Advanced Structural Analysis
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Module No. # 4.4
Lecture No. # 25
Matrix Analysis of Structures with Axial Elements

Good morning. This is lecture 25, module 4, Matrix Analysis of Structures with Axial Elements.

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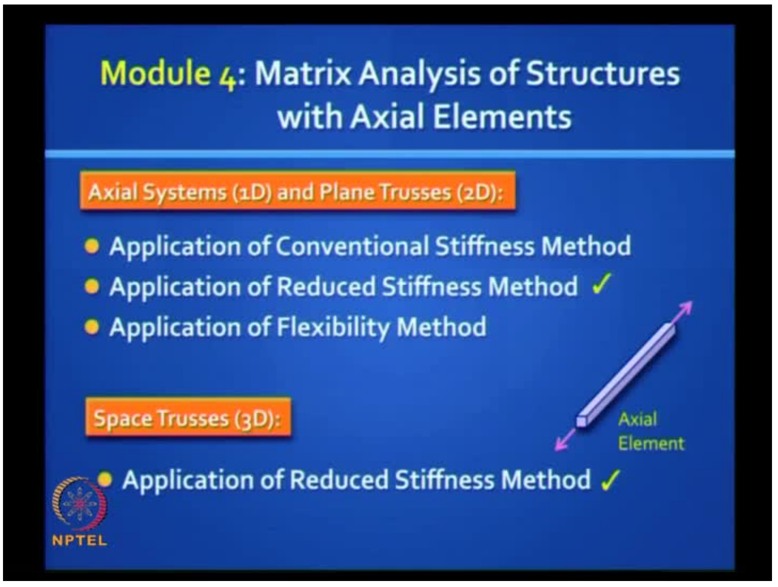

In this session, we will be completing application of the reduced stiffness method. If you recall, in the last class, we had shown how this can be applied to 1D systems and also plane trusses. We will do one more example of plane truss. Then, we will take a look at space truss.

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This is covered in the book Advanced Structural Analysis.

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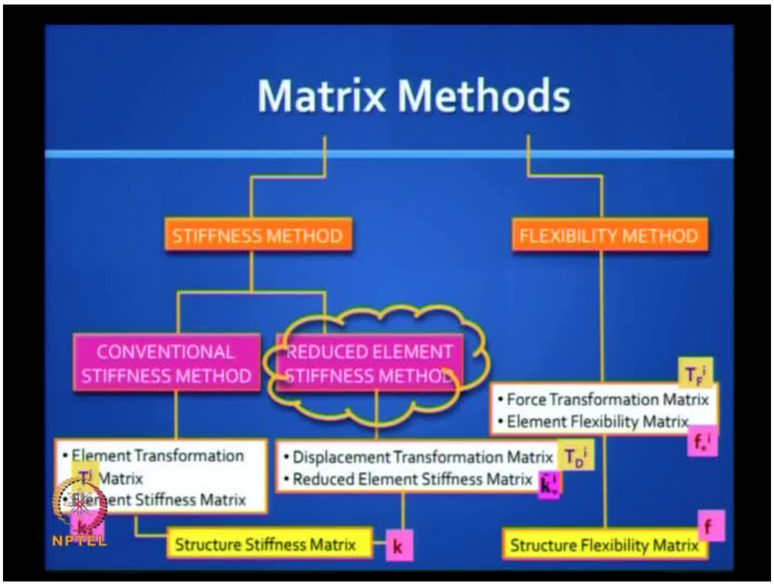

Reduced Element Stiffness Method

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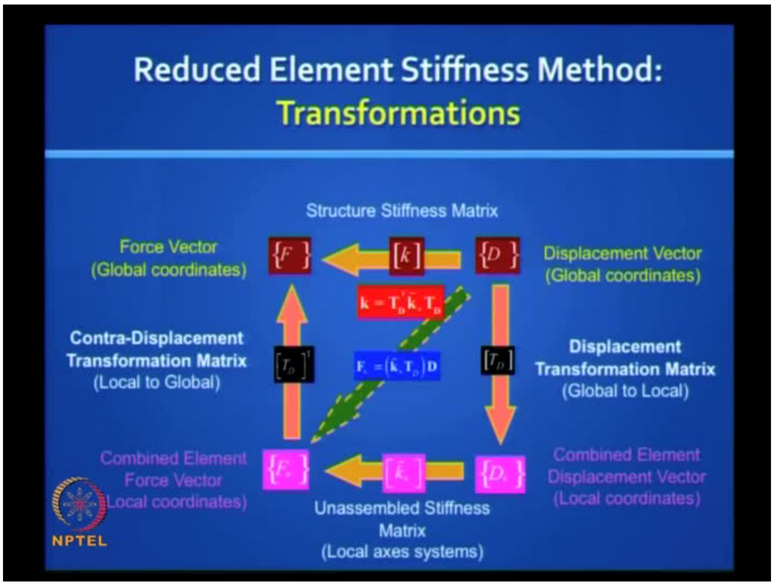

Remember this diagram. This is one way of saying the big picture, global coordinates, local coordinates, structure stiffness matrix, element stiffness matrix and the two transformations. In the stiffness method especially the reduced element stiffness method, your first task is to get the T D matrix correct. You remember that we had some difficulty when we did in the plane truss. Today, I will show you an alternative way of doing it.

Once you have got the T D matrix, you can partition it to T D A and T D R. If the problem does not require you to find support reactions and you do not have any support movements, then you can skip the T D R. It is enough to do T D A.

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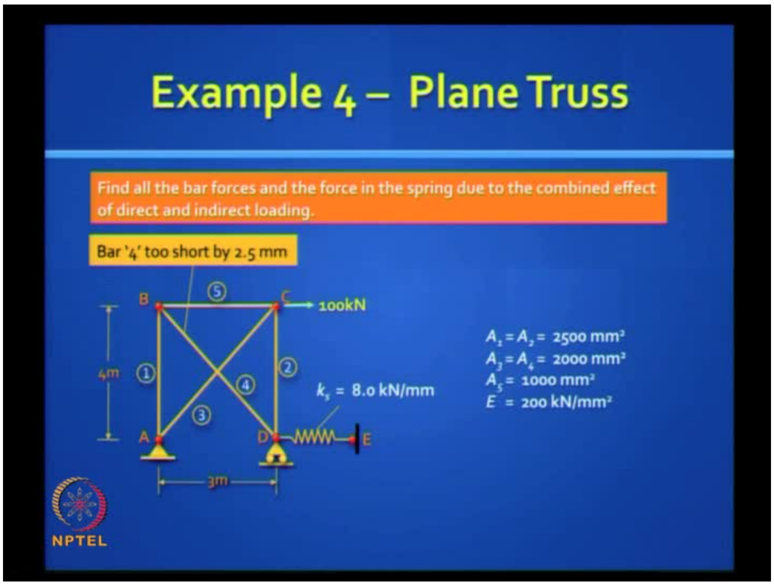

Remember: This was the problem we solved at the end of the last class.

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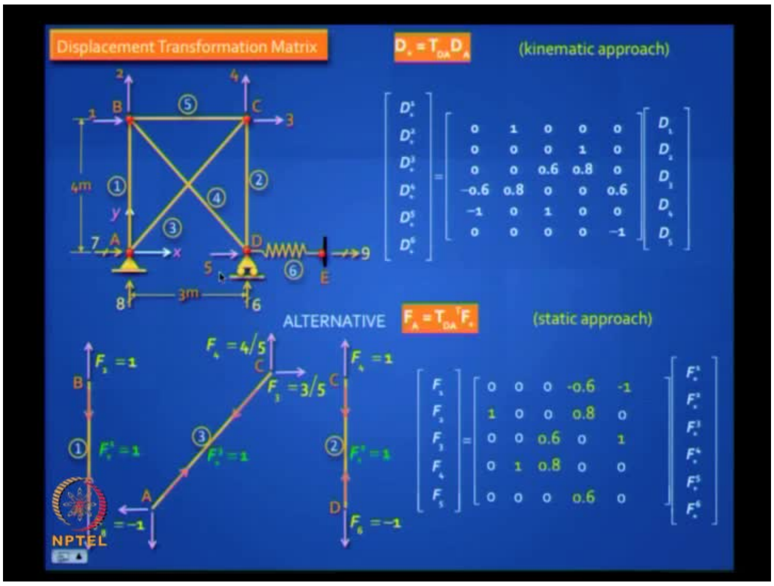

It took a while for you to get the idea of getting the T D A matrix using the kinematic approach. Finally, we got this. You remember? We did it step-by-step. This is actually a very efficient way of doing it because you can program it. You do not have to think. It will do it automatically provided you are clear about the start node, the end node, and the correct value of theta I x.

There is another way of doing this. Can you tell me that?

There is another perhaps simpler, but then you have to solve every problem manually. What would that be?

Force and step method.

You can use a static approach. How do you do that?

[Noise – not clearly audible] (Refer Slide Time: 02:42) We have studied T D transpose.

T D transpose; you are actually invoking the contragradient principle. Is it easier to derive the T D A transpose matrix and the T D A? Let us see.

In the static approach, this is the relationship (Refer Slide Time: 03:03) that we want. Please note: In the kinematic approach, what we are doing is we have the structure, in the structure, we are applying one global displacement at a time; arresting all the other displacements. Then, we are finding out what happens at the element level. If you look at the element displacement vector, what would these quantities refer to? (Refer Slide Time: 03:32) These are the element displacements. What do they refer to? What does D 1 star refer to?

Axial deformation in that;

It is just the elongation in the element 1. This has only one degree of freedom. Is it clear?

If you move the structure, the joint displacement one at a time, it is going to cause elongation in the different members. We are trying to figure out what these elongations are. Let us say you want to fill up the first column. You put D 1 equal to 1. You can actually draw a sketch and figure out what the elongations are; or, you can do it mechanically using the formula that we demonstrated in the last session. We assumed minus cos theta, minus sine theta, plus cos theta, plus sine theta for the sign convention.

Now, we are doing something different; something very interesting. I want you to understand physically what we are doing here. What is the size of the T D A transpose matrix for this problem?

5 into 6

5 into 6; so, there are going to be 6 columns and 5 rows.

Let us take the first one. I want to fill up the first column. That means I must apply F 1 star equal to 1. What is F 1 star?

It is the bar force.

It is the bar force in element 1. So, I must visualize the situation, where in this truss, (Refer Slide Time: 05:05) only this bar will have a force. Do not ask me how that force comes.

Let us say somehow you have only one force. Only one element has a force; no other elements have a force. Then, to satisfy equilibrium, you must be having some joint forces; only then, it is a self-equilibrating system. Can you fill up the first column of this (Refer Slide Time: 05:26) using equilibrium? Let me help you. You need to look at only bar 1 because bars 2, 3, 4, 5, 6 do not have any forces. So, will it not look like this? I mean I can draw the whole truss, but I might as well take it out separately. If that bar force in bar 1 has to be unity, then does not equilibrium demand that I must have equal and opposite forces at the two ends. This refers to the coordinate 2 (Refer Slide Time: 06:05) and this refers to the coordinate 8. It follows that if this has to have a unit value, F 2 must be equal to 1 plus 1 because 2 is pointing upward. F 8 must be equal to minus 1 and there are going to be no other forces in the truss. So, help me fill up the first column.

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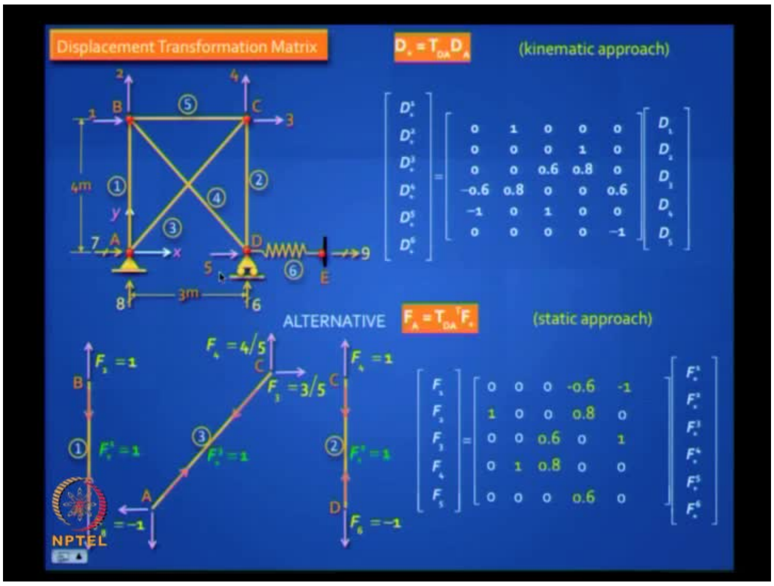

What will it look like? 0 1 0 0 0

Now, what about this F 8?

(Refer Slide Time: 06:40) [Noise]

If I want to include F, the restrain coordinates, then the size of the T D matrix will be bigger; I have to bring in T D R. Is it clear?

Similarly, can you fill up the next column?

0 0 0 1 0

That is the picture; 0 0 0 1 0. Got it? Third one?

0 0 0.6 0.8 0

So, third one. If I have to have a unit force in bar 3, I must have equal and opposite forces. So, very clearly, the component of this (Refer Slide Time: 07:18) along the x axis must be given by this force. The component of this along the y axis must be given by that force. At this end, I need not worry because it is going to a restrain coordinate. So, is it clear that F 3 and F 4; you will get like this (Refer Slide Time: 07:35).

Now, fill up the next one I am not going to draw anymore.

Minus 0.6; minus 0.6 0.8 1 0;

No, now, we are dealing with this bar 4 (Refer Slide Time: 07:49).

0.6 minus 1 0 1 0 0;

Please do it and give me the answer.

[Noise] (Refer Slide Time: 07:58)

We will take a look. In this bar, (Refer Slide Time: 08:07) if it were to have a force, it will effect 1, 2, 5; 6 is outside; 1, 2 and 5. So, what is F 1 going to be?

Minus 0.6

Minus 0.6

What is F 2 going to be?

0.8

Plus 0.8; what is F 5 going to be?

0.6

If you cannot do this, you are in trouble. It is easy; it is not difficult. Actually, you can generate using the kinematic approach and you can do a quick check using the static approach. What about next column? F 5; that is pretty easy. That is, you are going to fill up. If this bar were to have a unit force;

Minus 1 0 1 0 0

Minus 1 0 1 0 0; very good; you have become expert at this. Last one; sixth column? The spring alone will have a unit force. You will get only here (Refer Slide Time: 09:18).

0 0 0 0 minus 1

0 0 0 0 minus 1; great; got it?

Now, just check it out. This row here (Refer Slide Time: 09:32) in the T D A matrix is the column here in the T D A transpose matrix. You can check out all the rows. So, which do you prefer, the kinematic approach or the static approach?

Static approach.

That is because we have been brainwashed from the beginning to study equilibrium. Engineers are usually very strong in statics and very weak in kinematics because kinematics needs a little more exercise of your brain; you have to physically see the movements. Statics is more abstract. Is it clear?

However, it is good to know both techniques. However, when you are programming, just blindly do it by the method we did it in the last class. Is it clear? Now, are you absolutely clear about T D matrix, not afraid of it, comfortable with it? Imagine what you need to do in a space truss; it is really challenging.

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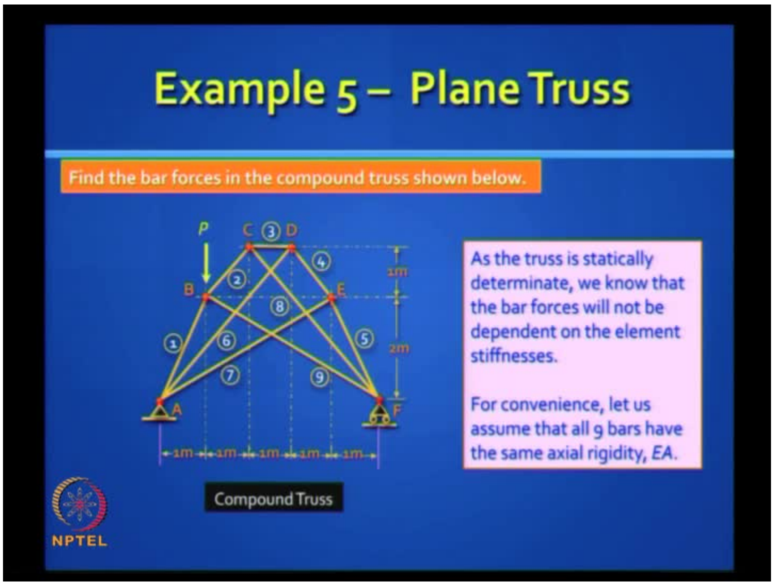

Let us do a tough problem. This is a compound truss. Remember: We have done this in basic structural analysis. Now, we are going to do it by reduced element stiffness method. So, how do we do it? As this truss is statically determinate; how do we know it is statically determinate? You can also do that check m plus r equal to 2J. We know that the bar forces will not be dependent on the element stiffnesses. Let us say that the question here is find the bar forces. So, no need to find the joint displacement. So, you have to assume the axial stiffness of all the members. If no input data is given you, you can conveniently assume that all the bars have the same axial rigidity E A. Will this change the answer?

Let us say every bar had a different E A. What will it change?

[Not audible] (Refer Slide Time: 11:30)

The displacement field gets affected. The elongations in the bars will depend on the axial stiffness or axial rigidity. The joint displacement will get affected, but the bar forces do not get affected. Why?

[Not audible] (Refer Slide Time: 11:51)

Because this is a statically determinate system. So, we can make this assumption. Let us do that and go ahead.

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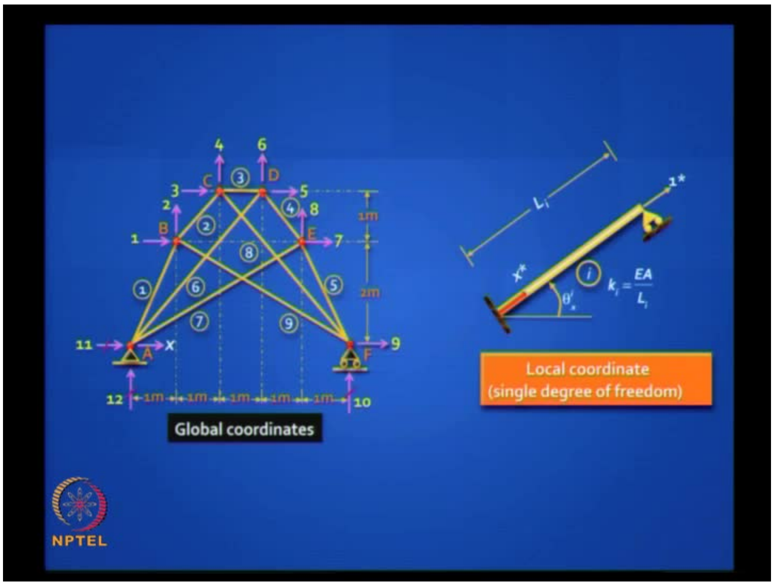

We will have to now put the global coordinates. How many active coordinates you think we have here?

9.

So, let us label them; 1 2 3 4 5 6 7 8 9. We have some restrained coordinates also; three - 10 11 12. What do we do next? Local coordinates; you have to draw that sketch and that is a standard sketch. Every element can be modeled like this. The axial stiffness is given by E A by L. In this case, we are assuming E A is constant for all the bars; just for convenience.

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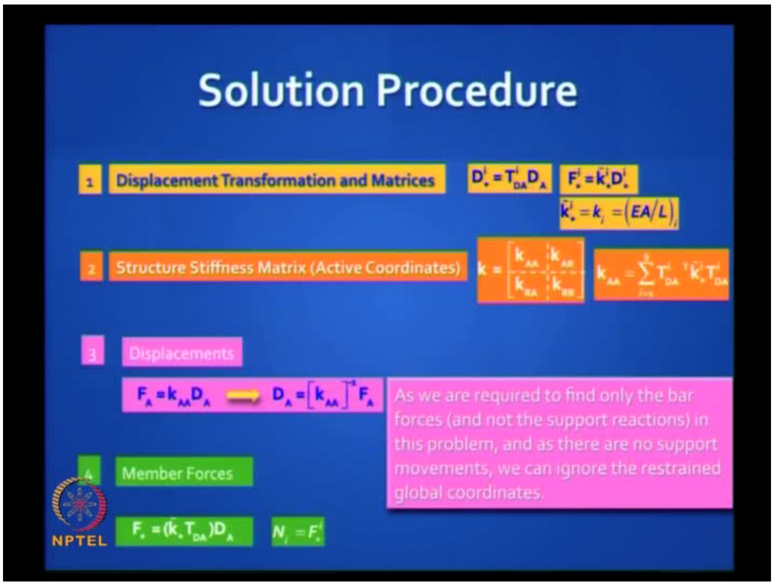

How do we solve this problem? Exactly the same procedure as we did earlier. Generate the T D A matrix, which is what you are going to do now. Structure stiffness matrix; you can use a direct stiffness method and get… All you need to do is K A A here; you do not need to fill the whole matrix because we just want to find the bar forces; very easy to do as we are required to find only the bar forces and not the support reactions in this problem. As there are no support movements, we can ignore the restrained global coordinates. Finally, you get the member forces. These are all standard; as we did in the last problem.

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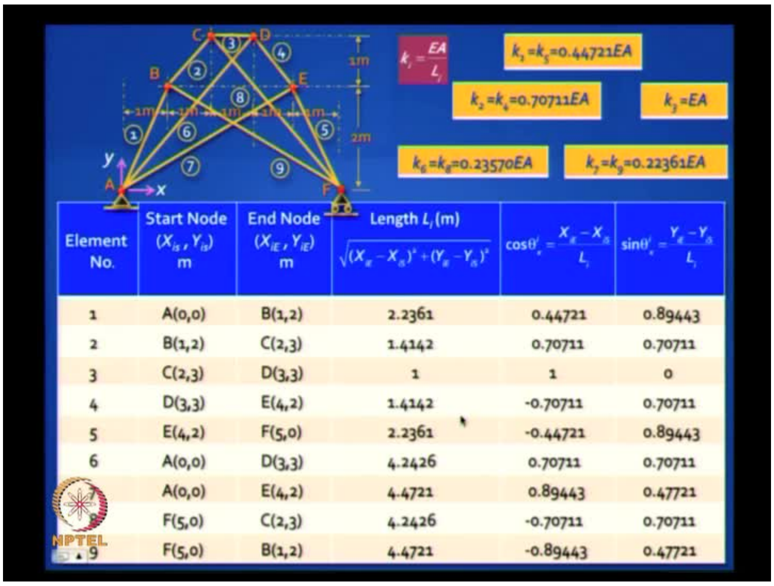

Here is another exercise for you. Fill up the T D A matrix with this help. Now, let us just see how we did this. This table is very helpful especially for programming. There are nine bars. So, I call them 1, 2, 3, 4, all the way to 9. Then, the bars are labeled here. I have to decide my origin. So, let me say origin is A. I write down all the coordinates A, B, C, D, E; everywhere. These dimensions are given to you. So, it is pretty easy to write this down.

Then, I have to decide on the direction for X star. That means I must identify the start node and end node. For example, bar 1 starts at A, ends at B; bar 2 starts at B, ends at C; bar 3 starts at C, ends at D; bar 4 starts at D, ends at E; and bar 5 starts at E, ends at F; I went sequentially. Then, bar 6 starts at A; can you see? (Refer Slide Time: 14:42) It starts at A, ends at D; bar 7 starts at A, ends at E; bar 8 starts at F ends at C; and bar 9 starts at F, ends at B. It is not very difficult to do; in fact, ideal for programming. That is how you identify the joints, the nodes, the elements, and the direction of incidence.

Now, once you have done this, you just have to put in this formula (Refer Slide Time: 15:16). It will generate the length. This formula is straightforward. You do not have to do any calculations; it will do it automatically. So, you will get all the lengths.

Now, bar 3 has a length 1 because it is 1 meter, but let it generate it automatically. So, you can write a small program; it will do it. Similarly, you can get cos theta and sine theta. Is this clear? You do not have to break your head over it; you can do it automatically. With this information, can you fill in the T D A matrix at least for element 1? For the first element, fill in the T D A matrix and by the way, you can also get the axial stiffnesses because it is given by E A by L; you have calculated L here. So, it works out to these values (Refer Slide Time: 16:10).

Sir, signs are interchanged.

Signs are interchanged?

Sir, in the fourth quadrant, cos theta should be positive.

Wonderful, cos theta for the fourth element;

It is right; from the end to the start; how does it work? What is the end?

4

4 minus 3; So, it should be positive.

Sir, in fourth quadrant, cos theta is positive.

Use your corrected value and let us get T D A. Give me T D A for element 1. Is the element 1 OK?

Yes sir.

Sir, except 4 and 5, everything is OK.

Expect for 4 and 5 everything else is OK; wonderful. So, 4 and 5 you correct it and you generate for element 1 only. You are right; the plus and minus got switched. Please get me the T D A matrix.

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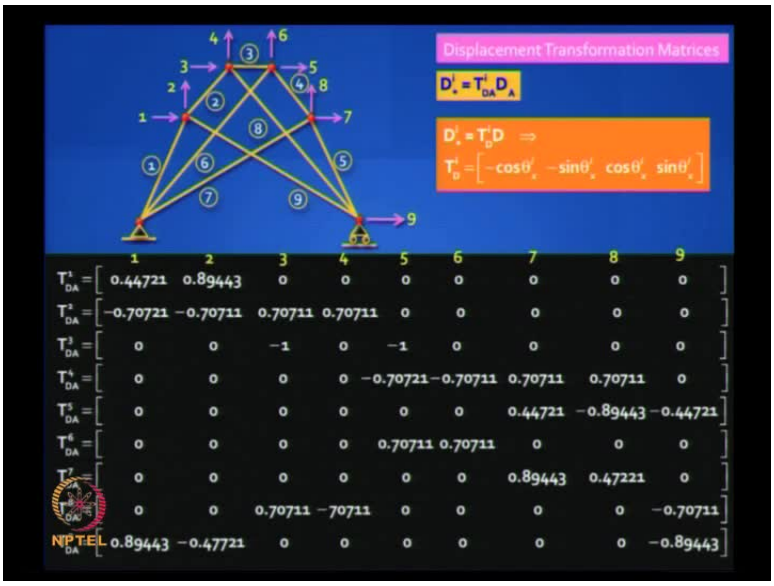

This is the formulation. This can be easily generated. For example, take the element 1. The element1 is connecting to 1 and 2 and something here; probably, 10 and 11 here. 10 and 11 is outside T D A. Which is the start node? Start node is here; end node is there. So, you will be using cos theta and sine theta plus. Does it match, is this correct? Let us just check (Refer Slide Time: 18:06) That is right; cos theta for this is this and sine theta for this is this. Is it clear? All the others will be 0. Like that it is possible to generate for all the other elements. Can you do it, will you work it out? If there is an error in the fourth and fifth, you correct it. Will you do it? Is it difficult, can it be done easily?

Let us proceed. I just want to demonstrate that you can do big problems as well just by programming and letting the computer generate all these results. However, you can also manually do it.

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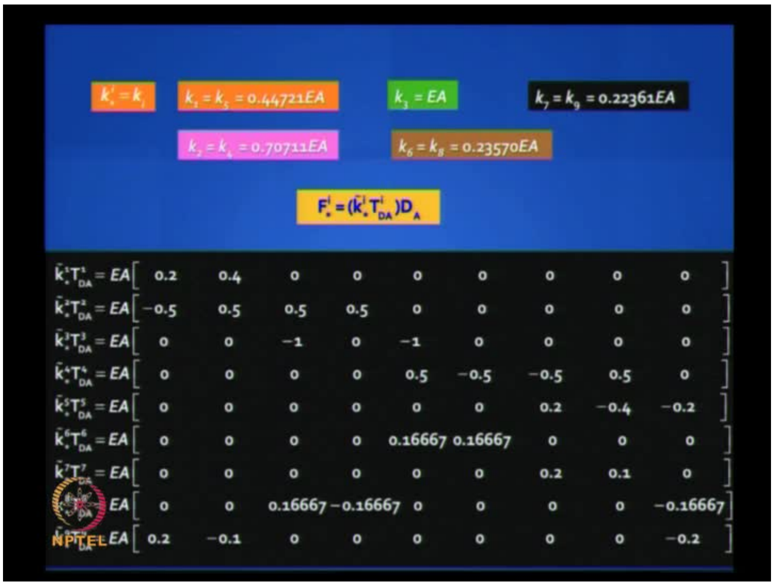

Now, you need to pre-multiply the T D A matrix with the k i star matrix. That is easy to do. You have already got k i star. What is k i star? k i star is 1 by 1 matrix. It is given by E A by L. We have already done these calculations.

Whatever values you got there just multiply them. The first row in the previous T D A matrix, you multiply with k 1; second row, you multiply with k 2. So, you will get these numbers. It is easy to do. What do you do next? Again you pre-multiply all of these with T D A transpose. So, you do that; you are right.

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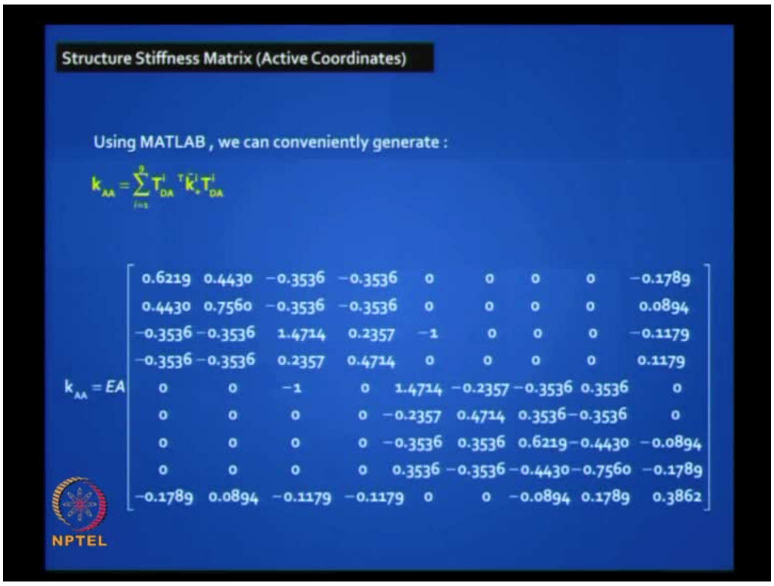

You do that, you some up over nine elements, and you will get the full structure stiffness matrix. Is it clear? You will get the full structure stiffness matrix, but if you make errors in the T D A with signs and all that, some elements in this matrix will have a wrong sign. However, you will notice one thing. All the diagonal elements will always be positive. So, please do this carefully; better not do it manually. Do it in MATLAB or in SKYLAB or whichever software you have. Will you check this out? It is a demonstration of how you can do it for a big problem. Obviously, such problems will not be asked in any examination because you just do not have the tools or the time to do it. However, in real life, you have such problems.

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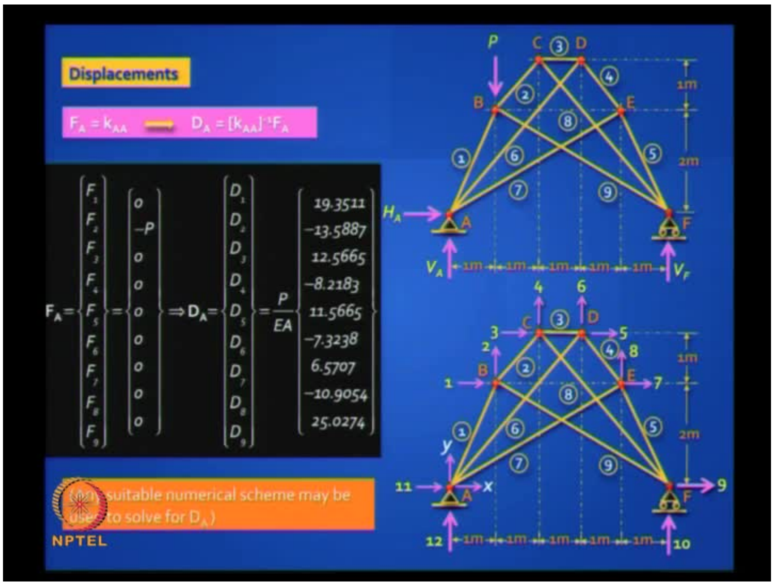

Then, what is the loading? The loading is a force acting at B and that coordinate matches with F 2. So, loading is F 2 is equal to minus P. Of course, you can handle any loading, but in this problem, we had just one isolated concentrated load. You have that stiffness matrix. You can invert it and do not bother to write it down; let the computer do it. Do double precision if you want so that you preserve the accuracy in the matrix; just pre-multiply it. Since you have a well-conditioned matrix, the inverted matrix will be very good. One way is to check it out; you do k inverse k, you should get an identity matrix with very minimal errors. Otherwise, if it is not so well-conditioned, it is safer to use some elimination techniques; or, you can use even Gauss elimination to do it; or, Gauss-Seidel. Those are all the options that you have, but I think it is good enough to just do the matrix multiplication. Once you have done this, you have got the joint displacements. What do you do next?

[Noise] (Refer Slide Time: 21:50)

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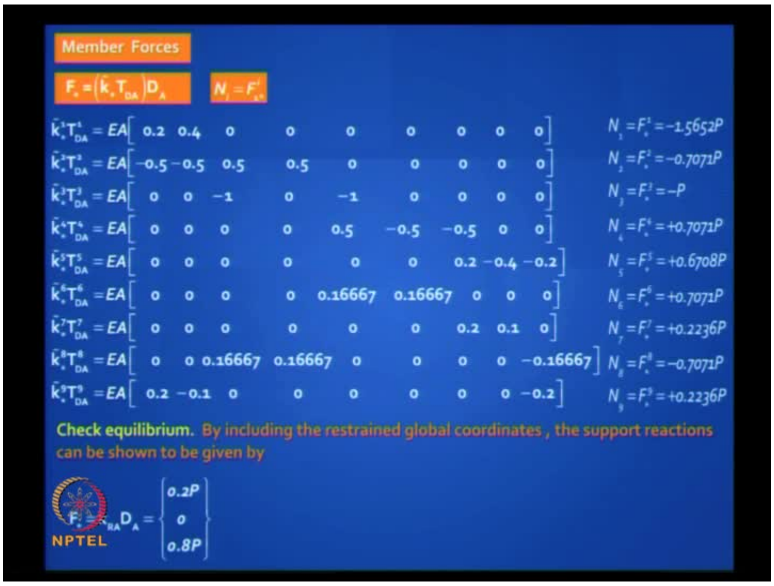

You have got the unknown displacements. [Noise] Then, you get the member forces. Remember: You had already calculated this k T D A. You have got those calculations; just multiply them with D A and you will get the bar forces. Check them out with the answers we got when you did the same problem because it is statically determinate. We used method of sections; they should match. That is the proof. The proof is that you are checking with the force method and saying whether you are getting the answers; that is it. We have solved a problem that is easier to solve using the force method by the more difficult stiffness method because with computer, the software will always do it by the stiffness matrix.

At least you are clear with the method while solving plane trusses?

You can check equilibrium by including the restrained global coordinates; you can also get the support reactions. It is easy to do. For want of space, we are not doing it, but we will get the correct answer.

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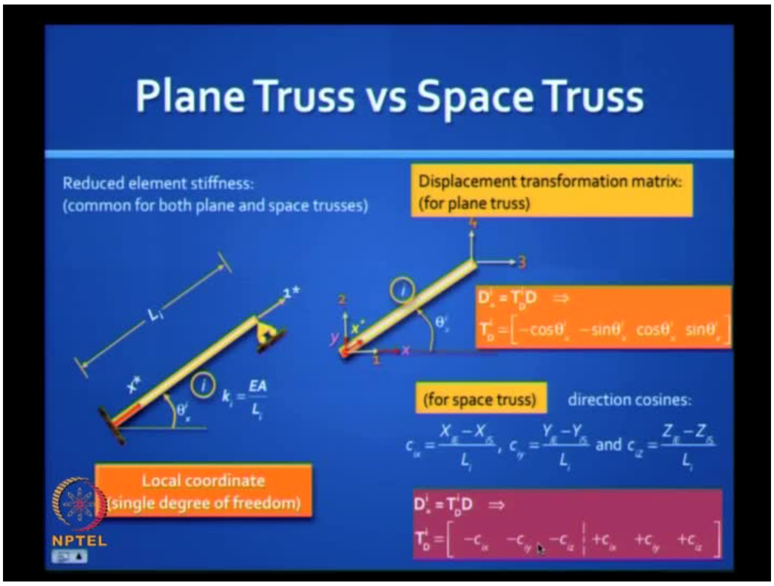

Now, let us jump into the most difficult problem of space trusses. One good thing is that the same element stiffness matrix holds good whether you are dealing with the 1D structure, 2D structure or 3D structure. The rank of that matrix is 1. All you need is the axial stiffness, but you have to worry about the orientations. This is because when it was an 1 D system, like a chain you had just the x axis; when you had a plane truss, you had x axis and y axis. Now, you have x, y and z. So, the local coordinates do not change, but the T D A transformation changes. Can you visualize how it changes?

For example, you are comfortable with a plane truss. You know that it is going to be… If you are trying to link the joint displacements at the two ends with the axial deformation of the element, it is always related by this - minus cos theta, minus sine theta, plus cos theta, plus sine theta. How do you think this will look when you are dealing with an element arbitrarily directed in space?

[Not audible] (Refer Slide Time: 24:16)

Will you use cos theta and sine theta? Are there more thetas than one theta? How many thetas you need to do?

Two; Three

There are three thetas. Let us say I have vector in space; I have x coordinate, y coordinate, and z coordinate. I have three thetas, but they are related by the norm that the direction…

[Not audible] (Refer Slide Time: 24:45)

That is right; cos square theta x plus cos square theta y plus cos square theta z must be equal to 1. Is it difficult to get the cos theta, sine theta? No, once you have the coordinates you do the same game. Can you guess what will be the T D matrix?

Minus cos theta x minus cos theta y minus cos theta z; plus cos theta x plus cos theta z plus cos theta y.

Brilliant, you hit the nail on the head; that is all.

For a space truss, write down the three directions cosines. You do not have to write theta also; you just need the coordinates because this is actually (Refer Slide Time: 25:35) cos theta x, cos theta y, cos theta z. Then, just like you had minus cos theta, minus sine theta; by the way, minus sine theta is actually a direction cosine with respect to the y axis. So, you need the three - the x axis, y axis, and z axis. You have minus; why is it minus for the first three and plus for the next three?

Start node and…

Because if you are looking at this corner, (Refer Slide Time: 26:02) I have not drawn a picture; you have 1 2 3; you will find that you are pushing the element inwards. So, the element is going to contract; whereas, at the other end, you are pulling the element outward. So, it is going to elongate. That is why in the first place, these were minuses and these were plus and the same argument holds good here. You have a doubt?

Yes sir.

Let me explain. This is the element. Let us first do the plane truss. Maybe you should be looking like this (Refer Slide Time: 26:43). This was the x axis; this was the y axis and the coordinate was 1 and 2; and you had coordinate 3 and 4. Now, we want to know what happens when I apply D 1 equal to 1 to this element and D 2 equal to 1? Do you agree? When I apply D 1 equal to 1, I prevent this movement and I prevent D2. This is going to reduce in length because I am pushing it. So, will I get a positive elongation or a negative elongation?

Negative Elongation (Refer Slide Time: 27:20)

Whether I push it this way or this way, I am going to get minus. However, whether I pull it this way or this way, I will get this member elongating plus. Does it make sense now? At the start node, for the direction, we assumed that we will get minus minus plus plus.

Now, what has happened is this is tilting in space. It is pointing like that. So, I have not only x and y, I have z. So, I have three coordinates – 1, 2, 3. Whichever way I push, as long as I arrest this degree of freedom and the other two degrees of freedom, I will always get a contraction. When I pull here (Refer Slide Time: 28:09) I get an elongation. Does it make sense? That is all you need to know.

Now, if you are doing conventional stiffness method, which we briefly discussed, it is little more complicated. However, in reduced element stiffness method, it is pretty easy. Clear?

Sir, tension coefficient method only applicable for truss, which is statically determinate?

You tell me

No

What is the tension coefficient method? It is just writing down equilibrium equations from joint to joint.

It did involve direction cosines.

It did involve direction cosines because the forces have components in all the three vectors. Actually, you have direction cosines, but you have got a certain number of equations; you are not looking at compatibility. So, obviously, it is applicable only for a statically determinate rigid structure. Luckily for us, most of our trusses including transmission line towers are by and large statically determinate.

You can also program it.

You can also program it; it can be done. However, standard software packages does not because it wants you handle both the determinate and the indeterminate. It will do… You know that the degree of static indeterminacy may be 0 or 1 or 2. The degree of kinematic indeterminacy may be 50 or 100. It will still prefer to solve hundred simultaneous equations because it is programmed that way; otherwise, you have a choice about the redundant. Is it clear?

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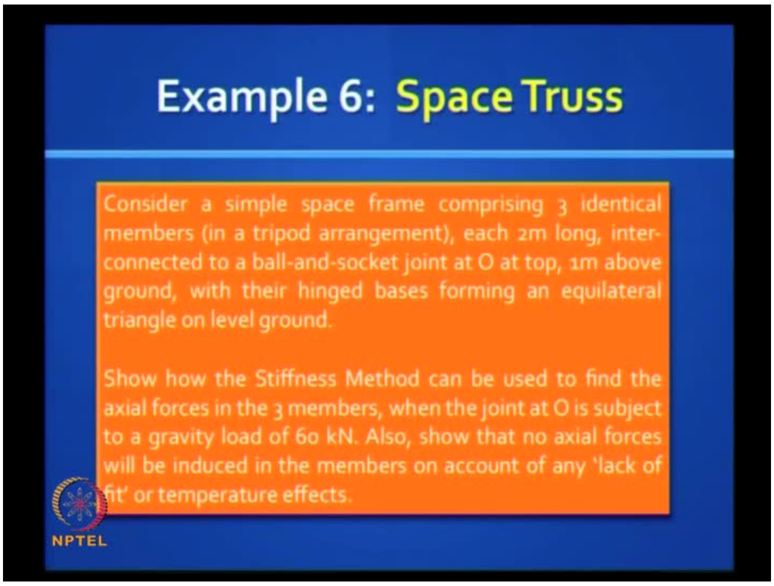

Let us take a simple problem. I want you to do it intuitively. You could be asked this problem even in your IIT entrance exam. So, I am going back to school giving you a simple problem. Consider a simple space frame comprising three identical members in a tripod arrangement.

Remember, you have been using a theodolite, which has got three legs. Let us say you keep it symmetrically and you have got a weight hanging from the ball; ball and socket joint on the top. Each leg is 2 meters long and interconnected to a ball and socket joint at O at top, 1 meter above ground, with the hinged bases forming an equilateral triangle on level ground.

Now, I have seen you people put some triangle there so that it does not move in the corners, when you use a theodolite. That is as good as providing a hinged base; otherwise, you have to plant it firmly on the ground so that it does not move.

Show how the stiffness method can be used to find the axial force in the three members, when the joint at O is subject to a gravity load of 60 kilo newton. Also, show that no axial force will be induced in the members on account of any lack of fit or temperature effects. Why would you not have any axial forces? Because it is a just rigid statically determinate structure. However, we have to demonstrate it using stiffness method. It is a simple problem. So, let us give it a shot.

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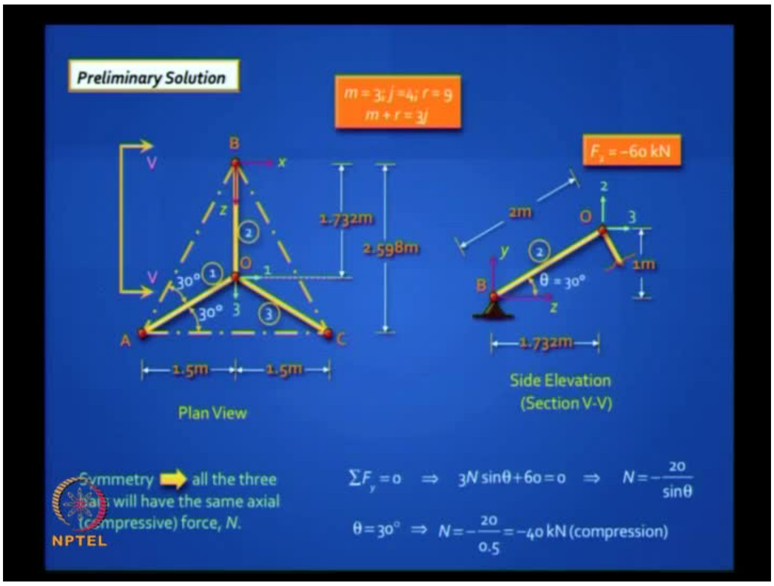

Here is a plan view. I have done the work for you. Here is a side elevation say section V-V. Can you understand what is going on? It is a triangle on the base; it is arrested here; arrested here; arrested here. They are pinned supports. This O is 1 meter above. A, B and C are restrained and I forget about the restrain coordinates. So, I have only three active degrees of freedom. Let me call them 1, 2 and 3. Does it make sense? The tripod 1 up, 2 along the x axis, and 3 along the z axis.

I have chosen this point b as the origin. This is x, (Refer Slide Time: 32:13) this is z, y is vertically above; this is y; got it? Is this picture clear? You can figure out how to locate these dimensions because you are given this length is 2 meters. If this is 2 meters, this angle will be 30 degrees because this is 2 and this is 1. So, 1, 2 and this will be root 3. So, this is 1.732. These dimensions can be worked out. That is the first thing you need to do.

Now, without doing any calculations except extremely simple ones, can you tell me what is the force in each of these legs? Will the forces be equal.

Yes

Yes, tell me the value. The total weight is 60. So, 60 divided by 3 is 20?

No Sir.

How do you get it? Common sense; how do you get it?

Vertical components.

The vertical components of the three axial force in the bar; each bar must be 20?

40 in each.

40 in each. Is it clear?

Yes Sir.

First of all, it is statically determinate. You have 3 members, 4 joints; it is restrained in three directions. So, m plus r is equal to 3 j. It is symmetric. So, all three bars will have the same axial compressive force N. 3 N plus 60 is equal to 0. So, N is 20 by sine theta. You can show that theta turns out to be 30 degrees. So, it is 40. You are right. That is how easy it is, but let us pretend we do not know the answer. We have to do it the hard way using the stiffness method. It is an easy problem.

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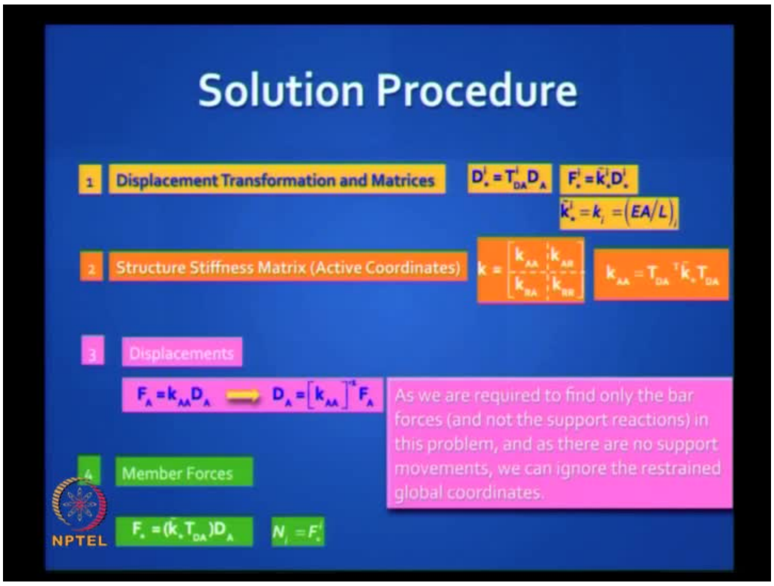

The procedure is as we did in the previous example.

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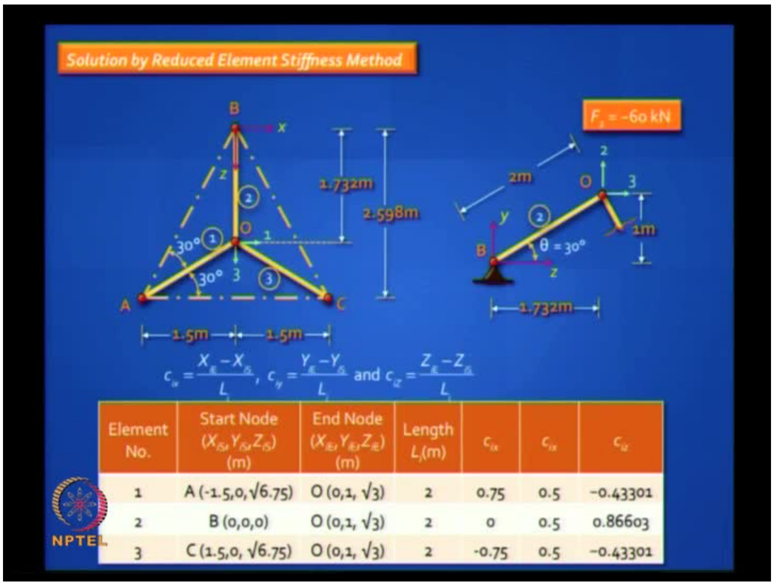

However, we are dealing with the space truss. This is the truss. Here again for the coordinates chosen, it is possible to write the start node, the end node, the C i x, C i y and C i z. Clear, you can do it? Then, the start node is arrested in all the three legs. So, the active coordinates are at the end node. So, the minus minus minus is not required here; only plus plus plus. Is it clear? That is easy to write down?

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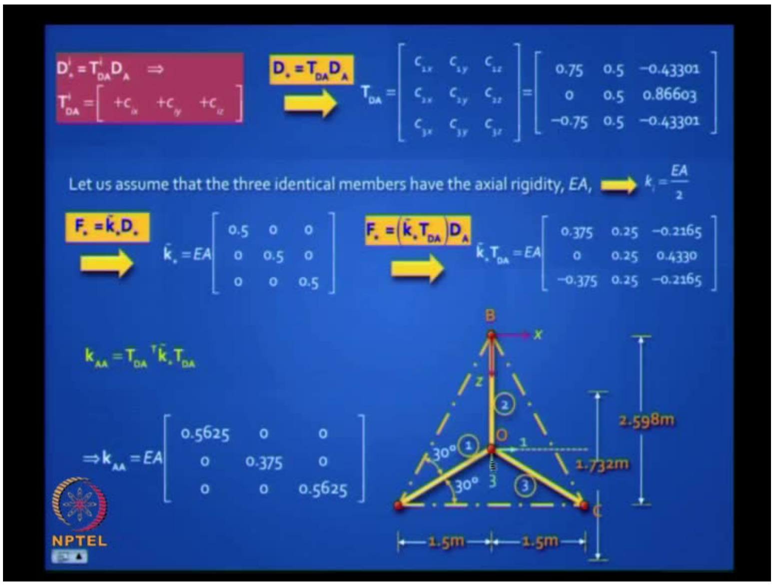

You can pick that out and that is what you get. You just have to pick the values that we got from the previous table; you have got the T D A matrix. It is straightforward; there is nothing in it. Then, what do you do? You have got your T D A matrix, then? You have to generate your stiffness matrix. How do you do that?

Let us assume all the three legs are identical; same E A value. The length of each is 2 meters. So, E A by 2; I can put it in the unassembled form; E A by 2, E A by 2, E A by 2; 1 by 2 is 0.5. So, I can write like this. Clear? This is k tilde star matrix. Then, I do the same thing as I did earlier. I post multiply by T D A. I have got T D A. What is T D A? This direction cosine matrix (Refer Slide Time: 35:52). I have got k star. So, I get this. Then, what do I do with this? Pre-multiply it by T D A transpose. Let the computer do all that; you have got this.

When you see an answer like this, you should pause a while. Why should you pause? You should think if you are a good engineer because you have got a diagonal matrix, which is rare. Did you make a mistake or does it make sense? You have to always pause. Whenever you get beautiful results, you must say why did not I figure it out? Is there some meaning? So, let us see carefully.

First thing, you have this tripod; 2 is acting upward. It is symmetric. If I arrest 1 and 3 and pull up 2, do you think there will be some reactions where I have arrested? Because they will be k 1 2 and k 3 2; or, there is no need to arrest. Those points are not going to move. What do you feel? It is symmetric. When I pull something up, will it have any displacement in the horizontal plane?

No.

No. So, it make sense. So, you have approved intuitively that k 1 2 and k 3 2 will be 0. So, that proves something. Now, you put a force if D 3 equal to 1; put a displacement here (Refer Slide Time: 37:29). You pull it. Do you think vertically there will be any movement?

No.

No. So, you would not get a force k 2 3. Agreed? Because vertically, there will not be any reaction. Sideways, will it be partial to one side? No. So, you will get k 1 2 equal to 0. So, intuitively, you know – yes, it is going to be a diagonal matrix. So, it make sense. Is it clear?

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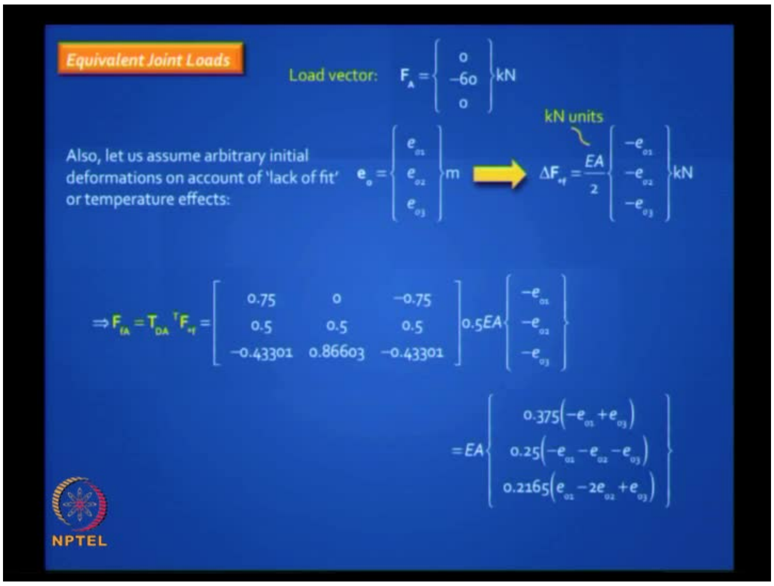

It is good to do these checks. Then, you find the equivalent joint loads. You get… Is this correct, the loads are 0, minus 60, 0? Then, what do you do? Now, let us add some more masala to this problem.

We will say let the bars have lack of fit. We want to show that the answers remain 40, 40, 40. Let us make it arbitrary. Let us say the three bars are either lack of fit or temperature effects, we have some initial values: e naught 1, e naught 2, e naught 3. Fine? Now, what will this cause in terms of forces? I will get fixed end forces. How do I find them out? Multiply it by minus k. I will get a compression and it is this value. Agreed? (Refer Slide Time: 39:00) Then, I have to shift this to the? Please pay attention. We have moved ahead. That you can sort out later.

We have to find the joint forces. How do we do that? T D A transpose. You multiply this out; (Refer Slide Time: 39:18) you get some forces. So, these are forces in terms of arbitrary e naught 1, e naught 2, etcetera. You need to prove that these forces are not going to add to the member forces. That is a great proof if you can do that.

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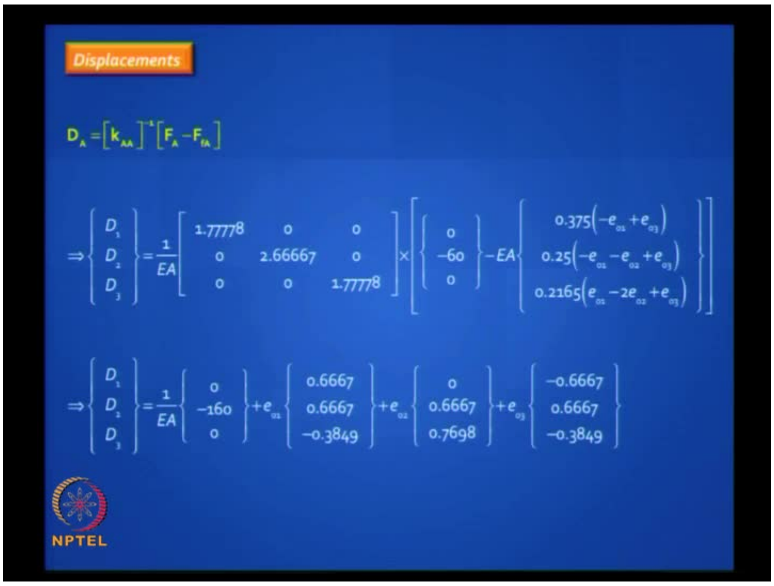

Let us see how to do that. Let us get the answers now. You have got the load vector now. I have inverted by k A A matrix, it will look like this. It is very easy to invert because it is a diagonal matrix. This is the load vector F A and this is the fixed-end force vector with a minus sign. With all that, the arbitrary e naught 1, e naught 2, e naught 3. When I actually do this multiplication, I will get some displacements; definitely, there will be displacements caused by these lack of fit issues.

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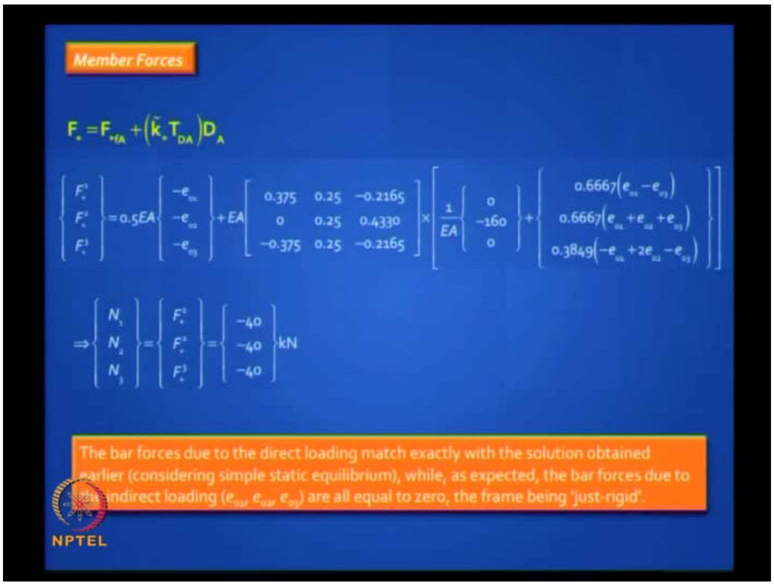

So, I will get D A; I will D 1, D 2, D 3. Now, what should I do with these answers? Put it back and get the bar forces. That is what you will do. Hey presto; you do not get e naught 1, e naught 2, e naught 3; they cancel out. You get only minus 40, minus 40, minus 40. This is a fantastic proof that in just rigid systems, indirect loading does not change the force field. It is a good demonstration. The reason is the structure is just rigid.

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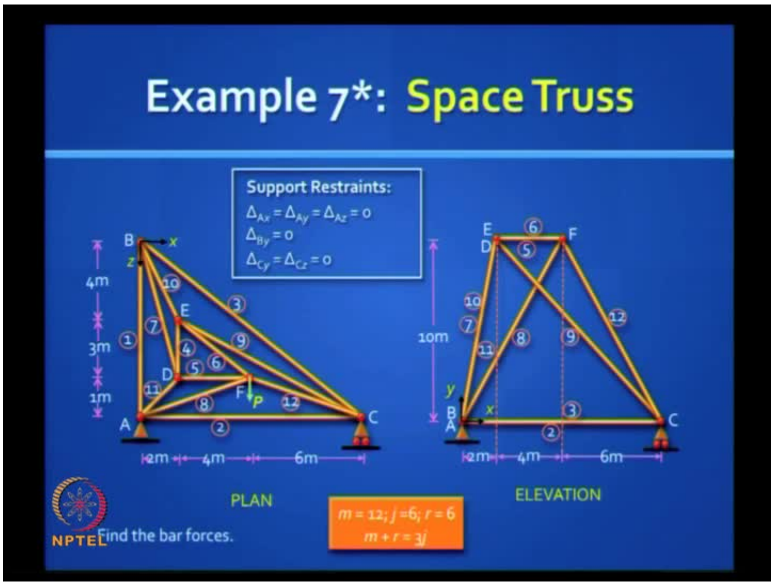

Now, this is a reading exercise. I put a star there. This is a complicated space truss. The triangle D E F is horizontal, but at this height 10 meter above ground. The triangle A B C is at the ground level and you got all these bars connecting them in space. I think we have done this problem by tension coefficient method. You have done it earlier; last year. Can you do it by stiffness method? If you can do this problem, you can do any problem because this is a small transmission line tower. The real one will be huge. So, if you can program this, you have done it. However, this is something I leave it and just find the bar forces. You can put the support restrains; you will find it statically determinate.

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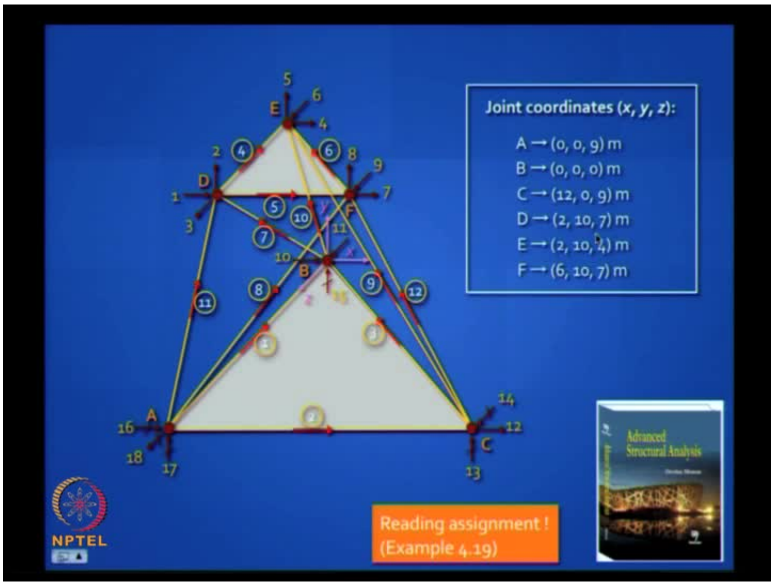

This is a reading exercise for you. You can go back to the book; it is a solved problem. Can you see the 3D picture? You have to identify the direction; the incidence of all the members. You have 18 global coordinates. This problem can be solved. Actually, we have solved it. It is demonstrated in the book; just go through it. Not to worry; in the exam, we are certainly not going to ask you these problems, but it is good. Those of you are interested you know that you can handle any problem by this method.

With that, we have finished the reduced element stiffness method. I hope you have got a good grasp of it. Only one topic left - flexibility method; we will go through it fast because really speaking, matrix method using flexibility method is not good for programming especially when we have intermediate loads. However, still we should know the concept behind it. So, we will cover it in the next class.

Thank you.