Advanced Structural Analysis
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Module No. # 4.2
Convex Hull Different Paradigms and Quickhull
Lecture No. # 23
Matrix Analysis of Structures with Axial Elements

Good morning, this is lecture number twenty three module four on matrix analysis of structures with axial elements. If you recall we started this topic in the last session and we introduced the conventional stiffness method.

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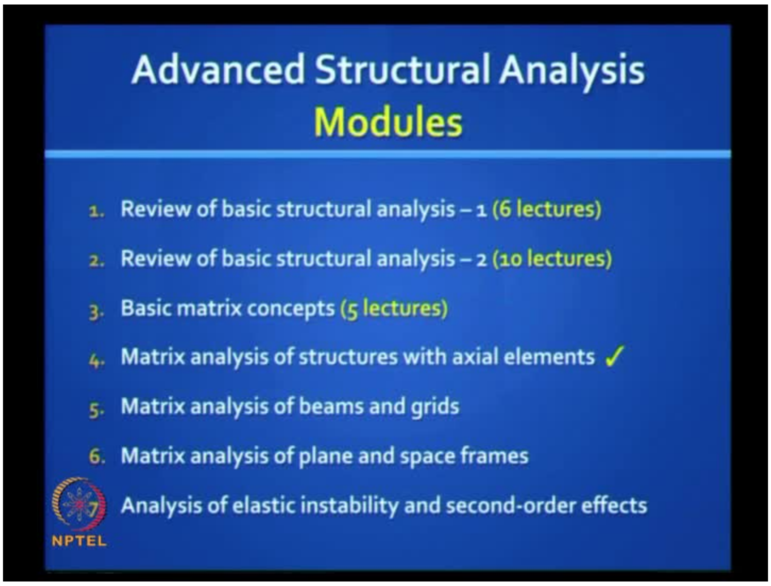

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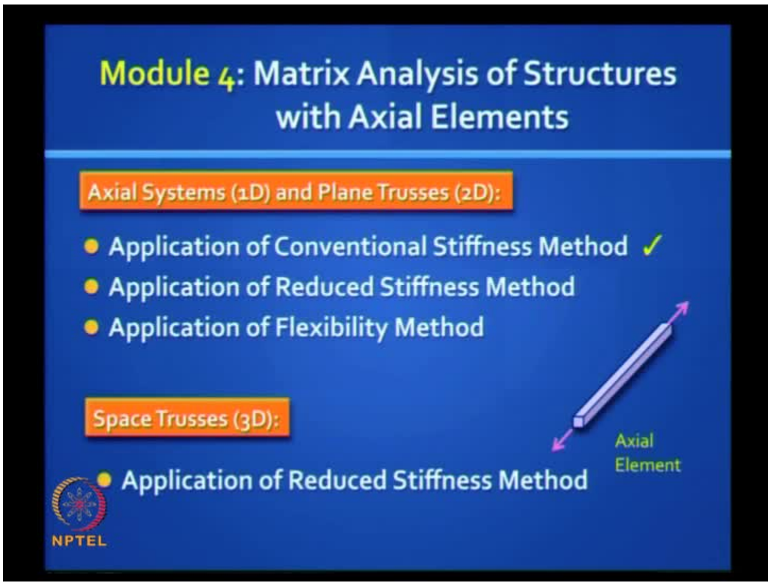

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We are going to continue with that method, and show how it can be applied to analyzing axial systems and plane trusses; this is covered in the book on advanced structure analysis.

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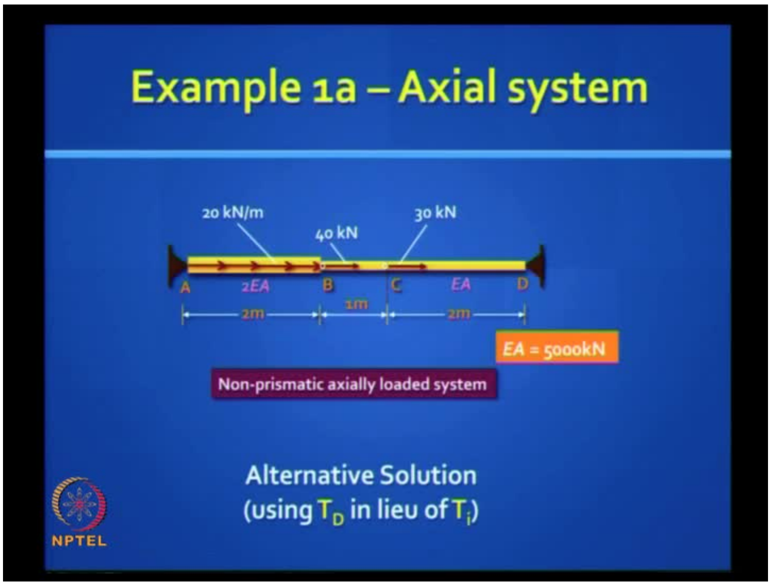

So, let us look at this problem once again, do you recall this problem, we started with this problem, we are now going to do the same problem using the displacement transformation matrix T D instead of the T i matrix.

Now, please pay attention and work with me, so that, you get the hang of this. This is actually easier to do by this method, because you do not have to worry about the linking coordinates, but you are dealing with larger matrices.

Of course for this problem it is not very difficult, because you have only two elements, but in real life problems big trusses it is little unwieldy, but it is good for manual analysis.

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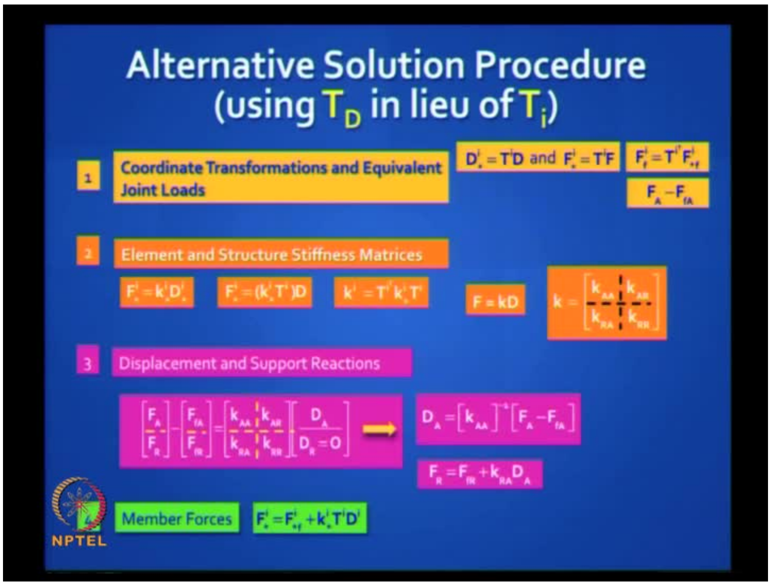

So, the procedure that we use, when we use a T i matrix is shown here, and just reproducing something that we saw in the last session.

What is the change that we need to do, in which of these steps do you think a change is needed when we use T D instead of T i. In the first step; first step what kind of change is needed.

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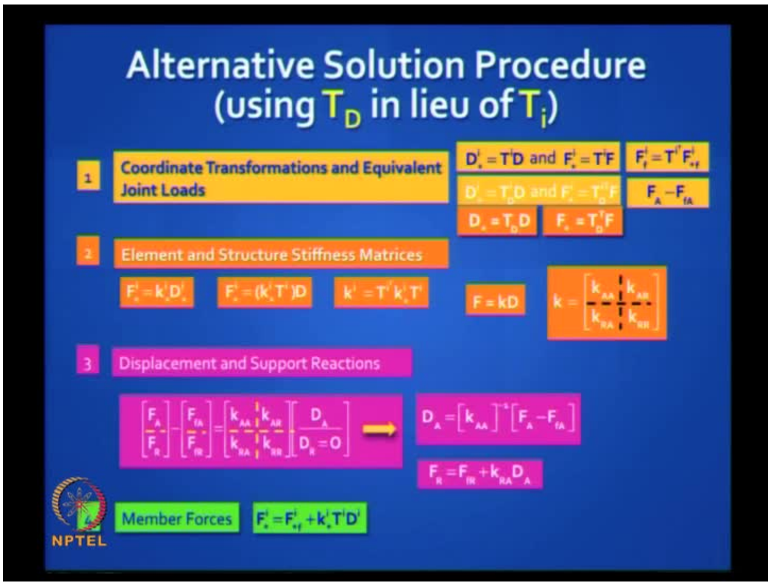

Well the transformation will look like this, D star is T D into D that is for the assembled one, and if you are doing it element by element, it is D i star is equal to T i D into D, and for the forces you have the contra gradient principle which means you have to take the T D transpose.

So, if you’re doing this method then you forget about the T i matrix, this is the replacement. What about the fixed end forces, what is a correction we need to do, you got the fixed end forces at the element level the local coordinates, how do you switch to the structure coordinates.

You have to pre multiply with what not that T i transpose, but, with T D transpose, so that is only other change everything else is the same. The, second step, will there be any change in generating the stiffness matrix.

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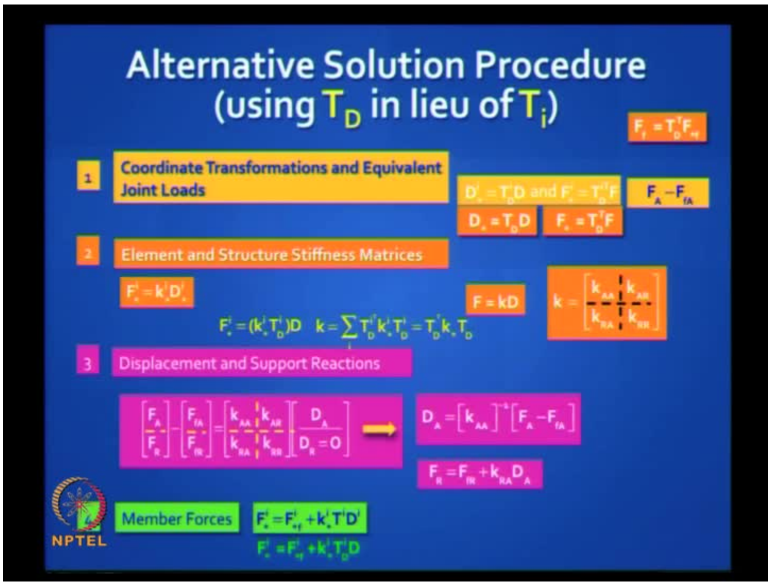

Remember in the earlier method you have to actually slots them with the linking global coordinates, you do not have that problem now, so it is simpler this is the transformation you will do, and you can sum up for all the matrices, we will demonstrate this

You end up with the exactly the same structures stiffness matrix, so it is just a choice for programming for large structures, T i is the traditional way of doing it, but for manual analysis of a smaller structures you can use T D.

What about the third step, will there be any change in this. No, third is identical, because there is no T i no T D coming into that, we are just solving equilibrium equations, so you solve the first equation, you find the unknown displacement D A, substituted in the second equation find the unknown reactions F R, then you find the member force. In a member force, is any change; there is a change, because T i is not going to be used, so the change is T D.

Now, I want to you look at this carefully, in this last step we actually calculate from the D i, the D i are those components of the displacement vector of the structure which are relevant for that i th element. So, the size of D i will be how much for an axial system element,

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That is for a plane truss element, for an axial element, one D element it is 2, 2 by 2, but here you do not have to worry about selecting those values, you can directly deal with the D vector of the global structure.

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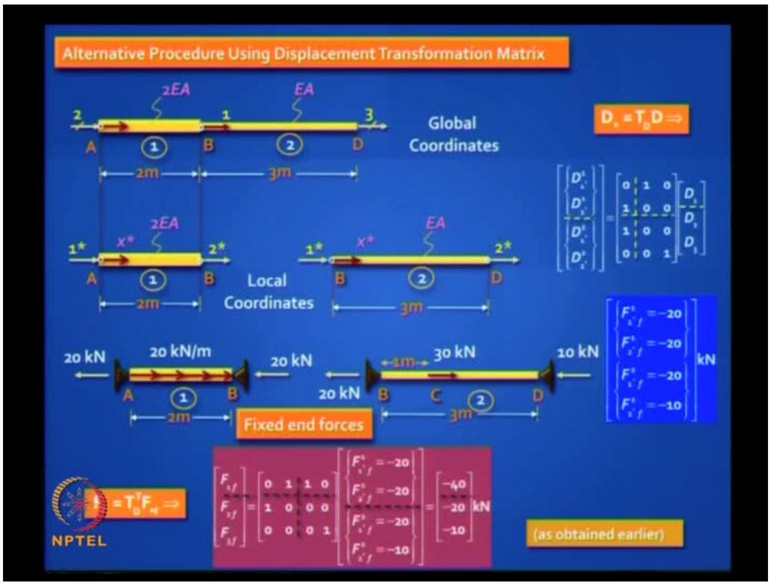

You will understand this as we work, so let us do this together, you have two elements, the global coordinates have 1 2 and 3, there is only one active degree of freedom that is number 1 2 and 3 are restraint. We just repeating the same problem we did in the last class, and the local coordinates are exactly the same as we did earlier, there is no change where i am just showing you this.

What is a first thing we need to do, we need to write down the T D matrix, can you write it down. How do you write down the T D matrix, give it a try, you can write down separately for the two elements, take the first element what is the T D matrix. For the first element if you pull D 1 equal to 1, what will be the reflection that you get at in the first element.

So let me help you with that, you need to write this, matrix; it will actually look like this, why do we say that, so take a look at this, this is your T D matrix to fill the first column and mind you there a two elements this belongs to the first element this belongs to the second elementwhen you to fill up the first column, you applied D 1 equal to 1 which means, if I put a unit displacement here what gets affected, well this gets affected and this gets affected, because they both connect to the same point.

Does this get affected, because you're applying only a unit displacement one at a time, does this get affected; no that is arrested, does this get affected, that get, so you say it is a simple logic

So, it follows that D 1 1 star is 0, because we do not get any displacement here, D 1 2 star is 1 the same as the moment to get that, D 2 1 star is 1, because it should match with D 1 equal to 1 and D 2 star is 0, does it make sense that is all.

Now you apply D 2 equal to 1 that means you filling up the second column, but you are put applying D 1 equals 0, D 3 equals 0, what do you end up with, if you put a unit displacement here only this gets affected, there is only 1 nothing else gets affected, all the others are 0, so does it make sense. Similarly, when you pull this third one you will get this. So, that it is actually easy, you can generate it using first principle, but you can also program it, because it will always be filled with ones and zeros.

So, you got your T D matrix, your next job you got T D 1, T D 2 put together in one box, is to get your fixed end forces. Now, in this method we done this calculation remember, can i assume that we accept that the fixed end forces as shown here, we did it in the last class, remember we had a long discussion.

How do you convert it to the structure, how did converted from local axis to global axis, so this is a local axis agreed, for the first element minus 20 minus 20 for the second element minus 20 minus 10 kilonewton, do you understand. This refers to this element, this refers to this element, minus means pointing towards a left, does it make sense we did this in the last.

Now, you have to switch from here to that is the global axis, how do you do that, actually you want to go straight to the structure how do you do that T D transpose, so you do that you already got T D, will you work this out yourself, tell me what the F f matrix will look like. There are two ways of doing it, doing it understanding every step which means you need to follow the class, whereas just copy what is written on the screen, when you understand it everything is clear.

So, I want you to raise question if you're not getting it, this step is very simple you got the T D matrix, take the transpose of that, and multiply that with this, do you get this that is it.

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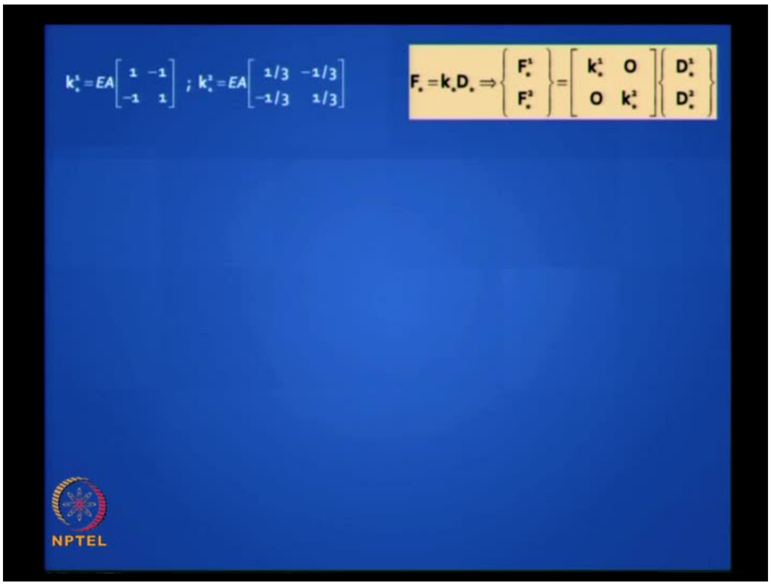

We got the same vector using the T i formulation this is the only difference, is it clear which is easier; this one or the previous one, this one I mean, because no thinking is required we just multiplying matrices. Let us proceed, what is your next step, we got the same answer, next step you write down your element stiffness matrices this is just a same way we did last time.

Can you generate the structure stiffness matrix by the T D formulation, what should you do next, if you want as put it all together and have the unassembled matrix you remember, you can, whatever you got here k 1 star you should put here, whatever you got here as k 2 star goes here, remember it is a diagonal matrix, they are uncoupled, how do you use this matrix to generate your final answer.

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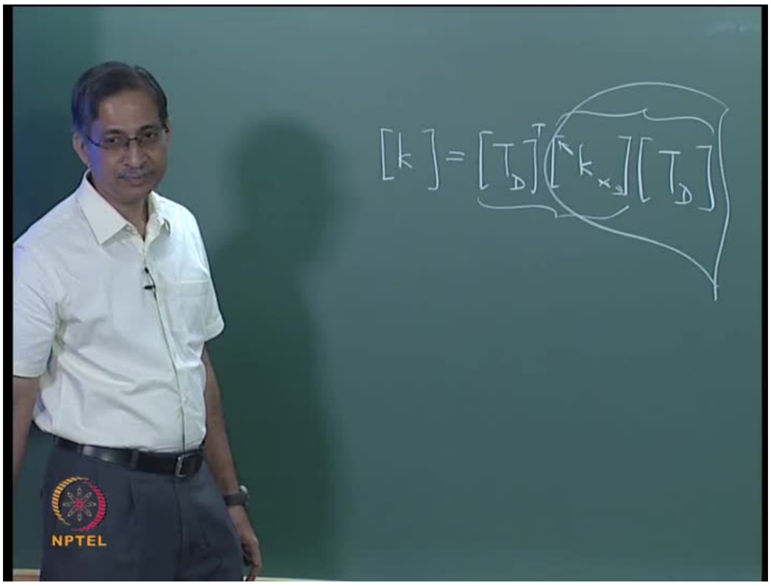

You’re not thinking, I want to generate the structure stiffness matrix k and I want to do it in terms of this matrix k star, which is a diagonal matrix as shown there.

TD transpose

That’s all.

So, I have T D transpose, now I want to raise an issue here, you need to do one these multiplications first, would you prefer to do this one first or this one first a or b.

Right side to do first.

Right side you would like to do first, why is that.

Because we using it like (( ))

Because, this quantity will come in handy later in calculating.

Tension

Calculating the bar force.

Bar force.

How does it happens.

Because the bar force (())

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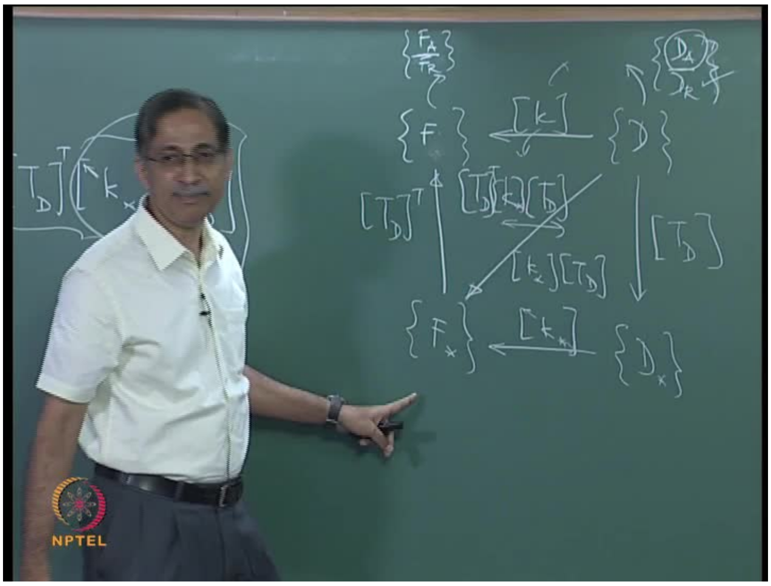

Well if you look at this picture, you have this structure; you have F star, you have F and you have D, and you got F star and you got D star, this transformation is given by what matrix T D, this transformation is given by what matrix.

T D transformation.

What is this principle called.

Contra gradient principle

What is this relationship.

k star

This is assembled or is this an unassembled matrix, so it is a diagonal matrix, because it has got as many elements as there are in the structure will be put all in an uncoupled manner.

What is this relationship k. So, remember this and this basically is mixture of F A F R and this is D A D R and similarly, you can partition this k matrix into k we will see that shortly, but at the end of the day you’re going to find out this, and this is known and is known to be zero.

You want to find the internal forces, these are the member end forces, so you have a short cut route here, after you note this you can get this directly by doing this transformation, k star into T D and then the next transformation k is given by this times this T D transpose k star T D.

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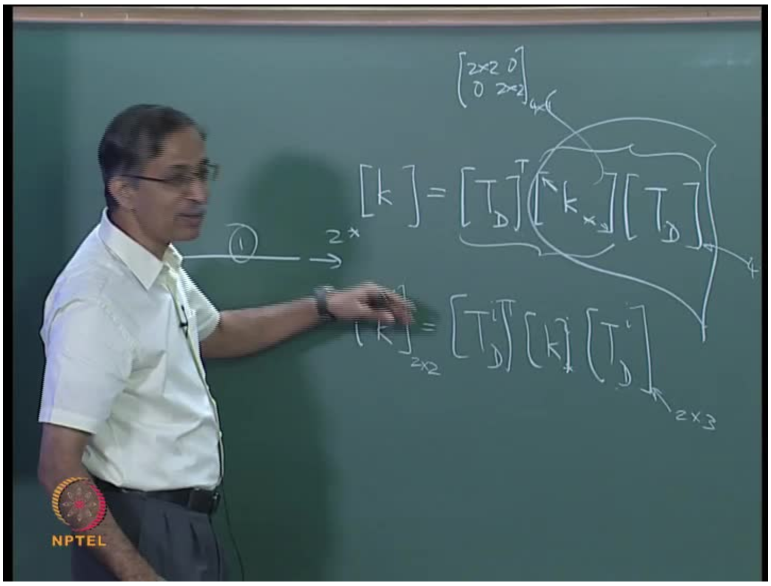

So, now you see why it make sense to do this calculation first and store it in your computer, because you can pick that up later when you find the member end forces, [is it clear]. So, you do it in two stages, first you calculate this then you pre multiply with that, you can do it in one short or you can do it in two steps or as many steps as you need as there are members.

So, you can do it at the element level, for the i th element this will be written as T D i transpose k i star T D i, so you got k 1 k 2 in this particular problem and what will be the size of this matrix for this particular problem.

What is its size, its 2 by 2, what is a size of this matrix, well it will look like this, you got a 2 by 2 there, you got a 2 by 2 here and say you got 4 by 4, so you got 2 by 2 here, what is the size of T D i.

4 by 2 4 by 2.

Now, what is the size of this, this includes all the elements what is a size of this.

Two cross.

That’s a one we did.

2 by 3, 2 cross 1, sir 4 cross 3, 4 by 3, 2 cross 1, T i D is 2 cross.

How many global coordinates are there.

3,

So what is the size of this vector.

3.

It is 3 by 1.

What is the size of this one.

4 by 1

4 by 1 if you assembling them all together. So, what would be the size of this.

4 by 3.

So, remember this is 4 by 3, but for each of them what will this be, this is 2 by 3, because each element has, you know the element looks like this, it has got 1 star and 2 star that is all it has got, you got element one likewise you got another element two, that is how it works does it make sense.

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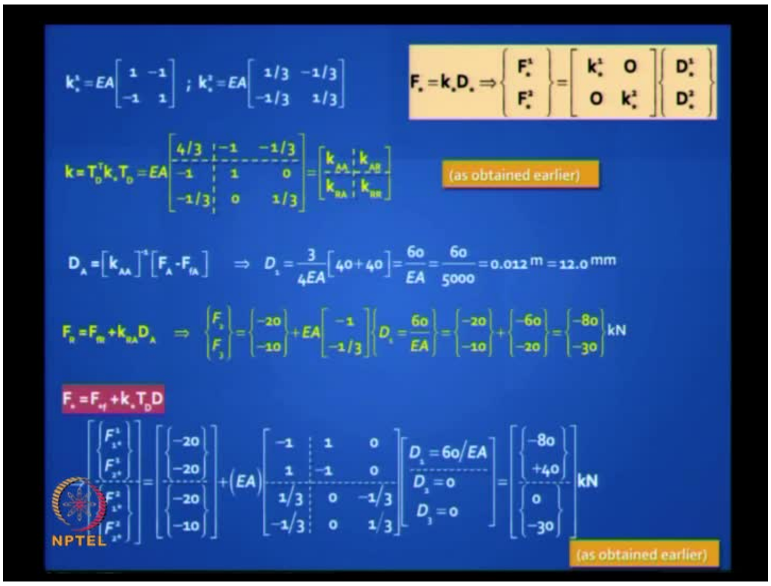

So, what you do, you do this and you add up the contributions for the two elements with the summation ends, so the two ways you can do this calculation let us see that. So, you can do it this way and you will get the same answer that we got earlier, or you could do it, well I have not shown that, you can do for each element and add up the contributions you will get the same answer. Now, what you do you do exactly what you did earlier so I can go fast.

You can find the unknown displacement, plug it back into the second equation and get your reactions then you get your member forces, this is where you can use the transformations you get the same answers.

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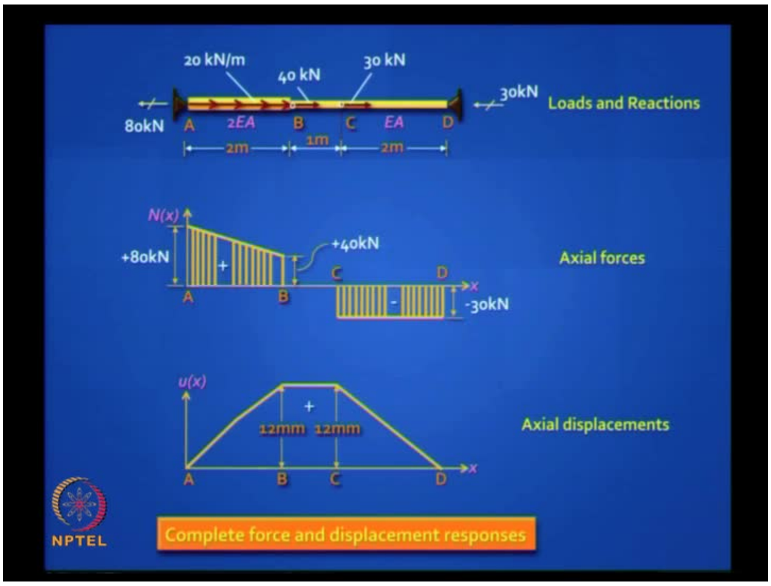

So, let us not spend time, you get this response which we got earlier. Can we move on, so you have two options either use a T i method which is the conventional method or use the T D method or better still learn to use both the methods, when we study the reduced elements stiffness method you will use only T D there is no T i.

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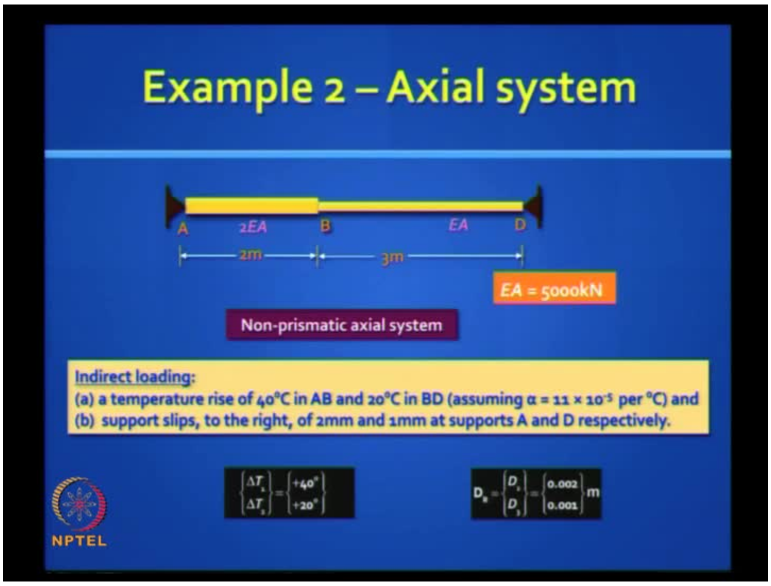

Second problem, take the same structure, now there is no loading, no direct loading and let us put some indirect loading, let us put two kinds of loading; one let us say, there is a temperature change, so you have a temperature rise of 40 degree celsius in element A B and 20 degree celsius in element B D.

What do you think will happen to those two members, because of this rise in temperature. They both go into compression, because they are not allowed to expand. On top of that let us put in through in some support movements, so you got support slips to the right of 2 mm and 1 mm at A and D.

The most complex problem is a combination of this and direct loading, but that you do for your assignment, will do it slowly how do you handle this problem, first how do you write down the input data in a vector form.

What is an input data. Well you got temperatures as you got the coefficient of thermal expansion, so you can write down your two temperatures delta T 1 delta T 2 and you know D R, D R is we will choose a same coordinate system agreed D 2 is 2 mm which can be written as point 002 meters and D 1 is point 001 meters both are positive, because your pointing from left to right, is it clear this is a given data.

How do you proceed, actually much of the work is already done, because you already generated your T I T D and you already generated the elements stiffness matrix, so it is here, your load is change that is all that is happen, so you do not need to do anylaborious work.

(( )) vector

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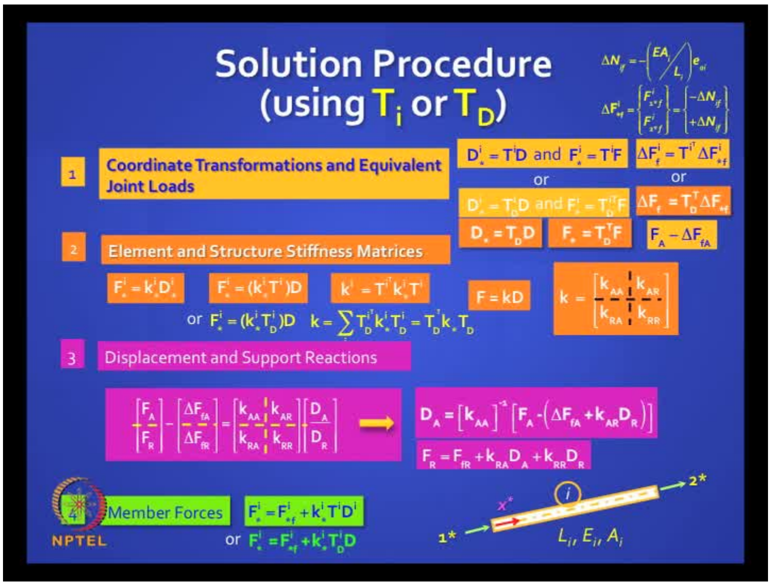

So, these are the steps first generate you i am going to show both methods T i or T D, if you’re doing T i write down the T i values first and generate the fixed end forces, how do you get the fixed end forces; take any bar, lock it is to ends that is a primary structure, allow the temperature rise to happen, that bar will go into compression.

The force in that bar is given by what, that is what we call delta N i F, that is a fixed end force in that bar, it will be minus, because it is going to going to compression, the axial stiffness is E A i by l i and E naught i is the thermal that is l alpha delta T for the i th element, is it clear. So, this is known and then do you agree that if this is going into force delta N i, that force has to be equilibrated by equal and opposite forces at the two ends.

So, can I write it like this on the left side plus delta N i F and on the right side minus delta N i F that is my delta F i star F that is my fixed end force, clear. This will give me compression, because N i is turning out to be negative, is it clear that is how you do it.

So, I can find out my fixed end forces, and I get my net load vector that is f a minus delta F F A, if you have this combined with intermediate loads then you have to add that part also.

If you are doing by the T D method, everything is same except you use T D and T D transpose, then you generate the structure stiffness matrix, you do not have to do any extra work, because we have already done this you can do the either value T i method or the T D method, then you apply the basic equilibrium equations. Please note in these equations, you have F A F R in this problem F A is 0, because no loads are given to you and you have this fixed end forces vector which we know how to derive, substitute those values and you solve for D A plug in the values of D A’s and you get F R.

Delta F i star, in that minus delta N i F and plus know sir minus and plus know sir.

You can write it in either way, find you must get it right either you can write it this way or you can put minus.

In that if we keep minus in the first one delta N., and here in delta F it is minus delta N and plus delta N know not matrix.

In that vector yes tell me.

Minus delta n and plus delta n.

Let’s look at, this is.

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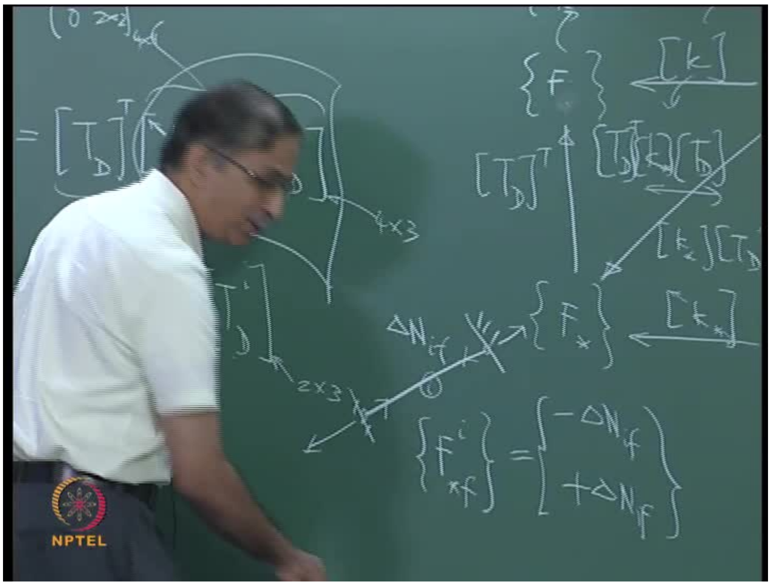

Please listen this is my vector, let me not pre judge the issue, let me say this has some internal force delta N i F, can is say that this is arrested; this is arrested due to some reason.

Let’s say I cool this bar, I reduce the temperature then it goes into tension you agree, so if I can shown the tension like that, which means it must be equilibrated by a force like this and a force like that, so my fixed end force vector F or the i th element, is the i th element star F what will it look like. It will look like minus Delta N i f plus delta N, so this is my first principle what do we have on the screen.

We, got plus or minus, so well caught that should be flipped, so ramesh that is a very valued point, so if it is a compression it will show up in delta N i inside, so please note this expression was correctly done, this should be minus and this should be plus correct as proceed.

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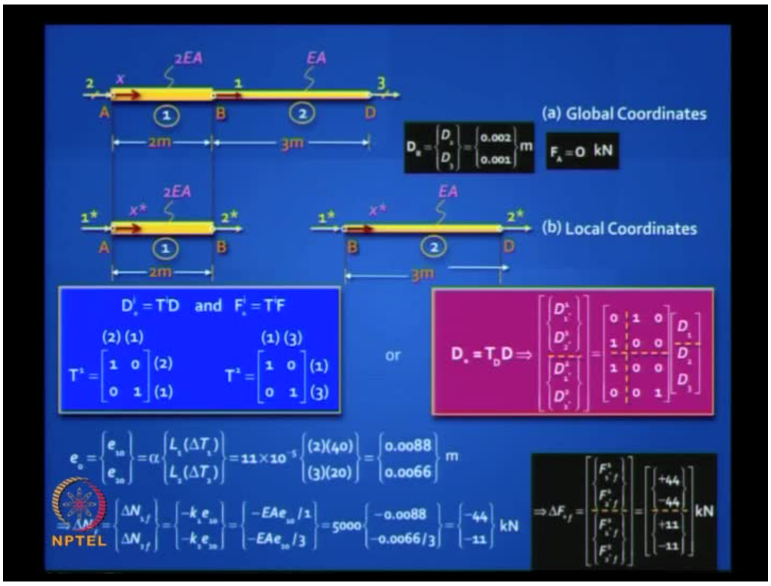

After this you find a member forces, you can do it by both methods, let us see how to demonstrate; so you got global coordinates, you got local coordinates, we do not waste time, this is a input data, you do not have an direct load vector, you done these two calculations either you do it by the T i method or you do by the T D method both expressions we have.

Now, we calculate the bar elongations it is a rise in temperatures, so E is positive, the step is straight forward L alpha T, L is known calculation as straight forward, what do you do next.

You find the axial forces they turn out to be compressive if this step correct now is where we need to do it correctly now tell me if what we done is right is it correct.

Its correct, because minus turns out to be plus and plus turns out to be (( )), finally you must be sensitive to the answer, when you see an element like this, the axial force is given by this value not by the other value, please note, so if this is moving this way you got tension, is it clear. So, N i in general is given by F i 2 star, this sign will always match.

So, here 2 star is minus, that means the axial force is negative, is this clear. So, the first element has an internal force of 44 kilonewton, the second has 11 kilonewton and both are compressive, this is before you started the analysis, that this is that kinematically determinant primary structure.

Now what do you do now. You have to release the active degree of freedom.

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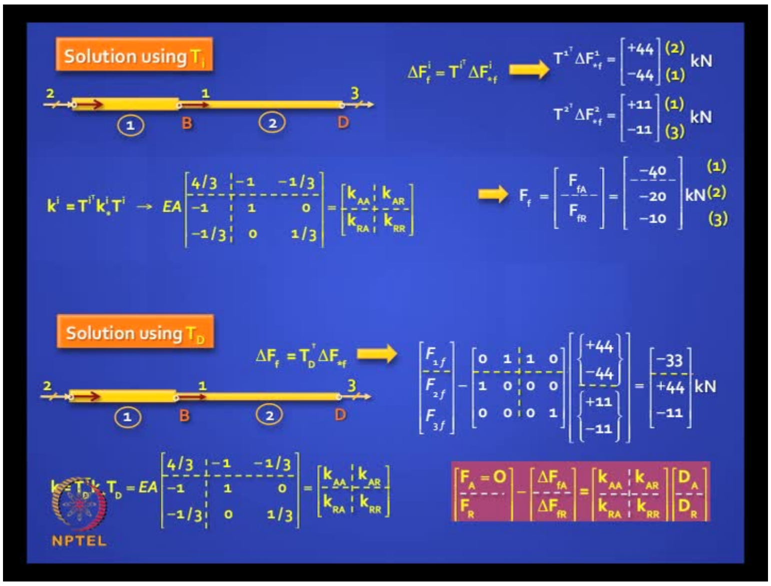

So, how do you do this transformation, well we have to move from element level to structural level, but so you do this transformation you have these matrices, but you put the linking coordinates, remember the first element is linked with 2 and 1 the second element is linked with 1 and 3, so I think that this is this should be 1 and 3 and when you add it all together you get 1 2 and 3, you can also do it intuitively you will get the correct answer, then you got the fixed end force vector, you have the k matrix, if you are doing the T D method it is direct, do not think you got the T D transpose, and you can directly get the answer, and then you use the same equation, you solve these equations.

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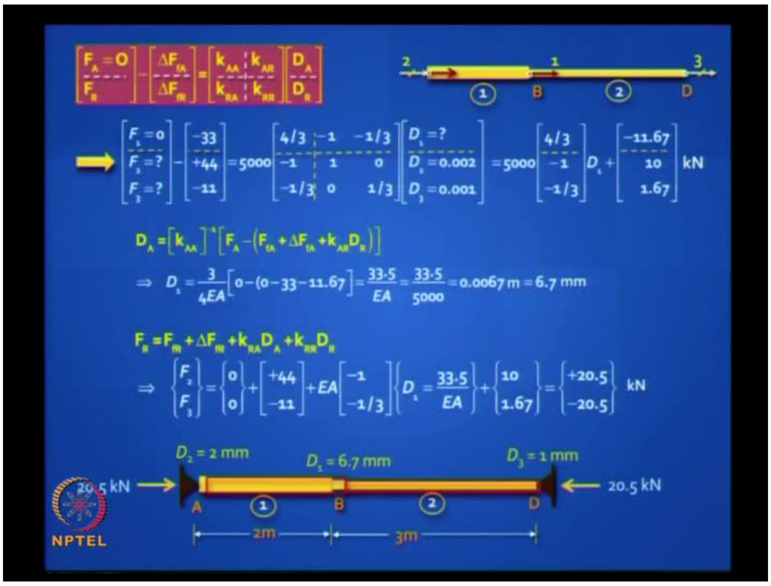

When you solve these equations you can find the unknown displacements, please note this is the equation, you got this stiffness matrix F 1 is the load vector, F A is 0, so there is no load here, you have a only fixed end forces here minus at 3 plus 44 minus 11, solve the first equation find out D 1, mind you have D R, D R is not 0, so you plug in the value find D 1 turns to be 6 point 7 mm, find F R you get 20 point 5.

What does this mean, plus is it compression or tension; compression, so it is very interesting you had two elements, the two elements had different lengths, their different cross sections, if you treated each one independently and you arrested the two ends and heated you got different forces, you got 44 in one you got 11 in the other, but, they actually have to have the same force to satisfy equilibrium, and that force you can intuitively guess is between 11 and 44, it turns out to be 20 point 5. So these are your reactions, after you get this what is your next step.

(( ))

What is your next step, well you can also see how it is moved.

We can find a elongations

D 1 is moved 2 mm, because that is given to you, D 3 has moved 1 mm it is given to you, and D 1 has moved 6 point 7 mm, so you can draw roughly a shape to scale to get an idea of the numbers.

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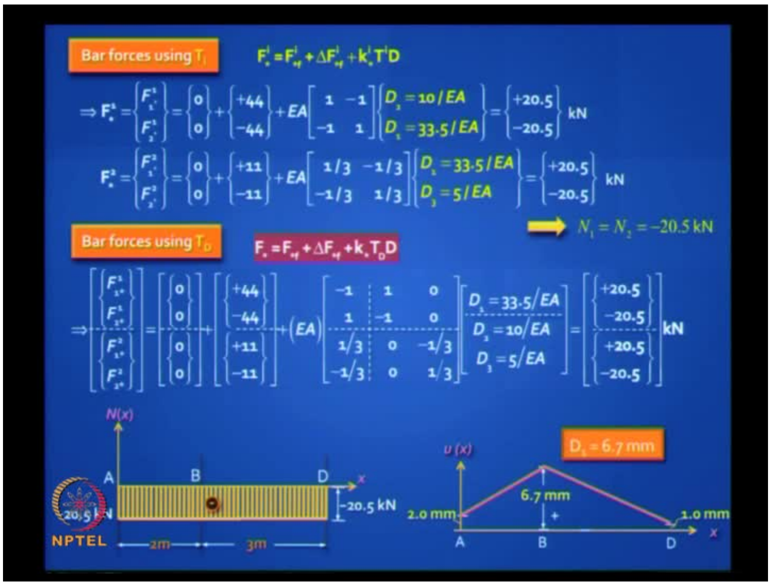

Then, we can find the bar forces either using the T i method or using the T D method, but you knew the answer any way, because you got the reactions, but, my (( )) go through this exercise you will get the same answers.

If you want you can plugged the variation axial force it is constant in both the elements is 20 point 5 and D 1 has a displacement of 6 point 7 mm D 2 has 2 mm D 3 has 1 mm it is a linear variation, is this clear, this problem is straight forward.

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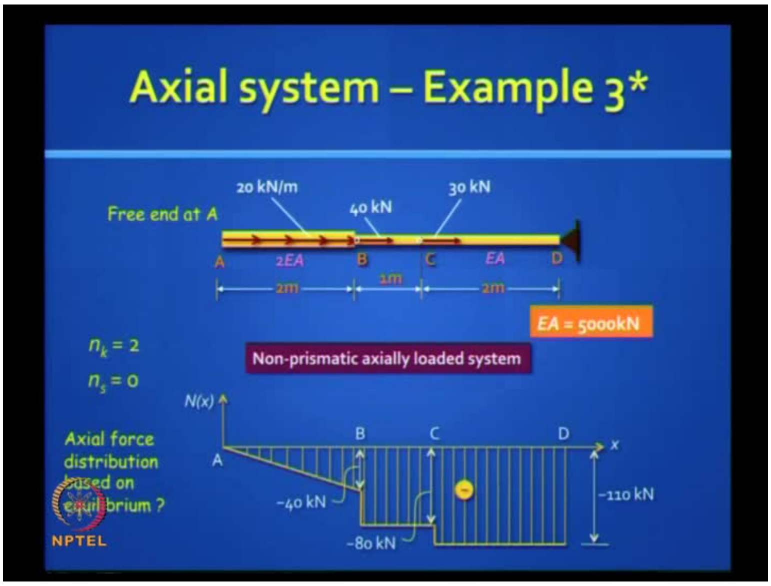

This is a problem when I put an asterisk it means it is a reading assignment, this problem is solved with the book; you can give it a short, what is a difference between this problem and the first problem.

You have a free end at A, so your active coordinates increase to 2, so degree of kinematic indeterminacy is 2, by the way what is a degree of static indeterminacy.

Under region

What's a degree of static indeterminacy give me a number.

Under region

You are saying it is unstable, but we are doing structural analysis, can you have equilibrium in unstable structures. Yes of course, you can.

So, we are asking to find the displacement end forces, may be you will be happy if i turn it to over by 90 degrees and make it vertical, so you have the feeling the whole thing is going to come down due to its self weight, but in this problem it is a weight less bar do not worry, so it is a suspended in air and you can still apply this load, so do not worry about that, for this loading do you have an answer, do you have a axial forces, can you find them out, sure you can, but it is statically determinant that I do not want you to forget your basics, can you quickly draw the axial force diagram, what will be the total reaction at D. Tell me the number.

110.

A 110 kilonewton..

Show me how the axial force varies from A to D, can you just draw it, because that is the end answer you get from stiffness method, so you can predict the answer in advance, degree of kinematic indeterminacy is two, but the degree of static indeterminacy is zeros, which means you can get the answer very easily, will you draw the diagram. Draw the axial force diagram, you are supposed to be experts with statically determinant structures, can you draw it, did you get this; good wonderful.

Now, the next question is in this stiffness method you will get displacements at A and B, definitely.

How do you get those displacements using the first method, with whatever little you learnt we did the review, how do you find the displacements.

(( ))

You apply.

(( ))

Unit load principle of virtual work, let us say I want the displacement at the free end at A.

(( )).

And what will be the axial force inside the system due to a unit load

Constant (( ))

So what is the formula for D A

(( ))

So, it is just the area of this diagram that we drew divide by E A.

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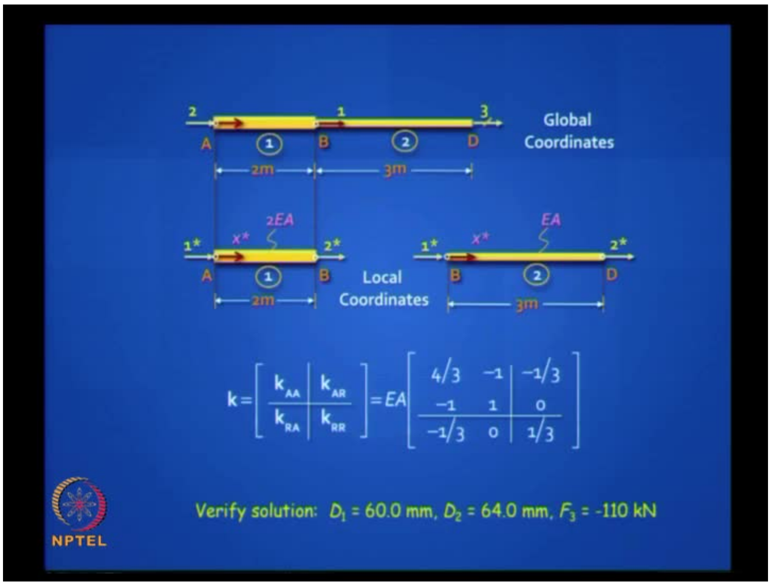

So, it should take you much time, you can check it out, but you have two degrees of freedom and I am not going to solve the problem, I am giving the answers check it out, the answers turn out to be 60 64 four just check it out.

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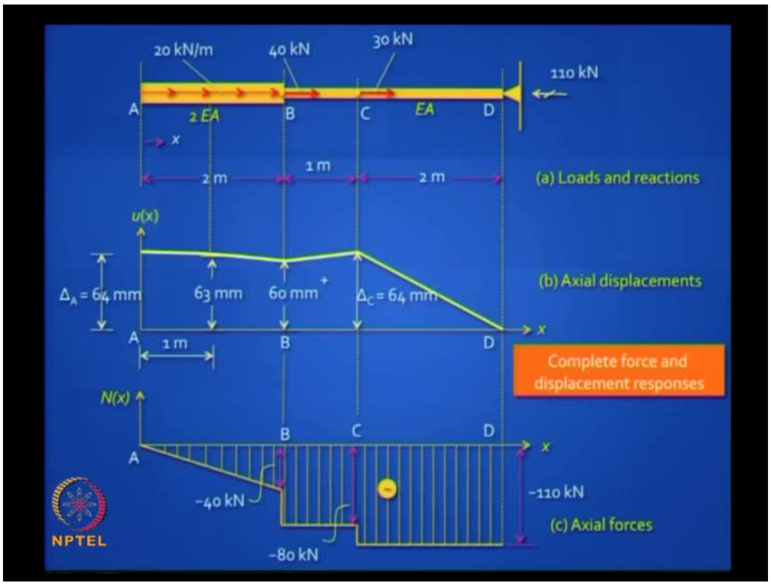

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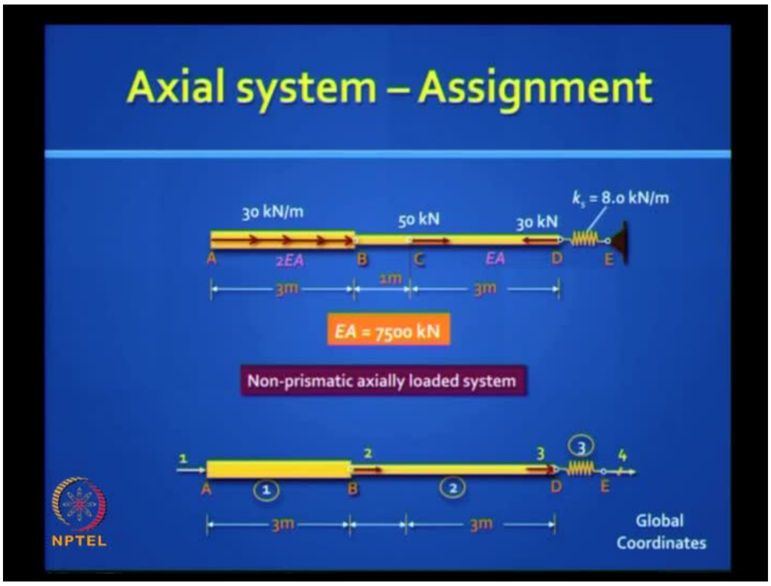

The problem that I have given you in the assignment is this one, this is the problem that you got in the assignment, it is a little more complicated, because now this is also statically determinate, so you can actually get the answers, the reaction is known

Here I put one spring at the end, will that complicate things, no do not tell me the unstable it may be, but I am asking you to find the forces, but what do you think will be, what is a complication, is there any complication. No actually you have been dealing with springs all the way the axial element is a spring element or the spring element can be interpreted as an axial element.

So, you will treat it as a third element, that is all, so choose these coordinates that I have indicated 1 2 3 and the 4 is restrained, and do this exercise, this is a problem subjected to some intermediate loads. Can you solve this problem, so go through the steps, do it both by T D and T i and you will be comfortable with this method of analysis. So, I think simple one degree, one dimensional systems you know how to deal with, can we move on to little more complicated problems we will take trusses.

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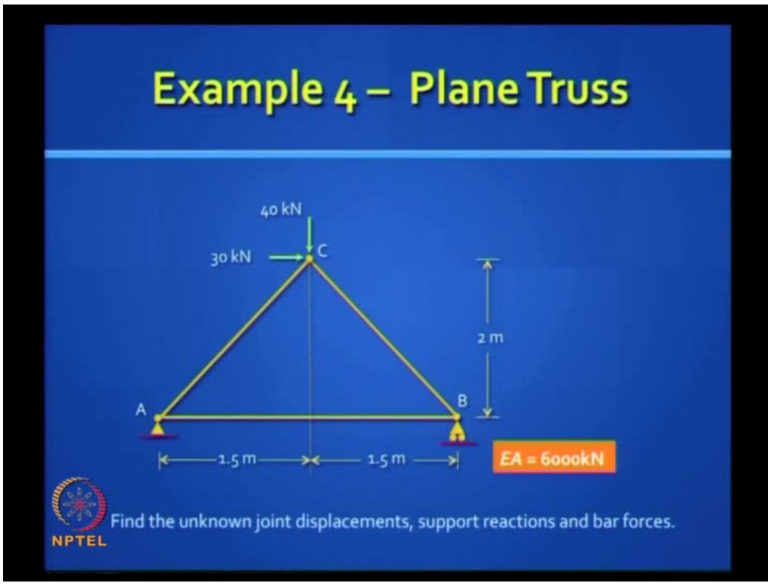

We will take a simple truss to begin with, if you have to do this problem which method would you normally do, no first what is the question to find the bar forces what method would you use.

We will use a force method why, because it is statically determinate you know the answersand you can also use the principle of virtual work the unit load method to get the displacement, but we do not want to do that method, we want to do it hard way, we want to do this stiffness method, because did you know that the computer will only do by this stiffness method.

Let's say you had a nice transmission line tower space structure, it is mostly statically determinate indeterminacy is very low, so the standard software will choose to solve hundred and twenty equations simultaneously, then do it by the force method, because you know that is how you programming.

So, this is a much smaller truss very easy to do, how will you handle a problem of this kind, find the joint displacements, support reactions and bar force, what is the first step well.

(( )) elements

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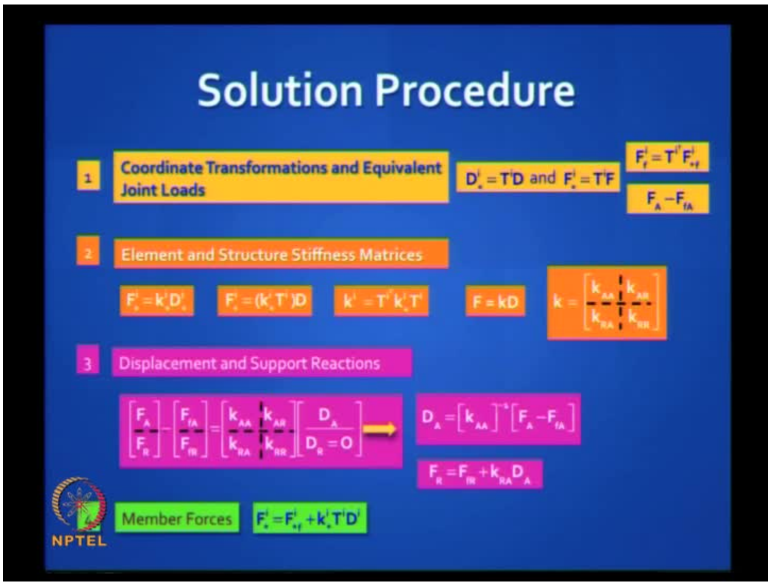

So, you go through procedure, find the coordinate transformations, next once you write the road map it is easy.

What is a next step the.

(( ))

The bandage of trusses is you do not get fixed end forces caused by direct actions, you will get the only caused by temperature changes or lack of fit, so say it is simpler what is your next step.

(( ))

You generate your stiffness matrices at this element level, at this structure level, what is the third step, apply the same equation so you will find that, you have really did this not much soon for innovation here, you are just playing around with the same set of equations find the bar forces.

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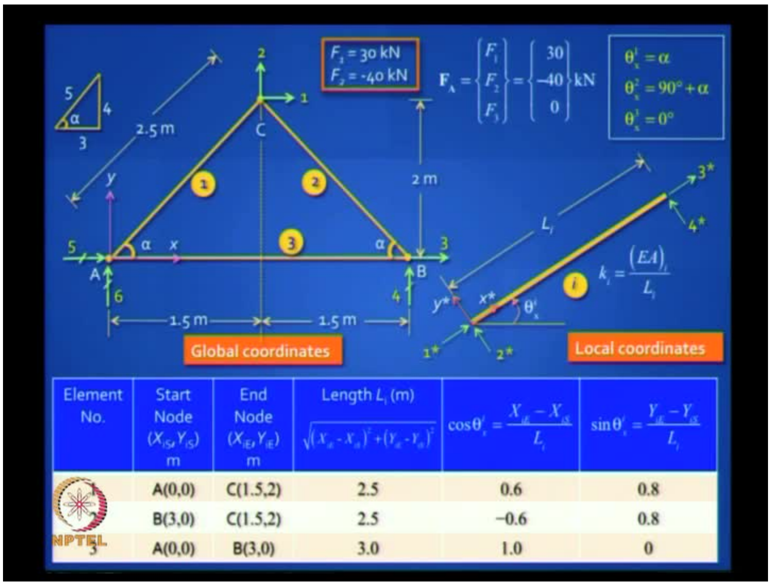

And you have the option of using either the T D or T i, let us choose the T i method and solve this problem, let us do it together it is a simple problem, but the steps of it you should understand.

You have three elements you can name them 1 2 and 3 and all the elements can have the same system of local coordinate, here how many degrees of freedom you have.

Four

Four, because it is a two dimensional system you have four degrees of freedom, we have discussed this earlier. Let us start numbering the global coordinates, where do you start.

(( )).

See you are right one two three those are my active coordinates, what is restrain.

How many restrain.

Three.

Three simply supported, so four five and six we can choose any one, the sequence is not important, usually after three you should put four near the three(( )) put it at B.

Now, what do you do, write down your T D matrix how would you do that, can you tell me what is a T D matrix for a inclined truss element.

Cos.

There you are, so your loads are given in this problem F 1 is 30 kilonewton F 2 is minus 40, so you can do it systematically handling as many members as you wish by writing down the element number, writing down the start node, writing the coordinates for the start node choosing some origin, usually the extreme left lower most location is treated as the origin, so you can write the look at the notations I have put start node X i s by Y i s for the start coordinate

X i e by Y i e for the end node, so your incidence is clear, your length can be computed using this equation, now I am doing it a generalize way you can do it for a any truss, and you can work out the cos and sin from the direction cosine.

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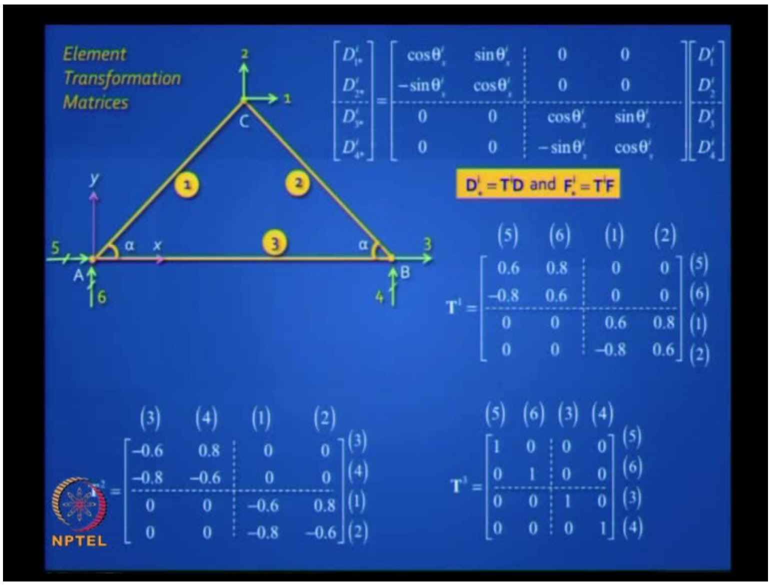

So, let us say give you bigger truss with 20 members, let the computer generate all these you just give it the table and it will do it in a minute, you have got this and using this you can generate your T D matrix as your rightly said this is how the T D matrix will look, then what, then you just generate it that is all.

So let us see the first element will look like this, because you already got the cosine and sin values from the previous stable, but now your job of putting the linking coordinates correctly starts, so you got this matrix by substituting cos theta and sin theta for the first element, but the first element is here and it is link to five six at the start node and one two at the end node, so you have to put those linking global coordinates here, we have discussed this earlier it is clear to you. Take the next one next one turns out to be like that, the linking coordinates are now for the second element, well it depends on which you chose as your start node.

So, let us go back, for the element two I could have chosen incidents from C to B, but in this case I chose from B to C, do you understand, I do not have to break my head and calculate theta A x

This will give me directly cos theta and sin theta from this formula, that is a beauty of this method, I get it automatically I do not have any difficulty, so any questions here, it is clean.

I do not have to manually calculate theta, it comes automatically when I give the coordinates X and Y of the start node and end node from these formulas which are standard formulas, I get automatically the direction cosine, and so it is a blind method no thinking, computer loves such method just all you need to write the algorithm, but do not forget you are linking coordinates, and when you program it, you must write down a root subroutine, which can handle this nicely, so that is why we got three four one two, and for the third element it starts here, the end is there, so it is five six three four.

So can you generate these matrices very easy what is your next step.

Element stiffness (( ))

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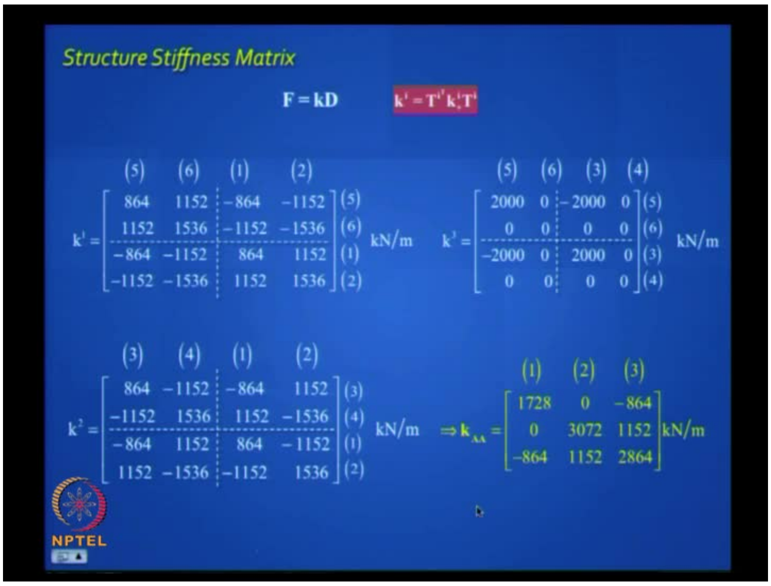

You generate the element and structure stiffness matrices; we turn this many times, the second row, the fourth row, the second column, fourth column will be 0 why, because they relate to shear forces which are 0 and there is no in independence between the first and third also, because one is the negative of the other.

The rank of this matrix is 1, so you can generate this, all you need is the axial stiffness of a three elements, so E A is constant for all of them only the length changes, for the first and second elements the length is two and half meters, and for the third element it is three meters.

So, 6000 divided by 2 point 5, and 6000 divided by 3. Now, what, now you can generate k 1 star and k 2 star will be identical, because they are symmetric and identical, k 3 star will be different, because the A by l values stiff you got them.

Now what do you do.

(( ))

Now you have to do this slotting business, so can you do it.

Yes sir.

Yes, we went through this exercise, you can do it; you first calculate k i star T i and put the linking coordinates, and put the units also its all stiffness is a kilonewton per meter.

You can do this, you got T i for every element, you got k i star for every element; can you not do the product, can you not put the linking coordinates, what is your next step, you premultiply this with.

Transpose

T i transpose, you do that and you get this, you get this, from this how do I generate my final answer, so what is a size of my structures stiffness matrix 6 by 6, 3 active degrees of freedom 3 restrained degrees of freedom.

So, let us just pull out k A A, how do I do that so I will show you step wise, look for where you have one and one, so you got one and one coming here, and one and one coming here, so pick up those values and add it up, so you have one and one here 864 and 864 will add up to 1728.

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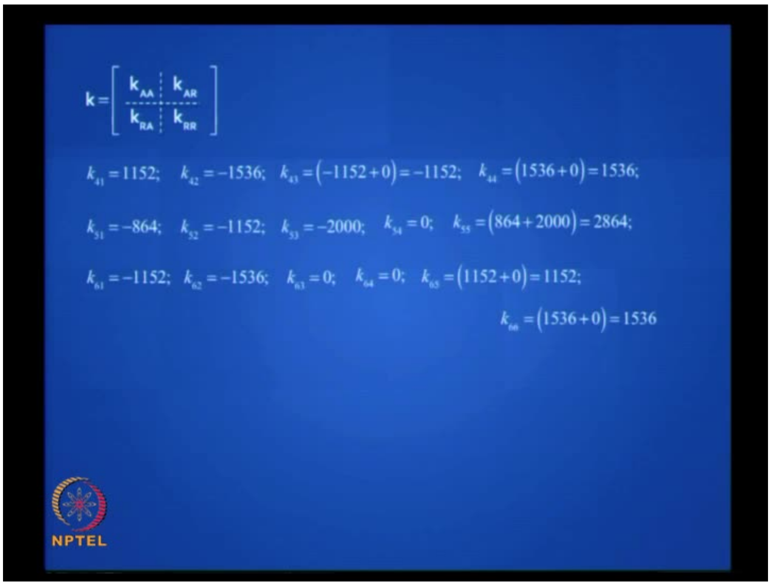

So, can you do thispicking up this slotting business we discusses earlier, and that is how you fill up it that matrix, and likewise this is what do you do in the first step, this is what do you do in the second, you can fill up all of them and then generate this matrix.

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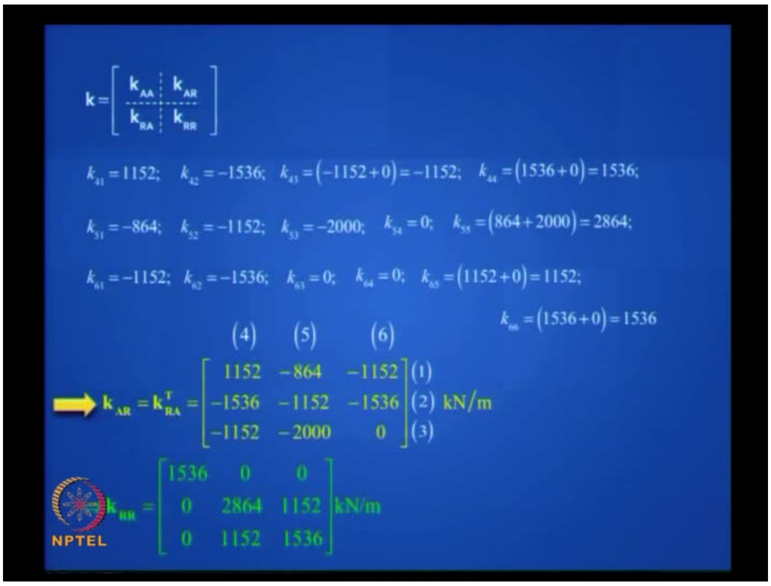

Similarly, you can generate all the others, and it is a symmetric matrix, so you need to find only one of diagonal term, and you got everything. Now, forget this loading we are ready to handle any loading on this structure including support settlements and temperature rise, lack of fit you are ready, and the moment you input the geometry of this truss the computer will generate this matrix and say give me the load and I will give you the results it is waiting.

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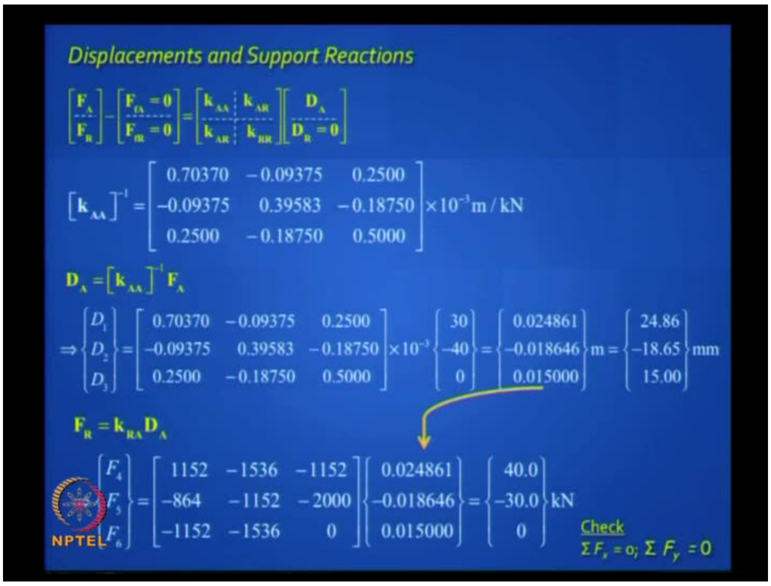

So, how do you do that in this particular problem, you have no fixed end forces, D R is 0, there is no supports settlements, so it is a straight forward calculation.

You can find inverse of your three by three k A A matrix, and mind you the units will be meter per kilonewton now, because it is a flexibility once you inverted it, then multiplied by the load vector you will get the answers those are here three displacements, then what.

Once you found D A, what do you do next.

(( )) not in terms forces complete you need to get the support reactions from the second equation, you got two equations here, after you get this apply that in the second equation find F R and you know those answers, because it's simple answers it is 40 and minus 30 that is statically determined then you find the bar forces.

You can actually check out sigma F X equal 0 sigma F Y at this stage which you should as an engineer, make sure equilibrium is satisfied.

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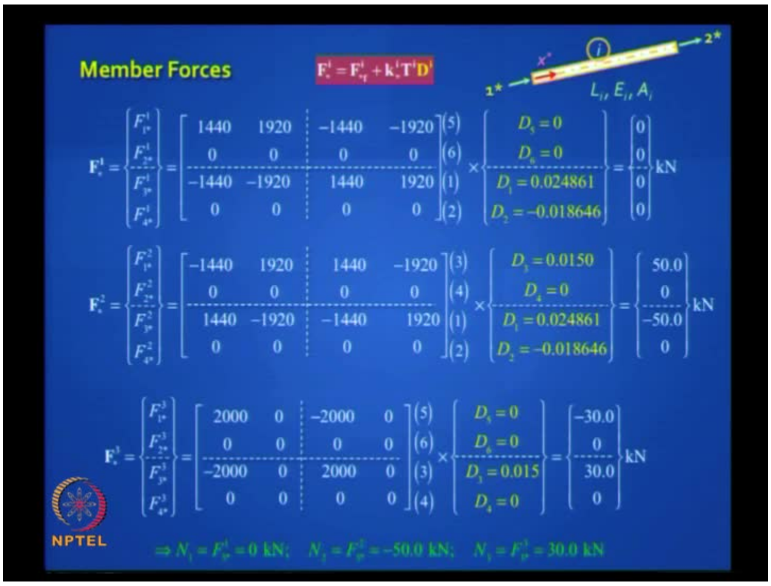

And, then what find the bar forces, how do you find the bar forces, there is no fixed end forces so T i transpose k i, here you have to be careful, because from that large displacement vector what is a size of a displacements vector 6 by 1.

You have to select only those four which pertain to the first element, so that is why I put a yellow colour here to remind you that you have do it carefully, so you take out those particular values 5612 from your D A D R and then plug that in, and low and the whole get zeros, because for this particular problem, for the loading you do not have any force in the first element. It is just an accident, because the loading was such. Second element you get, so what do you conclude from the second solution is it tension or compression, it is compression; because F 3 star is the axial force it is on the right side.

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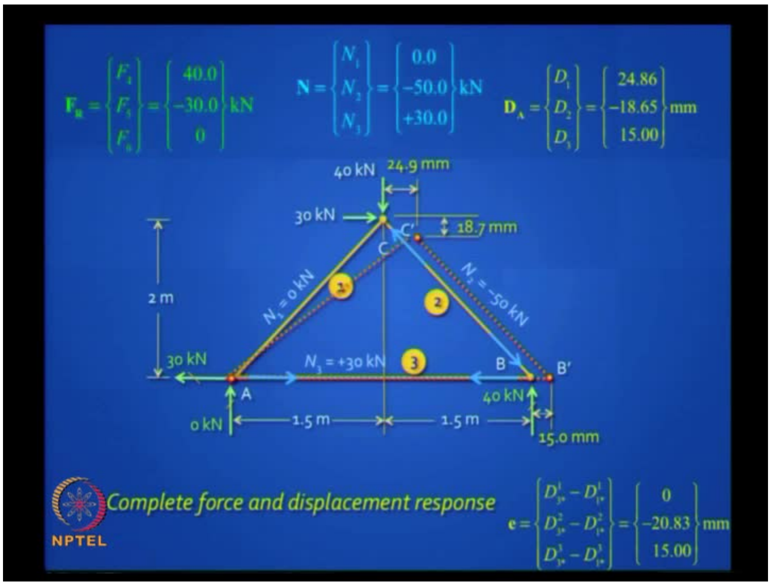

And see your shear force you always zero, you could predict in advance and the other one is plus 30 kilonewton, so that is how you interpret the axial forces, and you are ready your complete response is shown here, you can draw your reactions, axial forces you can mark, even displacements you can mark, you know the joint displacements you got you note D 1, D 2, D 3 write to some scale and you can even sketch this.

And, if you really interested you can also get the member bar elongations as E i is D 3 star minus D 1 star, if you really (()), you got everything you have got the complete force response, and the complete displacement responsefor this loading.

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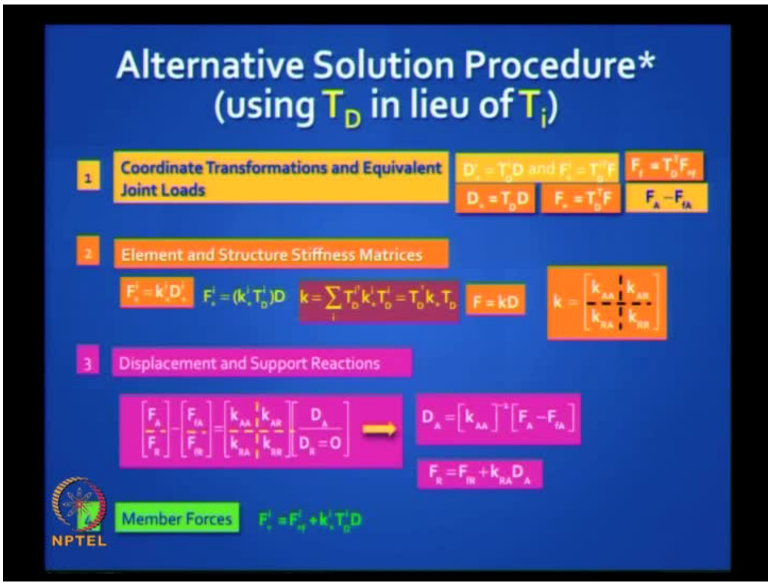

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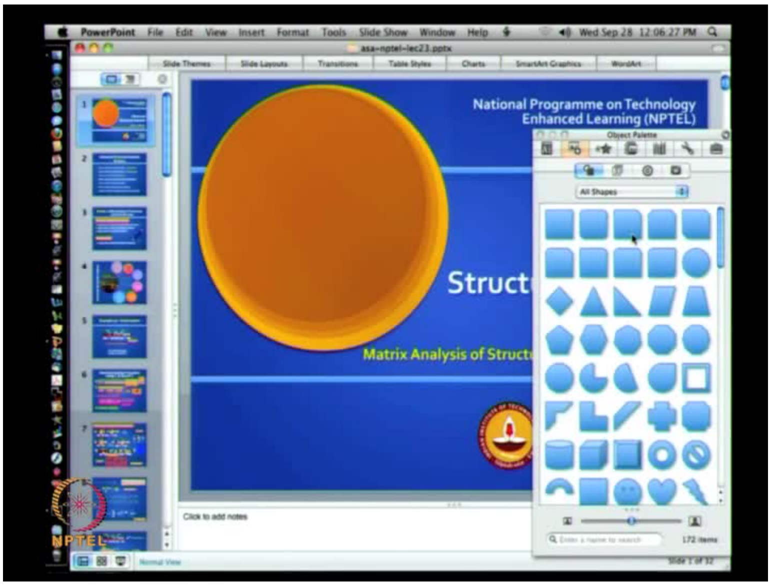

So, can I assume that you really understood, you can do the same thing with T D, can I assume that you know conventional stiffness method, please do that example problem, next class we look at the reduce elements stiffness method.

Thank you.