Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of Technology, Madras

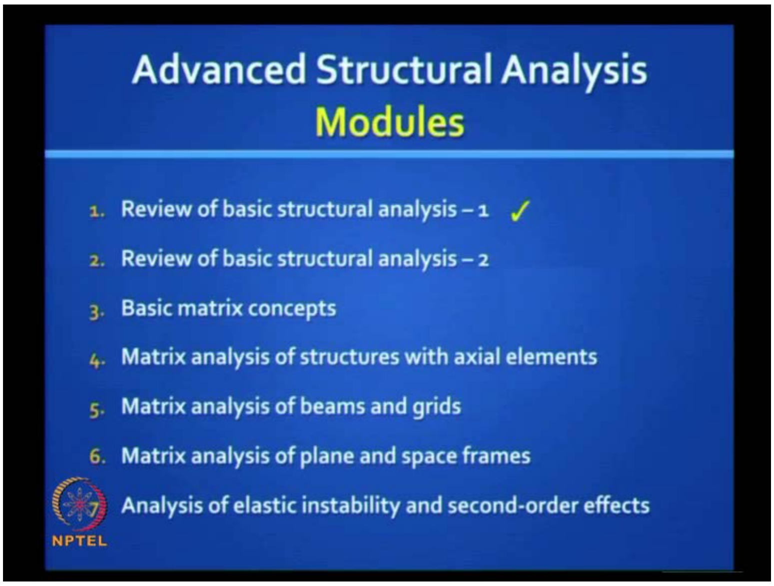
Module No. # 1.2

Lecture No. # 02

Review of Basic Structural Analysis-1

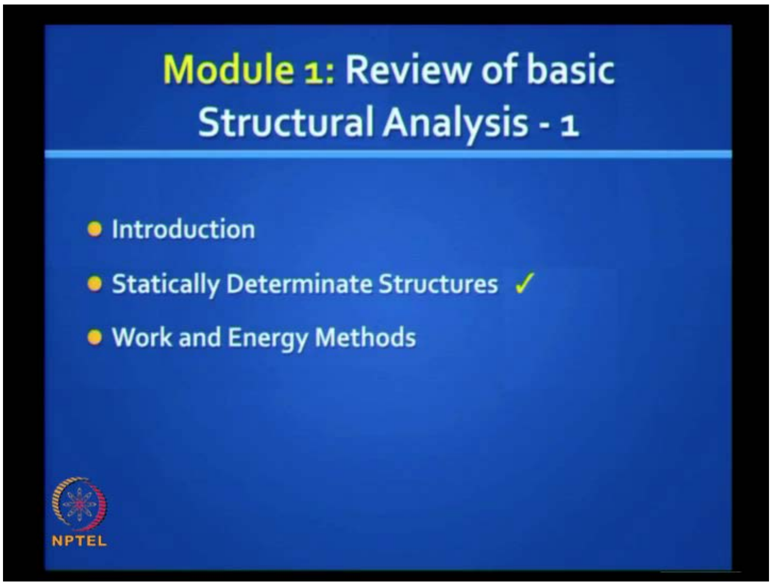
Good morning to you. We are now on to the second lecture in this course on Advanced Structural Analysis. We will be continuing reviewing basic structural analysis.

(Refer Slide Time: 00:26)



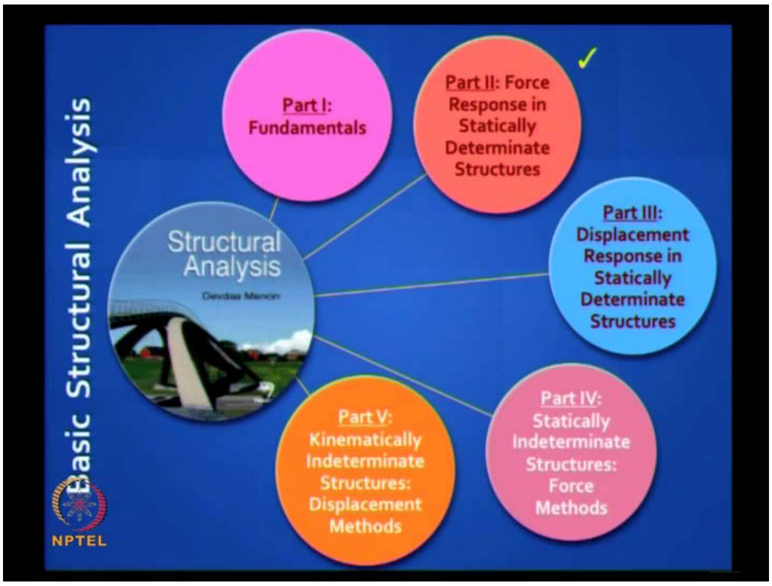
These are the 7 modules. We are on module 1.

(Refer Slide Time: 00:30)



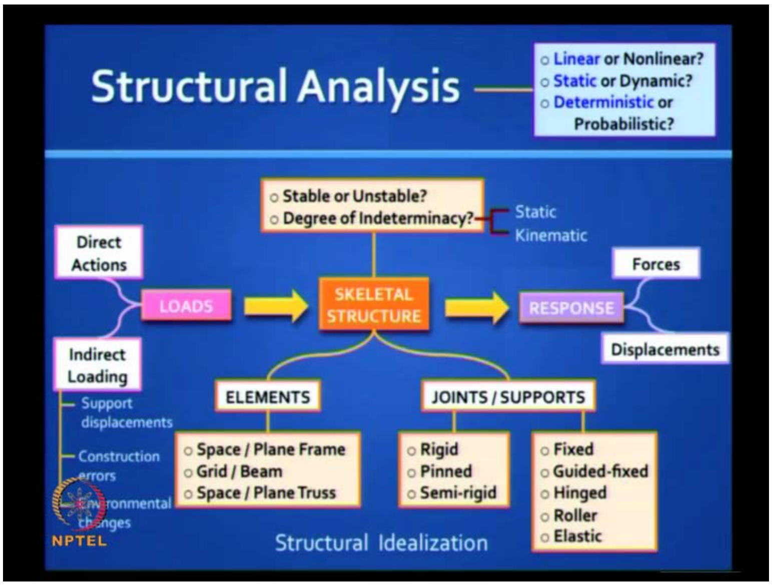
In this module 1, we basically have three parts; we covered the introduction yesterday. Today, we will look at statically determinate structures. In the next class, we will review work and energy methods.

(Refer Slide Time: 00:43)



As I mentioned earlier, the textbook that we will be consistently referring to is ‘Structural Analysis’, which is authored by me. That book has five parts. We finished with part I in the session yesterday. Today, hopefully, we will try to cover most of part II in a quick manner. Let us come back to the big picture.

(Refer Slide Time: 01:13)



We are dealing with a structure. We are actually dealing with a model of a structure. In this course in structural analysis, our structure will be a skeletal structure and we are removing the spatial elements from this structure. The structure is made up of elements and the elements could be space frame elements, plane frame elements, grid or beam elements, or truss elements. Of course, the most generic is the space frame element and all other elements are special cases of that one element. The elements are interconnected with joints. Those joints are modeled usually as rigid joints or pin joints – we have discussed these.

You can also have semi-rigid joints; at the boundaries of the structure, you have supports and it is important to identify them correctly. There are different types and the most generic is elastic. When you restrain movements, you can make it fixed or guided fixed or hinged or roller. On this structure, we apply loads. The loads are of two types: direct actions, which you indicate with those arrow marks or you can have indirect loads. There are three types as we discussed: you have support settlements, you could have construction errors, or you could have environmental changes. We are interested in finding the response of the structure, both the force response and to some extent the displacement response.

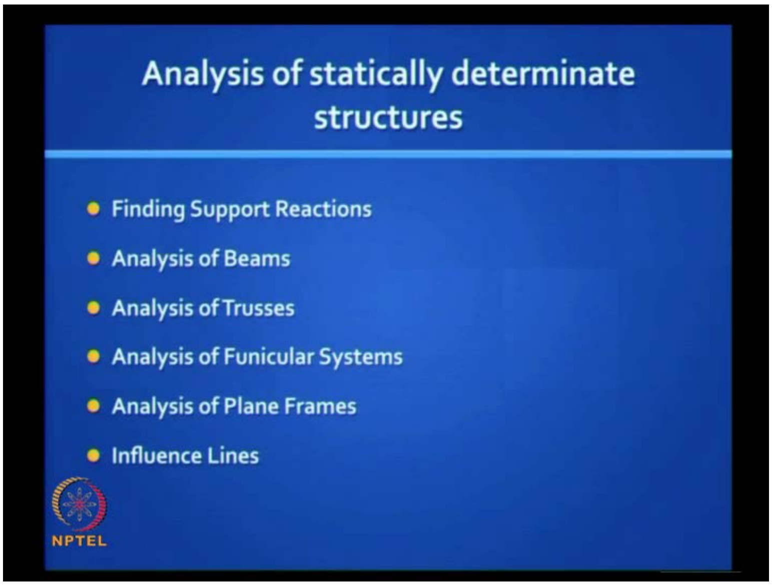
If you look at the structure in its entirety, you have two fields. You have the force field and some of the items in the force field come from the loads which you know, but many of the items you do not know and they come from the response; they include the support reactions and the internal forces. You also have the displacement field, which is related to the force field. Most of the displacements are unknown and so they are in the response side, but sometimes, displacements come as a kind of indirect loading and so they come to the load side. Now, most structures are indeterminate; they should be stable definitely and the indeterminacy could be either viewed for the purpose of analysis as static indeterminacy or kinematic indeterminacy.

You have a choice in your solution path of indeterminate structures. You could use the force method of analysis or the displacement method of analysis. If you were to do things on your own, manually without the help of a computer, then you would obviously choose that path which involves less effort. So, you have to look at the degree of indeterminacy, which is the degree of static indeterminacy versus the degree of kinematic indeterminacy and of course, you have to decide on what level you want to do this analysis.

Do you need to do a non-linear analysis, which is a little complicated as you know or is it enough to do a linear analysis? Do you need to do a dynamic analysis or is it enough to do a static analysis? Do you need to do a probabilistic or stochastic analysis, where you have uncertainties? You have uncertainties in the loads for sure. You also have uncertainties in the material properties in the structure and therefore, you will have uncertainties in the response. Or shall we assume that everything is deterministic? Well, in this course in structural analysis, we take the simplest form of analysis, that is, we assume that the analysis is static, it is linear, and it is deterministic. That is a starting point and it is good enough in practice.

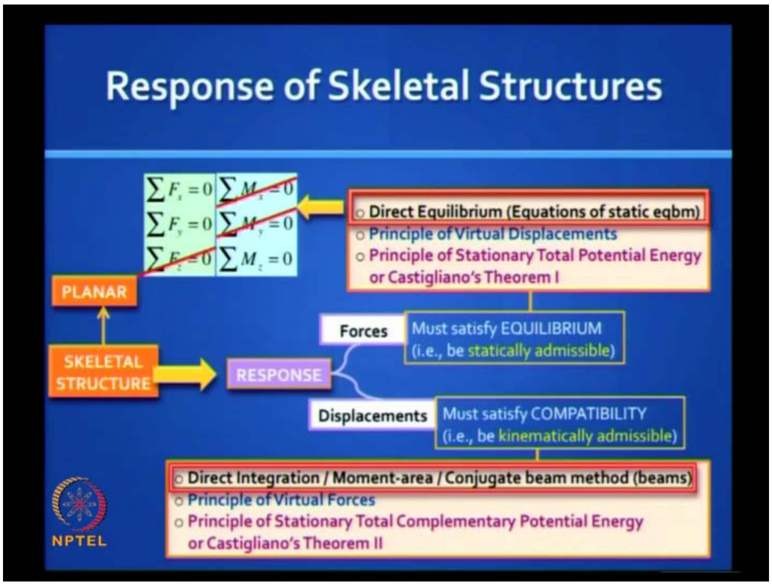
We should also know in advanced courses how to do non-linear analysis. We will get a taste of it in the seventh module of this course. How to do dynamic analysis? Often, we convert dynamic loads to equivalent static loads or we apply factors called impact factors or dynamic amplification factors to take care of the amplification caused by the dynamism; we take care of the uncertainties through load factors in design. So, if you are really skilled at analysis, you should reach to a level where you can do non-linear dynamic probabilistic analysis, but we are beginners and so we should first learn how to do linear static deterministic analysis.

(Refer Slide Time: 05:50)



This is the scope that is covered in analysis of determinate structures. We all begin with finding support reactions, which we know well enough. We should know how to analyze beams, find out bending moment and shear forces as well as find deflections and slopes. We should know how to analyze trusses, analyze funicular systems like cables and ideal arches, analyze plane frames, and learn how to draw influence lines; we have done all this. So, here, we will just do a quick review or quick overview of some of these topics.

(Refer Slide Time: 06:31)



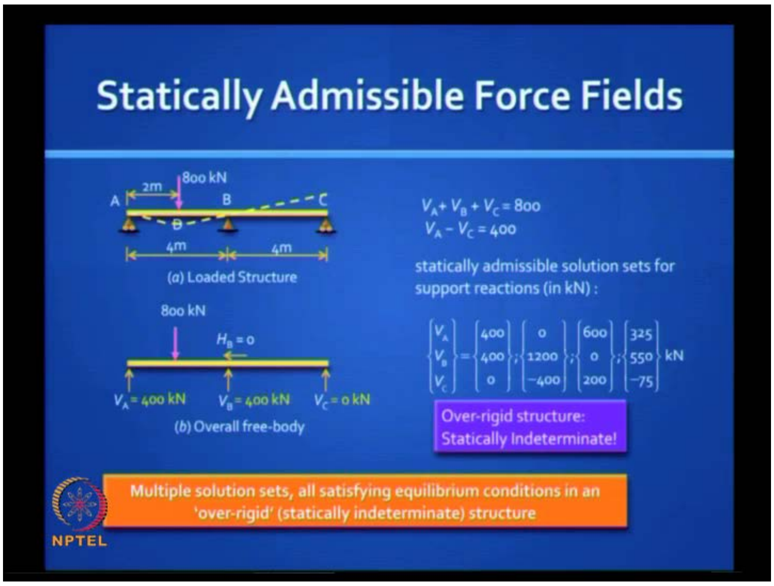
Now, if you look closely at the response of the structure, the force response and the displacement response, you will find that the correct response – the force response – must satisfy equilibrium, which means it must be statically admissible and the displacement response must be compatible. That preserves the integrity of the structure and the elements are interconnected the way they are supposed to be. If the joints are rigid, then, the angle between the connecting elements does not change. So, your correct response must be both statically and kinematically admissible. There is a third requirement; that is, the force displacement relationships should be satisfied, which in turn depends on the constitutive relationships in the material, the stress-strain relationships and so on.

You can satisfy equilibrium in a variety of ways. The way that we normally refer to is direct equilibrium (Refer Slide Time: 07:35). You are familiar with this; this comes from Newton's law. The resultant force on that structure is supposed to be 0 and if you interpret that in terms of scalar quantities in Cartesian coordinates, you end up with six equations of equilibrium: sigma F_x equal to 0, sigma F_y equal to 0, sigma F_z equal to 0, sigma M_x equal to 0, sigma M_y equal to 0, and sigma M_z equal to 0. This is called direct equilibrium; it is very straightforward and elementary.

If we are dealing with a planar structure and most of our example structures are planar, which means all the elements in the structures lie in one plane and the loads also act on that plane, then we can simplify and eliminate some of these equations. Which of these equations would you eliminate? ((.)) Let us assume that the structure is in the xy plane, and so, the z, f_z does not need to be applied. What else can you get rid of? M_x and M_y. M_x and M_y, because you are taking moments about the z-axis, which is in the plane of xy.

So, if you are dealing with a planar structure, for every free body or for the overall free body, your number of equations reduces from six to three. We will also look at simple ways of calculating deflections in beams. You are familiar with the direct integration method, the moment area method, and the conjugate beam method. In a subsequent class, we will look at the principle of virtual work, which can also be used to find the force response and the displacement response; we can also use the energy theorem to do exactly the same, but that is something we will reserve for later. Right now, let us do the first thing that we normally learn in structural analysis.

(Refer Slide Time: 09:41)



Here is the example of a two-span continuous beam. As you can see, there are four reactions possible: three vertical reactions and one horizontal. That horizontal reaction is clearly equal to 0 if you apply sigma F_x equal to 0, but you have three vertical reactions and you have only two equations of equilibrium left. You can mix and match the equations, but you have only two independent equations. If you apply sigma F_y equal to 0, it would turn out that V_A plus V_B plus V_C will add up to the applied load 800 kilonewton and if you take moments about the point B, for example, you can show that V_A minus V_C is 400. That is all that you get.

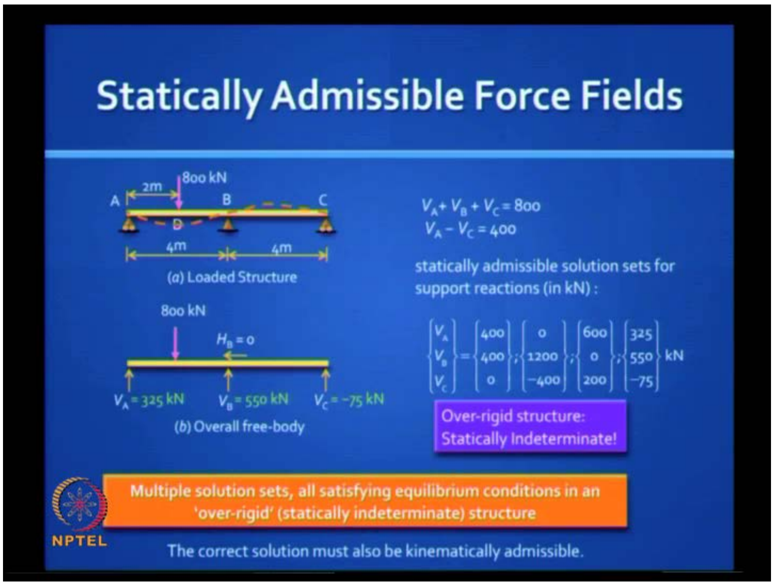
You can write down more equations but they will be linearly dependent. You cannot get…. This is not enough to solve the problem. You can imagine having multiple solutions. For example, I have shown you here four possible solutions (Refer Slide Time: 10:52). Each could be correct because all these solutions satisfy those two equations. We have three unknowns and two equations; so, you can have multiple solution sets.

Which of these is correct? I am sure each one of you in this room, about 70 to 75 students here, can come up with your own unique combination of these solution sets. How do I know which one is correct or closest to the truth? Is there a simple way to find out? Can I ask all of you to do some extra effort and then from that I will be able to judge which is closest to the correct solution? What is the additional quantity that I can ask you all to calculate? Compatibility. Compatibility is a big thing. Displacement, deflection, energy. Energy. If I ask you all to calculate the strain energy in the beam, once you have these forces, you have the bending moments; once you have the bending distribution, you can calculate the strain energy, which is an internal energy – internal elastic energy; so, you will all have different values of strain energy. Which do you think is closest to the correct solution? The one which is least.

You know, there are powerful ways of doing this, but the other thing to do as some of few suggested is to check whether you satisfy compatibility or not. Now, very clearly, the first solution for example, 400, 400, 0 will not satisfy compatibility because certainly V_C cannot be 0 if that contact is established between the support and the beam. The only way that could turn out to be a correct solution is if there is no contact, which means you will probably get a deflected shape which looks like this (Refer Slide Time: 13:02).

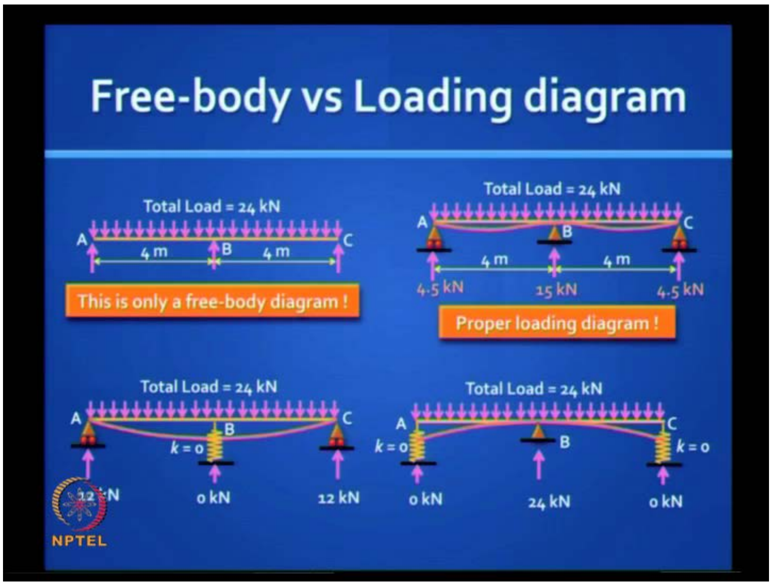
Can you see the first deflection shape – that dotted line? That means there is no contact. It is like there are supports only at A and B and the beam will definitely lift off from the support C. Then, you could get this solution; there is nothing wrong with that, but it is not kinematically admissible.

(Refer Slide Time: 13:24)



So, the correct solution which you will learn to do is one in which the beam comes back to the support C. You will find there is an uplift at that location and you have a negative force, which means a force acting essentially downward, and we can prove these numbers later. The correct solution must not only satisfy equilibrium, but it must also satisfy compatibility. You will find that if your structure is statically determinate, you do not need to explicitly satisfy compatibility; it gets automatically satisfied. It is enough to satisfy equilibrium and you can prove this. We will come to the proof when we study matrix methods subsequently.

(Refer Slide Time: 14:10)

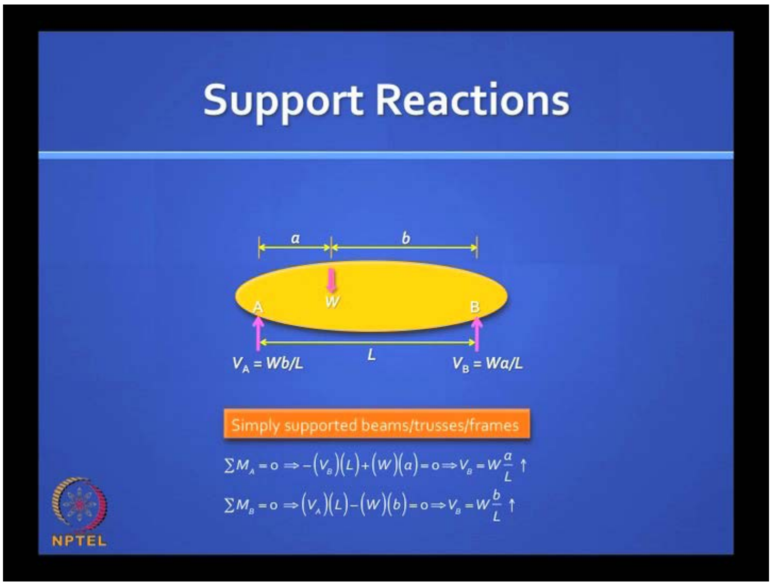


At this point, I want you to realize that when we draw diagrams like this, we tacitly assume that A, B, and C are support locations; that is, these are reactions, but this diagram does not reveal anything and it is necessary to know how to distinguish between a free-body diagram and a loading diagram. This is only a free-body diagram. If you want a loading diagram, then this would be the correct picture to draw (Refer Slide Time: 14:38).

You do not show the support reactions, but you show the supports. The big difference between this diagram and the previous diagram is that this diagram has with it an associated deflection diagram. That means whatever deflected shape you draw, you must show that the deflection is 0 at A, B, and C and that there is a slope compatibility at B. This is an important distinction to draw because for the same free-body diagram, I can have multiple loading diagrams.

For example, this is the correct solution for this problem, but look at this picture (Refer Slide Time: 15:23). Here, I put an elastic spring at the middle and if that spring stiffness is 0, it is a simply supported beam and for that free-body diagram, this is perfectly acceptable or you could have something like this (Refer Slide Time: 15:39). You do not have any supports at A and C, the springs are very flexible, and you still satisfy that free-body diagram. So, this is an important distinction to make note of.

(Refer Slide Time: 15:52)

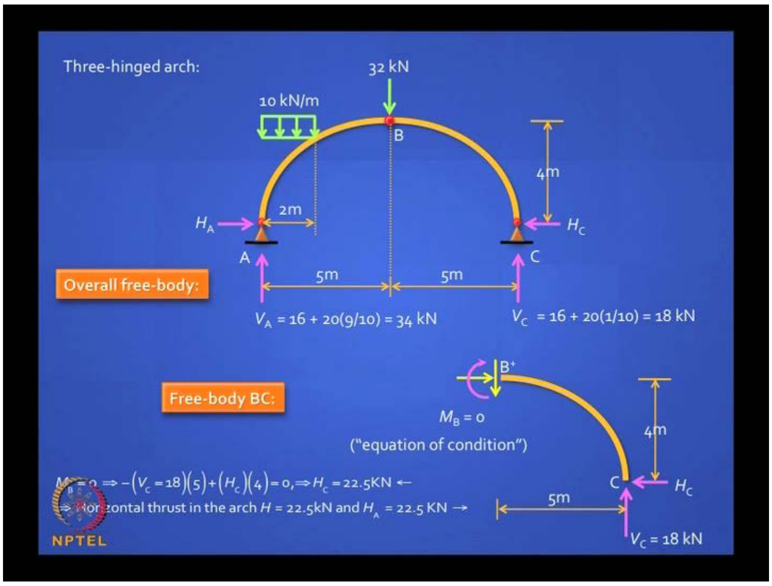


We start with support reactions. We begin with a simply supported beam. A simply supported beam is one in which you want stability. So, you need to have three constraints in a planar structure. Essentially, you have two vertical reactions and one horizontal reaction; if there is no horizontal load applied to that system, that horizontal reaction is 0. Essentially, you have two vertical reactions and you could have any loading. What I have marked here, W, is the resultant load.

Now, the structure that you put on top of the supports could be a beam, could be a truss, could be a frame, could be an arch, could be anything, could be a boulder like we have shown here, but the reactions do not change. The reactions are statically determinate and they are very easy to determine. If you mark the left side as A and the right as B and then if you take moments about A, you can find the reaction at B; it is very straightforward. If you want to find the reaction at A, take moments about B.

Intuitively, you know these results are correct because the load W will be shared by V_A and V_B in some proportion; it is in proportion to the distances. So, for V_A, the distance b by L, the ratio b by L is a fraction of the load W and for reaction at B, it is a by L. This is an intuitive, simple way of understanding a simply supported beam reaction.

(Refer Slide Time: 17:31)



Let us take a slightly more complicated problem. This is not simply supported. Here, you have two hinge supports, which means you have the possibility of a horizontal reaction coming in. This would be normally statically indeterminate or over-rigid, but for the fact that there is an internal hinge provided at the crown. What is this kind of arch called? Hinged arch. It is called as three-hinged arch. You give a hinge and earlier they did this deliberately. Why did they do this deliberately? To avoid indeterminacy.

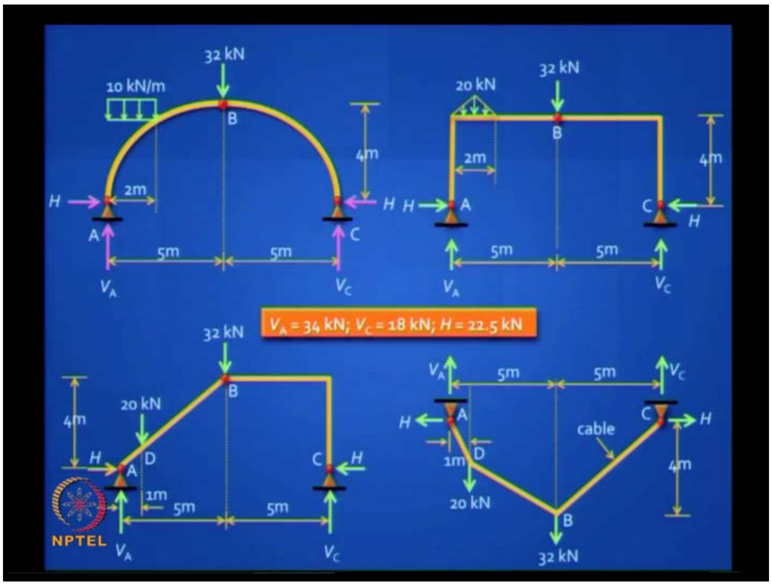
Why were traditionally people afraid of dealing with indeterminacy? How to compute this. Well, they did not know how to analyze indeterminate structures for many centuries, but even after they did, they still avoided it. In fact, if you take our own country, in India, most of the bridges even today are simply supported, they are not continuous, they are just rigid. Why is that? Support settlements.

It is because you have a possibility of indirect loading and unless you are absolutely sure that there is no relativity support settlement, which is possible if you are on very hard strata or you are on piles or you are able to handle temperature and shrinkage and other associated problems, then you can confidently go ahead with this because we have the tools to analyze these structures. That hinge at B gives you an additional equation. What is that equation that you will get? Moment at B is 0. Moment any point on that screen is 0. So, do not say…. The bending moment inside the structure in the arch at B is 0; there is no moment transfer possible.

To really take use of that condition, it is called an equation of condition.… Well, in any case, the vertical reactions are very easy to compute because that 32 kilonewton load in the middle will get shared equally and that 10 into 2, that is, 20 kilonewton load will get shared in a proportion of V_B by L, V_A by L. The vertical reactions are determinate. The horizontal reaction is what we wish to now find out.

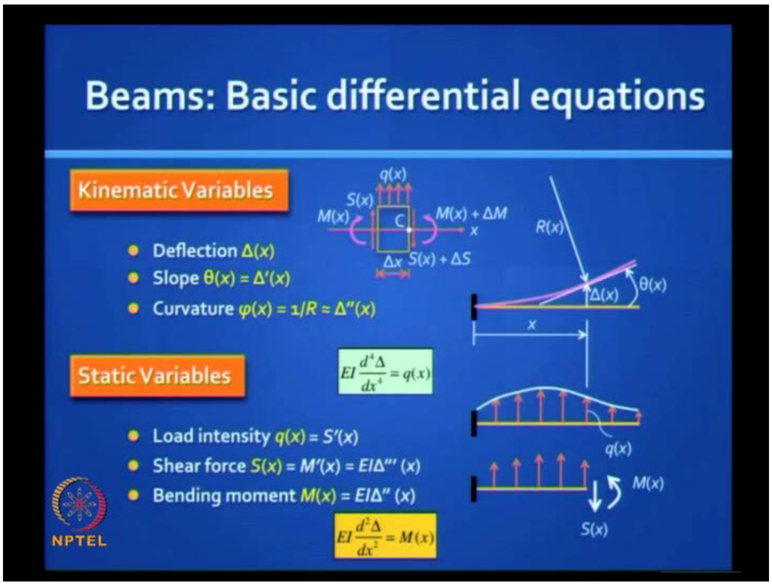
You take a free body; you can take either the left or the right. We are taking the right. A free body is more convenient because you do not have any loads to be shown in that portion and then you invoke the condition that the bending moment at B is 0. It is called an equation of condition and you can easily solve and get the horizontal reaction. This is the solution for a typical problem like this, but there is nothing unique about that arch.

(Refer Slide Time: 20:29)



We found that the solutions were V_A equal to 34 kilonewton, V_C equal to 18 kilonewton, and the horizontal reactions are 22.5 for this structure. Now, take a look at this structure (Refer Slide Time: 20:44). This is a three-hinged portal frame with the same height overall, the same span, and the same loading located at the same locations. You will find that these solutions that you have calculated for the arch also hold good for this frame and also hold good for this structure, where AB is inclined. As long as the loads are at the same place and they are the same and the distances overall are the same, you get the same reactions. So, that is an important point to note. When you have a statically determinate system, the type of structure is not important. It could be an arch, it could be a frame, it could be a beam and it could be a cable.

(Refer Slide Time: 21:34)



We finished support reactions by direct equilibrium. We go to the next topic which is beams and we will touch on the essential points. First, let us look at the basic kinematic variables. What are the kinematic variables which define the displacement in a beam? Deflection, slope and curvature. They are deflection, slope and curvature. If you take a curved shape of that cantilever beam as shown, we will assume deflection delta x to be positive if pointing upwards. It really does not matter, but for consistency let us assume upward deflection is positive.

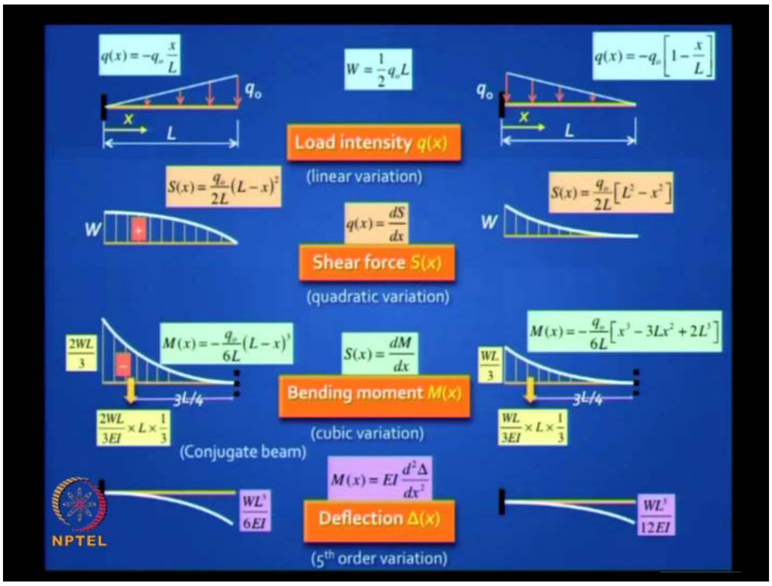
Then, the slope of this is called slope or it is also called rotation; it is a derivative of that deflection. The derivative of the slope turns out to be approximately equal to the curvature. The curvature is actually 1 divided by the radius of curvature at that point. You know that for small deformations, it turns out to be approximately equal to delta double dash; this is something we begin with.

Then, you have static variables. What are the static variables for a beam? You have the load intensity, which is usually given to you, shear force and bending moment. Again, these are the sign conventions we will assume. q of x is a load intensity usually measured in kilonewton per meter. I want you to understand that when the intensity change is along the span, you cannot afford to talk about meter. It is actually more correct to use the language newton per millimeter because it can change every millimeter, but it is equivalent to kilonewton per meter; that is important to note. That is q of x, the derivative.

How is it related? What is the relationship? What are the basic differential equations that relate these three quantities? Shear force is the derivative of bending moment. You can prove this by taking a small element of the beam and just applying direct equilibrium and you can prove that the load intensity is equal to the derivative of the shear force. The shear force is a derivative of the bending moment. What else can you prove? Bending moment is related to the kinematic variables. How? EI, flexural rigidity. That is right. If you invoke simple bending theory, the Euler–Bernoulli principle, M by I is equal to E by R. 1 by r is curvature and so, you can prove that bending moment is nothing but EI into delta double dash of phi, whichever you wish to say.

That is the relationship between static variables and kinematic variables. Actually, you can express all these static variables in terms of deflection. EI is called the flexural rigidity. It is a product of E, which is the elastic modulus assuming homogeneous elastic material, and I, which is the second moment of area; it is an area property. You can show that the shear force is EI into delta triple dash and therefore, q is EI into the fourth derivative of the deflection. If someone asks you what the basic differential equation for a beam is, you can either choose to explain it this way – EI into fourth derivative of deflection is the load intensity or you could say EI into the curvature is bending moment; both are correct.

(Refer Slide Time: 25:29)

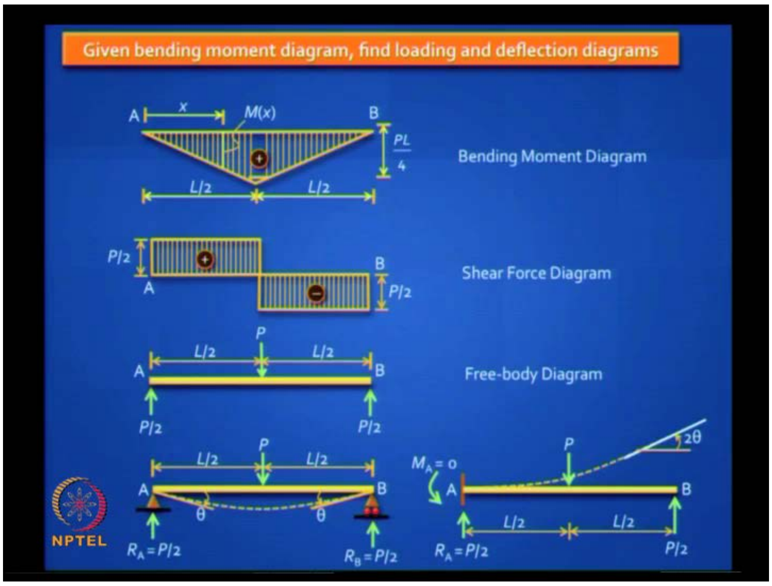


Let us quickly assimilate how these concepts can be applied to drawing bending moment, shear force and deflection diagram. I have shown here two cantilever beams with linearly varying load. You can write mathematical equations, the total load is W and you can write q of x. I put a negative sign because in the sign convention I have shown you, q is positive when pointing upwards, but if it is a gravity load, it will point downwards and so I put a minus sign there. Now, I am going to draw two shapes of the shear force diagram, which is related to the intensity diagram. This is one shape and this is another shape (Refer Slide Time: 26:07).

Let us say the top shape is a and the lower one is b. Which of these two shapes is correct for that first cantilever beam? a. a is correct. Why is a correct? ((.)) You have to look at the slopes; you have to look at that relationship q is equal to dS by dx. Let me use this pointer. You will find that in the slope of the shear force diagram – here, the slope is 0, it is tangential and it must be the load intensity which is 0, whereas in the lower case, there is a definite slope. Actually, this diagram belongs there for the other beam (Refer Slide Time: 26:59). You can see where the slope is 0, where the load intensity is 0. Where the slope is maximum, there actually the load intensity is maximum. It is a very easy way of understanding the relationship and you can write equations if you wish.

What about the bending moment diagram? Well, it takes its shape, the slope of the bending moment diagram is a shear force and the slope of the bending moment diagram is similar to the shape of the curvature diagram. All you need to do is to divide the bending moment diagram by EI and you have got the curvature diagram. You integrate the curvature diagram twice and you get the deflection diagram after you apply the boundary condition. Here, if you have a linearly varying load intensity, your shear force will vary quadratically, your bending moment will vary with a cubic variation and therefore, your curvature will also have a cubic variation. Therefore, your slope or rotation will have a fourth order of variation and your deflection will have a fifth order of variation. Actually, you can get these quantities by just integrating, but there is an easy method of finding deflections and that is a conjugate beam method, which we will look at shortly.

(Refer Slide Time: 28:24)



Normally, you are given problems where you are given the load intensity and you have to derive everything. You have to derive the shear force, bending moment, deflection, and so on, but it is good to tease yourself with a different kind of problem, where you are given a bending moment diagram. For example, this diagram (Refer Slide Time: 29:04). It is a simple diagram, a triangular shape and you are asked to predict the loading diagram, what load could have caused this. What load could have caused this?

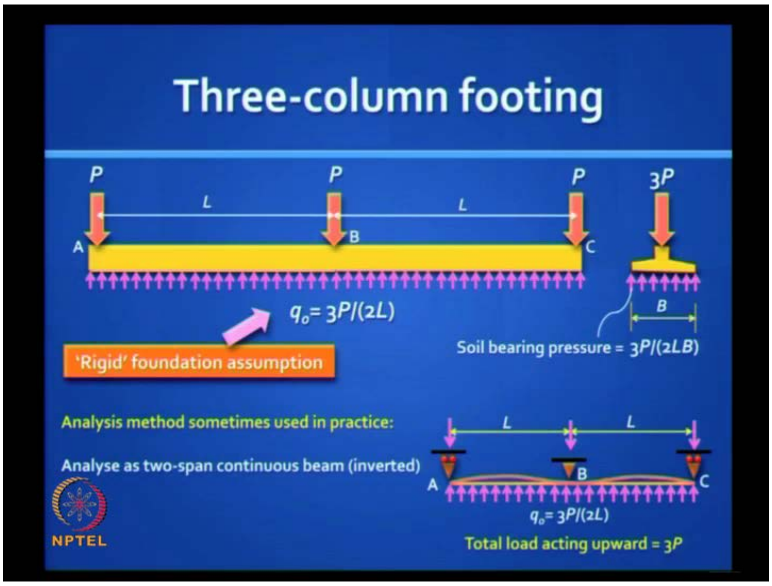
Simply supported beam with a concentrated load in the middle. Simply supported beam with a concentrated load is the standard answer everybody will give because from the bending moment diagram, you can pull out the shear force diagram; from the shear force diagram, you can pull out the free-body diagram, which is not the same as loading diagram and from the free-body diagram, you can pull out how many loading diagrams? Any number. The simply supported is one case, which is the easiest to understand and most familiar to us.

This is what you said. Can we have a cantilever beam, for example? Yes. Why not? You can have a cantilever beam here but this is not complete; you must also have this (Refer Slide Time: 29:41). So, the free-body diagram shows you three arrow marks: P, P by 2, and P by 2. It is your choice which of them you want to treat as loads and which you want to treat as reactions because the free body does not change.

You can have multiple loading diagrams having the same free-body diagram. It is interesting to take this one step further and see how the deflected shape changes. For example, this would be the deflected shape for the simply supported beam; it will be symmetric (Refer Slide Time: 30:15). What would it be for the cantilever? ((.)) You are right.

You will find the deflected shape for the cantilever will be something like that and these two deflected shapes are related. Why are they related? Same free-body diagram ((.)). You are dealing with a kinematic variable and so you must talk kinematic language; do not talk about bending moments and free bodies. Same curvatures – they have the same curvatures; at every point in that beam, the radius of curvature is exactly the same. All you need to do is to have a rigid body rotation of one diagram. The curvature does not change, but the deflections change and that is what you do. You rotate this first diagram anticlockwise by theta and you will get the other diagram. It is very important to understand these relationships. That is 2 theta at the free end of the cantilever.

(Refer Slide Time: 31:18)

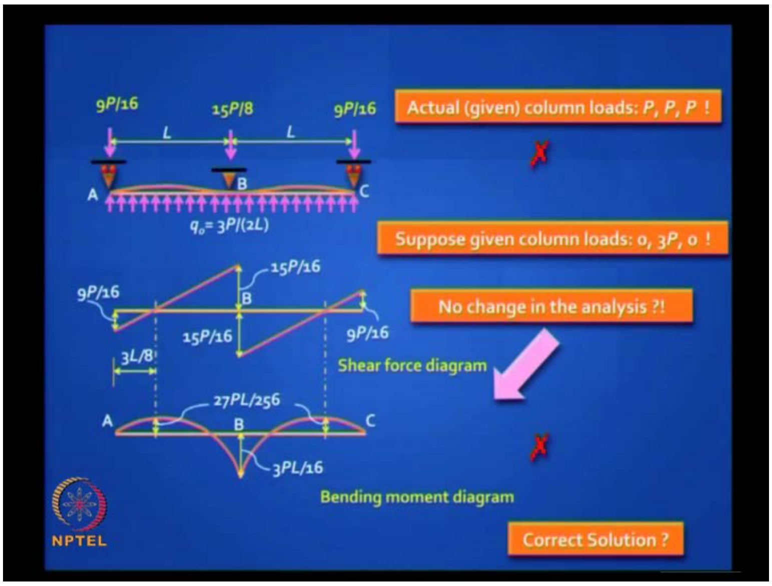


Let us talk about some other interesting practical issues, where a lack of proper understanding of basic structural analysis can lead to mistakes. Now, I am going to report to you some common mistakes that many practicing engineers make. Take a look at a footing like this; it is called isolated footing. You have a slab which you can treat like a beam and let us say there are three columns. Let us say it is a symmetric system, where you have three identical column loads P, P, and P; the spans are equal, L and L, and you have a pressure acting from below. That is the soil pressure and that soil pressure must be within the safe bearing capacity and the settlement must be acceptably low.

Longitudinally, this is called the rigid footing assumption. You assume that the soil pressures are uniform and so the total load is 3P; if the width of the footing is B and the total length is L, then the total area in contact with the soil is 2L into B and that is your soil pressure. If you want to treat it like a beam, then in kilonewton per meter, you can multiply that soil pressure by B again and you get a uniformly distributed load acting upward.

This is called the rigid foundation assumption. The assumption is that the foundation that is so rigid that you can ignore the deformations in it and it will move together as a whole. Now, the analysis is sometimes done by treating this as a two-span continuous beam and I will show you how. It is in an inverted beam and this is the model done. Do you find anything wrong with this? This is done by some engineers, that is, finally we have to design that footing. You have to find the bending moments and shear forces in that footing. You have this load acting from below and you analyze it as an indeterminate structure, a two-span continuous beam.

(Refer Slide Time: 33:39)



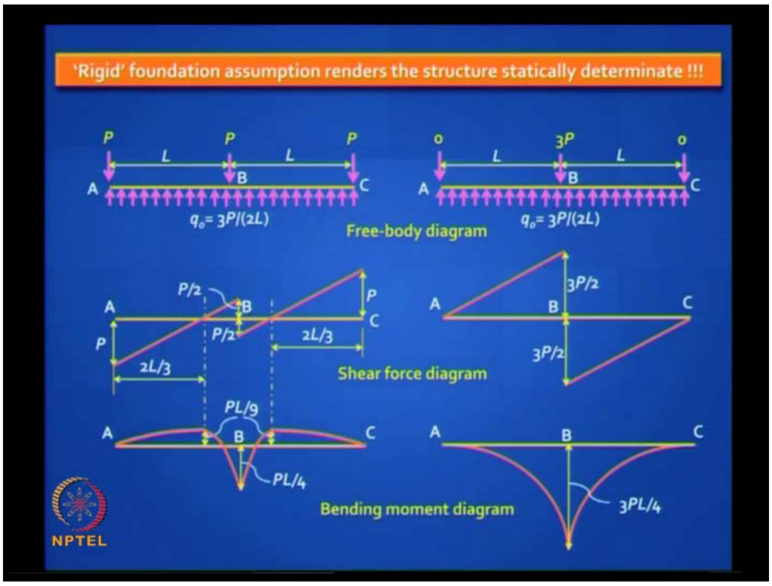
If you do that, I will show you the results – you will get a shear force diagram like that and a bending moment diagram. I have drawn the bending moment diagram on the tension side as our normal convention. It will look like that. Clear? Do not worry about those values, they are 100 percent accurate, you can check them. Is there anything wrong with this? Surely, they are not rigid. Actually, they are not rigid. You are right. Actually, the rigid foundation assumption is a big assumption but it is very commonly made and practiced.

Now, let us accept that assumption. For that assumption, is this result correct? Sitting on the ground, you are assuming a uniform pressure. 0 is assumed. It is assumed, yes. But it is actually the force. At B, there is a force ((.)). Well, this is how people do in practice. You have to give me a clear reason why this is on. Let me prove to you that it is wrong. Why is it wrong? Because if you work backwards and find the reactions at A, B, and C, you will get something interesting. You will get 9 P by 16, 15 P by 8, and 9 P by 16, but you began by saying its P, P, and P. There itself, you can see that there is an inconsistency. So, it is wrong, it is wrong.

Let us do something more interesting. Let us say the central column actually carried very heavy loads and you had practically very little loads on the two extremities. Let us take an extreme situation where instead of P, P, P, the loads are 0, 3P, and 0. If you were to do this kind of modeling, your shear force diagram and bending moment diagram will not change as long as the loading is symmetric. Agreed? That is something radically wrong.

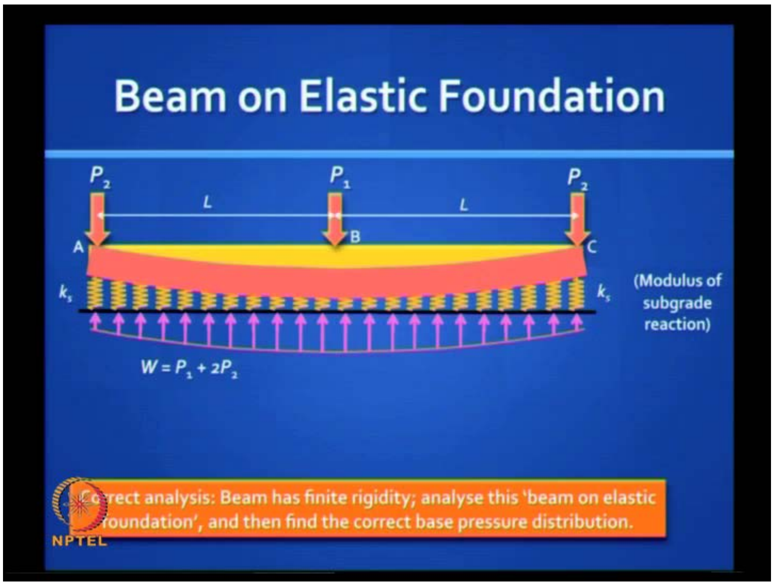
What is the correct solution? How do we handle it? You will be surprised, I have asked this question to practicing engineers in companies and they have not been able to answer. ((.)) That was for the exact solution. I do not want to do an exact solution, I am happy with a rigid foundation assumption. I just need the correct bending moment diagram and shear force diagram and obviously it must depend on the applied loads. You give me any combination of loads, P, P, P or 0, 3P, 0, I should get two different bending moment diagrams. How do I do that? Well, the answer is surprisingly easy.

(Refer Slide Time: 36:49)



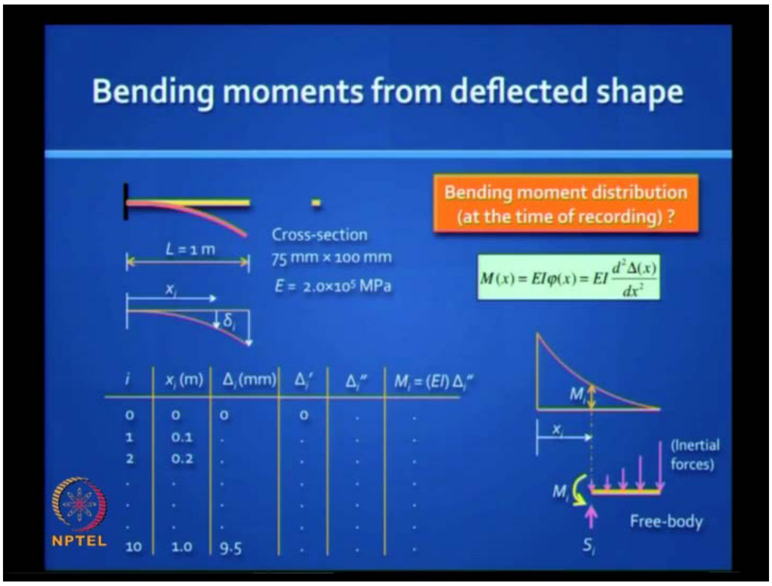
The answer is you do not need to have jumped into the assumption of a two-span continuous beam, because the moment you assumed the rigid foundation assumption, you have got a free body which is complete by itself. Can you not draw a shear force and bending moment diagram from the free body? This is the confusion that is there in many engineers. You mix up things – you mix up free body with loading diagram. Once you have got a free-body diagram that is all that you need; you have got the statics completely; you have got the shape of the shear force diagram; you have got the shape of the bending moment diagram; it is easy to draw; this is consistent. If it was 0, 3P, 0, you have got a completely different shear force and bending moment diagram. The answer is so simple. Once you make the assumption, everything falls into place. Do not complicate things which are assumed to be simple.

(Refer Slide Time: 37:51)



But you are right – if you want to do an exact solution, then you should put springs and you know there are assumptions; the soil has a modulus of subgrade reaction, which a soil test can do and you will find what the actual behavior is going to be. This is the behavior assumed: everything goes down by the same amount, which is not true. The actual behavior will be something like that (Refer Slide Time: 38:14). This is sometimes referred to as a beam an elastic foundation analysis. Let us not worry about this for the present.

(Refer Slide Time: 38:23)



Let me come to another important topic related to the beam. You can draw the bending moment diagram if someone gave you the loading diagram. Here is a situation where I have a cantilever beam whose dimensions are given: 1 meter long and the cross-section is 75 mm wide and 100 mm deep and the elastic modulus is also given to you. Let us imagine the beam is right here in front of us. Let us say, I pull down the free end and I release it; it will start vibrating. I leave the hall and go for a cup of tea. One of you takes a picture of it using a digital camera and first you have got a picture like that (Refer Slide Time: 39:12).

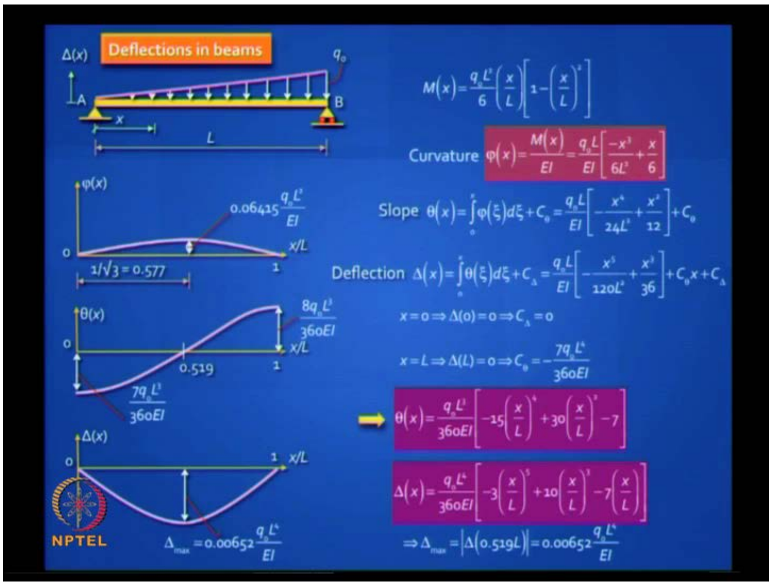
Does the beam have bending moments? Yes or no? Yes. Why? It is bending. Because it is bending, because you remember our discussions in the last class. I have asked the question very recently in other engineering colleges and they could not give the answer. Because it is bending, because the straight beam has changed its curvature, there has to be bending moments. How do you compute those bending moments? From the deflection.

How do you get it from the deflection? Derivative of deflection. You can digitize a diagram and get for different locations. Let us say you have ten locations on that beam. You can get the deflections and then you know that there is a relationship between curvature and bending moment, the second derivative of deflection. You can use any suitable numerical technique like a central difference formulation and get the slopes from this picture. From the slopes, get the curvature as the second derivative, multiply that with EI, and you have got the bending moment diagram.

This is a displacement approach. When you begin learning structural analysis, you learn the force approach: given some load, you can find the bending moment and shear force, but you can also use a deflection approach, the displacement approach. From the deflected shape, you can extract the bending moment diagram, which will look like this (Refer Slide Time: 40:40). The other question is can you not use the other method also? Surely, some loading is responsible for this bending moment. How do you find those loads? Those loads are imaginary loads and they were given a name by Newton. What are those forces called? They are called as the inertial forces; they are caused by the acceleration of the masses in the beam and they take a shape which is similar to the deflected shape.

If the beam is executing simple harmonic motions, then you know by definition, simple harmonic motion is one in which the acceleration is proportional to the deflection and so it would be that, but actually, it may not be executing simple harmonic motions and so, you have multiple modes and so on.

(Refer Slide Time: 41:30)

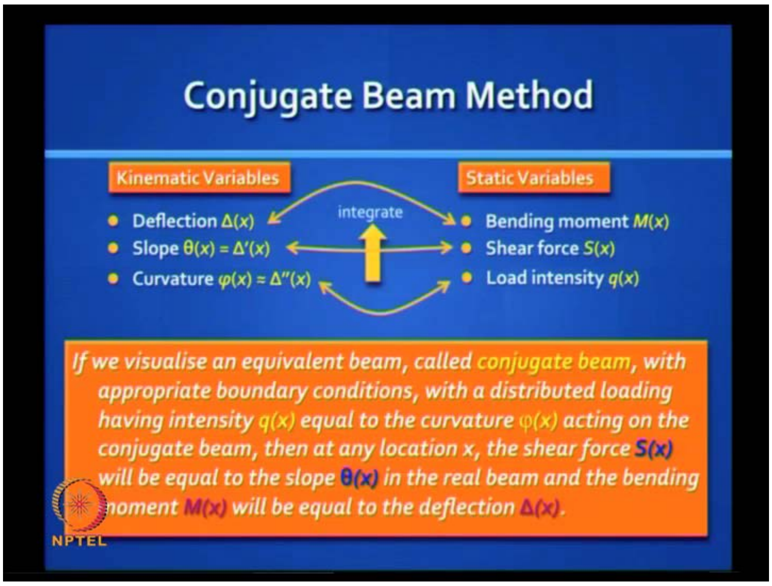


How do you calculate deflection in beams? Very quickly, let us say you are given this problem; you are given q_0, you surely can write an expression for the bending moment. Let us say you have done it correctly, I have given you the answer there. From the bending moment, you get curvatures by simply dividing that expression by EI. From the curvature, how do you get rotation? Just integrate it. Integrate it once more and you get deflection, but you end up with some constants; you will have two constants. How do you solve for those constants? ((.))

One at a time. Yes, apply boundary conditions; in this case, kinematic boundary conditions. At x equal to 0, the deflection is 0 and so, C_{delta} is 0. At x equal to L, the deflection is also 0, you get the second constant C_{theta} and plug it back into both the equations; you have got the two equations, but engineers are more comfortable by looking at figures. One picture can speak a thousand words goes the old Chinese saying and so you should draw graphs. If you draw the graph of curvature, which is like the bending moment diagram and will look like that, the maximum curvature is at the mid span; no, not at mid span. How do you know where the maximum curvature is? Well, where the moment is maximum. Where is the moment maximum? Where the shear force is 0. You can locate that point and find that curvature.

This is the shape of your rotation or slope (Refer Slide Time: 43:09). The slopes are negative on the left region and positive on the right region. This is the shape of a deflection diagram. It turns out to be negative because our definition of delta x is pointing upwards positive. You can locate the maximum deflection by finding out where the derivative of delta, in other words theta is 0 and you can write this expression; we have done this before.

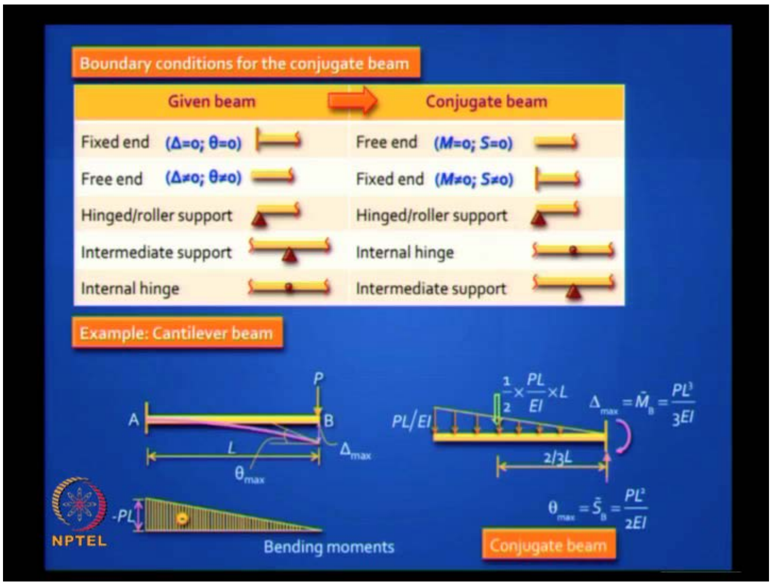
(Refer Slide Time: 43:39)



But practicing structural engineers want to be economical with their investment on time and they want to find short, fast ways of doing. One of the most powerful methods is the conjugate beam method. You have the kinematic variables and you have the static variables and you see that there is a definite relationship in each of these streams. If you travel upwards, if you integrate, you get from one the other. If you integrate curvature, you get slope; if you integrate slope, you get deflection; if you integrate load intensity, you get shear force; and if you integrate shear force, you get bending moment.

There is a kind of an analogy between these two streams and that analogy was put to good use in the conjugate beam method. If you have the same beam in which you apply the curvature diagram as a load intensity diagram, then the shear force in that beam would naturally correspond to the slope or the rotation and the bending moment in that imaginary beam would correspond to deflection. This is a simple concept and a brilliant solution to finding the deflection. Here goes the statement: if we visualize an equivalent beam called the conjugate beam with appropriate boundary conditions, with a distributed loading having intensity q of x equal to the curvature phi of x acting on the conjugate beam, then at any location x, the shear force S of x will be equal to the slope theta of x in the real beam and the bending moment M of x will be equal to the deflection delta of x.

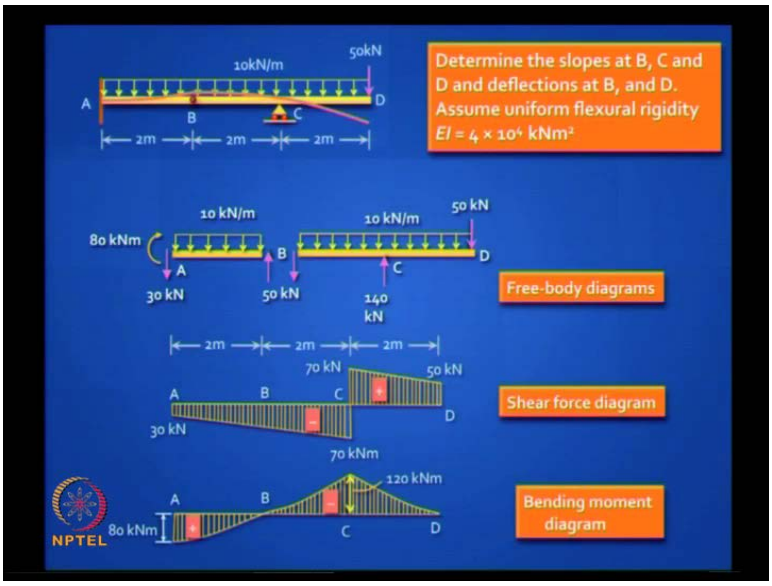
(Refer Slide Time: 45:21)



You can use that argument to prove the boundary conditions that you need to apply on the conjugate beam. At the fixed end, where the given conditions are the deflection and slope are 0, it would imply that the bending moment and shear force in the conjugate beam should be 0; they will always be 0 when you have a free end. You do not have the possibility of a concentrated load at the free end because the curvature diagram will always be continuous and it will never have singularities. The fixed end becomes free, the free end becomes fixed, and the hinged or roller will remain as it is – simply supported; the intermediate support becomes an internal hinge because you have a discontinuity in slope there and the internal hinge will be replaced by an intermediate support in the conjugate beam.

A very simple demonstration. Take a cantilever beam. You want to find the deflection and the slope at the free end – you want to find delta max and theta max. First, draw the bending moment diagram, then draw the conjugate beam, the free end becomes fixed and the fixed end becomes free. On that conjugate beam, put the curvature diagram, which means bending moment divided by EI. Then, you just analyze the beam and find the support reaction at B. You will find that slope at B is given by the shear force of the vertical reaction at B and the bending moment gives the deflection at B. Your deflection is PL cube by 3I, it is a standard formula, and slope is PL square by 2EI; very easy.

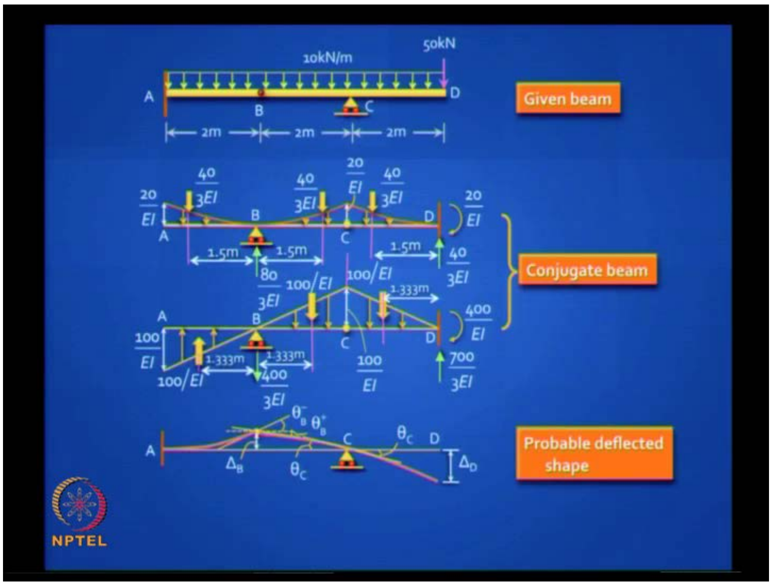
(Refer Slide Time: 47:10)



A little more complicated problem; here, I will quickly rush through it. It is little complicated because you have got propped cantilever with an internal hinge and you are ask to find the slopes at B, C, and D and deflections at B and D. It is difficult, but the easiest way you can solve such a difficult problem is the conjugate beam. Let us go through the steps.

What do you first need to do? ((.)) Analyze the beam and draw the bending moment diagram. You can also try drawing the deflected shape, it will look like that (Refer Slide Time: 47:45). Draw the free-body diagram. It is easy to analyze once you separate out – you know the parent and the child. You can draw the shear force diagram and you have got the bending moment diagram. Let us just accept it – you have got the bending moment diagram. Then, what do you do? That is the loading diagram for the conjugate beam. That divided by EI is the loading diagram on your conjugate beam. What is the boundary condition on your conjugate beam? Free end ((.)). A is free; at B, you have an intermediate support; C you leave in place the intermediate support; and at D, it is fixed.

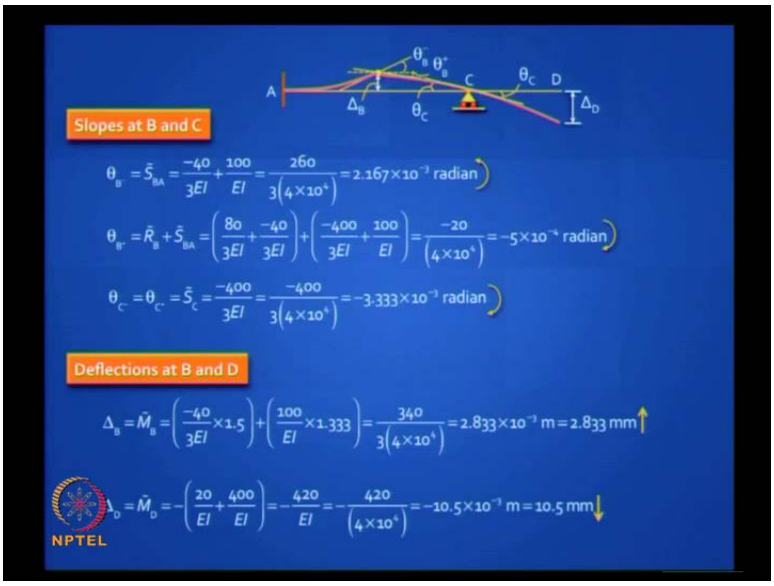
(Refer Slide Time: 48:30)



This is the given beam and this is the conjugate beam. In the conjugate beam, the boundary conditions are as shown. Now, instead of putting on that beam that big complicated diagram, it is good to separate out the curved parts from the straight-line parts, just for simplicity in calculation. If you do that and you do superposition, you can actually find out the centroids of the resultant forces and analyze it.

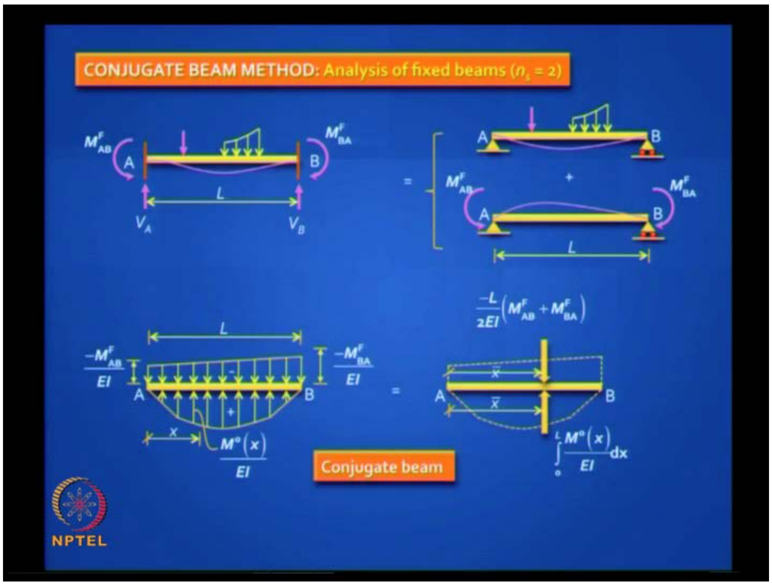
Look carefully at the values that you need to compute because at B, you will get two values of slope because the slope on left side and the right side are not going to be equal, but that is because you have a discontinuity in the shear force diagram in the conjugate beam because you have a support there. You can calculate all these very accurately.

(Refer Slide Time: 49:20)



Let us just accept this. This is something you should have learnt in basic structural analysis; I have just covered the concept here.

(Refer Slide Time: 49:34)



I think we are running out of time. Accidently, the conjugate beam method also gives a method for solving some types of indeterminate problems where the indeterminacy is not more than 2. For example, take this fixed beam. How do you find the fixed end moments in this beam? You can visualize this as a simply supported beam with the original loading plus the hogging fixed end moments. If you draw the conjugate beam for this, you will find that fixed-fixed beam becomes free-free and you can superimpose the sagging bending moment diagram called the free bending moment diagram divided by EI and the hogging bending moment diagram. If you divide by EI, you have got those two diagrams and all you need to do is to equate the resultant forces and apply the moment equilibrium, which means their centroids should match and you can easily establish simple formulas for fixed end moments. We will continue with this in the next class. Thank you.

(Refer Slide Time: 50:41)