Advanced Structural Analysis

Prof. Devdas Menon
Department of Civil Engineering
Indian Institute of Technology, Madras

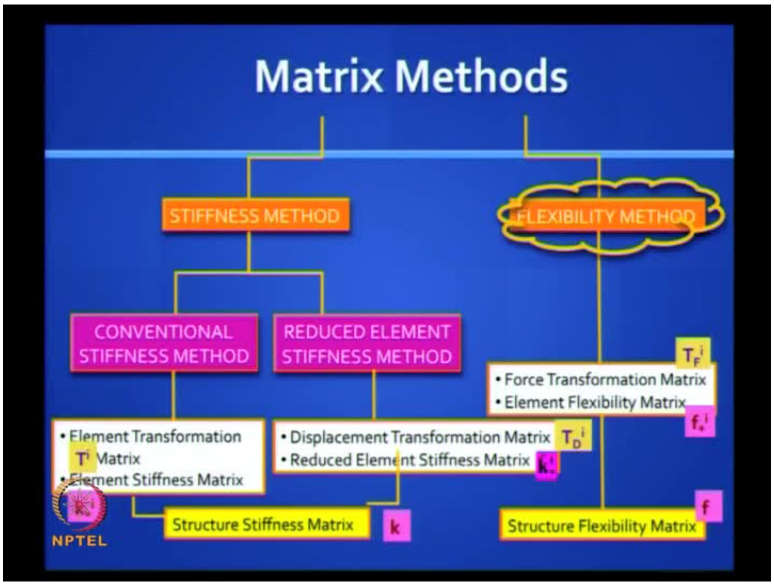
Module No. # 4.5

Lecture No. # 26

Matrix Analysis of Structures with Axial Elements

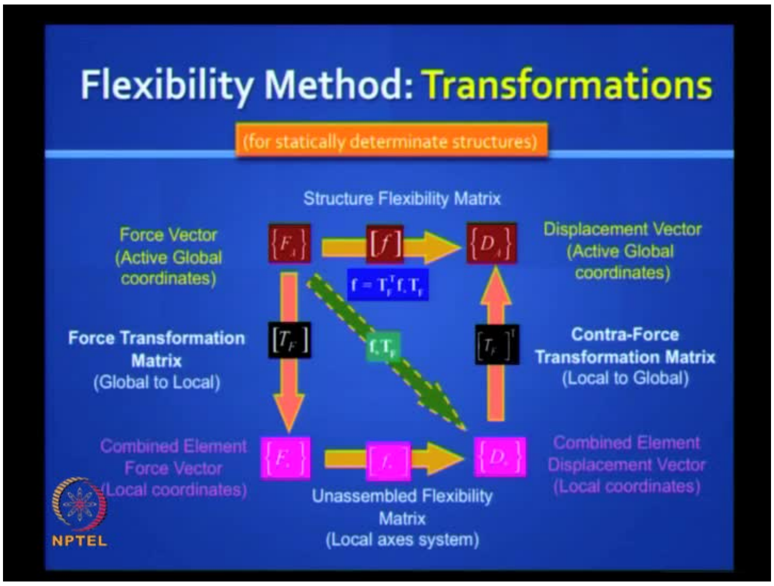
Good evening. This is lecture number 26. The last lecture in module 4, we will be covering flexibility method in matrix analysis of structures with axial elements. So, with this the fifth lecture in this module will be complete. Flexibility method, the space trusses application of reduce stiffness method we have already covered in the last session. This is cover in the book advanced structural analysis.

(Refer Slide Time: 00:49)



So, this is flexibility method. As you can see, we have already finished the 2 stiffness methods - the conventional stiffness method and the reduced stiffness method. Now, we will see how we can apply the flexibility method, which is little easy to understand. Now, that you have done the reduced element stiffness method.

(Refer Slide Time: 01:06)



So, let us refresh our memory about transformations. We know that there is a flexibility matrix at the element level which relates the element level forces to the element level displacements. Similarly, at the structure level, you can relate the structure level forces to the structure level displacements. Usually, in the flexibility method, we keep aside the reactions. There are ways of dealing with reactions, I will demonstrate in today’s lecture as well.

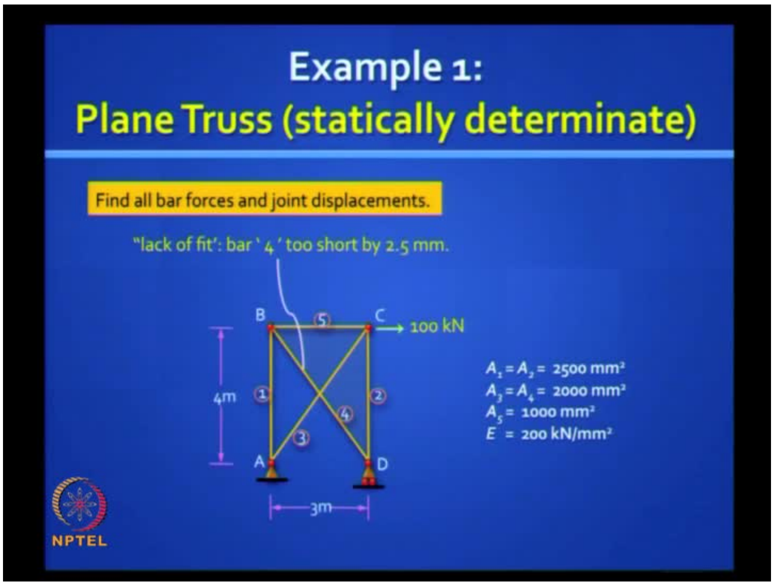
So, that is why we refer to only the active coordinate. So, the flexibility matrix f when you pre multiply to the force vector f, will give you the displacement vector D. So, these are the two flexibility matrices - one is at the element level, the others at the structure level; then you have transformations from the global coordinates to the local coordinates.

So, the first transformation is the T F transformation. Remember we had the T D transformation in displacement methods. So, here, we are moving on the left side; you can convert the load vector to the element force vector through a matrix called the force transformation matrix, and the principle of contra gradient - the contra gradient principle tells us that T F transpose gives us a relationship that establishes comparability that simultaneously establishes comparability. From the displacements at the element level, you get the displacement at the structure level. We have done this in module 3 but we will refresh in this module as well.

And this is the reason why in statically determinate structures, you do not have to explicitly satisfy compatibility; it is automatically satisfied when you apply static equilibrium equations, and of course, you have this diagonal which allows you to shift to the element deformations from the load vector directly.

Please note that the T F matrix is a unique matrix; it is a square matrix for a statically determinate structure and from the element flexibility matrix, unassembled f star, and D T F matrix, you can generate the structure flexibility matrix. In much the same way we did this in the stiffness method. Remember, we did T D transpose case k star T D and we got the k matrix; similarly, you are getting the f matrix in this fashion.

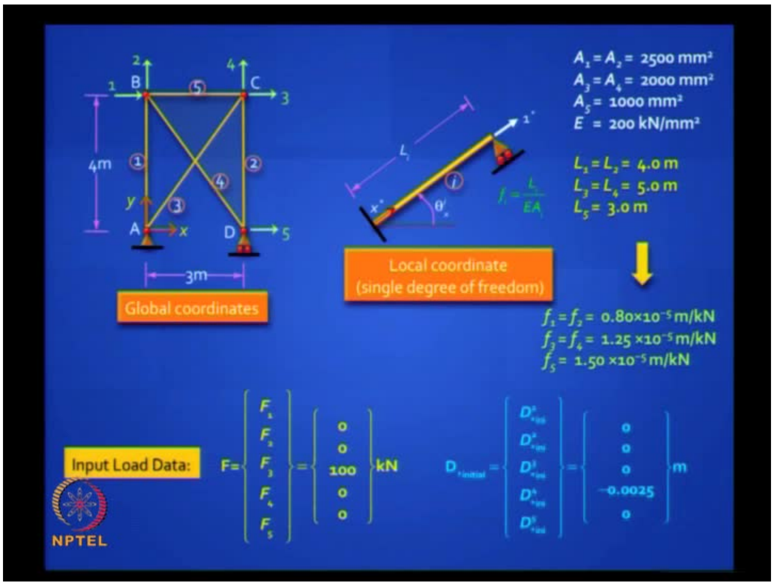
(Refer Slide Time: 04:17)



You will understand best when we do an example; so, let us do that. Let us take this example - this is a 5 bar truss or pin jointed frame; it simply supported, so, it is externally statically determinate, it is internally statically determinate. You can check m plus r is equal to 2 j, and you are given some loads, two loads - one is the direct action of a 100 kilonewton and the other is a lack of fit

So, bar 4 is too short by 2 and half mm. Question is very clear find out all the bar forces and the joint displacements, I mean these are the unknowns in the structure. You can also get the support reaction very easy. So, let us demonstrate how to apply the matrix method using the flexibility formulation for a very simple problem of this kind. How do we do this?

(Refer Slide Time: 05:23)



Well, first, you have to define the coordinates. Now, we can identify 5 active coordinates; we have done this earlier if you recall for the same example 1, 2, 3, 4, 5, these are the 5 coordinates at the active degrees of freedom, and the input load data says that F 3 alone is non zero, F 3 is 100 kilonewton, clear?

The direction is in the same positive direction as a coordinate 3, F 1 is 0, F 2 is 0, F 4 is 0 and F 5 is 0. We are also told that there is an initial displacement bar 4 is short by two and half millimeters which can be written as minus 0.0025 meter and that vector we refer to as the initial displacement vector at the element level, is that clear? This is a symbol D star initial and D 1 star is 0 but only D 4 star is non zero, it is a minus 0.0025.

So, you must be able to write down the input data in terms of the notations that we have developed for matrix method, is this clear?

Now, just look carefully, do you think these initial displacements will introduce any internal forces?

(( ))

No, because the structure is just rigid, the structure is statically determinate; the bar will be allow to have the length it wants to have. If it short by two and half mm, it will remain short by two and half mm; the all the other members will retain the lengths that they have been manufactured with and the joints will just move.

So, this is something that you know that in just rigid statically determinate structures, if you have a lack of fit or a temperature change, it does not introduce any internal forces, it does not introduce any support reactions, it only effects the geometry, you will get joint displacements, is it clear? That is, that is how it works in a statically determinate structure. If it is indeterminate and we are going to do that in the next problem, then there will be a tug of war, you know, some members do not want to change their lengths, while others want to and, you know, there is a compromise and there are internal forces created.

So, even before you started the problem, you recognize that it is only the 100 kilonewton direct action that is going to give you any internal forces. That 100 kilonewton will give you joint displacements, certainly you will get D 1, D 2, D 3, D 4, D 5, but the other load - the indirect load - will add on to the joint displacements, but it will not add on to the internal forces. So, all it is necessary to be good in structure analysis fundamentals, and then, use matrix methods only as a tool; otherwise, you would not be able to appreciate the solutions that you get.

As far as the local coordinates are concerned, you are dealing with the exactly the same element that we dealt with in the reduced element stiffness method. You have only one internal deformation and that is the elongation in the bar. You have an axial flexibility which is the reciprocal of the axial stiffness and it is called the flexibility - f i is equal to L i by EA I, is it clear?

And you have 5 bars, all the bars are prismatic; the bar areas are given; the lengths are known; it is a simple problem; the lengths are either 3 meter, 4 meter or 5 meter, the diagonal lengths are 5 meter. So, actually you can compute the flexibility values - f 1, f 2, f 3, f 4, f 5 - can be easily worked out, is it clear? And look at the units of flexibility, it can be either meter per kilonewton or millimeter per newton, it is a same, you know, it works out to exactly the same, is it clear?

We are ready now to proceed. How do we solve this problem? Generate T F matrix first then

F star (( ))

Then f star

F star into T F

F star into T F transpose get f then

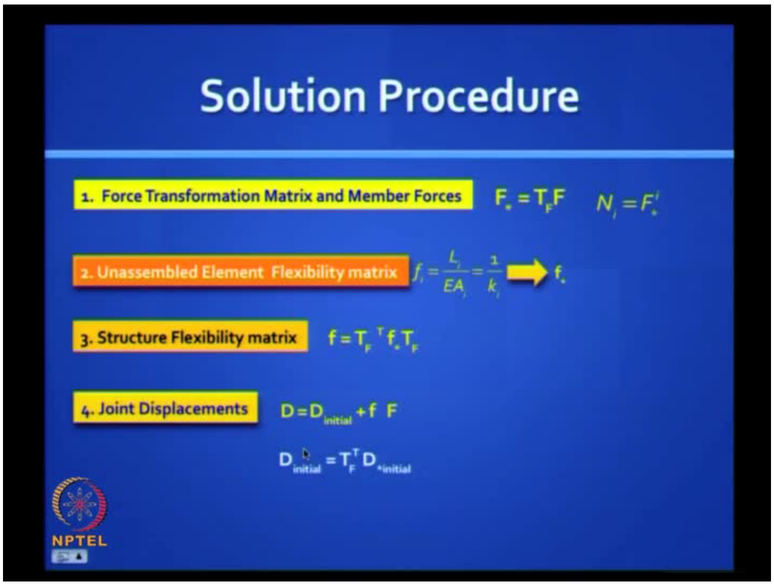
Get the structure

Get the displacements D A then

Then the force (( ))

No, you can get the, you see, that is a difference. You can get the forces directly because there is no redundant here.

(Refer Slide Time: 10:37)

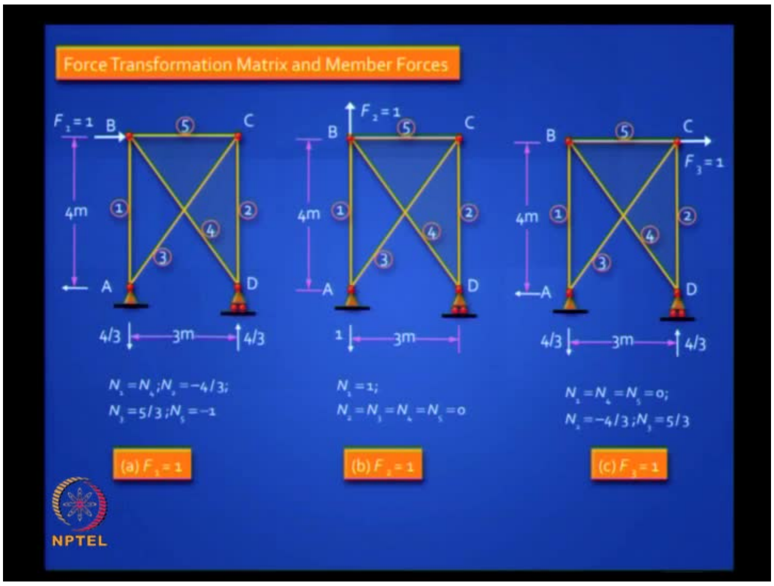


So, this after get the T F matrix directly, you can get the forces, so, that is it, very simple. First you find the T F matrix, then you write down the unassembled elements flexibility matrix. Form the T F matrix itself directly you can get the bar forces in that step itself, then the structure flexibility matrix, then the joint displacements.

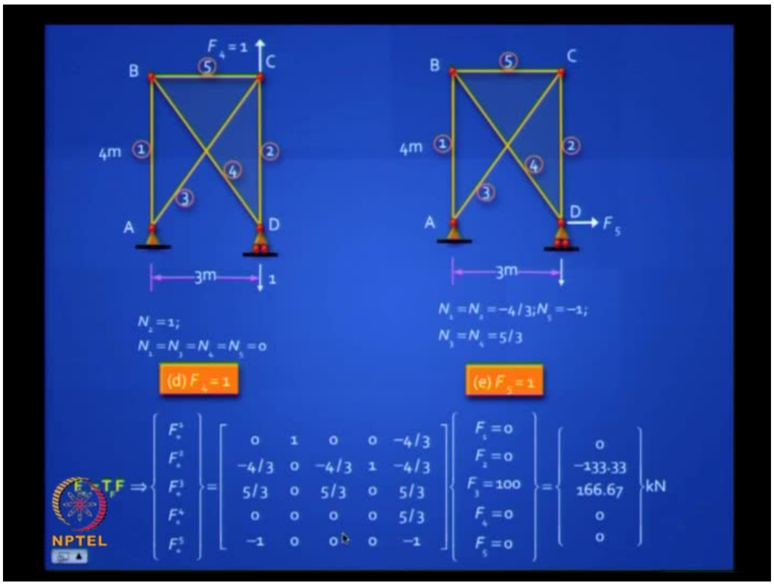
In the joint displacements, do not forget to include the initial joint displacement. So, the initial joint displacements you can get from the contra gradient principle. If the initial lack of fit is represented by D star initial at the element level, the effect that you get at the structure level is given by T F transpose that is compatibility, that is the beauty of the compatibility relationship, is it clear?

So, it is a very beautiful thing. You give me a statically determinate truss, let all the bars have different lengths, I do not care, but I should know how much different they are whether they are longer or shorter by to what extent. So, I have d star initial vector. If I pre multiply D star initial vector with T F transpose, I get all the joint displacements in one go, in fact, that is a terrific transformation, but it works only for a statically determinate system, because in, if it is in determinate, then the flexibilities of the bars will have a role to play. This is independent of bar flexibility. It is a very interesting and powerful property. Let us demonstrate.

(Refer Slide Time: 12:17)



(Refer Slide Time: 12:41)



So, the first thing you have to do is to analyze the same truss to different unit loads one at a time. Apply F 1 equal to 1, we have done this earlier, so, I am going fast, apply F 2 equal to 1, apply F 3 equal to 1, find out all the bar forces, can I proceed? We did this earlier, then you apply F 4 equal to 1, apply F 5 equal to 1 and fill up this matrix.

So, this is your T F matrix, is it clear? The first column refers to the bar forces, mind you this F 1 star F 2 star are nothing but N 1 N 2. So, for example, let us look at this picture shows the results of analyzing it for F 4 equal to 1, that will help us fill up this column, and this is very easy thing to analyze, because if you apply a vertical load, your only bar 2 will get effected and that bar 2 will have a force of unity.

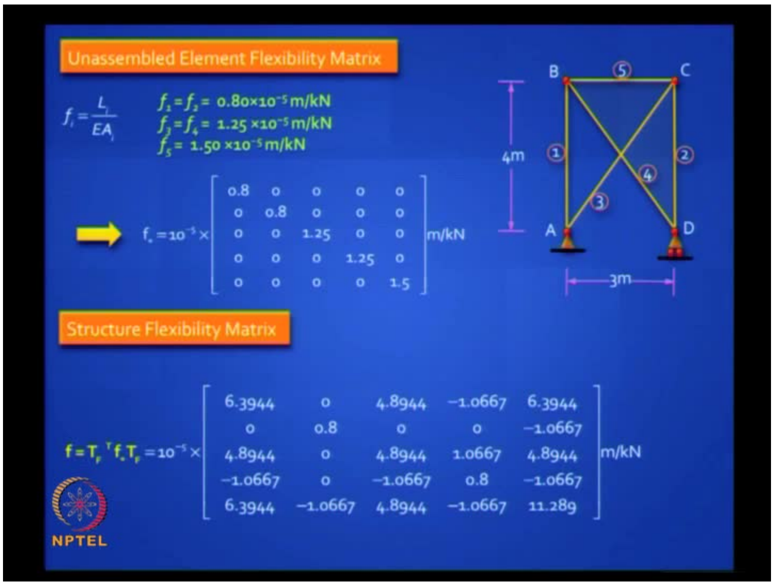
So, you can see bar 1 has a force 0, bar 2 has unity, bar 3 has 0, bar 4 has 0, bar 5 has 0, is this clear? So, this matrix is a property of the structure. Even before you apply the loads, you can generate that matrix and it is applying 1 unit load at a time.

The advantage of a matrix like this is that if you give me any load vector, I do not care what that load vector, F 1 could be minus 32.6, F 2 could be plus 21.82, F 3 could be anything, I get the answers in one shot. It is a linear transformation. In this particular problem, you had only F 3 having 100 kilonewton. I just multiply these 2, I get directly the final answers, is it clear?

You might ask why do I need to find out all these, I just need to solve for the single condition of 100 kilonewton, just one truss, that is what we do in normal structural analysis, but in matrix methods, we say that is right, you can do only one problem that way. We are given a solution which can do any problem. You change the loading I can still do it in jiffy by this matrix multiplication, is it clear?

So, this step is straight forward and the real power of this method comes when you deal with the indeterminate structure. So, am just giving one example of demonstration for a statically determinate structure.

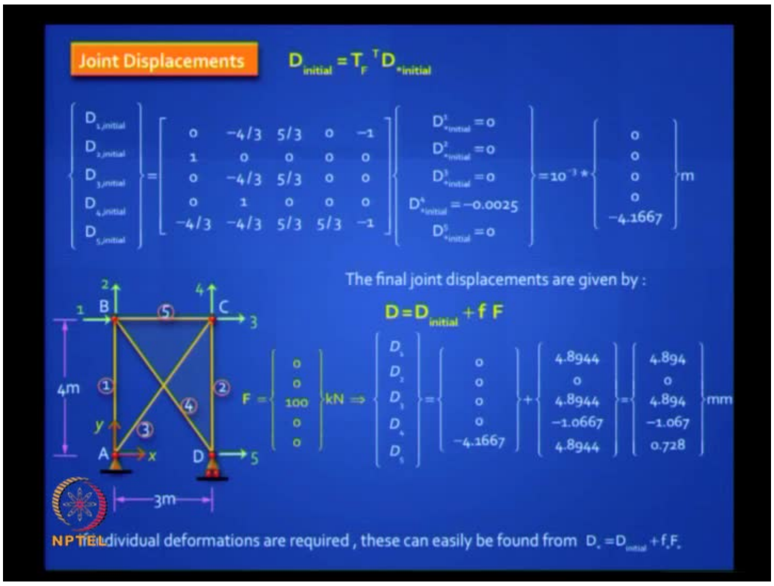
(Refer Slide Time: 14:56)



Then you have the unassembled element flexibility matrix. You got the flexibilities, just put them in a diagonal along the principle diagonal, you are familiar with that, so, just, this is f 1, this is f 2, this is f 3, this f 4 and this is f 5, that is your f star matrix, clear? 10 raise to minus 5 always avoid putting too many decimals inside your matrix, take it out; otherwise, you will run into errors. So, keep out, keep the significant figures inside your matrix and keep out the 10 raise to minus 5or plus 5 outside.

Then you carry out this product, and if it is a big matrix, then you do not do it manually, you do it through matlab or something. You have T F, you have f star, you can do this product in one shot, you get f. There is no advantage in doing f star T F first and then T F transpose afterwards, because in this problem, it does not make any difference, somehow get the f matrix.

(Refer Slide Time: 16:42)



Now, the f matrix is a beautiful matrix, it says give me any load vector and if I multiply that load vector, I pre multiply that load vector with the flexibility matrix, I get all the joint displacements caused by direct actions, not by indirect actions. So, give me any f 1, f 2, f 3, f 4, f 5. If you got the flexibility matrix, you will get the joint displacement. So, that is a next step, but before that, let us finds out the contribution of the initial the lack of fit.

So, this is what I have said earlier. This is T F transpose and this is your initial lack of fit vector, if I just do this product, I am getting the joint displacements. Now, it so turns out that if the bar 4 is changing length, only this roller is going to move, it is going to move inward and the reason is simple, the reason is simple. It is always good to get into the physics of the problem. Take the triangle A B C; the triangle A B C is not changing length, right? A B is 4 meters, B C is 3 meters, A C is 5 meters, so, leave the triangle in piece do not move it.

Now, C D is 4 meter, but with C as center and D as radius, I can draw an arc and I still retain the 4 meter, so, I move it inward so that B C is able to reduce length and get. So, that is a physical meaning. You can do it by a shear geometry and that is a reason why only one of these is nonzero, all the others are 0; that means the lack of fit in bar four is causing a movement of the joint D all the others - D 1, D 2, D 3, D 4 - are 0.

But the beauty about this particular transformation is let all the bar is have arbitrary lack of it minus 3 mm plus 2 mm. I will get the final answer without worrying about the geometry just by pre multiply. It is a very powerful matrix, have you got it? So, I have got the joint displacements caused by lack of it.

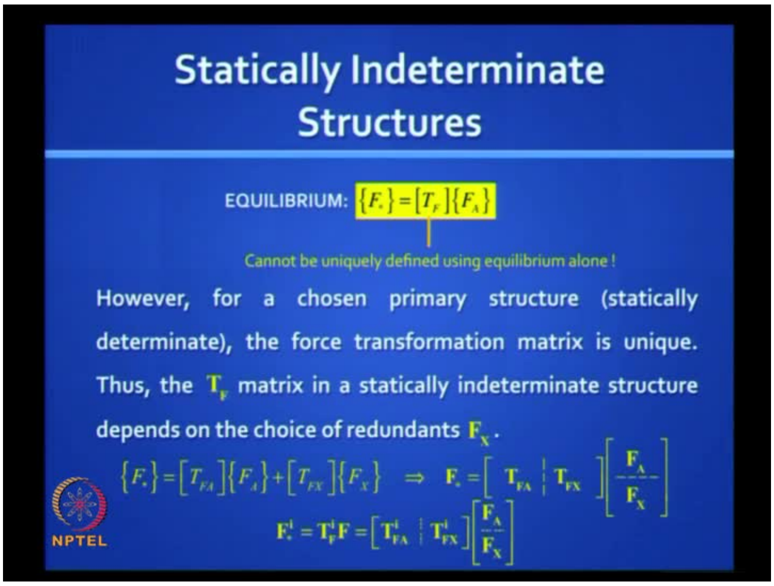
The final joint displacements is this plus what I got from the 100 kilo newton, by just you have to add it up. That is my answer, I finished the problem. Is there anything left that is still missing? Well, if I really want the bar elongations which normally nobody is interested in, I can find that also. So, the individual deformations are required, D star I can get as D star initial plus f star, is it clear? But this is normally not required, you just need the joint displacements and you need the bar forces. If you want the support reactions, they are very easy to do through equilibrium.

Shall we proceed? We now move to statically indeterminate structures and you realize that you have a problem, what is the problem? what is the problem with statically indeterminate structure?

We have a with this equation.

It just a redundance, redundance.

(Refer Slide Time: 19:51)



What is the problem with the T F matrix?

It is not unique.

It is not unique. Multiple solution are possible which are statically admissible. So, it cannot be uniquely defined using equilibrium alone. Previously that T F was unique; you apply F 1 equal to 1, the bar forces are unique; but you apply F 1 equal to 1, you have an indeterminate structure, you do not know the answers, you got it? So, we take advantage of the fact that a unique T F can be done, so, what we do is we reduce that structure to a statically determinate structure and play a games with that primary structure, is it clear?

We did the same thing in stiffness method, but there is a big different between stiffness method and flexibility method with regard to the primary structure. What is the big difference?

We have to choose a primary structure

Here you have multiple choice is possible. There you have to arrest the active degrees of freedom. Primary structure is a fully restrain structure, so, that is the difference here. For a chosen primary statically determinate structure, the force transformation matrix is unique. Thus the T F matrix in a statically indeterminate structure depends on the choice of redundant F X.

So, let us say I am an examiner, I have solve the problem with my choice of F X and the student use a different. I cannot compare my solution with the student solution, because we have dealing with different structures primary structure, but I can check the final answer, because my final bar forces must be the same, is it clear?

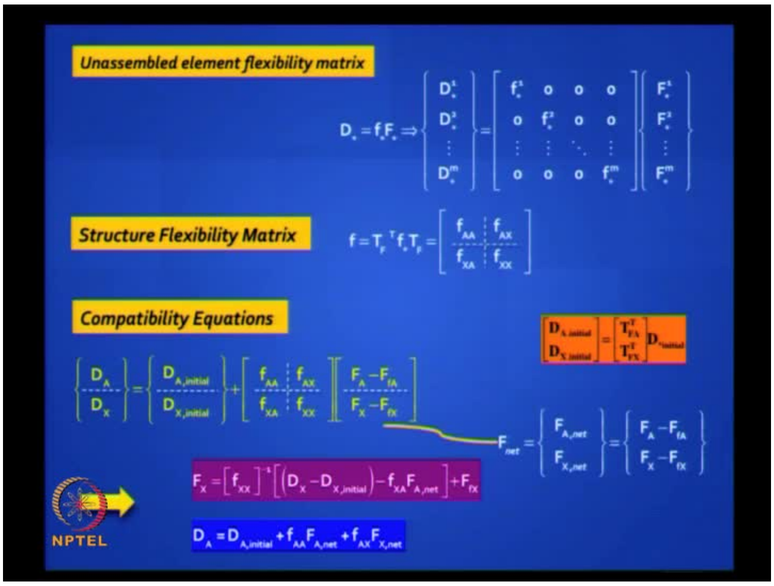
So, that is the trick here. So, what we do is we say that there are two force vectors - one is a load vector which I know and the other is the redundant force vector. So, I have a primary structure, I can apply the redundance has loads on the primary structure, you are getting it? The difference between F A and F X is F A is known. Somebody tells me what the loads are on the structure F A is known, F X is unknown. F X is x 1, x 2, x 3 depending on your redundant, is it clear?

Now, I have the primary structure; primary structure statically determinate. So, corresponding to the active degrees of freedom, if you know 1 2 3 4 5, I can write a T F matrix, that T F matrix I called T FA matrix when I am dealing with F A. When I am dealing with the redundant coordinates, I call it T FX. So, I can write my T F matrix as in a patrician form like this. T FA when I am dealing with F A and T FX when I am dealing with F X. T FA and T FX are both uniquely defined. By applying unit loads, I can generate T FA and T FX.

So, that is a property of the primary structure. So, if I look at this vector, this is known, I can find this out; this is know, I can find it out. F A will be given to me in the problem, F X I do not known, but if somehow I can get effects, I got the bar forces completely, is this clear? Is this clear? So, we have graduated from statically determinate structures to statically indeterminate structures, there is only one problem.

Someone has to tell me how to find out F X, because without that, I cannot solve it. Mind you this equation does not give me F star, does not give me the bar forces, because there is an unknown F X hanging around and that is because this equation is an equilibrium equation. Equilibrium alone cannot solve this problem, you need something more, you need compatibility and this I have written for all the bars put together, but in general, you can write it individually for different bars.

(Refer Slide Time: 24:17)



This kind of form is useful in, you will see let in beams and frames, when you have multiple degrees of the freedom at the element level. The unassembled element flexibility matrix is exactly what we did earlier, but for the primary structure which you have chosen, for the primary structure you have chosen, the primary structure will be different for different. Well, in this problem, it may not make a difference, but in general, your primary structure can be different. In this problem, it may not make a different.

So, structure flexibility matrix the same transformation, but now, this T F can be partitioned as T FA, T FX, and so, your f also the flexibility matrix, you can partition as f AA, f AX, f XA, f XX. The order of f XX will be dictated by the number of redundant which is nothing but the degree of static indeterminacy.

F XX and f AA will be square matrices, f AA and f XX will be square matrices, and f AX will be the transpose of f XA because of which principle? Because of Maxwell's reciprocal theorem, that directly comes in. So, you put together all you understanding. Remember, when we dealt with the stiffness matrix, we also partitioned but we partitioned as k a a and k a r, and k r a and k r r, where r was for restrained coordinates.

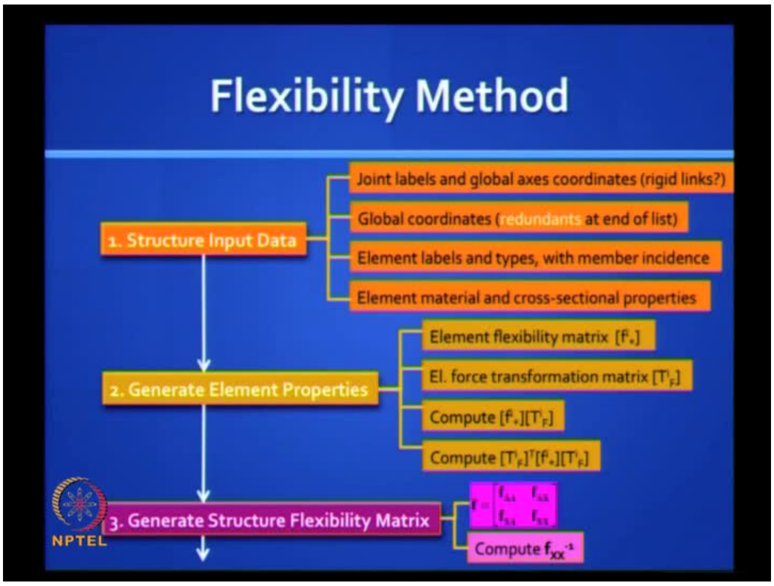
Here, you replace r with x, x is redundant coordinates. So, you have active coordinates and redundant coordinates in flexibility method. In stiffness method, you had active coordinates and restrained coordinates, is it clear? X is redundant coordinate. Now, what is the equation that you need to solve? It is a compatibility equation, without this, you cannot get the x. Now, you have to satisfy compatibility at the active degrees of freedom and also at the redundant coordinates.

So, this is pretty simple to understand D A D X is equal to D A initial D X initial. In case you have some initial displacements like this lack of fit problem that we have plus this flexibility matrix into what is some time refer to as a net load vector. Now, sometimes you have some fixed end forces not in trusses, but remember, the axial degree of axial element problem, you had some loads acting in the middle, we will do one problem of that kind, then you will have some fixed end forces which we have to bring in.

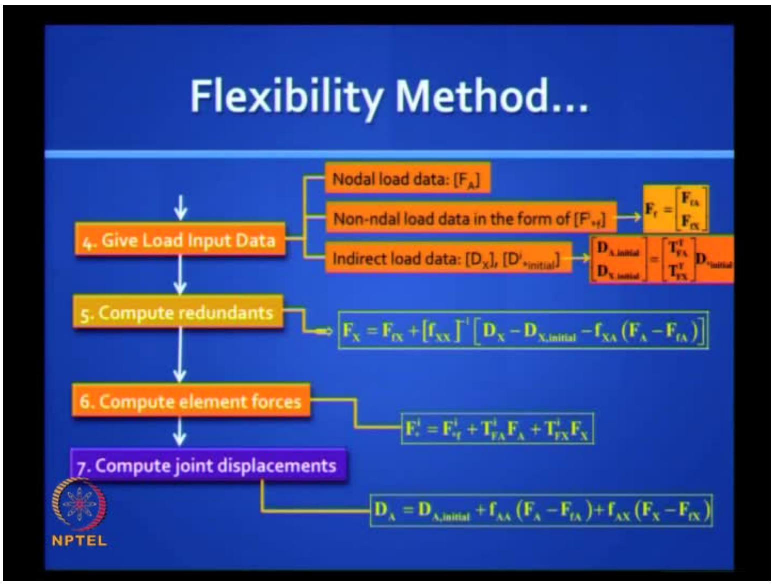
So, in such problems F fa and F fx will come, that is, so, you can get your initial joint displacements through this transformation both at the active coordinates and at the redundant coordinates, and you get your net load vector F A minus F, fa. Which of these two, you have two equations here - one related to D A and one related D X; which of them do you need to solve to find the unknown redundance? The first or the second? The second one, the second one .

So, solve the second one, you get F X; plug in that value of F X the redundance in the first equation, you get the joint displacements. Sometimes you may not be interest in the joint displacement, you just want to know the internal forces, then you do not even use the second equation, be happy with the first one, is it clear?

(Refer Slide Time: 28:10)

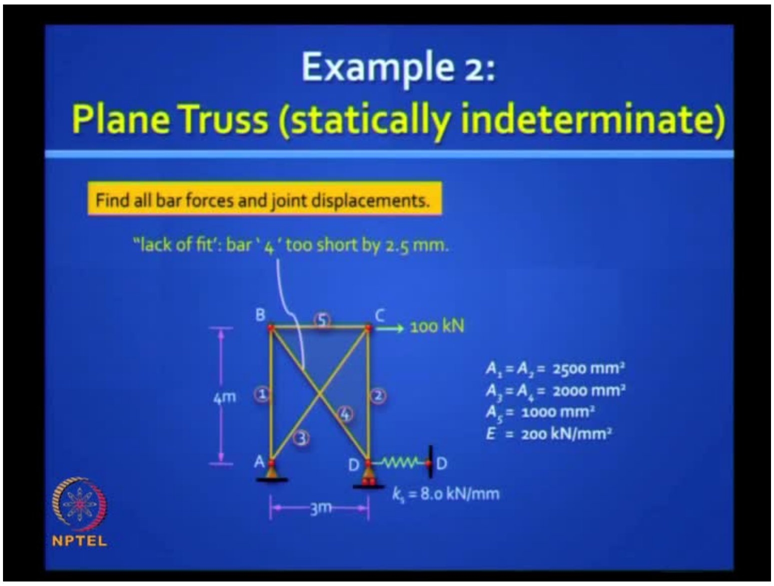


(Refer Slide Time: 28:46)



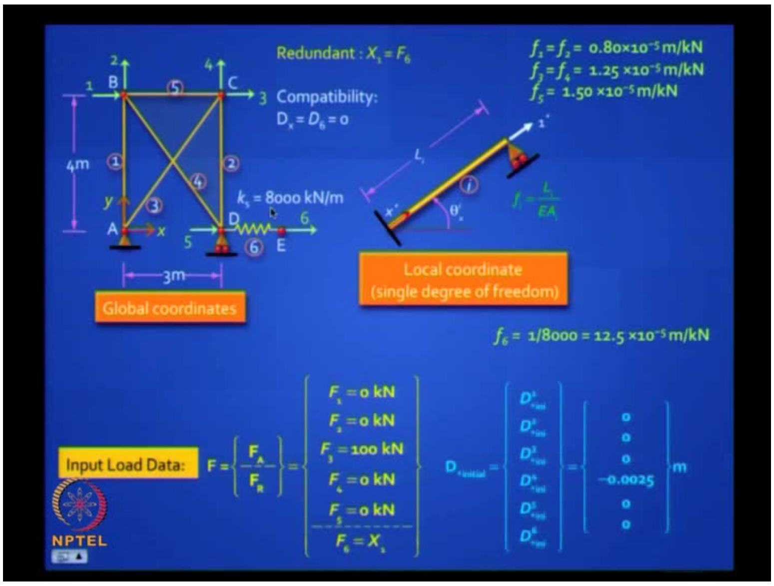
This is the flexibility method; it is quite simple, quite straight forward, the only thing is different people will have different solutions still the end. The final solution should be the same but the intermediate steps will be different. So, the steps are, if you want to program it, input the structure data, generate the element properties, generate the structure flexibility matrix, compute f x inverse, because you have to invert f xx. In case you are dealing with the statically indeterminate problem, input the load data, compute the redundance, compute the element forces, compute the joint displacements.

(Refer Slide Time: 28:54)



In statically indeterminate structures, you can compute the element forces only at the end. After you have got the redundance, statically determinate structures, you get it in the first shot itself, because you have the T F matrix complete. Let us demonstrate with the same problem. I brought an indeterminacy by adding one more element a spring and the loading is the same 100 kilonewton, same lack of fit. Now, this lack of fit.

(Refer Slide Time: 29:52)



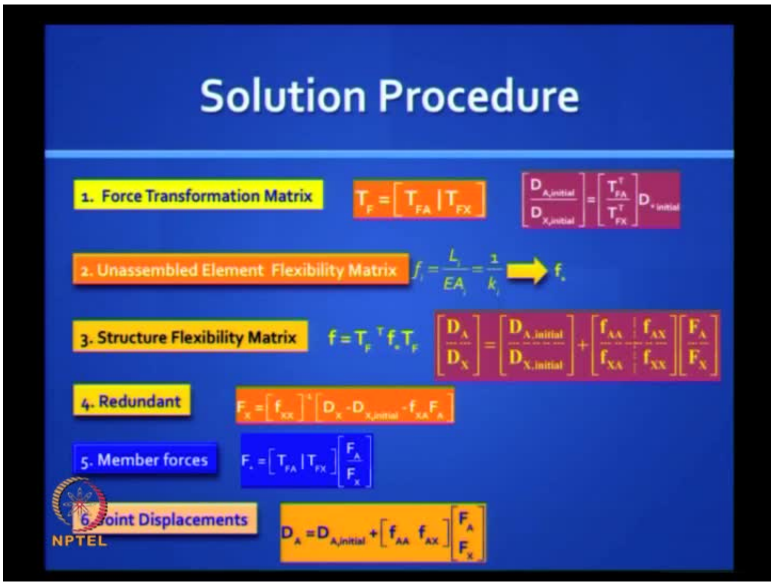
Now, you can see that earlier the roller could happily move inward by that amount. Now, the spring is not going to allow that, so, is going to cause some issues to the displacement field. We want to know all the answers in one shot using matrix method, is it clear? The question clear? How do we proceed? Well, we will define the same global coordinates, but we will choose the force in that spring as a redundant. The spring can be treated as another axial element. We have done the same problem by one of those stiffness methods.

So, F 6 is X 1, there is one the degree of static indeterminacy is 1 in this simple problem, and the compatibility requirement is D x is 0, D 6 is 0, what is it really mean? In this instance, it means that joint E does not move but you could have used it has a cut in the spring and then put the compatible condition is there is no relative separation between the cut ends either interpretation is fine. Right now we have remove the supported E and the reaction you get at E is happens to be the force in the spring, either interpretation is fine, is it clear?

Now, I have got a sixth coordinate which was not there earlier and I have a sixth element which was not there in the earlier problem, how do I proceed? Well input load data is exactly the same, only thing I have F x not F r, F x which is X 1 and the sixth element does not have any initial lack of fit. So, I added one more element and I added one more at global coordinate, that is the only change, and my element flexibility is as given earlier, these numbers do not change.

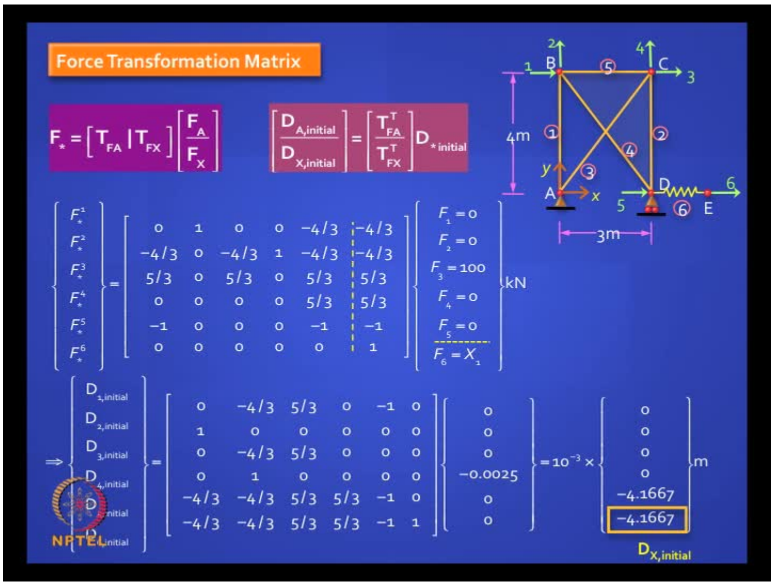
But I have to now introduce a flexibility of the sixth element. Now the spring stiffness is given to me, I just invert it, I got the spring flexibility, is it clear? So, it is very simple. When you do the stiffness method, you retained the spring stiffness as it is. Now, what do we do? First write down your T F matrix but now, you have T FA and you have T FX.

(Refer Slide Time: 32:00)



The T FA will be the same as the T F that we did earlier, because we had 5 elements, you have to add sixth element that is all, T FX will find out. Find out your initial displacements as we did earlier. Primary structure, you can find out uniquely. Generate the flexibility matrix, you have to add the sixth element; generate the structure flexibility matrix; solve for the redundant; find the member forces; find the joint displacements; absolutely straightforward stepwise.

(Refer Slide Time: 32:36)



Let us do it step by step. Force transformation matrix, I just need to do one more case that is I put F 6 equal to 1. If I put F 6 equal to 1, I get these bar forces, the rest of it I have already done. This is what I did earlier; only thing I added is sixth row. Why did I add a sixth row? Because there is a spring which was not there earlier, is it clear? That has six elements here, is this clear to you? And my force vector, I add this unknown X 1 as my sixth element, any doubt? Ask.

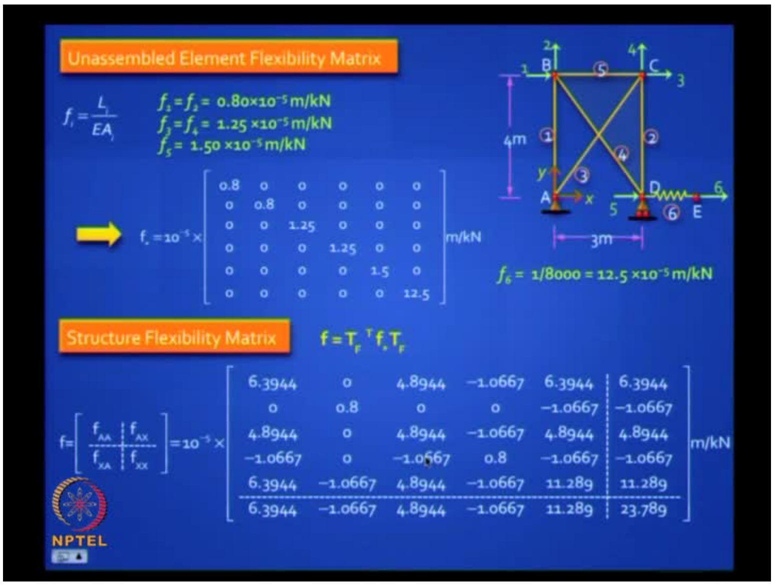
(( ))

Take this structure, if I apply F 1 equal to 1, will these bar forces change here? Will I get any force in that spring? The spring is free here by the way. You apply F 1 equal to 1, will I get any force in that spring? Unless I pull the free end of the spring. Even if I apply F 5 equal to one, that spring is just hanging loose there. Nothing was I so, do you agree that this is 0, this is 0, this is 0, this is 0, this is 0, (Refer Slide Time: 33:56) and it have a value when I apply F 96 equal to 1.

When I apply F 6 equal to 1, what happens? Plus 1, plus 1.

It should be plus 1, you get tension in that spring, and that actually what happens, it is interesting, then it is like F 5 equal to 1 for the rest of the spring, because I am pulling that end of the spring, it is as good as F 5 equal to 1. So, that is why this part of the vector is the same as this part of the vector, agreed. So, let us not go back, we know what we did earlier. We should write down the same vector. Then, what we do next?

(Refer Slide Time: 34:42)



We write down the unassembled element flexibility matrix. Everything is the same except for the sixth element. We have completed the axial flexibility of that element, so, that comes in as 12.5 into 10 raise to minus 5 meter per kilonewton. Then we do the same product and we get the stiffness matrix. If you compare this stiffness matrix to the previous one, what is common, can you tell me?

The previous one for the statically determinate structure was of 5 by 5 matrix. Now, we have a 6 by 6 matrix, what is common between that and this?

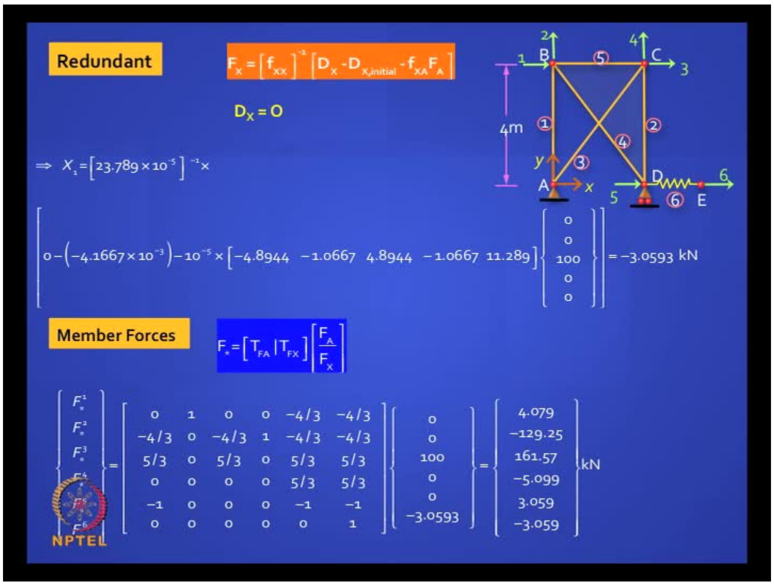
(( ))

F AA here

(( ))

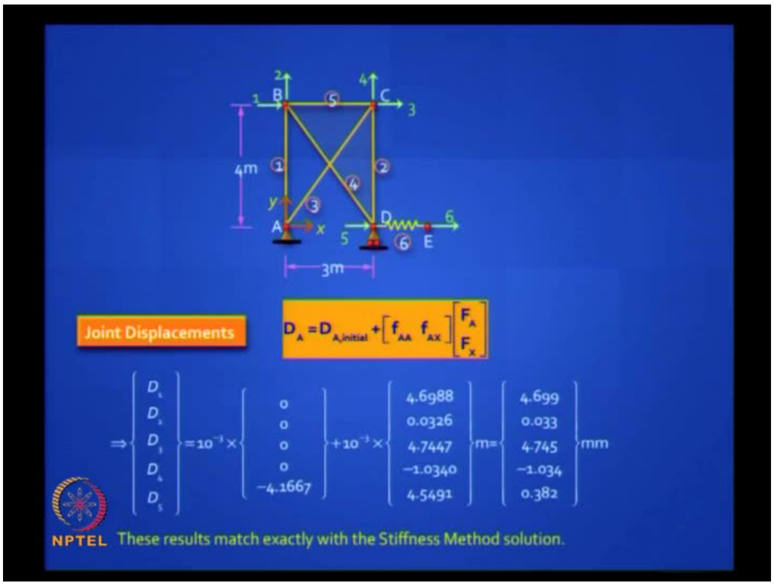
Is just same as the F earlier, is it clear? So, that is the only difference. So, you will get the same matrix, because you just have a sixth element, you have a sixth element, will proceed. Then you need F XX, you need to find the inverse of F XX which is just one divided by that value because it is a 1 by 1 matrix.

(Refer Slide Time: 36:03)



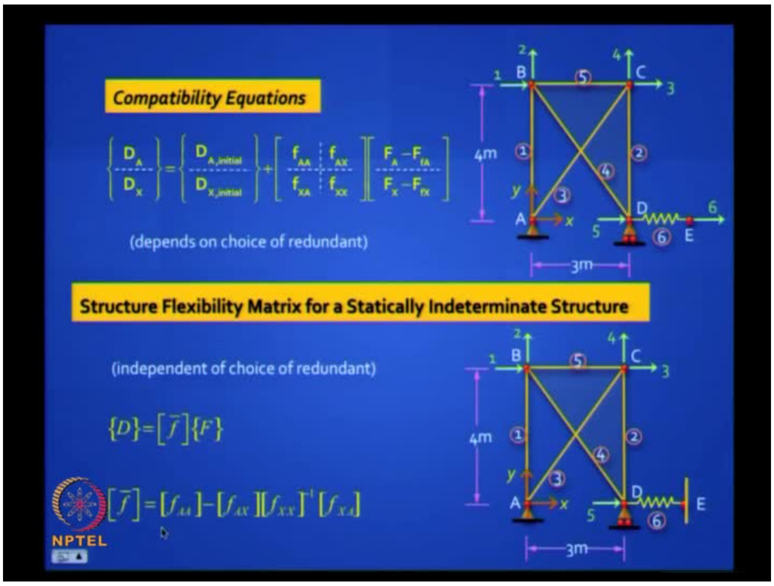
Then you find the redundant by applying the compatibility equation. A compatibility equation is D X is null vector, solve it, you get some value minus 3.0593. Mind you, in this solution, we are also including the initial displacement. Earlier, other forces were not affected by the lack of fit, but now, a force is effected by the lack of fit because it is a statically indeterminate structure.

(Refer Slide Time: 36:38)



Then what do you get? Find the member forces, now that you got X 1. You can get all the answers, is it clear? And you have to find the joint displacements and very easy, straight forward, exactly what we did earlier, but now you see, you have joint displacements everywhere, it adds up, and incidentally, we did the same problem by the stiffness method, you can just compare the final answers, you will find the match exactly, is this clear?

(Refer Slide Time: 37:15)



Statically indeterminate structures are not difficult, easy to do. Just want to raise 1 point – here, the compatibility equations depended on the choice of the redundant, and your flexibility matrix was a little complicated, but the actual structure did not have a redundant, the redundant was something you choose for your convenience.

So, what about the actual flexibility matrix of the structure? What will be the size of that?

(( ))

What is the size of here?

No, 5 by 5 by 5.

It will be 5 by 5.

It will be 5 by 5.

Can you find out what that matrix is from these values? So, that is interesting. It is just for your general understanding. You can have the flexibility matrix which I am calling f bar, which does not depend on the choice of the redundant, you know, I could have chosen any bar force here as the redundant but I do not need to. There are five active degrees of freedom in the actual structure, with out and I can write down the F bar matrix.

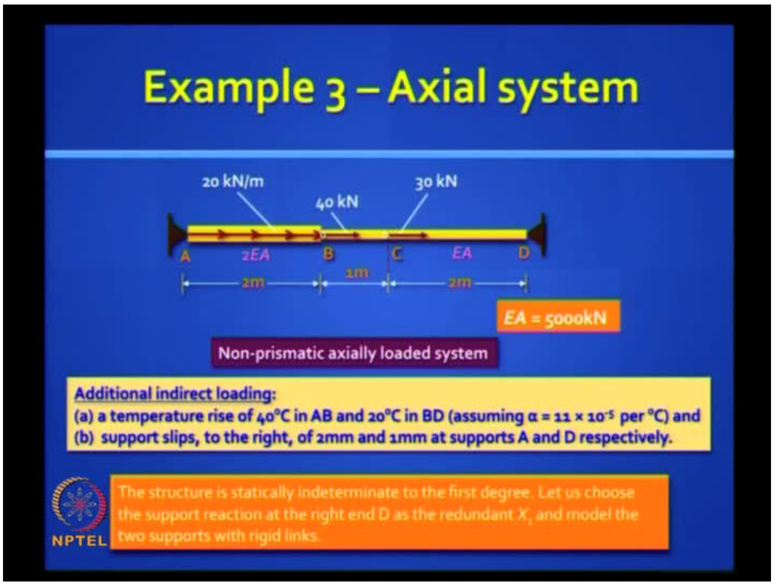
Can you tell me how to generate this F bar from this flexibility matrix?

Keep F 1 equal to 1 and use a compatibility condition all those things get that bar force.

Can you write an equation, can you write an equation?

So, it is a potential quiz question. Now, you all wake up. How to write using where ever redundant you have chosen? Can you write flexibility matrix for the structure? So, I will give you the answer, you can prove it its, but I leave the proof to you, you can do it, because once you got these, which means different student will have different sub matrices here depending on their choice of redundance, but when the carry out this product, everybody will be left with the same structure of flexibility matrix. So, the proof of this I will leave to you but it is very interesting.

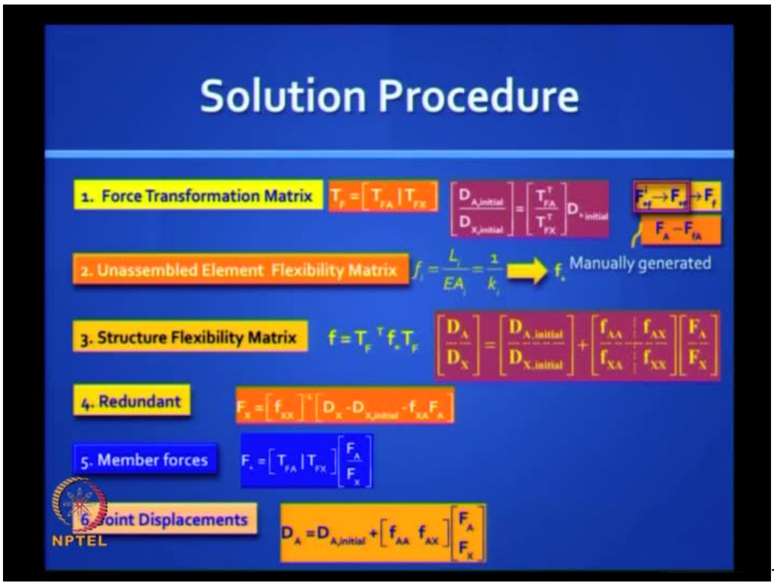
(Refer Slide Time: 39:37)



Last problem, let us quickly finish. We will do an axial system. Again I want to repeat this is more for our learning, more for using it manually. Flexibility method is really not suited for a generalize solution for big problems. So, we have done this problem earlier. Now, we will, we will do both loading simultaneously. You have a direct action, you have all those distributed loads acting plus we have additional loading. Remember, we did these two cases separately earlier, now we are doing it two in one.

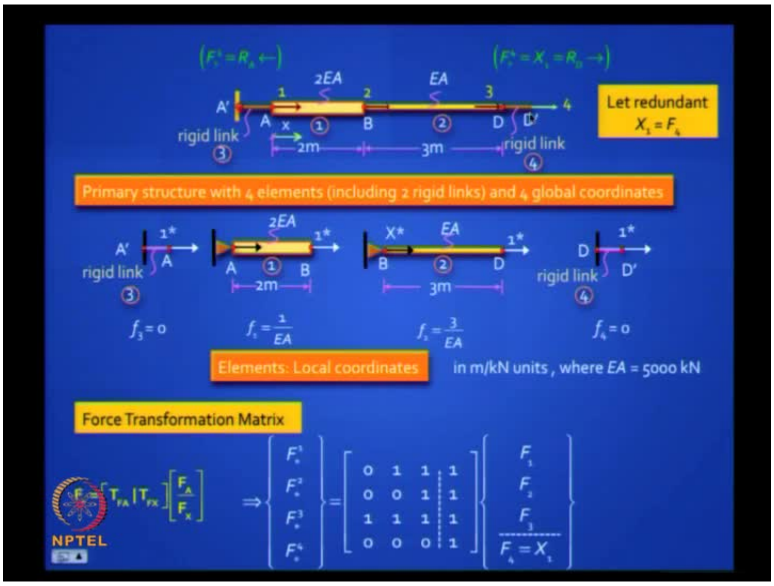
The structure is statically indeterminate to the first degree you can see, because if you move one end, it becomes determinate. Let us choose a support reaction at the right end D as the redundant X 1 and model the two supports with rigid links. I am going to demonstrate here how you can also find support reactions bringing an additional member called rigid links. Let me demonstrate that.

(Refer Slide Time: 40:48)



The procedure is exactly the same only thing, here, you know, you have, it is a pity that you have to barrow a stiffness method idea to deal with distributed loads because there is no other way you can handle it in a matrix framework, so, you have to find out fixed end forces, which means doing going back to stiffness barrowing that concept from stiffness method, and because if your fixed end forces are not constant in your element, you cannot even do a matrix transformation, you have to do it manually. So, those are the limitations of flexibility method. The rest of the procedure is exactly the same, no difference.

(Refer Slide Time: 41:32)



Now take a look. I have now introduced two elements here, elements 1 and 2 were there in the original structure. I have introduce here elements 3 and 4 and I am calling them rigid links because they are short links infinitely stiff. The idea is the force in that link is my support reaction.

So, I am able to access support reactions by converting them as internal forces in additional imaginary members if I want to. Normally you would not do such things, you would just leave it in peace, and, you know, find the internal force in that bar and then figure out what this support reactions, but if you really want the matrix method to give you the support reactions just as the stiffness method did, this is one type of way of doing it. Introduce additional members whose internal forces will give you your support reactions.

Now, here, my redundant is the force in at this end here which I called X 1, which is the reactions in the other support, and it follows that the internal force in this element 3. This internal force, it is positive. If it is tensile, do not you think it will give me the reactions pointing this way, this force, whatever, let us say I get 20 kilonewton, that internal force is my reaction R A pointing to the left, then only I get tension.

Let us say I get minus 20 kilonewton that means R A is pointing to the right, so, I must be able to interpret the physical significance of the internal force in terms of the reaction that I really want, because actually, there is no rigid link, I just created it. But here, the force the tension in the rigid link which also happens to be the redundant because it is also self-equilibrium, that will give me my support reaction R D pointing to the right, because it is tension, does it make sense.

This is just for your general understanding, do not worry too much, do not do, you know, problems in the examination doing all this, but if you really want to, you can handle support reactions, I just wanted to demonstrate.

So, you are now having four elements - element 1, element 2, element 3 and element 4 - and all the four elements have only 1 degree of freedom, it is an axial degree; it refers to the elongation in that member and the corresponding force is the axial tension in that member.

The four flexibilities are given. If you are dealing with the rigid link, what is the flexibility of the rigid link? 0 because the stiffness is infinity 1 divided by infinity and EA is given to you, so, you can write down those values. Now, you have to write down the force transformation matrix; now, write it down. I am not going to spoon feed you, write down the force transformation matrix for this simple problem.

What is the size of that matrix T FA, T A, T FX?

(( ))

How many elements are there?

4, right?

So, you need 4 rows, how many columns are there? 3

So, mind you, there are four global coordinates - 1 2 3 and 4; 1 2 3 belong to active coordinates, 4 belongs to redundant coordinates. So, the size of this vector is 4 by 1; the size of f star is 4 by 1 because there are 4 elements. So, the size of this is 4 by 4, give me that vector, give me that matrix 4 by 4, how do you get the answer? Apply f 1 equal to 1, so, it is very easy, it is like a chain. The clue is if I pull the free end of the chain, all the forces will carry the same tension, but if I pull the front end here, only this will carry tension, these will just move along like rigid bodies. They would not have any forces, right? Can you fill up? All of you please fill up the T F matrix from first principles T FA, T FX.

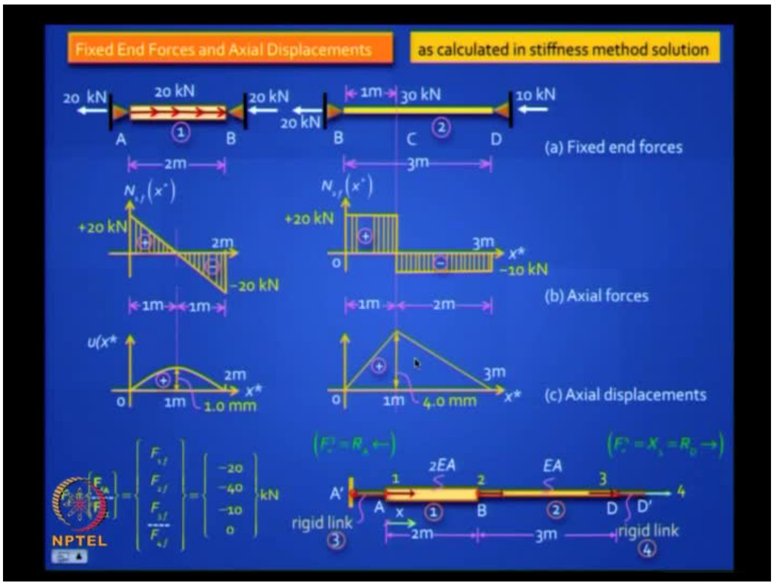
It is nice to do new types of problems which you never done before using very simple, have you finished? It will be just fill with ones and zeros. Have you all done it? Did you all get it? Rise your hands if you got. One solitary figure, let us do it together.

To fill up this vector, I apply f 1 equal to 1; which means I pull this end by 1. If I pull this by 1, which member alone will have axial tension number?

Third

Number 3, you would be you careful about the numbering because this element 1, 2, the rigid link is 3. So, the third only has one may be you put in top. You got it all wrong because your numbering, you should match your numbering. If you apply F 2 equal to 1, the first 2 elements will have unit forces 1 and 3. If you pull F 3, 1, 2 and 3; if you pull F 4, 1, 2, 3, 4. That is this makes is very simple. So, it is really easy in an in 1 D system, it is a chain, clear? Can we proceed?

(Refer Slide Time: 48:07)



Suppose, you did not want to find the support reactions it is a very simple problem 2 by 2. Now, this part, you have borrow bodily from reduced element stiffness method or stiffness method because there also you need this. So, remember we have done this. We have to find the fixed end forces, and after you done this, you have to figure out what the fixed end force vector is.

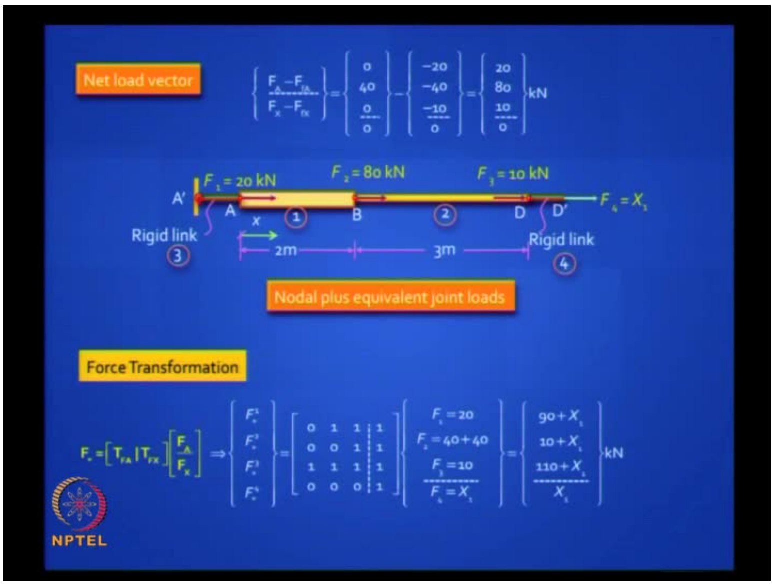
So, you look at these results and then manually you have to figure out what F 1 2 f is what F 2 f is F 3 F and what F 4 F is. So, look at this structure F 1 F will be the force in this coordinate, so, this is 1, 2, 3, 4, so, F 1 will be minus 20, you can see this direction; F 2 will be, F 2 will be minus 40; F 3 will be minus 10 because it is pointing this. See F 1 is pointing to the right, F 2 to the right, F 3 to the right and F 4 is 0, is it clear?

From this figure, the top figure alone pick up F 1, pick up F 2, pick up F 3, F 4 is outside. Any doubts on this? This has to be manually picked up. In addition, you must keep in store all these axial force distributions and the axial displacement distribution. Any doubts on this?

(( ))

We done this analysis earlier, we are just picking up manually F 1, F 2, F 3, any doubts? Shall I explain once more? Is it clear? Good.

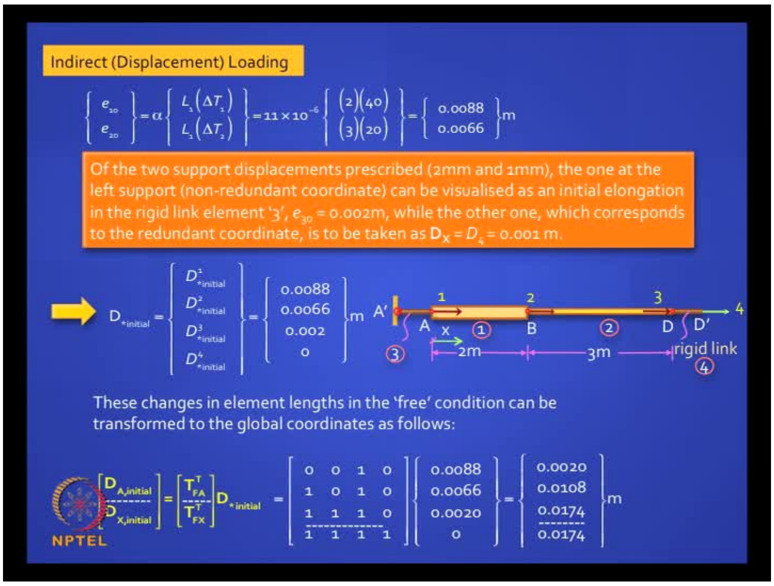
(Refer Slide Time: 50:07)



Then you need to find the net load vector. Remember, in the original problem, you had F 2 equal to 40 kilonewton, but now you have the fixed end force vector which you have to oppose for your equivalent joint loads. Remember, you have to put in the minus sin and then you get the resultant net load vector, what do you do next, what do you do next?

This is your, if only you knew X 1, you could get the forces. This is your force transformation. How to find X 1 compatibility? How to, what is compatibility?

(Refer Slide Time: 50:45)



D x is equal to 0.

Well, before that, there is an indirect loading, do not jump. The indirect loading was you had a temperature change in the 2 elements, so, it is like a lack of fit. In the primary structure, they will be freely elongate; this is the elongation you get. What did you need to do? You have to find the effect; they have at the joints by doing the transpose.

So, after 2 support displacements prescribed 2 mm and 1 mm, the 1 at the left support is a non-redundant coordinate, it can be visualized as an initial elongation in the rigid element 3. Do you understand? Remember, you had 2 elements, you add an extra element, you put 3 and 4.

Support the first one is, this is temperature loading but you had a support movement also. The temperature loading you can handle directly, but the 2 supports were moving. Remember A and D were moving; a moved to the right by 2 mm. It is like the rigid link 3 had an elongation, a lack of fit of 3 mm, are you getting it? The rigid element three had a lack of fit, it was too long by 3 mm; it is as good as a support moving three mm, is it clear?

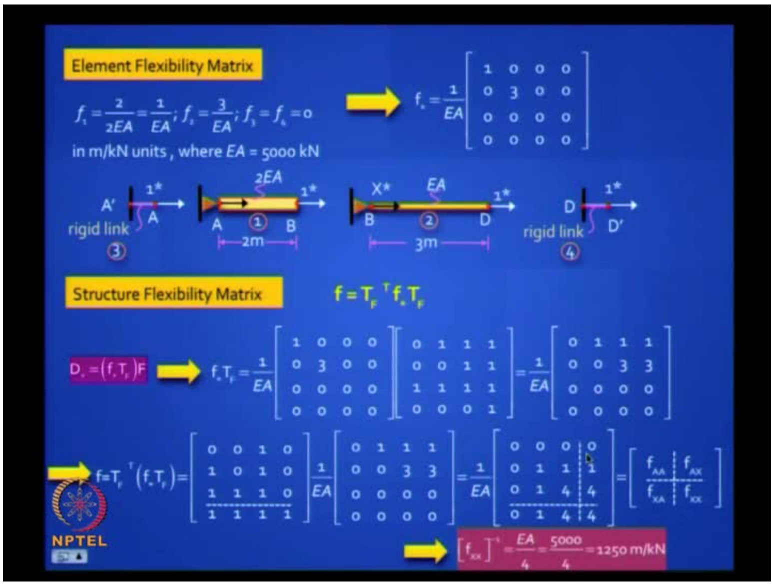
But the second one, this is a tricky one which corresponds your redundant coordinate, because that fourth rigid link, the right end of it is your redundant location; so, that will go to D X directly. So, there is a little catch in this beautiful problem - D X is D 4 is 0.001. So, if you want to write on the compatibility equations, you can convert these to the joint locations doing this transformation, please go through this example very carefully because it is got all kinds of complications in it, and you have to write the…

Where we have D 3 initial is zero point zero zero (( )).

You see, in the initial primary structure, what are the changes you get at the element level? Firstly element one has due to the temperature increases is increasing its length by 8.8 mm; element 2 is increasing by 6.6; element 3 is, is given, is increased by 2 mm; element 4.

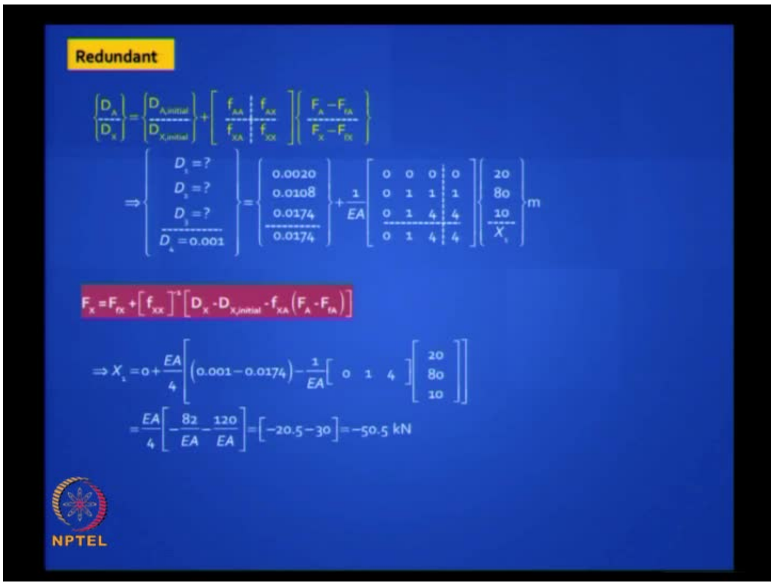
In the primary structure, that point is free to move; it is a redundant coordinate, so, you have to accommodated in D X in the compatibility equation not in the member elongation. We have done similar problems in method of consistent deformation. Remember, when we did the formulation, you had delta 1 L, delta D, so, you have to know which to put in the delta D column, which to put in the D X column, similar situations. In the D X column, the redundant location, you put directly the value; in non-redundant location, you put the other value.

(Refer Slide Time: 54:16)



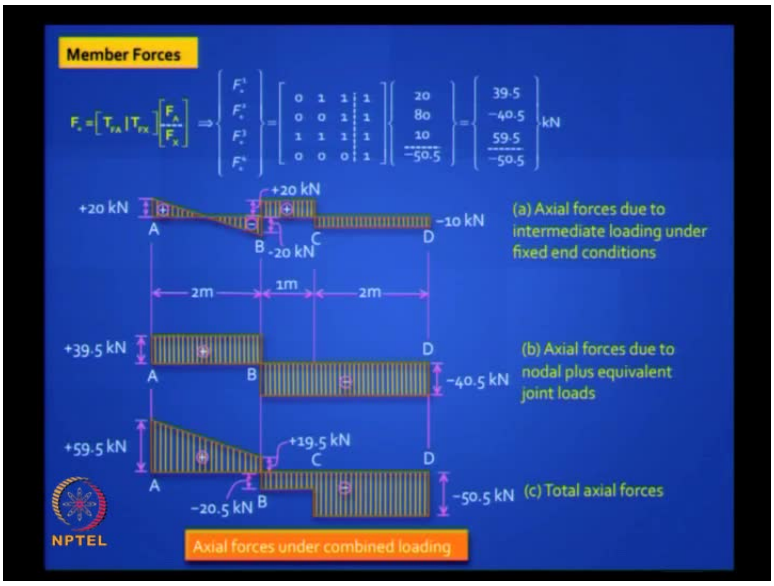
So, if you understood that, generate the element flexibility matrix, you have the T F matrix, you have the f star matrix, because all the elements these f 3 and f 4 are 0, so, the bottom rows are 0, and find out f matrix by doing this product. The procedure is clear to you, any doubts on this?

(Refer Slide Time: 54:38)



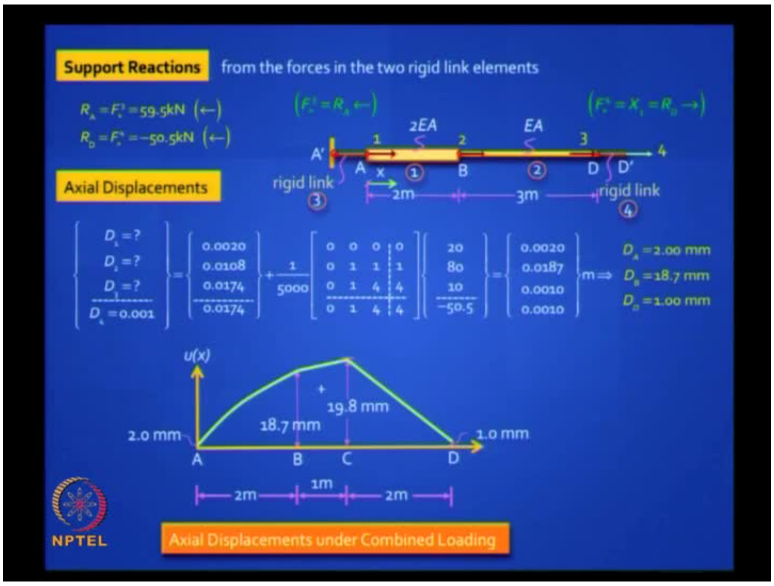
Just pre multiply and post multiply and pick out f XX, f XX is always at the lower right hand corner, always the f XX is located at the lower right hand corner, and this is your compatibility equation. Remember, the D X here, the final solution is 0.001 because the right support moves by 0.001, so, your primary structure, that is your D X, and this is your complete equation. So, the formulation of this problem is very interesting. If you make a mistake in the formulation, the solutions will be wrong.

(Refer Slide Time: 55:18)



So, you carry this out, solve for your redundant, you will find it is 50.5 kilonewton, find your member forces, you got the results. If you add up the 2 solutions which we did in the stiffness method, you will get this, so, it is a final check on this. So, are you comfortable with flexibility method? In your examination, I will ask you at the most only trusses because whenever you have intermediate loading and all that, flexibility method is not good. So, this you just read and put aside, do not break your head too much over this, but trusses it should handle any flexibility problem, is it clear?

(Refer Slide Time: 56:02)



Two types of problems - direct loading, second is lack of fit or temperature loading. Can you handle those two? Indeterminate structures plus you have an assignment problem which you can do. This is a little more difficult and you need not find support reactions using rigid links. Unless you are force to like here, here there was another need because the support was moving and there was no other way you could handle it, is it clear? You can find the support a reaction also, find the axial displacements, and get your final solution, is it clear?

So, we have completed one D structures space frames. We have done conventional stiffness method, reduced element stiffness method and flexibility method. Tomorrow, we will work with beams and grids. That last another 6 lectures.

Thank you.