Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

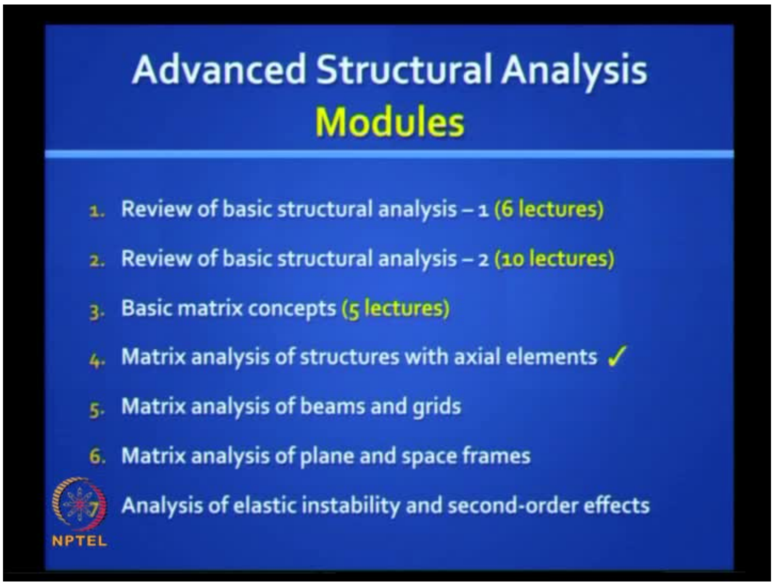
Indian Institute of Technology, Madras

Module No. # 4.3

Lecture No. # 24

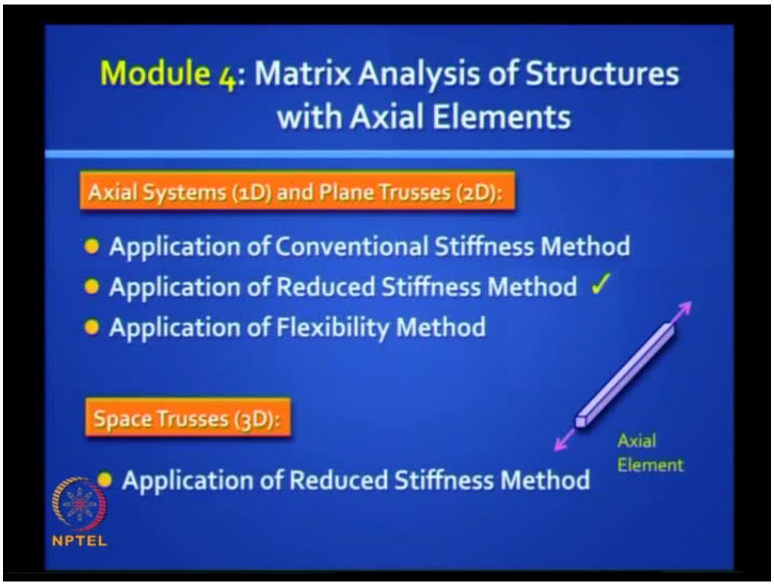
Matrix Analysis of Structures with Axial Elements

(Refer Slide Time: 00:16)



So, good afternoon. This is lecture number twenty fourth module matrix analysis of structures with axial elements.

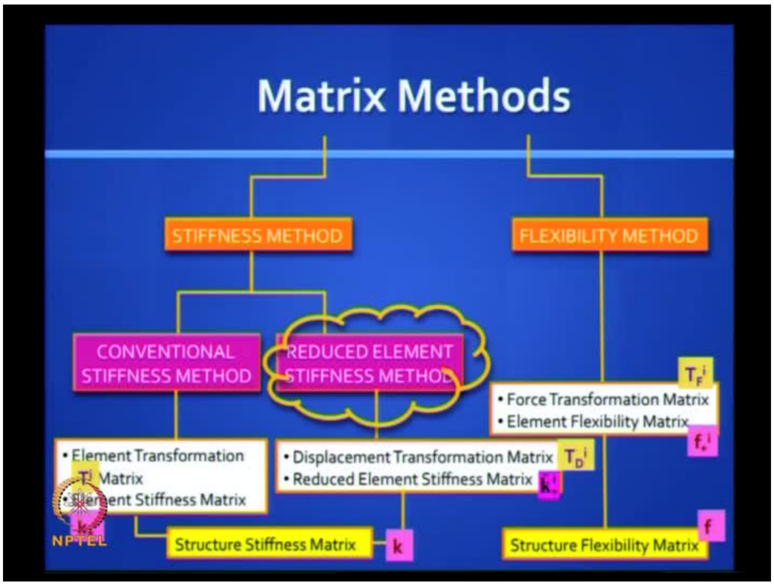
(Refer Slide Time: 00:24)



If you recall, we had shown how the conventional stiffness method can be applied to problems with axial elements. This included 1 D structures; it also included plane trusses, but space trusses we did not do, because we said we would, they are complicated and we would show how you can use the reduce stiffness method to apply to space trusses. We will cover this in two sessions. In today's session, we will look at the application of reduced stiffness method to 1 D structures and plane trusses.

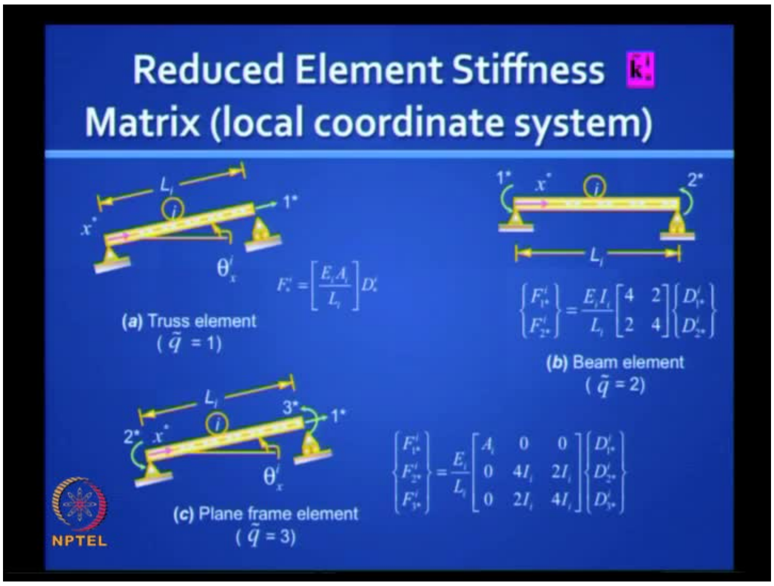
(Refer Slide Time: 01:13)

(Refer Slide Time: 01:19)



So, this is covered in the fourth chapter of the book on advanced structural analysis. This slide is familiar to you. We are looking now at the reduced elements stiffness method.

(Refer Slide Time: 01:26)



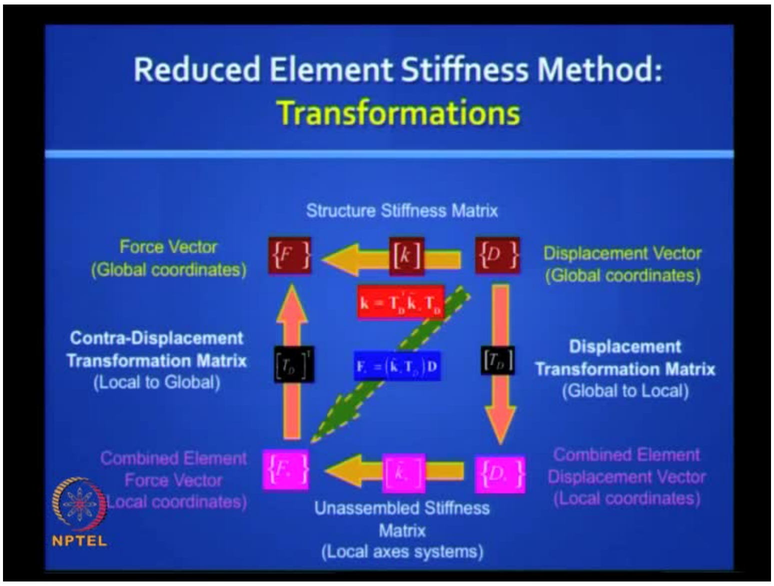
What is the need for this method? Well, this is not a method that is used in software. This is the method that is often taught in universities, because it is simple to apply; it is good for small problems, but it can be equally applied without any loss in accuracy to very large problems. It is not all that suitable for generalizing but it is worth knowing how these simplifications can be made.

Now, if you look at the genesis of this method, the conventional stiffness matrix for any element does not have an inverse, it is singular. Why is it singular? Its rank is not full, for example, typically if you take a space truss element in the conventional stiffness method, how many degrees of freedom do you have? What is the degree of, what is the size of the matrix? Space truss element. At each end, you can have three independent degrees of freedom; so, it will be 6 by 6.

So, you will be dealing with big matrices and it is difficult to do, but if you, what is the rank of that matrix? It is only 1. You see, it is, how simple it is to reduce the computational work and use the reduced element stiffness method. So, that is the real use of this method. Not only that, if you are comfortable with this method, flexibility method will be easy to understand, because the matrix that you get from the reduced elements stiffness method, if you invert that matrix, it is invertible. You will end up with a flexibility matrix.

So, we have done a review of this. Let us look at these applications. You can use it for a truss element. You have 1 degree of freedom, you can use it for a beam element, 2 degrees of freedom; you can use it for a plane frame element, 3 degrees of freedom and so on.

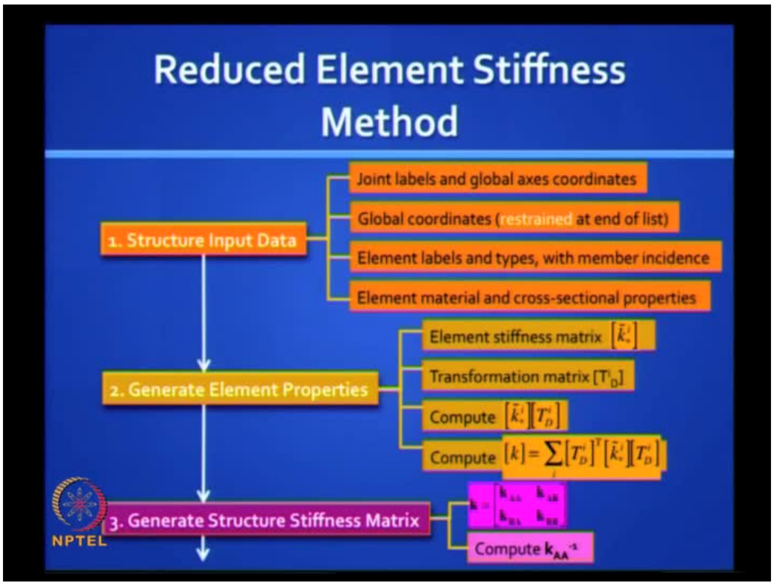
(Refer Slide Time: 03:54)



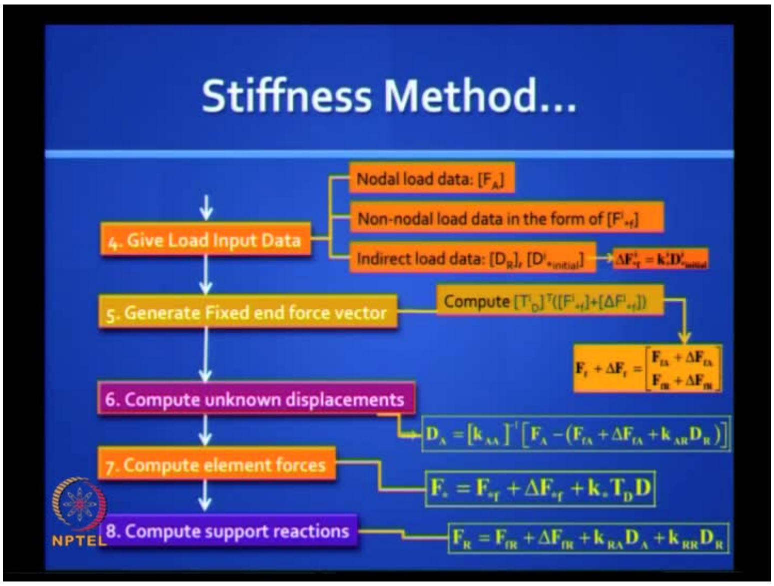
We are now looking at applications to axial systems. This slide again is familiar to you. Here, you need to first generate the displacement transformation matrix which relates the global coordinates, global displacements to the element level displacements. Then you have the, this matrix which is called the unassembled stiffness matrix in the local axis. What is a nature of this matrix? Remember, there is a tilde here; it is k star tilde; it is a diagonal matrices where you just lump together in the diagonal all the different elements stiffness matrices.

Then, you know that there is a contra gradient principle which shows a relationship between the element level forces and the structure level forces, and then, you have this transformation T D, transpose k star T D, which gives you the structure stiffness matrix. This structure stiffness matrix that you get using the reduced element formulation is the same structure stiffness matrix that you would get, had you done it by the conventional stiffness method. So, there is no loss in accuracy, it is just that you have the advantage of dealing with smaller sized element stiffness matrices.

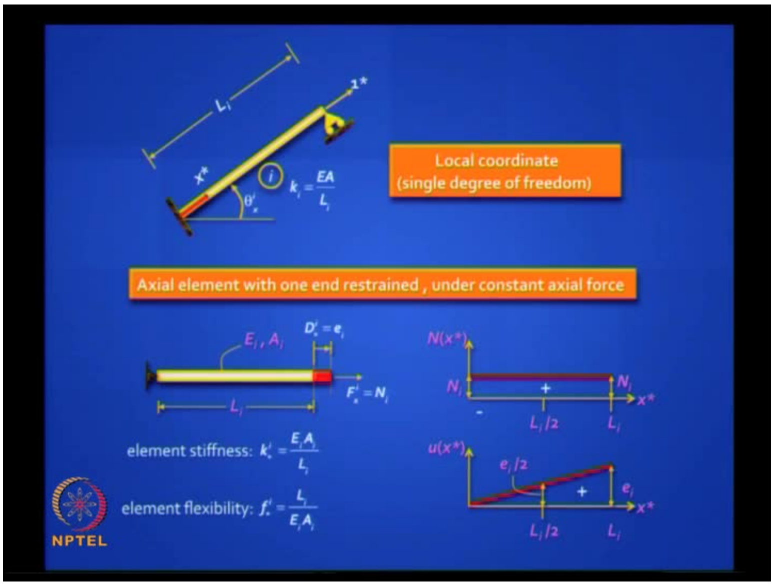
(Refer Slide Time: 05:19)



(Refer Slide Time: 05:22)



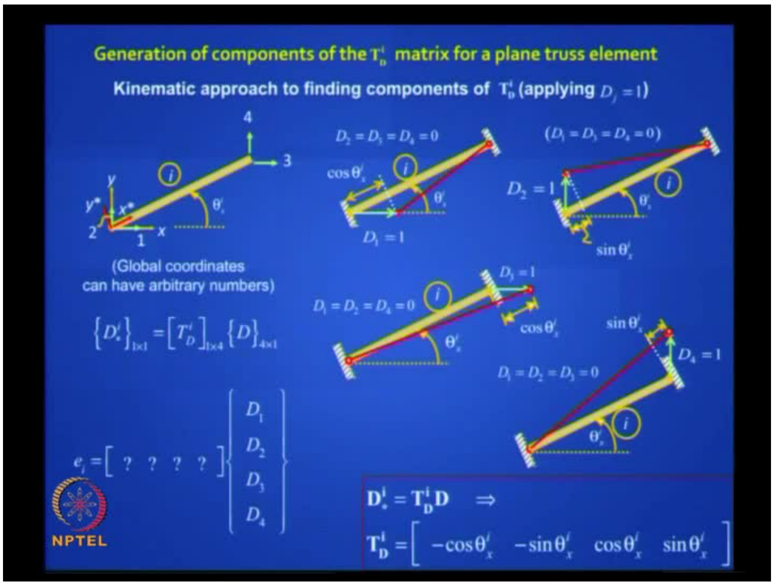
(Refer Slide Time: 05:26)



So, this procedure we have gone through earlier, but it will really make sense when we apply it to specific problems. So, let us take the simplest type of problem. Well, in all types of problem whether we deal with 1 D structures, plane trusses or space trusses, you are dealing with this element. Now, it has only one degree of freedom and you can interpret it, interpret that displacement as the elongation in the element. D 1 star is nothing but the elongation, the element. You can even eliminate that 1, because there is only, it is a 1 by 1 element stiffness matrix, but it is left to you.

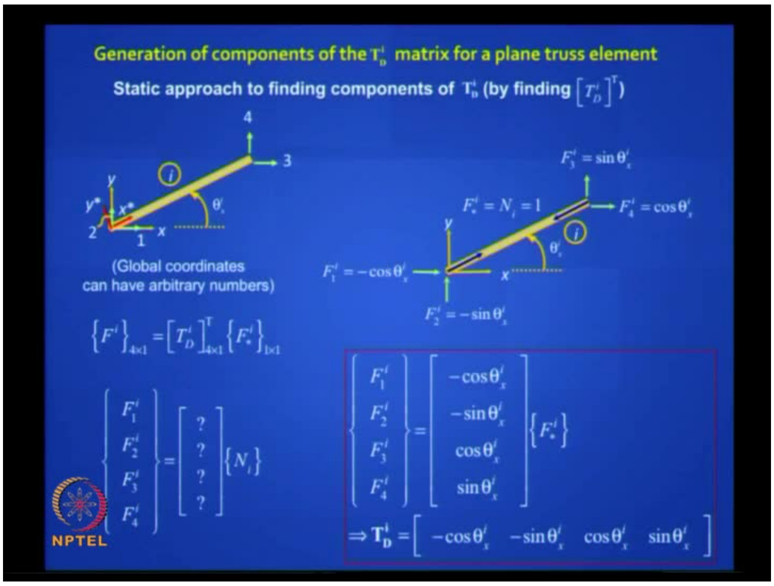
And the only problem with this element is you cannot discriminate between variations in end forces at the two ends. You can have only a constant element force in this as shown here.

(Refer Slide Time: 06:26)



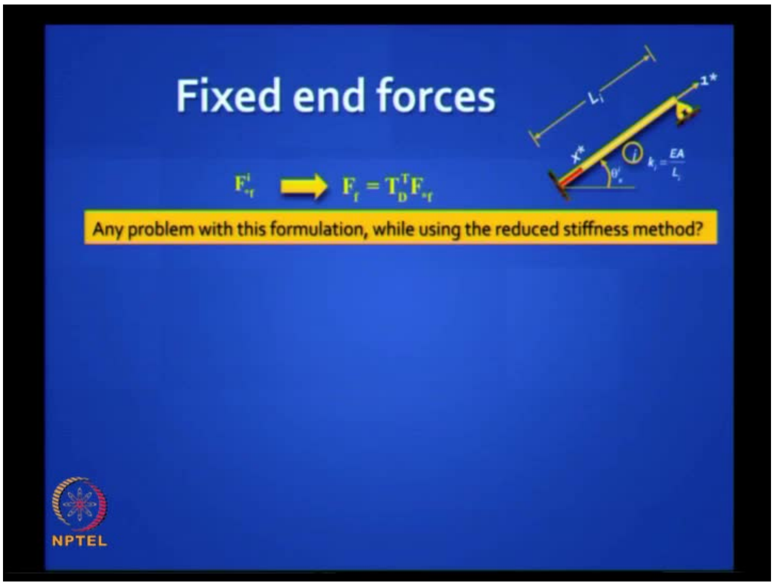
So, the first question that arises - how do you generate your T D matrix? You remember there is a kinematic way of doing it. You just have to remember its minus cos theta minus sin theta plus cos theta plus sin theta. Remember this, you have done this; that means you keep applying a unit displacement at a time in your structure and see what happens at your element level.

(Refer Slide Time: 06:57)



You can generate the same element stiffness matrix using a static formulation. Using the T D transpose matrix, you remember this. In this procedure, what we say is - if you have a unit force in the element, say unit tension here, then necessarily at the two ends you need external forces to satisfy equilibrium, and the relationship between the element level force and the end joint level forces is given by this matrix which is T D transpose, but you really do not need this since you know that it is minus cos theta minus sin theta plus cos theta plus sin theta.

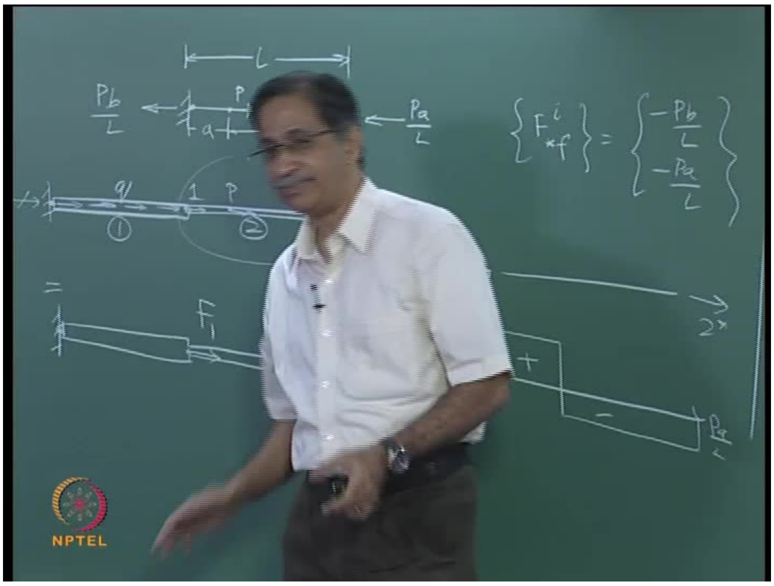
(Refer Slide Time: 07:32)



Next, if you have intermediate loads or say lack of fit or a temperature effect that you need to take care of in a truss or in axial element, we have to do this transpose. Remember, we did this in conventional stiffness method. Now, the question I want to ask you is - can we do the same thing in the reduced elements stiffness formulation or is there a problem? The first problem we did was two axial elements, non prismatic connected to each other with some intermediate loads in both of them. Then we reduce them to equivalent joint loads using this transformation.

We arrested all the degrees of freedom; found the forces. Then we did T D transpose and got the structure level force. Can we do the same thing here or do you see a problem? Is the question clear or we can do it? Will this T D transpose work? The answer is no. Now, can you tell me why it would not work?

(Refer Slide Time: 09:04)



Let me show you the problem. Let us take that same problem which we did. It was something like this. You had some distributed load; you had a concentrated load; you had 1 degree of freedom here 1. This was element 1; this was element 2. Let us say this load is q and this load is P. We somehow want to reduce this to a problem where you have only one force here which we called F 1. So, what did we do? We arrested all the degrees of freedom. Remember, you had 2 here and 3 here and we found that q into this length divided by 2. Is it force that you get at the two ends?

And P into b by L and a by L remember that. So, you got the element level forces and then we had to transfer them to the structure level forces. Take this for example - you have an element here; you arrest this degree of freedom; arrest this degree of freedom. If you apply a force here P, this distance is a; this distance is b; this length is L. We know that you will end up with the force here which is P b by l P a by L.

And so, you can write the, let us say this is i th element. You can write F i th element star f. We could do it. How do you write this? Supposing this had 2 degrees of freedom, if a 2 degrees of freedom, then you could have written this as, remember, in the conventional stiffness method, we had 1 star, 2 star, and so, it was possible to write this as minus P b by L minus P a by L; that was possible. You could accommodate two different values here.

But now, we have a problem, because we have unfortunately in the reduced element stiffness method, we have only 1 degree of freedom, and so, our element is like this. We say, we arrest this, this is pinned and may be you can. Since you are uncomfortable in the last class not having a support, there we will put a support there. And this is 1 star; so, I do not have two numbers here, I just have one, and what is the axial force in this? Due to this, I have a problem, because the axial force in this element actually has a variation.

In this region, I get tension in that region. So, remember, it was something like that. This is the axial force variation; this is tension and this is compression. This was P b by L P a by L, and so, I can have different values, different signs if I had 2 degrees of freedom, but here, I can have only one value, and though you, I have a uniform tension or uniform compression.

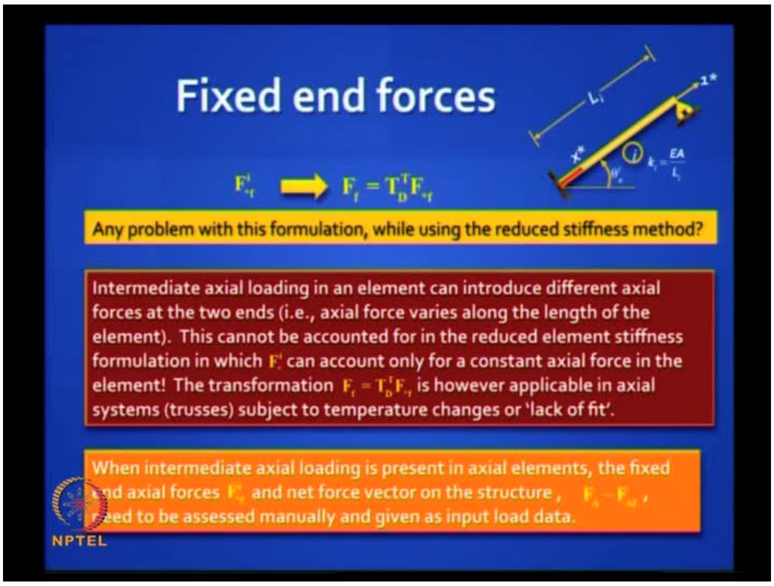
Sir, T D will be different for this.

Whatever I do, there is no way I can express a variation. This allows only a constant force in my element. This allows for a variation; so, that is a limitation. So, be careful when you use this formulation. Normally in trusses, you do not have a problem, because you do not have intermediate loads, but in beams, you will get; in frames, you will get; in axial elements, you will get.

So, yeah, then, you are doing extra work but if you had a u D L, you cannot treat it as two elements. You will have to treat this as infinite elements; so, it does not work. So, intermediate axial loading in an element can introduce different axial forces. At the two ends, that is the axial force varies along the length of the element. This cannot be accounted for in the reduced elements stiffness formulation. In which, F i star can account only for a constant axial force in the element. So, the transformation with T D transpose is not applicable in such cases, it is applicable however when you have a lack of fit or you have a temperature problem, because then, it will be constant.

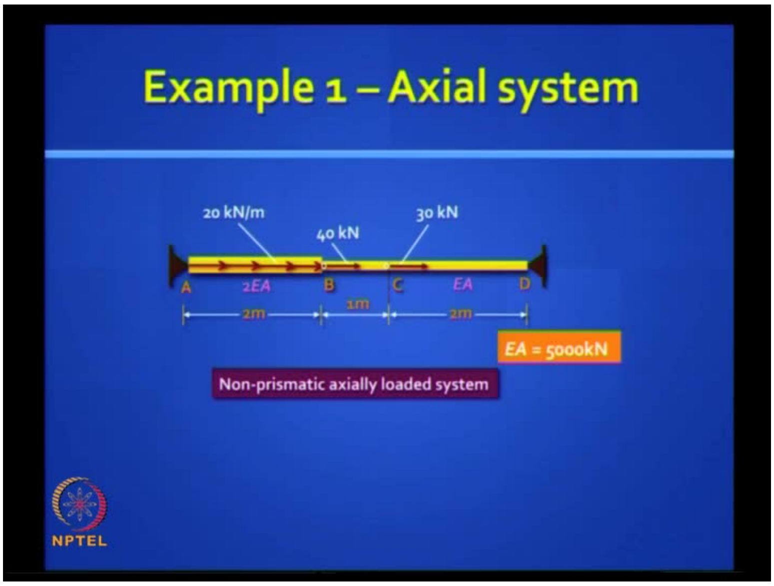
So, the problem is only when you have intermediate load, but otherwise, you can handle. So, in trusses, you can normally do it. In the latter case, when intermediate axial loading is present in axial elements, in the former case, when intermediate axial loading is present in axial elements, the fixed end axial force is F i star f and the net force vector on the structure F A minus F af need to be assessed manually and given as input load data. So, that is the only change.

(Refer Slide Time: 07:32)



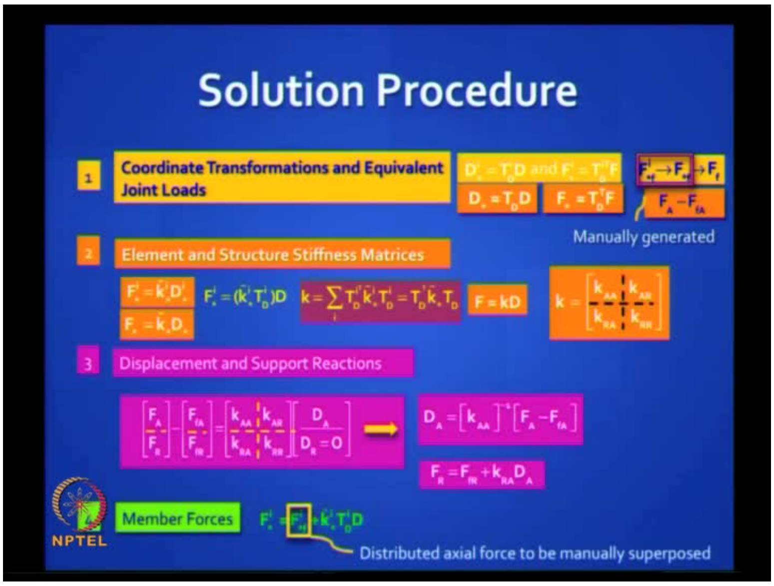
So, whatever you are getting in the structure, do not use T D transpose and shift from element level fixed end forces to structure equivalent joint loads. Do it manually. Let us demonstrate with an example.

(Refer Slide Time: 15:09)



So, we will take this, this very problem which we had already solved using the conventional stiffness method.

(Refer Slide Time: 15:17)

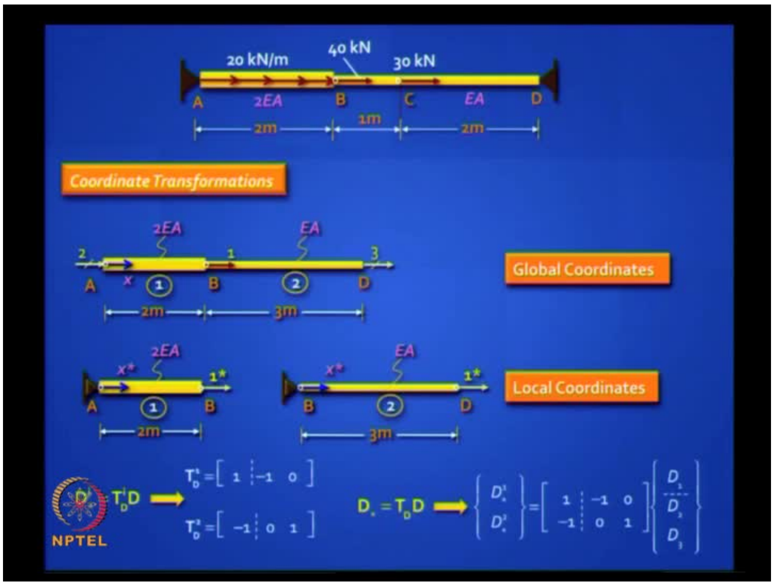


What is the procedure? Well, first, you have to find out the T D matrix. Then, somehow you must get the equivalent joint loads which we will soon demonstrate, but you have to do it manually; you cannot use T D transpose. Then you write down the element stiffness matrix for the elements and generate the structure stiffness matrix.

The structure stiffness matrix that you generate by this method will be identical to the one that you solved by the earlier method. You find displacements and support reactions. This part is common to both the methods, because once you got the structure stiffness matrix and you got the loads, you can find the displacements, then you find the member forces.

Remember, when you are using reduced stiffness method, use a different notation for your element stiffness matrix k tilde that little curly thing on top of k. That is how you distinguish. And again, remember, if you have a distributed axial loading, that is the part that you have to manually superpose. So, I want you do see this one problem and you will get to understand how it works.

(Refer Slide Time: 16:32)



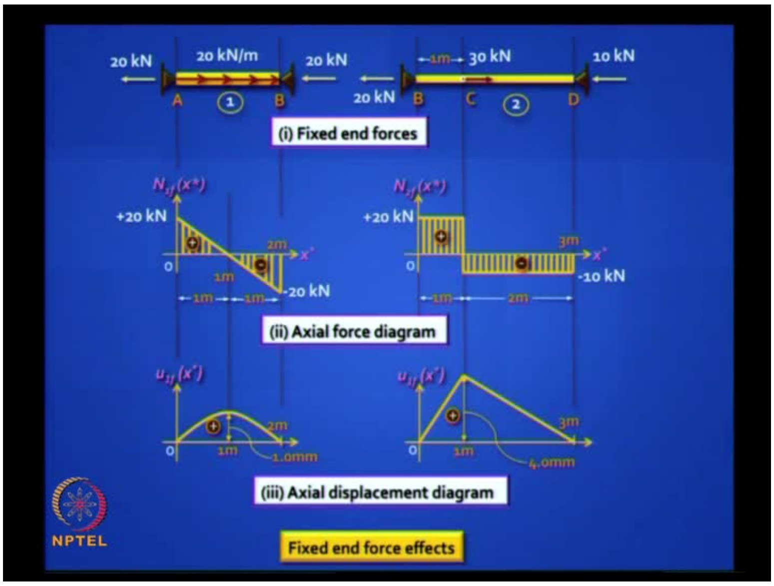
First coordinate transformation, we will choose a same global coordinates as we did when we did this problem by the conventional stiffness method. Only one active degree of freedom 1 and 2 restrain degrees of freedom 2 and 3. As far as the element is concerned, only one element coordinate 1 star, 1 star, whether it is element 1 or it is element 2. Can you write down the T D matrix for these two elements? Please try it. So, we are trying to find out what happens to the two elements when we apply D 1 equal to 1, when we apply D 2 equal to 1 and when we apply D 3 equal to 1.

Write down the T D matrix by the element stiffness method. So, I will help you. Now, this is what you will get. Let us see what is happening. If I pull this joint to the right, I get D 1 equal to 1 but I do not allow D 2 or D 3 to have a non-zero value. When I do this, what will be the elongation in member 1? It will be plus 1 or minus 1? Plus 1. What will be the elongation in, in member 2? Minus 1, it going to, it will going to contraction because you are pushing it to the right, this and you are not allowing D 3 to move.

So, this first column that I get, when I apply D 1 equal to 1, I get plus 1 as nothing but the elongation in element 1 and minus 1 in the element 2. Is this clear? Make sense? Now, you apply D 2 equal to 1 but D 1 is 0 and D 3 is 0, what will happen? Only this element will get affected. What is the moment in that? What is the elongation? Minus 1. So, you get minus 1 here and 0 here.

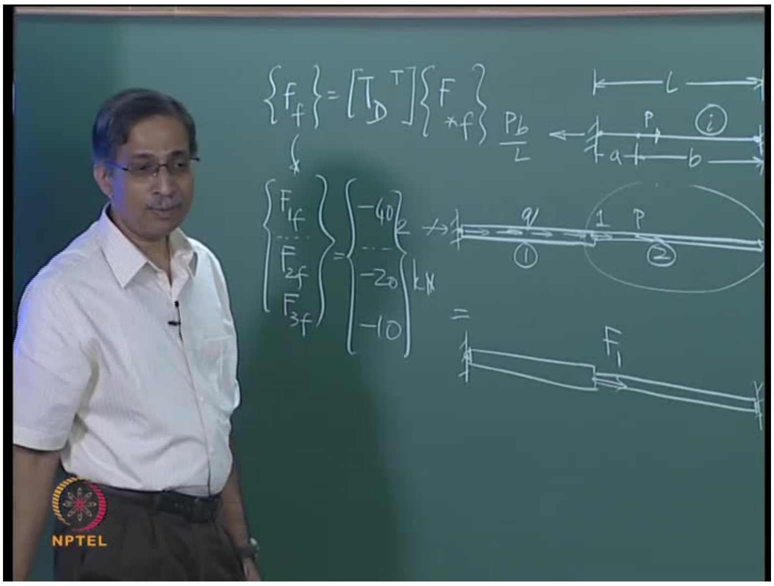
Lastly when you pull D 3 equal to 1, element 2 will get elongated. So, and element 1 remains unaffected; so, it is 0 1. Is this clear? It is as simple as that. In 1 D structures, you will have only 0s and 1s, no sin theta cos theta here, because theta is 0, is it clear? It is very easy; I hope there is no confusion here, it is straight forward. Next, so, you can put it all together in one matrix, you need not write T D 1 and T D 2 separately for small problems. It is nice to deal with one full T D matrix.

(Refer Slide Time: 19:52)



As far as fixed end forces are concerned, we have done this earlier. Remember, we had to convert the intermediate loads to the end loads and this way we have looked at earlier, remember this. So, I am not going to repeat this part, this part is well known, but here, we have this problem that I talked about problem of how to handle this situation. Well, you cannot handle it if you have only 1 degree of freedom per element. So, you forget about how to handle it in a mathematical frame work, just put it all together manually and tell me what F 1 and F 2 and F 3 are going to be.

(Refer Slide Time: 20:50)

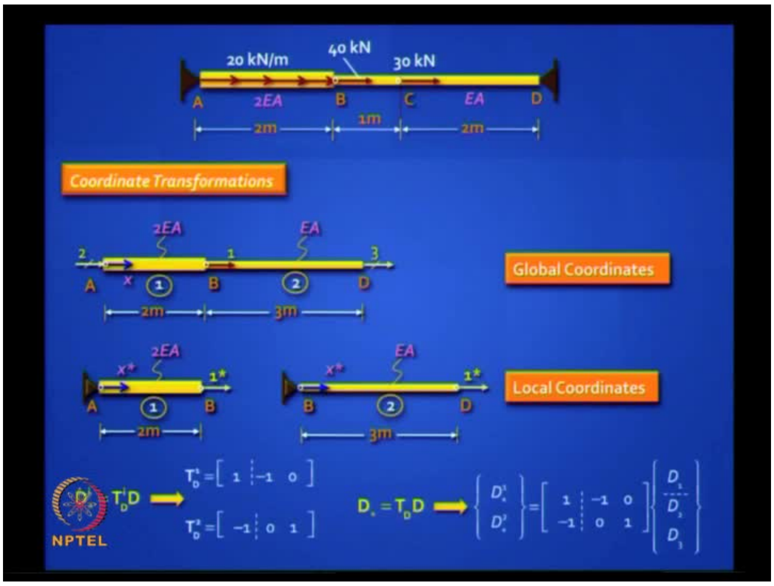


What are the equivalent joint loads due to the intermediate loading? Can you tell me what F 1 is F 1 f? See, all that we are saying is F f is equal to T D transpose F star f. You cannot do for reasons we have already discussed. So, do not do it and just give me directly this value, which means what is F 1 f and, you know, you can partition it like what is F 2 f? Can you give me these three values just by looking at the diagram? Tell me.

Minus 40 minus 20

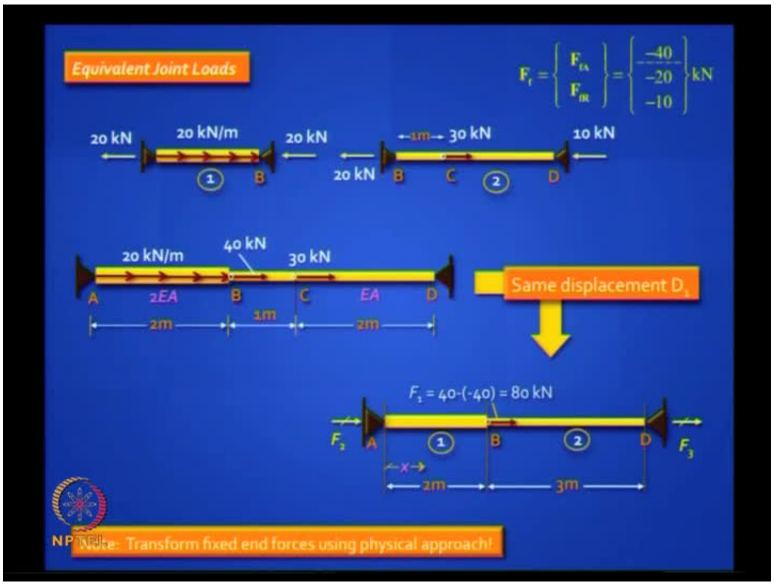
This is minus 40; this is minus 20 minus 10. That is it.

(Refer Slide Time: 16:32)



Now remember, there was a nodal load in the structure. Let us go back to the origin. Remember, there was a 40 kilo newton acting at F 1 so that you have to include.

(Refer Slide Time: 22:10)



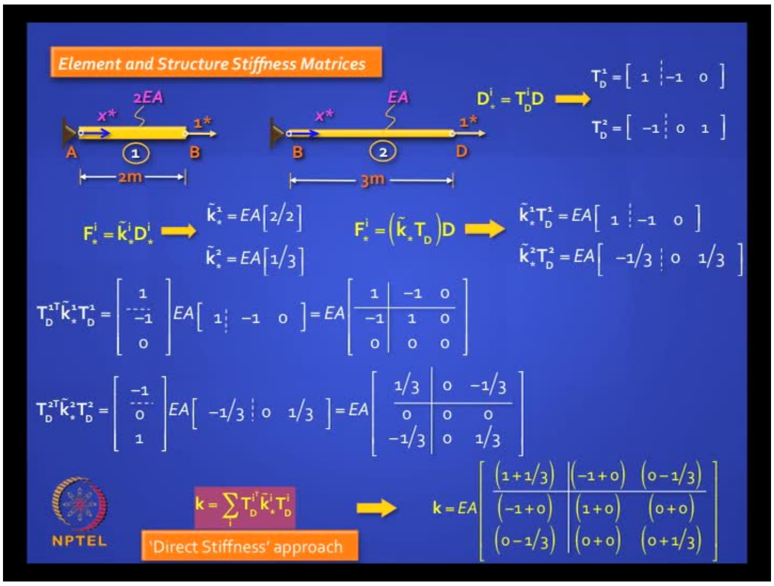
So, you have to do it the good old way and you will, you will write it like this. So, this is what we discussed manually do it. You will end up with the same displacements as you would get in the original structure, but now, you also have to add; F a is already there. So, we will use that later. You have to finally your net load vector is F a minus F f. Why do you put a minus?

People (( ))

Yeah because you have to reverse the load that accumulates at the artificially arrested locations.

So, that is why you have to the equivalent joint load manifests as a with the minus sign. We have done this earlier in the conventional stiffness method. So, this step is clear to you, you said it right.

(Refer Slide Time: 22:59)



Now, let us generate the element and structure stiffness matrix. How do we do that?

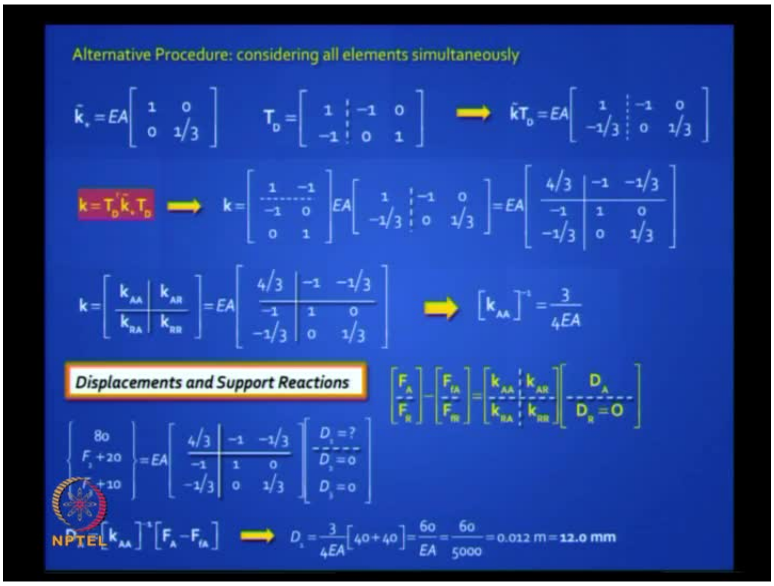
T d

So, you have got T D, you need k star. What is k star?

Well, you have two elements. For each of the element, the k is nothing but the axial stiffness which is E A by L. For the first element, E A is 2 E A and the length is 2 meters. For the second element, E A is axial rigidity; the length is 3. So, you got this. How will you put it all together in 1 matrix? Well, you can put it all together later. I will show in another way of doing it, but if you leave it like that, we can do this calculation. You post multiply k star with T D will you, will you get this? Because you just have to carried out that operation. Then you do pre multiply with T D transpose and you will get the final answer.

You do it for two elements. You understand each element is contributing to the structure stiffness matrix. So then, what you do? How do I get the total matrix? Just add, just add mean, you got this contribution. Add it up numerically and you will get the final matrix. Does this make sense? Very easy to do small matrices. You do not even need mat lab and all that, manually you can do this is, this clear; it is a simple problem. If you want to handle everything in one go rather than do superposition that is also possible. This element by element approach is sometimes called the direct stiffness approach.

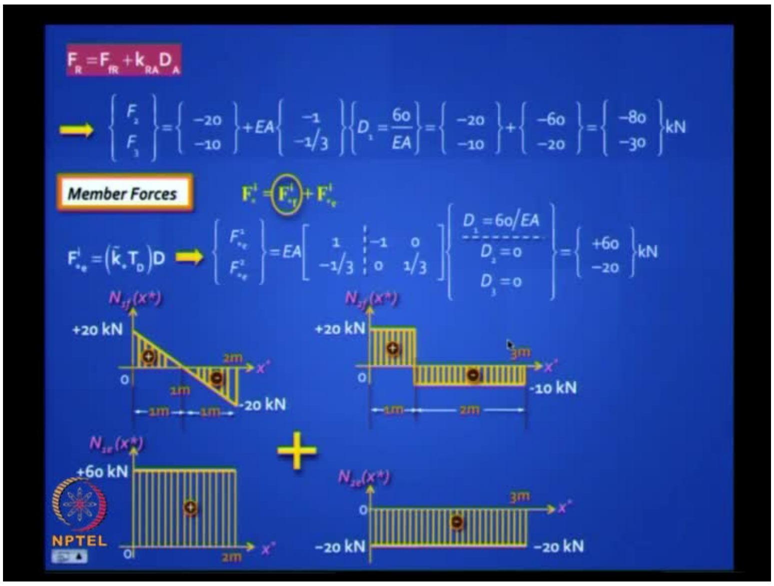
(Refer Slide Time: 24:50)



But you can do it alternatively by taking the full k star diagonal matrix, the full T D matrix and do the same operation. Go through these steps and you will get the same matrix, is it clear? Two ways of doing - do it element by element, add up their contributions in the structure stiffness matrix or do it in one shot. It is worth doing it one shot if your sizes are small; if sizes are large, you have say 20 elements. Then it is not that easy.

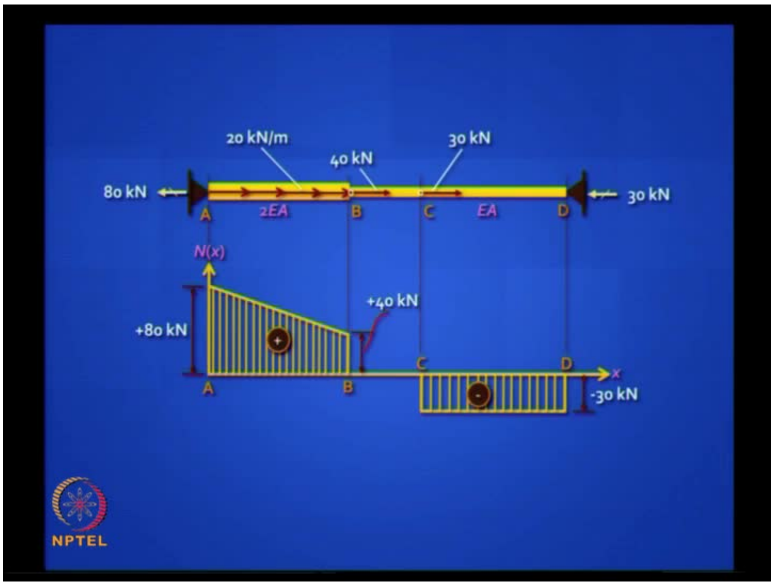
Is better do it element by element also your storage become much easier. So, we have got up to this stage. Now, mind you, you are getting the same matrix, you got earlier. In conventional stiffness method, you had more degrees of freedom per element. Now, you have much less and k AA is just 4 by 3 E A. Find the inverse of that and the rest of it follows. We have done exactly the same thing. I hope these equilibrium equations are familiar to you.

(Refer Slide Time: 25:59)

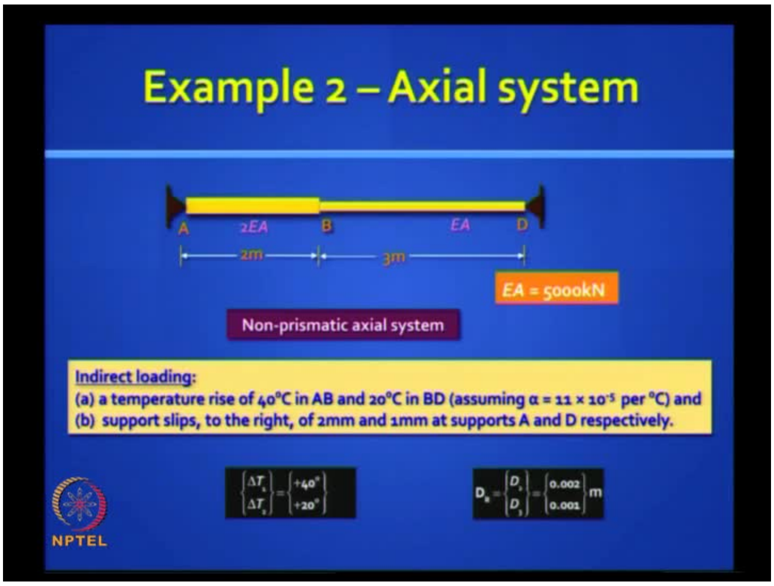


You solve the first equation and you will get D 1. Remember, we got 12 mm last time. It is the same thing. This part from here onwards it is similar to find the member forces, of course, you have these are the support reactions. You get the support reactions also. How do you get them from the second equation? Remember, use the first equation to solve for D A and use the second equation to solve for F R. We did this earlier; it is nothing new. So, you do that and you will get the support reactions, then you get the member forces it is here rather you have to be careful, because you have to do it manually, because you are getting two different member forces. So, you do it carefully and you have to add. See, you will get this constant force from your matrix analysis. We got this these values -60 and 20, but you have to add what you got the distributed force which you got manually.

(Refer Slide Time: 26:59)



(Refer Slide Time: 27:04)

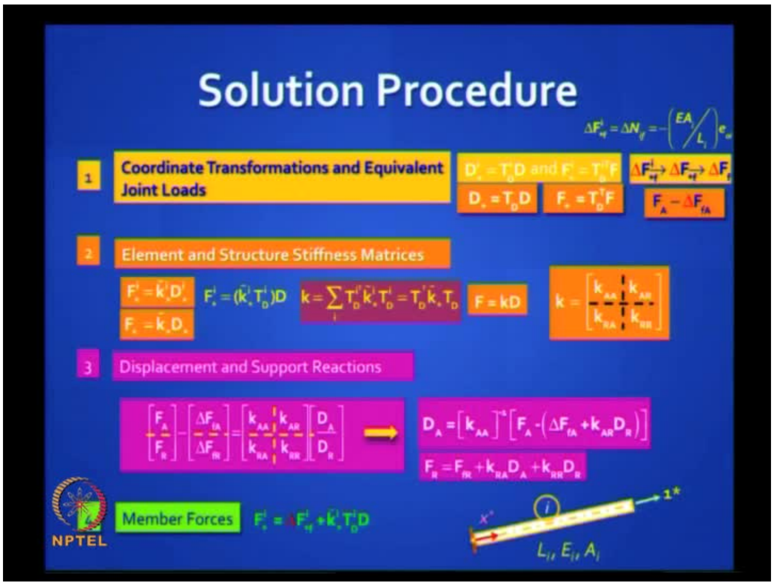


And then, that is how you get the total axial force distribution which will look like this. It is the same answer that we got earlier, is this clear? And you can also get displacements. Second example, this also I think we did by the conventional stiffness method. Here, any questions. Here, there is no axial load, there is only indirect loading. You have a temperature rise of 40 degree celsius in A B and 20 degrees in B D. The coefficient of thermal expansion is given to you.

You also have support slips to the right of 2 mm and 1 mm at A and D. Same problem we did earlier. Let us do it by this method. Here, can you use the T D transpose to get your equivalent joint loads? Yes, because you have a uniform force, there is no variation; so, here you can conveniently do. So, you must use the right technique at the right place and not get mixed up.

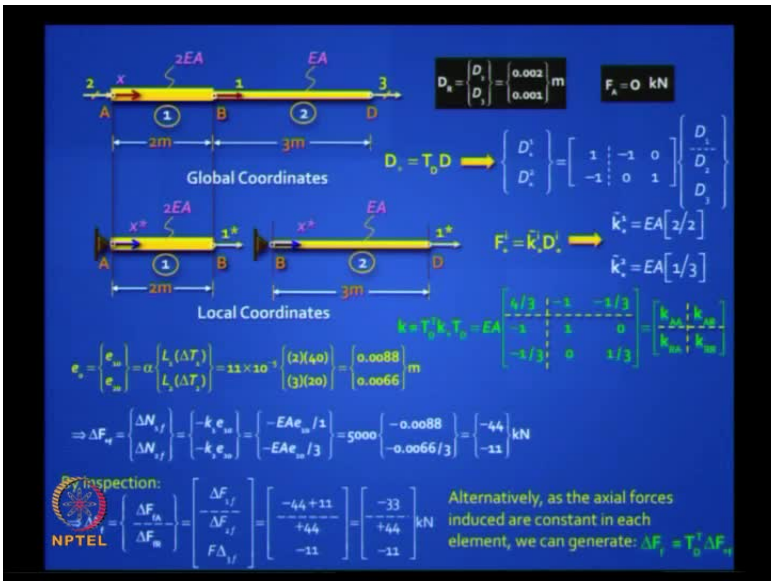
So, the input data is this. It is indirect loading, change in temperature and support slips. So, how do you deal with the change in temperature? You take a primary structure do not allow any moment and you will find there. When you raise the temperature, you will get some axial compression in both the members that, that gives your internal forces and that is how you work out.

(Refer Slide Time: 28:21)



So, the steps are similar. Only thing your axial force will have minus E A by L into E and the steps are exactly identical. We put a delta for delta F f, because we are saying that when you have indirect loading, this is an additional effect that we include, and so, we put that capital delta, is it clear? Rest of it is same.

(Refer Slide Time: 28:47)

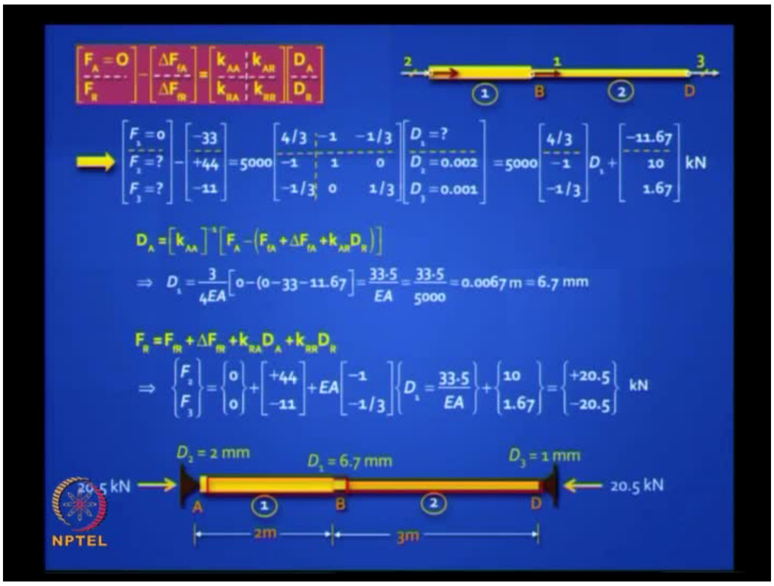


So, you have the same coordinates, we do not waste time. We have already generated the T D matrix; it is the same structure and the same k matrix. You have already done the stiffness matrix, so, only the loading that is going to change. So, how do we take care of the loading? First, find out the free elongation L alpha delta T and that elongation is fully prevented in the primary structure. So, is this step clear? You get that much, that is e naught. Now, if you have a lack of it problem, you get e naught directly. You do not have to do this step of L alpha delta T, you got e naught.

And then, you know that this will induce axial compression of 44 and 11 kilo Newton. We got these numbers last time also remember. In the first element, you get a 44 kilo Newton compression; second element, you get 11 kilo newton compression. So, you can do this calculation directly.

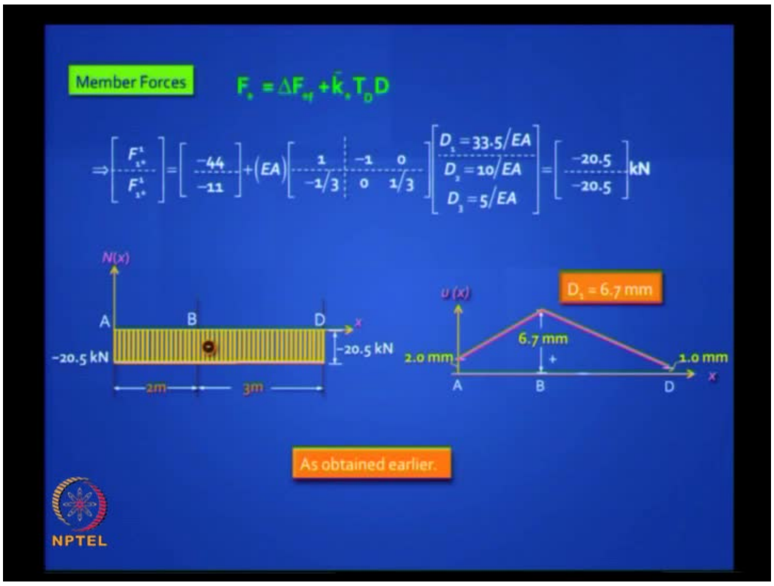
Always use your common sense, do not use formulas. You should get a feel for the problem. Yes, this, this makes sense, do not put plus 44 and plus 11 and you have only 1 force per element, because you have only 1 degree of freedom. Is this clear? This you can handle. Then you can like we did by inspection, fill up this or you can do it by T D transpose. It both will work, you can check it out, but many case, you have 44 and 11, they will add up together at D 1 will end up with minus 33 and you will have 44 and 11. Is it clear? D F 1 F 2 F 3 will have these numbers.

(Refer Slide Time: 30:34)



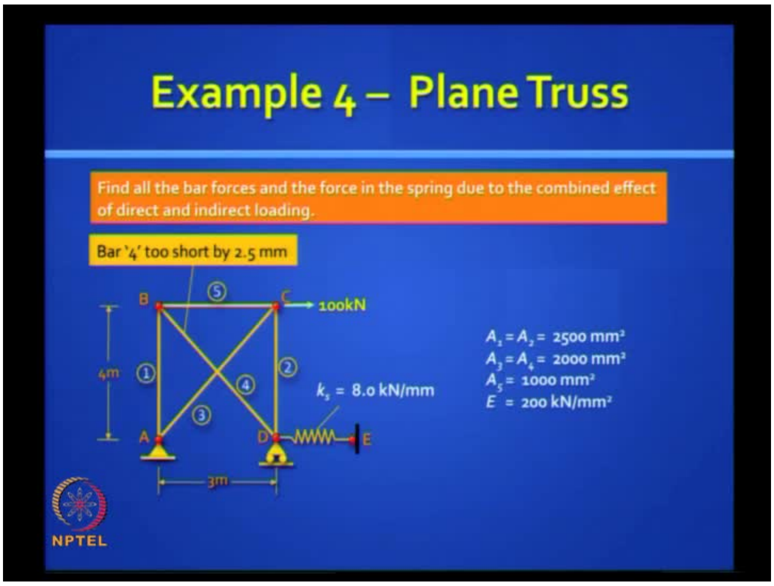
Thereafter, it is a straight forward using matrix methods, you solve this. You will find D A, we get 6.7 mm. Find the forces. Remember, we did this problem, we got the same answers 20.5 kilo Newton, and so, you can get, you got a reactions. If you want to see how it will change shape, this is how it will change shape - the left end moves to the right by 2 mm, that is input; the right end moves. So, the right by 1 mm and D 1 has moved 6.7 mm.

(Refer Slide Time: 31:06)



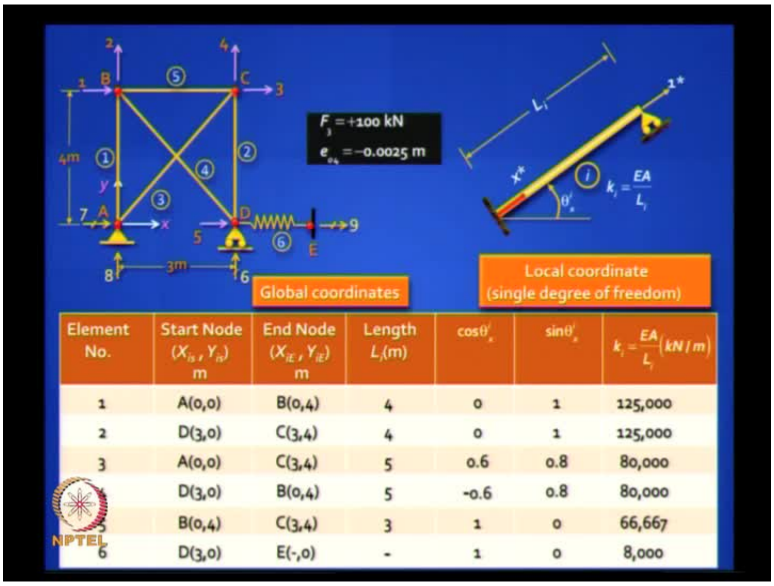
And if you want to find member forces, same procedure, that is it. You got everything; you got the internal forces. It is a constant internal force even though you have a non-prismatic element; because that is the only way you can satisfy equilibrium. Both element will push each other and the forces 20.5 kilo Newton, this how the displacement. Are you comfortable with this? You have to do a similar problem in your assignment.

(Refer Slide Time: 31:40)



So, as obtained earlier same solution. Now, let us take a truss. Let us do it together. Did we do this truss by the conventional stiffness method? No. So, we are we are doing it first time, just take down this truss. You have five bars and you are given an applied load of 100 kilo newton at C and you are told that the bar 4 is too short by 2.5 mm. Say it is a complete problem. How do we do? Let us do it slowly together. First, tell me what are this? How will you express the input data given? How will you express the loads given? What is a first thing you need to do? You have to assign the coordinates.

(Refer Slide Time: 32:48)



Element No.	Start Node (X_{is}, Y_{is}) m	End Node (X_{iE}, Y_{iE}) m	Length L_i(m)	$\cos\theta_x^i$	$\sin\theta_x^i$	$k_i = \frac{EA}{L_i}$ (kN/m)
1	A(0,0)	B(0,4)	4	0	1	125,000
2	D(3,0)	C(3,4)	4	0	1	125,000
3	A(0,0)	C(3,4)	5	0.6	0.8	80,000
[illegible]	D(3,0)	B(0,4)	5	-0.6	0.8	80,000
5	B(0,4)	C(3,4)	3	1	0	66,667
6	D(3,0)	E(-,0)	-	1	0	8,000

So, shall we do that? So, how many active degrees of freedom do we have?

Four, five…

You have a roller support which can roll. Roller support can roll, agreed? So, 5 if you said 4, you got it wrong, 5. You can number them anyway you want, but usually along the x direction 1 and along the y direction 2. Then so, you have got 5 global coordinates. These are active, what about the restrained 1s? So, you can mark them also. Now, let us see the question is what was the question? You got 3 only know, you got 4. You are right. The spring force will also push at the right end; so, you got 4. You have 6, 7, 8, 9.

Now, mind you this is the essential a manual kind of method. The reduce elements stiffness method you have to use a little bit of your brain to help you solve the problem. The support reactions are very easy to calculate. So, if you had to find only the bar forces, you need to actually generate only the k A A matrix. You do not need the k A R; you do not need k R A k R R. So, if you go back to the original question, the original question was just find the bar forces. So, let us do the minimum work required to solve this problem. So, what do we do now?

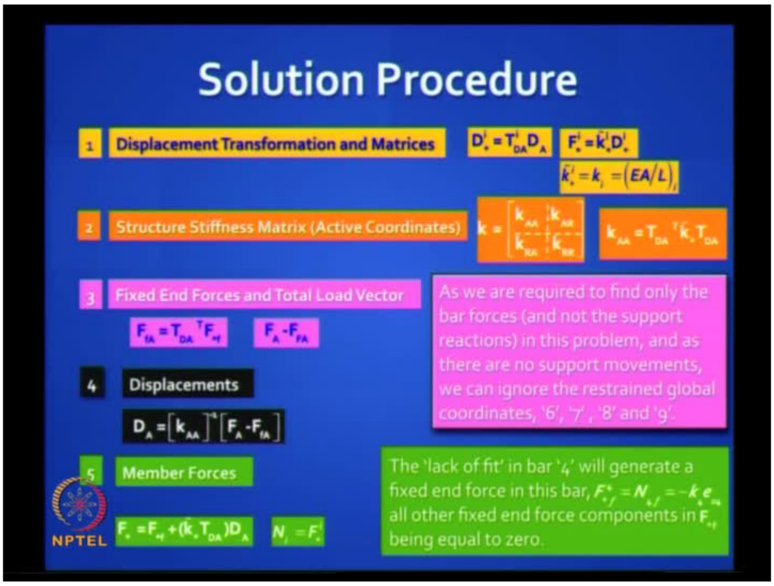
Now, we have to deal with each element. So, you need the local coordinates of the element, and first, you write the input data. What is the input data? F 3 because we choose 1, 2, 3, 4, 5 active coordinates, only F 3 is non-zero. It is a load, it is plus 100 kilo

Newton agreed. Then, you have an initial elongation, because the bar 4 was given to be too short by 2 and half mm. So, how do you write that? e naught 4 is minus point naught naught 25 meters. Does it make sense? That was given to you. Now, the local coordinates, just draw a general picture like that. That is a typical local coordinate, is it fine? 1 degree of freedom per element and you can put a theta and you have covered all the 5, all the 6 element.

The spring also you can treat as a sixth element. What is the difference between the spring and truss element? Absolutely no difference, it can also take axial tension. So, it is just to frighten you, do not get frighten that curly thing. You can straighten out and just put an element there, call it number 6. There is nothing great about that. Now, what do we do next? You have to generate the T D matrix. How do you generate the T D matrix? We generate the T D matrix for this truss. You have how many elements? You have six elements. Generate the T D matrix. Well, may be it will help us if we make a small table. This is especially useful when you have large trusses.

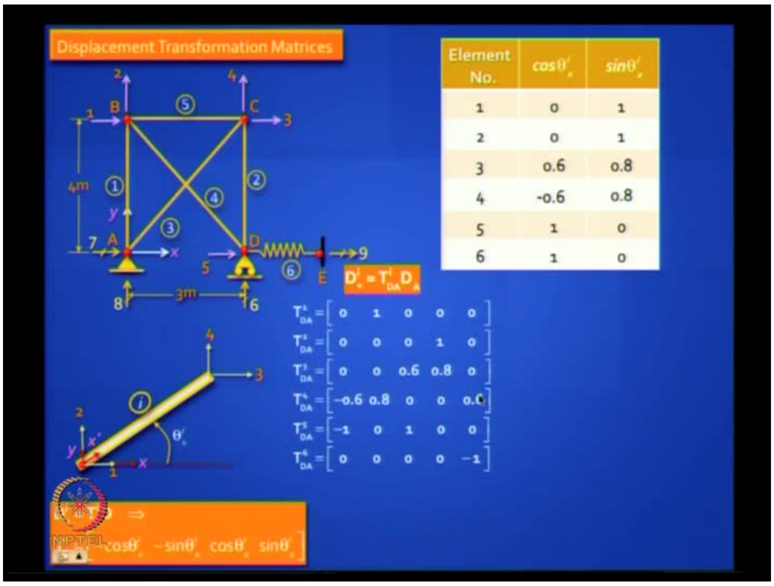
But in this problem, you could have directly done it. So, it is, this is a conventional style. Identify the element number. The start node give it some coordinates. The end node, the length of the element and cos theta can be easily written as you know that x end node minus x start node divided by length of the element. So, let us say we have done this exercise and the stiffness also is given E A by L. So, I am giving you this table, because that is easy to do. With the help of this table, now generate the T D matrix. You can pick up cos theta sin theta. From here, will you do that?

(Refer Slide Time: 37:51)



While you do that, let me review the procedure. First, you have to get the T D matrix. It is enough to get T D A, because we are not interested in the reactions and all that; T D A is all that you need. Then, you generate the structure stiffness matrix. Find the fixed end forces. You have as we are required to find only the bar forces and not the support reactions in this problem, and as there are no support movements; we can ignore the restrained global coordinates 6, 7, 8, 9. Find the displacements and find the bar forces. The lack of fit in bar 4 will generate a fixed end force in this bar, which you can calculate and all other fixed end force components in F star f will be 0. This all obvious from the problem.

(Refer Slide Time: 38:39)



Now, you want to get the T D matrix. Let me help you with this. You got this. All that you need right that big table. I made it simple. Generate the T D matrix, T D A matrix. Again to help you, this is the formula. If you have the two ends and you had coordinates 1 2 3 4, actually the coordinates will be different. They will be numbered arbitrally, but you have got cos theta sin theta. Can you now using this information, generate the T D A matrix? It is not difficult. Let me give a clue and let me close it.

You have six elements. For each element, what is the size of the T D A matrix? Why do you say 4? This is for the whole structure. How many active degrees of freedom are there? 5. So, it is going to be 1 row 1 by 5 generate. It is 1 by 5 and there are six elements. So, totally it will be 6 by 5. I want to see that 6 by 5, you have to do it. You got all that sin theta's and cos thetas is here; just move your brain.

Some of you are too tired? Wake him up. Please do this. You want me to show you the answer? I will give it a shot. We do not have too much time, I will give you one more minute, but if you do one problem, you will see that a it is very easy, not (( )). The axial system was very easy because you had only 0 1. Now, you have this cos theta sin theta come into play, but there is a method in the madness. Your axial T D matrix, what is the full size if you include T D A and T D R? How many columns will you have?

You will have 9. 5 belonging to active degrees; 4 belonging to the… So, e you will have mostly 0s and you will have this 4 non-zero quantities cos theta minus cos theta minus sin theta plus cos theta plus sin theta. So, each row will fill up four locations, the rest should be 0, do you understand? I will give an example. Take this element, element number 1. Element number 1 will have values in the first row because this is the D 1 is there. The second row, the seventh row and the eighth row, of course, 7 and 8 do not come in T D A. So, it will fill up only in 1 and 2. What values will you put for 1 and 2? Will you put minus minus or will you put plus plus?

Plus plus

Why?

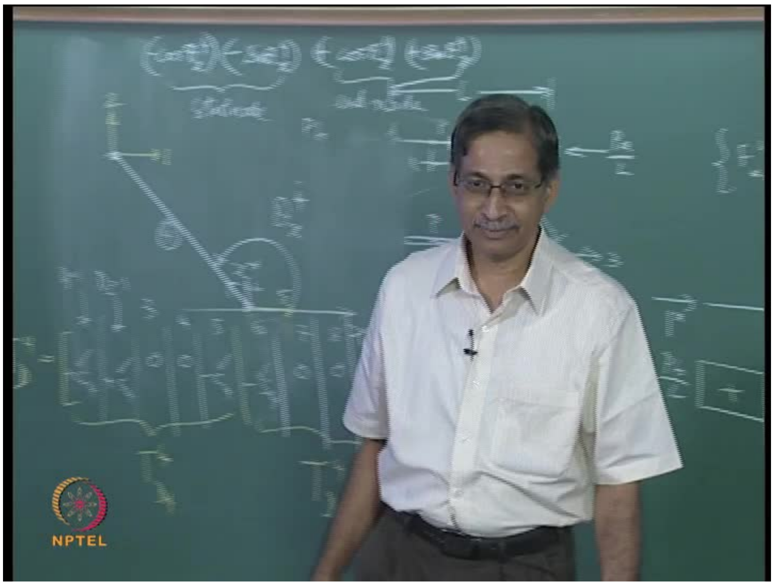
1, 2, 3, 4

No, it depends on you angular incidence.

Which is your start node? Which is end? Look at this element. In this element, my local x star axis was pointing from here; that means this left end was my start node; this right end was my end node. You get what I am saying? So, you have to identify which is start and end, then only you can do it; so, which means you have to go back to this. So, let us go through it. Once more element 1, the start node is A; the end node is B, x star Local x star is moving in that way. Element 2 from D to C; element 3 from A to C; element 4 from D to B and element 5 from B to C and element 6 from D to E, is it clear? Because if you reverse, then theta change. Do you understand? So, let us go through the solution.

This first row, let us take, let us take element number 1. Element number 1 will have only positive values, it will not have negative values, because these two will go to 7 and 8, 7 and 8 do not come in T D A. So, you just copy down 0 1 to the first and second. Let us look at the fourth element. This is the fourth element. Fourth element starts here and goes there. So, 5, which is 5? 5 is active; 5 is active, and here, you will put 5 and 6. So, 5 will be minus cos theta and 6 will be minus sin theta. So, minus cos theta is this. You get minus of minus is plus 0.6, got it? Did not get it? Listen careful. Let me explain on the board. This is only tricky thing. Once you have got this, everything will moves smoothly.

(Refer Slide Time: 45:36)



This was the element number 5, 5, was it 5? 4, element number 4. This was the incidence of x star; this was the incidence of x star, which means that if this is my positive x axis, this is my definition of theta 4 x. So, this is start node; this is end node. Now, what are the structure coordinates here and here, here and here? What are the structure? What are those number? What is this number 5? What is this number 6? But it is kind of a restrain coordinate. What is this number?

1

1. What is this number?

2

So, if I had to write T D for the element 4, I have to fill up a vector like this of which the first 5 will correspond to T 4 D A and the next will correspond to T 4 DR, got it? Now, so, this has got 5 parts, am I right? So, 1, 2, 3, 4, 5. So, here I will fill up for D 1, here, D 2, etcetera. So, this is 3; this is 4; this is 5; this is 6; this is 7; this is 8; this is 9 if I may remove this. Now, how do I fill up which will be 0 and which will not be 0? I have got this is 6 and this is 5.

So, let us be very clear. I will have a non-zero value here. So, let me put a tick here right D 5 D 6. These will be there and I will have 1 and 2 and the rest will be 0. So, let me go ahead and put the 0s so that my life becomes easier. Now, I have to decide what to, I have put in these number, I have to put in minus cos theta this is 4 x minus sin theta 4 x plus cos theta 4 x and plus sin theta. This is how it goes. These two will go to the start node always, that is how it is defined and these two will go to the end node. You just go back to the derivation.

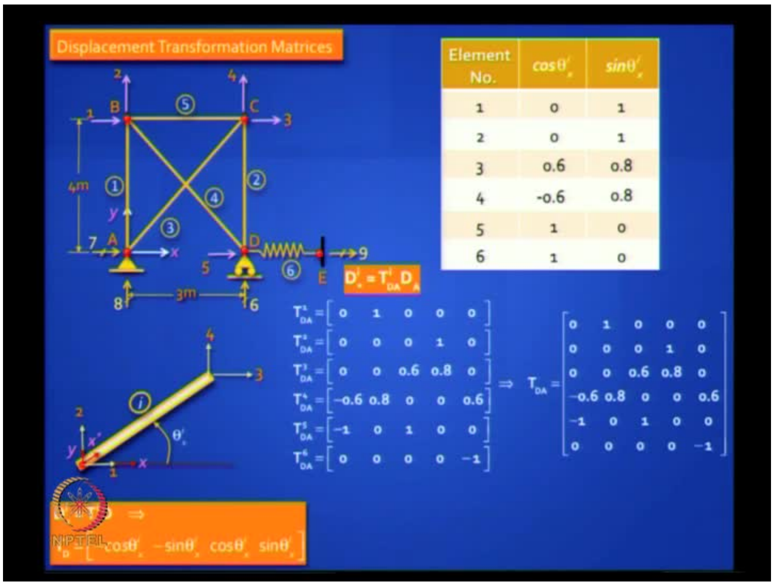
Now, so here, I should put. So, let me simplify and put minus C 4. Here, I should put S 4; here, I should put plus C 4. Does this make sense? Tou can program it, you will do it automatically. Now, do these answers match? Because take a look here, C 4 is minus 0.6 S 4 is plus 0.8. Does it makes sense? Now, this will fill up the whole vector, that is T D A and T D R. If you want, you can fill T D R but we do not really need T D R. So, can we fill up this T D A matrix? Can you do it correctly without making any error? Any doubt, ask me.

In second element, what is the start node?

In the second element, it is going up D to C or you can choose anything you want. How to, you may, is there a golden rule? There is no golden rule. I like going from bottom to top. If you want go down from top to bottom that is all. There is nothing (( )) in all that. Let us, I think I went through that. For first element, from here to there; second element from here to there; third element from here to there; forth element from here to there; fifth element from B to C. Now, you are asking why not switch everything upside down. You are welcome but be consistent.

So, in the first table, you do it correctly, but an incidence is very important. Otherwise, you cannot use blindly these formulas. Otherwise, you figure it out yourself. You can also use the T D transpose and use the static method and get it. But this a kind of blind method which will always work. Have you understood? Can I proceed? Clear? Just go through this problem carefully. If you get it once and you solve the assignment problem, you will understand.

(Refer Slide Time: 51:39)

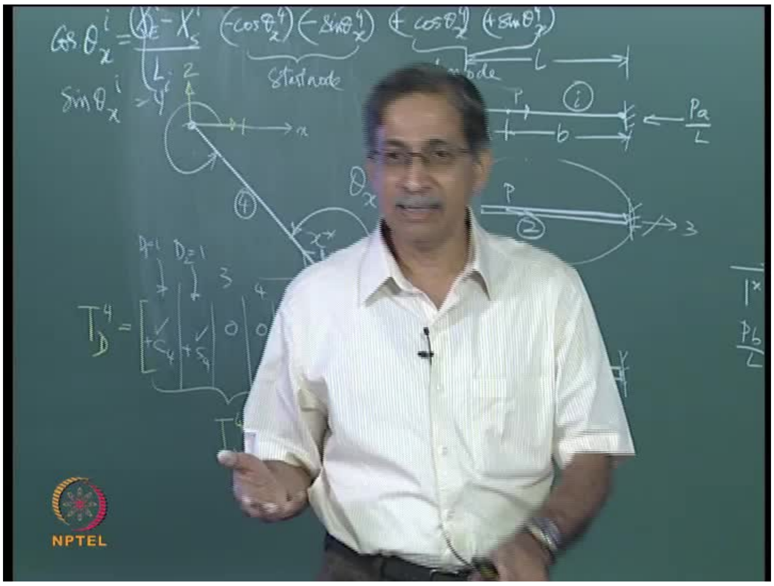


So, I can put it all in one box directly like this, yeah.

(( )) basically what is the order we can go from left to right

You can go any way but your cos theta, your theta will be defined with respect to the positive x axis. Supposing this was my start node, my theta would have become like this. That is a definition of orientation, is not it?

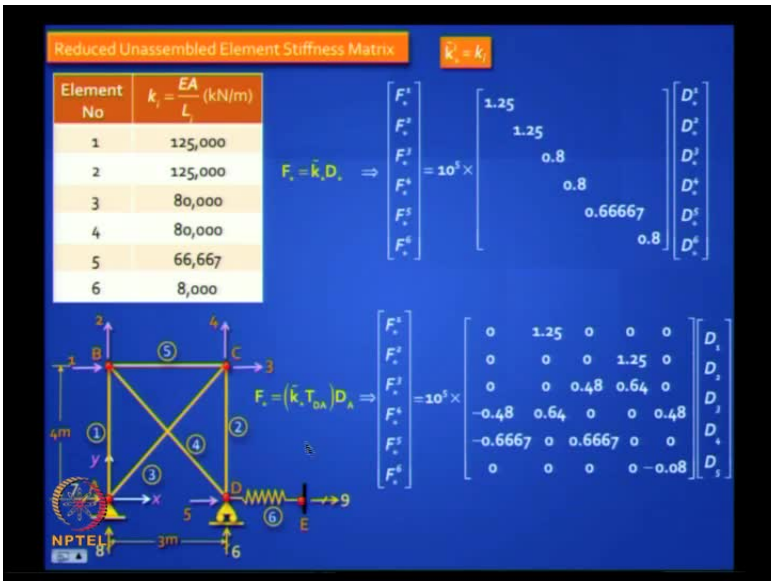
(Refer Slide Time: 52:15)



So, see this was my direction; this was my x star axis. Supposing I choose this as my start node, then my theta x would be this is my positive global x direction. It will go like this. This becomes theta x which is different from this theta x, got it? See, we are talking of direction cosines. So, we said cos theta i x is always x n node of the element i minus x start node of the element i divide by the length of the element i. this is the direction cosign definition.

Similarly, sin theta will be with respect to y axis. This x will be replaced by y. So, it is easy to program, you do not have to worry. You do not have to break your head about minus and plus, it will do automatically and beautifully, but once you decided on the direction of incidence which is your start and end, you better stick to it, do not change, do not change your game plan in between; so, it follows very nicely. Can we move on? We will just finish this.

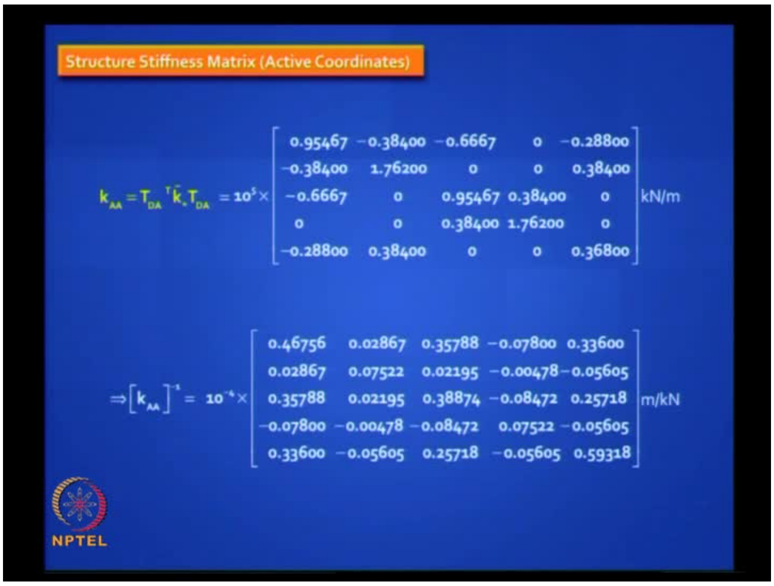
(Refer Slide Time: 53:27)



So, now, you have got the T D matrix, please do it correctly, but it is not difficult. Please do a couple of problems you will get the hang of it. Then you can assemble your, you got the stiffness matrix, you got E A, you got L, you can all put it together this way. It is a diagonal matrix. If I have not filled up anything else, it means it is all 0. So, this is my k tilde star matrix clear, because you will get it from your axial stiffness for the six elements. Then what? Then I have to generate my structure stiffness matrix. There are two ways of doing it.

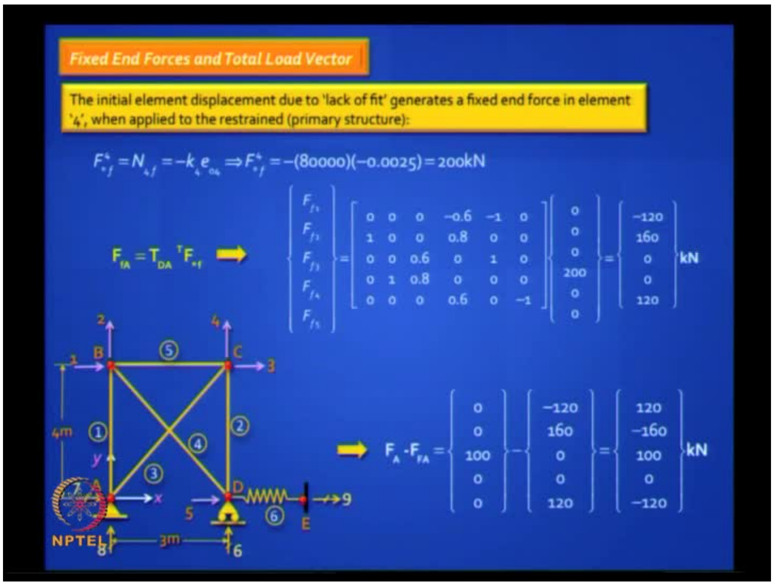
It is actually T D A transpose k star tilde into T D A. I can do the latter part first or the former part, first is better to do in this way, because this comes in handy to find my member forces. So, all this is best and using matlab or some you know, but if you want to do it manually, you can work out; it is not all that difficult.

(Refer Slide Time: 54:40)



So, can, once you have T D A and you got this, can you do this product easy? You get this, then you pre multiply it with this. In one shot, you got the final structure stiffness matrix. It is a 5 by 5; it is a square matrix; it is a symmetric matrix. Now what? Now Invert it or you can use gauss elimination. Now, you are ready to handle any loading that can come on the structure. Any loading can be handle. This is the property of structure. So, it is ready. Once you have given the geometry, give the E A by L values. You can program your system for you to generate the elements stiffness, structures stiffness matrix.

(Refer Slide Time: 55:22)

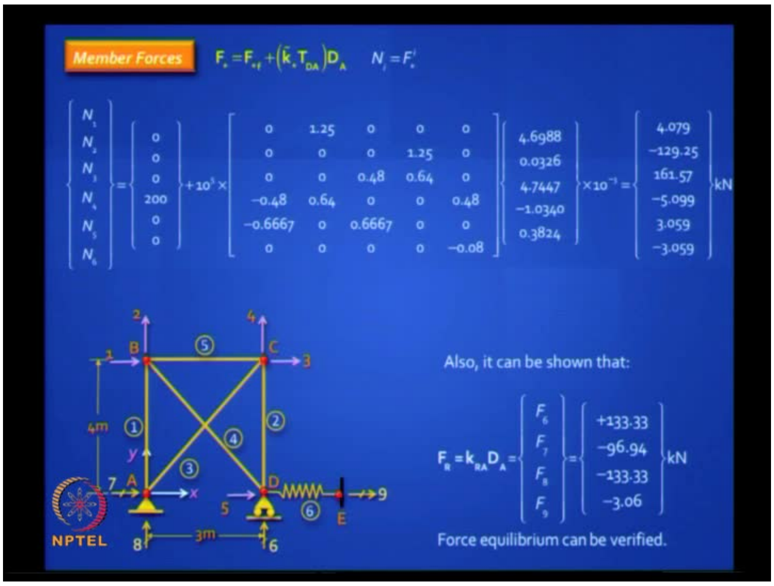


Then, you find your fixed end forces. Here, one of the elements is too short. So, remember, how to do this? See, the axial force is an element is always minus of in k e naught k is k. For that element axial stiffness, E A by L is given, e naught 4 is given; so, you get 200 kilo Newton minus of minus is plus, which means if the element was too short in the primary structure, you have to stretch it to fit to the two ends. That is why you will get tension initially.

Does it make sense? Physically does it make sense? Why? You get a positive quantity 200 kilo Newton. The element came too short. In your primary structure, the ends are locked in the original configuration. So, you got a stretch this element to lock it there; so, you will get a tension which is nothing but axial stiffness into that initial shortness in length, got it? So, you will get this F star f. Then, if you do T D transpose, mind you there only one non-zero F star f you will get. You are converting what happens to the joints? You know this; this has to pass on to the joints in the structure.

You do not have to break your head and figure out what happens, just do T D transpose T D A transpose. So, it is, it is switched over from the element level to the structure level, is it clear? We did it for the axial system. The same logic holds good for the, for the structure, and then, you have a equivalent joint loads. Remember, the original node loading is 100 kilo Newton at F 3, but you have this extra loading coming from that lack of fit. So, this is F e minus F FA. This is called the final total joint load vector F A minus F FA. You got lot of loads now.

(Refer Slide Time: 57:26)



Only one of them is not 0, that is the F 4. Then, what do you do? Then, you find the unknown displacement. You got the load vector; you got the structure stiffness matrix. You inverted the k A matrix. You do this operation, you will get the answer. You got the answer; you got the, then, what do you do next after you got the solutions? You got the unknown displacements, but that is not what you really wanted, you wanted bar forces.

So, to get the bar forces, you remember you already computed this and you are ready. You got the initial bar forces. There is one 200 kilo Newton already there, this F F A remember, and then, you do this operation, you get these answers, and if you watch, you can also find the support reactions if you bring in T D R into picture, that is it. One problem understand fully and you can handle it. Tomorrow I will take a really bigger problem - 10 bar truss, a compound truss just to demonstrate and we will close. We will do a space truss. Thank you.