Advanced Structural Analysis

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Module No. # 3.3

Lecture No. # 19

Basic Matrix Concepts

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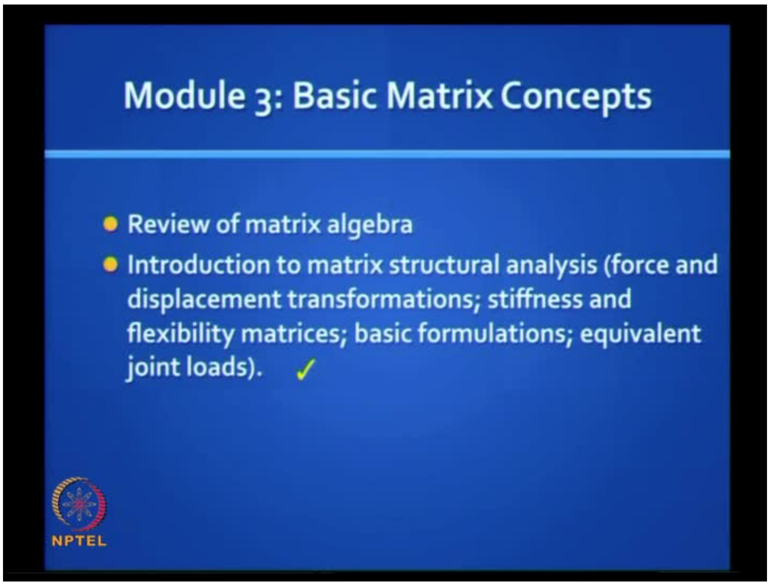

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Good afternoon. This is lecture 19, module 3, Basic Matrix Concepts. We were doing the third lecture in this third module. If you recall, we had just started the second topic of Introduction to Matrix Structural Analysis; this is covered in chapters 2 and 3 in the book on Advanced Structural Analysis.

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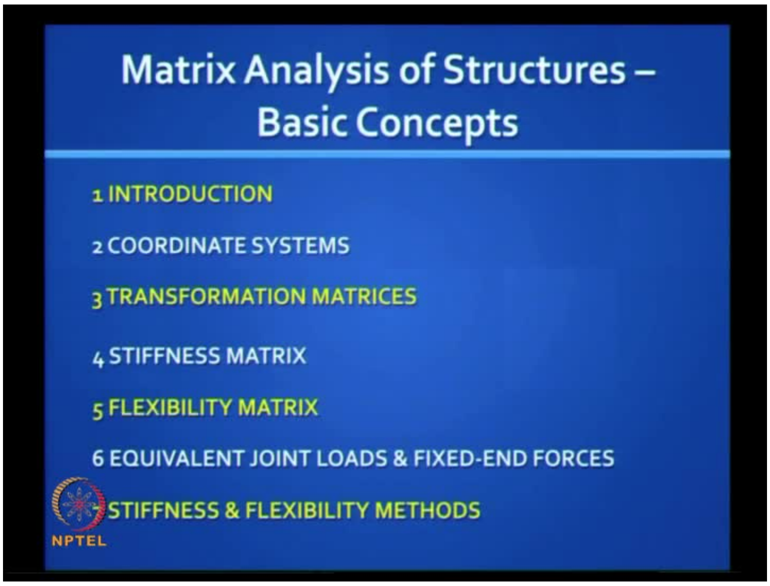

We had covered the introduction on basic coordinate systems. And in this session, we will look at the transformation matrices and stiffness matrix.

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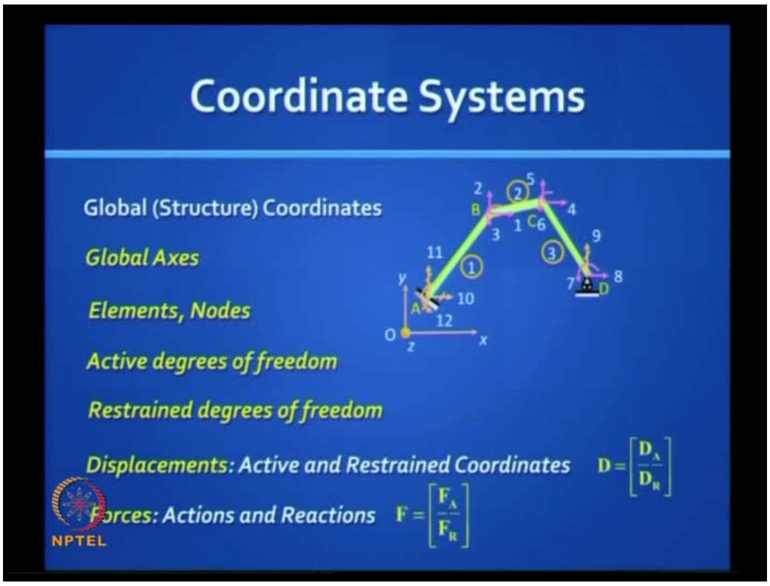

This is to refresh your memory. Please see what the real task for us is - we want to be able to make the digital computer do structural analysis. We want, first of all, to have a framework in which we show the structure geometrically, which means, we have to identify the elements and nodes with reference to some Cartesian coordinates.

We have to identify the active degrees of freedom, restrained degrees of freedom and that is how we defined the vectors D and F - D is the displacement vector, F is the force vector; the subscript A stands for actions if you are referring to forces and it also stands for active degree of freedom if you are referring to displacements.

So, F A the sub matrix F A really is the load vector and F R is reaction vector - R stands for reactions and R also stands for the restrained degrees of freedom. D R is the restrained displacements; the displacements are usually 0, unless you have some known specified support settlement.

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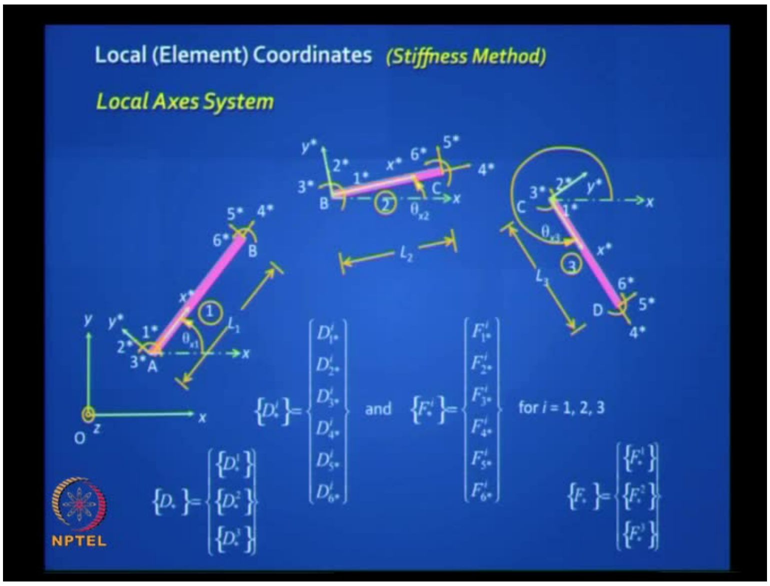

Next, you will recall, we defined the - local coordinate - local axes. In this particular plain frame, for example, you have three elements and we have the local coordinates - the local axes. You have 6 degrees of freedom per element, so you have the element displacement vector, the element force vector and we use some superscripts and subscripts to clarify.

You remember the star is what we use when we want to refer to the local system and the superscript we put i referring to the ith element, so here i can be 1, 2 or 3. And you have six different displacements and six different forces. We can assemble all the elements together into one overall combined element displacement vector D star. In this case, what will be the size of D star? 18 by 1, because D 1 star itself is 6 by 1, D 2 star is 6 by 1 and D 3 star is 6 by 1. So, this is convenient to do when you are dealing with small frames; when it is a large frame we do not do this. Similarly, you have F star.

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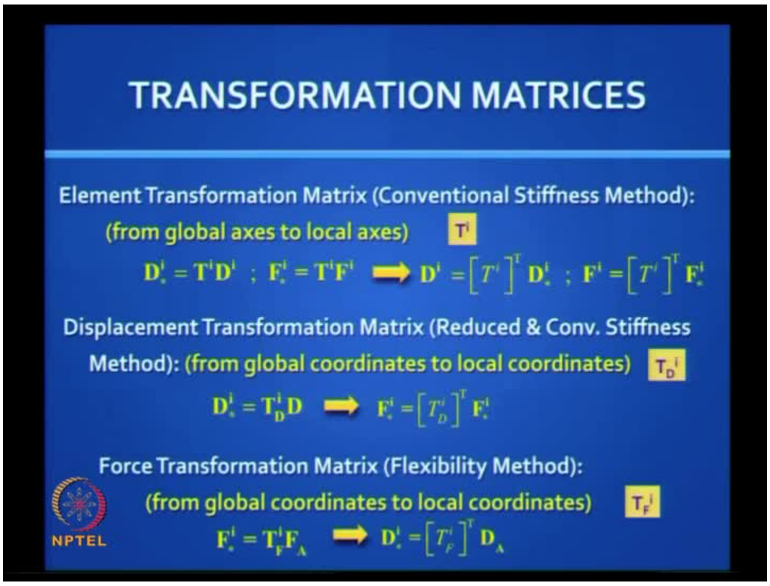

Now, let us look at transformation matrices. You need to know how to switch between the structure and the element, so that you have some relationships. This is a linear transformation, so you can transform from D i to D i star.

This is a standard transformation. T i is called the transformation matrix - the element transformation matrix - and the same T i holds good not only for displacements but also for forces.

This is the conventional stiffness method. The interesting thing about the transformation matrix is that, it is an orthogonal matrix which has this beautiful property of the inverse being equal to the transpose; so, it is very easy to do the reverse switch, that is, you can move from the local coordinates to the global coordinates. We will see how this can be applied; so, this is one major type of transformation we do. You would have studied in school coordinate geometry, so you know about what happens when you shift the axes, you rotate the axes and so on, so it is something similar.

There is another set of transformation we will see later, where you can do what is called a displacement transformation matrix. This is used especially in the reduced element stiffness matrix - we will see this later. This can also be applied to conventional stiffness matrix.

When you do the flexibility method, you have the force transformation matrix; you can do the switch from the global to the local coordinates. So, we will see these as we go along. You will find that with increasing familiarity, you will understand everything, but it is good to plant the seeds early. We will keep coming back to these slides and your understanding will get strengthened.

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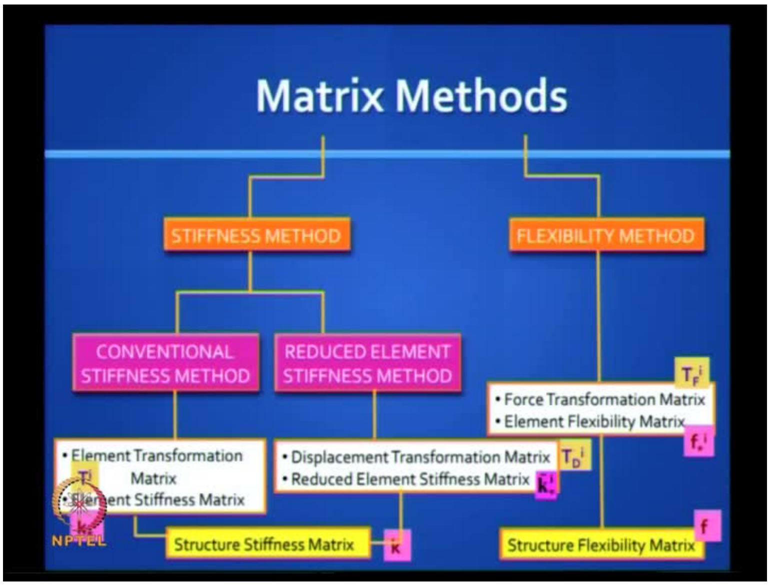

So, you can work both from global to local and from local to global. Just to give you a big picture view of the different methods we are going to study, the so called conventional stiffness method is the way software’s work in computers. This is how it is done by the computer; this is the master method very generate dealing with slightly big matrices. I am not sure if this is what is taught in many Indian universities in matrix methods, because what is taught is the simplified version of it, which is more convenient to do manually.

So, we are going to refer to that as a reduced element stiffness method. It is equally accurate, but here you have less flexibility; it has some limitations. And here, you deal with smaller size stiffness matrices; the number of degrees of freedom is less. We will discuss this in the next class. The stiffness matrix you get here has an inverse and that turns out to be the flexibility matrix.

So, you have the third method that we need to study; it is called the flexibility method. In software packages, you do not have the reduced element stiffness method and you do not have the flexibility method.

So, in many advanced courses; they do not waste time with these methods. They just cover the conventional stiffness method. In fact, many modern text books do only that. Traditional text books cover everything, but sometimes in many universities, I think in your gate exam, for example, they will cover the reduced elements stiffness methods without calling it so. They will still call it a stiffness matrix method, but it is actually the reduced version of the method and you learn flexibility method.

What we are trying to do in this course is to cover everything. But for manual use, to demonstrate in the class and so on, it is probably easier to use the reduced elements stiffness method - as compared to – but we will do both, because we like to know how the black box in the computer works; so you should see the big picture.

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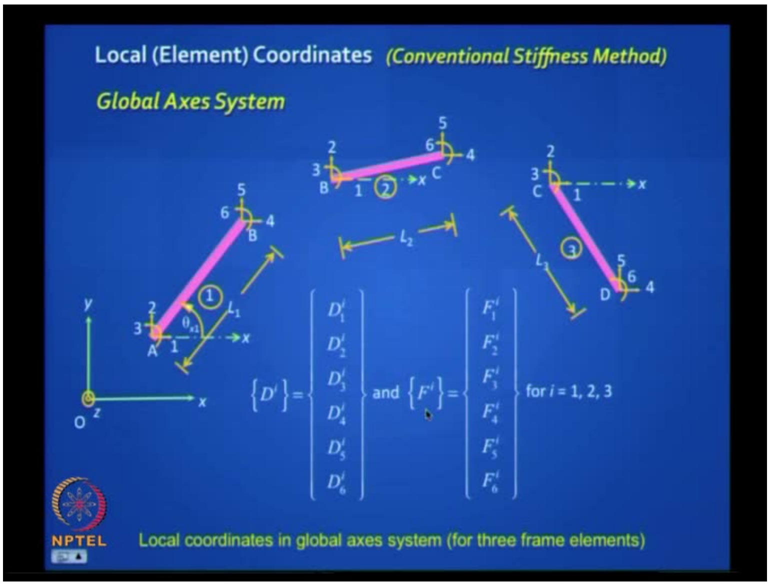

So, we begins with the conventional stiffness method. So, let us take a fresh look at this conventional stiffness method. You can express the degrees of freedom both along the local axes which what we saw earlier, but you can also do it along the global axes.

So, for example - 1, 2, 3 instead of pointing towards the local launcheral axes 1, we can point towards the global x axes - can you see that? So, this is a big change - that means we do the transformation here itself, so I am not putting the star anymore. Remember - 1 star, 2 star, 3 star - there the 1 was pointing along x star - this way - 1 was along that and 2 was the normal to it. Now, 1 is parallel to the global one, and 2 is parallel to the global y axes and you will find the 3 does not change.

Why does not the 3 change? Because the vector pointing outward from the x, y plane is the same, so that does not change and so there is no transformation needed there. So, it is like you do the correction at this stage itself - at the element level. If you can define 1, 2, 3 like this, you can also define 4, 5, 6 and similarly, for all the 3 elements. Got it? So, this is another way of doing it and this is the transformation that we are trying to do.

And so I am now defining element displacement vectors and element force vectors - they are still element, because the i comes here; i refers to the ith element. This is for i equal to 1, 2 and 3. The big difference between this and the previous element displacement and force vectors is in designation how do I make the difference? That star is missing; the asterisk is missing - have you understood?

Now, all we need to know is how to switch from the local axes system to the global axes system and then the job is done. How do we do that? Let us take a look.

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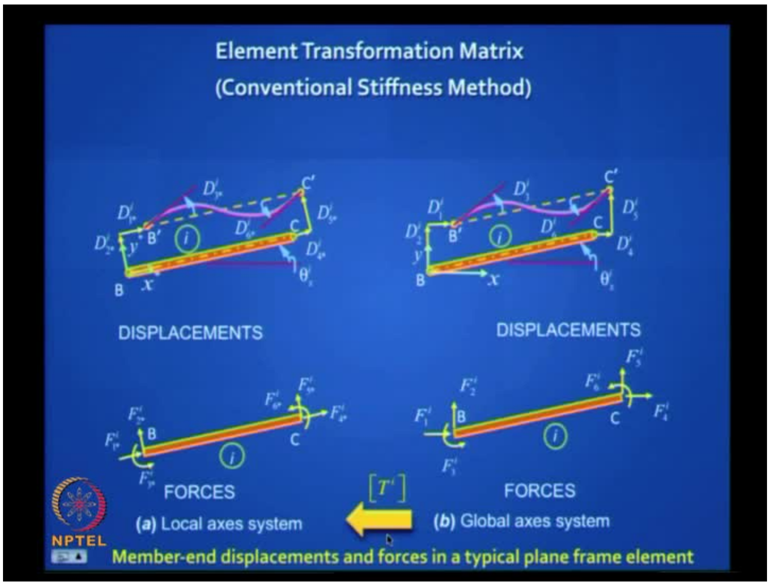

Let me explain once more. Please listen carefully and pay attention. This is your This is what we did in the last class. Here, you have 1 star, 2 star, 3 star and so on - 1 star, 2 star here for sure. If this joint has moved here, the displacement along the local x axes is D 1 star with the i - i stands for the ith element. And this moment normal to it is D 2 star. And this change in angle anti clockwise is D 3 star. Does this make sense? You are clear about the physical meanings of and Similarly, I can define D 4 star, D 5 star, D 6 star. Is this clear to all of us? Mind you, this theta i that I am talking about here is not referring to a displacement; it is referring to the original inclination of the member prior to loading with reference to the global x axes. Is that clear? So, this is what we mean by the local axes system.

What do we mean by the global axes system? This is a similar representation of the element end forces. Do you understand this now? Now, if I have a force acting this way, I call it F i 2 star, along long surely x star axes I call it F i 1 star, a moment acting this way I call a F i 3 star. These three are conjugate with these three - can you see that? The directions are the same - one is a set of forces, the other is a set of displacements. The same here at the other end - this is the end node, this is the start node. Does it make sense to you? So, this kind of description of element level forces is what we call the local axes description.

We have another description - the global axes. Let us see how that looks. Now, I am taking the same element and I am taking the same displaced configuration; but I am choosing to define the displacement D 1 D i 1 as parallel to the x axes - you see, D i D i 2 as parallel to the y axes and D 3 i does not change - it is same as D 3 i star; these two do not change, you can see there is no change. But, these two change. Likewise here, this moves horizontally - D 4 I - D 5 - and this D 6 does not. Do you get the hang of it?

Similarly, here the component of the force along the x axes - global x axes - if F 1 along the y axes, global y is F 2 and the moment is F 3, F 4, F 5, F 6 - is this make clear?

With this, graphically, I am trying to show you what we mean by defining the element vector and displacement vector - force vector. You have two choices - one with the star which tells you we have to align yourself along the longitudinal x star axes, the other one is you align yourself along with the global axes. Is it clear?

Now, there is a relationship between these displacements. They have defined it this way - the boss here is central government that means the global axes is your foundation and from that you switch to the local axes. You do it through a - transformation - linear transformation using a standard matrix called the T i matrix - it is a transformation matrix. Can you work out a relationship of this matrix? What is the size of this matrix?

2 by 2 3 by 3 sir 3 by 3 sir 2 by 2

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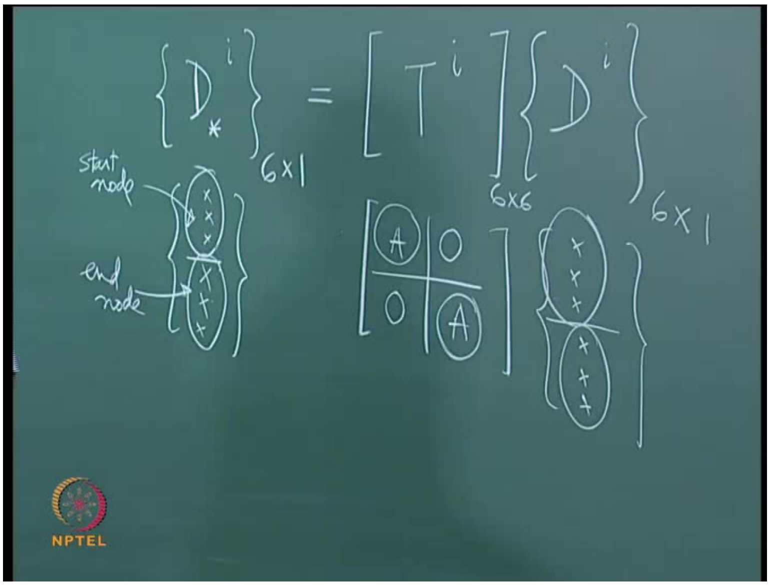

Wait, you have to understand the question first. What is the size of this - for the ith element?

There are six degrees of freedom. You are right - three comes from one end, but the element has two ends. You have a head and a tail and what we are saying is this you get by transforming - this is a transformation matrix. You are transforming something else - what are you transforming?

This is also 6 by 1 - this matrix is the transformation matrix which I call T i, obviously, this has to be 6 by 6. But there is some nice property about this matrix. What is the nice property? Apart from that, this matrix will look like this. Even this you can partition, this you can partition, this belongs to the start node, and this belongs to the end node. Because you are following stiffness formulation, will there be any relationship between this and this? They are independent. Actually, this is independent with this and this. And this whole thing is independent with these, so you will get some 0's somewhere here. So, Which will be 0? So, the half diagonal matrices will be null matrices and whatever relationship you get here - let us say this is A, this will also be A - does it make sense?

Let me explain. You got 3 by 3 here. Now, this is related to this through this transformation; this will be related to this through a similar transformation. Isn't it? Does this make sense?

Yes, but once you have a transformation, they are not independent. These two are dependent, these two are dependent, what is their dependence? This transformation. You are just rotating the axes, obviously, there is dependence there. Can you tell me what A will look like? It is a functional direction cosine and it is not difficult to work out, so it will look like this.

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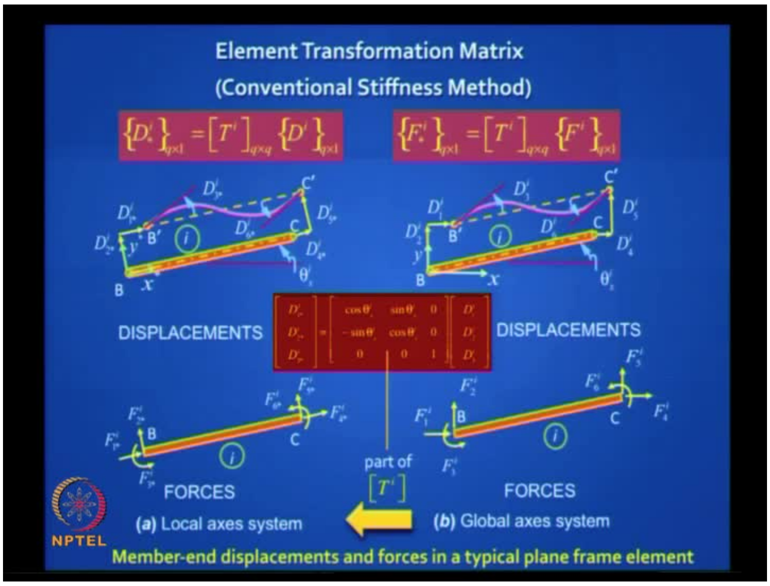

Along the longitudinal axes of the element, that is the axes x star, the displacement at any point along the element will vary. The displacement at the extreme left end is our unknown; we are calling that D 1 star. At the end node, we are calling it as D 4 star. D 1 star in D 4 star are independent, that is why we called them degrees of freedom. Any pending doubts? You will get the hang of it soon, this is quite straight forward. Have you got it? So, this is your A matrix and it is easy to establish.

Now, let us take a look at these relationships. This is exactly the relationship we wrote there. This is the fundamental transformation matrix used in the conventional stiffness method and it is very easy to remember. You have a cosine and you have a sine; everything is positive, except, the second element for obvious reasons. You have cos theta and minus sin theta. Why is it minus sin theta? Because it goes to the negative side.

I hope you can work this out, it is straight forward. We will do it in great detailed when we get into the actual problems. Do we get this? This is our A matrix; this is for the start node and this is for the end node. Everything is the same except that 1, 2, 3 gets replaced by 4, 5, 6. Does it make sense?

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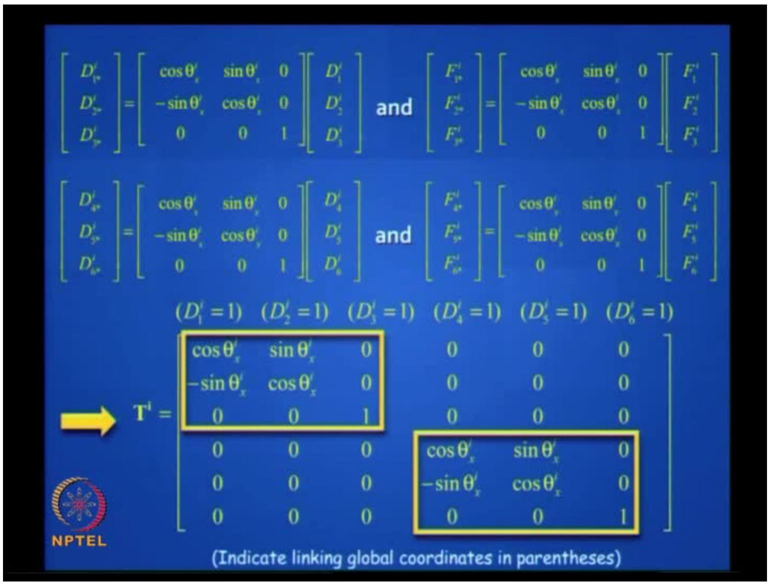

So, if you want to put it all together it will look like this, clear. Standard transformation - the property about this is - it is an orthogonal matrix. Now, why did we align 1 star, 2 star and 3 star along the global x, y and z axes? Why did we make it 1, 2, 3? What do we achieve by that? Think in terms of displacements there. Any displacement or the entire vector space will (( )) That you can span even with the local coordinates.

Let me give you a clue. First, we began with the structure. In that particular frame, how many displacements did we have? We had 12 - D A and D R. We had D 1, D 2, D 3 all the way to D 8 - they were active degrees of freedom; D 9, D 10, D 11, D 12 were restrained degrees of freedom. How are those linked with these? So, you have a think in the stiffness language. In the displacement methods what is fundamental? Displacements or fundamentals. And how do you link the state government with the central government? How do you link the element displacement with the structure displacement?

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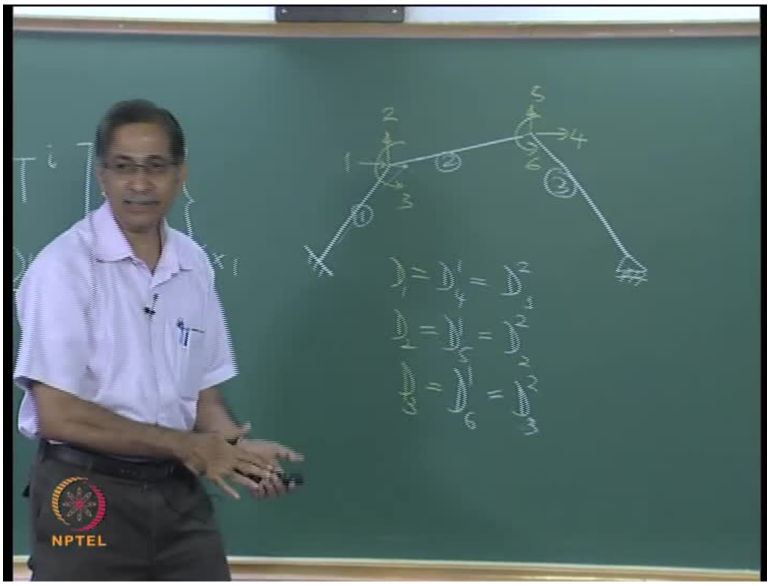

This is very important; it is very simple. Remember, this was the first element, this was the second element and this was the third element (Refer Slide Time 22:39). The structure was kind of fixed here and there was a roller here. Let us look at the active degrees of freedom. If you remember - if you remember - why do not I join them? This was element 1, 2 and 3 - what were the degrees of freedom?

We said this joint, we start the active degree numbering here, we said this is 1, this is 2, this is 3 - remember, this is for the structure. Now, what is D 1 equal to? From the element point of view, how will I write? See, you now know all the definitions. It is exact equal to D first element, this element and fourth - Needs little thinking.((Plus))

This element moves here, this joint also moves there - where is the plus coming?You have to get out of statics where you do plus, to kinematics where you do not do plus. A conjugal relationship, say between husband and wife, works as long as there is no separation; so, do not separate out. This is compatibility - do you get it? In defining these displacements, itself you are defining compatibility. What is this equal to? What is the second element? D 2 1

Now, you got the hang of it; so that is why we are playing this game. See, there is a big difference between trying to figure out what somebody else worked out and trying to figure it out yourself from the first principles. Let us say you have to find a way to make the computer do it; it is wonderful.

Well, let us let us continue - what is D 2? What is D 2 equal to? D 1 5 equal to D 2 2. What is D 3 equal to? D 1 6 equal to D 2 3. You are getting the hang of it now; so on and so forth, we will not waste our times. So, this is 4, 5, 6 etcetera. Got it? That is the idea; the idea is you do the correction early stage, and then you are satisfying compatibility straight away.

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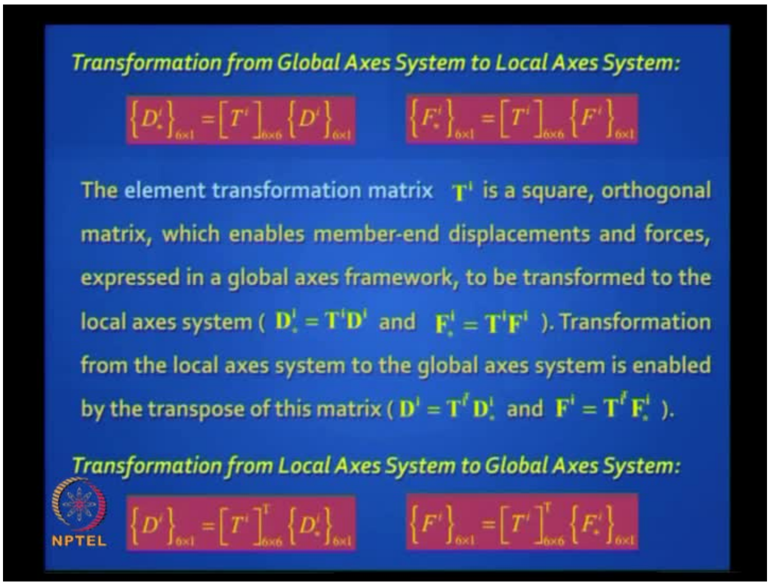

These are the fundamental transformations. The element transformation matrix T i is a square, orthogonal matrix, which enables member end displacements and forces, expressed in a global axes framework, to be transformed to the local axes system. Transformation from the local axes system to the global axes is enabled by the transpose of this matrix, which is equal to the inverse of the matrix; you can switch back and forth.

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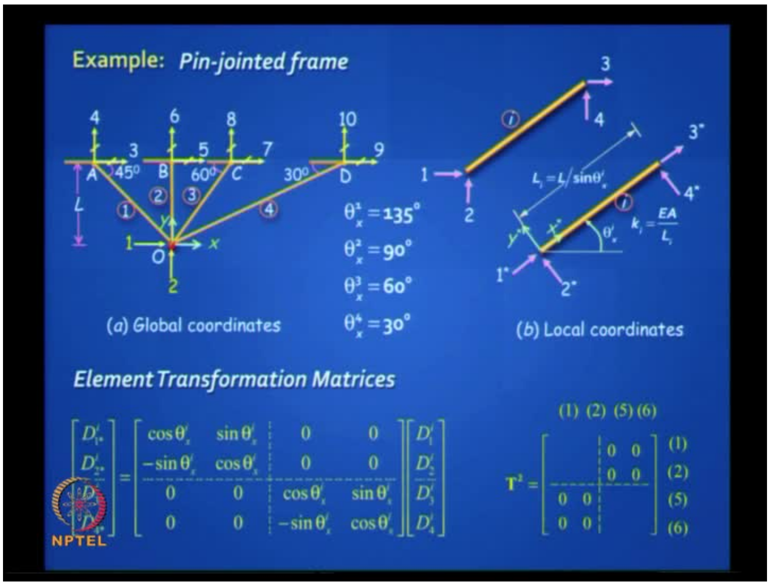

Let us give an example. This is a pin-jointed truss - pin-jointed frame. How many active degrees of freedom, do you think there are? It is a plane truss (Refer Slide Time 26:44).

Where can you apply loads? Only loads; A, B, C, D are all supports. So, two active degrees of freedom; these are coordinates 1 and 2 - one along the global x and one along the global y. Then, how many restraint degrees of freedom do you have? (Refer Slide Time 26:44). Why do say 4? Because it is along the member force acts, so for each member only one at A 1, B 1, C 1, D 1.

He is strictly right, in the sense; you know the direction of your support reaction in this case, because they have to be aligned along the members in this particular problem. But if they say, you let go a little bit; it does not matter, you will finally get the same answer. So, do not be too rigid, let us accept what they say. Let us say 3, 4, 5, 6, 7, 8, 9, 10, but we know the reactions are not independent and then, you are bringing equilibrium. I am doing stiffness method, I know only compatibility. Until I bring in equilibrium, I cannot say that that the reactions are related; so, let us leave it like that. Is this fine?

What about our element? All four elements I reduce to one element; so, why should I draw four pictures - I like drawing one picture. All four, I cover with the angle - theta x i. If the length – you know this height of that truss is L - I can write an expression for the length of all four members as L divided by sin theta. Got it? I need four values of theta, they come from that truss; these are those four values I am measuring with respect to the positive x axes. Got it? Ultimately, I want to generate the stiffness matrix, but let us begin with the transformation matrices.

So, my local coordinates are 1 star, 2 star, 3 star and 4 star, got it? And 1, 2, 3, 4 are like this - without the star; 1, 2, 3, 4 is along the global axes system and 1 star, 2 star, 3 star, 4 star is along the local axes system - by now you know?

Now, how do I write the transformation matrix? Is it clear till now? I have taken a very simple example. How do I do? Well, I also have the stiffness, let us say. Does this make sense? It is the same A matrix we got without the rotation component, which is 1; so, you have cos theta minus sin theta sin theta cos theta - does this make sense? This transformation is straight forward. Again, half diagonal is null - sub matrix - is it clear?

Let us apply it. No doubts? We have done an example of a plane frame transformation and we are doing it in the plane truss. You can do a beam later. How do we proceed? Now, this is important. The size of this matrix is 4 by 4 - there are four elements for each of them; so, there are four transformation matrices and for each of them it is 4 by 4.

So, if you take, for example, this element 2, we want to fill here and here; but we can straight away mark the linking – linking what? Global coordinates. See, you identified this element as 1, 2 and it is matching with this 1, 2. This is 3, 4 but, for the global coordinates this is not 3, 4 - this is 5, 6 - are you getting what I am saying? Let us write to help understand what we are doing. For that particular problem - this is 5, 6 - this is 1, 2 - this is global coordinate.

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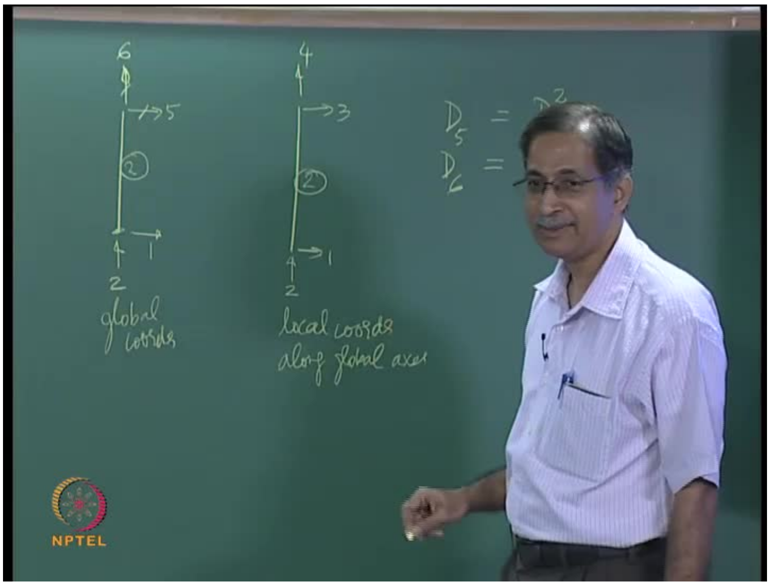

This is 1, 2, 3, 4 - what is this? This is local coordinate along the global axes that means I already aligned it in this direction. Now, D 5 and D 6 in the structure must be equal to - how do I write D 4? Like the same way, we wrote here. For this second element 2, how should I write that? In that second element D 2 4.

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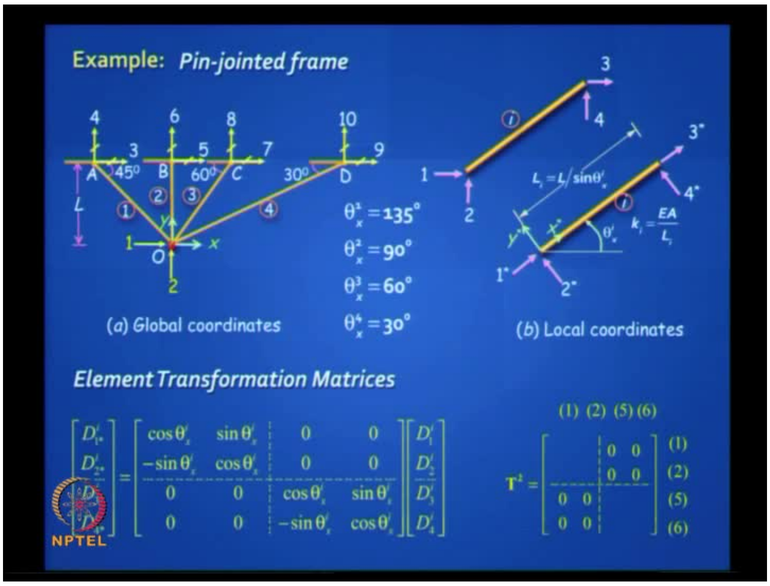

I do not write so much. For convenience, I do it in brackets here - the parenthesis here - you get it? 1, 2, 3, 4 is local - 1, 2, 3, 4 this way - so, 1, 2, 5, 6 is global coordinate - this is helpful for me. This is a clue for me that the displacements that I get at the element level or the same other displacements I get at the global - no plus here - be careful; it is exactly equal.

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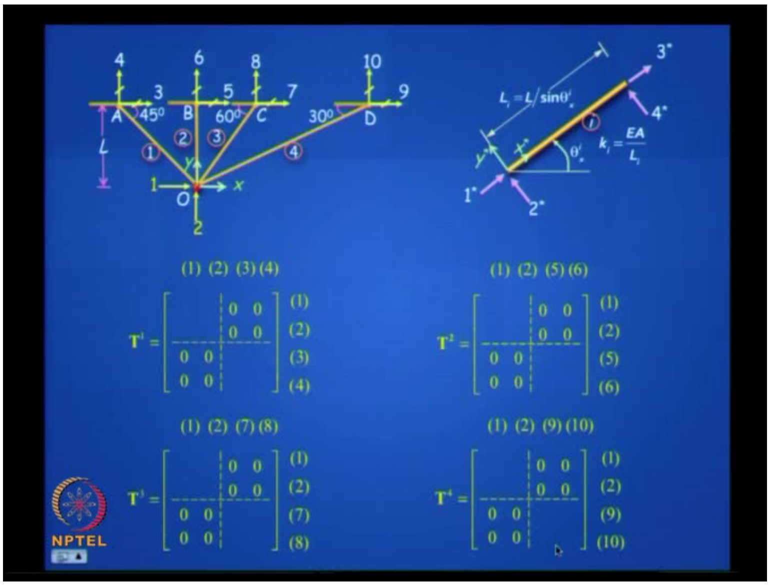

So, it helps me assemble my structure stiffness matrix from my elements stiffness - is this picture clear to you? Like this, you have to play a game. You can similarly take for T 1, T 2, T 3, T 4 and I need to plug-in the empty boxes - how do I fill in those empty boxes? sin cos that is easy to do in a tabular format; I can do it effortlessly - is this clear?

Let us go through this once more. For element 1 - 1, 2, 3, 4 are the connecting start and end nodes - 1, 2, 3, 4 - 1, 2, 3, 4. For element 2, we have already finished. For element 3, it is 1, 2, 7, 8 - 1, 2, 7, 8 which match with the local coordinates 1, 2, 3, 4 and for element 4, it is 1, 2, 9, 10 - does it make sense?

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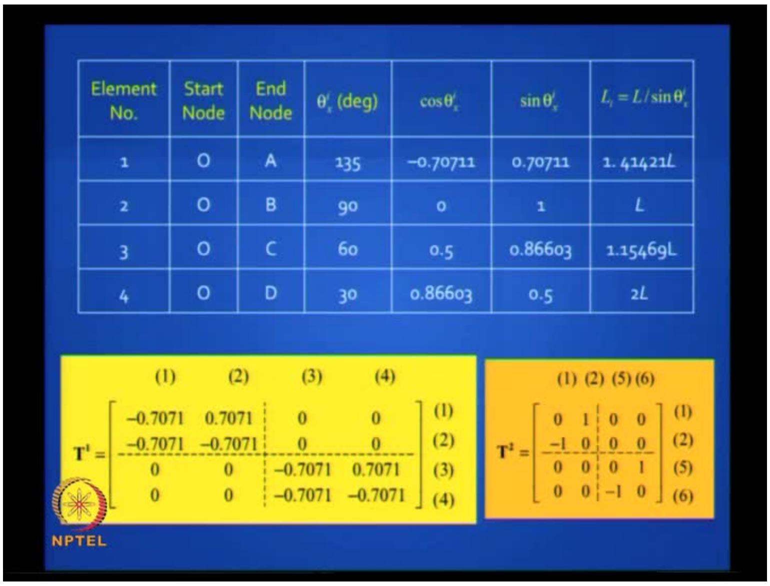

Now, you can do this neatly in a tabular fashion, mark your start node, mark your end node, mark the theta value, calculate cos theta and sin theta which are also called direction cosines and get the length - in this case it is also function of sin. And just plug-in those values - cos theta, sin theta and all that values. You will understand - all these are very easy to do - it is mechanical, the computer will do it effortlessly; matlab you can do easily. So that is how you generate - generate what? Transformation matrices.

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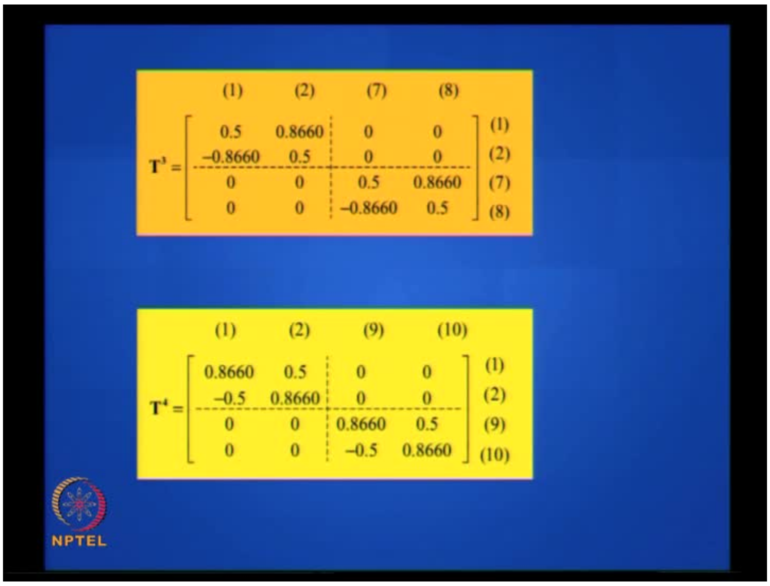

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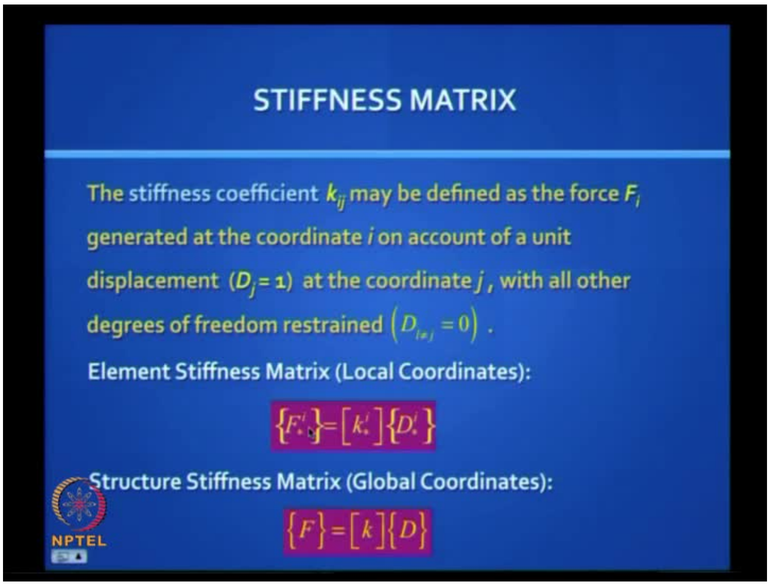

Now, we look at stiffness matrix. We already know the definition K ij; it is called the stiffness coefficient. It is the force F i generated at the coordinate i on account of a unit displacement D j equal to 1 at the coordinate j, with all other degrees of freedom restrained; you are very familiar with this definition.

In matrix forms, it looks like this in the local coordinate system, for each element - for the ith element it looks like this and for the entire structure, it looks like this - got it? For each element, it will look like this and for the entire structure it will look like this. Here, you do not have a star, you do not have I, because all the elements contribute to this matrix; this is for an individual element - is this clear?

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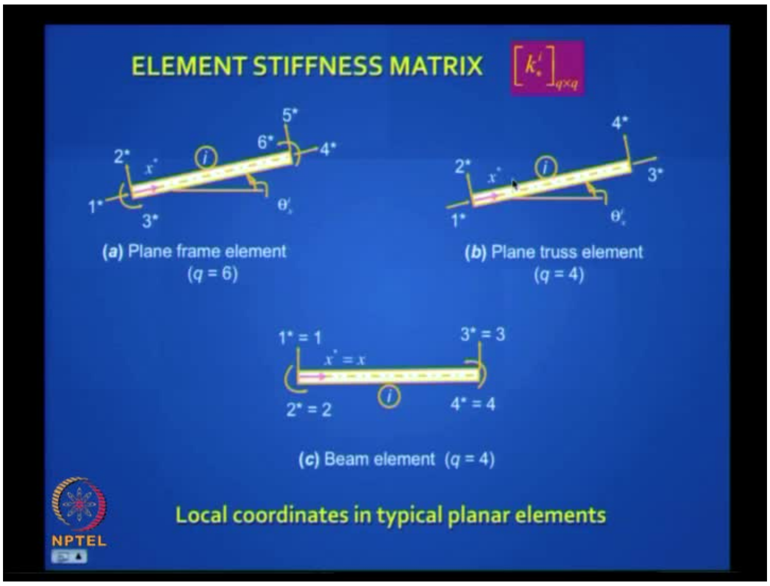

Now, let us look at typical elements that you may get. Let us keep it simple, so we look at a plane frame element with six degrees of freedom - we have already done that; plane truss element - how many degrees of freedom? Four degrees - 1, 2, 3, 4, because you can have four different displacements; and beam element, we assume that it actually does not move, so you have only a translation, a deflection and the rotation, four degrees of freedom, got it? 1, 2, 3, 4 - we will keep looking at these three elements again and again. Obviously, the plane frame is the most generic, because it is a sum - it is a combination of the beam and the truss.

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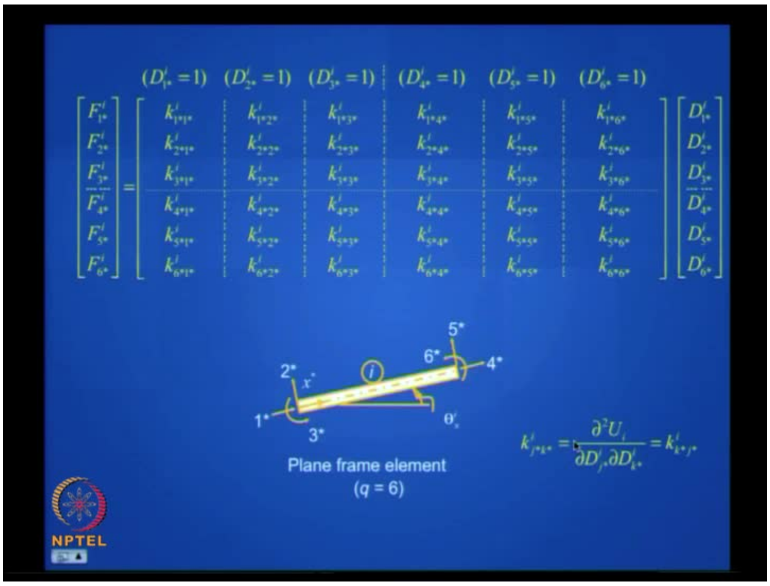

The stiffness matrix for a plane frame element will look like this (Refer Slide Time 36:55). Let us not fill in those, but you know the meaning of this. It is a 6 by 6 matrix relating F i 1 star to 6 star, D i 1 star to 6 star, so it is a 6 by 6 matrix. It is going to be a symmetric matrix, it is a square matrix and how do you fill these? You already know a physical approach - you can give one displacement and arrest the others; you can fill these up, but you can also do it using many other techniques including the energy method. We will see all this later. I am just introducing notation and am giving you a framework on how to do matrix analysis.

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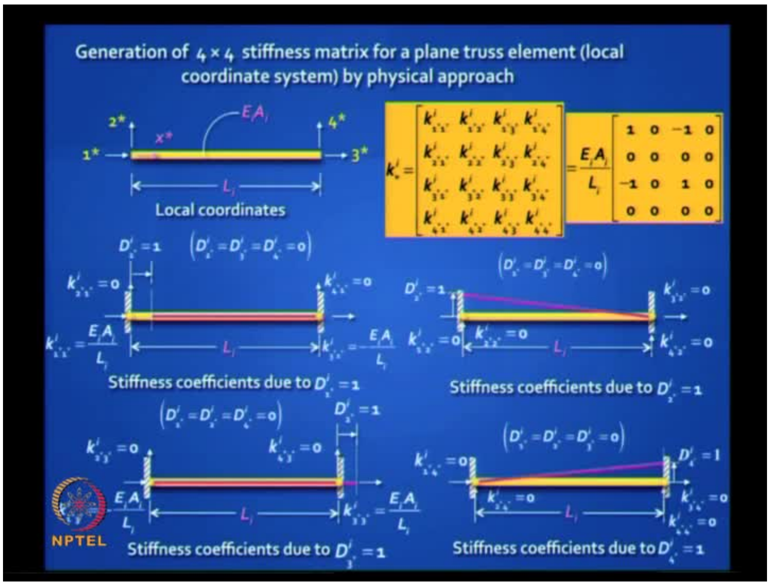

Let us take one example of something you have never seen till now. I want you to generate right here, right now in the midst of your yawning at 5.30 after doing a lab; I warn you to wake up and generate the 4 by 4 stiffness matrix for a truss element - all of you.

How to do this? It is going to look like this - it is going to symmetric, so generate it by the physical approach that means you do not need any calculations. So, what is the first thing you will do? You have to draw a sketch where you apply only one unit displacement - which one will you apply? So, D 1 star equal to 1 - can you draw a sketch? It will look like this - you just have to push it horizontally.

You have an element D 1s and all the others are restrained. What is the force you need to push it? E A by L - axial stiffness. What is the reaction you get at the other end? With a negative sign - why? Because it has three opposites, we have chosen 3 star positive.

Now, what is k 2 star 1 star? You see this symbol - does it make sense to you? We have got this value that is E A by L; we have got this value which is minus E A by L - what is this value? And what is that? Why is it 0? Because there is no reaction, there is no resistance, there is no reaction, there is no problem, so those are 0.

Next sketch is going to be tricky. Now, you apply D 2 equal to 1 – ((D 2 star (( )) - draw it. Give me the four values. In other words, help me fill up this column, because we have already filled up the first column. In the second column what are the values? First, draw the sketch and then tell me the values. This is interesting; it looks tough, but actually it is extremely easy. Can someone tell me the answers? 6 E i by - where is the i coming? This is a truss element. All are 0 - all four elements in that second column are 0? No, it cannot be all 0 - which is non-zero? 3 2 and 4 3 are 0 - all of you get 0.

You see, all you have to do is to lift it up, like a rigid body. But why not displacement? That member has stiffness only along its own axes - he gave the correct answer. All you do is to lift it up - it will move like a rigid body, because you are allowing that moment and it is spin jointed. See, the first thing you learnt in a truss member was that you will have a force only in that member and there is no shear force - is there no shear force in a truss member? So, how can you have a non-zero?

I have a member like this - please look carefully - I have a member like this - and I want for you - this is left end and I want this to go up. No problem, this is a rigid body motion, got it?

This is something new. You got a matrix with the second row 0. It is very easy to do the third one; fourth one is another set of 0's - so, what does the matrix actually look like? Fill it up; it has got lot of 0's in it.

Sometimes your stiffness matrix looks like this, which is why, the reduced element stiffness method said - why should I put lot of 0's? I do not need those 0's - so, it becomes smaller. Have you got the hang of this? Is this clear? But, then the question why should we have such a matrix? How is it helping us?

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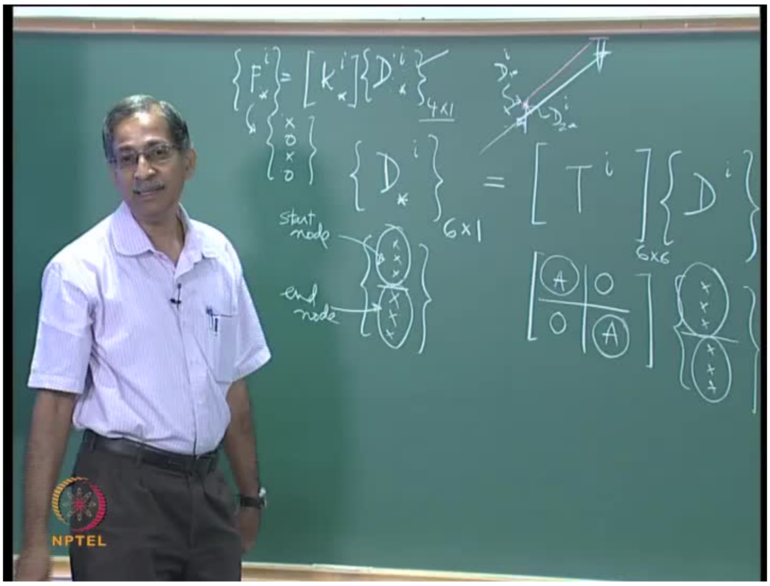

We are saying D i star into K i star is equal to F i star. I can have an element in a truss like this - please listen - I can have an element in a truss like this which takes a shape like this. Certainly, I have four values for these displacements, so the components of this 4 by 1 can be independent. No question about it. This is a valid vector and the components of these are independent. But, in this vector you know that it can have forces only along this direction. So, what does that mean?

((By the way, this will be one value - is it clear?)) So, this will be D i 1 star, this will be D i 2 star - etcetera; so, these four values can be independent and that is why we call them degrees of freedom. But, these are not independent; in the sense - of the four values, the second will be 0 and the fourth will be 0. And even worse, these two are not truly independent, because the resultant of them must be aligned along this direction - is it clear?

We will see it is related to the rank of the matrix. What is the rank of the stiffness matrix? 1 - it is 1 - you can see that this can be reduced to 1.

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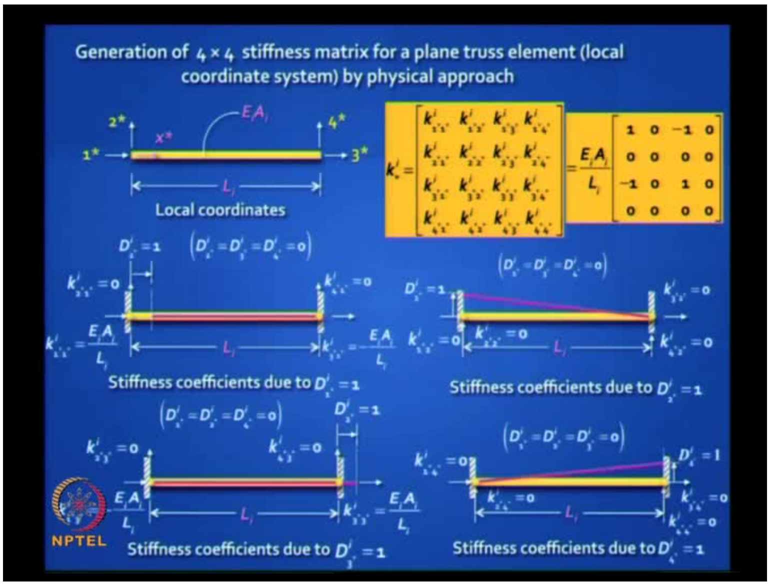

All that we learnt in concepts of matrix algebra are very deeply meaningful and they have a physical meaning as well. So, we need this matrix, it is helpful and we need this; we want the bigger picture and no matter what we do, we should get correct results. We know that even after you multiply, you should get 0's; obviously, if the second row and fourth rows are 0's, you will end up with 0's - you have too; so, this has a physical meaning.

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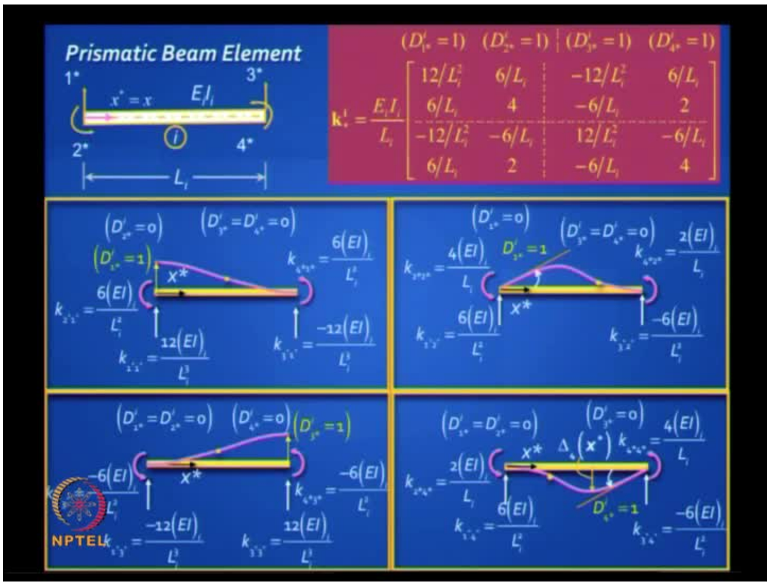

For a beam element, without much ado, the stiffness matrix will look like this (Refer Slide Time 45:45). We will study it in depth later. You already have a clue as to why it will look like that. You can break it up into four parts - we will study it in detail later; do not worry too much about it.

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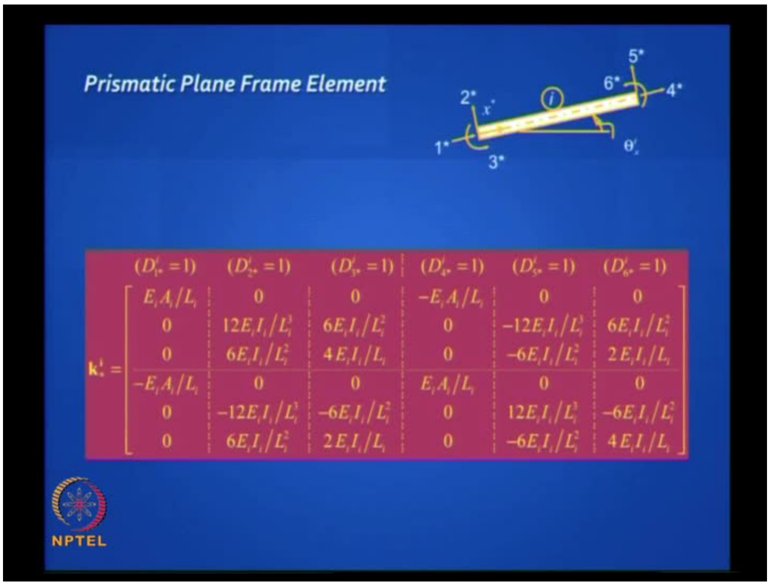

You can fill up this matrix. Here, it is a function of E I - remember 4 E I by L, 2 E I by L, 6 E I by L square, 12 E I by L cube - that is all that you can get. If you have a plane frame element, you will have a combination of the truss and the beam; we will see this later.

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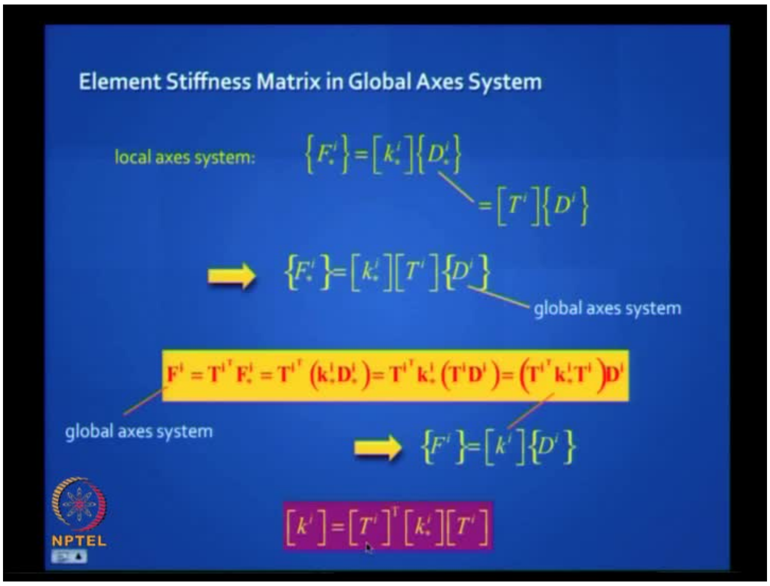

This is 6 by 6; right now it is an introduction, you should know that you have these elements stiffness matrices. Now the challenge for us is we have got the transformation matrix, we have got the elements stiffness matrix, they are properties of the dimensions of the element, length of the element and the E and A or E and I values, how do I get the elements stiffness matrix in the global axes system? Not for the structure, I am still sticking to the element.

In the local axes system, this is what I have got - I have got K i star; what I want is - this. You see how I am getting to do it. This D i can be written as T i D I - got it? This star. Now, if I do that I am able to get into the global axes system and F i is also T i transpose F star I - remember? You plug this in there and you get F star i is K star D i star from first principles. When you substitute here, you will get it in this form - this is a standard form - this is a standard transformation we do.

So, this is how you are able to derive the element stiffness matrix without the star, that is, the element stiffness matrix aligned in the global axes system. Once, you have the transformation matrix T I, which you got - it is just sin and cos and maybe 1, and you have the local element stiffness matrix - these are fundamental principle in matrices - is this clear?

Now, look at look at this transformation. This transformation tells you that this is definitely symmetric; so, when you pre multiply and post multiply with the transpose of matrix, you will end up with another symmetric matrix. This is something that can be done effortlessly in the computer. So, you generate the T i for all the elements and generate K i star for all the elements; do this exercise you got straightaway and you align all the elements in the direction of the global axes system.

To give you a parallel, you have these national parties in the central government - different political parties. I do not want to give any names. They will all have their little groups in the state government also; they are all aligned - already aligned. They will only vote for them, so it is something like that where you got the groups at the state level.

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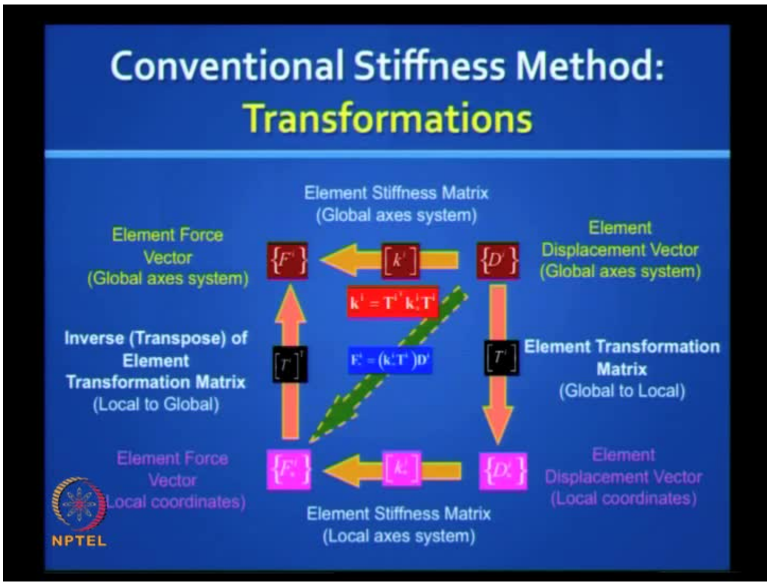

Let us take this one picture which is a beautiful way to understand it. I want you to draw it. I am going to draw a playground with four corners. At the bottom line - You see how I have done; it is very easy to do it. I am putting the forces on the left side and the displacement on the right side; so, the first transformation that I have is the element displacement vector in the local coordinate with the star. If I have this and if I pre multiply with the element stiffness matrix, I get the element force vector in the local coordinates - is this clear?

This is the meaning of the arrow. I am not writing any equations; I am symbolically showing you this first transformation - it is a linear transformation. In other words, if you give me this and I have this, I pre multiply this with this - I get this. That is why the arrows are pointing this way, so this transformation I know – clear?

What is the other transformation you know? (()) Global - no multiplying, we are playing a game; we are not deriving any equation. This is the other one - this is aligned along the global axes system; so, I have D i without the star, I have K i without the star and I have F i without the star. This is in global axes system; this is in local axes system - is it clear? So, both the arrow point this way; this is the element stiffness matrix in the global axes system and this is the element stiffness matrix in the local axes system - is it clear?

Then, what is the other relationship I have? I have this relationship. From global axes system, using my element transformation matrix, I get the element displacement vector. Now, I have got this nice play ground with four corners; so, let say I have this vector - can I straight away go to this vector? How? I do not walk along this corridor; I just cut across straight here. How do I move from here to here? ((T transverse Ok)) Even before that - is there a relationship between this and this? Yes, there you are - that is the inverse of the ((transpose)), so your playground - your corridors are now ready.

Now, let us do the diagonal. Does it make sense to you? Is this easy to remember? Those matrices are not easy to remember; play ground is easy to remember. This is a nice sketch; I want you do keep drawing it till you master it. Four corners are clear? And the four transformations are clear? They are all symmetric matrices; they are square matrices, so are these terms clear to you?

Now, let us take the diagonal - how will it look? It will look like this - this is equal to this times this - this is equal to this times this; therefore, this is equal to this into this times that - that is a shortcut - got it?

What is this in terms of this? Just take this and pre multiply by that. Instead of going round along the corridors to reach here, I take a straight shortcut from there to there and obviously, it will look like that - is it clear?

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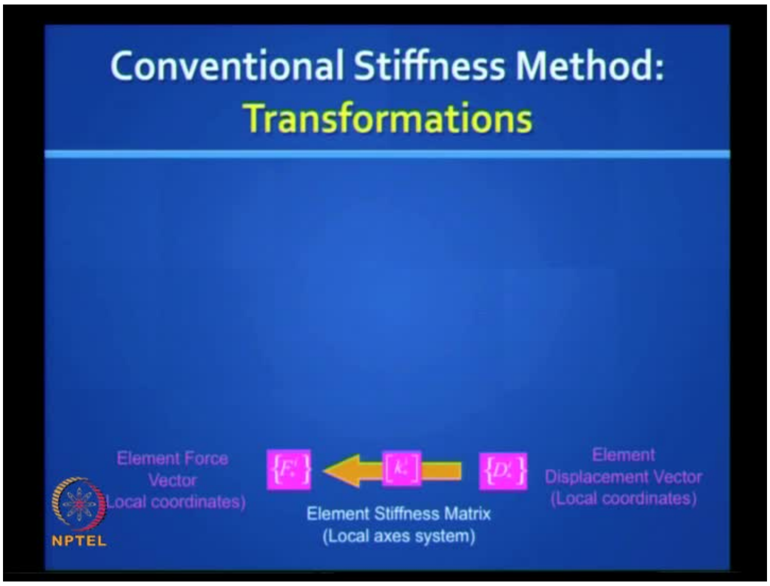

This is the first of many diagrams like this we are going to draw in this course, but they are the easiest way of remembering the relationships. Let us play this game once more. We began with this, then we did this, then we did this, then we did this (Refer Slide Time 53:31) - are all four crystal clear?

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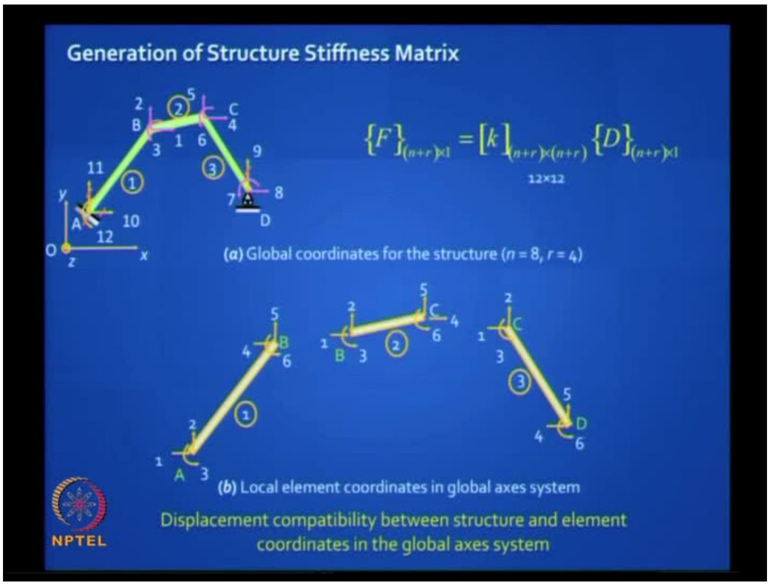

Then, we said take the shortcut from D i to F i star like that and then, we said my objective is to get this in terms of all the other jokers; so, how do I do this? Straight away like that - got it? That is all; so, you have got all the relationships to do the direct stiffness method. Let us just demonstrate with one example; then, only you will understand.

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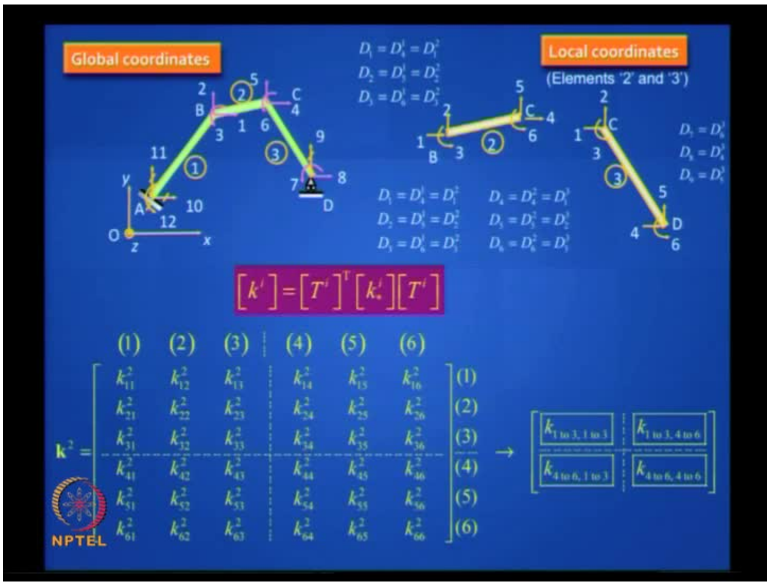

You have got this problem. You have got 12 degrees of freedom - 8 are active and 4 are restraint. These are all element level degrees of freedom aligned in the global axes system and these are the compatibility relationships; you yourself spelt it out very nicely. We need to do this transformation - how do we do it?

I want you to see how - you have got the 3 T i values, you got the 3 K i star values; you do the T i transpose K i star T I, you got the 3 K i values - what are those K i values? You got the element stiffness - the computer does it effortlessly - for all the three elements.

Now, the big question is how do you assemble the structure stiffness matrix? You have done that - you did that and you got this. Take the first element – in the first element, the start node is matching 10, 11, 12 of the global coordinates and end node is matching 1, 2, 3 of the global coordinates. So, I am writing 10, 11, 12, 1, 2, 3 and I am saying I have already got this - I have got these - is it clear? Do you understand the meaning of this?

I have assembled this - I have assembled this K 1 - how did I assemble this K 1? I did the transformation that we saw in the last step; I am now moving from state government to central government. I have got three states; all the three states have already been aligned.

The stiffness matrices are ready - K 1, K 2, K 3 is ready - how do I put up the big matrix? What is the size of the big matrix? No, for the whole structure; no, there are 12 degrees of freedom - 12 by 12.

It is like this (Refer Slide Time 54:31); please listen carefully. I got my three elements - in this, I got a 6 by 6 matrix; in this, I got a 6 by 6 matrix; in this, I got 6 by 6 matrix - three elements. All of them, somehow, must fit into my 12 by 12 matrix of the whole structure. You have to collect boxes from here and put them into the main 12 by 12 box - the right box ((goes)) to the right slot. If you do not do that, you will miss out; so, there must be a clever way of doing it - how do I do it? That is what I marked here. I have a 12 by 12 matrix and you can see nicely they come in packages of 3 by 3; so, the big 12 by 12 - I have 3 by 3; 3 by 3; 3 by 3; 3 by 3; 3 by 3; 3 by 3; 3 by 3; 3 by 3 and like that - I have got all these slots.

Now, you just need to match these linking numbers you got 10, 11, 12, 1, 2, 3, 10, 11, 12, 1, 2, 3; so, this matrix I can make it look like this - got it? This is 10, 11, 12 global; this is 10, 11, 12 global, so this should go into the main structure 10, 11, 12 global; it goes straight there, because the stiffness has add on.

You studied this in displacement method. If you have two elements and the contribution of these to the structures stiffness matrix is something and this they add on to the joints; we have done that understanding earlier. You have got an element stiffness matrix contributing - like a donation coming from the state party to the central party, so it should go to the right slot - are you clear? So, this will go to 10 K 10 to 12, K 10 to 12; this will go to - this part is 1, 2, 3, 1, 2, 3 - so, this will go to the upper left corner. Please listen carefully; this is interesting, because once you get it - you get it.

See, look at this matrix - this matrix is 1, 2, 3, 4, 5, 6. Unfortunately, my start node is 10, 11, 12 and my end node is 1, 2, 3; do not blame me for that - that is the way I did it. I did it, because we decided that active degrees of freedom should come to the top of the list and restrained degrees to the bottom. For my element level, the upper left corner became 10, 11, 12 - I mean it was not my choice, it happened and the lower left became 1, 2, 3; but, I have to make these boxes and keep it ready. Does it make sense? Then, there is a cross from 1 to 3 to 10 to 12 and 10 to 12 to 1 to 3 - is it clear? Does this make sense to you? If it does not; go back and read up from the book.

Take the next slot. The next element is - this is easy 1, 2, 3, 4, 5, 6 - nothing to worry, it will go to the right slot effortlessly; no need to play around.

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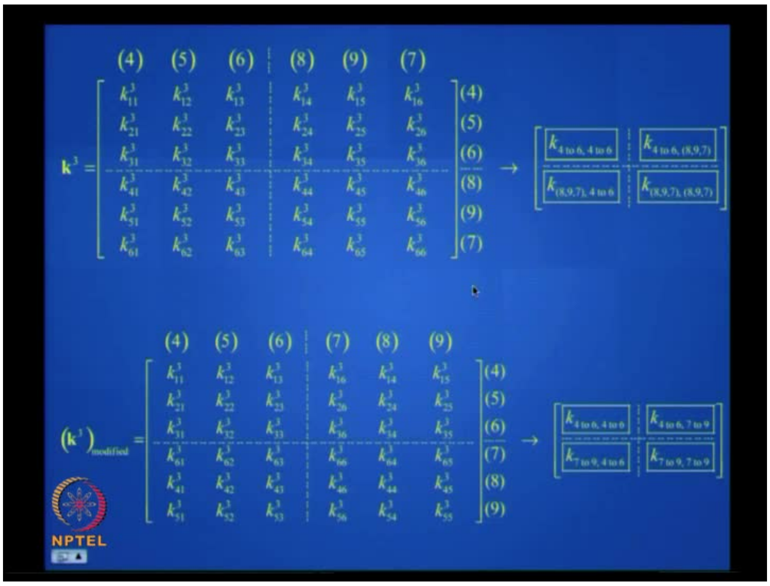

And the last one is 4 to 6 which you need to modify, because you got a mix up of 8, 9, 7. Sometimes, this happens in real problems, so you have to rearrange the rows and columns and you get 7, 8, 9.

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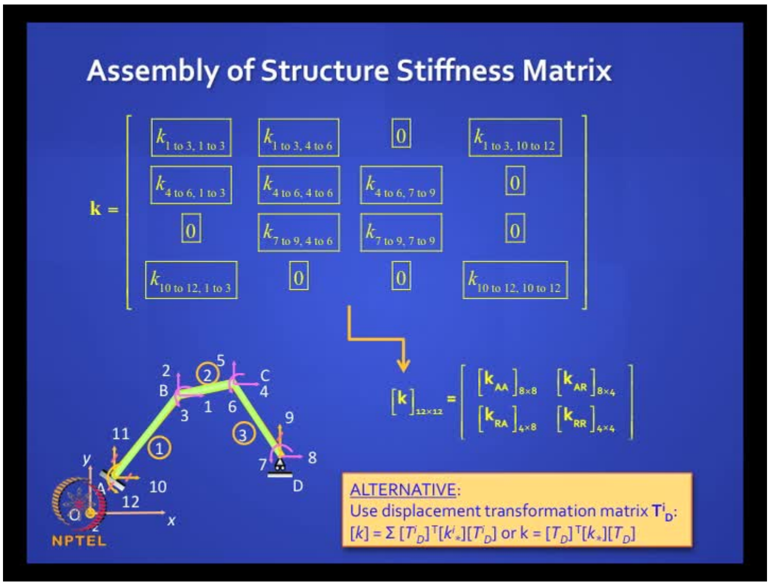

At the end of the day, this is what you need to do; this is your 12 by 12 matrix. From the first K 1, you get this, you get this, this, this (Refer Slide Time 60:16); you have to appropriately put the boxes - does this make sense to you?

There will be some 0’s and this is my K A A; this is my K A R, this is my K R A and this is my (()). If you have understood - very good; if you have not understood - not to worry, we will pick up this later.

This is just in introduction; we are not actually solving any problem, but we are trying to see how you can make by proper planning the computer do everything for you.

We will carry on from this point in the next class.

Thank you.