Advanced Structural Analysis
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Module No. # 4.1
Lecture No. # 22
Matrix Analysis of Structures with Axial Elements

Good morning to you. This is lecture number twenty two in a new module, the fourth module of this course on advanced structural analysis

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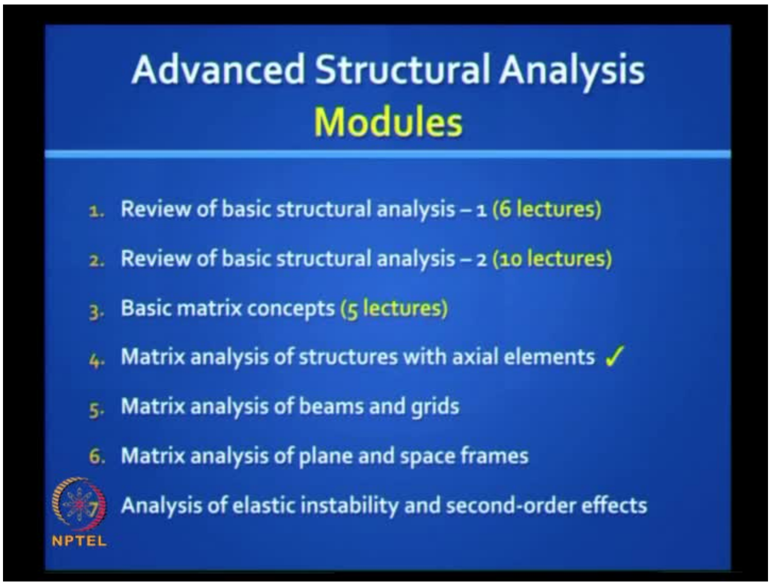

With this module we apply matrix analysis of structures to problems with axial elements. If you see we have finished twenty one lectures. First three modules have been completed.

The third module which we did in the last five lectures, gave you the big picture view, but I am sure you would not have fully understood. That understanding will sink in as you start applying these concepts to actual problems. So, in the next three modules we will be applying the concepts we had broadly shown, in this particular module we will deal with the class of structures commonly referred to as pin jointed frames, so they are usually also called trusses; you have space trusses, you have plane trusses and you occasionally encounter situations which are one dimensional, the one dimensional problem you have something additional which you do not encounter in plane and space trusses in terms of loading, what could be that.

No, in plane and space trusses which have pin jointed frames, the loads are assumed to be applied always at the joints, so you have nodal loads, but in axial elements; you can have intermediate loading, you can have distributed loading, so then you have to bring in fixed end forces and so on. So, we will look at that category also which normally is not studied, so that is what we are going to do in this module.

In the next module, module number five, we will look at beams and the space structures linked with beams in a horizontal frame work called grids, where you have torsion also. In the next module, we will look at the larger type of structures, which involve rigid jointed plane frames and space frames, that is the most complex and we will also include issues such as, how to account for internal hinges, how to account for even axial deformations, shear deformations and so on.

So we will build it up slowly, we will begin with the first level, the easiest matrix analysis of structures with axial elements.

Now, axial elements can go in to 1 D structure, 2 D structures and 3 D structures. In 1 D structure and 2 D structures we will demonstrate application of the conventional stiffness method, the reduced stiffness method and the flexibility method, all three we had covered in the third module, and we will also see how these methods can be used to space trusses which are little complicated, but in terms of actually solving problems, we will apply only the reduced stiffness method.

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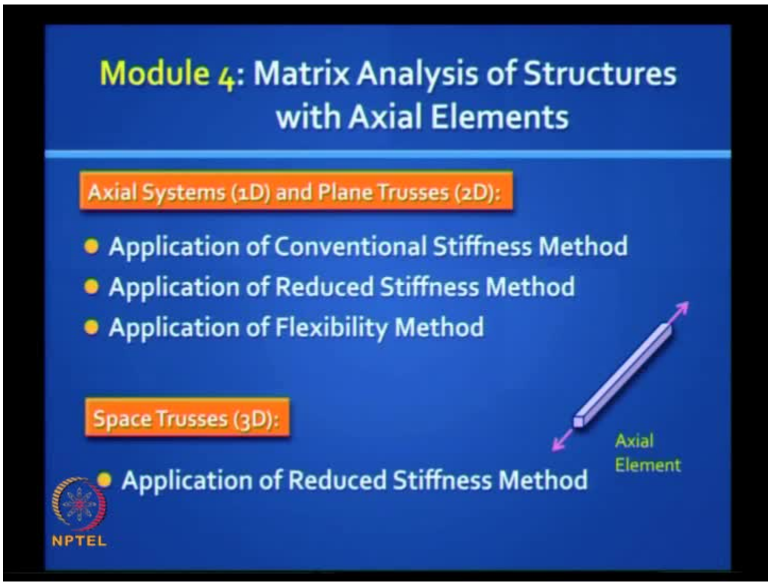

So, in all these elements, regardless of whether the structures 1 D 2 D or 3 D, remember that element can take only axial forces, and that simplifies the analysis.

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In this session we will look at conventional stiffness method, all this is covered in this chapter on structures with axial elements in the book that I wrote called advanced structural analysis, for a full understanding you must necessarily prefer to the book, because these subjects, these topics take time to sink in and you need to go back in refresh your understanding.

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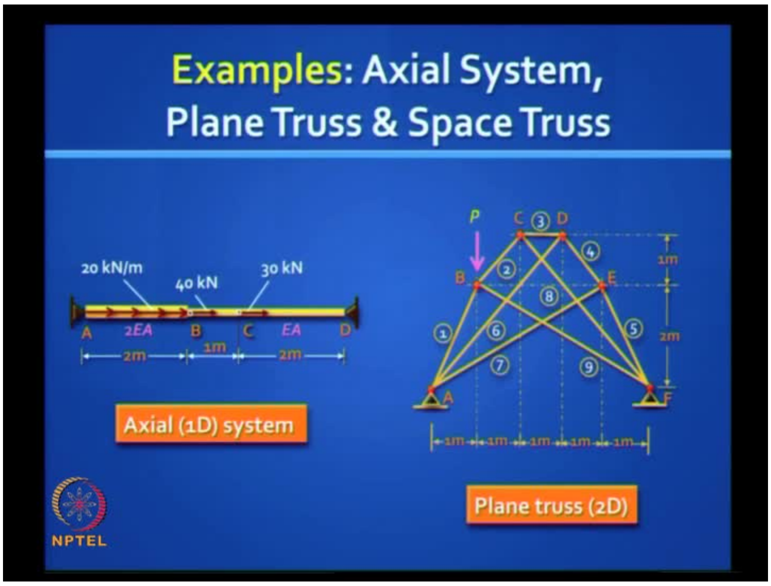

What is the type of problems that we encounter, so I will just show you three types of problems; this is one example of a kind of problem you will encounter with 1 D systems.

You can have two elements or three elements or any number of elements, you can have distributed loads, you can have intermediate loads, that is one category, second category is a plane truss.

All this can be done quite easily using matrix methods and we will demonstrate step by step, I am not asking you to write a generalized program which can be use by others, but let us just do it step by step manually, and when we encounter matrix operations; such as transposition, inversion, solving simultaneous equations, we will use some standard software like mat lab, so that is the idea.

We are those of you are having an interest in programming you can take it a step further in your project and write programs in whichever language you wish, but that is a you know latest step. Right now, we will do manually step by step the process of matrix analysis.

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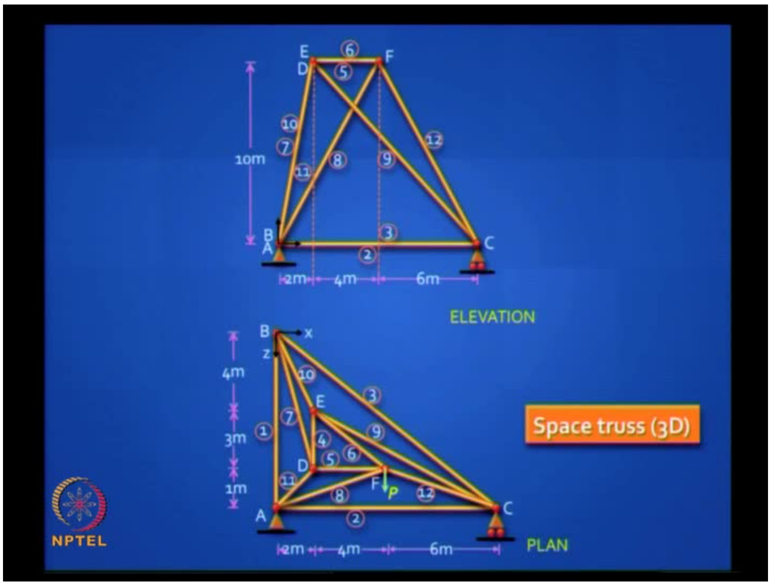

Finally, we look at complicated structures like this, it is a space truss I have shown the plan and elevation, large number of elements and how to make the digital computer solve this problem, is very interesting, so this is the last thing we will attempt in this module.

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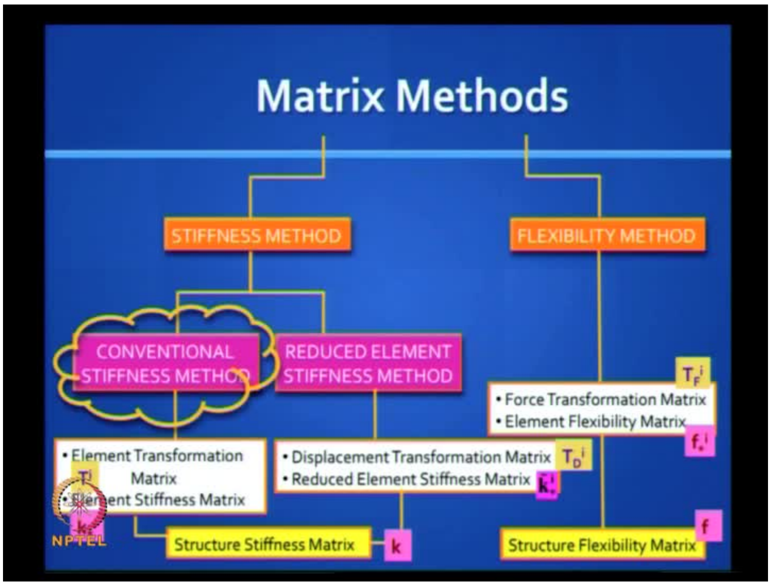

So you have seen this picture earlier the three types of methods that we will use, two of them is our stiffness methods. We will now look at the conventional stiffness method, later we look at the reduced element stiffness method, and finally we look at the flexibility method.

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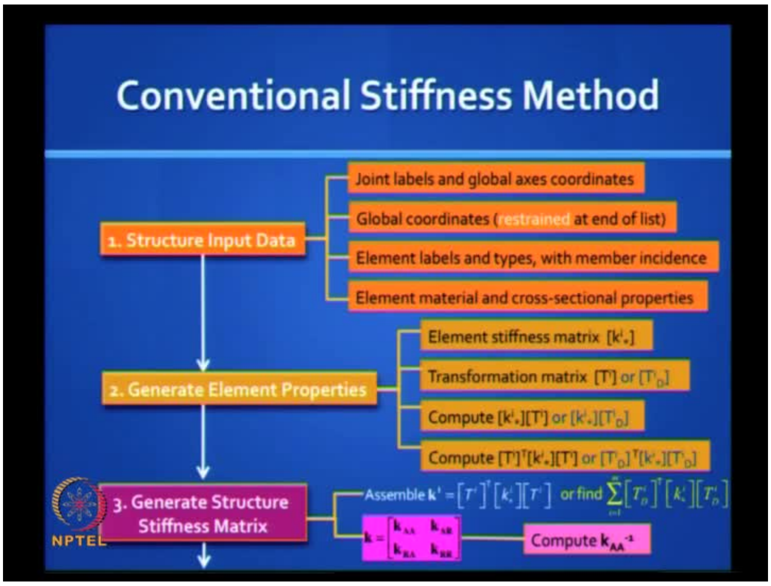

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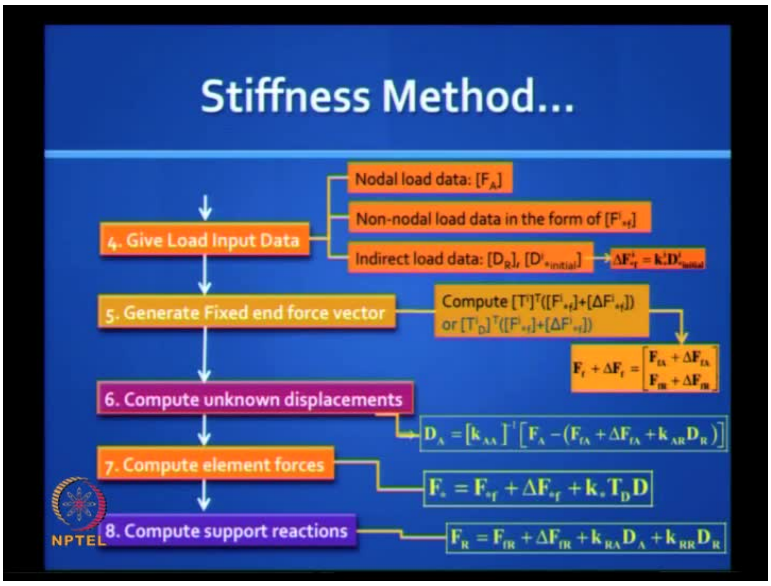

In the conventional stiffness method, you know that we have to go through different steps, when you want to program it, first you have to input the structure data, then you have to generate the element properties, then you must generate the structure stiffness matrix from the elements stiffness matrices, then you must give the load input data, generate the fixed end force vector if required, then you compute the unknown displacements, you compute the member element level forces and compute the support reactions, all this can be done through matrix analysis.

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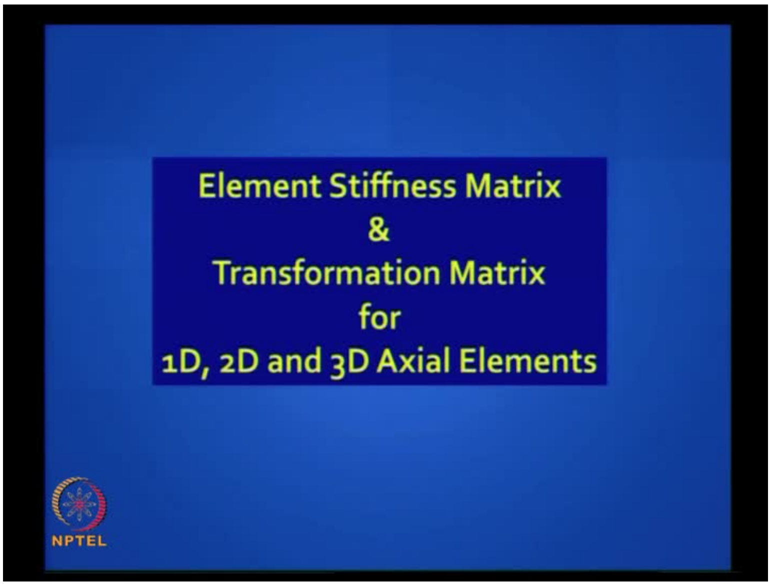

So, now let us see how to do this, so we will go back to refreshing or understanding of how to generate the element stiffness matrix and the transformation matrix in the case of 1 D 2 D and 3 D axial structures using axial elements.

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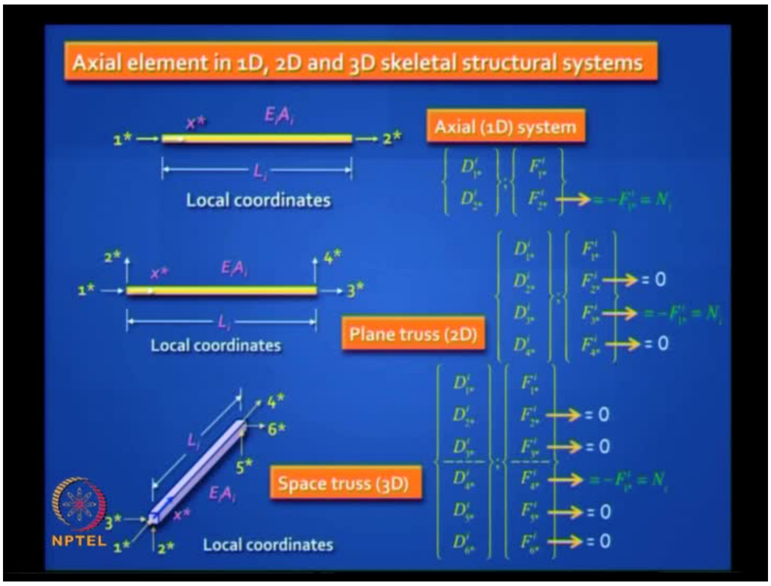

So this is a typical picture of a 1 D axial system, an element used in a 1 D axial system, you agreed that there only two degrees of freedom here, we have not looked at this picture in the earlier module.

You have two degrees of freedom, they are both axial degrees of freedom, the left node can move and we will call that one star, and the right mode can move we call that two star. So, this is the element displacement vector, this is the element force vector. Are they really independent, the two components of the vector, In D they are independent, D 1 star and D 2 star need have no relationship with each other but, F they are not.

Can we say something more .

They are interdependent.

They are interdependent and actually they are equal and opposite to each other, and they are equal to the internal axial force in that member, is this clear.

Next we apply it to plane trusses, so this is the plane truss element, here you have four degrees of freedom we have seen this in the last module, and you are familiar with this, here again the displacement vector has independent component, that is how we identify the degrees of freedom, but the force components are again inter related, can you say something about those F2 star and F4 star are zero, why are they zero.

So, what do those force components refer to, shear forces, so you do not assume that axial elements have no resistance to shear, so they are zero and equilibrium demands that F 3 star should be equal and opposite to F 1 star.

Finally, we deal with the space truss element, here again you can prove that the shear components are all zero, and the axial force at the two extreme ends are equal and opposite, and that force is a single anode. This is why the reduced elements stiffness method is so easy to apply, because you do not deal with so many different quantities, you deal with one axial deformation in an element and one internal force in the element, but here this is the way it is done in the conventional stiffness method.

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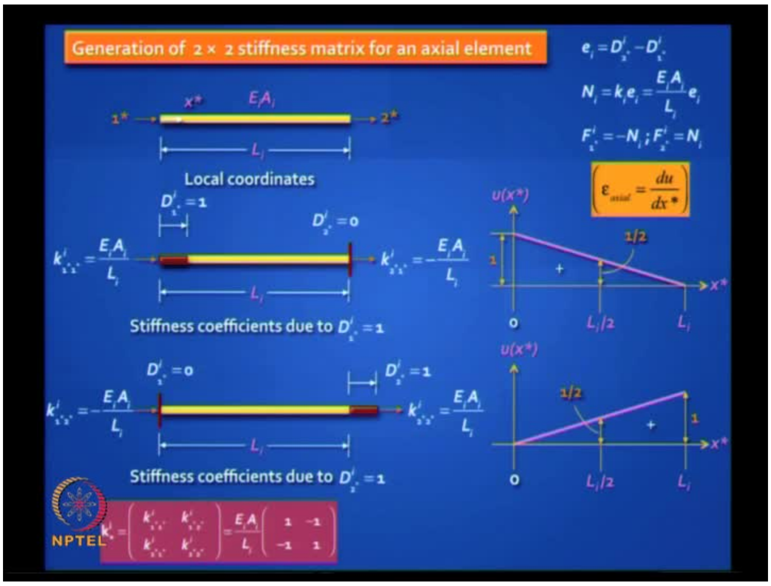

Now, let us generate quickly the stiffness matrix and the transformation matrix for these three types of elements, for this element you know that the elongation e i is a difference between the two displacements, and is actually in terms of tensile elongation its D 2 star minus D 1 star, you agree.

The if the elongations known and its elastic behavior, then the axial force is related to that elongation, and that is simply the axial stiffness E A by L times the elongation, and this must be equal to the end forces, so do you see those relationship, they are well known. One more thing; we have said that there only two axial degrees of freedom, what about displacements within the element, let us say the displacement at one end is D 1 star, at the other end its D 2 star, at the middle of the element what will the displacement be.

zero

D 1 star plus D 2 star by 2

D 1 star plus, so you can do linear interpolation. What is the basis for doing linear interpolation, is it an assumption or is it exact, can you explain further, why not quadratic, you can still have linear elastic behavior with. So we are talking of the axial deformation u of x, axial displacement u of x.

If we take a section then the force will still be equal, and it will have N 1 x N 1 of x which is still F 1 so for that.

We are talking of deformations not forces, [its modeless same]

N is equal to k e so it is a linear relation

That is talking about the overall behavior of the element, we are talking of a point say

That depending in any section and find a displacements

We can call u of x, because it is in the x.

Ok so that will be proportional to analyze.

Why will it be proportional.

Because, it is linearly elastic.

No, that is not the answer.

It is proportional directly e i, because it was,

You are right, but you are not getting the name N x is k e of x.

Those, are all integrated over the entire link, we are now taking a sections somewhere finding how much such that point move.

The clue is given in what I have shown here, I said the u is related to strain, the axial strain at any location is d u by d x star, and if the axial force is constant, at all values of the length axial, then you bring in hoop's law, because force is directly related to stress, stress is directly related to strain. So, force is directly proportional, the internal force, the axial force is directly proportional to the axial displacement that is the relationship, is it clear.

So, you will find that you have a linear variation of the displacement when you integrated, strain is constant, so axial displacement will vary linearly, and you can show it, for example, take this example if you apply D 1 star equal to 1, which means you push the left end by a unit quantity, you will need to apply a force and that force will be equal to the axial stiffness, it will be positive, it will be E A by L.

You will develop a reaction at the other end equal and opposite, so k 2 star 1 star will be minus E A by L, is this clear this is obvious. What, about the deformations, how will the axial displacement vary, it will vary like this and this is exact. I am now plotting u of x, u of x star which means any point here I can interpolate the displacement from this end and this end, it will be maximum here and its value equal to unity, because it is going to be equal to this quantity, and here I prevented the moments of this linear, its linear because of this relationship, and not because of any other reasons.

Likewise, if now I pull the other end and arrest the first end, I will get another set of relationships and the displacements will look like this.

So, this give me a good picture and this is a reason why we limit the degrees of freedom to the ends of the element, because we know everything, we know the full displacement field.

And what will be the stiffness matrix here, it is a two by two matrix, the element stiffness matrix, what are its components, it is simple E A by L, so there are no zeros here, it is a full matrix, but it is symmetric, all stiffness make sure that it seems there is no but there, you know all stiffness matrices are symmetric and square, no question, but there is something special about this.

It is singular, why is it singular, that is another way of singular, she says the determinant is equal to zero that is right, why is the determinant is equal to zero, because the rank of the matrix is only one, why is the rank only one, because the force are not independent, because this element cannot resist any load unless you arrest some degree of freedom, you catch hold the one end and then pull the other end, then the system is stable. So, there are many ways of understanding what is mathematically apparent.

Now, the next question is, for such a problem using the conventional stiffness method, you have the local axis system and you have the global axis system, remember what we did earlier, so what will be the nature of the transformation matrix T I, remember T i.

It’s going to be a two by two matrix, what will be the special quality about that matrix. Do you remember the T i matrix.

Cos theta and sin theta

But, that is the general formula, here in this simple example what is the.

It’s an identity matrix, why is it an identity matrix, because x star and x are the same in this problem, theta is zero. So, these are the easiest kind of problems you will get, you do not have any inclination theta x as you would get in a truss or in a frame, so in the line element, in a system where you’ve line element, the problems are very easy, transformation matrix is always identity matrix, so its simplifies your calculations tremendously.

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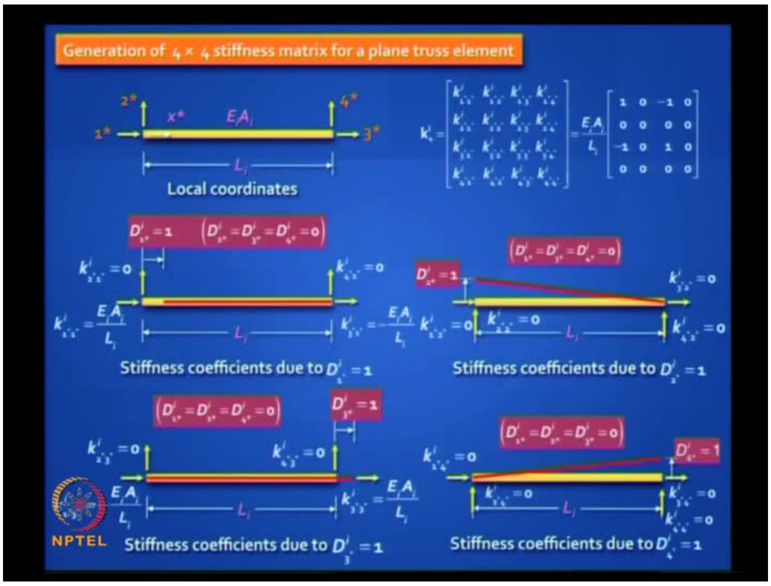

Now, let us look at the 4 by 4, we have done this earlier, so can I go fast on this, you know the stiffness matrix, and I think we did this in the last sessions, so any questions on this.

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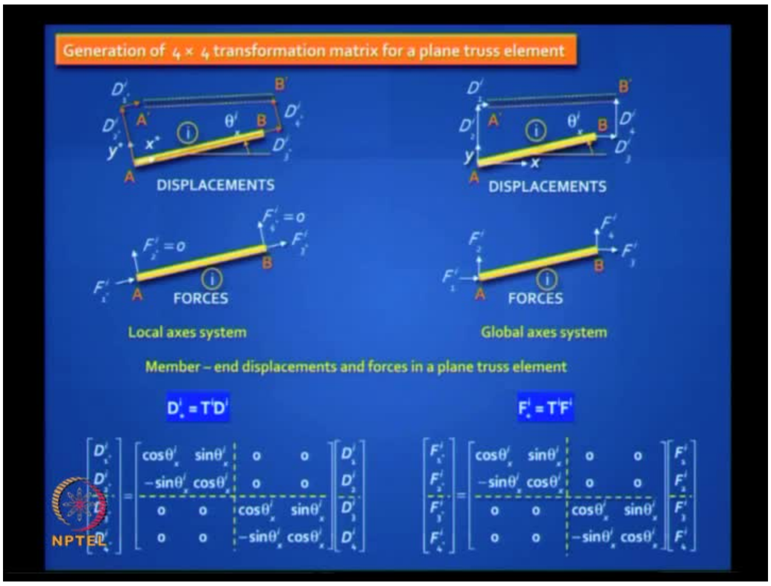

But, let us refresh our memory on the transformation matrix, you got a four by four stiffness matrix, in that stiffness matrix remember there were zero columns and zero rows, the second and fourth rows and columns were zero, because they correspond to the shear forces, they are rigid body displacements.

But, in this, you remember this, here what is a nature of the transformation matrix, here you will have theta coming into play, what is it going to be, you will have a negative sign somewhere.

Minus sign.

There you are, so these are the relationships, you will get a minus sign in this second term, we have done this, so shall I go ahead.

These are the tools with which you can do matrix analysis, once you have the tools ready you can do it.

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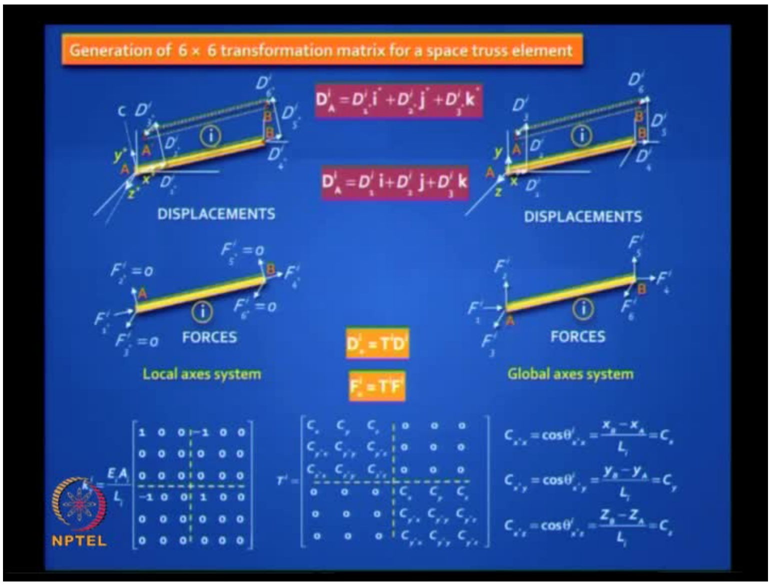

Lastly, and this is the most difficult of the lot, if you have a 6 by 6 space truss element, how would you do the transformation matrix.

Column should be 2 3 5 6

No, this is really complicated, because you have an element in space, you have x y z and you have x star y star z star, so what is your direction cosine matrix look like, how many numbers do you need to deal with. Is it just theta x theta y theta z. No, because you need to project one on all the other three, so you need to know it at least in principle, because this needs a lot of thinking and clarification, I want you to read the book for this if you are interested, but to show you what you are likely, you have to use vector notation and it works out like this.

The stiffness matrix is very easy to do, because you have only E A by L and mostly they have full of zeros, but the transformation matrix will involve many angles x star with x, x star with y, x star with z, y star with x, y star with y, y star with z, and z star with x, z star with y, z star with z, so it is a little messy, but that is why you need to program this more carefully when you do it, but this is little advanced and will keep it outside the scope of what you need to know, but in principle you should know what needs to be done, and this is where the orientation of your, let us say i section makes a lot of difference, whether you are pointing this way or the other way it will all have to show up in these, because you can get everything wrong if this matrix is not well done.

So, this is little tough, but you will see, this reduces to such a simple thing when you do it by the reduced element stiffness method. So, we will actually do problems only by the reduced elements stiffness method when we deal with space frames.

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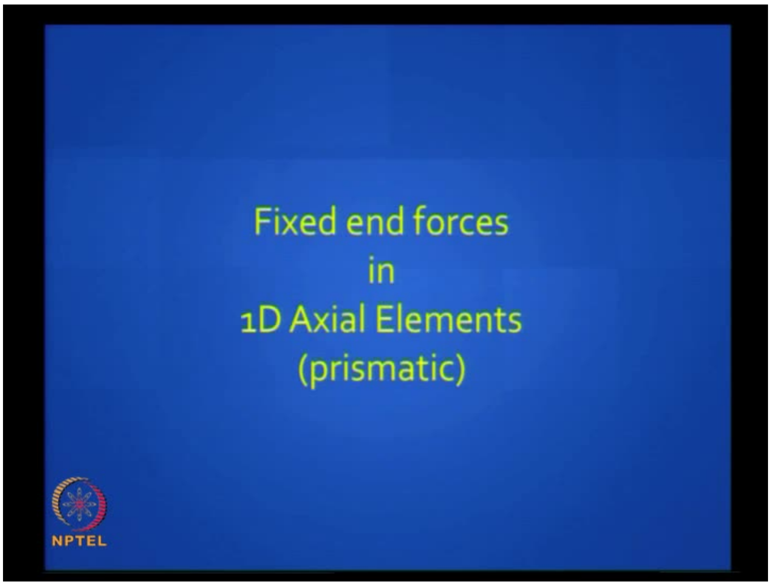

But, if you want to do a generalize program which anybody can use, then you have to necessarily look into these direction cosines, theory is over.

Now, we get in to problem, but before we do the problems, you will encounter intermediate loads in one dimensional axial system, so let us quickly finish that and then will be ready.

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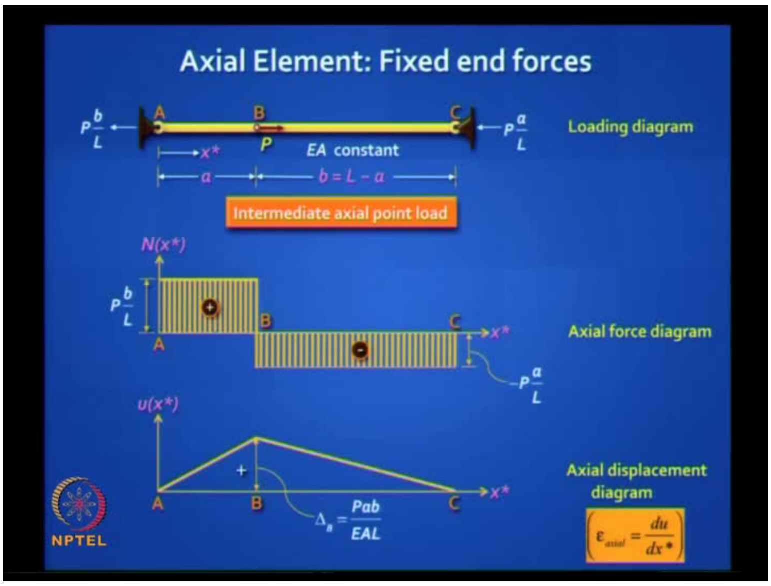

So, let us say I have an axial element with an intermediate load P, located at distance A B. If I have a situation like this, one of the options I have is I split in two elements, so it is still becomes a nodal load, but that is an extra effort. If i still want to manage with one element, then I must have an equivalent joint load, I must pass on this load P to the two ends those are the fixed end forces if you wish.

What do we mean by fixed end forces, you are familiar with fixed end moments in beam, there you arrested the rotational degrees of freedom, here you arrest the translation degrees of freedom, and you if you pull it there you will get some reactions at these two ends. Can you guess what those reactions will be?

AB by A + B

At, the left end .

P b by L.

Some fraction of P will go on to the end A, some fraction will go on to the end C, which will be more in this case if A is less than B.

Left end, it will be more.

Left end, it will be more, right end, it will be less.

Will it be kind of linearly vary, as you would in a simple supported beam with the considered load, how do you prove it, you are right this is not statically determinate, you cannot do it like in a simply support beam, how do you prove it.

.Elongation of AB is equal to shortening of BC

Elongation, that is well said, elongation of A B will be equal to shortening of B C, so how does this prove.

Mathematically

In terms of stiffness zest, which element is stiffer A B or B C; A B, and the stiffness is add up at B, you can prove it either by the force method or the displacement method. Incidentally at the end of the last class, one of you asked the questions remember we try to compare the flexibility method with the stiffness method, and the first point we wrote was, what are the fundamental unknowns. And we said the fundamental unknown in the stiffness method is displacement, because the stiffness method is also called a displacement method. You have to first find displacements then only you will get, you may be interested only in the internal forces, but you cannot find the internal forces until you find D A, so your fundamental unknowns are displacements in the stiffness method

In fact there are other names for the stiffness methods, stiffness method is called displacement method, it is also called equilibrium method, because you use equilibrium equations to find the unknown displacements. Likewise, the unknowns, well everything is unknown in any of these methods, but the primary unknowns, in the force method really you are, you may not be interested in finding any displacements, but you are forced to find displacements, because you have to bring in comparability equation.

So unknowns are redundants and the redundants are forces. So you will find the other names for the flexibility method are force method and compatibility method, because compatibility is a governing equation.

Lets come back here, you can prove iti leave the proof to you, you guessed it correct P b by L and P a by L and if you want to plot the axial force distribution as one of you rightly said A B will go into tension, which I show positive, and B C will go in to compression.

So, this is about we call an axial force diagram, and if you want to plot the deflections, the displacements how will it look.

Triangle kind of.

Perfect, you get a triangle and you can prove it, if you know the total stiffness of the joint, you can prove how much it will move.

Why will it be a triangle, you are right, would you guess it or,

Because a B moves more and,

No, established a relationship.

A and C are fixed,

What is the relation we discussed a while back, it is all depends on the strain; the axial displacement comes by integrating the strain. The strain is directly proportion to the internal force end.

So, if the internal force is constant, the displacement vary linearly which is what we have done, there is no shear force and bending moment here, like that shear force in bending moment, is it clear.

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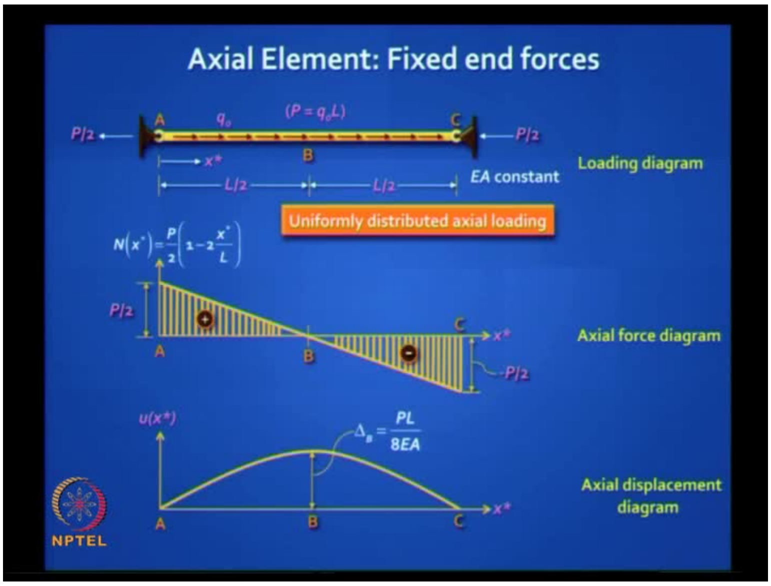

So, that is the argument, and you can prove that it is going to be P a b by E A L, with this you knows everything about the fixed end force here. The other case you should look at is, if you have A U D L, what'll be the reactions at the two ends, very easy, half of the total load P. So its P by 2, P by 2, what will be the variation of axial force, it will be linearly varying not like the previous case, and how do you think the deflections will vary; parabolically, and you can prove that the deflection of the middle is P L by 8 E A, is this easy for you, you can also remember the formulas for deflection, that is all you need to know. Of course you can have all kinds of intermediate loads, but let us limit to these two simple cases, store it in your memory and be ready to handle problems.

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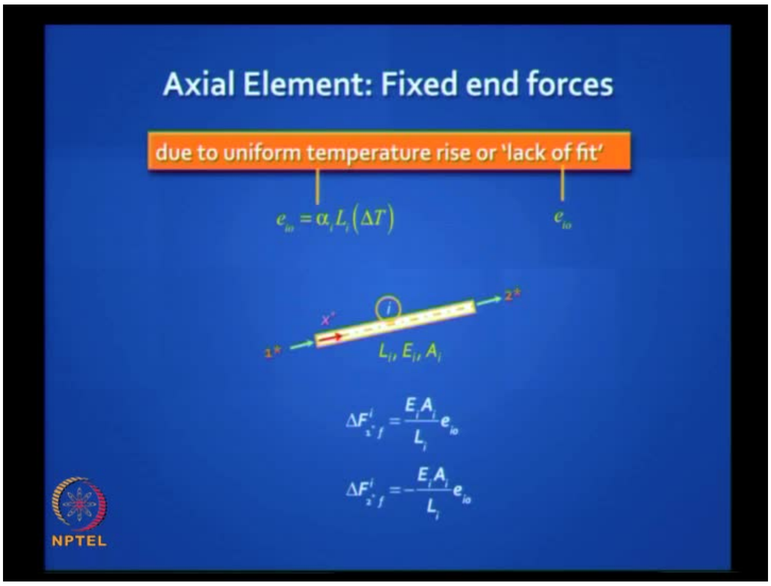

There is one more type of fixed end force you can get in stiffness method, due to environmental effects, so it could be either cause by change in temperature along uniform temperature along the whole bar, you should not heat locally in one point in the bar. Let us say the entire temperature in the bar is raised by 20 degree celsius or cooled, that is delta t

So, we use a notation e i naught to refer to the change you would get under free condition, that mean no restraint and that is given by the coefficient of thermal expansion alpha times, the length times, a change delta T clear.

Or you could get e i naught directly in a lack of heat problem, let us say a bar is given to be too long by 5 mm, and e i naught will be plus 5 mm, is it clear.

So, here again in the primary structure when you prevent all the movements, you will get an internal force that'll be a fixed end force, it is a constant force throughout the length of the member, what will it be, axial stiffness times, so I hope you are familiar with this.

Please remember one point e i, naught, you have to judge whether it is tension or compression and put the sign appropriately, we will demonstrate this.

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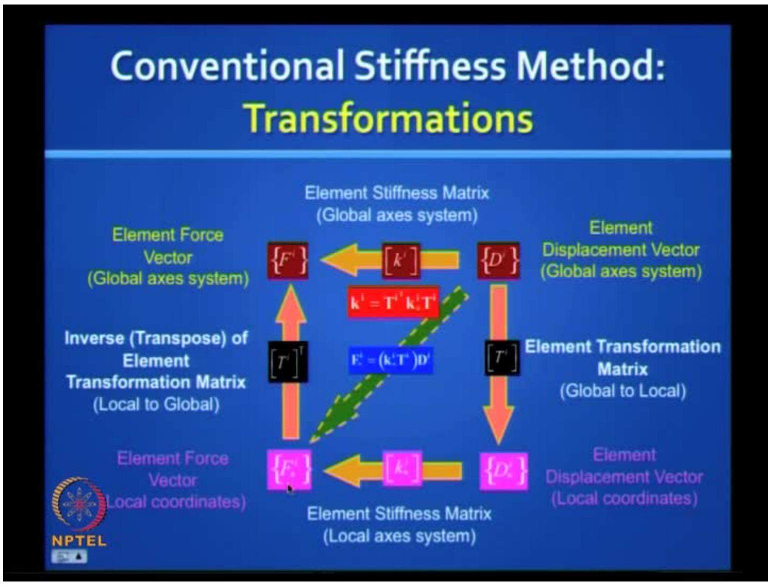

This slide I am going back to, you know you do not have to mug up any formulas, remember this play ground everything will fall in place, this is how we generate. Look at this, the steps are very clear, first identify the coordinates, develop the T i matrix, develop the element k i star matrices, do this operation first then do this operation, put the linking coordinates and assemble your structure stiffness matrix.

First, assemble your matrix element wise and then assemble the structure stiffness matrix, after you solve this equation and get the unknown displacements, directly use this to get your membrane forces.

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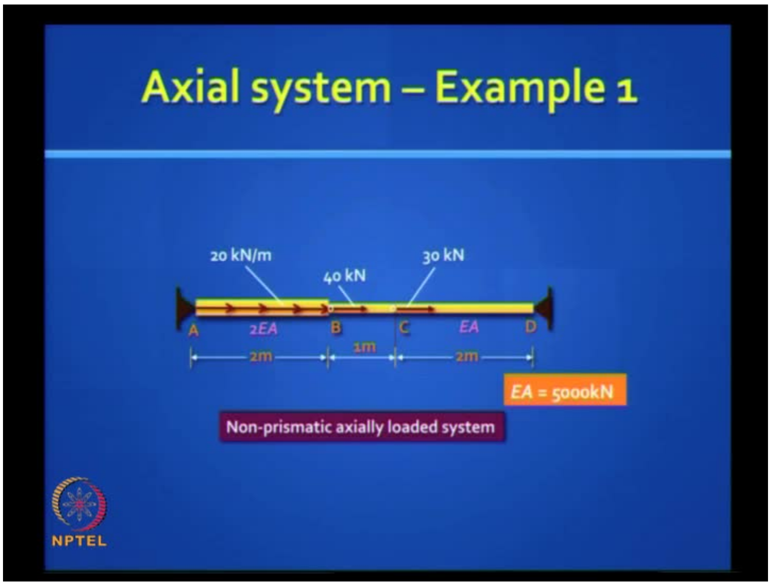

So, let us demonstrate this with some examples, we will have time only for one example. Let us look at this, this is a problem, now I want you to, actually do it with me, we will work it out so that you understand.

This problem you have two intermediate loads, you have two elements, you can make three elements if you wish, but that is a little extra work, you can have A B D C and C D as three elements in which case only in A B you have intermediate loading, but let us take advantage of the fact that we know how to deal with that 30 kilonewton load also, we will have only two elements A B and B C B D and what is the question.

The question asked is, get the complete force field and displacement field, which means find out all the unknowns, find out the support reactions obviously.

Find out The variation in the axial force, find out the variation in the displacement, usually that is something not done in practice.

So we are interested only in the maximum displacement if at all, with that you know everything, before we start this problem, can you predict the variation on in axial force and the variation in displacement.

Just scribbles something in your paper, we will check out whether your intuition is right just scribble something, how will n of x vary with the x, and how will u of x vary with x along this full length of this non prismatic axial system.

Where do you expect there to be tension in which section, A B will be fully under tension what about C D, C D definitely in compression.

What about B C, B C is compression.

A B, half tension full tension, well somewhere there is a transition from tension to compression, will it exactly be at B.

No.

Could be we do not know, because you have a constant loads of forty kilonewton acting at B.

So 40 plus 30, 70, so we cannot.

You just sketch something and am not going to look at it, keep it and we will check it out later; also draw the displacement, they u of x, you can draw it separately for the three elements.

Now, let us proceed how do we solve it, so first lay down the map, you help me fill up this, what is the first thing you should do, I am asking you how will you do it using a computer step by step. First you write down the degrees of freedom and label them, and then you should do the transformation.

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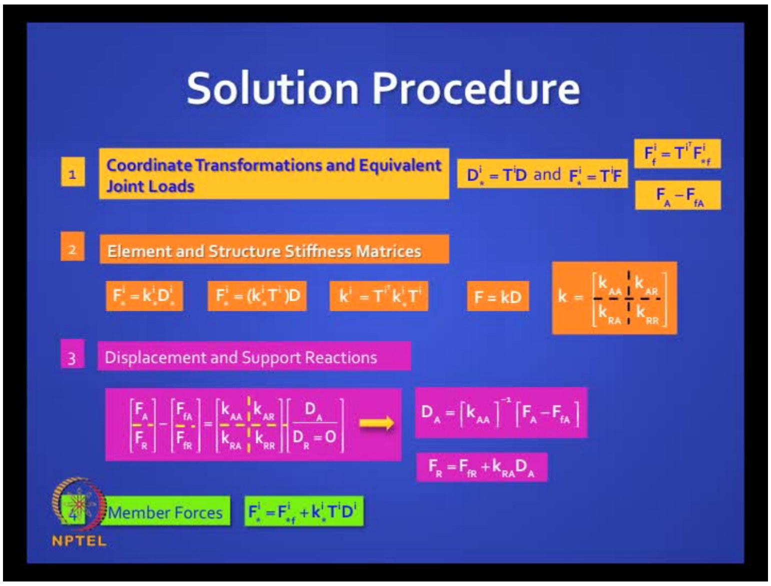

So, this is the first step, you should identify the global coordinates, local coordinates, do the transformations and find the equivalent joint loads.

All the input should be done, so you need to find out the equivalent joint loads, write down your T i matrix, you can use a T i matrix to find your equivalent joint loads, this is the first step. What is your next step.

Sorry e i there is no i here, e has no matrix e is constant, k i, use the right term element stiffness matrix and generate the structure stiffness matrix.

You have to bring into this form, how we do it we will see, you can do that transformation. Now, you are ready to solve the problem, what is the next step you need to do.

You’ve got the equation so, you find the solution find the displacements and the support reactions you have two equations, you can solve up those two equations.

You will find that in these two equations, you know that there are no support movements given in this problem so this is a null vector this part.

Take the first equation, invert your k a matrix after you found it there, find out your equivalent loads at the active degrees of freedom, find the unknown displacements used that to use apply in the second equation and find the support reaction.

It is very simple, very straight forward, but you have not done any problems so this is your first problem, and finally you have to find the member forces, how do you get the member forces, well you have first intermediate loads, so you have fixed end forces then you remember a diagonal path, from the displacements directly you can get the element.

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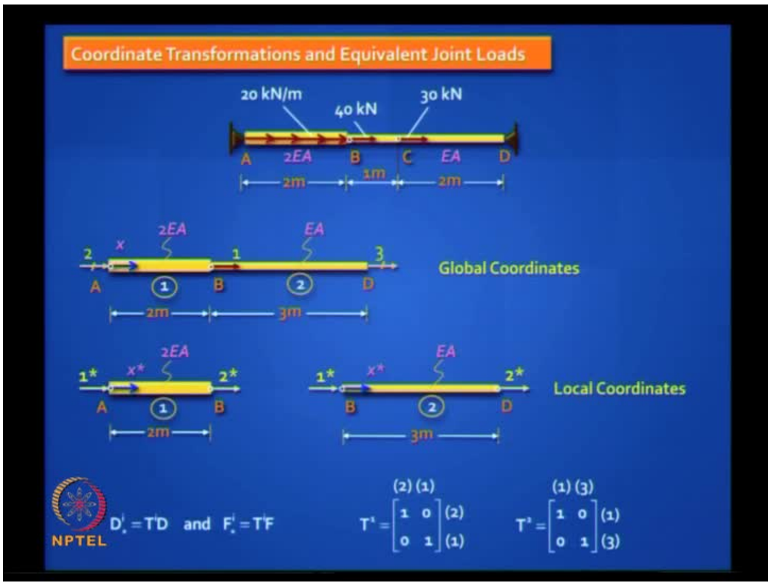

So, this is all, nothing more to be done, let us do it by step by step; first write down the global coordinates, well we are very clear there is only one degree of freedom.

So we will label that one, then we have two restrain degrees of freedom, one here and one there. Let us point it all in the same x direction we call it two and we put a slash to remind us that it is a restrained.

Please do it, so you got three, in all problems do it step by step in a simple elegant way you will find this subject is then very easy to follow. In, the beginning make it a habit to write down neat sketches, so that one look at and you will understand what is going on.

Can you now attempt labeling the local coordinates, so you have to separate out the two elements and do the local coordinates.

That will look like this agreed element one, element two, write down the length local x star axis, two degrees of freedom from one star two star.

No, questions very clear, what is your next job, you have to do coordinate transformations, how do you move from local axis to global axis and put the linking global coordinates in brackets, remember even if you do not remember with this exercise you will understand.

So, I will show it you, since it is the first time this is what we need to do and it will look like this, remember we said the transformations are all going to be identity transformations. Because x and x star are aligned in the same direction, nothing to worry. So, both T 1 and T 2 will be identity matrices, only thing you have to put the linking global coordinates here.

Now, you see the local coordinates for one left end is one star, end is two star, but this one star matches with two global and one global.

So that is why I wrote 2 1, 2 1, in the second element one star two star matches with 1 and 3, that is all you need to do, is this clear. Is this step clear, because this is how you link the local coordinates with the global coordinates, shall we move on.

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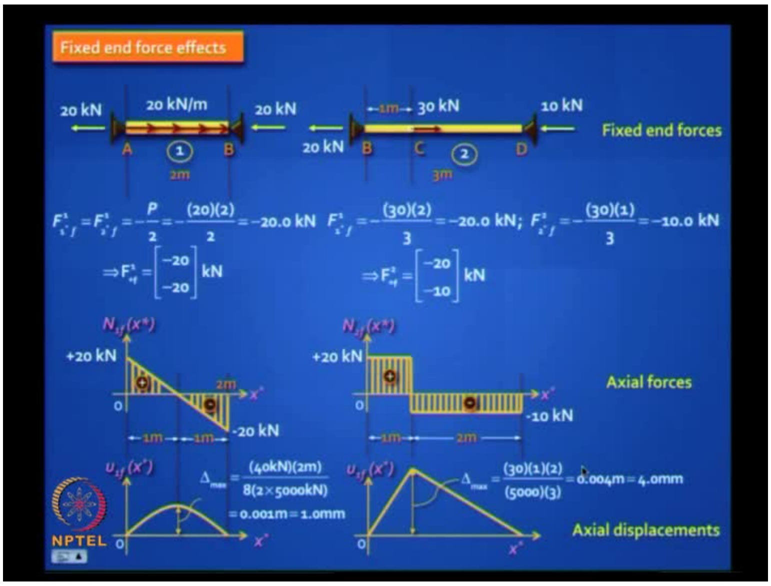

Now you need to find the fixed end forces, can you tell me what the fixed end forces are, what are those reactions you get in the first element, what you think this will be, tell me the answer.

20 and 20

20 into 2 is 40, will be shared equally, so you get 20 20 there, and here how much of 30 will go here.

Twenty.

Two thirds, this is one meter and how much will go there; 10, and how do you label it, well you can write this, these are the notations we use.

The first element fixed end forces written as F 1 1 star f, that is this one and this is F 1 2 star f, mind you everything pointing in the local x star direction is positive, so they will both be returned with the minus sign, you guessed 20 correctly, but you should also put the minus sign.

So, both these put together in a vector form is F 1 star f dash, is this notation clear to you, can you similarly create F 2 star f, what is F 2 star f, minus twenty minus ten, that is it. So, no more explanation you can do it, got it. What do we do next, well before going ahead, lets also for our own understanding, see how the axial forces and axial displacements look, can you give it a short, what will be the axial force variation, well it will be like this, we just discussed it, this U d l will give me an a axial force variation linear, and it will give me an axial displacement parabolic, and the value of the maximum displacement here is P L by 8 E A.

Now, here the E A is 2 E A, mind you in this problem, so you have to write two times E A, E A is given as 5000 kilonewton, you get point naught naught 1 mm, which is 1 mm you get a 1 mm axial, this is a very small displacement makes sense.

This one you have an intermediate load, this is a variation of axial force you have a triangular variation of axial deformation, and you know this is P a b by L E A, so substitute those values to get 4 mm, is this clear.

You do not need to actually do this in most problems, but we want to have a complete understanding, so we want to know exactly what is happening in this structure, is this clear. So, this is what happens due to the fixed end force vectors.

Now, what do you do next, you have got this 20 20 and 20 10, you have to pass it on to the global coordinates.

How do you do that, will you proceed, let us see how it works out.

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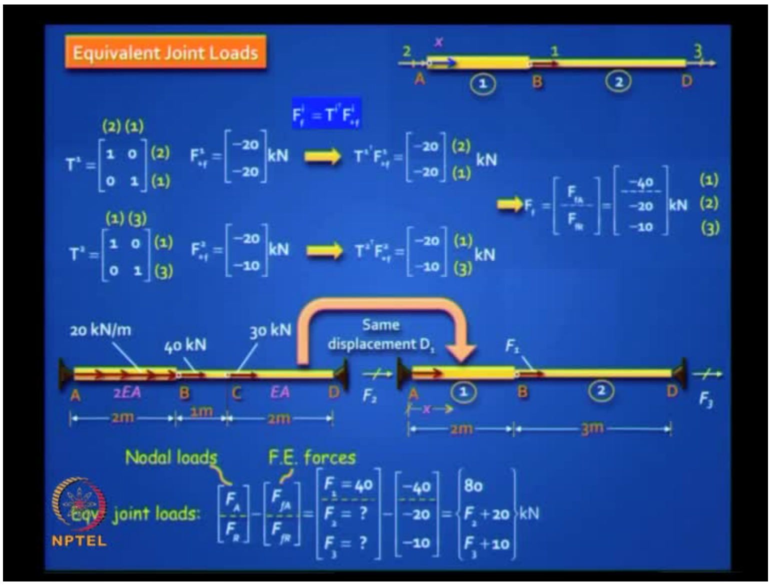

So, we are going to equivalent joint loads, what is the idea of equivalent joint loads, you want to get rid of the intermediate loads, and replace it with something like this.

What is a criterion by which it is really equivalent, a displacement; no you will never get the same displacement field. No, you will never get the same displacement variation; it is the same as the displacement field by the way.

What is this equivalence guarantee that both will have the same what, same no not throughout, at d; that is all your guaranting, the same displacement d 1, is it clear, that is the idea, this is the base for equivalence, so how do you do that.

Well you do it, first noting down what you already done, you got that two transformation matrices, you got the fixed end forces vector. What's the transformation you do to get the fixed end forces, you are now moving from local to global, so what is the transformation you need to do, T i transpose that is all you do.

So, if you look there, you have to do this, just pre multiply by T i transpose which is no big deal, because the transpose of an identity matrix remains an identity matrix. So the only purpose of doing this, you will end up copying the same thing is, now you are bringing in the linking coordinates, is it clear.

With the help of these two can you identify what your fixed end force vector is. At the coordinate one these two will add up, that is what you are doing in the slot wise pushing, and at two and three it is node, so you can use these two and get your fixed end force vectors does it make sense to you, that is it.

Now, this is what you do, your actual problem is you have the original nodal loads, you have now the fixed end forces which you now need to oppose, then only you will get equilibrium satisfied, so you put a minus there this is these are called equivalent joint loads.

This will add up to some quantity in which the first is 40, no in the original load there was a 40 kilonewton remember at b in the original problem there was a 40 that is this 40.

But, you have some reactions here due to that 40 that we do not know, and then you have from this you have 40 here, so you put that 40 there with a minus sign, and you also have some reactions minus 20 minus n you put that there, and you will get this is your final answer.

This is a one way of doing it, but you can do it using your own logic you will get the answer.

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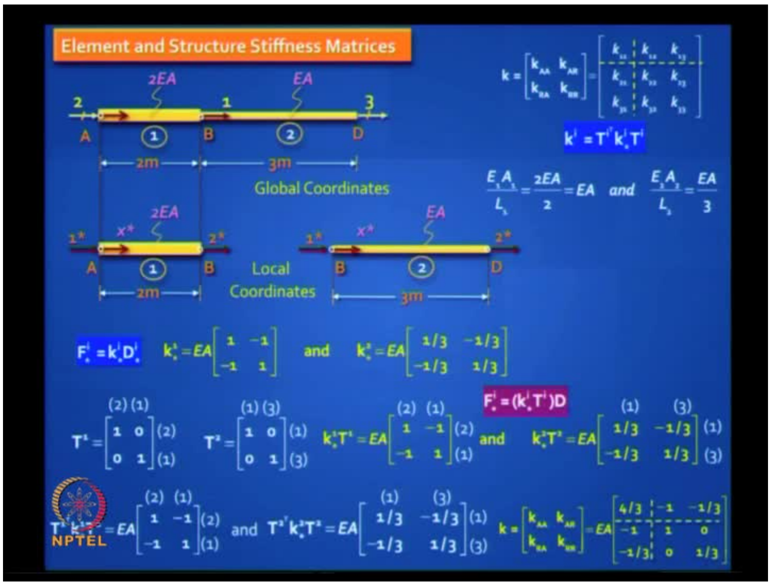

So, let us complete this now you have to generate the element and structure stiffness matrices, that is easy you know the formula. So you know that the E A by L values of each of them are known, because E A is two times for element 1, so is this not easy to do, this is the first one, this is E A by L, E A by L is this, and this is 1 by 3, does it make sense, plus 1 minus 1, it is easy to do.

Then, you do the transformation, you have T 1, T 2, you first do, see you need do this, but it is better to do this first, because you will need thisto get the internal forces. So do this which is actually very easy, because you're multiplying with the identity matrix, but now you are putting the linking coordinates, and again you multiply with T 1 transpose you will get that, so finally you got these values.

What do you do next, you have now converted the element stiffness matrix from local axes system to global axes system with the linking coordinates, and you have to now put it into the slots.

Remember, this matrix 2 2 will go here, do you understand 2 1 will go where, 1 2 and 2 1 will be symmetric we input them either way, so you have to put it in the correct boxes, will you tell me, you do this yourself, fill up this matrix for me.

This 3 by 3 matrix using this transformation put it in the right slots, that is little exercise you do and we will stop.

You just have to add what is k 1 1, well k 1 1 from the first element is 1, 1 into E A, from the second element it is 1 by 3 E A, so it adds up to 4 by 3.

What is k 2 2, k 2 2 from the first element is one times E A, and from the second element there is nothing. So, can you do this, it is a simple addition exercise you will end up with this. I do not want to push you, are you comfortable till this stage, we will pick up the solution in the next class.

Because, you need to find the reaction, I want to do it slowly, are you clear up to this stage or shall we finish it straight away.

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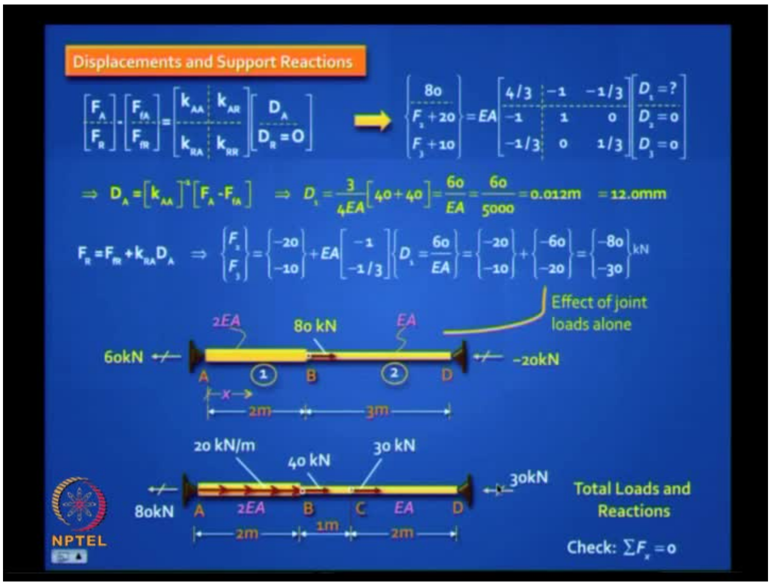

Now, you have these equations you just substitute those value solve, how do you solve this, what is the size of your k a matrix, three by three k a a matrix, this matrix.

One one.

One why, because here only one unknown active degree of freedom, so it is very easy to invert it, you solve that first equation you will get the answer D 1 is 12 mm, after you get the answer what did you do, you plug it into the second equation find the support reactions, and if you want to interpret the support reactions, you will find that due to the joint load affects alone you get this 80 kilonewton is passed on a 60 and 20, and due to the complete response that is you add up these two quantities, you get 80 and 30 and you can check equilibrium, check sigma f x equal zero.

Please go through this exercise it is quite simple, what is left now, you almost done, you need to find the internal forces, how do you do that fixed end forces plus k i T i times D you have to be careful about D, I will explain how.

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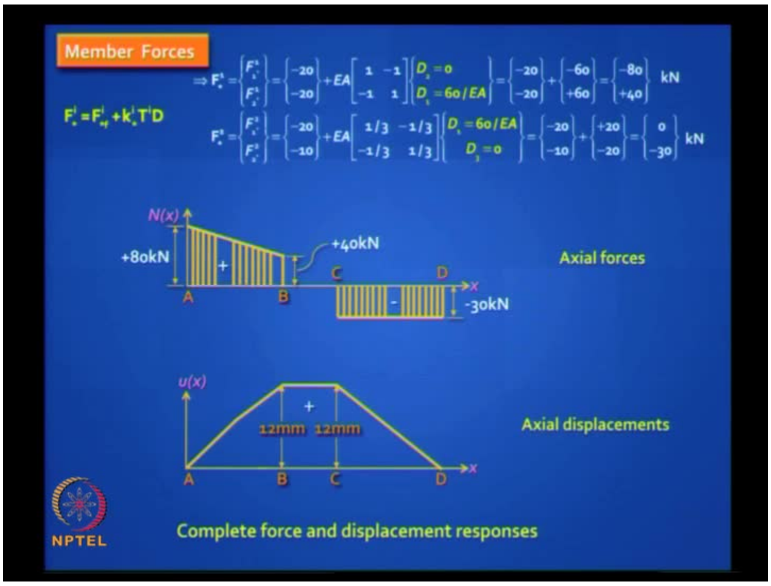

So you have the fixed end forces you remember we calculated minus 20 minus 20 minus 20 minus 10, you add e at these two level, but remember you have to put the linking displacements, here you have to put D 2 and D 1, and here you have to put D 1 and D 3 that is the only thing you need to be careful about.

When you do the other method the T D method you do not have to think, it will automatically do it, and here you have to put the correct linking values, this is the 12 mm which can be written as 60 by E A then when you solve it you will get these answers. Once you got them you got the axial force distribution. Is this what you drew in the beginning. I asked you to guess, so you see, you have to check your intuition, you actually get zero axial forces in the middle in this particular problem, and your deflected shape will look like this.

Ok so this is a complete solution, we will see how you can do the same problem using T D matrix and the choice is left to you which way you wish to do in the next class thank you.