Advanced Structural Analysis

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Module No. # 3.4

Lecture No. # 20

Basic Matrix Concepts

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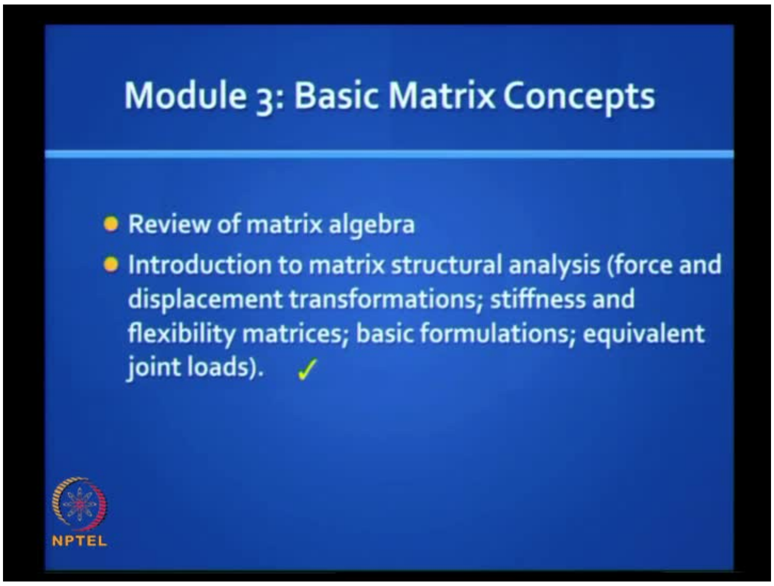

Good morning this is lecture number 20 in our course on advanced structural analysis we are continuing with this third module on basic matrix concepts, as applied to structural analysis.

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This is covered in the chapters 2 and 3 of this book on advanced structural analysis.

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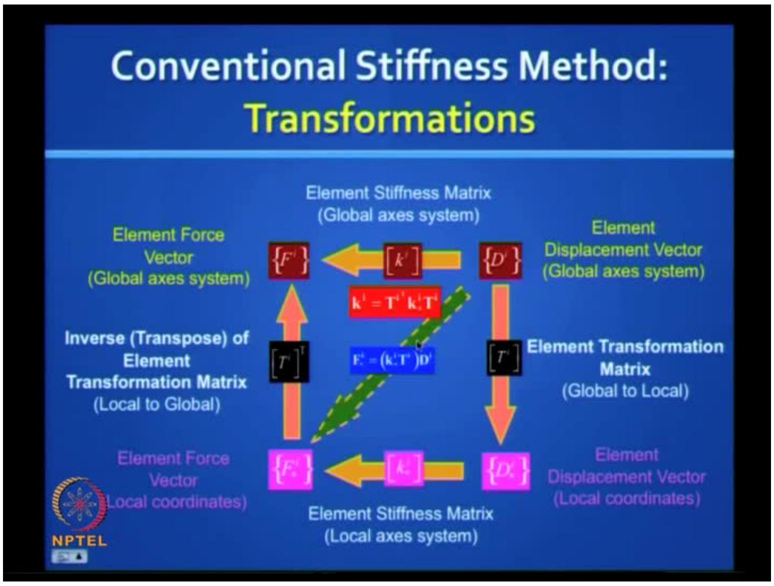

You will recall in the last class, we gave a big picture view ah this picture really shows how you can do various transformations to finely arrive at the arrive at what what is the end result of this? You will get the element stiffness matrix of every element aligned along the global axis ok.

So the end result of these transformation is this K i is equal to T i transports, K star i T i after this, your next job is to assemble the matrix and get the structure stiffness matrix ok

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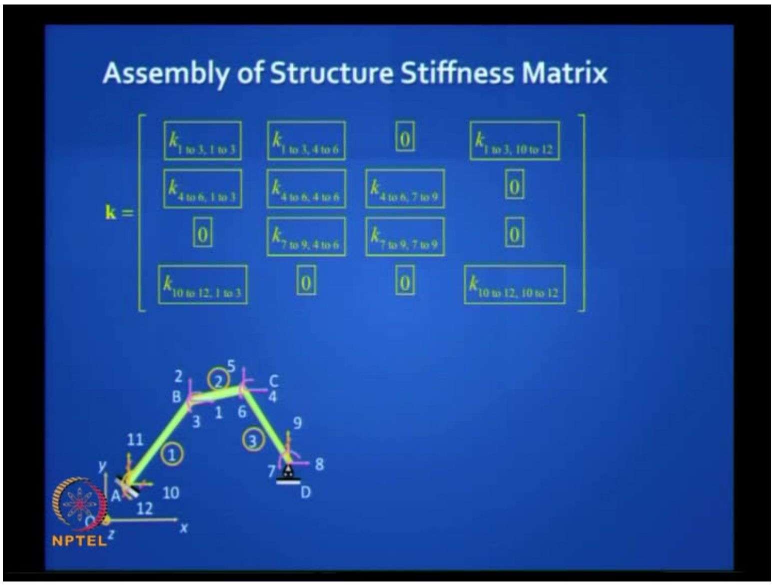

So that assembly is where some of you had a difficulty so, remember we had some difficulty understanding so, let me tell you a story ok.

Ah, there was a housewife who wanted to buy ah one of those gadgets or mixer, come grinder, come juicer, come so many other things so, she bought it online and this pick packet reached her house. She was very excited because, she paid through credit card and all that and when she open the box it looks beautiful there was a brochure but, it all came like this matrices in small pieces and she was supposed to assemble them because assembled is very big, the unassembled is very small so, for packaging purpose is came in small pieces.

So, she was postgraduate student so, she thought she could easily put it altogether. So, she opened the manual and she spend one hour trying to put it altogether and did not work, just like you did not get the hang of this in the last class.

So, she kind of gave up, anyhow frustration she left the house and went out. When she came back few hours later, she was amazed to see that, the matrix was well assembled already the and it the you know it connected to the socket, plug and she just put on the switch and she heard a beautiful whirling sound.

Oh my god, how did this happen and she looked around and she realize there was a made servant in the house, who is absolutely the illiterate. She was wondering, did she fix up this wonderful gadget and she called and, she said did you do this? she said, she smiled hesitantly and said yes, but you do not know how to read or write and she replied yes mam, when you do not know how to read or write you have to use your brain.

Ah so, your real challenge is you are literate, you know how to read or write, you can look at solved examples in text books. If they say step 1, step 2 you know what what all that means but, really putting things together on your own, you need to be little illiterate and start using your brain.

So, here we have the challenge of both knowing how to read and write and also trying to fit it all together. So, let us do this one more time, looking through the eyes of the maid servant ok.

So you have three elements there, for each element in the last class; we generated the T i matrix, ah first we generated along the local coordinates, then we transformed it and put it along the global axis but, we put some linking coordinates right.

So, we have got those readymade stiffness matrix boxes and we just have to put it at the right place in the big matrix. The structures stiffness matrix, what is the size of that? Its 12 by 12 and we have these small three 6 by 6 matrices, which we can again partition into 3 by 3, 3 by 3, 3 by 3, 3 by 3 and we need to put the right 3 by 3 box in the right slot.

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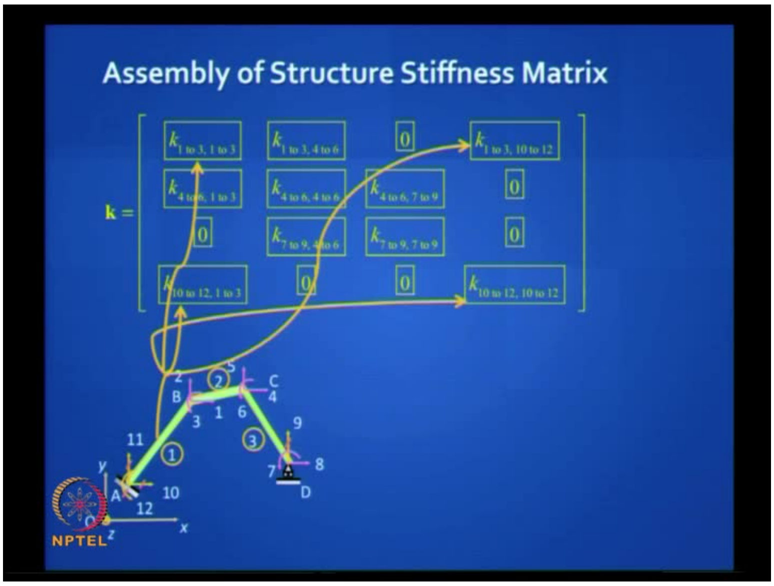

So, all you have to look at is, connectivity. Let me help you, take the first element, the first element is connected to nodes 10, 11, 12 and 1, 2, 3 so all you have to do is, to pick up the appropriate boxes and put them into the slots as shown here, does this make sense? It fills up only those four slots.

The contribution of element one is only to these four slots 3 by 3 slots and it is direct contribution. You just have to show it there, you do not have to play with it, you do not have to multiply with the change sign, nothing just put it there. Can you do it? That is all you have to do, do it intelligently right.

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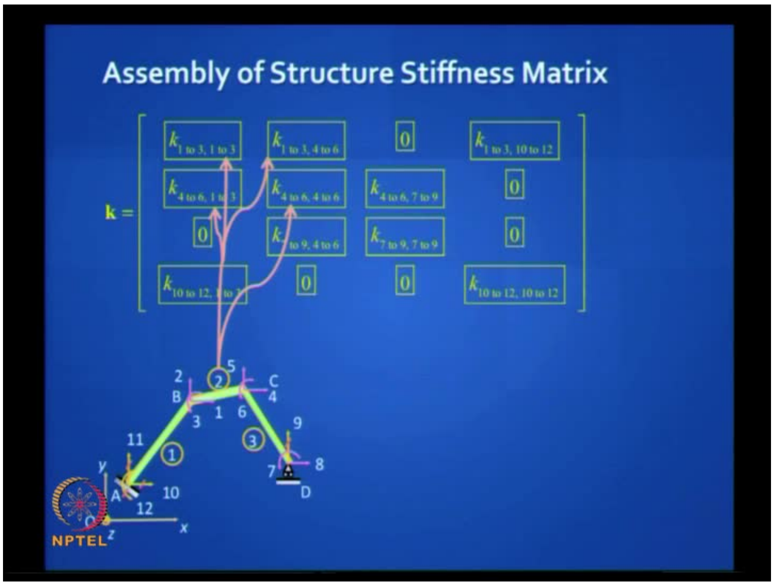

Ah element one has no contribution elsewhere in that big picture in that global, take element two, where do you think it will fit in? It will fit in that simple, the top corner because 1, 2, 3, 4, 5, 6 element coordinates match with, global structure coordinates 1, 2 so that is simple and the last one, last one will connect 4, 5, 6 and 7, 8, 9. Does it make sense?

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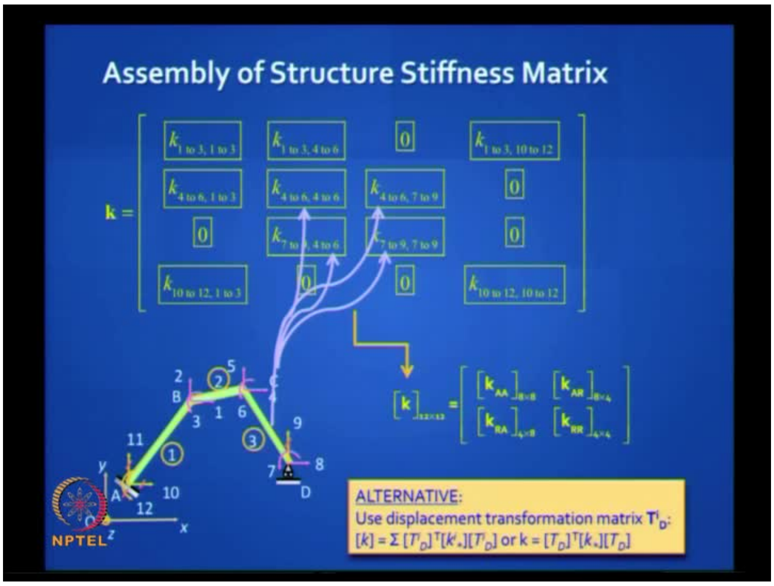

That is all, and in some places like 1, 2, 3 you will have two contributions, you just algebraically add them up, because stiffnesses get added up. Does it make sense? That is all you have to do and when you do an assignment of course, we will make it a small problem you get a first hand feel of how to do it yourself ok. So, there is no way of learning this accept doing it by yourself, like the maid servant did it in our little story.

Finally, after you got this matrix and we will see that there are many other ways of getting the same matrix. There is the reduce element stiffness method, there is another method, which we will study soon of getting this matrix because, this has the big advantage that you are playing with little boxes, no matrix is more than 3 by 3 and you are finally.

So, your storage is very elegant economical, only thing you must program it well, so that it goes to the right slot. There is a crude way of doing it; we will look at that, for beginners, that you will be more comfortable with that. But, when you do programming, when you want to make, a matrix solve a very large problem, you cannot do that; that is too unwieldy. This further you can partition to the active coordinates, you have eight active degrees of freedom and the restrain coordinates. does does this make sense? Ok and the alternative is to use the, what we call the displacement transformation matrix T D which is what I want to show you next ok.

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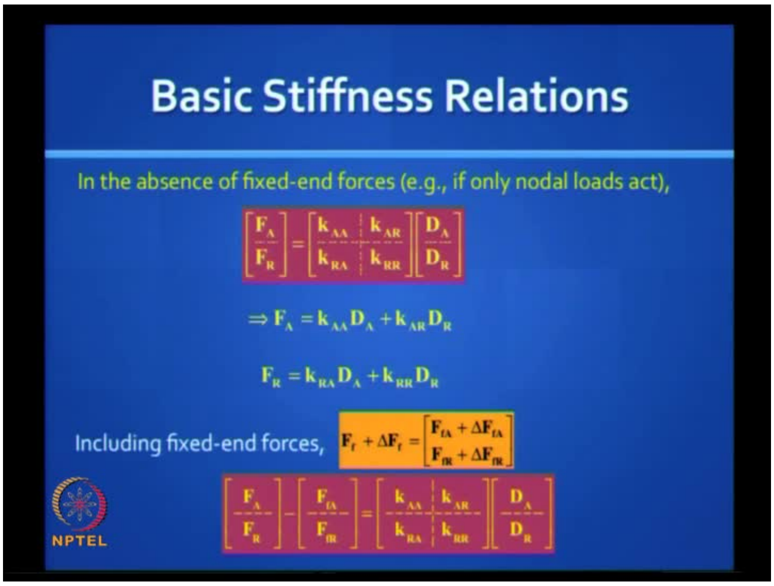

But ah, before that let us say, you got the stiffness matrix, you can write relationships like this, basically F is equal to K D at the structure level but, D itself is subdivided into active degrees of freedom, which you do not know you do not know D A displacements but, you know D R displacement right, they are usually 0. On the left hand side, the force vector F A is known to you, that is a load vector acting at the, F R is the reactions you do not know right. So, you can expand this into two equations, because a K matrix can be subdivided and they will look like this. Does it make sense?

So, you need to solve the first equation, find out the unknown displacements D A, plug it in the second equation, you get the support reactions here, from these from the global picture. Then, the unknown displacements D A you should use, using equation similar to your slope deflection equations and get the member level forces, does it make sense? this is the big picture.

(Refer Slide Time: 07:38) Now, that is what you get in a truss, because there are no intermediate loads but, in a beam, or in a frame, or even in an axial system; like a chain where you have intermediate loads, you have to add the fixed end force effects right, which you have some introduction to already, so you have to modify that equations slightly ah we will look at that in detail later ok.

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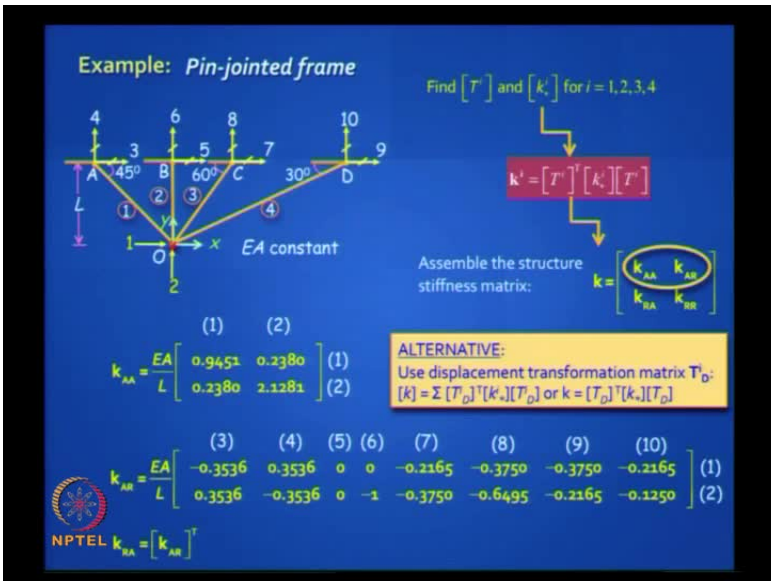

Then let us go back, let us take another example, let us take this frame which we looked at ah yesterday. How do you go about doing it, first you find T i and K i star, we have done this, what is a next step? Finding K i finding K i by doing this transformation which, in mat lab you can do effortlessly for each element. Then, what is a next one? No, no you this is at the element level. Next is assembly, this is where you had difficulty right. We assemble it to this form and then, you can solve for whatever loading you get.

So, mind you up to this stage, you are dealing with the property of the structure and you say, now give me the loads I can handle it but, the computer does this in a in a jiffy, the moment you feed in the geometry of the structure in the material properties. It is ready It is ready to handle any loading because; it is already going to work out this properties effortlessly if your programming is good.

For example: in this structure you do it this way, ah let us look at K A A, it will be a 2 by 2 matrix, because you have only two active degrees of freedom and K A R for example, with look like this but, in this method you have to always keep track of the linking global coordinates which I have shown here. 1, 2 are the active degrees of freedom and 2, 3, 4, 5 all the way to 10 are the restrain degrees of freedom and K R A will be the transpose of K A R and you can calculate K R R.

You will never need K R R unless, you have support settlements because that is D R is usually 0, the alternative is this using a displacement transformation matrix, this is for people who are not very comfortable you know putting it in slots and all that. If somehow you could have a robust method, where you do not have to keep track of this linking coordinates then, you have got a blind way of doing it ok.

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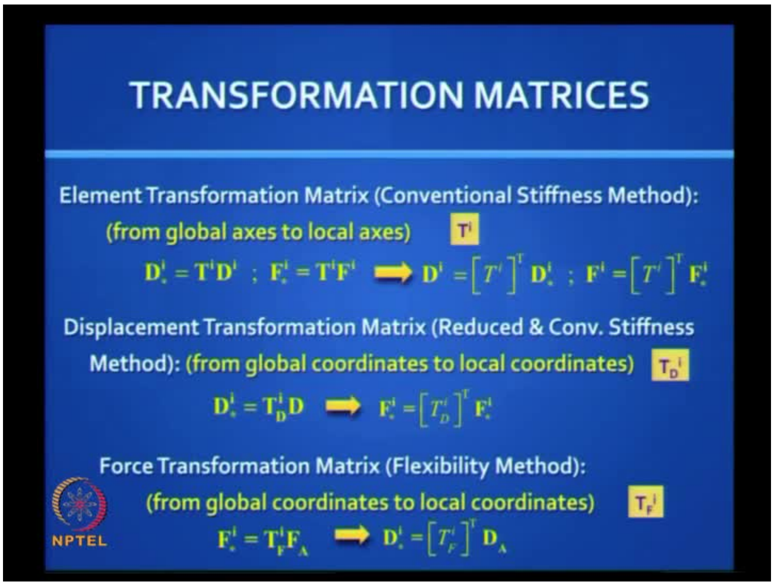

Not suitable for programming large structure but, ok at at your level to do manually and I I think that is the way it is taught in most universities. But, that is not how algorithms are built in software packages so, let us look at that. Ah remember, I told you about three transformations, this one we finished, you studied this, we know how to do the transformation using T i.

The second one is using the displacement transformation matrix, this is (( )) when you do the reduce elements stiffness method, which we are going to study next but, you can use the same idea in conventional stiffness method and in the next class, we look at flexibility method where, instead of a displacement transformation matrix you have the force transformation matrix, it will all fall into place.

Now, the inverse of those matrices helps you switch from the element level the local level to the global level and there is a principle, ah you do not need to do the inverse there is a principle called the contra gradient principle, a beautiful principle which will look at short[ly]- right.

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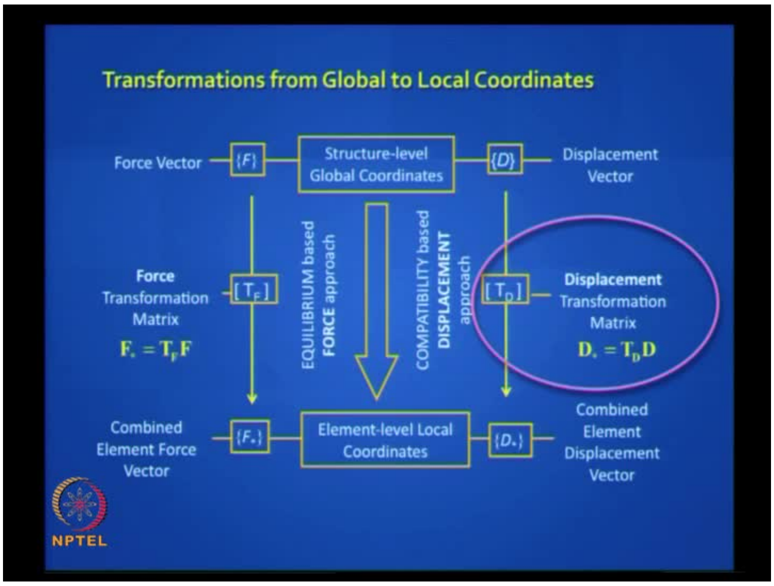

So, let us look at these transformations, look at this now, I am talking about structure level coordinates here and element level coordinates here right I have combined them, so F star really means F 1 star, F 2 star you know I can put them all together neatly.

So, does this make sense? and I am saying, if you give me the global displacements they are linked all in one go to the element displacements through a matrix, which I called the T D matrix. T D matrix is the displacement transformation matrix and obviously, it must satisfy compatibility. Likewise, in the flexibility method there is something linking this to this, that must satisfy equilibrium and what would be the good name for that transformation matrix, force transformation matrix ok, we will study that later. For the time being, we look at the displacement transformation matrix and see how it works.

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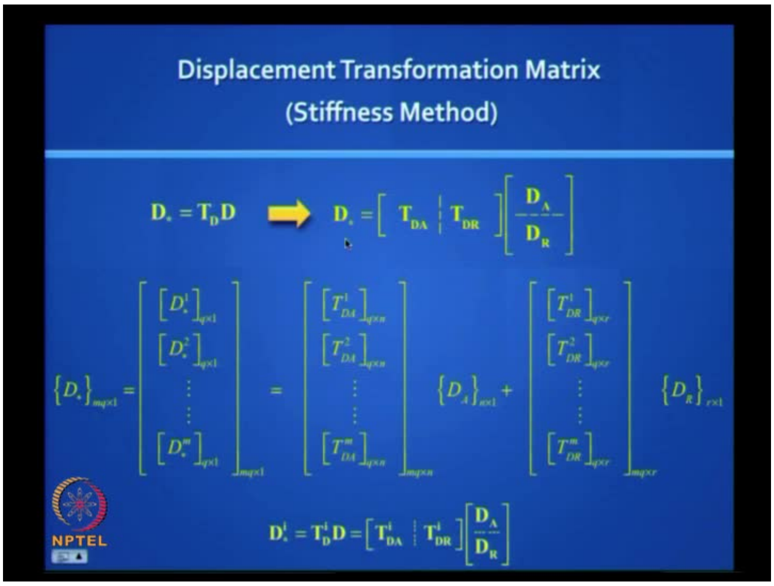

So, since we know that the displacement vector is really made up of D A and D R then, I can again partition my T D into T D A and T D R, does it make sense? T D A into T D R ok and if you expand it, it will look like this. Do not get worried about those size of those matrices, it is just how it works out? How it falls into play? Now, I can do something here this D star is actually including all the elements in the structure. It is too messy sometime, so I just look at one element. When I look at one element the i th element, it will look like this D i star and this T D now, becomes unique to that elements so, it is T i D which can be partitioned as, T i D A and T i D R. Does it make sense? this also you can do, it is actually all the D i stars put together, that actually make up the D star ok.

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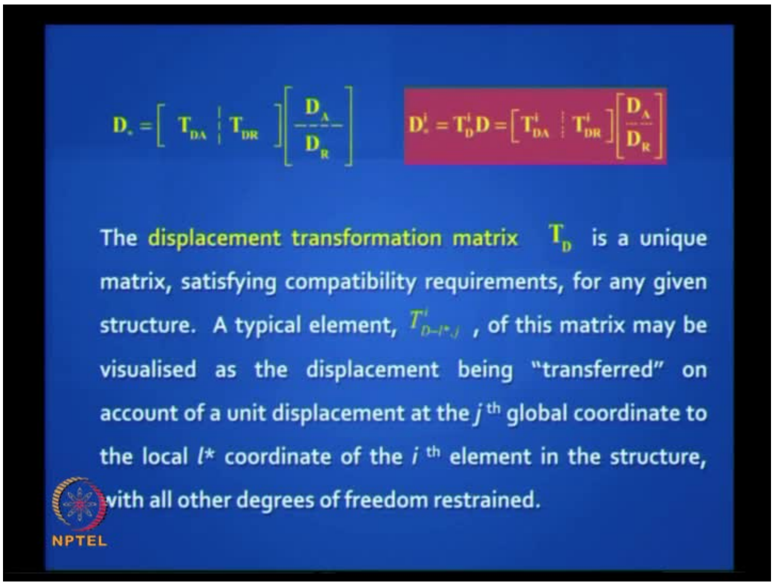

Now, let us take a look at an example: but, before that a definition, the displacement transformation matrix T D is a unique matrix satisfying compatibility requirements for any given structure and a typical element D i T D, T i D at L star j of this matrix may be visualized as the displacement being transferred on account of a unit displacement at the j th coordinate to the local L star coordinate of the i th element in the structure with all other degrees of freedom restrained, that is quite a mouthful but, basically you get what I am trying to say. Do you remember? When you have two subscript, the second subscript goes to the cause and the first one goes to the effect ok.

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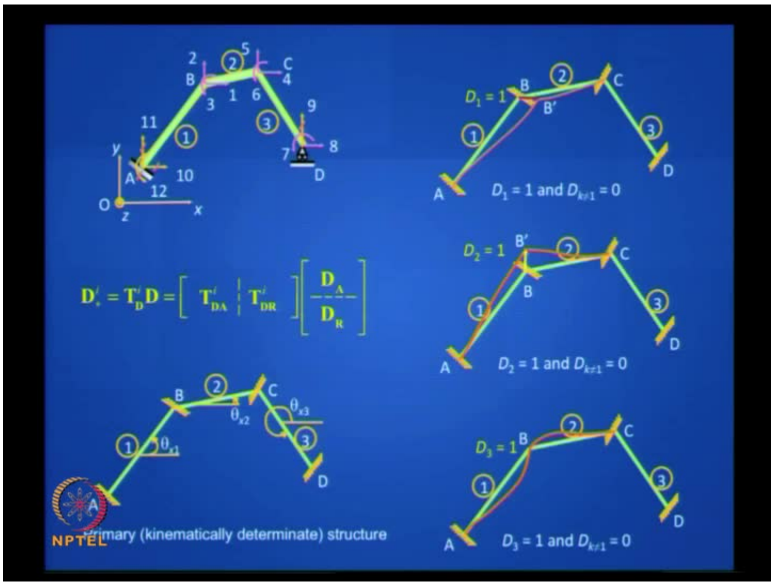

Let us just try to understand what that means, take this same structure ah, let us see what it means for the i th element. What should we do? Well really, we are trying to first arrest all the degrees of freedom. So, you got the kinematic structure; all degrees of freedom arrested then, one at a time let us say D 1 equal to 1, you get a shape like that. We are trying to see the effect of this in each of the element stiffness matrices, each of the element displacement vector that is what we are trying to see. In other words, if I apply D 1 equal to 1, which elements will it affect? It will affect first and second, the third will remain unaffected, if I apply D 2 equal to 1, likewise something like that will happen.

If I apply D 3 equal to 1, that will happen so, you you already have an exposure of doing this, you can actually do it from first principles, using the physical approach because, we did a few examples earlier right. But, we the computer has no feel for any physical approach, it has to do it mechanically, blindly and that too is possible.

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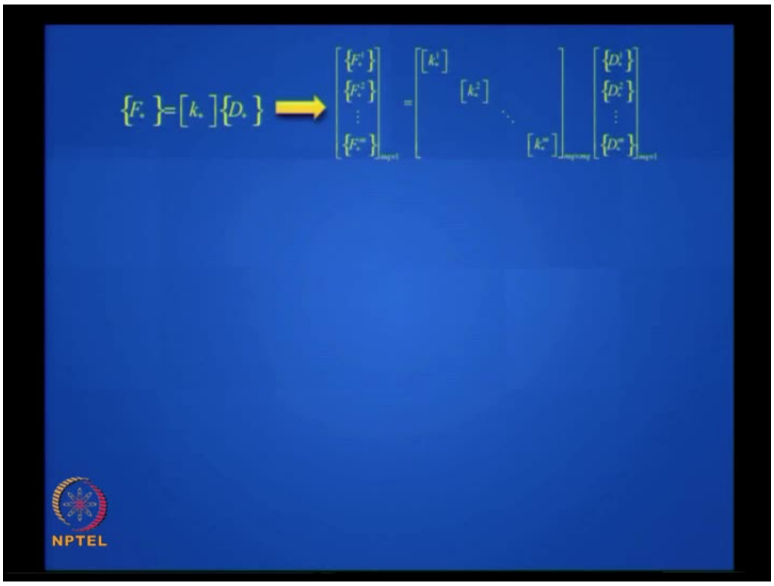

Ah before that, let me talk of this matrix K star, which is sometime refer to as the unassembled stiffness matrix ok. Ah let me give you a simple example: let us take a truss.

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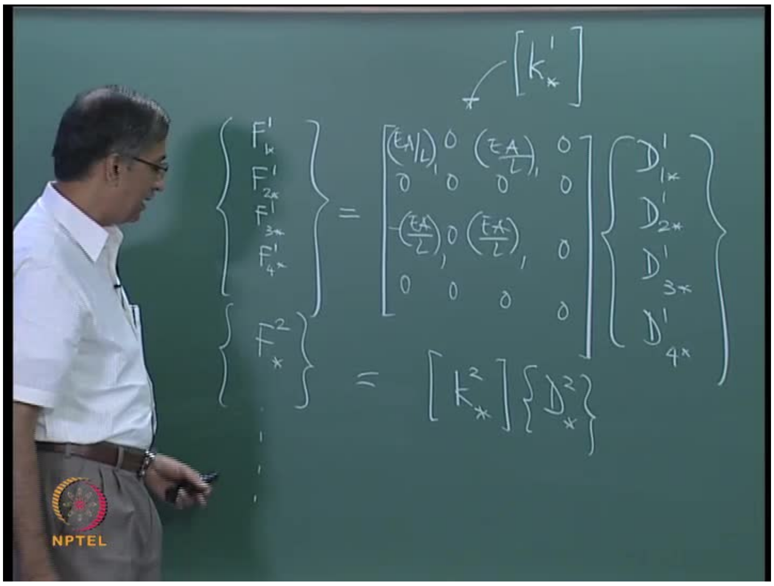

Let us say the truss has four elements, so I have F first element. There is only 1 because, ah ok 1 star, 2 star, let us take the first element F 1 3 star, by now you know that the truss element is four degrees of freedom. We worked out this stiffness matrix in the last class right. So, this is F four star got it. This is related to to D 1, 1 star D 1, 2 star D 1, 3 star D 1, 4 star through a relationship, which we are going to refer to as, K 1 star right. This is the element stiffness matrix 4 by 4 and you remember, some rows are 0 which has 0 second and fourth. So, because they refer to shear forces in a truss element, the shear force is 0. So, we know that these are anyway going to be 0 and so, the columns also will be 0 right and the non zero values what are they, they are very easy to remember.

So, the principle diagonal will have E A by L of that element 1, ya minus comes to the half diagonal elements right. Now, if I take the next one, which i will put is F 2 star ok I get another another 1 like that right is equal to K 2 star into D 2 star, does this make sense to you? Similarly, I get let us say there are m elements, I can write this matrices right.

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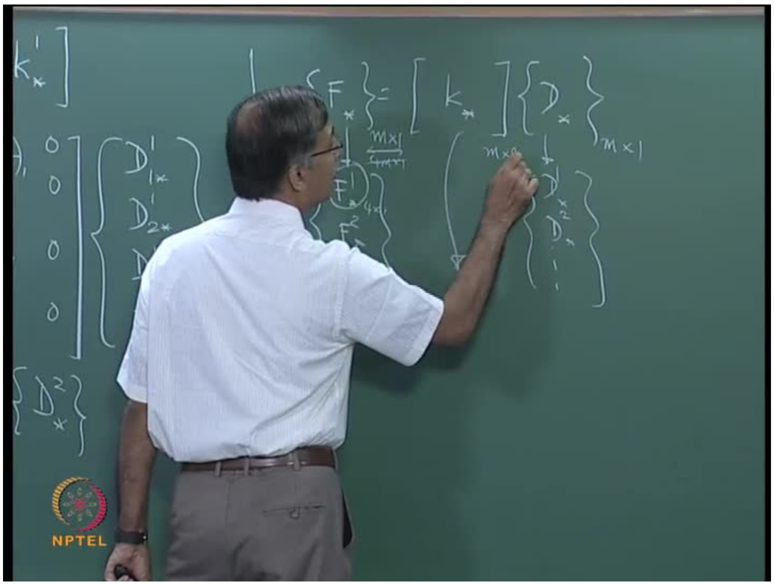

Now, if I want to relate all of them to a single matrix ok. So, this is really meaning F 1 star, F 2 star etcetera, this will be right. How will this look like? Let us say, this is dimension m by 1, m members this will be m by 1 right and each of them each of them will be 4 by 1.

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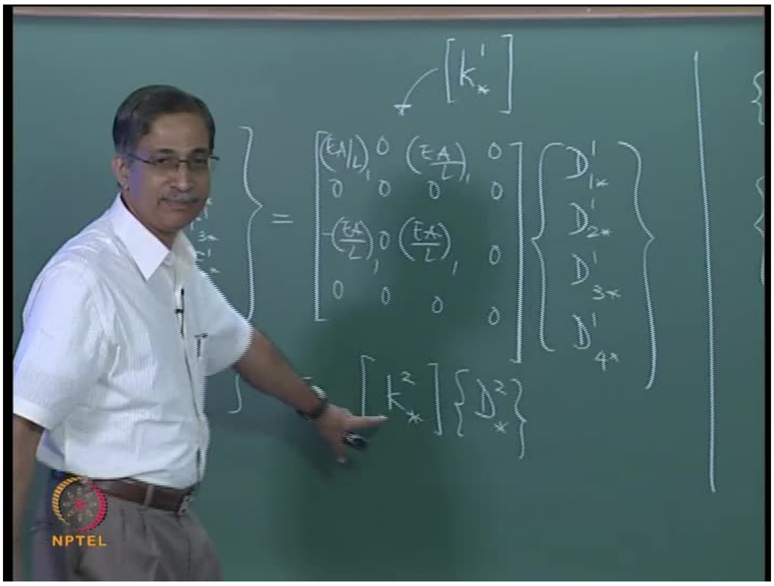

So, the actual size will be 4 m by 1 right. Let us look at, the sub matrices and not worry about this ok so, we do not worry about this. What will be the size of this matrix m by m? What will it, how will you make it? How will you arrange it? You got all the individual elements matrices, how will you arrange this? No (()).

Sir k one k two k one star k two star.

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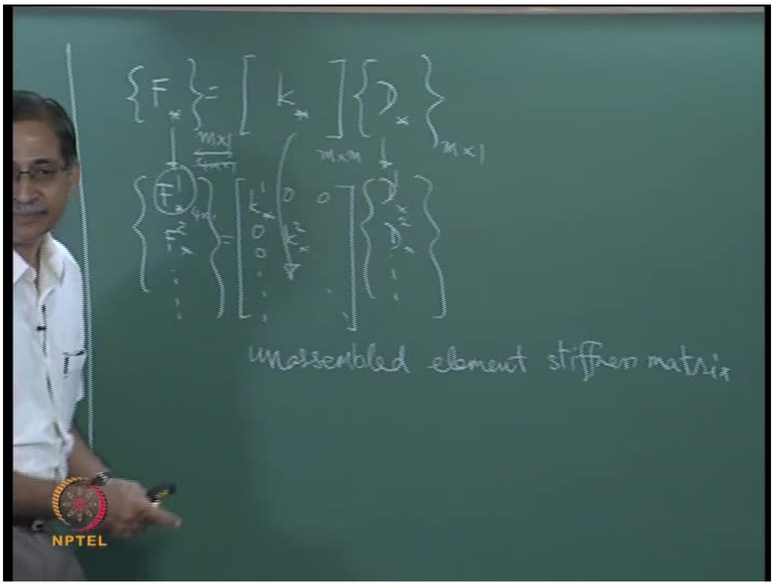

Wonderful, see the element stiffness matrix of one element has no relationship with the element stiffness matrix for the other element. So, if you really want to put in a matrix form, there is only one way you can put it. This will be a diagonal matrix where, you have say K 1 star here, K 2 star there, all the ways here and you have zeros everywhere else, is it clear? Because, then only I get this is equal to this times that, which is this, this is equal to this times that and nothing else. So, this is sometimes referred to as the unassembled, because you really not assembled the structure you just put together the all the element fellows together but, it is not the structure stiffness matrix. So, this is called the unassembled element stiffness matrix does it make sense?

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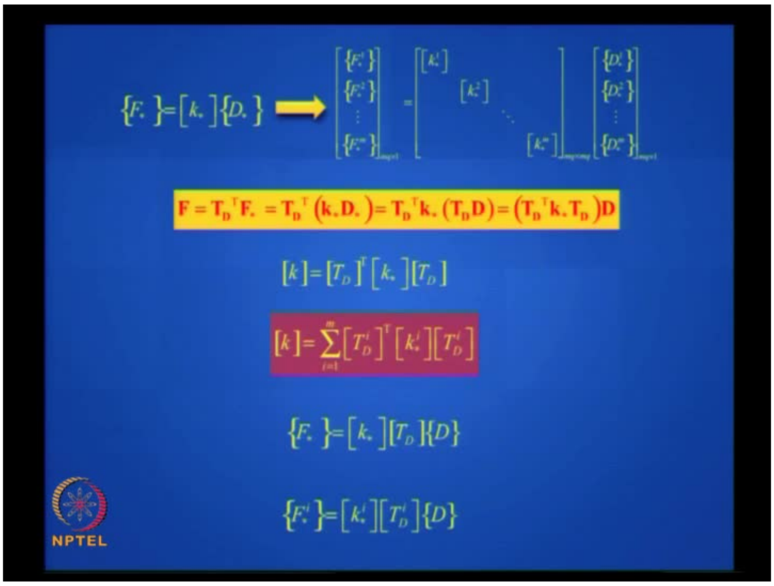

That is what shown in the slide here ok, so it is always the diagonal matrix and if you do the transformation here and see it is a simple transformation ok. I need to explain this transformation ok, this depends on the contra gradient principle which I showed you but, I have not proved it, i will prove it little later. Let us accept this principle that, the reverse works this way and so you can prove that it takes a same form, this form is familiar to you, because we did T i transpose K i T i, you have a similar transformation here. So you see, you get the same structure stiffness matrix from the displacement transformation matrix at this manner.

So, let us say you get T D, let us say you got K star, you got K by by this method ah and if you do not want to do it for the whole structure, you want do it element by element that is also fine but, then you have to sum up for all the elements. I want you to do understand that, unlike the previous case where we dealt with K i, which was always limited to the size, it is in square matrix size 6 by 6. This K is a structure stiffness matrix size is as big as the global degrees of freedom, you will understand when we do an example and then, ah you can have these relationships, after you find the displacements, you can get the number end forces.

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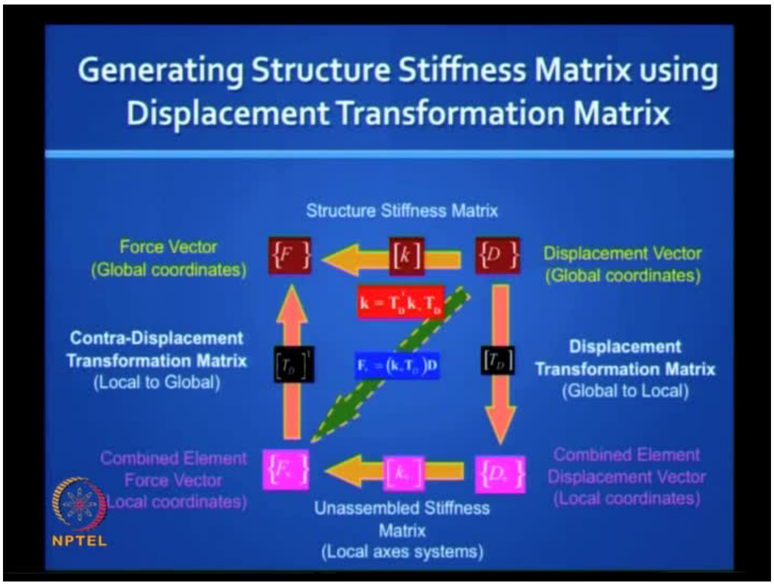

So, let me put it in a different way. If I want to generate the structure stiffness matrix using this approach, the displacement transformation matrix. First, I create this relationship at the element level putting all of them together, so it is unassembled. Is it clear? Exactly, what I have shown on the board.

Then, in the real structure I know this is what I want ok. Now, my playground is different, now my playground is on one side, the goal poles will have the structure level coordinates, on the other side, I have all the element level coordinates put together in an unassembled manner but, all of them are there. Then, my T D will take this path right because, D star is T D into D, I have not partitioned D A, D R, but you can do that. Is this clear?

Let me ask you, is this step clear from here to there? Is this step clear from here to here? This your conventional stiffness matrix. Is this step clear from here to here? Ok that is how to get this T D is what we will see shortly but, once you got T D ah you have the same game going on, that will be T D transpose we will prove that it is so, right now you accept it as it is and then, you have this short cut along the diagonal and so you will get this.

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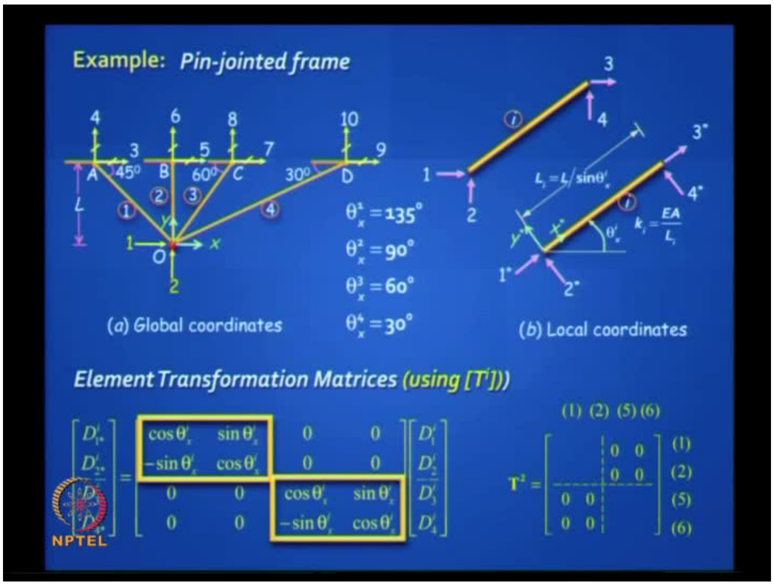

So, you know this also establish the same relationships. Let us let us see this problem which we did in the last class, I am just showing you to remind you that we did this using T i. How do we do the same thing using T D? So let us see, remember in T i those where our non zero elements; the transformation is very simple, you had a cos theta and a sin theta and a negative in one of those ok.

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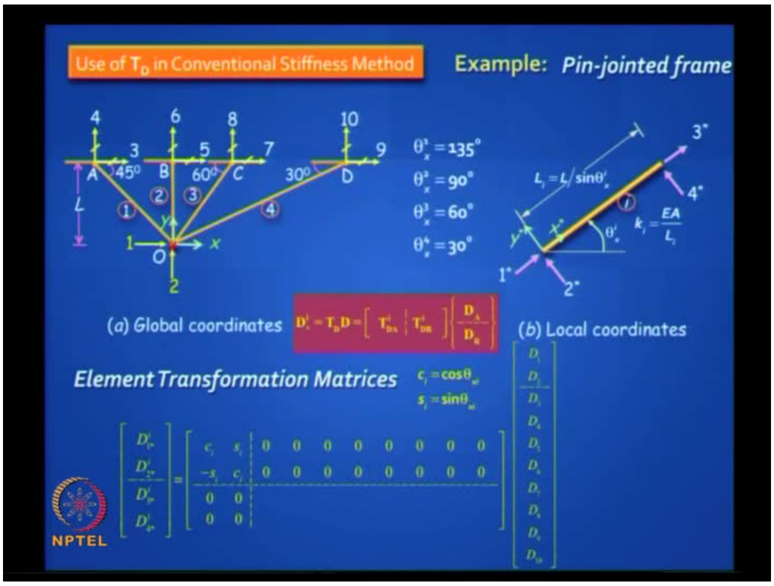

Let us, look at the same problem and here we are using the T D in the conventional stiffness method. What do you need to do? Well, I am not going to the global axis, I am sticking to my original 1 star 2 star 3 star 4 star, I am not doing any switch at this stage. Then, I need to develop this matrix right. It will look something like this ok, first of all, in this truss element; I have four degrees of freedom so, do you understand D i 1 star all the way to D i 4 star. This is by T D matrix and on the right side, I put all the elements that are there in this all the coordinates in the structure, global coordinates. There are two active coordinates and there are eight restrain coordinates right. So, the size of my T D matrix for any element i, there are four elements is 4 by, 4 by 10, 4 by 10 in case, are you getting it? It is simple, it is 4 by 10.

Now, how do most of the elements are zero. Let us prove it, now how do I fill up this column? When I if I want to fill up this column, what should I apply? D 1 equal to 1 and everything else 0. If I want to fill up this, what should I do? D 2 equal to 1 everything is 0, you want to fill up this D 3 equal to 1 everything else 0 right.

(Refer Slide Time: 24:50) Now, do not you agree that, this box will be the same as the box we got earlier, because it is a same transformation. Cos theta, sin theta, minus sin theta, cos theta right, because we are aligning to the global axis because, the global axis are pointing x in you know horizontal x and vertical y.

(Refer Slide Time: 25:16) So, I have taken a short form, I have said let C i be cos theta i, because I need not write so many letters and my matrix looks smaller and sin theta, I write as S i. So, do you agree this is so but, there is one more box of C i, S i where should that come, where depend on which of the four elements you are dealing with right. Let us take the first element, this set of C i, S i minus S i, C i will come some way here, where will it come? Will it come here? Will it come here? Will it come here? or Will it come there? Well check the coordinates, the element number one is linking 1, 2 and 3, 4 so this is 1 is not it, this is 2, this is 3, this is 4 because that is what the global coordinates point to.

So, where will it come? It will come there, for i equal to 1, so that top corner is common for all of them, because 1, 2 is common for all the four elements, in this particular example. What about for, i equal to 2, the second element; where will it go? i is now you see it is by your just kicking around a foot ball, what about i equal to 3? i equal to 4, that is it that is it.

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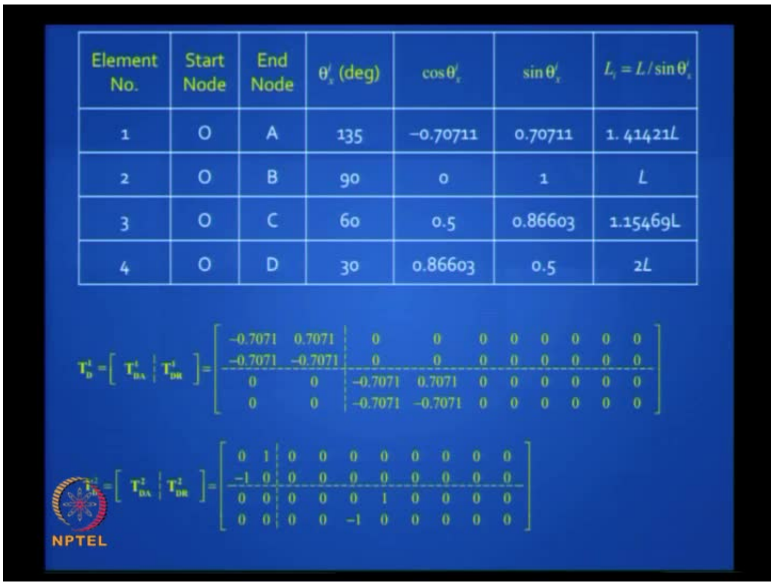

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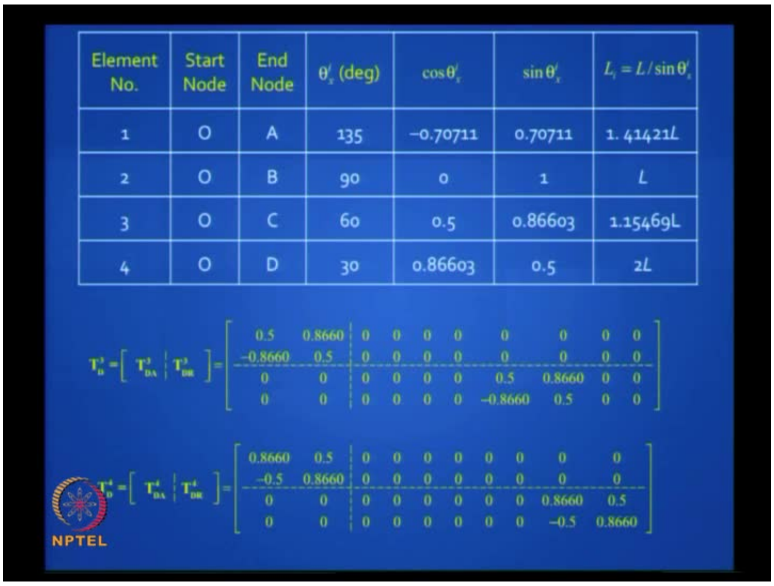

So, you got four T D matrices, for all the four elements K star is very easy, what is it? It is a same way, it is this one right. You can play the game and and now, you can actually work it out, it is very easy to do it

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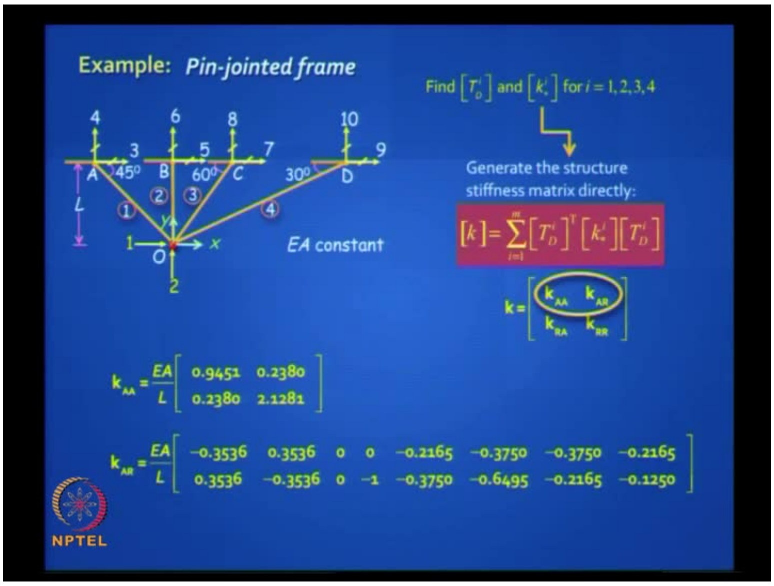

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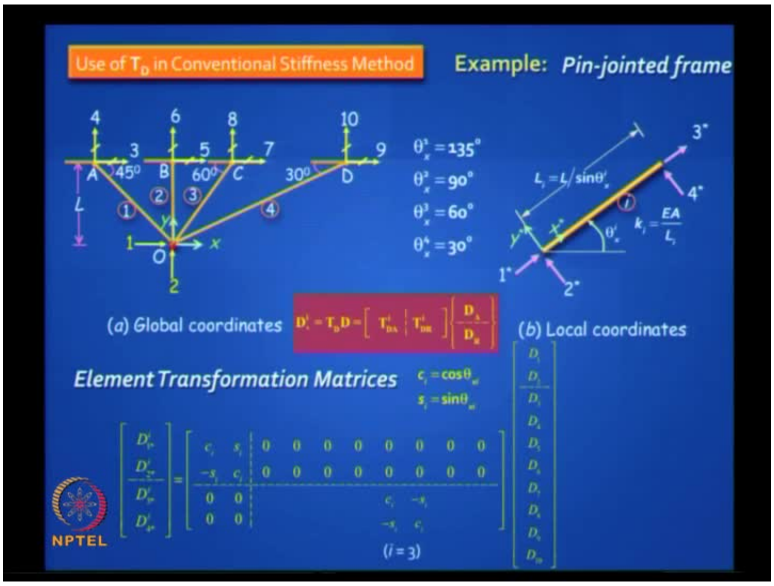

And find T D and K i as we pointed out, generate this, sum it up and you will get to exactly the same form right sure ya no no sorry sorry that that should be signs sign that is a same box which is going, that minus should not have been there agreed.

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It should be all, it is a same box well pointed out, thank you. Good, you students should have sharp eyes to catch errors right.

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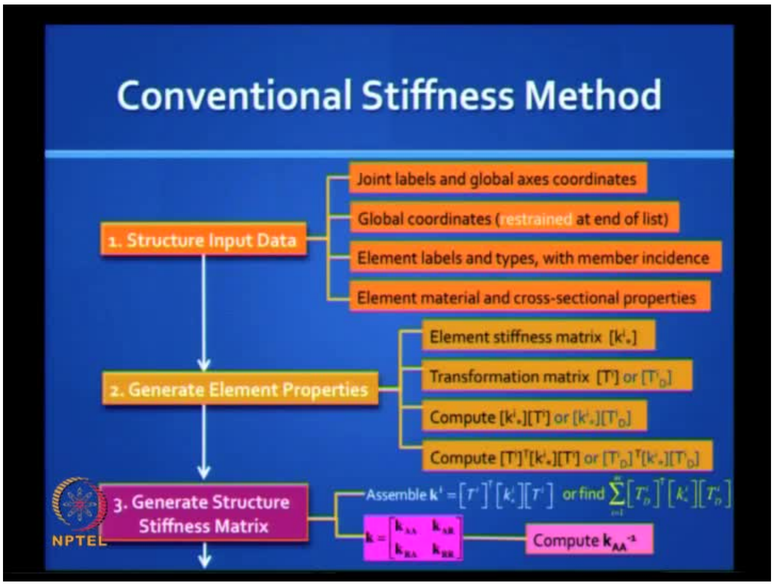

So let me sum up, because we are just showing the big frame work, we are not solving any problems. What do you do in this stiffness method? Well, your first job is to make sure that, the computer able to deal with the same structure that you have in mind.

So you have to join, you have to give information regarding the joint labels, the global axis coordinates identify the active and the restrain degrees of freedom, put them at the end of the list, give them labels and identify the incidence which is your start node, which is your end node and give the element material and cross sectional properties. In fact, if you take any software that is the first thing you need to do, you input it and once you get the coordinate, it should it should draw that structure for you to see and confirm and say yes, this is in fact, in the big software packages, you not only see the line you can do what is called an extrusion.

Let us say, you are given an eye section, you have to make sure that the eye points in the right way, I mean it it does not get flipped over to all that come, you can see clearly. So, sometime we make errors in the input but, you should be able to see using some visual C plus plus. I am not asking you do any of that but, then this is the first step.

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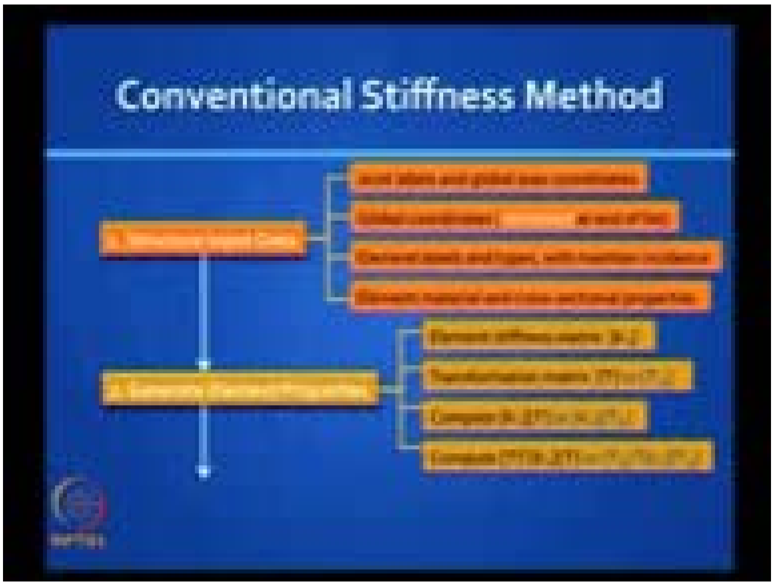

Next step; what do you think is the next step? After you have given the input, that is come much later I mean I am not saying what you have to give you are right you have to give the loads, what this going on inside, the moment you give this it is not waiting for loads. It is trying to be ready to handle your loads know so, by the time you keen the loads it would have done its homework, what is the homework? It will generate the element properties right. So, we have done that the element stiffness matrix, the transformation matrix, either you can do T i software packages do only T i but, let us say humans will do T i D, because now you are not comfortable putting those brackets and put serving them into box.

So, T D is the blind way, you know we are really covering all the global coordinates. So, what if most of the elements of that T D matrix is 0 ok then, you do the product end (Refer Slide Time: 29:51) so that, you are able to generate the structure stiffness matrix and there are two ways you can do it and finally, you find that its K A A that you need to invert, because only that forms the matrix, which will help you get the unknown D A.

So, you do not have to invert the whole K matrix for example: the truss problem only K 1 only 1 and 2 those coordinate. So, K A is really small and ah when you learn in the university, they teach only that K A A, so you do not see K A R, you do not see K R A, you do not see K R R, so you do not get confused. Now you get confused but, then you are trying to see the big picture and trying to see what is really going on, you are trying to capture the complete displacement field, the complete force field and you decide, what do you want to do?

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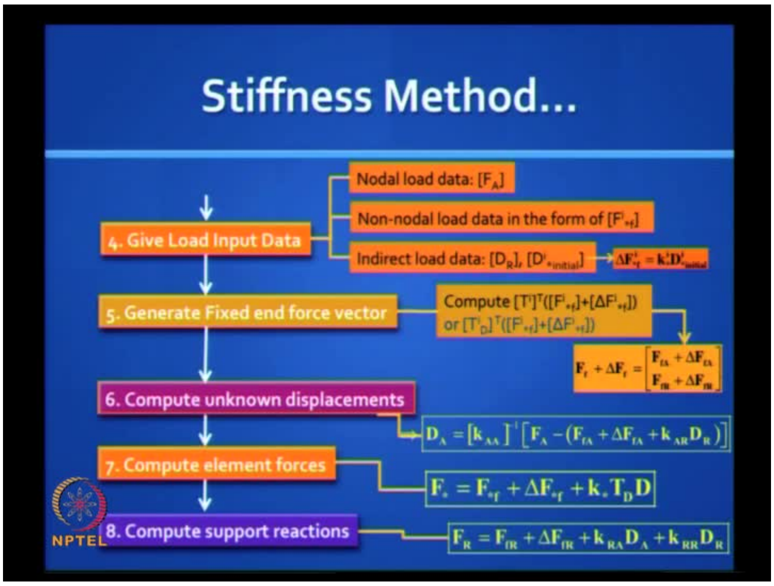

Next, you put in the load data ok and this we will see later you generate the fixed end force vector, you find the unknown displacements by using the inverted K A matrix, compute the element level forces, compute the support reactions, you got everything and all this happens in a jiffy at the press of a button but, inside the black box ah, we are trying to see what is inside the black box and how is it working?

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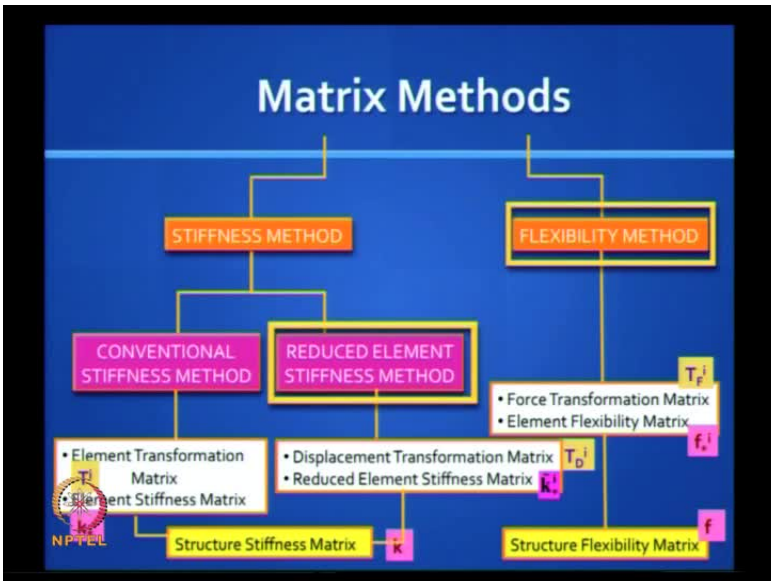

So remember, we have got a some the idea of what the conventional stiffness method is, we are now, going to look at the reduce elements stiffness method and we will also look at flexibility method in the next class ok.

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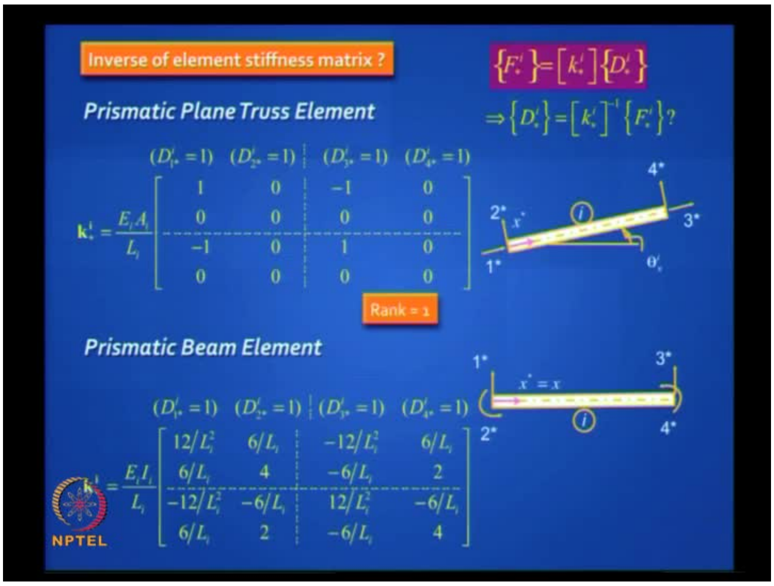

But, something is common to these and I want you to understand. In the last class, we talked of three element stiffness matrices, one was for a truss element same as the one I showed on the board, second was for a beam element and but, we said there is a problem with the rank of these matrices, what is a rank of the first matrix? 1 ok. So the question is, can you find the inverse of the element stiffness matrix? Is it possible? No no no so, is there a flexibility matrix for the element stiffness matrix obviously, because it is it is a it is singular, it is rank is not full so, you cannot invert it, does it mean there is no element stiffness matrix.

Ah I mean no element flexibility matrix, because we we have studied that the flexibility matrix is the inverse of the stiffness matrix, but clearly inverse is not possible so, what is wrong? Or what is right? Why physically, why is it not possible? That is the answer to a different question, why are you not able to find a flexibility matrix for these elements? The answer is is actually ridiculously simple but, I want you to say and mathematically it turns out. So so, your intuitive hunch is justified by a mathematic now, you do not have the intuitive hunch. You try to see the mathematics hence, and give me the the hunch behind it. Why cannot you have a flexible? Why cannot you invert this? What is a reason? I will give you a clue ya I will give you a clue.

Sir just because.

The give you the clue is, if you take the flexibility matrix, what does it tell you? If you give some forces, you can find the displacements right. If have a structure and I push it, the flexibility tell me, how much the deflection is? Now, here it will not going to tell me, what the displacements are? Why? you know the everything.

Sir.

Because, there is something when you look at those pictures, which should make you nervous, when you built such structures. Why are you not nervous? You should get nervous when you look at those elements, ah support see that is just hanging, floating in the air ya, how can you build structures like that, if you push them they will just, this is not outer space right. So, you do not get any unique results. These are unstable structures, the way we have drawn them right. That is the simple reason, they are unstable structures.

They you have rigid body moments so, you have independent displacement vectors which is fine but, you do not have independent force components in the force vector. That is a simple answer but, there are there is more to it. So this rank of this is one what is a rank of the second one ah two (())

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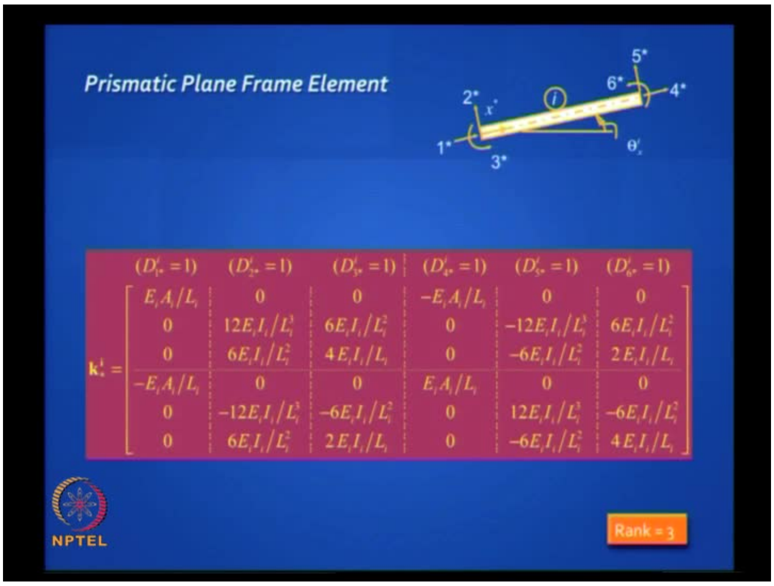

And what about this one, the plane frame element.

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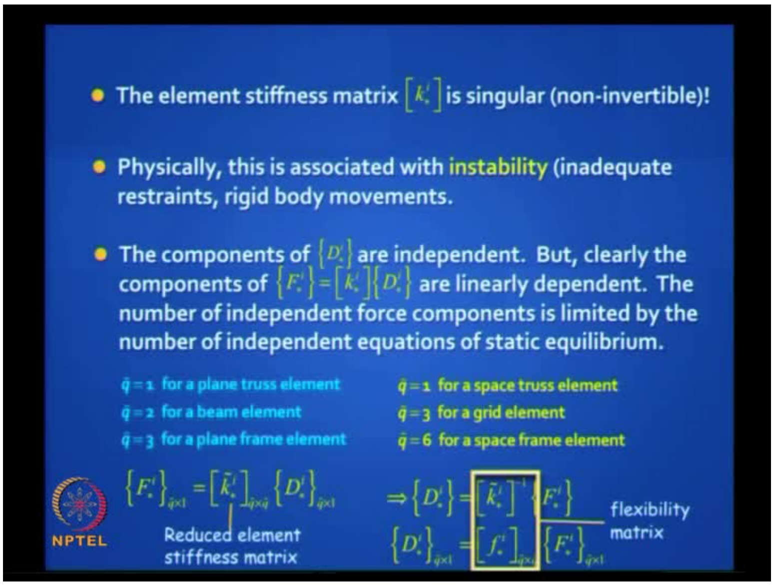

One is for the truss, its 3 ok why it is 3? You should figure out by, you can multiply you know they are all dependent, they are not independent ok. So, here is a summary of it, the element stiffness matrix is singular, non invertible. Physically, this is associated with instability, inadequate restraints, rigid body moments. The components of D i are independent but, clearly the components of F i star are linearly dependent. The number of independent force components is limited by the number of independent equations of static equilibrium so, take the truss element.

Theoretically, you have four forces but, only one equation of equilibrium right. So, that is why the rank is one that is why the rank is one so, it it makes lot of sense so, I am using the notation q for the conventional stiffness method, where q is four for the truss, plane truss element but, it reduces to 1 so, I put a q tilde that sign is called tilde q. Tilde is 2 for beam element q tilde is 3 for a plane frame element. Now, I want you to fill in the blanks. What about a space truss element? 6 by 6 is what you get, if you follow the conventional stiffness method but, if you want to what is a rank of that matrix? One chocolate for (()).

Four one six.

The plane truss 4 became 1, face truss 6 will become wrong wrong 1, argument is simple plane truss what is a equation of equilibrium? 1 because, it I tilt it and put it in space, it is still 1. Ah so you did not get it. So, it is beautiful so, who would manually ever dream of doing the conventional stiffness method to do a space truss, when the reduced element stiffness method offers you such a dramatic, drastic reduction in the quantum of work involved. So that is why, it is good for manually to study the reduce element stiffness method.

So, for a space truss it remains 1, for a grid element it is 3, for a space frame it is 6, 12 become 6 so I bring a new notation here ok the tilde is my notation for telling you that, this stiffness matrix this element stiffness matrix is different from the stiffness matrix, we talked about till now. This is what, this is dealing with the reduce size of the matrix, this has a has a has an inverse because [of] the rank is full here. This has an inverse and that inverse is called the flexibility matrix of the same size so, it is beautiful where, that is why it is good to look at the reduced elements and flexibility in one go because, the size of all the matrices are identical. Does it make sense? Ok.

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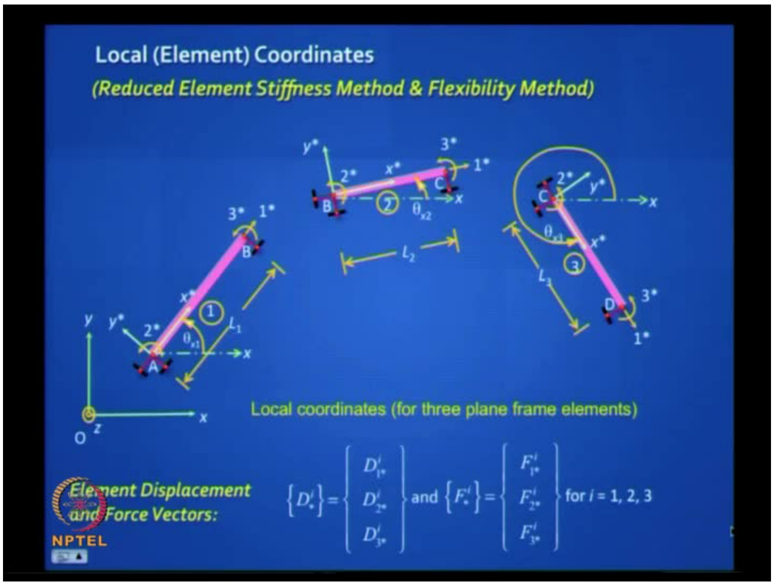

Next, so next let us look at this problem, we did this earlier right. Now, I have given three supports ok, I can give any three independent supports, you see these things and I shall made it simply supportive. These two rigid link say that, this joint cannot move, this rigid link says, it can move horizontal it is like a roller support but, it cannot move vertically.

Now, this element is stable. If I apply loads on it, it will resisters loads. So, that is a kind of element you are going to deal with from now on and actually you should be more comfortable with these visually because, they are familiar to you and you know they they you cannot play around with them, they are not mechanism, they are good solid elements. But, I have chosen simply supported, you can choose a cantilever also, you will get the same results but, we will choose it simply supported. Is it clear?

Now, how do I mark my degrees of freedom? Early I did, 1, 2, 3, 4, 5, 6 for each, how do I do it now? How many do I get for each element?

There is in one force for the first element.

I have an axial degree of freedom because, I am not assuming inextensible. If I assume inextensible that goes, so that is my 1. Then, I have two rotations so, 1 star, 2 star, 3 star much better than 1 star, 2 star, 3 star, 4 star, 5 star, 6 star right. This is the reduction; it is a reduced element stiffness matrix we are dealing with. Is it clear? Would you like to do this or the other one? This when you are doing problems for yourself, that when you want to sell your software and make money ok.

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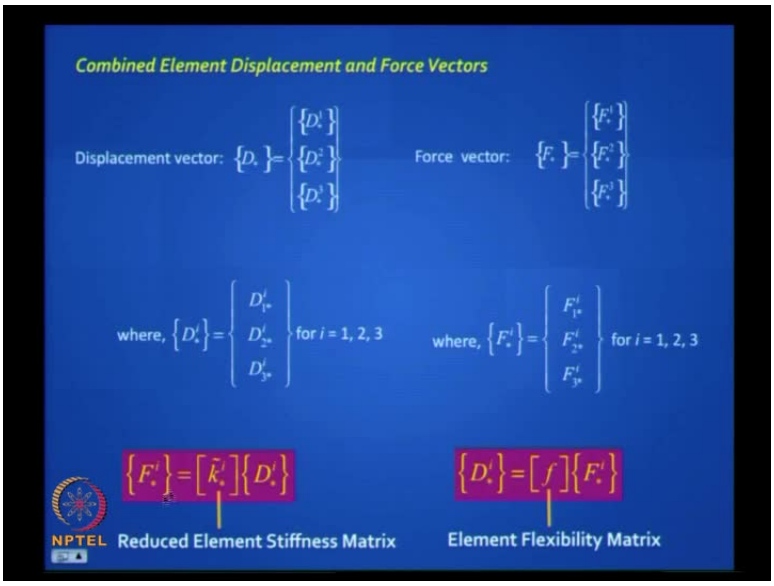

So, let us proceed so you have 1, 2, 3 everywhere right. Does it make sense? Ok these are my local coordinates, now I can put it all together and then, I get my combined element stiffness. So, for each of them it is like this got it, that 6 became 3 that is all much easier to handle and there are three elements and I can put them all together and that is my combined element stiffness matrix. Does it make sense? Everything that we did till now, accept the size is dramatically reduced and you put a tilde to make sure that, you are clear. So, I have both, I have displacement vector, force vector combined and when I I put the relationship between them, it will be the diagonal unassembled stiffness matrix or I put it the other way I I I get it in this fashion. Now, each of them at the element level will look like this ok.

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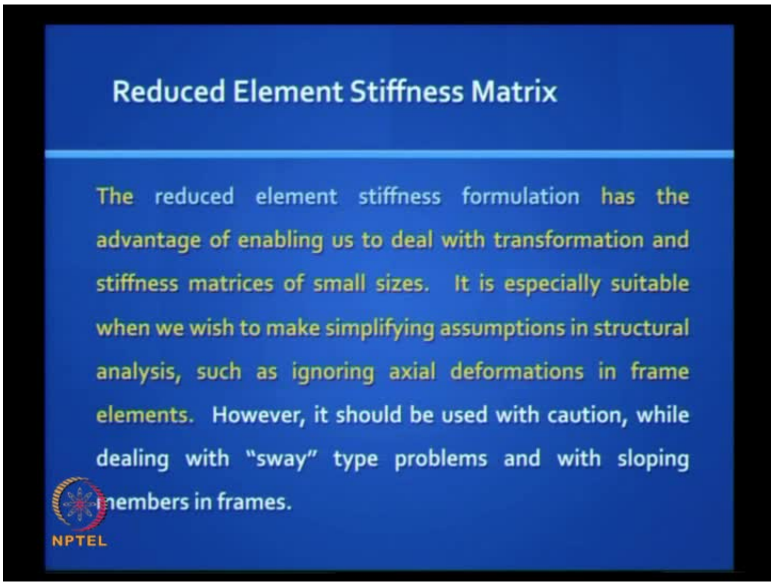

The reduced element stiffness formulation has the advantage of enabling us to deal with transformation and stiffness matrices of small sizes. It is especially suitable when we wish to make simplifying assumptions in structural analysis, such as ignoring axial deformations in frame elements right. However, it should be used with caution, when dealing with "sway" type problems and those stretchers problems; we have seen where the columns are sloping. There we have to use extra brain work not worth it might will use the software we have doing it ok.

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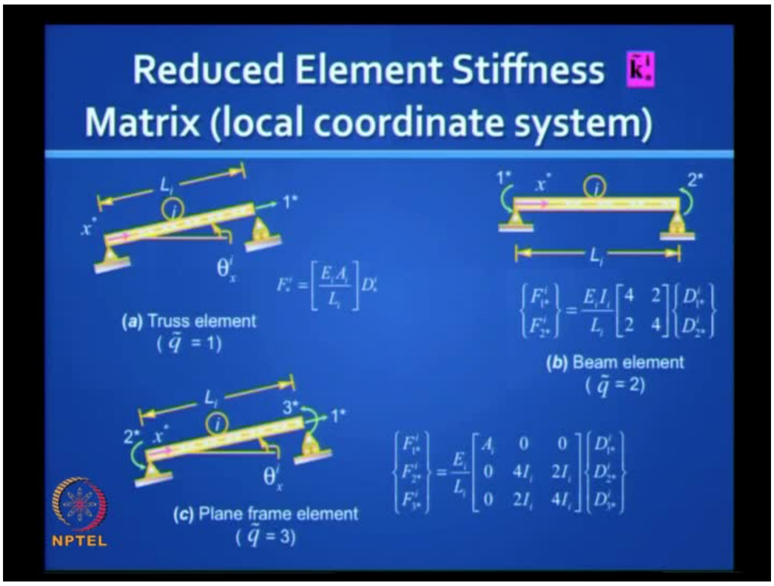

So, you must use the right instrument, for the right occasion. You do not use an elephant to drive a small nail so but, we end up doing that. So, let us look at the different types of reduce element stiffness matrices; take this, it is a 1 by 1 matrix right only one degree of freedom. What do you think the degree of freedom is? Well, I have I have shown this is simply supported by; I can pull it only in one direction. What is the displacement D 1 star for this, physically what does it mean? It is just the elongation.

So, that is easy for us, it is what we called E i earlier. Ah now, we are comfortable, we are connecting with, what we studied in structural, Structural analysis is actually very simple and you have to do not, you should not lose that simplicity. So now, we realize that is not it much better to deal with this, than with that but, you have to deal with this, when you dealing with large problem. So, you have to know both ok. That is your truss element and we call that axial stiffness, what about your beam element? To you are familiar with this, the value will be 4 E i by l 2 E i by l right, what about the plane frame three, it is nothing but, the the truss element and the beam element put together and we assume that the axial stiffness is independent.

We ignore any axial flexure interaction, that is what we do in first order analysis, will study later in the seven module there is an interaction, because it behave like a beam column right. Now, we pretend there is no interaction, not only we, all this have software packages also happily pretend that, there is no interaction but, actually you have something called a P delta effect which we will see later, that is called second order analysis. Does we make sense? Ok

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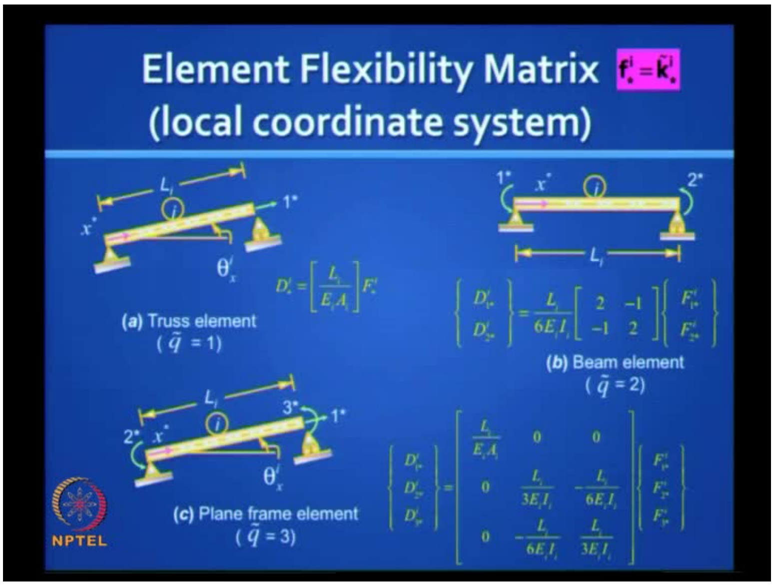

What about the flexibility matrix? That is very easy, I put it this way, I flip it over and I call this axial flexibility and look at this this makes sense, remember if I apply a moment here, what is a rotation I get there? L by 3 E i and what do I get here? L by 6 E i with an opposite sign that is what this matrix tells you. So, it is not difficult and by the way, it is an inverse of the previous one and similarly, this works out.

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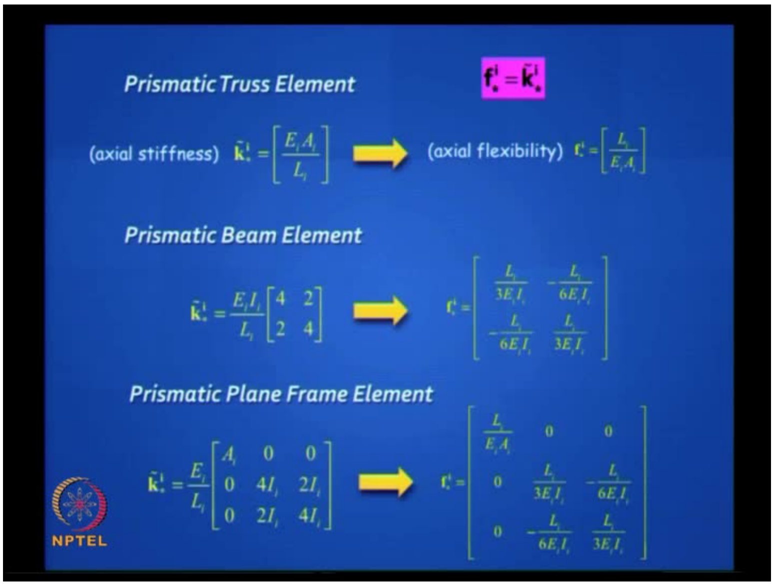

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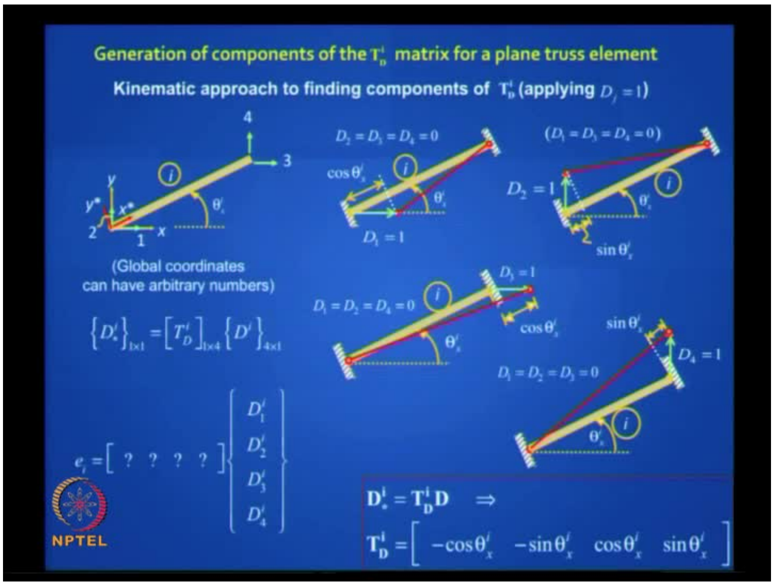

So, if you want to put it all together. These are the three values for stiffnesses and these are three values for flexibility, very easy and it you know you already learnt this and manually, you can do this but, you should also know the other one. Ah how do I generate the components of this matrix for a plane truss element? I will I will end with this definition ok.

So, let us take a truss element like that. How do I get the T D matrix for this truss element? Well, now I realize there is only one degree of freedom though though in my global structure, I may 1, 2, 3, 4, now the 1, 2, 3, 4 I have shown for convenience, it may actually be 5, 6, 21, 22, in the actual structure. So, those are happening there and but, the elongation is given by, how much these move, so this 1, 2, 3, 4 is arbitrary now, basically I am I want to know, if these joints move by some known amounts, how much elongation do I get? That is a meaning of of this T D i, T D i is just how much elongation the i th element got it.

So, the elongation due to D i 1 actually, this this i can be removed. It can be just D 1 because, it is a global thing. How much is that? So, you do you did not put i here ok, this i can be removed but, it is linked to the other i, in the conventional stiffness method ok.

What are these values? How do you work them out? Well, let us do it from first principle you you put D 1 equal to 1 ok. How much will be the elongation, the component of that that is cos theta, will be plus or minus elongation remain as you put D 2 equal to 1. It will be, it is cannot you see this length of the element is reducing, it be sin, it will be minus sin theta. If we take the other end and you pull it, horizontally and you lift it up right, that is your transformation matrices. Does it make sense?

That is the answer to these question marks, we will stop here. Have you understood today's class? A little better understanding of yesterday's class right, so you will find that, with subsequent classes this knowledge will get solidify but, you have to do some reading on your own. Thank you.