Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of Technology, Madras

Module No. # 5.3

Lecture No. # 29

Matrix Analysis of Beams and Grids

Good morning. This is lecture number 29, module 5, Matrix analysis of beams and grids.

Today, we will complete the application of conventional stiffness method and we will introduce the reduced elements stiffness method.

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This is covered in this book, which you need to refer to Advanced Structural Analysis. We will continue with conventional stiffness method, remember we did different types of problems. We first took a non-prismatic fixed beam. Then, we did continuous beam, remember three span continuous beam, which also had an overhang and we did not take any shortcuts. We have dealt with different boundary conditions; we also dealt with internal hinges, we also dealt with intermediate supports. Now, here is some more types of supports that you can get you can get the extreme end, as hinged or roller and you can get a guided fixed support.

Now, it is possible to take advantage of reduced degree of kinematic indeterminacy. We will do that in the reduced elements stiffness method, but in the conventional stiffness method, we do not take any shortcuts. Actually, it is quiet easy to deal with any situation, you have to assign the appropriate boundary conditions.

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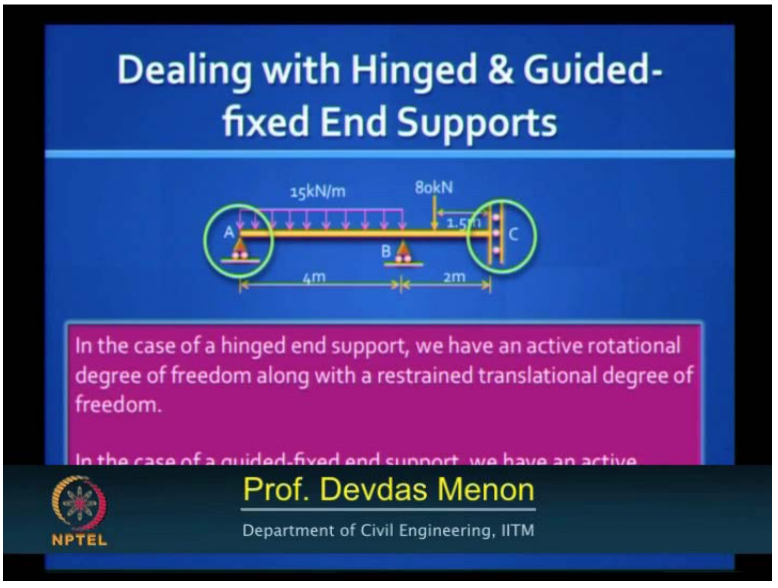

In the case of a hinged end support, as you see at the end A in this example, we have an active rotational degree of freedom, along with a restrained translational deflection degree of freedom. At support C, guided fixed end support, we have an active translational degree of freedom, along with a restrained rotational degree of freedom.

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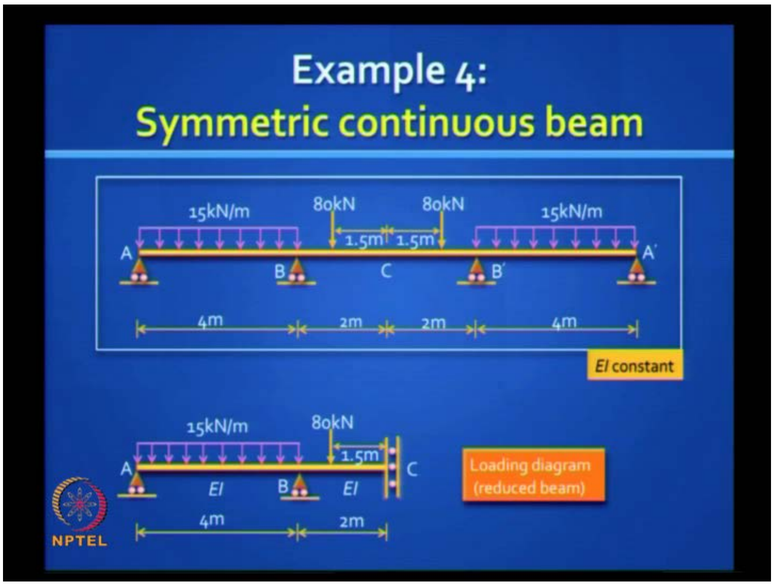

We have done this example earlier, remember, it is a symmetric continuous beam; we can take advantage of symmetry; we cut the beam at the middle at C and and insert, what kind of support, do we insert there and guided fixed, guided roller support, as shown here. And let us see, how to analyze this.

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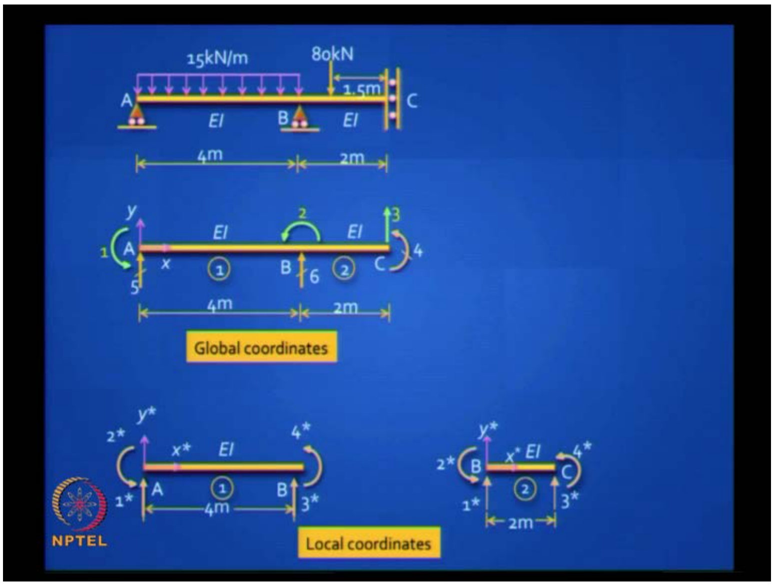

What are the active degrees of freedom? How many of them do we have? Yeah. three.

three.

three active and the method of analysis exactly the same, so 1 2 3 we start with the left end rotation at A, rotation at B, deflection at C- these are the three active degrees of freedom.

Then, the restrain degrees of freedom are 4 5 6, as indicated there. As far as the local coordinates are concerned, just you have two beam elements, each has four degrees of freedom and if you had to write down the…… In this particular problem, do we have any any nodal loads? No, they are all distributed loads, there is no support settlement given now.

You have to write down the transformation matrix. What does it look like? Can you try the two transformation matrices? Well, they are identity matrices. You just have to assign the linking global coordinates.

What are the linking global coordinates for number one?

5 1

6 2

You have to do it in that order: 5 1 6 2, for element one: for element two:

… 6 2 3 4 …

…6 2 3 4. You must know the sequence, because the the translation must match, with the translation. Do not mix up the rotation.

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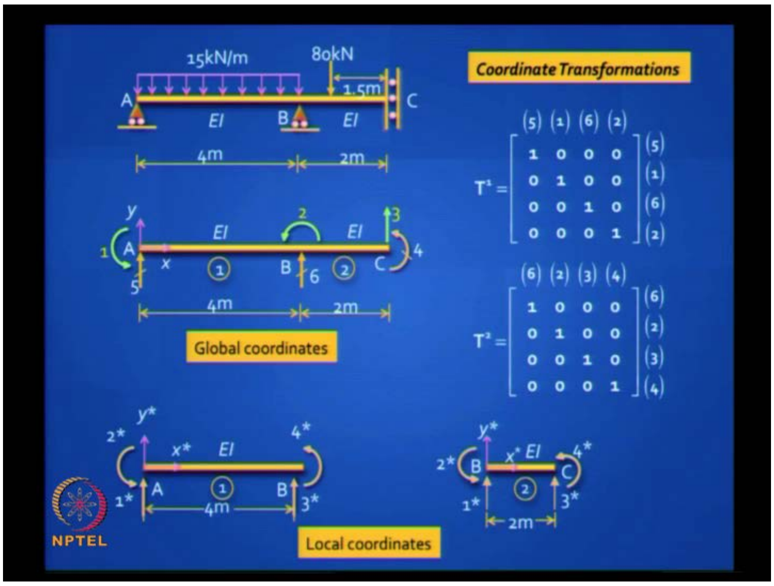

So, is this, ok 5 1 6 2, 6 2 3 4 - that is all-, very easy to do.

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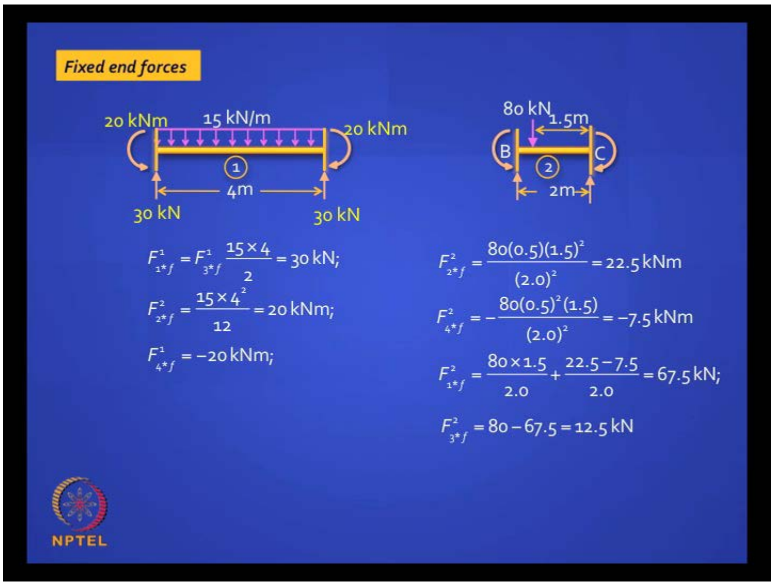

Next, you have to find the fixed end forces. We have done this problem earlier, so I think we can go quiet fast. You can work out the fixed end forces. Mind you we are not taking shortcuts, so you are arresting all degrees of freedom that you identified, as active degrees of freedom.

For example, in the second case, C is arrested. The the rotation at C is arrested and these are simple fixed end moment calculations, that you can do. If you recall, we did this problem by slope deflection and moment distribution method. There, we took a shortcut; we took advantage of the fact, that, there was a guided roller support there and we modified both the stiffness and the fixed end forces.

We do not take those shortcuts in conventional stiffness method. We will take that shortcut, in reduced element stiffness method, because this is, the normal way computers, software solve these problems. We just want to stimulate, what the standard computer program does, is it clear.

So, we really do not care, how many degrees of freedom there- are, we just want to correctly identify the degree of kinematic indeterminacy. We do not want to take shortcuts, of the kind that we did earlier. But, if there is symmetry in the structure, definitely it is worth taking, dealing with half the frame, so you can work out these. These are very straight forward.

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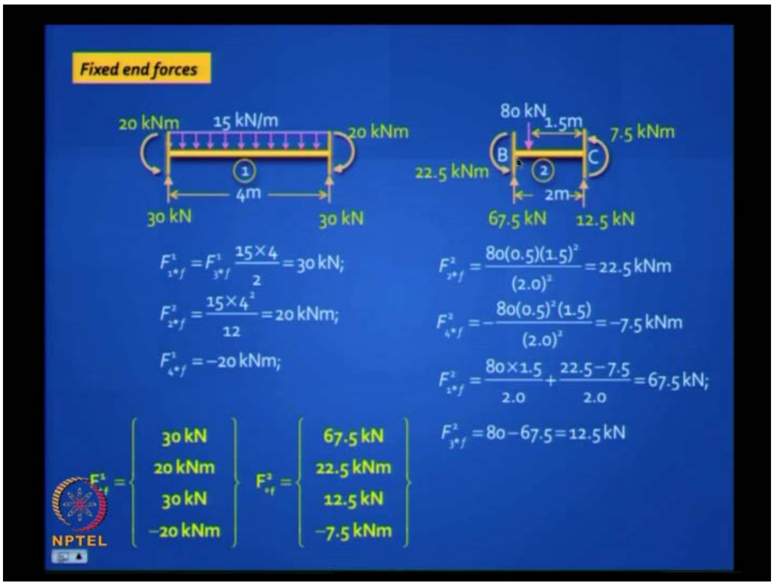

Under constrained load, the formula is minus, not minus, because now it is plus minus- since- you are dealing with the conventional stiffness method, where anticlockwise is positive. So, it is W a b squared by L squared, you know that formula, you can apply all that and you can get the fixed end force vectors.

One more important difference here, you need to get the vertical reactions as well, which is something- we did not do, when we did slope deflection and moment distribution method. Those are easy to compute, in the first case, since, it is symmetric. Just take the total load, 15 into 4 divided by 2, you get the two reactions. When you have an un-symmetric load, you get the shears; not only from the load, but you also get from the lack of balance, between these two loads.

In this case, because this should this should be both should be anticlockwise. This should be anticlockwise and the difference, divided by the span, you should add up.

Yeah, this should be clockwise, you are right. This should be clockwise, it is an error shown there but the vectors are correctly shown. Pre-multiply, why do we do this exercise? Because, we want to find out, what the equivalent joint forces are in the structure.

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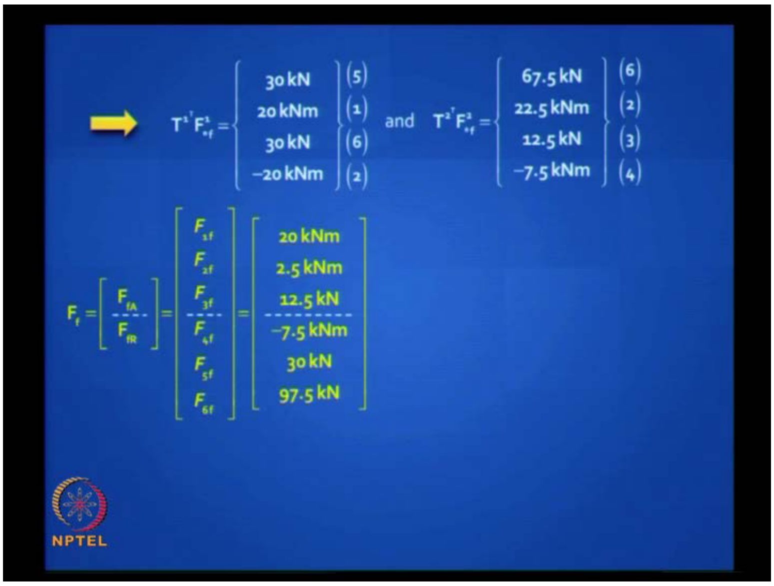

So, you are pre-multiplying by an identity matrix but you get along with it the linking coordinates and then you just you have done the slotting; you just assemble them properly. You have six degrees of freedom: three are active, three are restrained. Corresponding to one- you have 20 kilonewton meter, corresponding to two- you have minus 20 and plus 22.5, which add up, 2.5 five and so on and so forth. So, this is this can be easily assembled. Is it clear?

This is your fixed end force vector. What do we do next? You find the net, the resultant load vector, which is F_A minus F_f. F_A in this case, is zero, it is a null vector, because you do not have any nodal loads, in this particular problem and so, you can find the resultant force. You can even, draw a sketch to tell, exactly what is happening.

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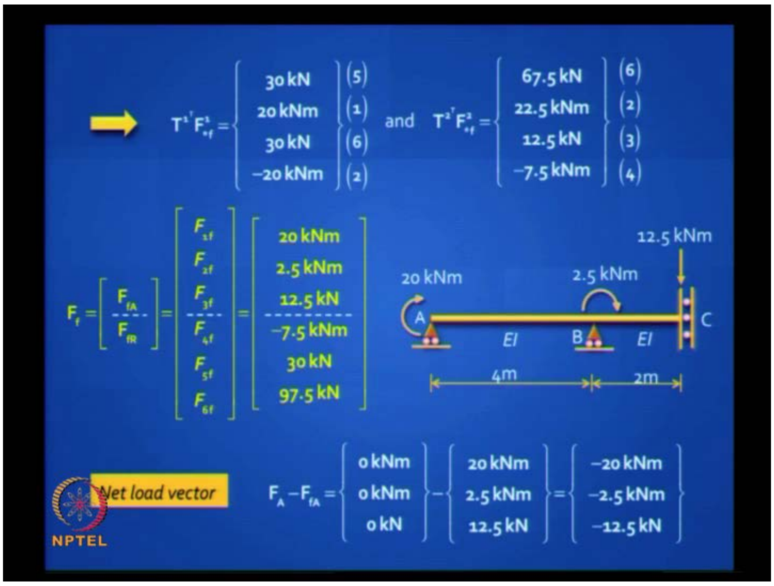

This is your resultant force vector and to remind you once again, the displacements you get, at the active degrees of freedom: D_{1}, D_2 and D_3 from this problem. From this, nodal load problem will be exactly equal to, the displacements you would have got, with the original intermediate loads. Is it clear? That is the equivalence, we are trying to establish.

It is very effectively done, because you can do matrix analysis, only if you have dealing with nodal loads. Because, matrix had only vectors, which have discrete entities, so your coordinates are limited.

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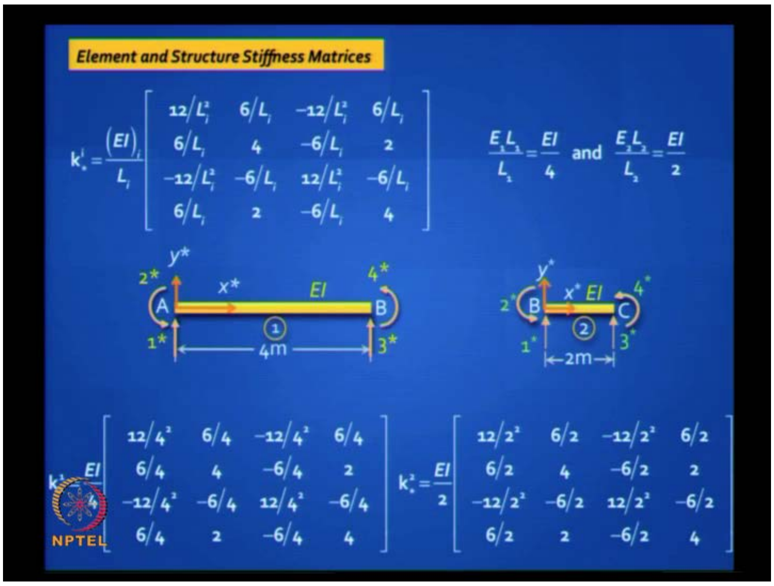

Next, you generate the element and structure stiffness matrices. Element stiffness matrix is standard, there, you have two elements. All you need is EI and L. In the first case, actually EI is constant, for both the elements. L is 4 meters for first element, 2 meters for the second element. So, plug in those values, you can get $k_{star}1$ and $k_{star}2$. Very simple, straight forward and we have done this problem earlier by moment distribution method.

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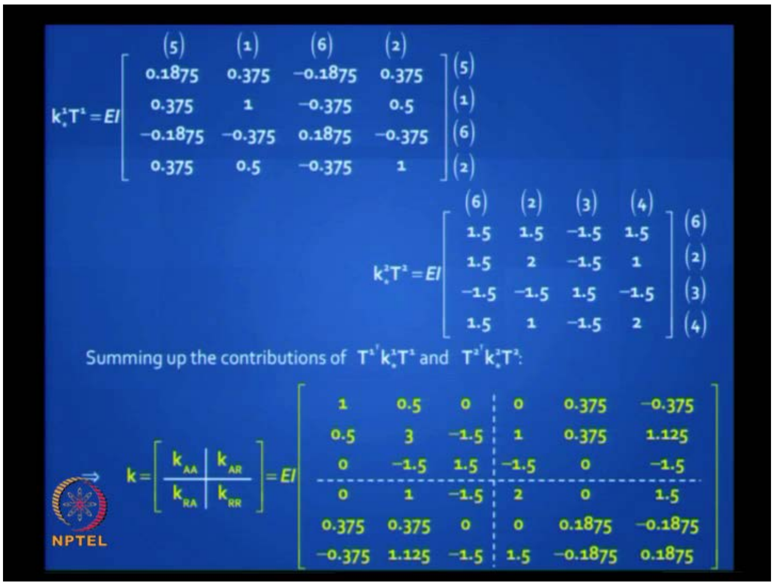

What do we do, next? We have to assemble, the Structure stiffness matrix. To do that, we have to convert, from the local axis system, to the global axis system, through the transformation- T_i transpose k_{star} into $T_{i.}$.

So, you do this first and then you, actually, you get the same thing again, when you pre-multiply by T_i transpose, you got the linking coordinates, then, you have to do the slotting, which I will not explain. You will finally, end up with a, full 6 by 6 matrix: the top upper corner is always k_{AA}, the bottom lower corner is k_{RR} the upper corner is k_{AR} and the lower is k_{RA}.

You should write a program ideally which will generate this automatically. You should just take a look and check there, it is a symmetric and it is the diagonal elements are all positive. You are You are set to handle any loading. this And this is what the computer does, the moment you give, the geometry of the structure and the material properties, it is ready, with the stiffness matrix and then it is waiting for the loads. You also got the loads.

Now, you these are your equilibrium equations. If you solve the first equation, you see how familiar this whole thing is now, nothing. I mean once, you have got the method; you got the solution.

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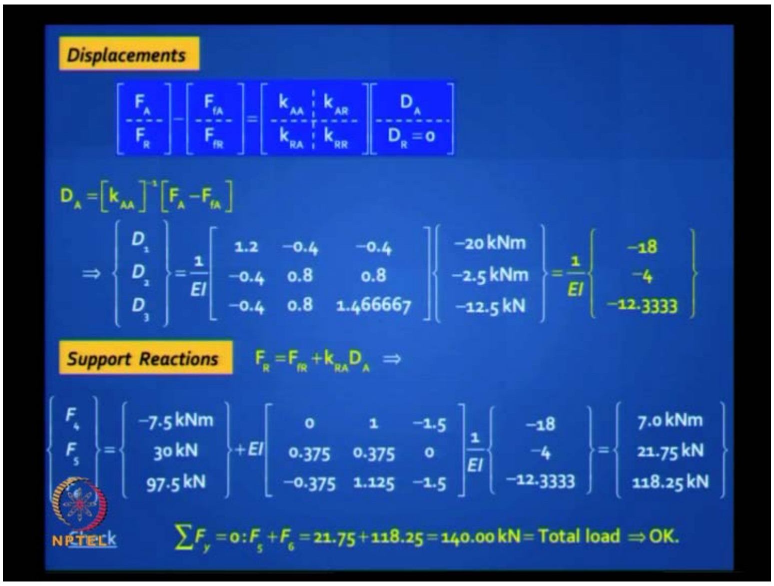

To solve the first one and you will get some solution, in this problem, EI was not mentioned. So, if you plug in the value of EI, you will get the answer in millimeters and radians.

What do you do next? Find the support reactions, how do you do that? Take the second equilibrium equation and plug in the value of D_A in this problem, D_R is the null vector. You will get some answers and what should you do after getting these answers? No, before that, you must check equilibrium; you must check equilibrium and it is a simple check; you can do many checks; you can do moment equilibrium check. The least you can do is, you just add up the total downward forces, must add up to total upward forces and it is exact to the to the third decimal place, in your solution.

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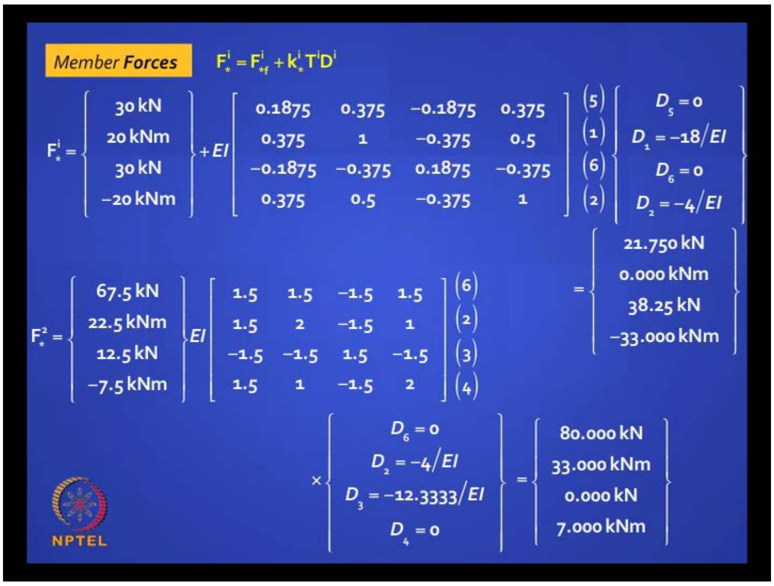

Then you find the member forces, you are familiar with this, here you have to be careful about one thing, what is that? You got there is a F_{star}i f -missing. What is that you need to be careful about?

The D_i you have to be careful, because if you go back, you got displacement, D_1 D_2D_3. When you go to the element coordinates, you have four degrees of freedom here. You must get the correct D_i relevant to to the element that you are taking into consideration.

For example, the first element, this is a fixed end force vector, which you have already got, this is k_{star}1 T 1 which you already calculated and stored in your computer. Now, you recognize, there are linking coordinates of 5 1 6 2. So, you have to put D_5 D_1 D_6 and $D_{2,}$ some of them are restrained.

For example, 5 and 6 are restrained, so they are zero. You have to pick up D_1 and $D_{2,}$ from the solution that you generated from your D_A vector. This is the only thing, you have to be careful about, at the rest of it follows. Is it clear?

You have to select those displacements, from your final displacement vector, which has D_A and D_R, which are matching which are which are matching you are an element and that comes nicely, with the linking, coordinates which come along with k_{star}i T i.

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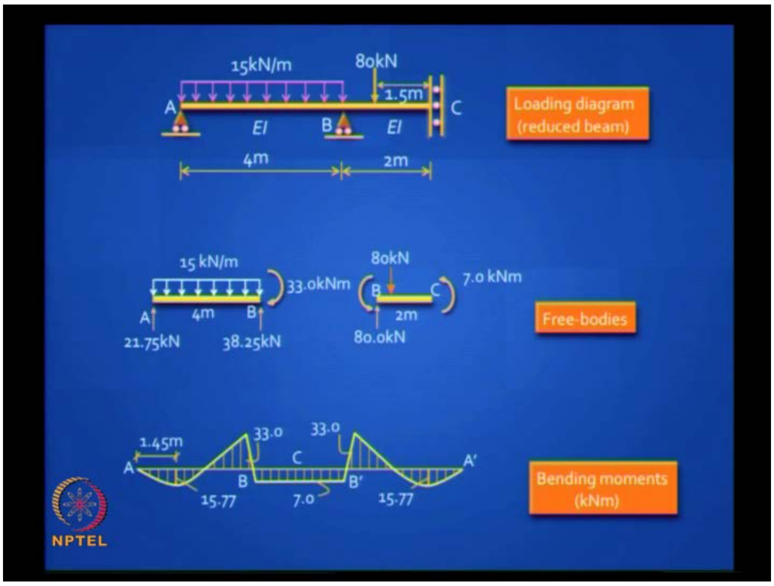

Do a similar exercise, for the second element and you get some moments and then you can draw your free body diagram, bending moment diagram. This is exactly the same solution, we got by the methods, we did earlier. Is it clear?

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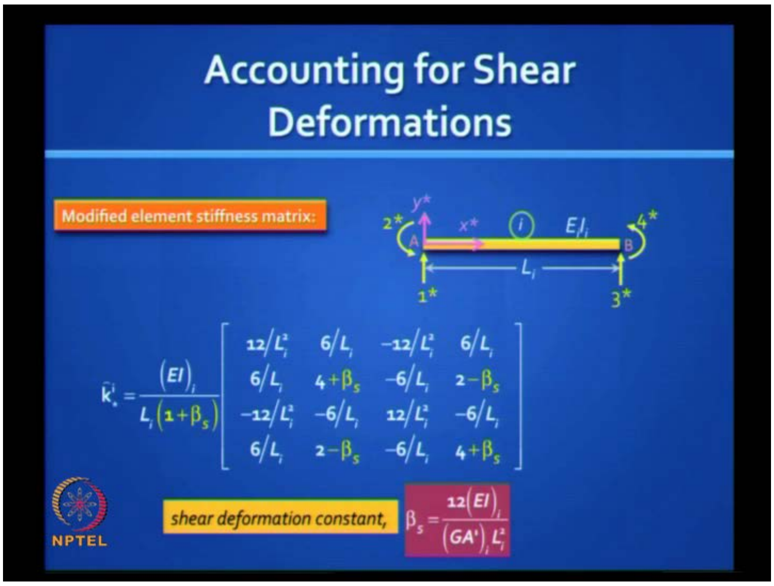

One last topic, before we move onto, reduced element stiffness method. Sometimes, it becomes necessary, to account for, shear deformations. when When do you think, you need to count for, shear deformations?

The length is very small.

When, the span to depth ratio is is low.

In normal beams, the span to depth ratio is, how much? Well, 10 to 15. If it is above 8 or 7, you would not make any significant error, by ignoring shear deformations and normally, well proportioned beam belong to that category. Occasionally, for various reasons, possibly for architectural reasons, you might require, require a deep beam. So, if you got a deep beam, even if you do not have a deep beam; if you still include shear deformations, you're only going to get a more accurate solution.

So, that deep beams are those beams, in which the span to depth ratio is roughly less than two and half, between two and half and six or seven, you have the the the middle range, which is between, shallow beams and deep beams.

You encounter such situations, especially, when you deal with shear walls, in tall buildings. So, you know shear walls are the lateral load resisting system, a part of the lateral load resisting system, in a multi storey building. The shear wall is essentially, a vertical cantilever but, it is connected to different floors, through the horizontal framing plan. You you may have beams; you may have slabs and essentially, if, it subject to and the lateral loads come from, those diaphragm is at the slab levels, in the different storied building of a structure.

Now, if the shear wall is slender, in the sense, it is it is like this, (Refer Slide Time: 17:24) width is small and the stories height is large, there are many stories and essentially, it is going to behave, like a flexural element.

It will be the deflection shape, it will be like that. But, sometimes you encounter shear walls, which are very wide and stocky, the height is less. So, when you try to push that shear wall; you'll find that, the deflection that you get, has two components: one is flexural, the other is shear. Remember, we did this exercise earlier.

Shear deflection in cantilever beams and the shear deflection can be comparable to the flexural deflection. In a cantilever subject to a constant load p at the free end, what is the deflection? p L cube by 3 EI. But, there is also a shear deflection that you get. Remember, we did this, which has g a dash coming into the picture, in the denominate. So, in such instances, shear deformations have to come in. Now, this kind of beam, which you include shear deformations was was originally, described in detail, by Timoshenko and sometimes it referred to as Timoshenko beam.

It is an energy formulation. I am not giving the formulation here, but you can go through it in literature. You will find that, you have to modify, your element stiffness matrix, by including a quantity, where you have a ratio of the flexural rigidity, to the shear rigidity. Now, that is called, the shear deformation constant beta$_s$.

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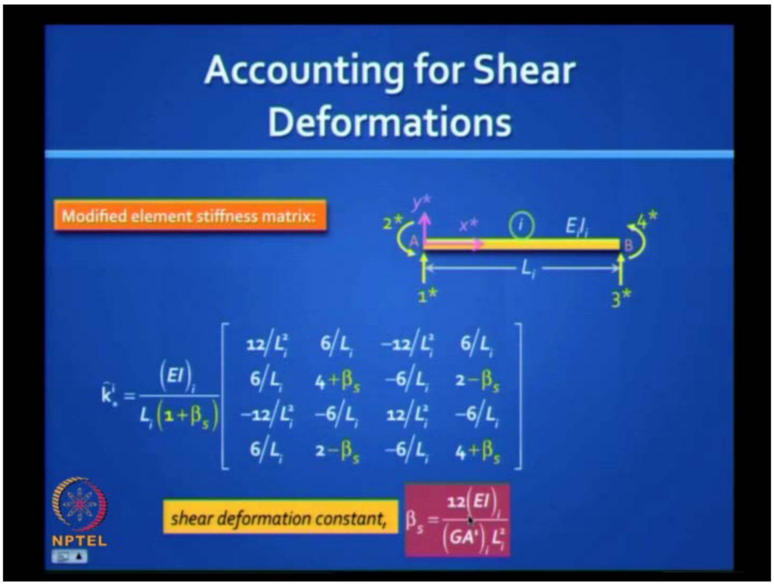

So, you have EI and you have G A dash, I, A dash. G is the shear modulus, which is related to the elastic modulus, through the Poisson's ratio. You know that.

A dash, is not the gross cross sectional area, it is the reduced cross sectional area, to account for the fact that, the shear stress distribution, is not uniform. .

So, there is a form factor that comes into play. So, this is a non-dimensional factor, that is why, L square also comes in the denominator and we will see, how what's the variation of this factor? How it affects the analyses? But, you are and sometimes this formula is useful. You can easily run this and check, what is the error I make, if, I ignore shear deformations; by including shear deformations, in any beam problem.

This can be also used, when you do frame problems. So, all you need to do, is to modify this term outside, by this factor: 1 plus $beta_s$ in the denominator and wherever you have 4 EI by L, you have to add 4 plus $beta_s$, into this constant; where you had 2, you have to put minus $beta_s$, that is all you need to do.

So, four terms, in your coefficient matrix get affected and all terms get affected, because this is common outside. Is it, clear? So, this is a correction, you need to do and check out the solution.

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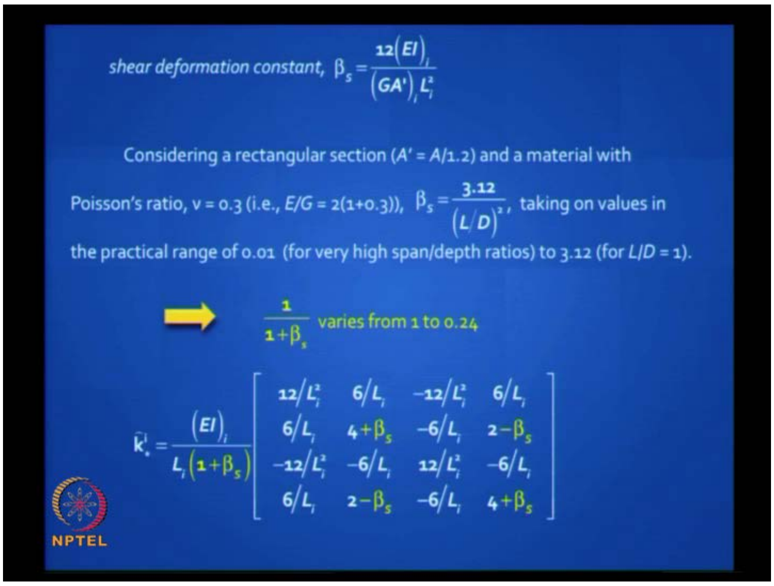

Considering, a rectangular section and a material with Poisson's ratio, mu equal to 0.3, taking on values in the practical range of 0.01, for very high span to depth ratios, 2 3.12, for L by D equal to 1. Normally, you would not get a depth of beam, which is less than the span of the beam. So, the variation of 1 divided by 1 plus beta_s is between 1 and 0.24 in practice.

So, that is that is a kind of range, that you get obviously when 1 by 1 plus beta_s is goes to 0.24, it is going to make a big difference but, those are very rare on exceptional cases. We are not going to do any problem but, if tomorrow you are need to include it, in a real life situation, please make use of this matrices. With this, we have covered completely, the conventional stiffness method.

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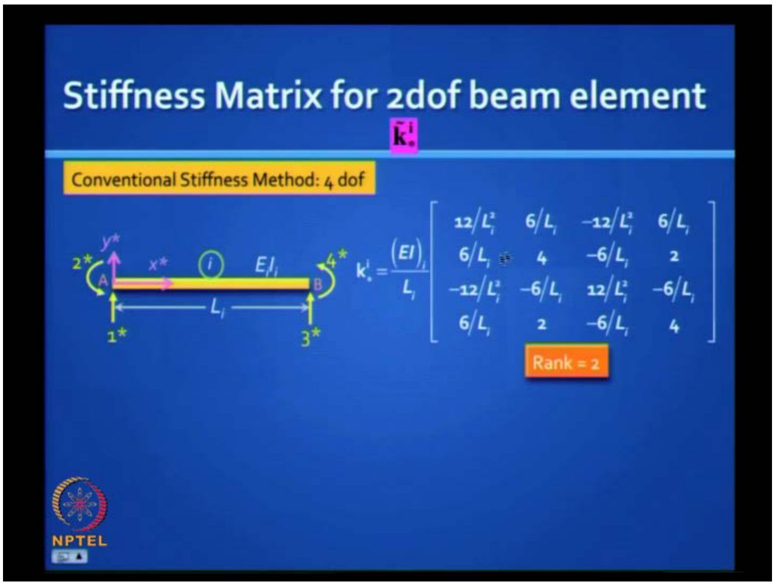

Now, we will look at the reduced element stiffness method. In this reduced…. In the conventional stiffness method, we have this stiffness matrix. I am not including shear deformations.

What is the rank of this matrix?

3

3

2

3

2. Rank is 2. It is not 3; it is 2. You can easily make out. Is it, not? Take a look. You can play around with rows and columns and two of them, will you know find the reduced form but more easily, you can see, why is it? Why is the rank not 4 but, 2? What, is it, 0.2?

There are only two equations of equilibrium are available, that is why, you can't you cannot find the inverse of this matrix. It is a singular matrix, but in physically, what does it, mean? when you When you find, the rank is not full? What is it, point two?

It depends equations, sir. The other forces physically …

What does it? We have discussed this earlier.

The other forces are depend…..

It is unstable. It can have rigid body motion. See, you can have a flexibility matrix, only for a stable structure. You can have stiffness matrix for an unstable structure that, at the element level, not structure. At the structures to be stable, otherwise your structure stiffness matrix will become singular. This is very interesting. The element stiffness matrix is singular but, your structural stiffness matrix will not be singular, unless you take the full k matrix. You have k_{AA}. k_{AA} will not be singular. Because, you are arresting some degrees of freedom and you have to arrest a certain minimum number, for it to be stable. Is it clear?

Now, what is the to make this stable, what should, we do? You can arrest, any of these two degrees of freedom. If you make it simply supported, it is going to be stable. So, that is the element: we will use; we did this for axial element, remember. We caught hold of one end, we said, we can pull only the other end.

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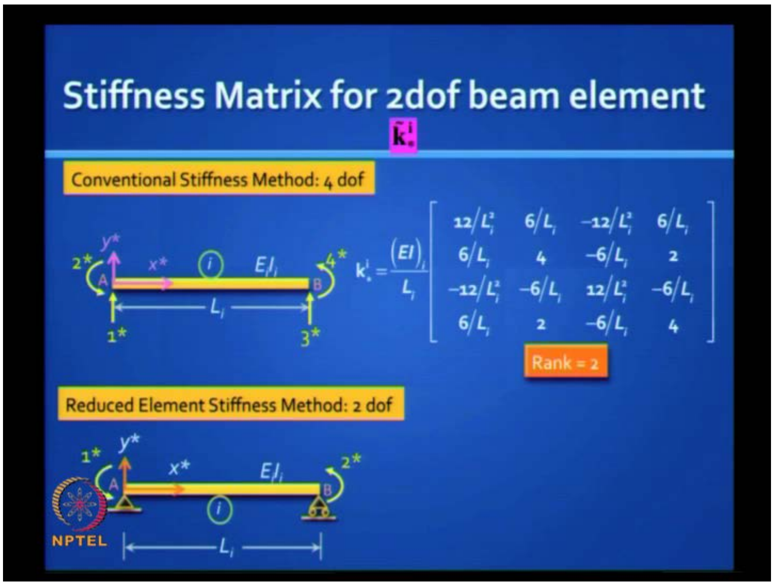

Here also, you arrested, when you have a simply supported beam. So, from now on, in the reduced element stiffness method; this is the the this is the element, that we will use for the element.

You can, it is a good question. Can you use a cantilever? Please, do so, most of us like, simply supported conditions and we are already familiar with, you know 2 star and 4 star. The 4 EI by L 2, EI by L, is now, child's play, for all of us. So, for shear convenience, we have used simply support, but you can also use, cantilever. In the book, I have discussed this. In the book, I have also used cantilever as an option, but I do not want to confuse you. So, go by simply supported, but you can also use this.

Now, can you write down, the element stiffness matrix, for this, straight away? It is a 2 by 2 matrix and we will use a symbol k tilda, what is it?

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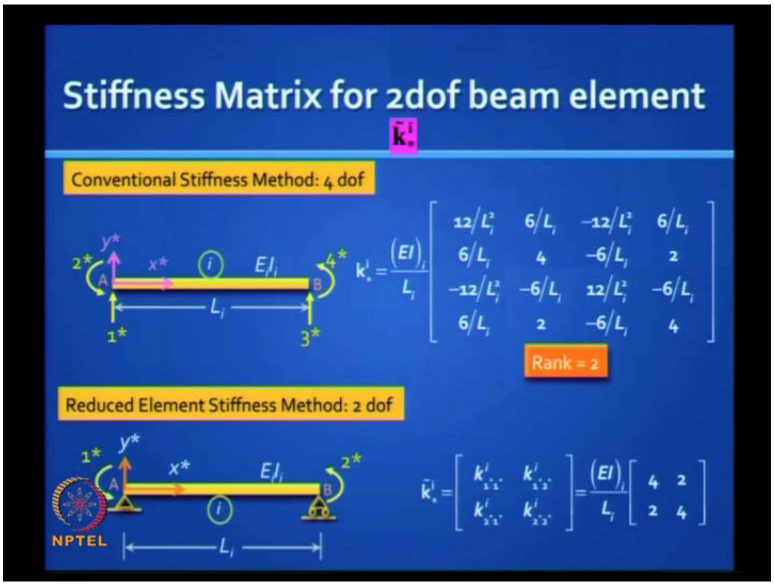

Can you extracted from the the full, four by four matrix? Can how How do you, extract it? What should you remove? what should you remove? What should you remove, from that, 4 by 4 matrixes?

The translation column.

Translation column.

Just knock off the first column, third column, first row, third row and whatever you left with. That is an easiest, you can do. Whatever, you are left with, is your reduced element stiffness. Let us do it in, a way, that you will appreciate better, let us, use a physical approach and derive the same thing.

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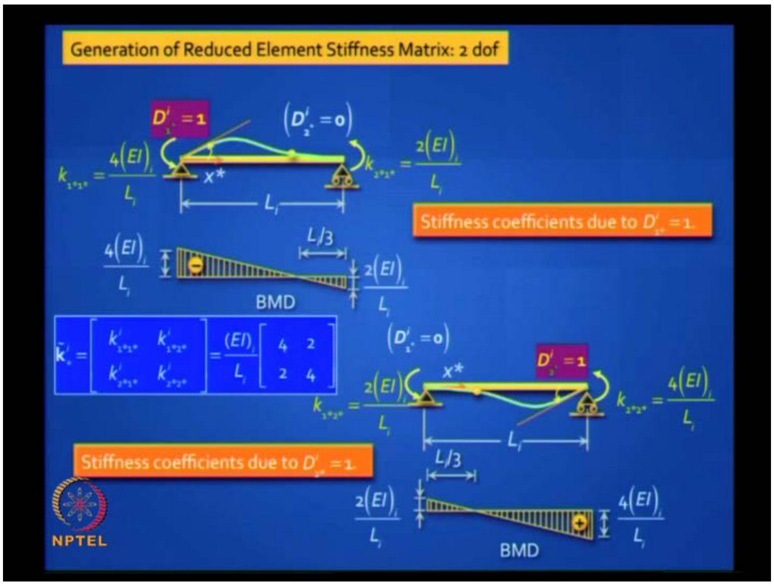

So, what do you normally do, to get the first column? You apply D 1 star equal to 1 and the moment link with that is 4 EI by L, you have a carryover moment at the other end, which is 2 EI by L. That is all, you need to worry about. Do not worry, about the reactions.

If you want to, the deflected shape will look like that; the point of contra flexural will separate the beam and the ratio two isto one. Bending moment diagram will look like that.

If you now, do D 2 star equal to 1, you get a similar figure and from this, you can generate, your element stiffness matrix. Is it, clear? Very easy, to re… and very easy to remember: 4 EI by L; 2 EI by L; 2 EI by L; 4 EI by L, no minus signs, here. Clear? easy.

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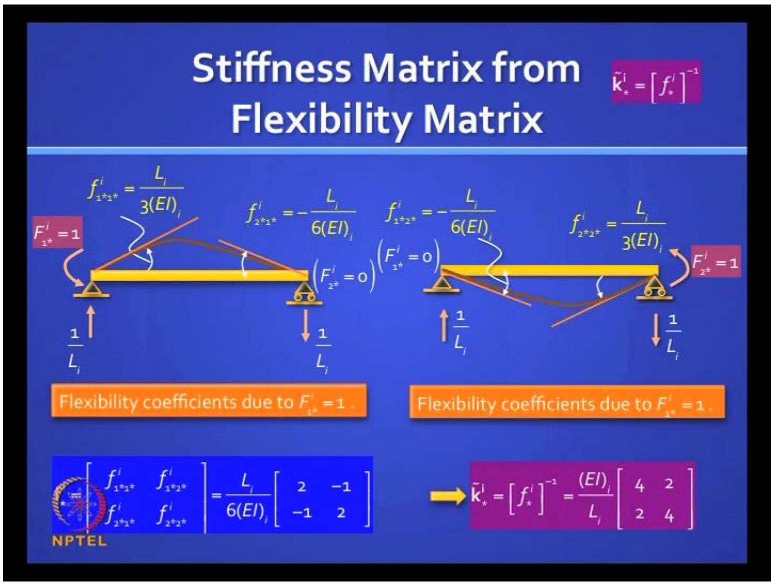

Now, you can also do it from flexibility matrix, because you get, if you invert the flexibility matrix, you get the reduced element stiffness matrix.

In an axial element, it was very easy, because it was 1 by 1. It was EA by L or L by A. Here, it is not 1, it is 2 by 2. Let us, do this, you remember. If I take at the same simply supported beam, apply $f_{star}1$ equal to 1, the deflected shape will look like this, if I use conjugate beam method or some such method, I can show that this angle here, $f_{star}1$ is L by 3 EI and the angle here will be half this, put it in the opposite direction, it is minus L by 6 EI. If I do, F 2 star equal to 1, it look like this. So, what is my flexibility matrix? It is going to, look like this. Here, is negative sign comes.

If I take the inverse of this matrix, I get that matrix. You can work both ways; you can generate the flexibility matrix, from the stiffness matrix or you can, but, you can do it only, with the reduced element stiffness. Is it clear?

Now, let me ask you a simple question. How can you use a two degree of freedom element? How can it handle the deflections, in your structure?

Remember, your global coordinate system: It has not only rotation; it also has deflections: with this 2 by 2, can you handle it? It is an important question, to ask? Is it clear?

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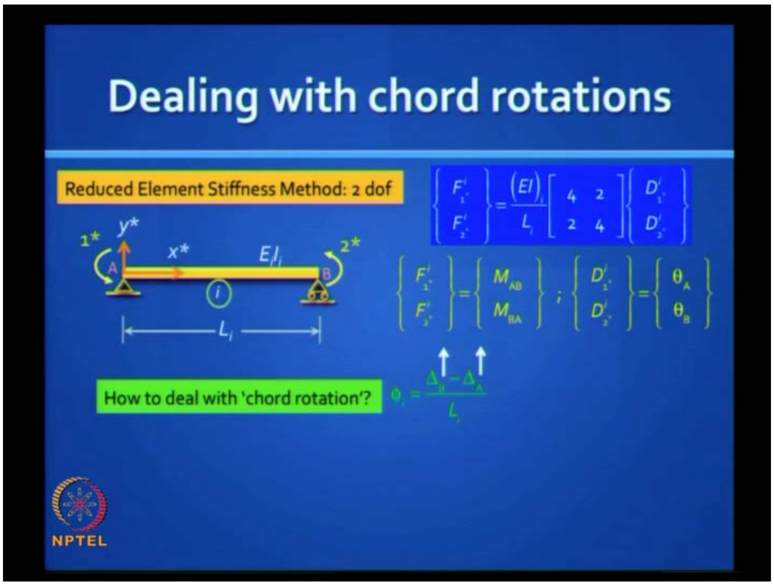

How do we, How did we manage in slope deflection and moment distribution method? Chord rotation. That is right, we use chord rotation. Normally, if you do not have chord rotation, your interpretation in this element AB is, MAB is $F_{star}1$, MBA is $F_{star}2$, theta $_{A}$ is D_{star} 1 and theta $_{B}$ is $D_{star}2$: these are flexural rotations.

If you have chord rotation, that means you have relative movement, differential settlement. Now, we switch, our sign conventions, we say that anticlockwise chord rotation is positive; Vertical reflection upward is positive, not downward. So, DB minus DA both is acting upward. If DB is more than DA, we will have an anti clockwise rotation and phi will be positive. Is it clear?

How do you deal, with this? What happens in a beam, when you have an anticlockwise chord rotation? You get clockwise end moments, so how how can we take advantage, of this? Your stiffness matrix does not change. So, how do you accommodate, that is right. you have to you cannot touch the left hand side. You play with the D_{star}. You have to somehow accommodate, the chord rotation in your in your, displacement vector, how do you do that? So, you have to do it in the, displacement transformation matrix. So, how do you do that?

Theta a minus 3i by….

Theta $_{A}$ minus 5i. So, that is what, you do.

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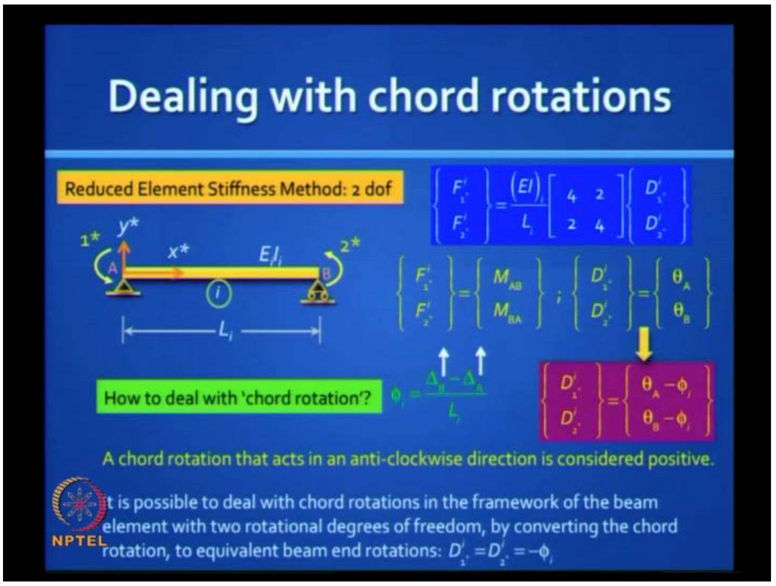

So, it is a clever trick. If you have an anticlockwise chord rotation, just, recognize that the effect it has, is that of a clockwise flexural rotation at the both the ends.

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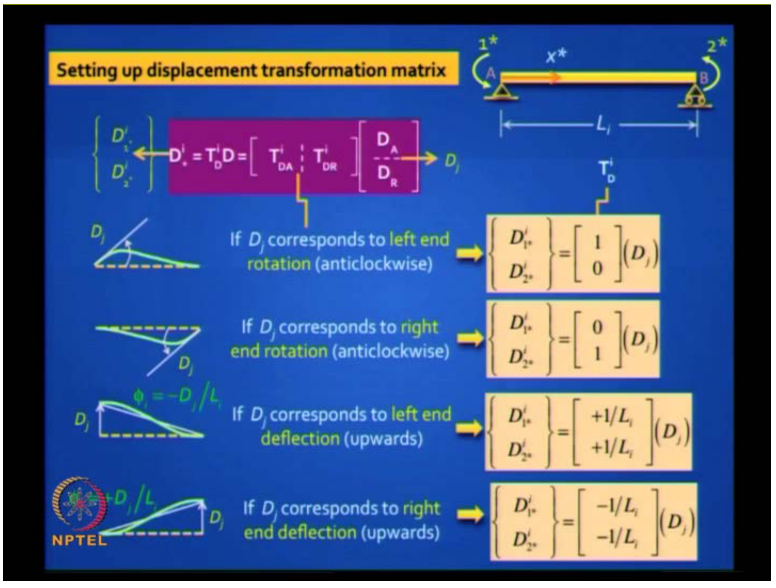

Let us demonstrate, this once, because most people, who have done this, have really not fully understood this. I hope, you understand, why we are doing it? You can go back to our earlier sessions and see how we did it.

So, let us say, you need, you have some, displacement transformation matrix to generate for for this element. Let us do it, component by component; let us say, from you global coordinate system, you have some D_j, that D_j could be a translation; could be a rotation; usually, there will upward or anti clockwise. Got it? You want to write down, the effect of that, on this two degree of freedom element.

There are four possibilities. Let us look at them. Two possibilities are shown here, there are pretty easy. Let us say in your, continues beam structure, you had a rotation D_j anticlockwise. It will either, go to the left or it will go to the right.

So, do you, agree. If D_j corresponds to the left end rotation, obviously $D_{1\ star}$ will be one time D_j and $D_{2\ star}$ will be zero. If D_j corresponds to the, right side of your element, anticlockwise, then it will be zero one. This is clear. This is straightforward. There is nothing special about it, the problem comes, when you have a deflection. So, let us look at that. Let us say you had a deflection, in your structure, in your, continues beam that means your element is going to deflect like this, all other coordinates, all other displacements being arrested. So, how will you write, $D_{1\ star}$ $D_{2\ star}$ for this, if length of the element is end.

See, I wrote something here, you have to write something similar here. For this situation, what will you write here? What is the first component? What is the second component?

D_j by L

NO

0 minus D_j by L 1 minus D_j by L

No

No, D_j is a deflection.

1 by L is your chord rotation. 1 by L is a, because D_j is a deflection. You do not put chord rotation in the global coordinates; you put only deflections. Get it straight.

So, you put 1 by L, now, the question, should you put plus 1 by L or minus 1 by L? So, you have to think, if the left end is going up, then you having a clockwise chord rotation, so you have anticlockwise end moments. So, you should put, plus, because anticlockwise is positive. Well done. The last case is, when the right end goes up, that is it. Is it clear?

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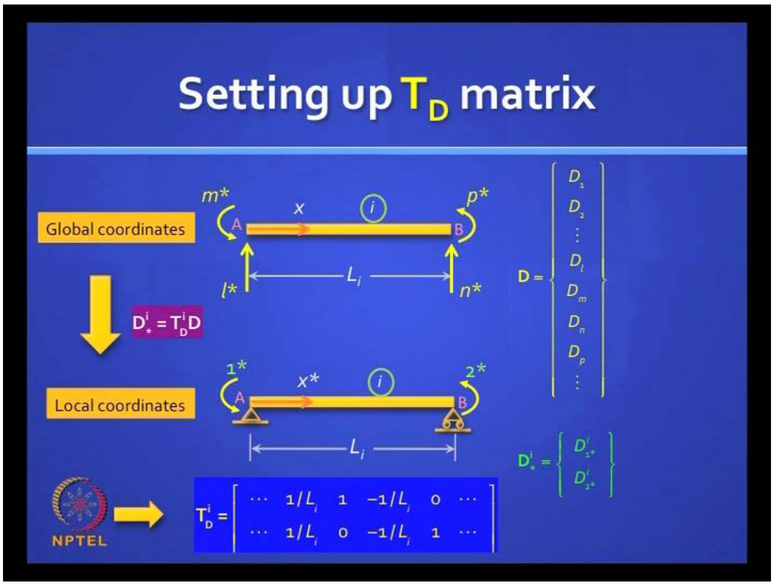

Just to seal, your understanding, let us take a look at this situation. I have shown an element here, which is a part of continues beam. The coordinates are l m n p, it could be anything. So, this part of continues beam and this could be active degrees of freedom, restrained, I do not care. So, they let us say, they come together like this-D_l D_m D_n D_p, at the global level, at the element level I have only 1 star and 2 star. The element coordinates are $D_{1\ star}$, $D_{2\ star}$ and you need to do this transformation. Can you write down T D_i, in terms of L?

So, we are talking of this segment, you will have four terms: 1 2 3 4. Tell me, what those four terms? Are you getting it? What are those four terms?

1 by L 1 0 2by L

one erratum

1 by L 1 by L 1

1 0 1 by L

this is this portion This portion, let us try to figure out.

It is very easy. If I lift this up, that is my first coordinates, D_l by, unity. Do, I get a clockwise chord rotations?

Yes, sir.

Do, i get a clockwise chord rotations?

Yes, yes, sir.

So, I get anticlockwise end moments, which are positive.

So, the equivalent rotation is positive. Got it?

Number two: If I put $D_{m\ star}$ equal to 1, what do I get? I get a unit rotation, on the left end. So, its remains positive and I do not get anything here, $D_{2\ star}$ is zero.

Then I have $D_{n\ star}$ going up by unity, what is my chord rotation? Anticlockwise. So, what are my end moments? Clockwise. Clockwise means, it is minus.

So, do you get the hang of it? That is it. Because, once you have got this, we can proceed ahead. There you can deal with any problem, but it is only, this little bit, you need to understand, the procedure is the same, you have to generate, these matrices. The procedure is identical.

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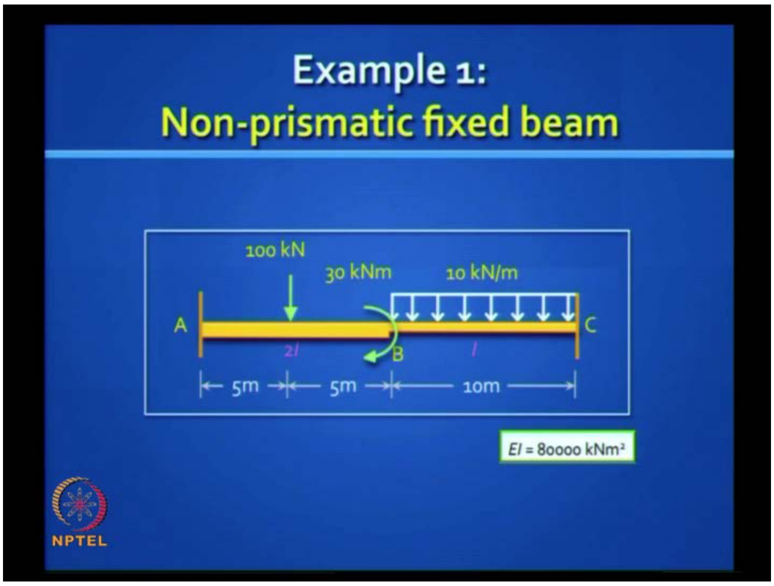

Let us demonstrate quickly, with one example. This example, which we have already done, by the conventional stiffness method.

So, find out the T D matrix, manually generates the fixed end force vector. Remember, we had problems, when you had distributed loads in the axial system. Why, because it can handle only a constant one force. Here, you have a distributed loading, so you, you the two end moments, may may not be the same.

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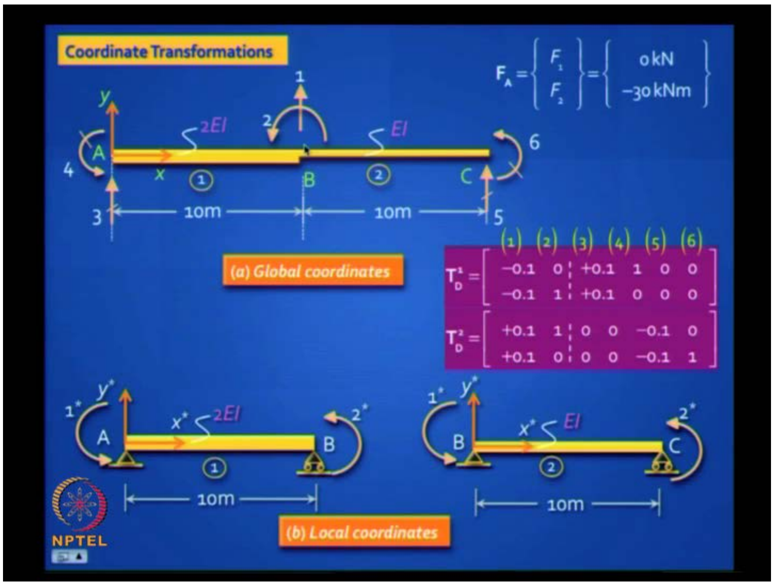

We will demonstrate this and the other rest of the procedure same, fixed end forces is only, manually supreme force or generally….So, coordinate transformation, if you look at global coordinates, these are the same coordinates; we used in conventional stiffness method; we are not changing here. Same coordinates and local coordinates are what we have just discussed two degrees of freedom, write down the transformation matrix.

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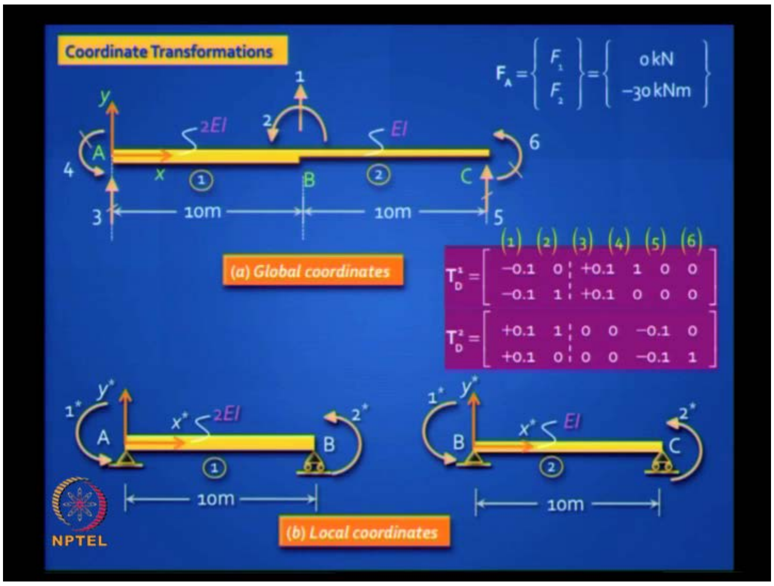

Write down the transformation, there are two matrices; you have to write T_D1 and T_D2. You can to put altogether, also.

So, let me help you, first of all the given loading is, you have a nodal moment 30 kilonewton meter. So, this is what you get. Let us do it together. Listen carefully. D_1 equal to 1, this goes up, it causes an anticlockwise chord rotation in element one and causes a clockwise chord rotation in element two, so when, you have anticlockwise chord rotation, you will get, clockwise end moment. So, you should put a minus sign.

So, it is minus 1 divided by 10, so you get minus 1 divided by 10, for the left one and for the right one, you get plus 1 divided by 10. Is this clear? You all got it.

Am just extending what we did, so you fill up the first column, for which covers both T_D 1 and T_D 2, second one D_2 equal to 1, you have a rotation here. The right end of this will have 1, the left end of this will have 1.

So, 1 1 0, 1 1 0. Is it clear? Clear.

Now, you move to the restrain coordinates, 3 equal to 1, if you lift this up, it affects only element one. You have a clockwise chord rotation element one, so anticlockwise that is positive. So, you get plus 0.1, plus 0.1, nothing happens to the second element.

Number four: If you give a rotation A it effects only D $_{1\ star}$ 1 in the first element, nothing else happen to 1 0 0 0.

Number five: You lift this up, you have anticlockwise chord rotation in the second element, clockwise end moments, so equivalent flexure rotations will be negative 0.1, 0.1, in the second element, nothing happens. Is this clear? and so on, so far.

Can you do this, correctly? You have done. So it is a powerful technique of dealing with deflections, converting them to equivalent flexural rotations, through the displacement transformation matrix. We do not touch the stiffness matrix, so if you got this step, correct? It is easy.

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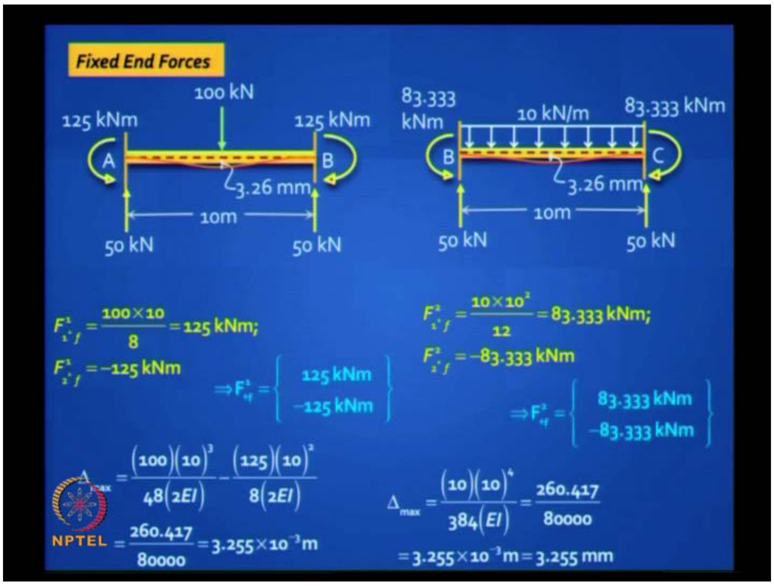

Fixed end forces, we have done earlier. So, you can find the fixed end forces, find the deflections, if you want, we have done this, earlier. Put the fixed end force, (Refer Slide Time: 42:30) in a vector form. You got this diagram, so, this you have to directly get. In conventional stiffness method, you could assemble it, doing the contra gradient principle.

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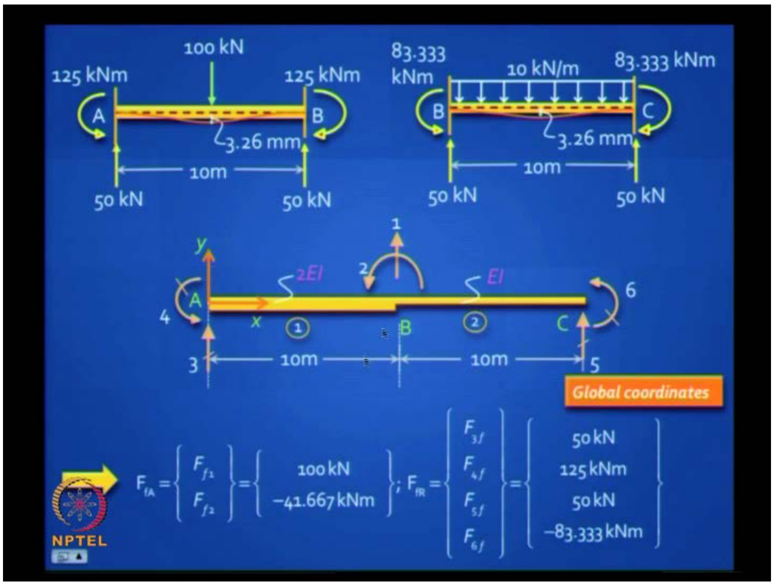

You can put, T_i transpose. Here, you cannot do that. Here, you have to do it manually, that means you draw these sketches and then you figure out. I want to find out F_1. So, F_1 will be 50 plus 50, 100. Is it clear? I want to find out F_2. F_2, is this coordinate that will be, 83.33 minus 125. So, minus 46.

I want to find F_3, that is, this force that is plus 50. I want to find F4, that is plus 125, anti clockwise I want to find F_5 , that is plus 50. I wanted to find F_6 that is (( )). Is it clear?

You have to do it by inspection. No, contra gradient principle, when you are using the reduced elements stiffness method, but this is ok, in fact this method is, more for manual use.

Yeah, that is nice. We also, do it here by the way. Here also, we write F star. the this does not change but, we have only 2 here. There we had a 4 by 1, we including 50 50 also. We did the T_i transpose and we got the linking coordinates and then we did slotting. It was all done mechanically, here you cannot do that. You have done it with your eyes, open. Is it clear?

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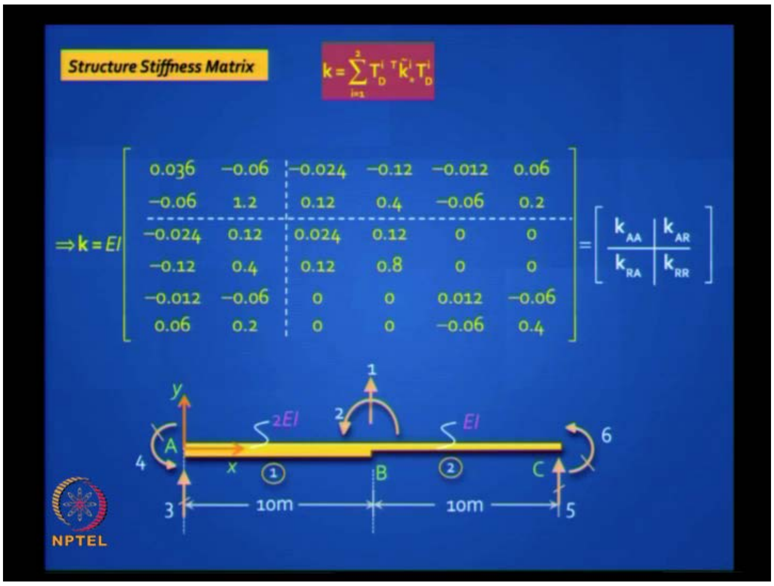

Then find the net load vector, from now on, its same as conventional stiffness method. Because, you you got beyond the element, so this part is common and of course the element stiffness matrix derivation, you have to do carefully.

So, you have two elements, span is same, El is changing. Very easy to write down: 4 2 4 2, 4 2 2 4 EI by L. What did you do after you get a K star, you do K star T_D. T_D you have already got, do not have to write any linking coordinates, because of all the global coordinates, in one go. Then, what did you do? You pre- multiply this with T_D. You just sum up, you got the same structure stiffness matrix, which you generated, in the conventional stiffness method.

You manage with less coordinates, then this solution is, ditto, same as what you did in conventional stiffness method. Because, you have to invert the K, you have to find so, that we are just reproducing, what we did earlier.

What you do next? Find the support reactions; same equation. You can check equilibrium. Then last step, member forces you have to do little carefully, you have that you have done this, you remember, when you computed the structure stiffness matrix, you first did K i star T_D i, that comes in handy here. Because, that is what you inverse here. .

And plug in those values will get the same answers. Please, go through this problem solved in the book also, exactly same solution. Is it clear? So, we demonstrated one application, we will do all the other applications, as well in the next class. Thank you.

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