Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of Technology, Madras

Module No. # 5.1

Lecture No. # 27

Matrix Analysis of Beams and Grids

Good morning, this is lecture number 27. We are starting a new module, module 5, matrix analysis of beams and grids.

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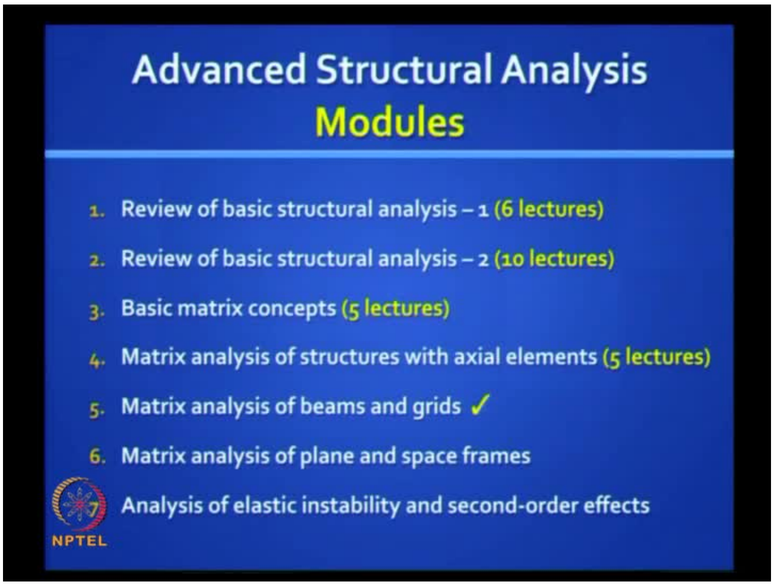

So, if you recall, we have finished 4 modules. In the last module, we showed how the stiffness method and the flexibility method can be applied to structures with axial elements. Now, we look at beams and grids.

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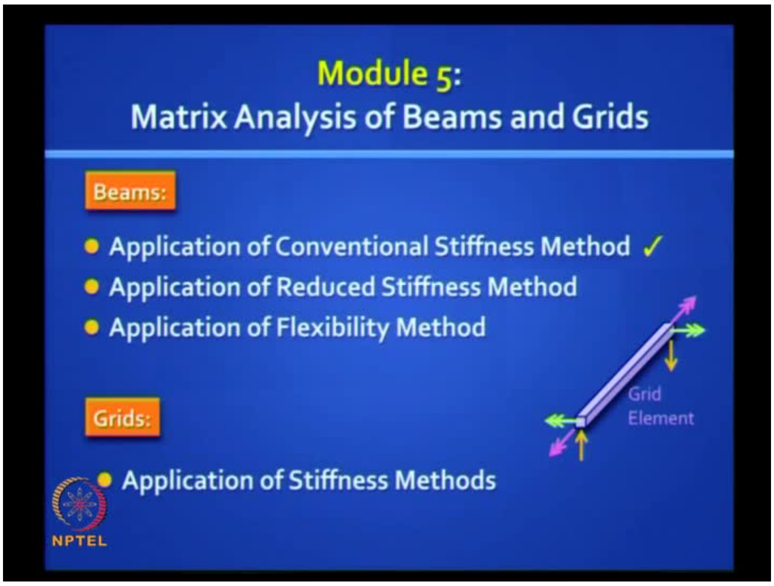

Here, again we will show how the conventional stiffness method, reduced stiffness method and the flexibility method can be applied to beams and we will restrict the application of only the stiffness method, especially the reduced stiffness method to grids.

What is a difference between a beam element and a grid element? What is a difference between beam element and grid element?

I have shown you a beam element here. There are only 2 internal forces or force resultant, that any section. One is a shear force; the other is a bending moment. The bending moment here is shown in the vertical plane and you get shear forces only if you have a variation in the bending moment. The grid element is this element plus something extra, what is it? Torsion, that is right. So, that is a grid element. Grid element is the beam element plus some torsion; we will look at this at the end of this module.

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This is covered in the section on beams and grids in the book on advanced structural analysis.

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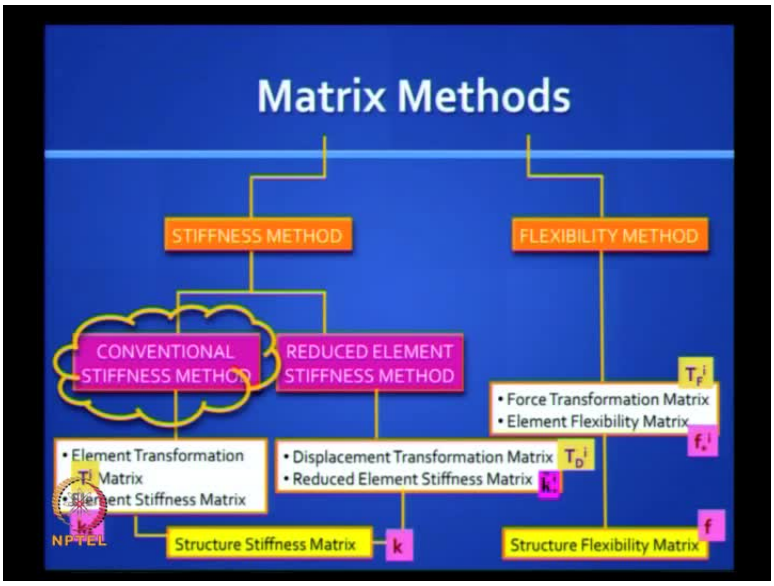

So, as in earlier cases, we have 3 methods: conventional stiffness method, the reduced element stiffness method and the flexibility method.

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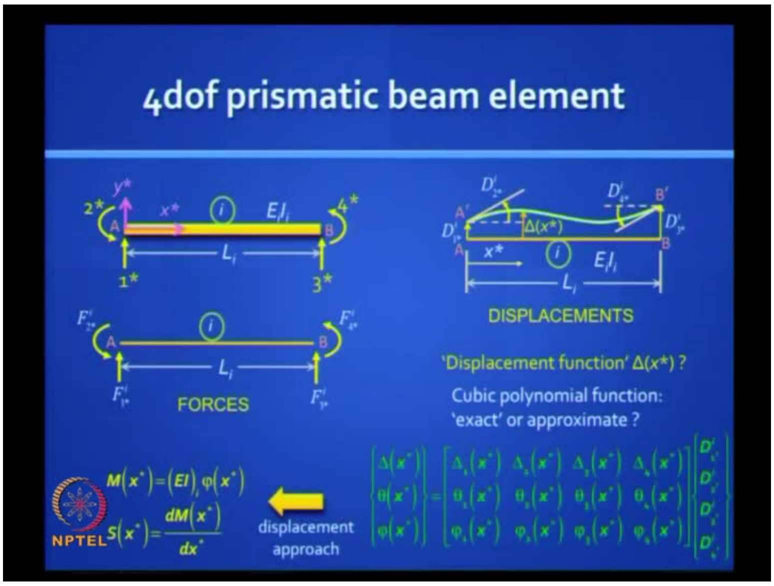

We will begin with the conventional stiffness method. It becomes easier now because you already have a taste of this. We saw how it was applied to trusses, but now with higher order elements, you will find it, the work is, yeah, the work is little more because element stiffness matrix itself becomes larger. Imagine a space frame element; it is a 12 by 12 element stiffness matrix, so we will keep that to the end, we will move slowly.

Here, you have 4 degrees of freedom; we have looked at this element earlier. Now, this is a prismatic beam element, length is L i in the ith element, flexural rigidity is constant EI, can you write down from 1st principle from the physical approach, can you write down the 4 by 4 element stiffness matrix for this element?

I will show you how this can be derived in many ways, but you already know the answers; you can try drawing the 4 sketches, just write down, that 4 by 4 element stiffness matrix. For the sign convention we are following here 4 degrees of freedom. 1 star and 3 star in the local coordinate system refers to translations or deflections at the 2 ends; 2 star and 4 star refer to end moments or rotation slopes. If you recall, when we did the displacement method initially, when we looked at the historical development of this method, when we looked at slope deflection method and moment distribution method, the sign convention was anti-clockwise positive, sorry, clockwise positive.

Why did we switch to anti-clockwise positive in the matrix method? Because we wanted to follow the laws of vector algebra or Cartesian coordinate system, we choose as X, Y, Z and i cross j must be equal to k, so that is why we did the slight switch, but do not get confused.

Also, if you recall, when we did slope deflection method and moment distribution method, we did not really include the deflections as in a slope deflection equations as unknowns, except when they were known chord rotations. But in the stiffness method, we include everything except in the reduced element stiffness method. So you have a 4 by 4 element stiffness matrix and I want you to generate it on your own.

Well, you must realize that we have restricted the degrees of freedom only to the 2 ends of this element, is that justified? If you take any location X star in that beam, that point inside that beam, that has a deflection delta X star, can we write the delta X star in terms of D 1 star, D 2 star, D 3 star and D 4 star as shown? Only then, only then we are justified in limiting the degrees of freedom to 4. Can we do this? How do we do it, how do we do?

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Well, in a classic displacement approach we do not know moments. Moments are at the tail end of the derivation, we do not know moments. So if I have to go by this argument, then it follows, that the displacement function delta X star can be a polynomial, but of what order?

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It does not depend on loading, this is displacement method.

If I know D 1 star, D 2 star, D 3 star, D 4 star in terms of these 4 end displacements, can I write an expression for delta, that is a displacement approach? I can, I can write a polynomial equation, yes or no, what will be the order of that polynomial?

Let me make it even simpler. Let us say, you are doing a laboratory experiment, you are trying to discover some relationship between Y and X, X is your independent variable, Y is your dependent variable. You are able to do 4 experiments, you play with the value of X, you have X 1, X 2, X 3, X 4 and you get 4 values of Y: Y 1, Y 2, Y 3, Y 4. When you try to plot and you hope, that you get a smooth relationship.

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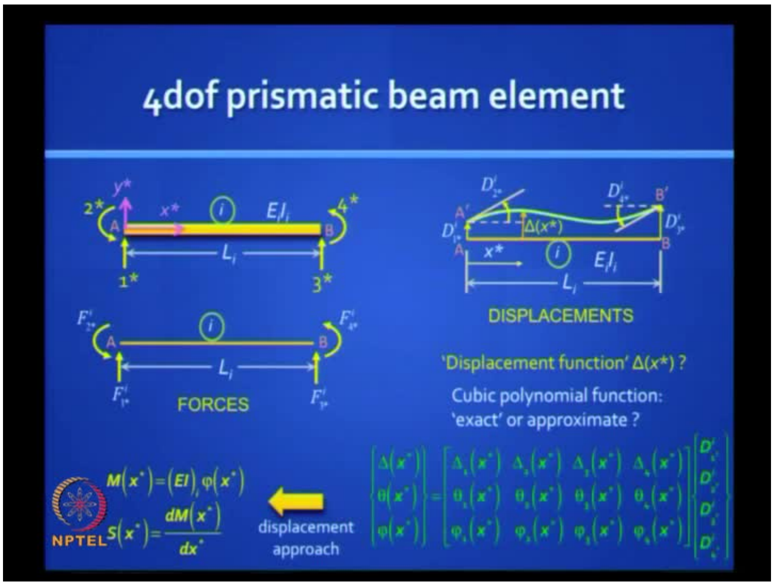

If these 4 points are exact, you would be able to get some curve; you can try to fit a polynomial. What is the highest order of polynomial you can fix with the data that you have? You have 4 points, if it is a straight line, how many points do you need to, so you got 4 points? So, it is a 3rd, it is a cubic polynomial, so this is a kind of thinking that you need to develop.

Here, you have, you want to write an expression for deflection at any location within the beam. Delta as a function of X star, you are making a statement, that all I need to know are the end moments. That means I need to know the deflection and the slope in the direction shown at the 2 ends. With that information alone, I can write down an expression for delta. So, if you do by the displacement approach, what is the order of the polynomial that you can do? It is going to be a cubic polynomial.

The next question now we ask is - is that true or is it approximate? We are now digging a little deep into the subject, you will find, that finite element analysis, which is something you will study in an advanced course, use such displacement functions to develop the theory.

You have a complicated structure, you break it up into small parts, they are called finite elements and you try to reduce your degrees of freedom to the moments at the ends of the element, and you have, you have to interpolate to, to get information within the element. And if you are lucky, you got an exact formulation, in which case you get an exact solution.

Now, let us talk about finite element analysis of a plane frame. You have beam elements or let us say plane frame elements. The usual assumption is, if you have an approximate displacement function, the finer you make your mesh, the more accurate your results. But is that what we do when we do a plane frame analysis or even a space frame analysis? Is it enough to take just the beams and columns from the beam column joint, from one joint to the other joint and have just 1 element? Or do we need to divide it into, say, 10 elements, one single beam or column into 10 elements in the interest of greater accuracy? No, you will find in practice people just limit it to single element and they get the exact results, which means, the displacement function on which you are deriving your stiffness matrix from 1st principles must be exact.

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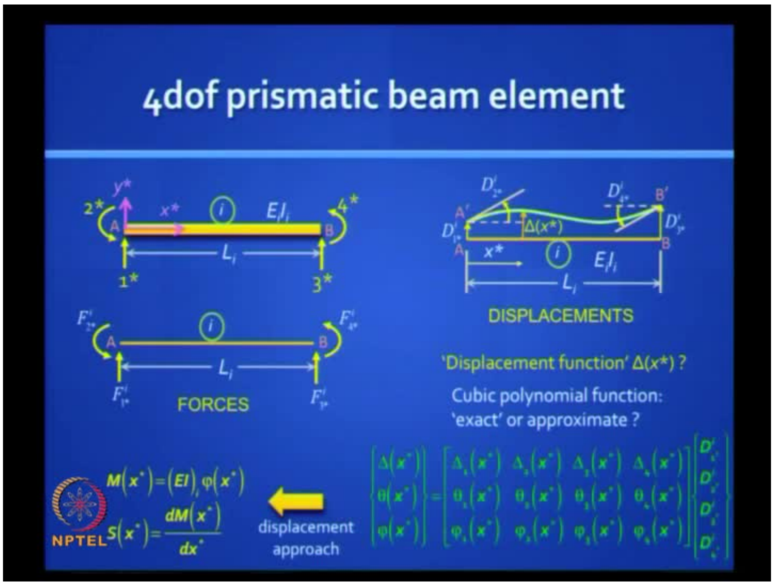

Why is the displacement function exact in the case of a prismatic beam element? Incidentally, we can do a similar derivation for the truss element, we have a linearly varying displacement function because the strain is constant if you have a constant force, so that is justified here. How is it justified?

So, I want you to think, you have to finally bring it to bending moments. In the slope deflection method of analysis, we were happy with the end moments being unknown and we write the end moments in terms of slopes and deflections and we get the end moments and then, we get the moment in between, at any point, by interpolating the end moments because your beam is reduced for a simply supported beam, agreed.

So, what is a variation of bending moment? That linear, as long as you do not have intermediate loads, so we are trying to get rid of intermediate loads in all displacement methods through equivalent joint loads. So, your bending moment can have 2 different values, here also you have F 2 star and F 4 star, they need not be the same, so we have a linear interpolation between the 2 end moments to get the bending moment at any section, can you use that?

Curvature is linear.

Curvature is linear, so slope is quadratically varying and so deflections, so there is a, you are dealing with an exact function; good.

So, if you could write the deflection at any point, the slope at any point X star and the curvature at any point in terms of those 4 end displacements using some functions, which are called displacement functions, sometimes called trial functions in finite element analysis, because in a plate element for example, you do not have it exact. So, it is a, it is a good approximation, that you make, so it is called a trial function. This is no trial, this is exact and we will prove it.

So, those are displacements and these are the forces and there is a relationship from the displacements, you can get an expression for curvature. Then, if you have a cubic variation for displacement, the curvature will have a linear variation. If you multiply the curvature at any location by the flexile rigidity EI, what do you get? You get bending moment, this is a displacement approach and if you take the derivative of the bending moment, you get the shear force.

And so, if you apply the boundary conditions at the 2 ends, 2 bending moment and shear force, you get F 1 star, F 2 star, F 3 star, F 4 star, so there is a beautiful relationship. Let us work on this.

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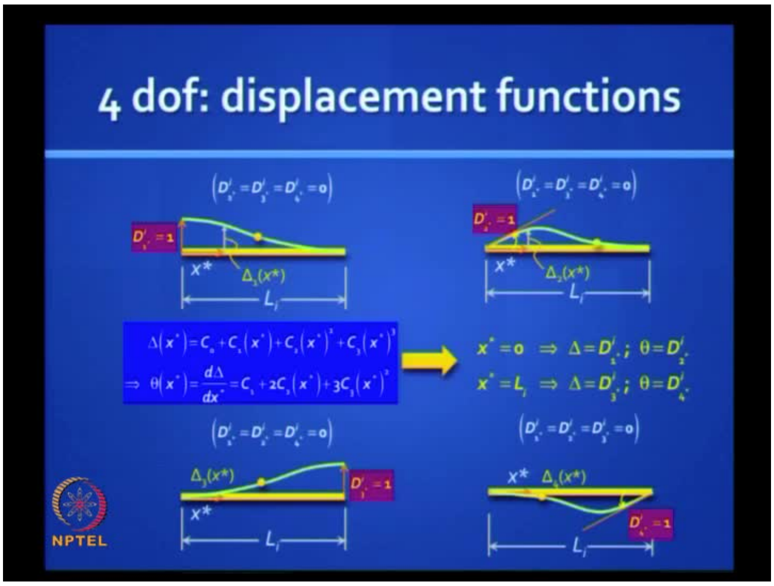

So, let us get the physical meanings of the 4 columns in your stiffness matrix. In the 1st column you apply D 1 star equal to 1 and you arrest the other degrees of freedom, this is a picture that you get.

The next one you apply D 2 star equal to 1, we have done this before, so that is a picture you get.

Next, you apply D 3 star equal to 1; D 4 star equal to 1; you get those same deflected shapes flipped over, either laterally or vertically.

From this, can you pull out the definitions of stiffness coefficient? Well, you can, but first you write down an expression for deflection. You have 4 boundary conditions, write down an expression for slope, apply the boundary conditions and you can actually get equations for these deflected shapes. I would have shown you 4 deflected shapes and I, making a tall claim, the claim is any arbitrary deflection in that beam, can be obtained as a weighted average of these 4 shapes. Let us prove it.

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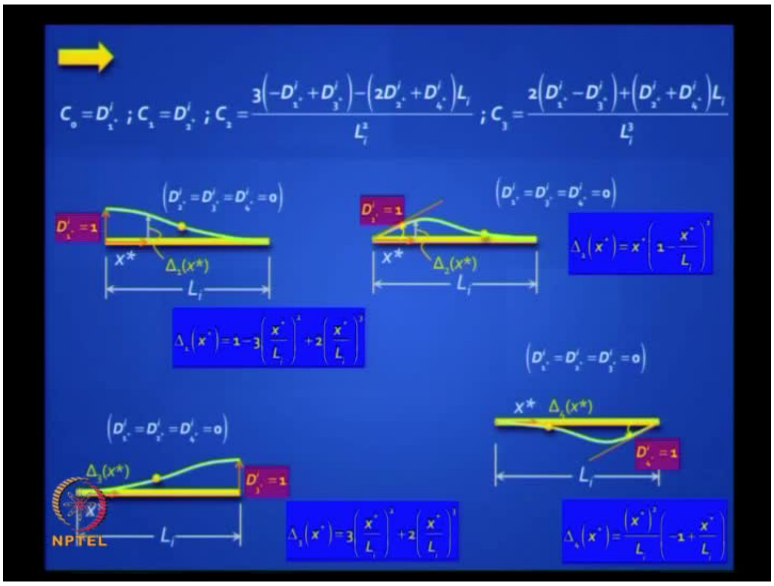

Well, you have, you can substitute the boundary conditions. Which boundary conditions? That at the 2 ends, when X star is 0, the deflection is D 1, D 1 star and when X star is L, the deflection is D 3 star; that is the definition of deflection.

Similarly, you can write for the slopes and so you can actually go through 1st principles. I am not asking you to do it; I am just giving an introduction to what you will need to do later in finite element analysis. You do not need to do it, but this is the theoretical background, you can actually generate these equations, not bringing any statics in to the picture. We did not even bring flexural rigidity here, shear geometry, shear curve fitting. I have 4 expressions, all of them are cubic, they, technically they are cubic Hermitian polynomials.

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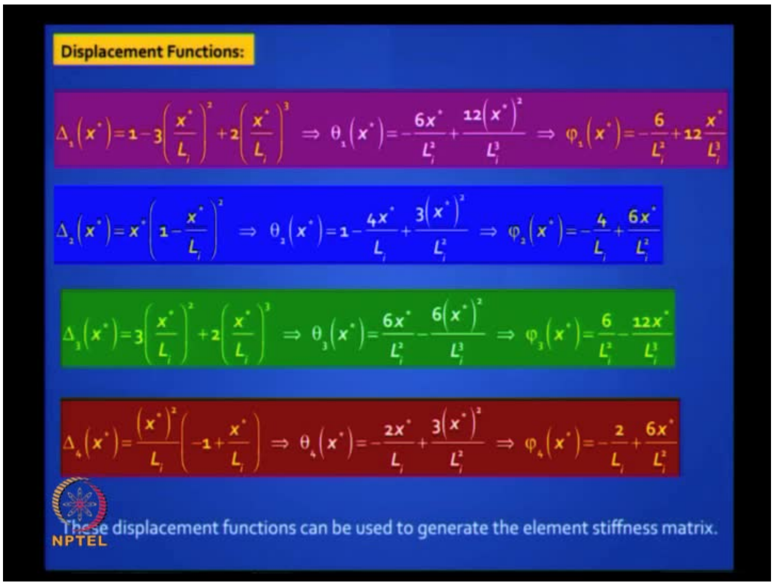

And these are those 4 expressions, you do not need to memorize them, but you should know that they can be derived from 1st principles by simple mathematics using the polynomial function. You have 4 constants, 4 boundary conditions, either the deflection of the slope is 0 or 1, plug them in, solve the, for these equations. You will get, so you get an expression for delta; you get an expression for theta; you get an expression for phi. The curvature, curvature will be varying linearly; slope will be varying quadratically and deflection will have a cubic variation. These displacement functions can be used to generate the element stiffness matrix from 1st principles.

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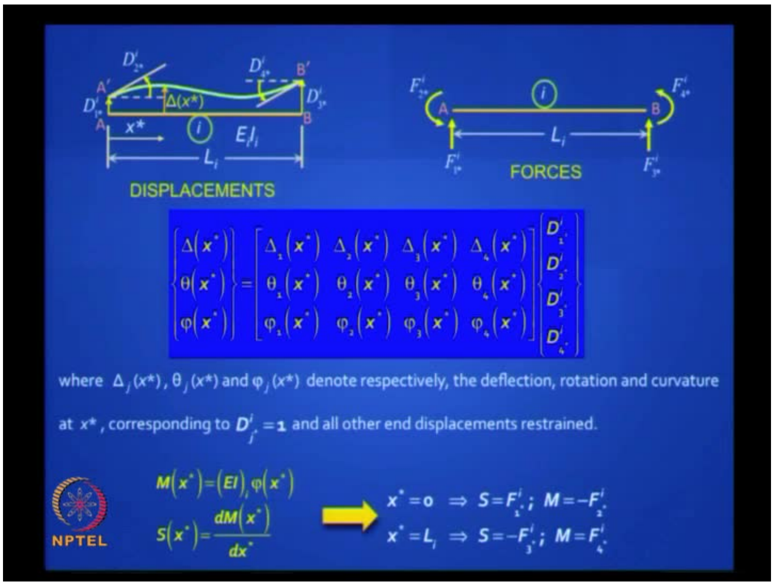

How do we do that? So, you, we have these functions and we have these relationships for, from relating bending moment to curvature, plug in those boundary conditions in terms of X star equal to 0, F 1 star will give you a shear force, F 2 star will give you a moment and so on. So, if you plug this in, you can actually generate the solution.

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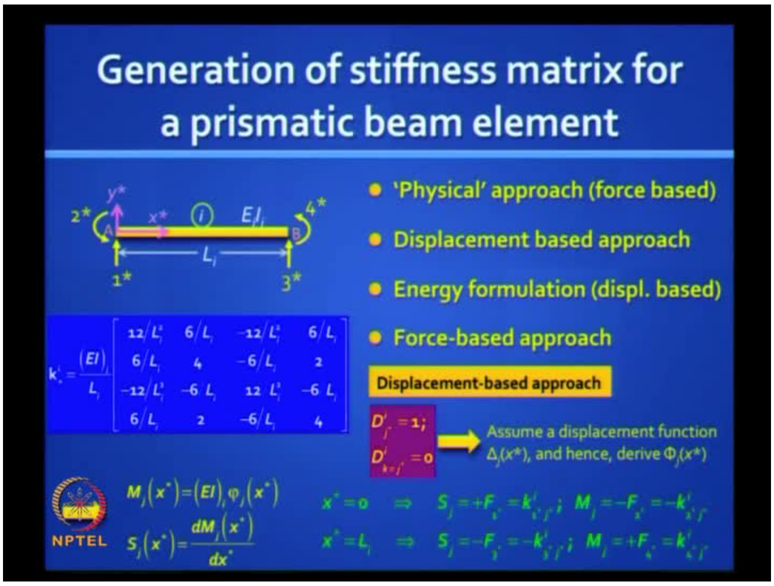

So, there are many ways of doing it, this is one way. It is, it is not the best way, I mean, for people who are very comfortable with statics, but let us see. You had a non-prismatic beam element, a tapered beam element, then such methods will be very effective.

So, there are many ways of doing it, one is the physical approach, which is something that I would like you to do. You, you know exactly how it is going to behave; you can write down the stiffness values. The other is a displacement based approach, which we just took a look at; 3rd is an energy formulation, which is also displacement based and the 4th is the force based approach. At least, I want to cover the theory behind these 4 approaches, but this is what you would get in the physical approach. Let us check it out.

Take the 1st column, the 1st column is what you get when you apply D 1 star equal to 1, so let us look at that. If I lift this up by unity, but I do not allow a slope there and I do not allow any deflection near any slope here, you can visualize the behavior, you get, you get a chord rotation. When I lift this up, I get a chord rotation, I will get a chord rotation; what is the value of the chord rotation?

Chord rotation is only joining the 2 ends with the straight line that is the meaning of chord. You are right, flexural rotation is 0, chord rotation, so it is 1 by L is a chord rotation, clockwise or anticlockwise? The chord rotation is clockwise; if I have a clockwise chord rotation, what are the end moments? I get anticlockwise, which means they will be positive. And what will be the value of those anticlockwise chord moments, 6 EI by L square.

You remember we derived all this and that is the reason why F 2 star in this 1st column is 6 EI by L square and F 4 star is 6 EI by L squared clockwise, is positive in this new sign convention and if these end moments are known, then the shear forces are the sum of these 2 divided by L. You will always, so you will get 12 EI by L cube and the diagonal element will be positive, the half diagonal will be negative.

Now, you take the 3rd column. In the 3rd column, you lift up D 3 star, what is the chord rotation you get? Anticlockwise minus 1 by L, so you get clockwise end moments, so the, both the end moments will be negative, that is why you get minus 6 EI by L square; minus 6 EI by L squared shear forces will be plus and minus. Remember, the diagonal element will be always positive of diagonal.

So, I have given you a simple technique, where you do not need to do anything but use your brains and fill up the, can you fill up the 1st and 3rd, get it.

The 2nd column let us see. 2nd column is, imagine you are giving a unit rotation here, can you see, remember the deflection shape, it is a simple shape, you arresting all the degrees of freedom. What is the moment that you need to get that unit rotation? 4 EI by L. What is the carry over moment at the other end? 2 EI by L l, so both are positive; 4 EI by L here, 2 EI by L here, and they are both anti-clockwise, so we will have a couple, we will have a couple. What is a shear force you get? 4 plus 2 6 EI by L square and you should know, which is positive and which is negative and likewise, you can finish the 4th column.

The 4th column you give a unit rotation, this is a physical approach and I want you to be strong in the physical approach and it is exact, you know that.

Another interesting thing you can notice is, this is a beautiful square symmetric matrix, so square symmetric matrix from the physical approach force based we can derive it, but we can also derive it from the displacement based approach. Assume the displacement function plug in those boundary conditions, which we discussed, you will get the same answer.

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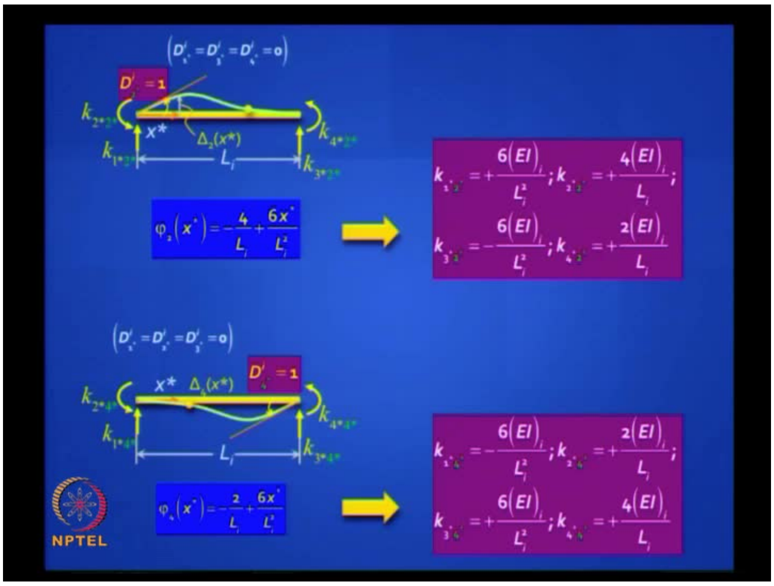

So, these are the definitions of k 1 1 star, 2 2 star, 3 1 star, 4 1 star. I have put in green color, green color; the 2nd subscript because that is where I am applying the unit displacement, is it clear.

So, it is, you remember when I have k i j, the cause is always in the 2nd term, the effect is at the 1st term location, so when I apply D 1 star equal to 1 here, this 1 star will appear in all my stiffness coefficients in the 2nd term, and the 1st term matches with the coordinate, that I am dealing with. Is this clear? This is a physical, meaning, I have the curvature from my earlier derivation and I plug in those boundary conditions, I get these answers. If I do the same thing for D 3 star equal to 1, which is the same deflected shape flipped over, I get the same equation from 1st principle, by the way without, without… How did I do this? You do it, I have done it, it is there in the book.

You have got the curvature equation, you put X star equal to 0, you will get the 2 end moments and X star equal to L, plug in those boundary conditions, there you have stiffness coefficients. You are finding it difficult? See, this we have derived, if you multiply this with EI, what do you get? Bending moment, put X star equal to 0, you will get the moment here, sagging positives. You have to put the correct sign, put X star equal to L, you will get the moment here, you derive, you take the derivative of the bending moment expression, you get the shear force in that expression; put X star equal to 0, you will get this quantity. And put X star equal to L with the minus sign, you get this quantity. That is the way to do it in the displacement method; did you get it?

I just want you to get the hang of the theory for future applications, got it. So, this is the displacement approach. We calculate the bending moments and shear forces not in beginning, but at the fag end after we get the curvatures. And so, you can do this derivation; for all the 4 cases you get exactly the same designs.

So, we have done 2 alternative ways of deriving this element stiffness matrix, one is the forced approach. Physically we understand what is going on, we just write down those values.

Second is, we pretend we do not know statics and we know only geometry, we know only kinematics, we derive shape functions or displacement function. Take the derivative of it, get the rotation; take another derivative of it, get the curvature multiplied by EI, get an expression of bending moment, plug in the static boundary conditions. Take the slope of that expression of bending moment, get an expression shear forces, plug in that 2 extreme values, put the boundary conditions, I get the same.

So, it is beautiful and this is how it is done for difficult problems. If you want to do 1st principle displacement based approach, this is the basis for that.

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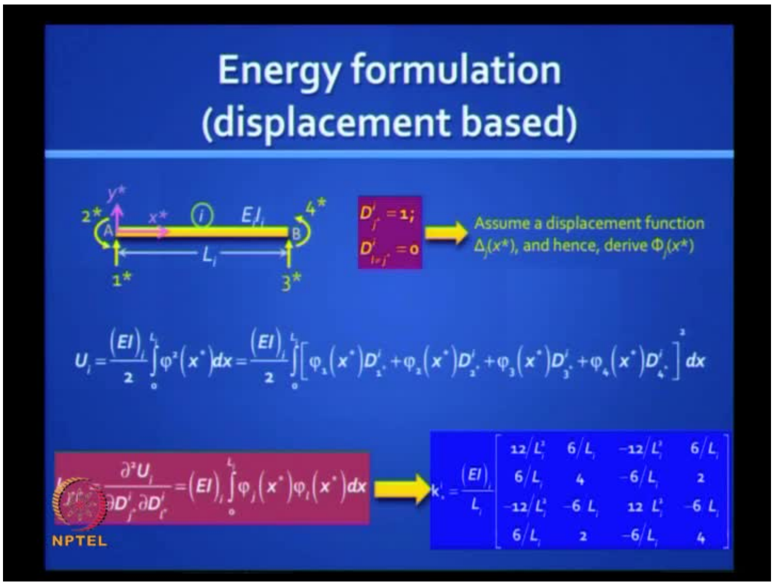

There is an energy formulation, it is discussed in the book, you can read it in the energy method. You have to write an expression for strain energy. Once you have a displacement function and in this case you have got an exact displacement function, you got an exact expression for strain energy.

Now, you know that you can pull out the stiffness coefficient from the strain energy by the mixed partial derivative. You do this; you will get the same matrix. This is, and by the way, in finite element analysis, this is commonly done, this is the energy formulation.

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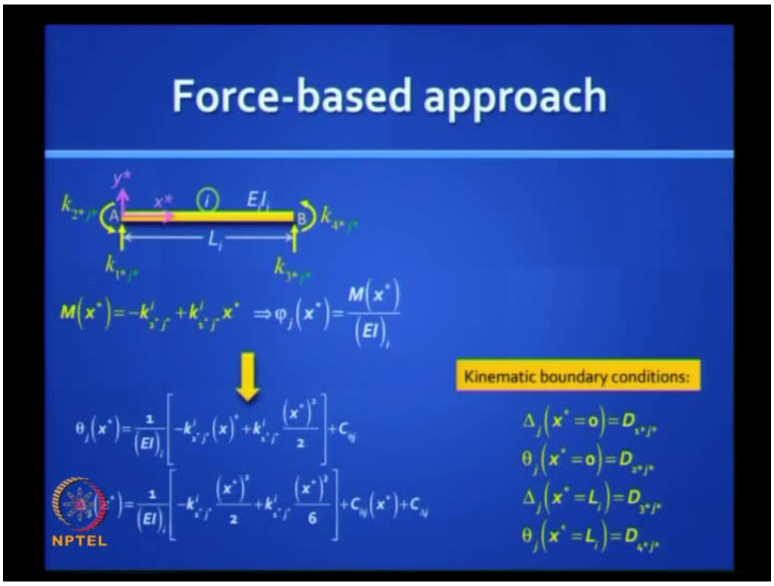

And lastly, you have a force based approach, where write down an expression for bending moment at any section considering of free body. Can we do this?

Take a free body and in terms of a stiffness coefficient, can we write an expression of bending moment? See, your support reaction is k 1 star j star, where and your end moment here is k 2 star j star. If I cut a section here, the sagging moment expression, will it not take this form? That is first principle.

Then I have, I derive an expression of curvature by dividing my bending moment with EI and from that curvature I integrate, I get an expression for slope. I integrate the expression for slope; I get an expression for deflection.

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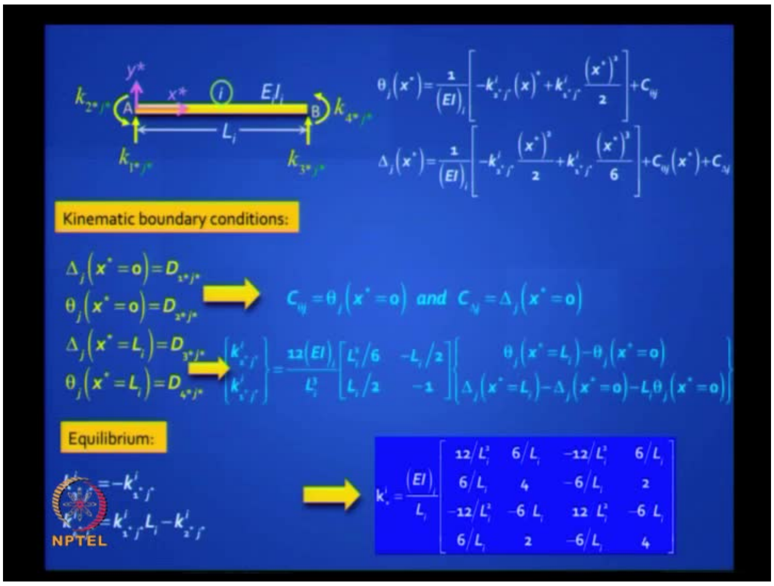

I apply my kinematic boundary condition at the 2 ends for each of those 4 cases, you know I keep doing this, apply the boundary conditions, put equilibrium, I will get the same matrix.

I am not asking you to do all these 4 methods. I will be happy if you remember the physical approach, but now see the power of understanding the structural behavior. First principles you can have, you have alternative paths to dealing with the same problem and the wider and deeper your understanding, the more you will enjoy this subject, is it clear.

We have just added, put together all that we have learnt till now, that for a simple beam element, we would not repeat it for the space frame element, that it is the same logic. Is it clear to you?

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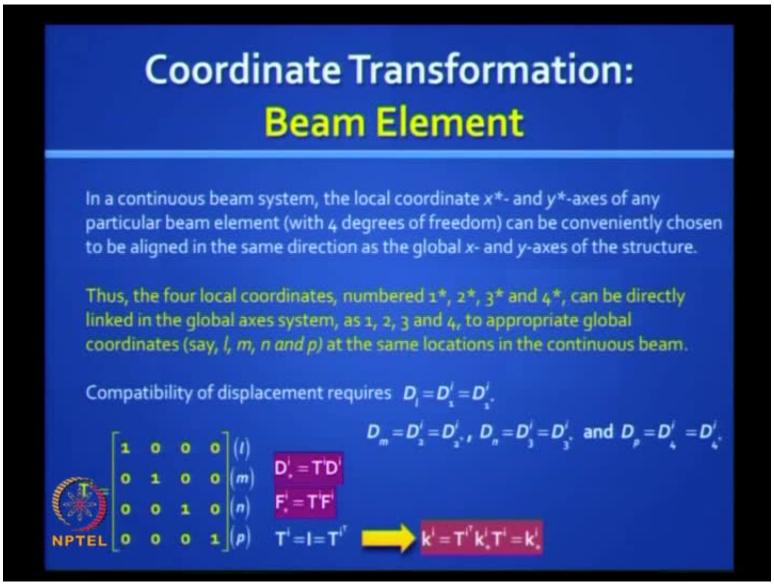

Now, it is time to play games. We have to show how we can apply this knowledge to solving problems as far as coordinate transformation is concerned. In the conventional stiffness method, you do not have to worry at all because what are the kinds of beam problems you get?

They are usually fixed, non-prismatic, sometimes continuous beams, beams with overhangs. The beauty is, all of them are in one line, so it is like your axial element, but it is not one-dimensional, why not?

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Yeah, the definition of a 1-D element is not only the element should be in one line, but the… So, in axial forces you have 1-D, so you, a beam is a planar element because you have to bring in X and Y coordinate, so loads are in the X-Y plane, is it clear.

So, in a continues beam system, the local X star and Y star axis of any particular beam element with 4 degrees of freedom can be conveniently chosen to be aligned in the same direction as the global X and Y axis. So, it is like, so like you are, so what is the advantage of this? Your T I you still need it, but it is easy. What is the T I matrix? It is an identity matrix.

Thus, the 4 local coordinates, numbered 1 star, 2 star, 3 star and 4 star can be directly linked in the global axis system as 1, 2, 3 and 4 to appropriate global coordinates, which could be LMNP or whatever. So, the compatibility is as shown here and this is how you will write the transformation matrix. It is an identity matrix; you have to correctly write the global coordinates. In this case, I have shown it as LMNP, it could be whatever you get in the structure

So, the beauty about the identity matrix is, the transpose of this matrix is also an identity matrix and so, if you do the, if you try to convert the element stiffness matrix from the local coordinates to the global coordinates, you get back exactly the same matrix, except along with it you will get the linking global coordinates, which is crucial for you to do the slot wise adding, is it clear?.

So far you see, how when you first read about this slotting and all you found it difficult to understand, but slowly now things are falling in place, but we went slowly. We first did axial element, which has only 1 degree of freedom, 2 degrees actually in the conventional stiffness method. Now, you got 4 and bigger ones are on their way. So, slow and steady**.**

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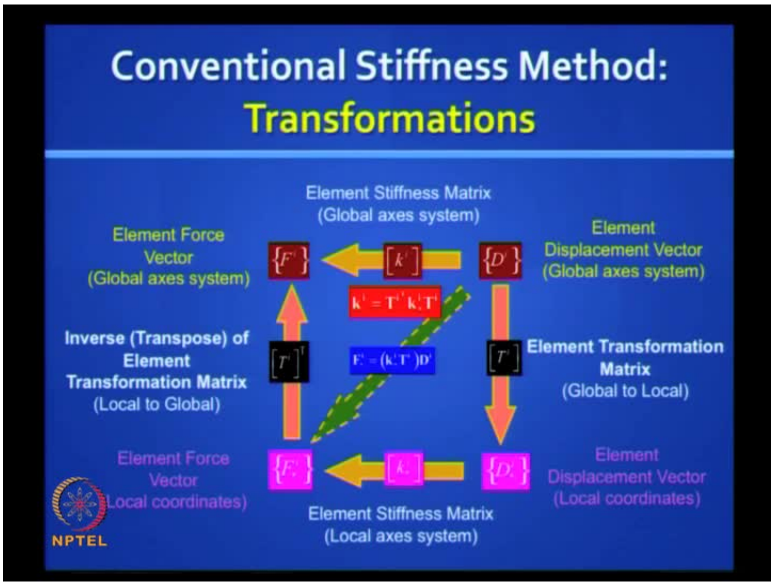

This is now a familiar playground, we have played soccer here quiet a number of times, we are familiar with all the goal posts. You have that element level, you have the element level at local coordinates, element level in global coordinates, you are familiar with these transformations.

This is the basic transformation, T I from global to local and then, if you take the transpose of that, you get from, from local to global and then you have the diagonal. And then you have the, this is the transformation to get the element stiffness matrix in the global and then, all of them you should correctly put together and assemble the structure stiffness matrix.

There is another method called, that using the displacement transformation matrix, where you directly deal with all the global coordinates in one go, but would be dealing with big matrices. We will show both, but when you are doing programming, you are dealing with large structures; this is the way to do it.

In your examination you have the choice, whichever like you can do, but whenever possible, do not do the conventional stiffness method, reduced element stiffness method, you know, you will deal with much smaller matrices; you can finish the problem much faster.

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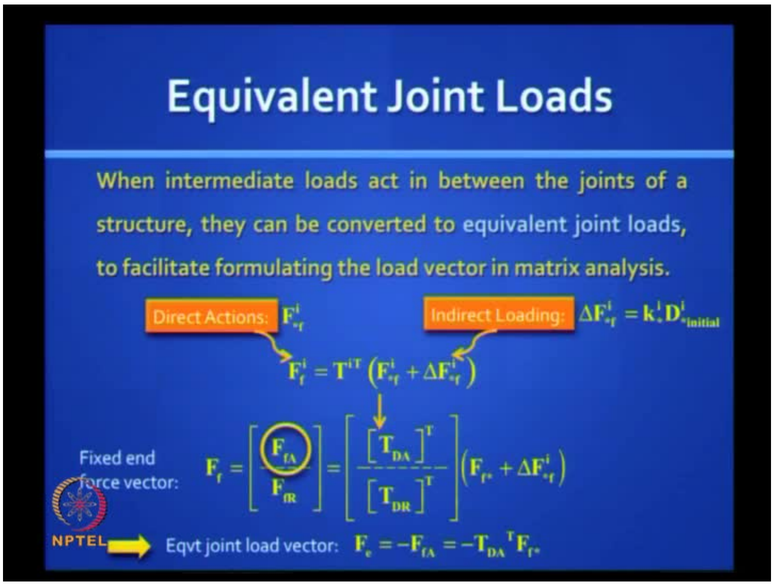

Then, you need to deal with equivalent joint loads. We are comfortable with beams because we have already done slope deflection method, moment distribution method, we know how to find fixed end moments; only thing, we are now using a word fixed end forces because you also have to find the shear forces. Earlier, we just found the end moments, now you need also the shear forces, so these are equivalent joint loads and you know how to…

You have another possibility, sometimes you have indirect loading. Support settlements will cause indirect loading in reduced element stiffness method, not in this method, but you could get not so much in, in beams, but you could get in grids, you could get in plane frames. You will have temperature effects, creep effects, shrinkage effects, so you have to add any additional moment, that you could get from indirect.

In beams you would not get in grids, you can in plane and space frames, you can, then you know how to do the shifting, from the element level coordinates to the global level coordinates, that is all.

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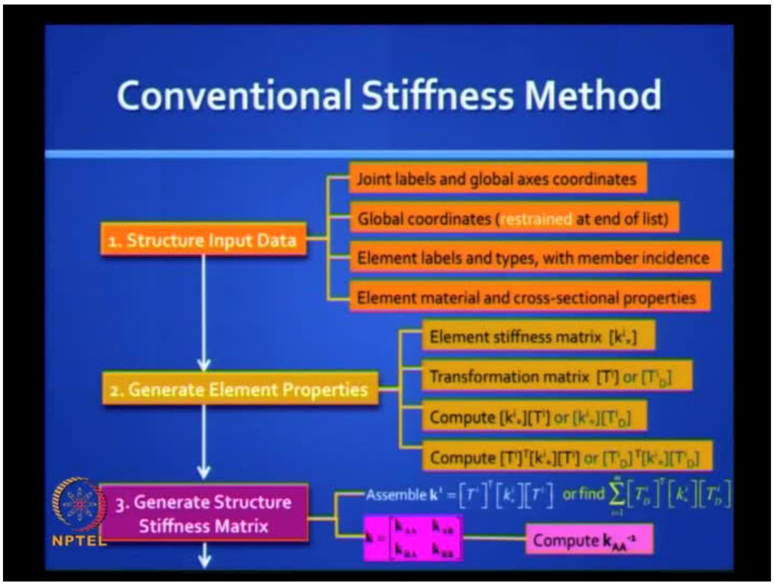

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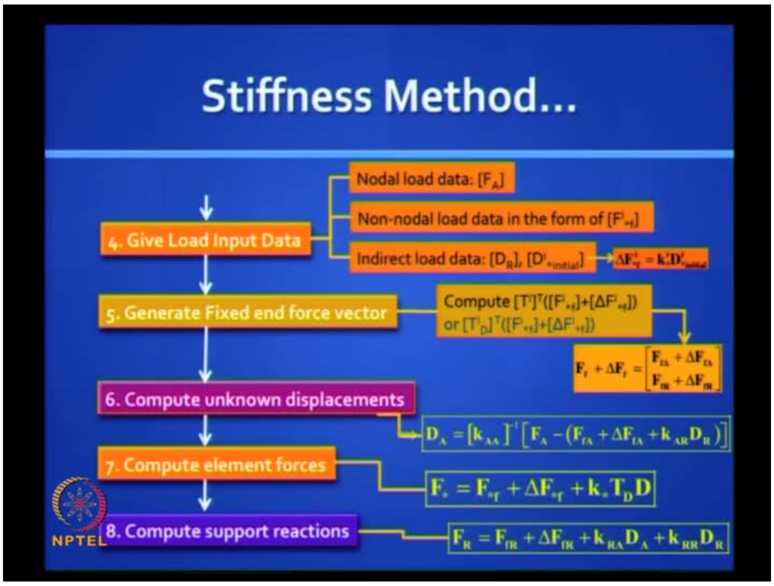

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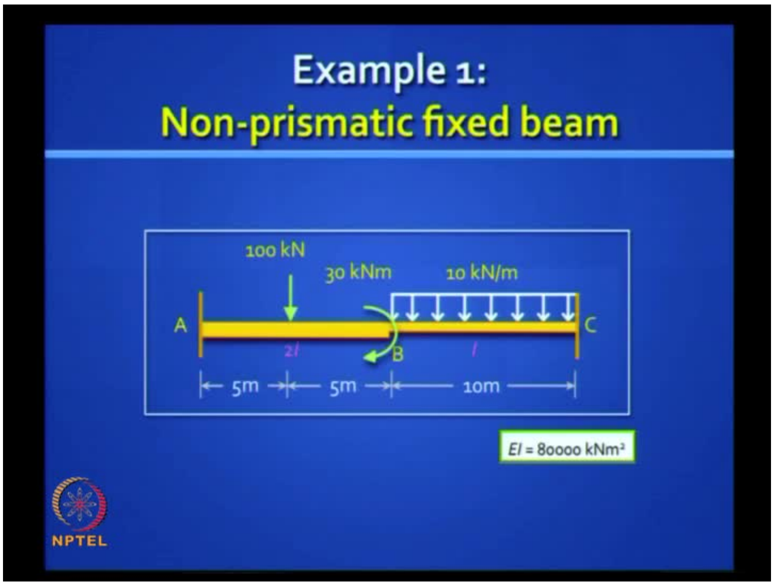

And you can work out the equivalent joint load vector, so this is familiar terrain; we know the steps involved, let us just apply.

So, we will do this together one problem and we will do it by both methods, using T I and T D in one go. So, let us do it together. This is a fixed, non-prismatic beam; both ends are fixed against translation and rotation. And you do not know the fixed end moments for this problem because it is non-prismatic. It has got all kinds of loads including a concentrated moment in the middle.

The question is, can you draw the bending moment and shear forces diagram and finally, if designers want, that also if possible, give us a maximum deflection here and there. So, that is the complete problem, I do not need to write a problem statement because you know, what we need, how do we proceed?

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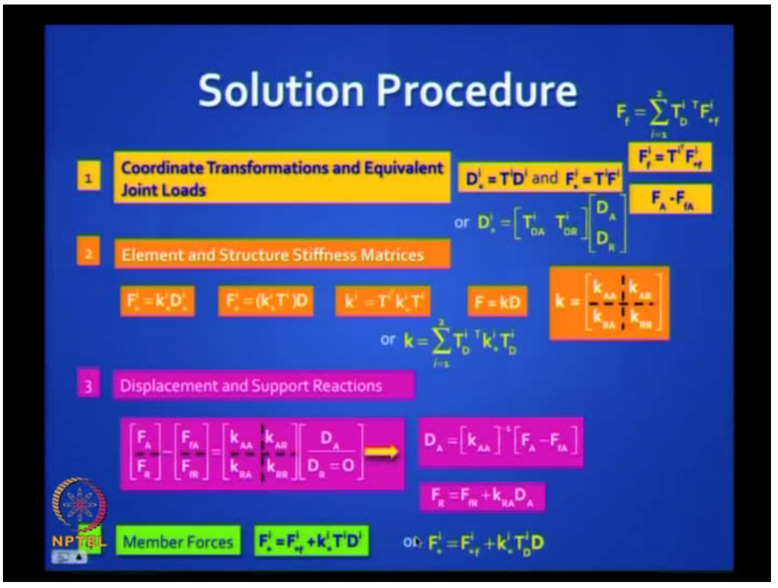

First, you have to locate the nodes solution procedure. As before, we have to write down the global and local coordinates, number them, do the transformations, find the equivalent joint loads.

You have 2 options, one is the conventional transformation matrix with the identity matrix or you try the TD matrix and you have TDA and TDR. And if you are using the TD formulation in the fixed end forces, you have to do using the summation for the 2 elements.

Element and structure stiffness matrices, I am just reproducing what we did yesterday or the day before for axial elements, this part is also familiar, even this is familiar, we have done this, these equations are familiar. Here, DR is 0 is O, it is a null factor because there are no support settlements in this problem, but if someone gave you support settlements, you say no problem, I just add it here. It is not at all an issue support settlement or rotational slip is no problem in this method because you can handle it in DR, so not a problem. But we have kept the question simple here.

Then, you find out, from the 1st equation you solve and get the unknown displacements; 2nd equation you get the support reactions and you use the unknown displacements to get your member forces, is it clear.

Method is same, only we are switching from axial element to beam element. The procedure is identical; if you are using TD matrix, there is a slight change. The slight changes between the TI and TD show up in the first transformations. In assembling your structure stiffness matrix it does not show up here and it shows up finally, when you get the member end forces, otherwise the method is identical.

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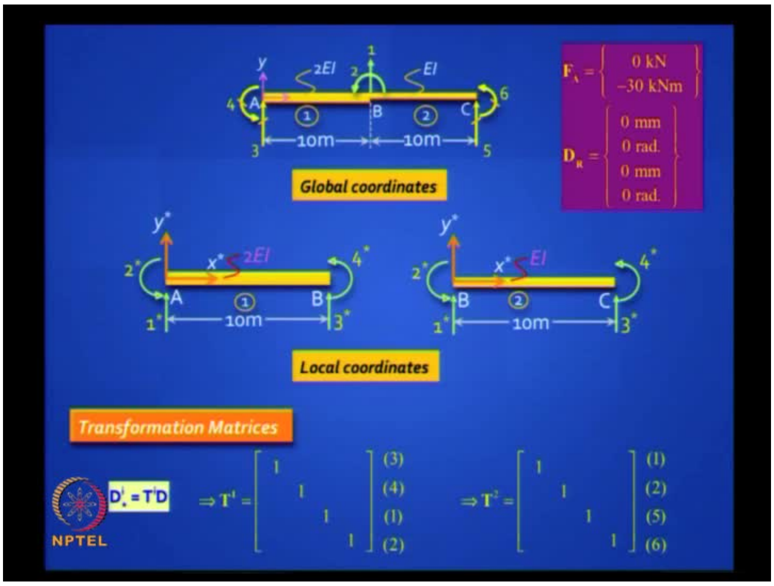

So, let us take our global coordinates. As usual, we will give preference to translation in the, at the active degree of freedom, so 1 and 2. Is it clear? 1 and 2 because only joint B can move and we have separated out 2 prismatic elements and 3 and 4, in that order are the support reactions restrained at the, restrained coordinates, and 5 and 6 are also very easy to do and we will stick to this. If you want to be a little different, you put 4 and 5, 3 and 4 here and 5 and 6 there, it really does not matter.

Now, what do we do? Local coordinates, but before we proceed, let us put the input data given. You are told that there are no support settlements, so DR is a null vector. You are told, that there is one nodal load you have a concentrated moment. Remember in that problem, 30 kilo-Newton meter acting clockwise, so you should put minus, F 2 is minus 30 kilo-Newton meter, F 1 is 0, is this clear.

And you do not know F 3, F 4, F 5, F 6, you have to find them out where you have support reactions; so far so good, clear.

Now, local coordinates. You have 2 elements, both of them have the same span, 10 meter, they look alike, except the EI value is different, you have 2 EI for the first element and just EI. Do not plug in the EI values straight away because it gets eliminated, but if you want you can include it, so 4 degrees of freedom; very easy to do.

Now, what is your next step? Write down your T I matrix for the 2 elements, do it, they are all identity matrices, but I want to see the linking coordinates, that is all; write the 2 transformation matrices.

What will be the linking coordinates for the 1st one? 3, 4, 1, 2 and for the 2nd one?

1, 2, 5, 6

You said it; that is all. Anybody can do this, clear.

Can we proceed? You just write the identity matrices and fill up. You can fill up in the row as well as the column, but the column is good enough, it is a square symmetric matrix; is this clear?

You have done the 1st step, if you want to do by the T D matrix, what will it look like? Can you give it a shot, T D, because let us do both methods parallel.

What is the size of the T D matrix? T D, again you can subdivide into T DA and T DR.

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That is right, now you fill it up this is the alternative approach.

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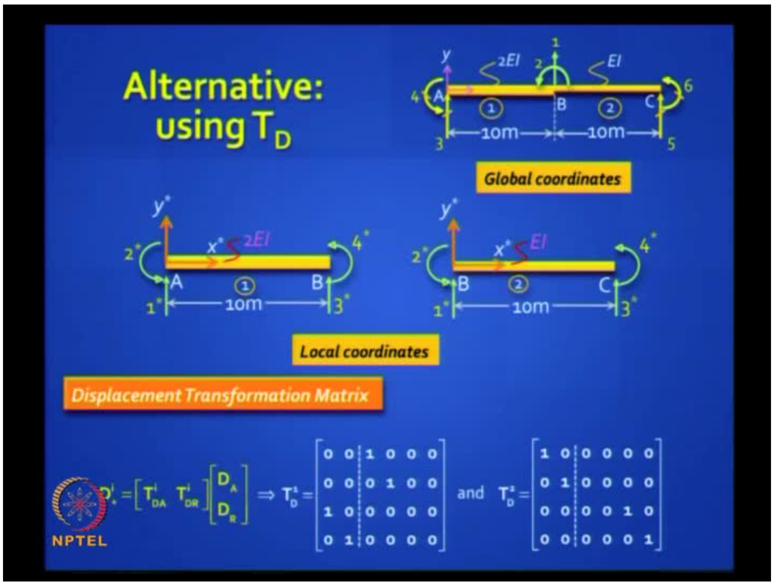

So, look at it this way. If I apply D 1 equal to 1 and I do not allow any other moments in the structure, what will this effect? This will affect this displacement D 1 equal to 1. In the global structure will, will cause this 2 also go up; D 3 star in the first element will also be 1 and D 1 star in the 2nd element will be 1 and the rest will all be 0.

So, that is, that is it. So, when I put D 1equal to 1, D 3 star will be 1 and D 3 star will be 1, the rest will be 0. If I apply D 2 equal to 1, D 4 star in the 1st element will be 1 and D 2 star in my 2nd element will be 1 and the rest will be 0. Does it make sense to you, D 4 star D 2 star, likewise, so?

And I have done a partition here because I am separating out the active degrees from the restrained degrees. Is this clear, have you all got this? Easy to do.

2 approaches, the advantage of this approach is, you do not have to worry about the slotting, the matrix multiplication takes care of everything.

Let us see both the approaches, clear and you should know both. Can I move ahead?

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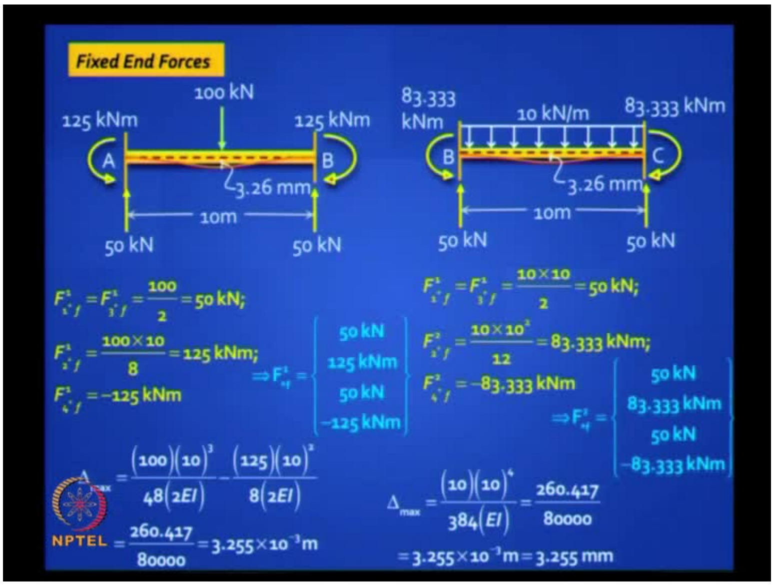

Next, you have to find fixed end forces, which you know. You know the formulas, so first the reactions, 100 kilo-Newton shared equally 50-50 and w l by 8. Remember, when we did slope deflection and moment distribution, the left was minus, right was plus, now it will be reverse because anticlockwise positive. So, be careful, the left is plus and the right is minus because you have a downward moment. Is this, those calculations are clear?

You can fill up the first fixed end force vector, do the same thing and you can also get the deflection if you want to. In your examination do not worry about this, but if you really want to, you know the formulas for finding deflection in fixed beams.

For the other beams, similarly you can work out w l squared by 12, you can and the vertical reactions you can get 10 into 10 is 100 kilo-Newton acting downward, resisted by 50-50 up. So, you get the 2nd vector also. Is this clear?

So, you have got the element level, fixed end force vector using the concepts that you are very familiar with. You can also get that deflected shape in this case. Coincidentally, both the peak deflections turn out to be the same, but that is just incidental.

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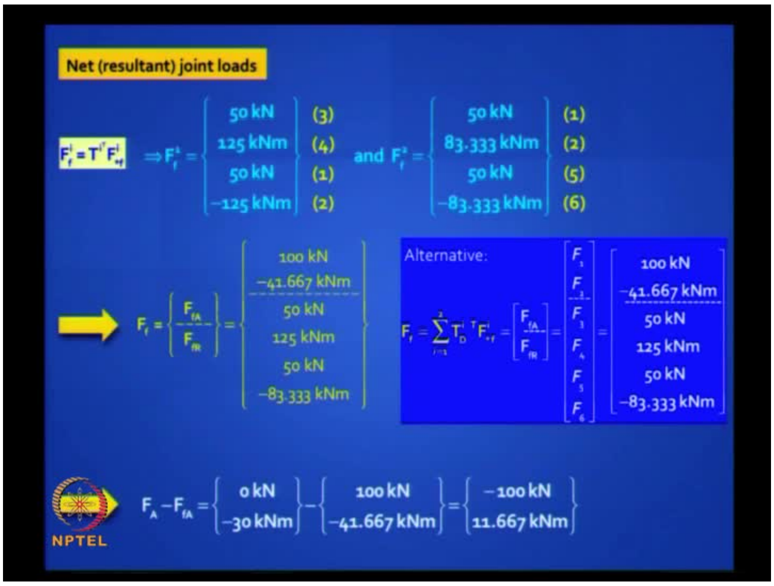

So, you have got the fixed end forces at the element level, you have to transfer them to the global level, so you have to just bring in those linking coordinate, which we did. Remember, 3, 4, 1, 2, 1, 2, 5, 6? The numbers do not change because you are multiplying with the identity matrix.

And then, then, now you do the slotting, so 1 and 2 go here, 1 and 2 go here, so these will add up and you will get this. 3, 4 go here and 5, 6 go here, does it make sense; does it make sense? That is, that is something you have to do. This is a slotting thing; we have to put the right…

Once you shifted to the global coordinates, global axis system, you are matching the element with the structure, does it make sense, clear.

If you want to do the T D approach, that is straight forward. You got your T D, take the transpose pre-multiply; you will get the same solution without doing any slotting. So this is in a sense easier, but you are dealing with bigger matrices.

Then, find the net load vector, which is the nodal load vector minus the fixed end force vector. Remember, this is your final nodal vector. Why are we putting minus? Because you artificially restrained those joints, they have accumulated, you have to let go of those restraints to get back your original loading condition, so you got the net load vector.

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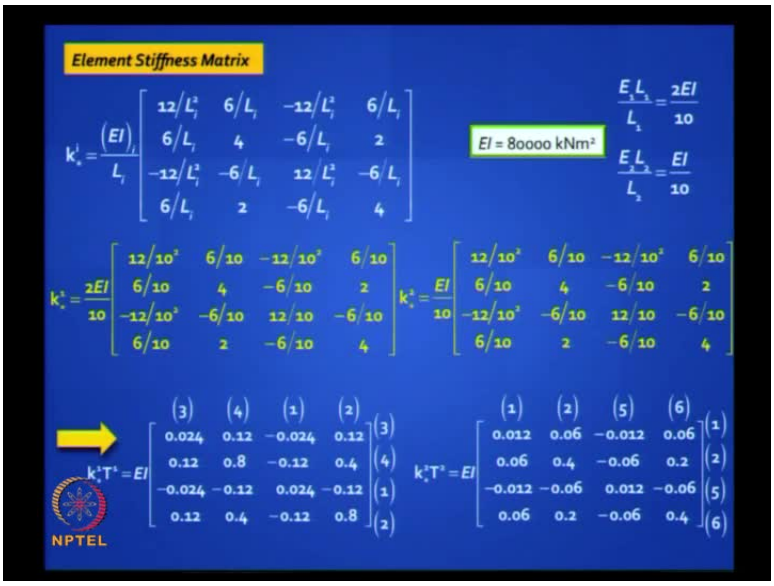

Element stiffness matrix, we know the formula, we have derived it, just plug in the values. In this case, both spans are 10 meter only, the EI value is changing, EI value for one is 2 times the other. So, can, you can write down these 2, k 1 star and k 2 star are cleanly obtainable from the formulas, both are prismatic only, EI is changing, so the inside part is the same and the outside 2 EI by 10 and EI by 10.

Next, you pre-multiply or you post-multiply the element stiffness matrix with the transformation matrix, now you bring in the linking coordinates.

Actually, the transformation matrix is identity, so you would, really this is no effort at all, I just substituted the values of L here, that is all. So, you get this.

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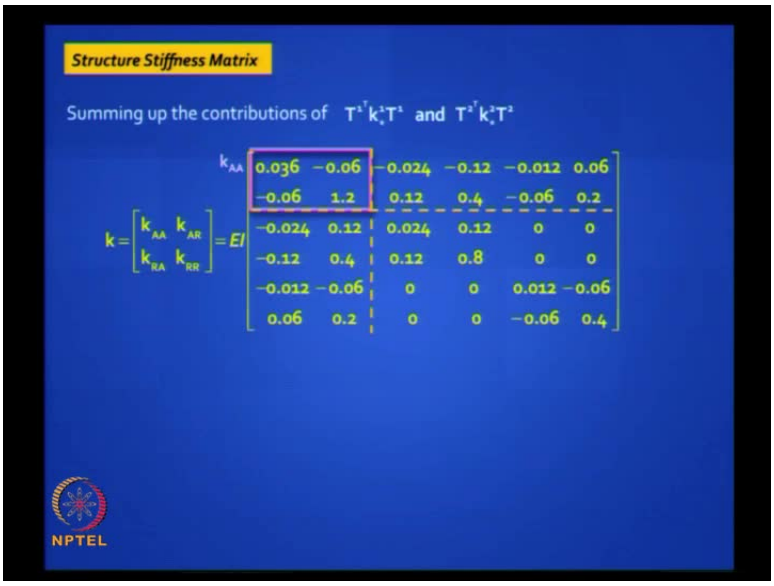

And then, when you pre-multiply the whole thing with T I transpose and you do the slotting business, you will get the full 6 by 6 matrix, is it clear, which you can partition as k AA k AR k RA and k RR, and k AA is of importance because this is the one you need to invert.

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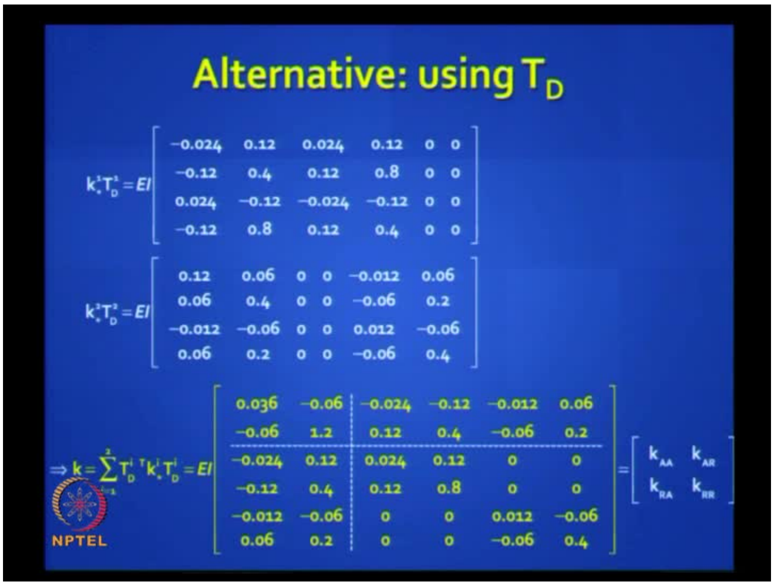

If you want to use T D, do this and just add up the contributions from the 2 elements, you will get exactly the same matrix. Is it clear?

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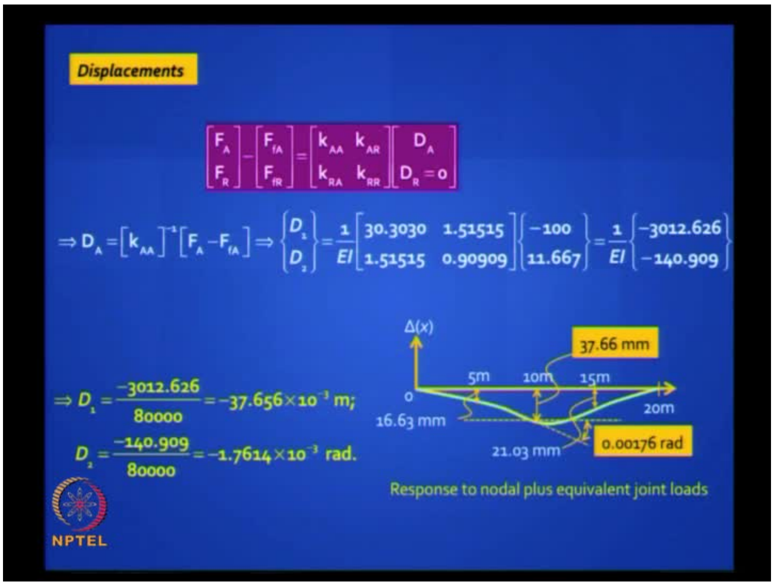

You have 2 alternative paths, then you substitute in your equations, find out your unknown displacements, plug in those values. If you want you can look at the deflected shape, D 1 is minus 37.7 millimeters, so minus means it went down and should have gone down anyway because the loads were all acting downward. And you had a, D 2 is a rotation, minus means it is actually clockwise not anticlockwise.

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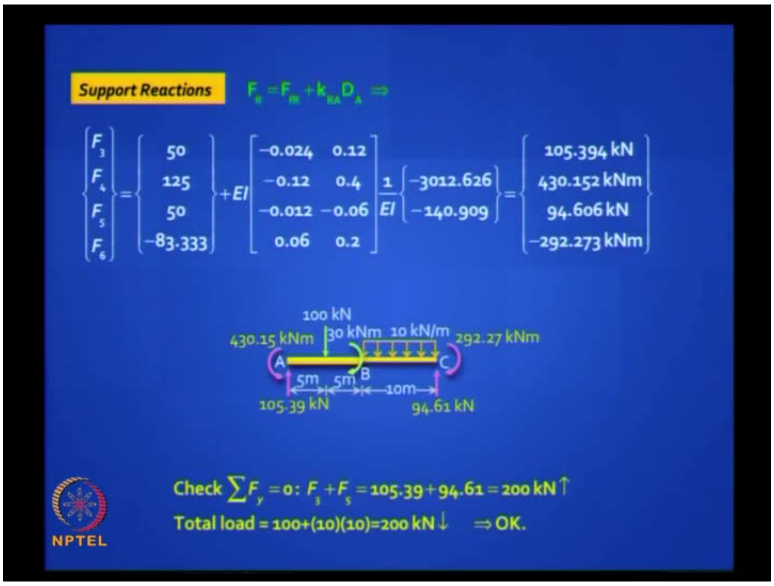

Then, you find the support reactions, the 2nd equation and write the correct units. The force is kilo-Newton; the moment is kilo-Newton meter.

Draw a sketch and take a look, whether the answers make sense, does it satisfy equilibrium; do a simple check on vertical equilibrium. This is something you have to do even when you use software because sometimes you make big mistakes. So, it is a quick check, you find total downward load is matching the total upward load.

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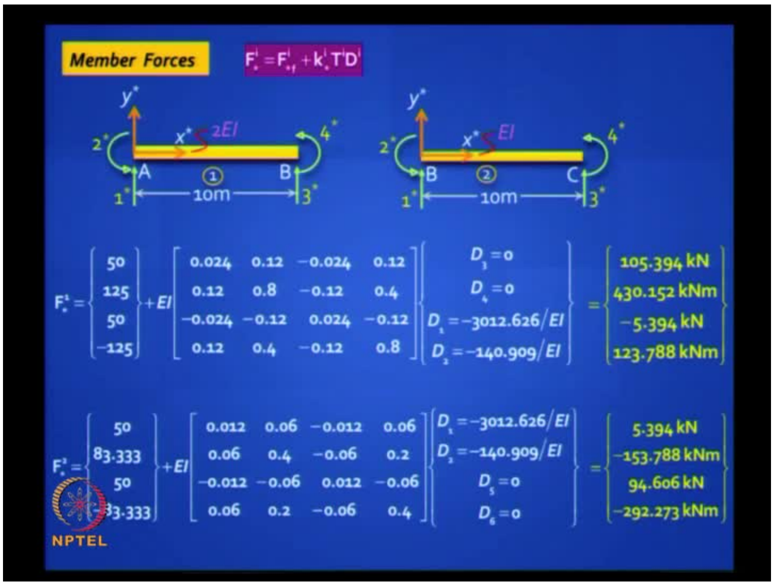

Find the member forces by using those equations to…

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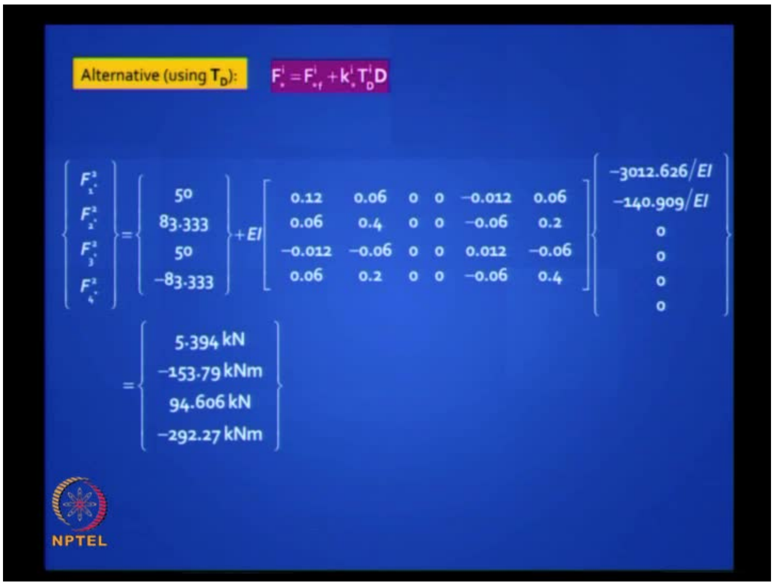

So, you have got 4 answers for each element, 2 end moments and 2 forces and with the help of that you can use T D and get exactly the same results.

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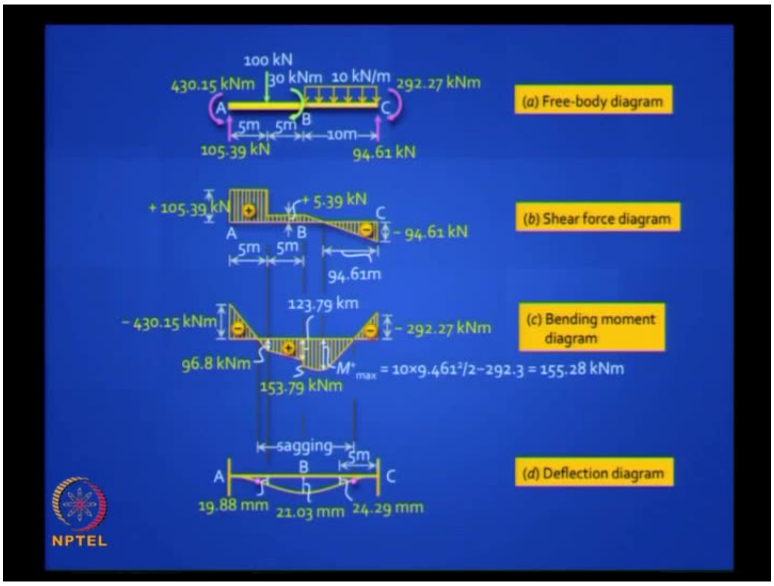

You finally draw shear force diagram, bending moment diagram and deflection diagram, which I think you are familiar. So, simple and we will, you can do more such problems of this kind.

Thank you.