in moment distribution method, we will now do it in matrix method.

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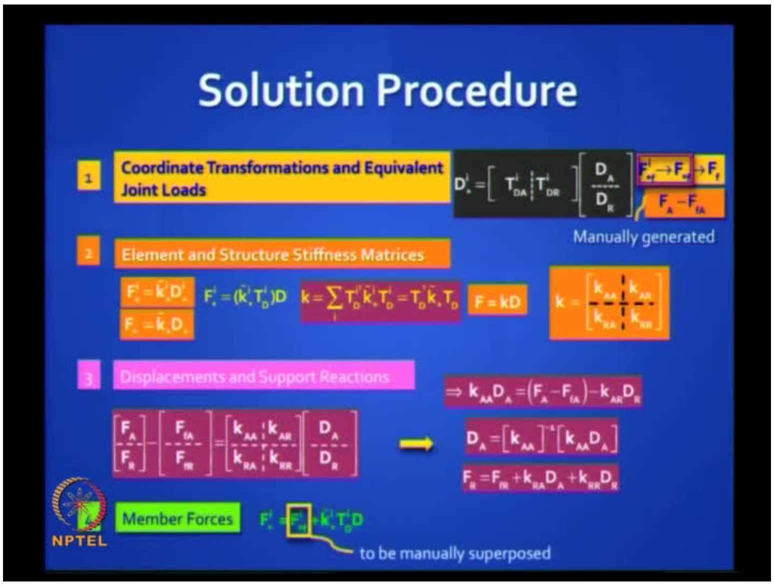

So, the first thing is, the procedure is quite clear. You have to do the coordinate transformation, write down your displacement transformation; matrices - you have to figure out the fixed end force vector, you have to do this manually, because you have a problem in the reduced element stiffness method, because you do not have a constant fixed end moment, it can change. So, you have to do it manually; you have to get the net resultant force vector F A by F fA.

Then, of course, you generate the stiffness matrices in the usual way and you solve the equilibrium equations, find the unknown displacements, use the disclosed active displacements to find the support reactions, at the end you find the fixed end forces. And if you recall all previous discussion that last equation what is it really represent in terms of what we did earlier, they are nothing but slope deflection equations; remember, I said m a b is m f a b plus 4 EI by L.

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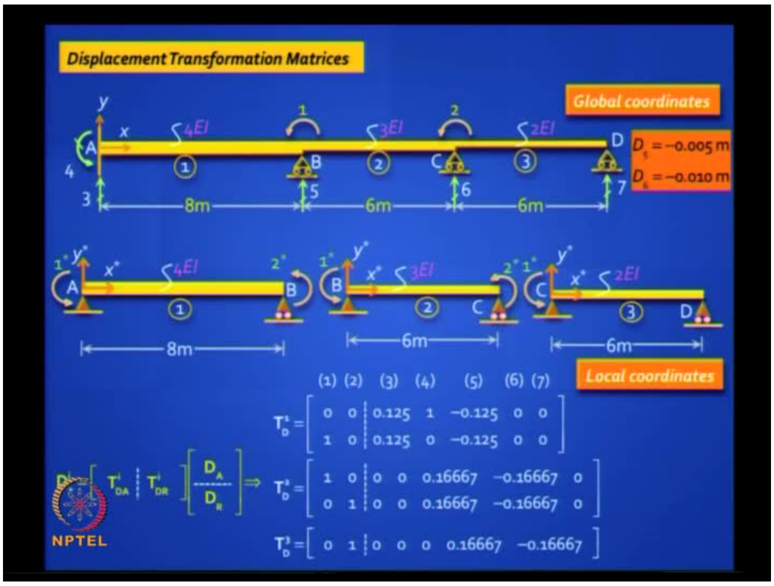

So, all that, it is a matrix formulation of slope deflection equation. So, let us proceed, the first thing you need to do is find the coordinate transformation. So, you got global coordinates here; we agree that, we will have only two active degrees of freedom: 1 and 2. At the end D, what should you do? Leave it in piece, do not a put curled arrow there and give it a number, because we are not interested in that rotation; we do not want to know theta D, because we know the bending moment at D is, in this case 50 kilo Newton meter.

We will bring all that later, but the first step is, if you want to work with reduce kinematic indeterminacy, do not include that as an active degree of freedom, but you have restrained degrees of freedom 3 4 5 6 7 - the usual convention will do that.

Why are we putting the restrain degrees of freedom? Because we want to find support reaction, that is a real (( )). Also if you have, in this case we also have that, what do we have? We have support settlements in this problem; so, you have to bring that in the picture. And let us look at that we have two support settlements, remember, B and C go down by 5 and 10 mm respectively.

So, you will label that as D 5 equal to minus 0.005 meter and D 6 as minus 0.010 meter. Next question, are there any nodal loads in this structure F A? You will find that there were only distributed loads and there was some constant load in the middle of that beam. You had a nodal moment at D, but it does not have a number, so how do you deal with that? You understand the moment at D, that becomes in a load which you have to account for in the fixed end force vector for the element three, that is the only catch; you have to do it carefully and then you can solve the problem.

And we did it in slope deflection moment distribution method, so let see how to proceed. These are the element level local coordinates, we have three elements; and for each element, we usually have two degrees of freedom expect for the third element. The third element has one degree of freedom; please pay attention, 1 degree of freedom and you have to be careful about that.

So, please write down the transformation matrix for… what will be the size of that matrix? It is going to look like this, at least get it right. So, for the first element, what is the size of the T D matrix? How many rows will there? Two rows for the first element. How many columns will there? There will be seven, because there are seven coordinates, of which two belong to T DA and five belong to T DR. Can you fill up? Similarly, you will do it for the second element; and for the third element, you can do it all in one go, the third element, it is a just one row.

So, you give it a shot very easy; if you apply D 1 equal to 1 in the main structure, what do you think will happen? Most of those numbers will be 0, except D 2 star for element 1, and D 1 star for element 2, that is it; so, I have given the solution here.

Let us go through it carefully, if I apply D 1 equal to 1 here, you agree that the second element in the… but we have 2 elements here: one is element in the matrix and you have the actual beam element. So, when I see element, I am referring to the element in the matrix; the second element in the first beam element will be 1 and the first beam first element in the second beam element will be 1 and the rest will be 0, clear.

Now, you apply D 2, that means, this one D 2 equal to 1; you agree this will get effected and this will get effected, which will show up as 0 0 0 1 1, clear. Next, we apply D 3 equal to 1, which means we are lifting up the left end A and restraining all the other coordinates; this will result in a clockwise chord rotation in the element 1 alone.

Clockwise chord rotation of magnitude 1 by 8, 1 by 8 is 0.125. And if you have a clockwise chord rotation, you end up with equivalent anticlockwise positive end rotations. And that is why, you write plus 0.125 plus 0.125 for the first element, and you write 0 0 0 4 for the other element and so on; so, I am sure you can derive for all the others.

Let us look at the last one; let us say D 7 equal to 1, if you lift up the last one, only element 3 will be affected, agree? Only element three will be affected; you have a clockwise or anti-clockwise chord rotation? You have a anticlockwise chord rotation and the equivalent flexural rotation will be clockwise, which means negative and it will be 1 divided by 6; that is why, you have in the last row, you have minus 0.1667, but all the others are 0, is it clear?

Can you all do this correctly without making any mistake? Do you have any doubts on this? This is the first step, if you get it right, you will get everything right.

Global coordinate 4

Global coordinate 4, yes.

Global coordinate 4 is, ya, yes. You have a unit rotation, it will affect only beam element 1.

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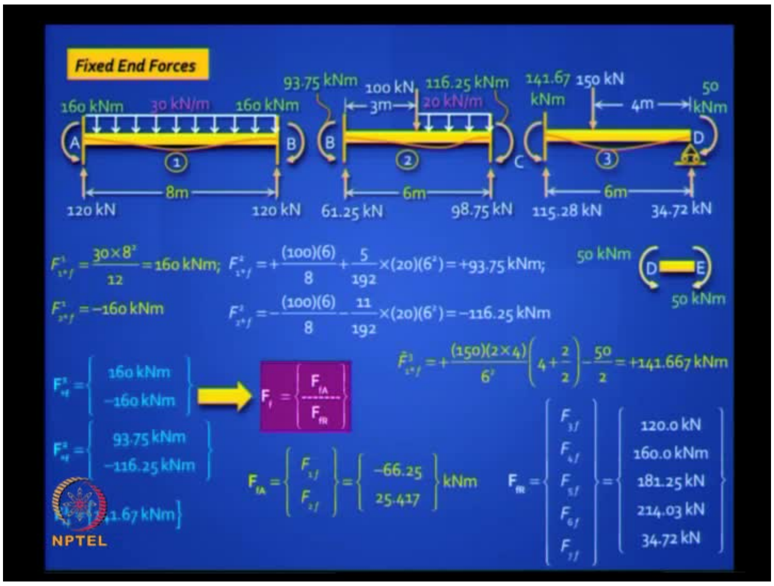

You have to be careful when you read these rows. This is 1 0 0 0 0; it is my fault I did not align all the rows correctly all the columns correctly, is it clear? No mistake, everything is fine.

We will proceed. Now, we have to find the fixed end forces; we have done this problem earlier, so can I go fast over this? You can work out the fixed end forces. Remember, you also need to find the vertical reactions. Why do we need to do that? Not only the fixed end moments, also the vertical reactions, because in the global coordinate, they pile on to those vertical forces in the coordinates you identified. So, you need to do that little extra exercise, but that is easy to do; we have done that earlier.

Take the second element, we have done this before. Remember, here the only thing to worry about is the formula, when half the span is loaded with the u d l, for which you may have to look up a table, because you may not remember it. Remember, it is 5 by 192 W L square, and 11 by 192, that is the only form, but we have done this earlier.

What about the third one? The fourth element, you do not worry, because you have already dealt with it. In the third element, you have to be careful, because the end D is now roller, and you are going to apply a 50 kilo Newton meter at that end D clockwise as a load; please tell me, what is the fixed end moment you get at C? Please calculate and tell me, because this is where you can go wrong, this value; that is F, for the element 3 1 star; calculate it and tell me, you have done this, when you did slope deflection method moment displacement; do it once more.

Well, due to the 150 kilo Newton load, for a fix-fix beam, you know the formula - W A B square by L square, for the left end, and for the right end, it is W A square B by L square; and you need to release that, so half of it gets carried over to this side; so, that part you can deal with. What about that 50 kilo Newton meter? Half of it gets carried over with the correct sign, so that is all you need to do.

I hope you have understood, do not go wrong here? Because we are taking shortcuts, shortcuts can be taken only by intelligent people, otherwise you get lost in the woods. But the work load reduces enormously, we have reduced the indeterminacy. Any doubts on this? We have done similar problems earlier; keep the science intact correctly. What do we do next?

We write down the fixed end force vector, you can put it all in 1 box or you can keep them separately as I shown here; that is, these are only the end moments corresponding to that degrees of freedom. Typically, two degrees of freedom per element except for element number 3, where you have only 1 degree of freedom.

From here, manually, you have to build up the fixed end forces, that act on the main structure. Finally, we want that, we want F fA and F fR.So, we can get that we need… so, this is the fixed end force vector in the global coordinates, F fA correspond to the 2 coordinates - 1 and 2, remember 1 and 2, 1 and 2 are here these two moments and these two moments.

So, you just algebraically add up those two moments, you will get F 1 f F 2 f. The net moment that you get corresponding to the fixed end moments at B and C; that is, F 1 f and F 2 f. And what is F 3 f, F 4 f? You can pick up from here, remember, this was F 3 f.

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So, that is plus 120, F 4 f is plus 160, F 5 f is a summation of 120 kilo Newton and 61.25, 181.25, F 6 f is likewise summation of these two and F 7 f is 34.72. It is easy to do, but you have to do it with your eyes open manually. Now, let us proceed; well, not really manually, you can also program it, if you do it skillfully.

So, you can find the net load vector, which is the nodal force vector; in this case, it is a null vector, there are no nodal forces in this problem, and F fA which we got in the last step. You have to put a minus sign, because you have to satisfy equilibrium. And you will find that, you are actually getting you have reduce that complicated problem; it was simple problem where you apply only two nodal moments in the direction shown. And this is equivalent to the original problem, in the sense, you get exactly the same theta B and theta C and that is the beauty of the displacement method; such a complicated problem, you reduce to a simple problem, where you have only nodal forces.

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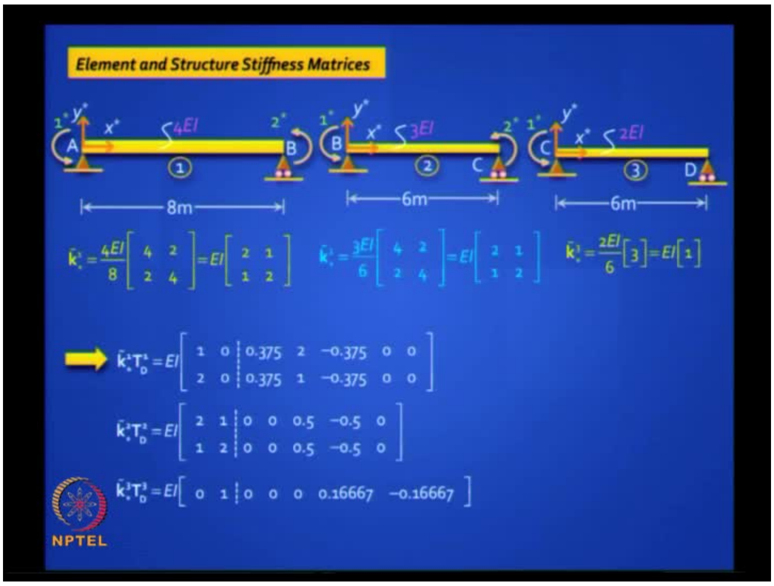

Next, generate the element and structure stiffness matrices; this is easy. Remember 4 EI by L 2 EI by L, so put the right EI, in this case EI is 4 EI; in this problem, L is… so easy to write. Next element, 3 EI and L is 6, so we see is very easy; please write down for the third element, because third element is different; there is only one, it is a 1 by 1 matrix; the EI here is 2 EI and the L is 6, but you have to it is 3 EI by L EI into 1, it is easy.

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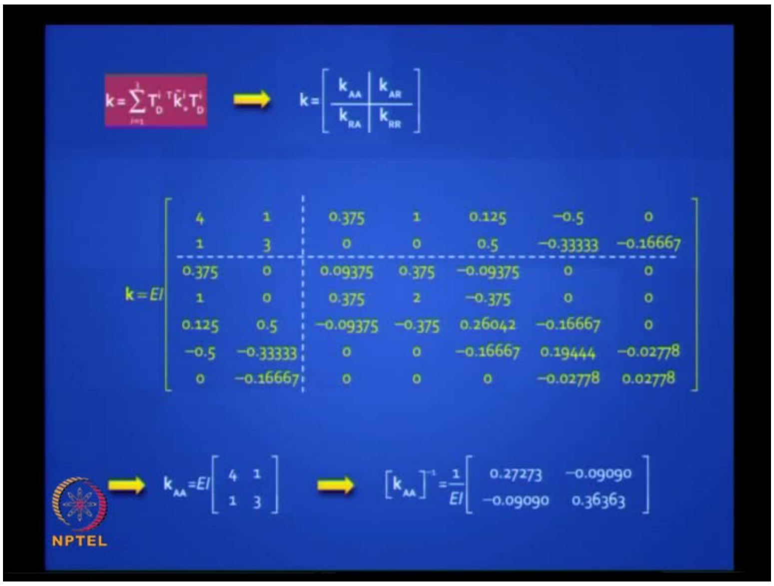

Now, can you assemble? Well, you first do this product; we have done this before, let us not waste time, you have got the T D matrix in the beginning; you understood how to do it, just pre multiply that with the k i star matrix, get this, then what do you do again? Pre-multiply this with T D transpose and they just add up everything, add up mechanically, no slotting you required here.

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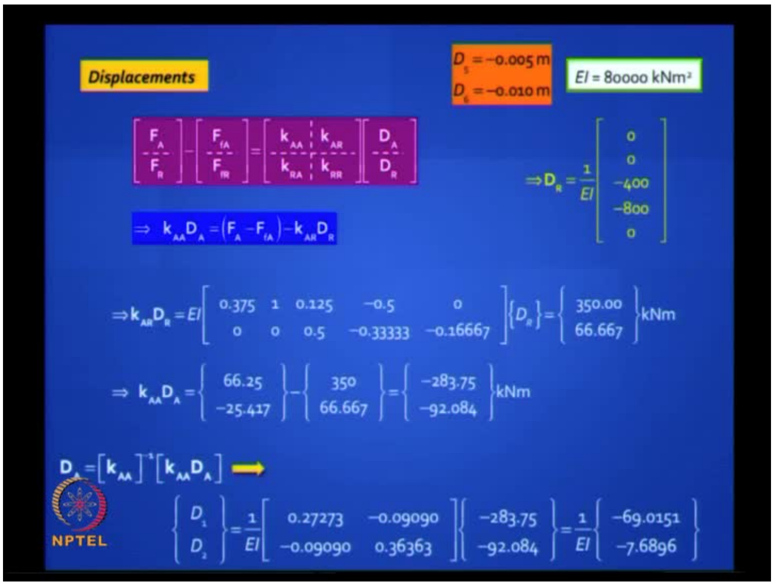

That is a beauty of this T D matrix; not to reduce, when you use T D even in the conventional, you get this advantage. And the top upper corner, on the left side, ease of k

AA and that is your real; you do not worry about the size of the whole matrix, it is 7 by 7; it looks scary, but we want to only invert 2 by 2. That is k AA, you can pick it out and inverting it is easy and you are ready to take any load, but the loads are also known, these equations are well known, it is always good to find out k AA D A directly. In this case, you have support settlements.

So, D R is given as, you know we did this earlier, minus 0.005 meter minus 0.010 meter at 5 and 6 coordinates, but EI is 80000. You have an option either to plug in 80000 in the stiffness matrix or you try to get rid of EI by converting these deflections in millimeters to equivalent EI terms, one side; in other words, if you multiply these two with 80000, you can write this as 1 by EI into minus 400 minus 800. In short, if you take this product, you will get minus 0.005 meter.

It is a clever trick to do, so that you do not you leave your stiffness matrix in piece, that EI gets canceled out. And there are five elements here, because there are five restrained coordinates; you know that there is no rotational slip or settlement anywhere else, so it is 0 0 minus 400 minus 800 0.

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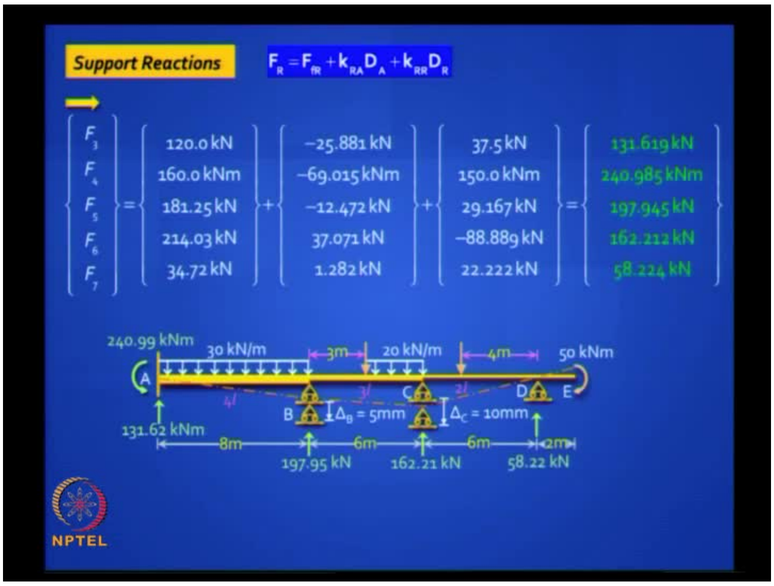

Now, you are all set to apply and solve; you first take care of the D R k R, you can do this step wise or you can do it one shot; I leave it you, finally, you get the answer - you get the unknown displacements. And we have solve this problem earlier by the hard conventional stiffness method, you get if you check up you get exactly the same solutions. Now, you need the support reactions; this is where you have to look at the big k RA and k RR matrices, which we generated in… but we done this in the conventional stiffness method, so this step is same to that, similar; there is no difference. So, you should get exactly the same answers, because only D A was unknown here, the rest we had derived earlier in the conventional stiffness method and you will find you are getting the same reactions.

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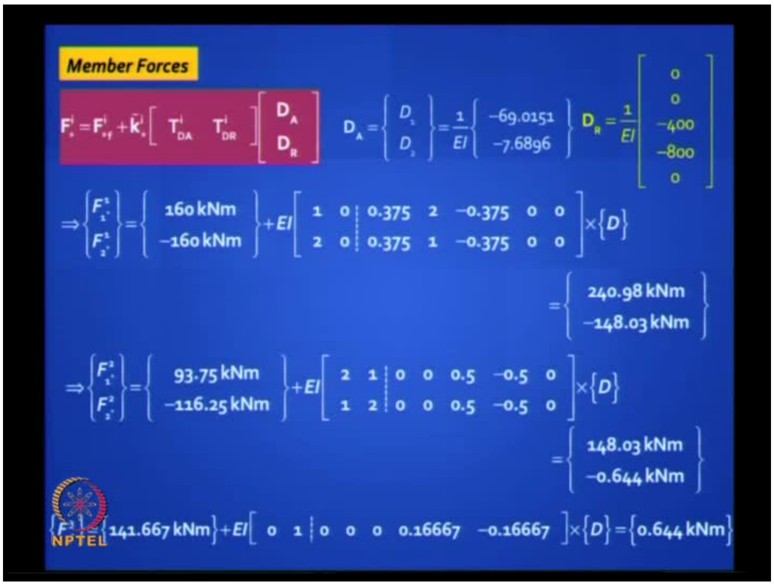

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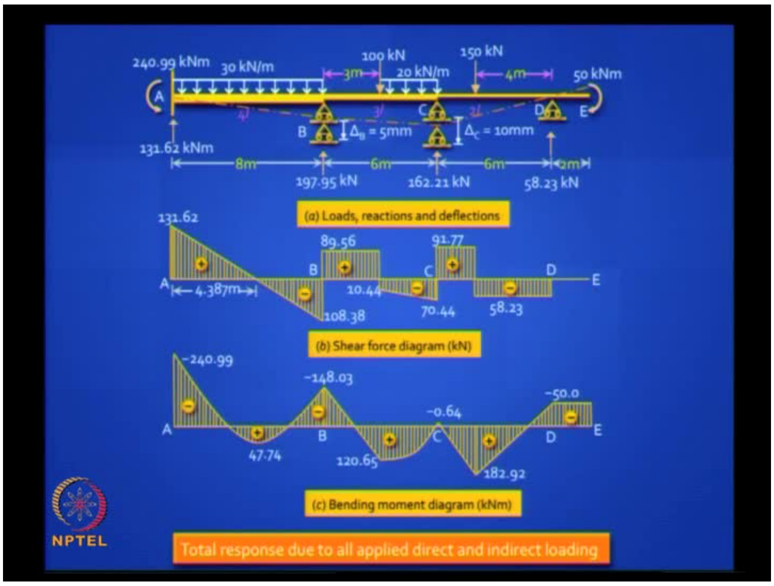

Last step, member end forces; so, remember those slope deflection equations. So, you note all the displacements, both the active and the restrain. and for the first element, plug in those values; second element, third element, you get the same answers as we did earlier.

Finally, you have to draw the same bending moment and shear force diagram, is this clear? It is a powerful shortcut method; instead of solving for five unknown displacements, we solved for only two; our interest was to get the final solution to design this beam. So, we need to know the bending moment diagram and shear force diagram and we have got it. We will do the same problem tomorrow using the flexibility method and you can decide which is the easiest to do.

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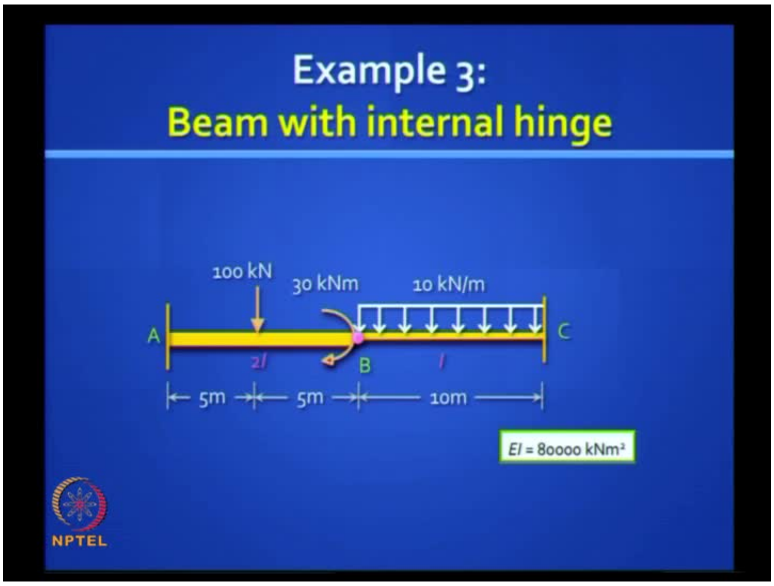

We have not done this in moment distribution or slope deflection method. How to deal with an internal hinge in the reduced element stiffness formulation? You will find that it is easier to do this here than in the conventional stiffness method, because there you have to artificially create a clamp and all that; here, life is easy, this is all you need to do.

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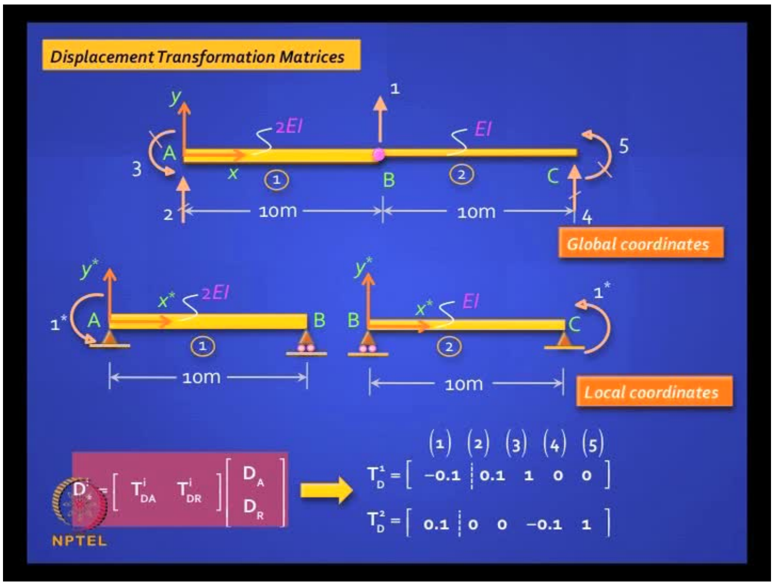

First of fall, whenever issue a hinge avoid putting any global coordinate their active or restrain, because we want to take a shortcut; so, remember, earlier we had a coordinate 2 here and we had a clamp there. Now, we just leave it in piece, there is a hinge there. So, that is what we have to note; there is no coordinate corresponding to rotation at B, because there is no unique value for rotation there; both the ends can move and you do not care with how much they move, because you just want the bending moment and shear force diagram. You are not looking for the absolute value of the rotations there, that is a real idea. Ideally, we should show a hinge also at B to remind us.

The other coordinates are clear? Two; so, the coordinate 2 now shifts here, 2 3 4 5, so one active degree of freedom, 4 restrain degrees of freedom - same methodology. This is beautiful, because you have only one degree of freedom for each of the two elements and the stiffness is 3 EI by L. Can you write down the T D matrix for this? It is very easy, one row for each.

Fill up this matrix, so it is going to look like this. Let us check it out, if I lift up B by unity and do not allow any other moments, then I have an anticlockwise chord rotation 1 by 10 for this element, which will give me… should it be minus or plus? Minus, because anti-clockwise chord rotation is clockwise equivalent flexural rotation; so, it is minus, and for this, I get plus, because it is the opposite, so that is how I fill up the first column.

Second column, how do I fill up? Second column is simple; I will lift up these, if I lift up these, I have a clockwise chord rotation for this; and nothing for the second one, so I get anticlockwise positive 0.10. Third one, this will give me 1 here, and 0 here and so on, and very clear, simple, easy to do, ones you get the hang of it.

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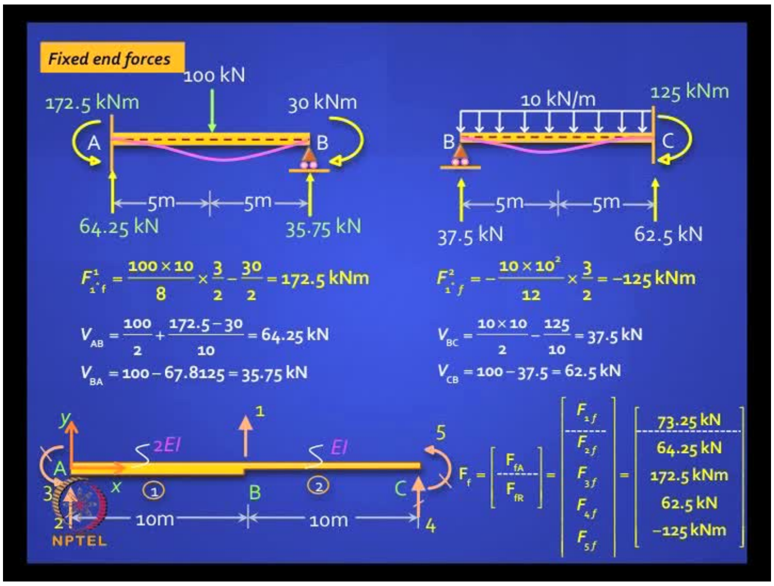

We will move ahead, fixed end forces. Here, remember, in there was a nodal moment 30 kilo Newton meter to the left of the hinge, so can you.. so you have to find out the fixed end forces and we have done this before, is this clear? Due to this 100 kilo Newton load, you get a fixed end moment here, which is W L by 8 into one and half times, because of the prop cantilever effect; I hope you remember that.

And it is anti-clockwise, so it is positive, but you have a clockwise end moment of 30 kilo Newton meter given as a load in the problem, but you are taking it at the element level and so half of it gets carried over with the same sign and it is clockwise, so you should put a minus sign here, is it clear? Once you calculate this, it is easy to calculate your vertical reactions. There is a reaction here, there is an arrow that should be here, you get the reactions; we did this earlier also.

Similarly, you do it for the next beam. There is a hinge, but there is no nodal moment at B; this is a straight forward problem, W L squared by 12 into one and half 3 by 2, find the vertical reactions.

Now, can you assemble the fixed end force vector? Yes. can you find out F fA and F fR? Well, it is easy, do it by inspection. Corresponding to this coordinate 1, you have to add up this reaction and this reaction, right; it adds up to 73.25. And similarly, you can fill all the other fixed end coordinates: 1 active and 4 restrain, is it clear? Actually the procedure is similar to the earlier procedure.

So, we do not have time to actually sit and write down all the numbers, but that is something you need to do, when you go back home and refresh your understanding, prepare for the exam but the procedure, is it clear?

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(( ))

We got fixed end moments at the element level, yes. You are going back to… you cannot do that in reduced element stiffness method, got it? You are right, you can normally do it and you should it in conventional stiffness method, but do not ever think of doing it in reduced element stiffness method, because reduced element stiffness method has reduced coordinates, it cannot handle this complexity; you have to do it manually.

So, you have to look through inspection, is it clear? Please remember, because this is a common mistake people might make, you cannot do it.

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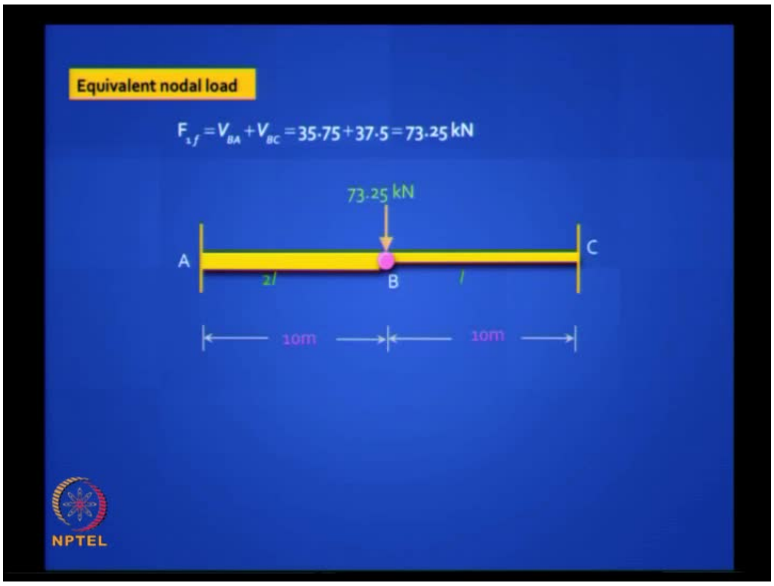

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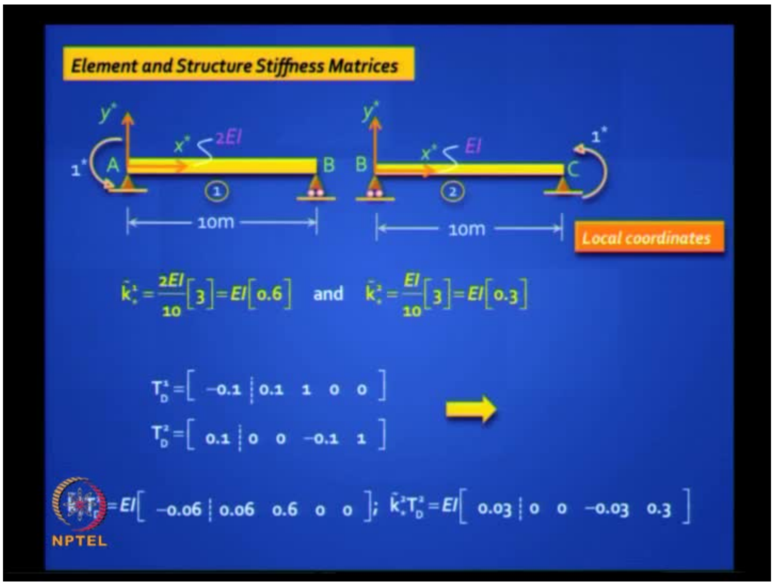

So, you have an equivalent joint load and this is what we have to analyze and that is easy. Let us generate the element stiffness matrices, we have 2 elements; single degree of freedom EI 3 EI by L, easy to do, you got those numbers, then you know the T D matrices.

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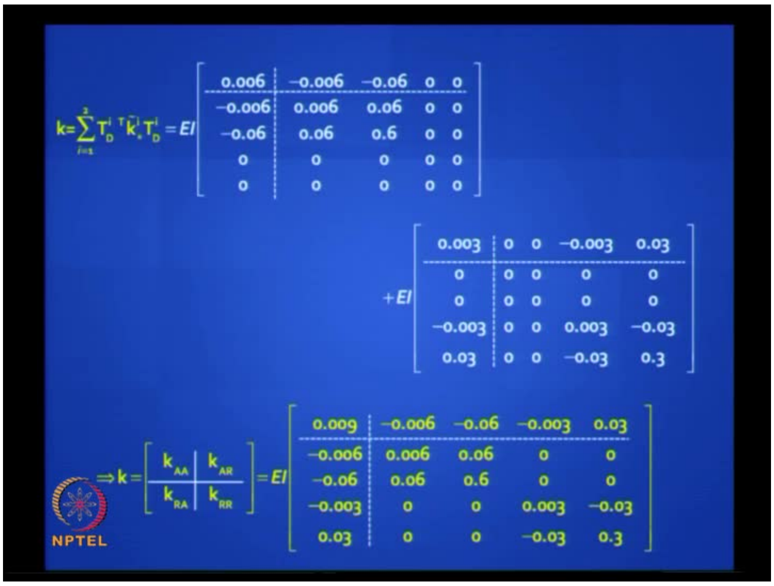

So, T D k star T D, you can work out. And then, do the summation, you will get two contributions: the first contribution comes from the first element, second contribution comes from the second element; and you just add up, you will get the final stiffness matrix k A is 1 by 1; very easy to do.

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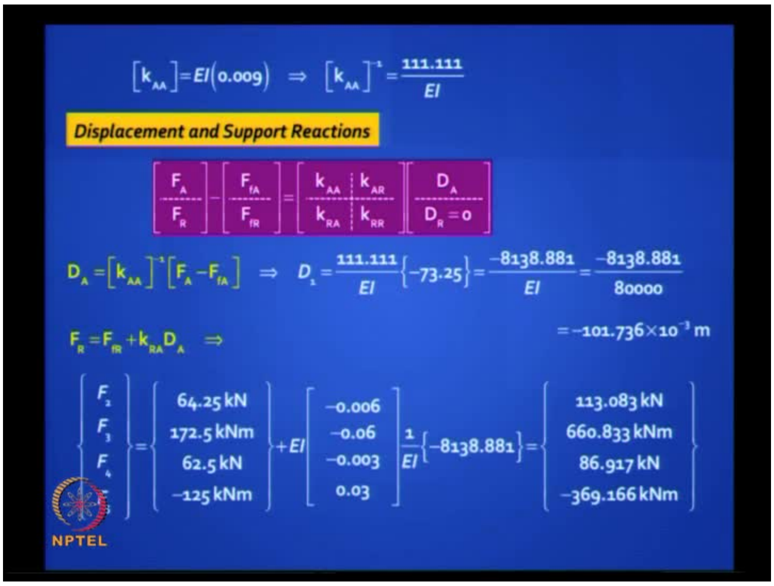

You can find the inverse which is just the reciprocal; same formulas, these are now looking very familiar, there is nothing great in all this; solve for the unknown single unknown displacement D 1 in the middle; remember, we did this problem, we got 101 mm in the conventional stiffness method.

You will be also noted that, when the hinge was not there, the deflection was very small; the hinge made the whole system very flexible, the deflection shot up. By a large amount, find out the support reactions, they match with what you solved earlier using the clamp, but this is much easier.

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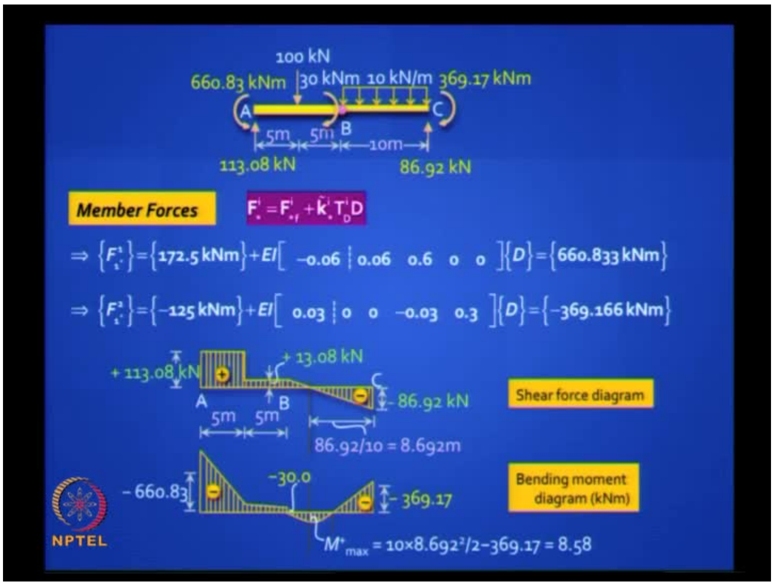

One unknown here and there you had to bring in the clamp. So, next member forces, same procedure, same member end forces, same shear force diagram, same bending moment. So, we are really doing the same problem by all the method, so that you get a clear idea and it is an easy check. And the check is even better, when we do by flexibility method which is the completely different approach and you are getting exactly the same solution.

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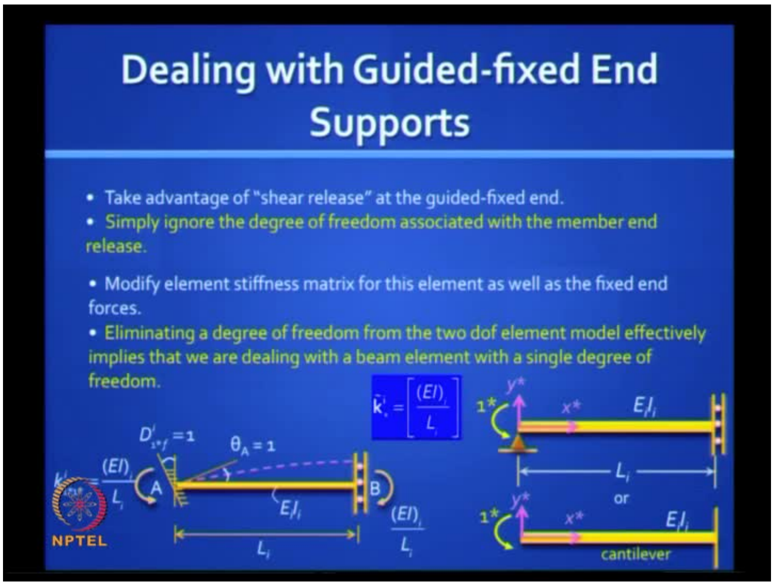

Last problem and this has to do with guided fix supports. And this is a little, it requires little understanding and please listen carefully. So, here, you have what can be called a shear release, it is not a moment release, because in a guided fix support; there is no shear transfer from the beam to the support. And like in the previous case, you ignore the degree of freedom associated with the member end release; that means, the translation you ignore we did this in moment distribution method, slope deflection method, do the same thing.

Modify the element stiffness, what will be the element stiffness for a guided fix support case? (( )) remember those three magic numbers; your 4 EI by L, 3 EI by L, which one you will use?

3 EI by L.

That is all. Only one number to remember, you remember it very well. You can prove it, it is a cantilever behavior and that is the element that you can assume. This is here, we will take Ramesh's suggestion, remember, in the beginning, he said, why do you always take a simply supported beam as the element, why cannot you take a cantilever.

Here it is ideal to take a cantilever, because the behavior is exactly like a cantilever. So, my suggestion is you can take this element, but it looks so complicated, right. Why do not we take a much simpler element? And that simpler element is this, simple cantilever fixed at the other end and you know that the stiffness is the same.

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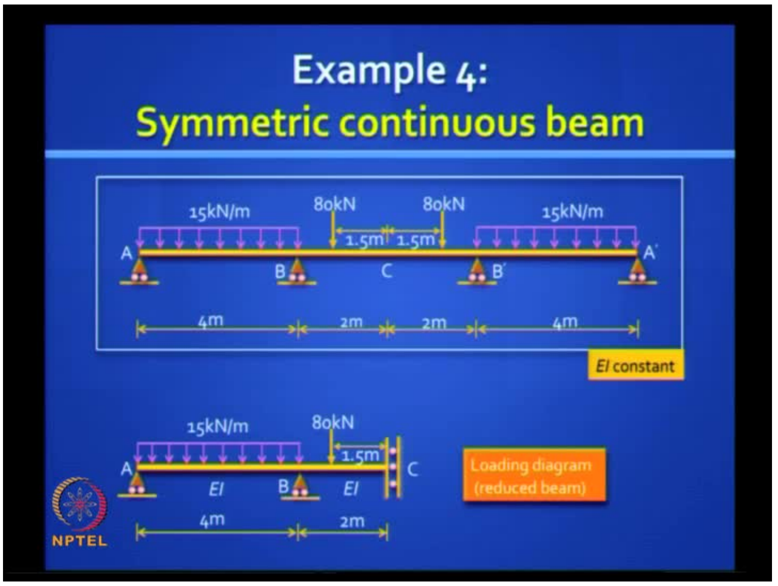

There are two ways of getting EI by L. So, we will use this in our symbols, so that our understanding is simplified, is it clear? So, let us demonstrate this with this same example which we did earlier. So, we can take advantage of the symmetry, cut it into two and you get a guided fix support there, what is the degree of indeterminacy here - kinematic indeterminacy?

Two normally, which two…

(( ))

Do not say A B; tell me theta, delta, which one?

Theta A and delta B.

Theta A, theta B.

Theta B, delta C.

(( ))

Well, strictly speaking, theta A, theta B, delta C, but the whole idea of demonstrating this is to take advantage of… so, there are two advantages you can take: one is delta C you do not worry about, the other is theta A also you do not worry about, what do the previous examples? Far ended hinge.

So, you guys should wake up; whenever you get a nice shortcut, take it; from three, we have reduced to one, single unknown, what is that single unknown? Theta B. Do you want to do a 3 by 3 matrix inversion or 1 by 1 matrix 1 by 1? So, be intelligent; do one, the procedure as the same.

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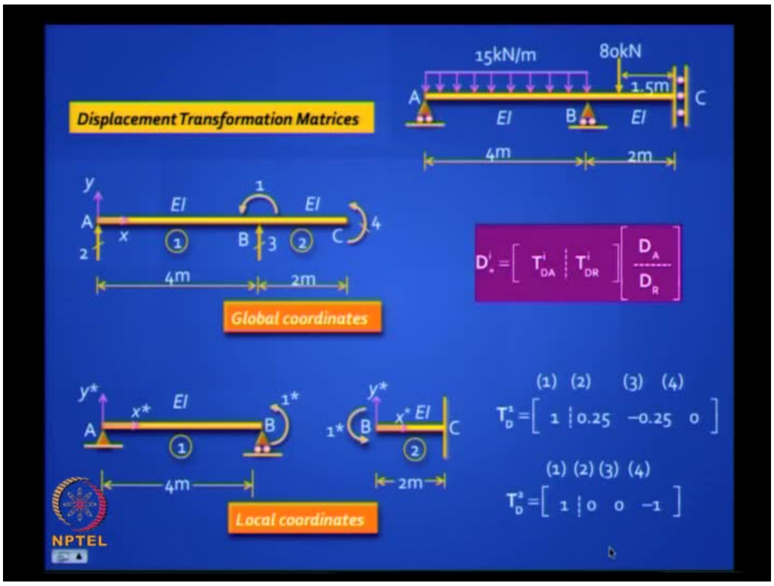

Take a look here; this is the reduced beam problem, these are the global coordinate, just 1 is it ok? Just 1, and 2 3 and 4 are restrained in the global coordinates. Local coordinates just 1, but simply supported I have shown here and cantilever I have shown here, is it ok? 1 star, 1 star, you know the stiffness; for the left one, it is 3 EI by L; for the right one, it is EI by L, that is it.

Write down the T D matrix. What will be the size of the T D matrix for each of those elements? 1 by 4. So, you have to give me 8 numbers. I have be a covering all types of problems, all types of complexities, all types of shortcuts.

So, let us check it out; if you lift up, if you put this rotation 1, it will effect this and this, right. So, you get 1 1, both anticlockwise positive. You put D 2 equal to one; that means, you lift up this end, which is simply supported; you get a clockwise rotation in element 1, which means anti-clockwise equivalent flexural rotation. 1 by 4 is how much?

0.5

0.5, plus 0.25, but nothing happens to the second element, so this is 0. If you lift up the middle here 3, left will undergo anticlockwise chord rotation of 1 divided by 4, which means minus, because it is a clockwise equivalent end moment 0.25 whereas nothing; and for the right end, (( )) now you have to be careful.

In a cantilever, translations do not create any problem. In a cantilever, remember, the it was guided fix here and support here. If you have support settlement, it is not going to create any problems; you have to visualize it. So, this is where you do not make mistakes; your T D value will be 0, because this is a different ball game, right, because D is meant to deflect D; B is free, nobody stopping that deflect; let it deflect, it is a rigid body moment, the whole thing will go up.

So, you have to be careful; this is where you recited from memory, it did not work here. You have to always use your brain, that is the only thing. The last one is item number 4, anticlockwise moment , here you have to be careful. If you give an anti-clockwise here, at the other end you will have an equivalent clockwise opposite moment.

So, this is the only thing you have to be careful about; may be, you might like it in the old style. Let us try to generate the T D matrix for this guided fix support case from first principles.

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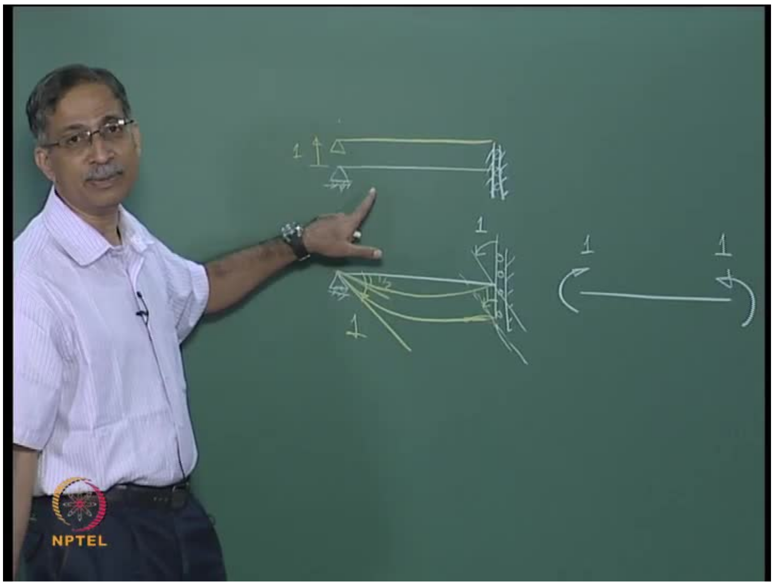

So, you have guided fix support here, prop here and we first take the case where you have a movement here. If you have a moment here, it will just move like that, the rigid body movement. So, in terms of rotations, you do not have any rotations, so that is why the rotation value is 0. In the second case, we are dealing with a rotation an anti-clockwise rotation at the right end, which means this rotation is now unity; and we need to draw the deflected shape and figure out, what is the rotation at the left end.

We will take the same boundary conditions and you must recognize that there is a possibility for the beam to deflect at this end, because you have a guided roller support here. So, this is little tricky, because you might be tempted to draw it like this, in which case this angle will be half this angle, which is not necessarily true, because this can deflect.

So, let us take some help from the contra gradient principle and let us see what happens. If in a beam element to satisfy equilibrium, you have a moment - the unit moment - acting here. To satisfy equilibrium, you also need an equal and opposite moment acting here one.

So, the T D value should be minus 1 and the minus is quite clear, because you have an anti-clockwise rotation here, you have a clockwise rotation here, so it is certainly minus, but it is not minus half, its minus 1, which means this has to deflect some more and that is not a problem. So, let say this goes down to a point there, you can get the desired deflection, still maintaining this change in angle is 1 here; and from half this, this increases to 1, so that is a physical explanation. And the answer is absolutely crystal clear, it has to be minus 1.

So, there are two explanations, why you need to put. Minus 1, when you are dealing with this case; and you have to put 0, when you are dealing with this case, because there is no rotation happening in this case. Fixed end forces, can you find out?

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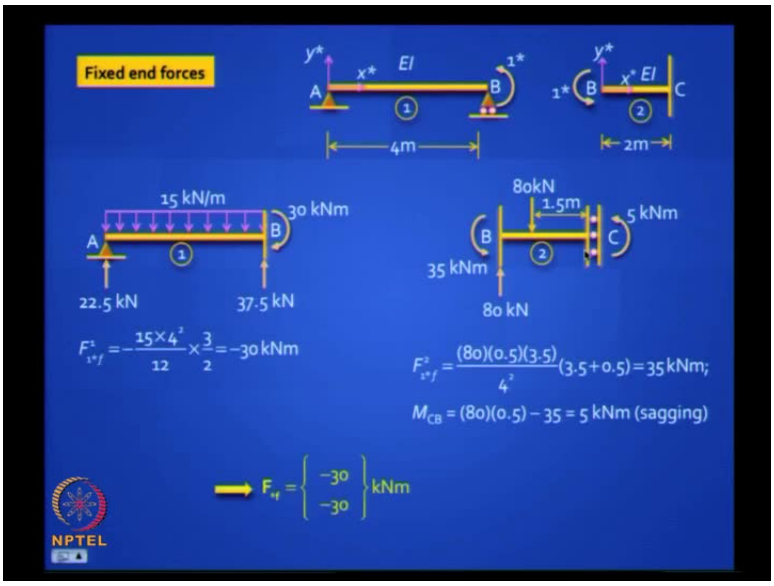

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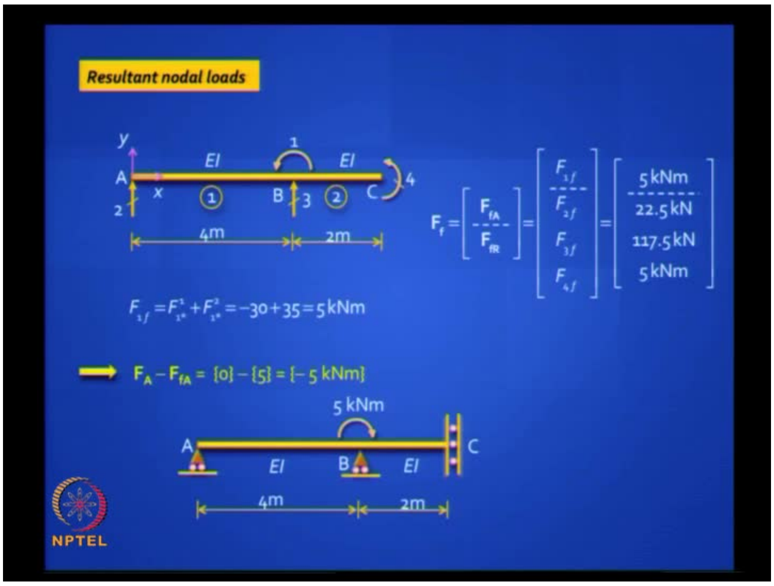

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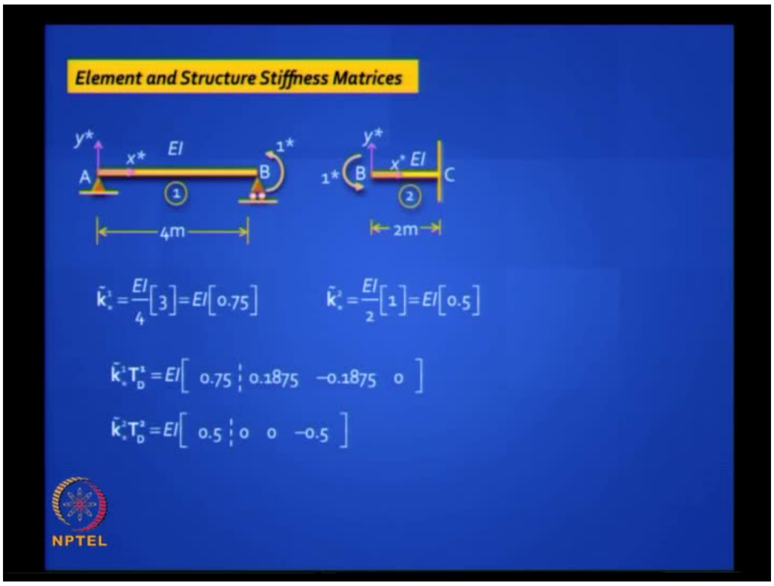

Let us look at fixed end forces; fixed end forces for the first one is 30 kilo Newton meter, and for the second one, we have done this problem earlier, you get 5 kilo Newton meter. You can add up and get the resultant force vector, which is 5 kilo Newton meter; and you have this matrix, 5 kilo Newton meter and substitute…

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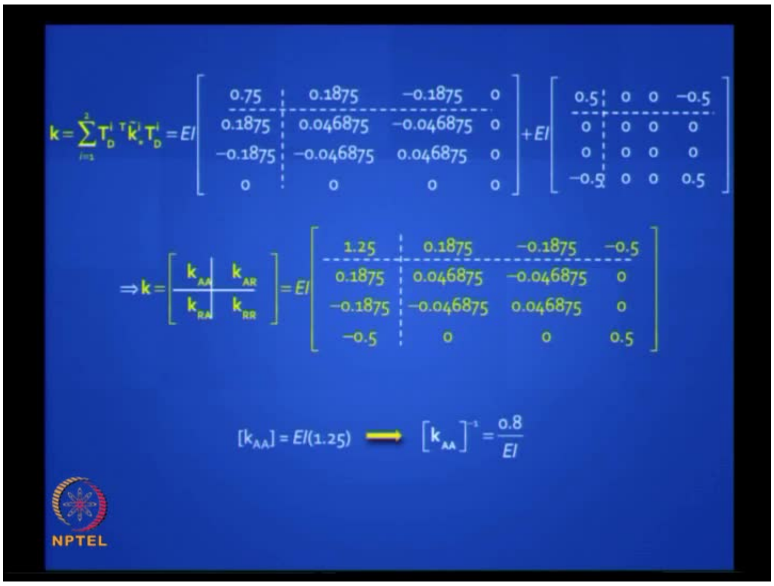

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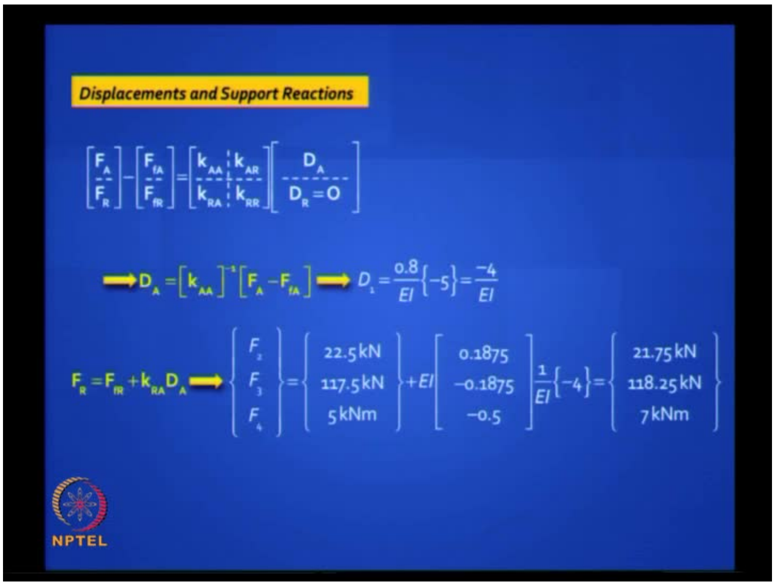

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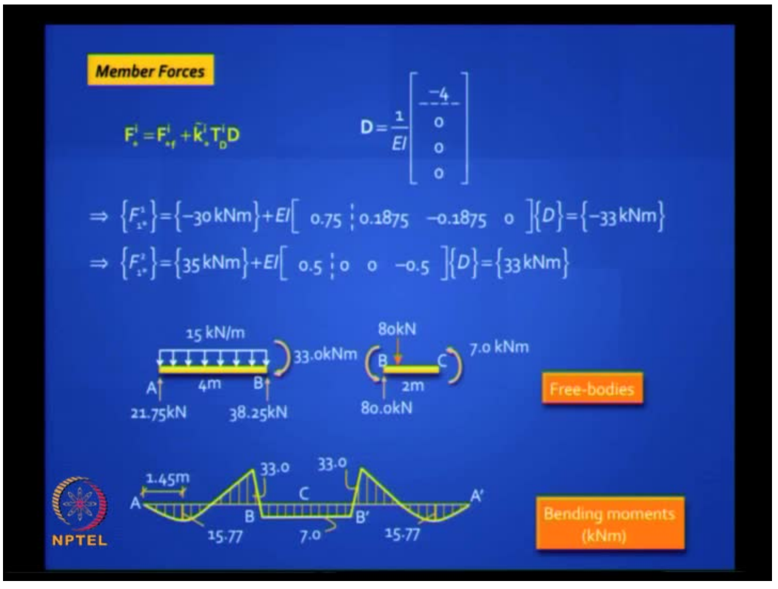

You get 3 EI by L for the first element, EI by L for the second element; and plug in those values, you will get the stiffness matrix. And find the inverse, same procedure, find the unknown displacements, its minus 4 EI 4 by EI. Find the support reactions, you will get exactly what we got earlier; member forces, that is it.

So, be careful about this particular shortcut, but you can cross check with your previous examples. When you have doubts about T D, check with T D transpose, because you are very comfortable with equilibrium, right.

So, but we have covered all the possibilities that you can get. So, we have now reached a stage where we have finished conventional stiffness method, we did four examples; we have finished reduced element stiffness method, we did the same four examples.

Now, what is left is flexibility method. We will do only two examples; we will finish it tomorrow. And the last topic left is, very interesting topic – grids; we are going into spatial structures, we will take some examples and do it; you have an assignment to do three problems, one of them is a grid.

Thank you