Advanced Structural Analysis

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Module No. # 3.2

Lecture No. # 18

Basic Matrix Concepts

Good morning to you. This is lecture 18, the second lecture dealing with the third module on basic matrix concepts in this video course on advanced structural analysis.

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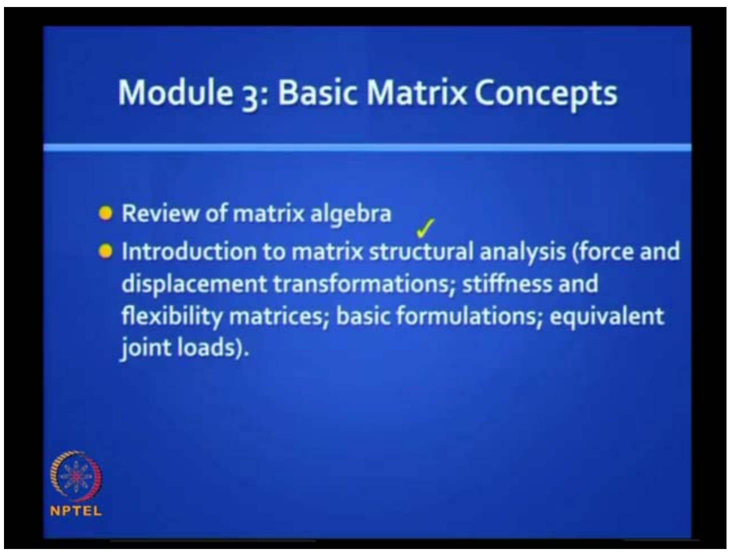

So, we are doing the third module, which we started yesterday. There there is one topic remaining in review of matrix algebra that was dealing with Eigen values and Eigen vectors, we will quickly cover it. Then, we will start looking at how matrix concepts can be applied in structural analysis.

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So, this is covered in the second and third chapters of this book on advanced structural analysis.

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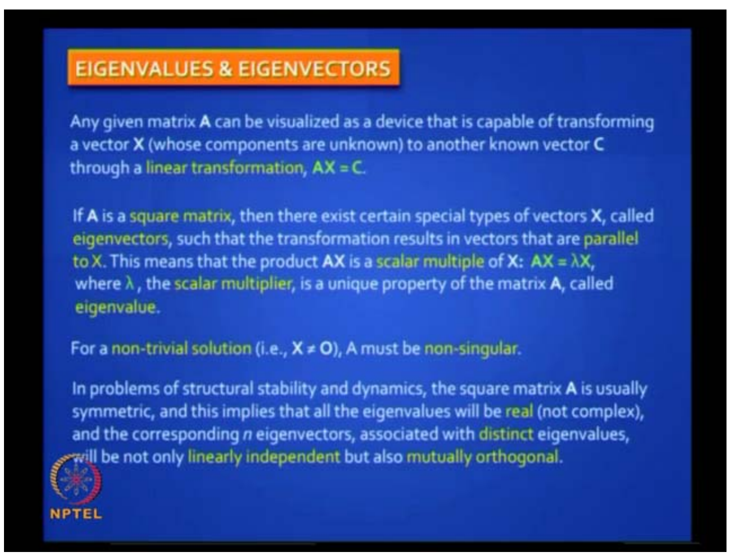

So, Eigen values and Eigen vectors are something you all studied while learning mathematics. What are the applications of these from your own experience, so far in mechanics, for example, have you ever applied it? You learn some technique in Math’s; have you ever applied it? Never?

Solving simultaneous.

Solving simultaneous that is again in mathematics, but you do not have any applications in engineering, in mechanics. Well, think about stress analysis. Locating, finding principle stresses, locating principle planes, you have an application there. Think about buckling; you studied buckling in mechanics of materials; you deal with an Eigen function there; the buckled mode shape is an Eigen function. So, it really has a wide variety of applications. And one of the interesting applications is in solving simultaneous equations; you can end up diagonalising the coefficient matrix A. How does that help making the coefficient matrix diagonal?

Other element multiplied by .

Sir, directly we get.

You get directly; there is the solutions become uncoupled. They no longer need to be solved simultaneously; you can solve one at a time. So, that is the ultimate way of solving these equations. If somehow, you can reduce the coefficient matrix to its diagonal form, which we can do with the help of Eigen values. So, I quickly go through this. If you recall, we said that any given matrix A can be visualize as a device that is capable of transforming a vector X, whose components are unknown to another known vector C, which is your constant vector through a linear transformation and that is the meaning of this equation AX equal to C; it is a transformation. You are transforming one vector X to another vector C.

Now, where do Eigen values come here, an Eigen vectors come here? From whatever you learnt so far. What does an Eigen value transformation do? What does it do in a physical sense? What will be the end result after you do a transformation? You get the same; you would not get any identical vector you get a…

Spiller times.

You will get a vector, which is a scalar product of the original vector X, what is it physically signify?

It is a vector, which parallely to

That is right. That is right you get a parallel vector. So, if A is a square matrix, then there exists certain special types of vectors X called eigenvectors such that the transformation results in vectors that are parallel to X; very simple. This means that the product AX is a scalar multiple of X, and that multiple is often designated as a lambda is called as scalar multiplier. And it is a unique property of the matrix A, and the word Eigen comes from German, which really means one, unique.

For a non trivial solution, you see one solution is when X is null vector, the physical example that you can appreciate is the case of buckling. How do you get the buckling load, the critical buckling load? Well, actually if you remember you must have done the Euler buckling analysis, you write down an equilibrium equation, assuming it to a buckled, and you find if try to find p critical, which would facilitate that buckled shape.

So, you will end up writing an equation, where two solutions are possible; one is axis, the displacement vector is a null vector that means, there is no buckling, the second is there is buckling. When you have a null vector, it is called a trivial solution. In the case of the column, it means that the column remains straight remains unbuckled. And at a certain critical load, at what is known as a bifurcation point, two alternative equilibrium positions are possible; and it will prefer to buckle; so it cannot take any further load after that. So, that is how important it is.

Now, to find a non trivial solution, it is important that the matrix A must be non-singular. In problems of structural stability and dynamics the square matrix A is called is usually symmetric, which makes our life easier; and this implies their all the Eigen values will be real, not complex; there are situations where you you have imaginary numbers coming in, but not in our real life problems in structures. And luckily for us, the Eigen vectors are this going to be distinct, and they will be linearly independent, and also mutually orthogonal that is a property of the eigenvector.

So, in stability problems, the Eigen values correspond to the buckling loads; you have and the corresponding Eigen vectors and they refer to the Eigen functions if you are I dealing with the continuum. They represent the mode shapes of buckling right. So, you have the first mode called the fundamental mode; then you have this next second mode, third mode and so on. In practice everything buckles in the on the first mode. We will study this in greater detail in the seventh module, but for now we cover the maths part of it.

And in dynamics, what do you think the Eigen values represent? In vibrations of strings, you may have studied in school; you have the first mode, the second mode, the vibration and so on. Is it not? So there, the mode of it is called a natural mode, the mode represents the shape, and it is similar to the shape you get in buckling; say the flexural vibrations in a beam. We have the first mode, the second mode, third mode so on. It is a shape that you get; and what is the Eigen value represent corresponding to any particular natural mode? They are called natural frequencies, and you probably heard a of the phenomenon of resonance in the excitation coincides with the natural frequency; then you have large amplification in the response. So, that is how important it is to study Eigen values.

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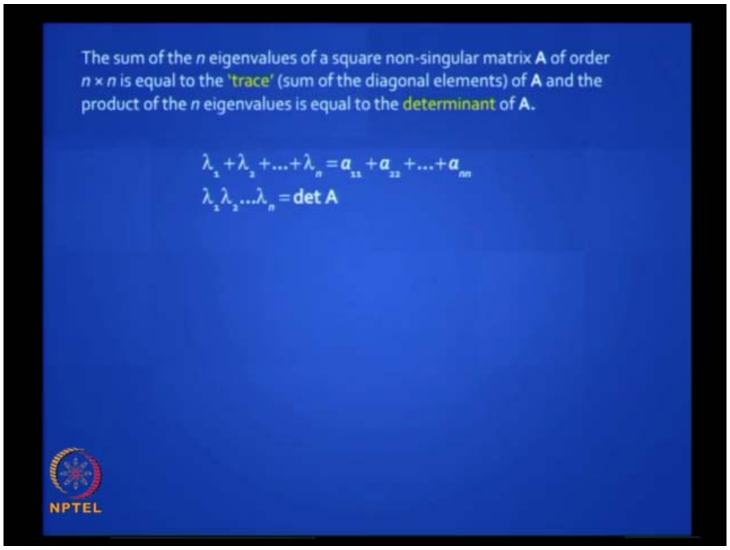

There is some properties at I think you know, the sum of all the n Eigen values of a square non-singular matrix A of order n into n that means, you have n Eigen values is equal to the trace, which is the sum of the diagonal elements of A, and the product directly gives you the determinant. So, there is a, the properties of the matrix A are linked to the Eigen values and the Eigen vectors.

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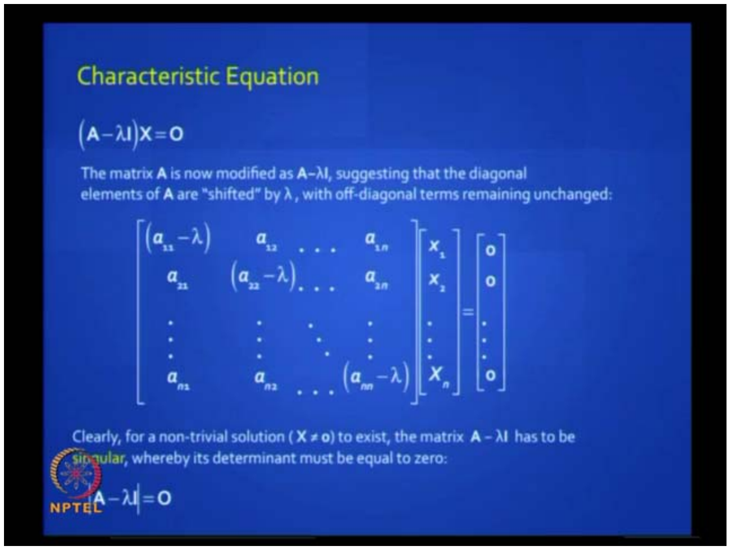

So, you can take a simple example. If the Eigen you know, you can prove this that these two properties are real. When you probably come across this it is called characteristic equation, which is what you get, when you want a non trivial solution. The determinant of this matrix will will have to be 0, which means the elements get shifted by lambda. You remember you must have done this exercise, and you need to solve, you shifted the lambda X for the left side, so now you have you have a homogeneous solutions.

Clearly for a nontrivial solution, A minus lambda I has to be singular, which means the determinant must be equal to 0. So, this is how you studied; and you end up with the polynomial equation of order n in terms of lambda, the roots are Eigen values and the corresponding next vectors, which will not have unique components you know, they will since a rank of this matrix is less than full, you will have linear dependence in the components of the Eigen vector.

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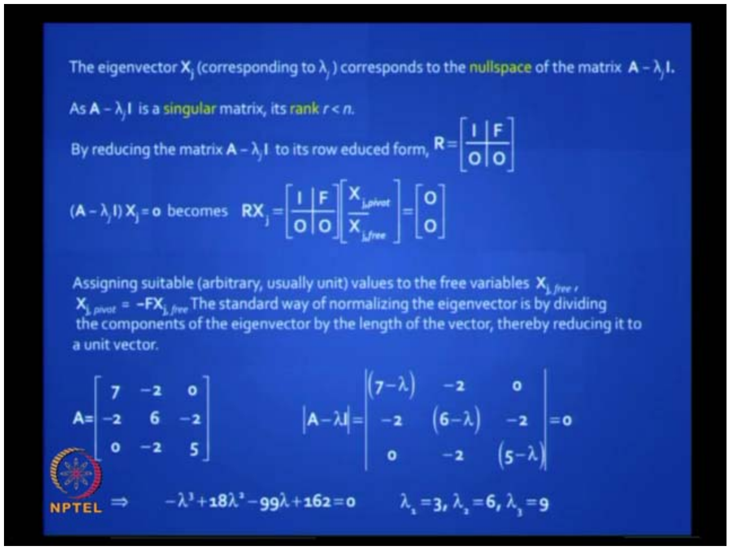

So, there are many ways of doing this and you probably studied some of them. One convenient way is to reduce this A minus lambda I to its row reduced form. And then you can easily find out using the technique that we discussed yesterday you can identify the Eigen vector. Take an example, and you done lots of these problems in think in your earlier semesters. This is the, what you do? You get cubic equation in this case, which you can solve and identify the three Eigen vectors; you put them in ascending order. So, the first Eigen value is a lowest, this gives the 3 6 and 9.

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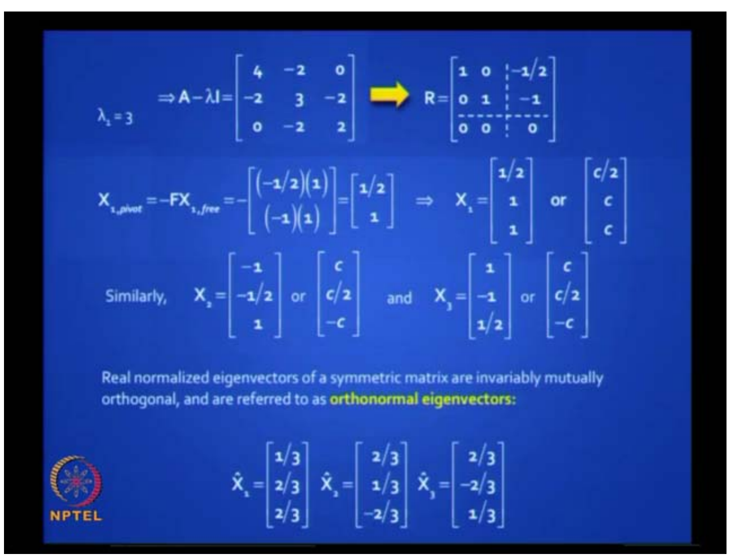

If you take one of them lambda 1 equal to 3, if you locate its row reduced form, it is easy, then to find the Eigen vector. This is not the only way you can do it; you probably learnt it in some other way. But finally, you will find that it is c by 2, c and c; now c can take any value. So, for example, in a pinned pinned column subjected to an axial load; the Eigen shape is a sinusoidal shape. The sinusoidal shape only relates every points displacement to every other points displacement, but it does not fix the amplitude that is what is reflected here; c can take any value, c is like the amplitude of that wave.

All that it guaranties is that all points on that wave have a definite relative amplitude. So, that is that is the beauty of the mode shape; the mode shape does not dictate the absolute value of the deflection. Similarly you can do it for all of them; and you can normalize these Eigen vectors, you studied them. Commonly what we do is we take one of them, the the the top one you make it to unity and and scale the others. But there are other ways of scaling and once you normalize it you call it an orthonormal Eigen vector.

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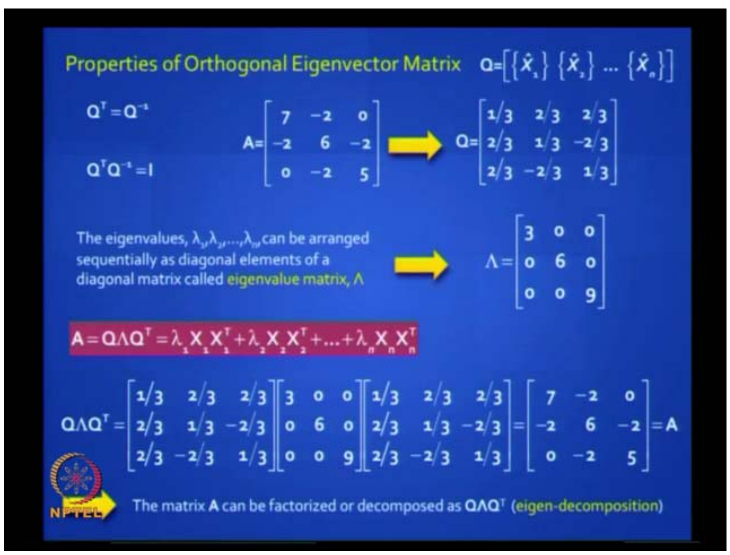

So, a vector like this, you can put together, and you can get a matrix; and that is called the orthogonal Eigen vector matrix. And it has certain beautiful properties; one of the properties of this matrix, which you see here, is that the transpose will be same as the inverse. So, the finding inverse is very easy, just take the transpose of it. For example, once you have identified Q for this matrix A, if you take the transpose you will get it, and you can also put the three Eigen values together into a diagonal matrix, and that is called a Eigen value matrix, it will look like that.

And then if you do a little trick; Q that symbol is called capital lambda u Q capital lambda Q transpose will give you back the original matrix A that is a fantastic property. And what is the advantage of this property? This property helps you diagonalize the matrix A, because your A is a original coefficient matrix. So, if you do that, you get back A, this matrix A can be factorized or decompose into that form, and that that procedure is called Eigen decomposition. These are the techniques used for solving simultaneous equation essentially.

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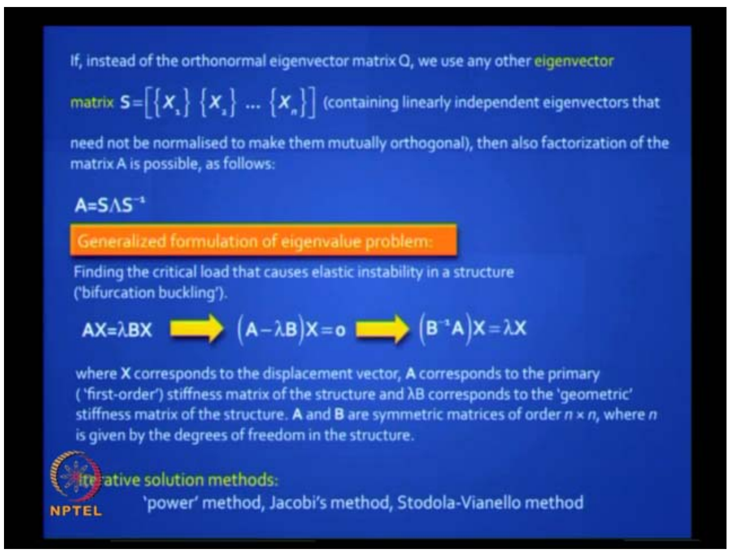

And if we do not choose the eigenvector, we choose any other I mean we do not use the orthonormal, we take any eigenvector. You can choose any eigenvector, then you will end up with the symmetric matrix, and then also factorization is possible. Commonly, we when we apply to real problems of finding buckling load and so on, we have slight shift in the formulation. Instead of saying instead of saying A X equal to lambda X, we we sometime encounter the form A X equal to lambda B X.

We will see this how this happens later, in which case you get A minus lambda B into X equal to 0, and you need to solve this equation. And typically X corresponds to the displacement vector A corresponds to the primary stiffness matrix. So, there is a strong relationship between the stiffness matrix, and the mode shapes, and the eigen values and lambda B corresponds to the geometric stiffness matrix which is something we will study in module 7. Just want to show you that mathematics is a very powerful tool in structural analysis, and it is good know at least conceptually what is going on.

You need not do any eigen value long hand calculations, the computer will do it for you, you can use those tools, but at least broadly you should never forget the background what is going on under the black box. So, that are many iterative methods in our times we have to really manually solve all these, and learn all the techniques and find out which is the most efficient.

So, there are there is power method Jacobi's method, Stodola-vianello method, and so on. And commonly we have to do this whenever we studied buckling instability or dynamics, where you have multiple modes to be consider.

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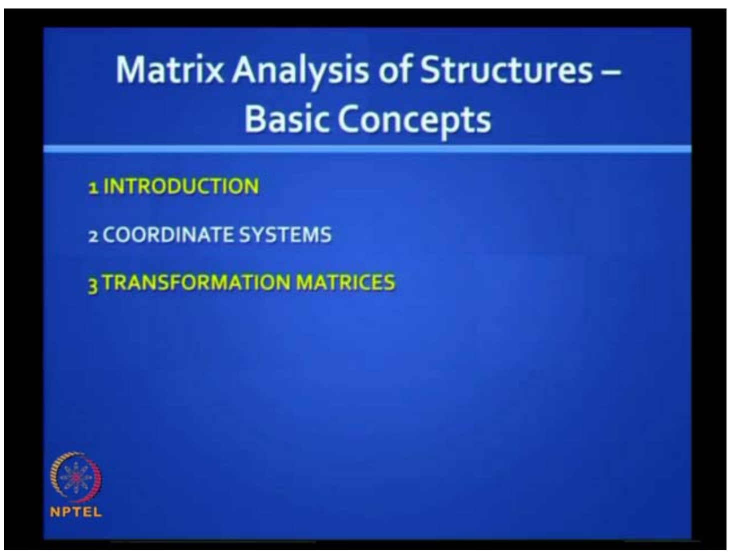

So, with this we conclude math of the math's part it, which is just a refresher. Now, we get into how to apply matrix concepts to analyze the structures.

Now, remember we have 7 modules – 2 modules was basic structural analysis which we do manually. This third module is basically introductory, it is the fourth, fifth, and sixth where we directly apply matrix methods to problems. In module 4 we apply to trusses an axial systems, module 5 we applied to beams and grids grids are three-dimensional.

In module 6 we applied to plane frames and space frames, in module 7 we do instability and and second order analysis. You will find that, these concepts really start making sense only when you start applying them. So, what we are going to do from now is only a kind of introductory, we kind of planting seeds which will take a little time gestation period for them to germinate inlet. So, once you start applying, it will become very easy. So, if you known fully understand at this stage, do not worry it all fall into place as we go on. So, the topics we are going to cover is introduction coordinate system, and these looks simple, but you realize at the first difficulty you encounter is how to convert all that you been doing on paper drawing a truss or a frame how to pass it on to the computer.

So, you need to have a coordinate system. You need to have a framework in which you can put that structure, and identify everything that is going on in the structure. So, that is how important coordinate systems is then you need transformation matrices. What do you think these mean? Essentially, you will find you need two coordinate systems, when you run a country like India, you have a central government the federal government, and you have state governments right and you have to have a some control between the center, and the states.

Likewise in a structure - the structure is one like the country the nation state India. So, the structure must have a federal coordinate system, we called them global coordinate systems. So, the entire structure is located in a global framework with one x, y, z, but then when you get to the element level; the element may be inclined and located anyway. You have you it is you better to go in for a local coordinate system, the state government; and transformation is all about switching from the local to the global or global back to the local. So, that is very beautiful it is it is really transforming - it is a linear transformation from one system of Cartesian coordinates to another system.

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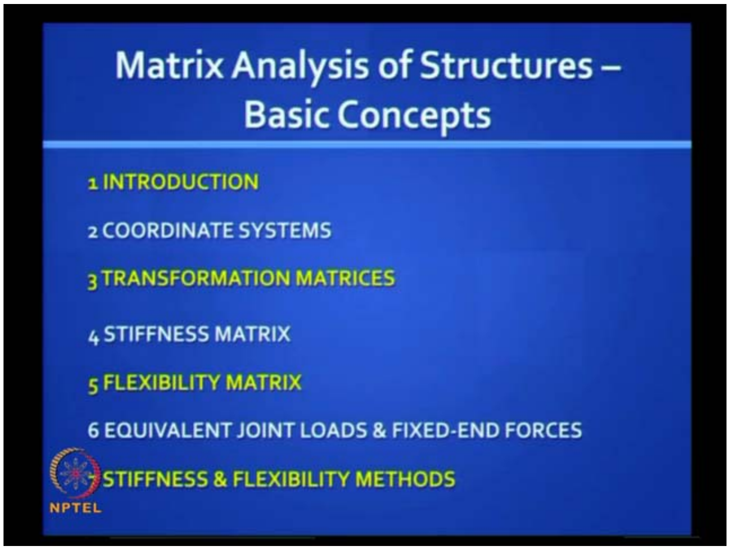

Then we need to look at stiffness matrix again here, you can define the stiffness matrix at the element level. And you can define the element system matrix at the structure level. What you think is a difference?

the entire structure.

Like what is going on in your hostel where we group the Andhra group, Karnataka group and you know the Kerala group, and then you know the Maharashtrian groups and so. They are all element level groups, but we need national integration. So, we need to connect the groups together, and make make a sleeve harmoniously, if you do not do it harmoniously you will have separation. You have lack of compatibility which is the cause of much stiffen the world today right.

So, similarly in a structure unless the members, they can have their independence, but unless they operate together harmoniously in the interest of the overall structure, you will have problems. So, similarly you have to understand stiffness matrices, flexibility matrices both at the element level, and how it all fits in at the structure level. And you already studied this whenever you encounter intermediate loads, because you want to limited degrees of freedom, you have to pass them effectively to the joints. So, you need in equivalent joints loads, and you need to find fixed-end forces.

Now, we will also look at three different types of methods, you have a stiffness methods which again branch out into two which we going to call conventional stiffness method, and reduce stiffness element - stiffness formulation, and flexibility methods; we will see this shortly.

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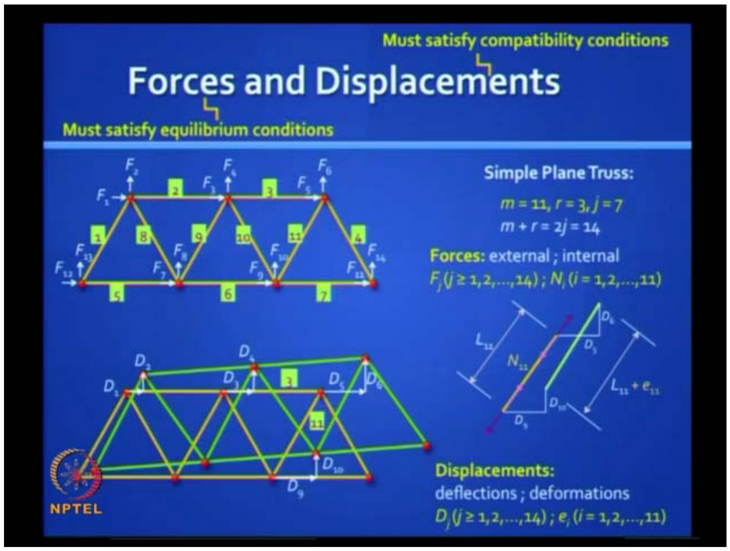

But first let us review, what we have learnt already I have shown this picture earlier, if you take a truss identify it is degrees of freedom you will find that they are D 1, D 2, etcetera. And the at the corresponding locations in the same direction, you can identify forces we call them F 1, F 2, F3 so on. These are. So, called external forces and joint displacements, where they effect the structure as a whole, that is the central government, but at the element level, you pull out one of those truss members. An element has its own internal force which we will locate at the ends of the element.

We have two forces at the two ends, and somehow these forces are link to F 1 and F 2. So, actually you you have a transformation there itself; how do you convert joint forces on the structure to element level forces in different elements. You see you can see the potential, and the basis for that must be equilibrium. Similarly, every member in this truss can elongate the two joints can move compatibility demands, that whatever moments you get at the element end levels must match the moments you get in the structure.

So, 2, 3 elements joining at one point will have the same moments in the global coordinate system, but in the local you have to resolve them into the relative components; that is compatibility. And structural analysis is all about getting a good grasp of these knowing what is happening in your force field, knowing what is happening in your displacement field, and defining the laws that relate these fields. Once you have got a good big pitch understanding, you can program everything, the computer will handle everything for you. We are lucky we are dealing with linear analysis. So, the transformation they are all linear, there is absolutely no problem.

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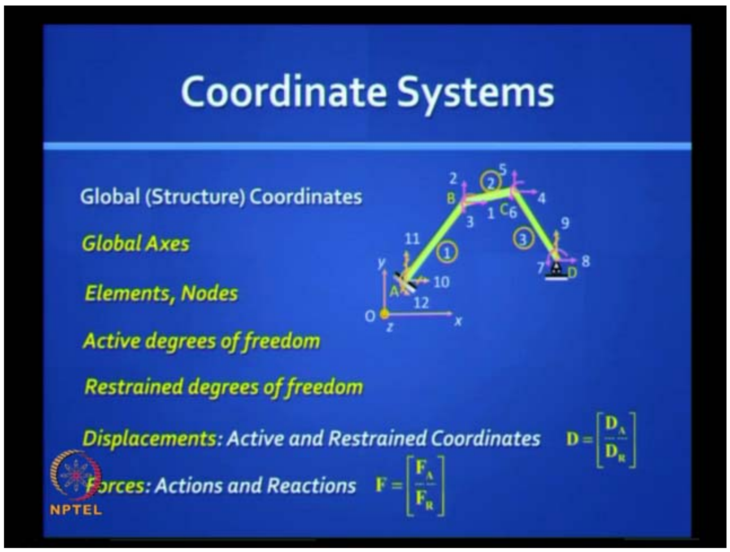

So, let us begin with the basics of coordinate system, let us say I have a structure like this. I have made it it I have made it a little complicated in the sense your inclined column, which no other more complicated types of problems you can get. And I have a structure here, it is a plane frame I first need the computer to to to kind of get get hold of the structure, and be able to draw it back to me and and so, that I can confirm that this is exactly what I want.

So, I need to have a coordinate system first to identify the joints in the structure right. So, I first choose an origin in this case, I could have chosen usually you choose the extreme lower most left end, but you can choose anywhere you know that plane. So, I have put it somewhere there, and that is my origin and I have x and y; conventionally x is pointing to the right, and y is pointing upwards, and that automatically defines z as the american says zee.

You have to follow the rules of vector algebra, which means i cross j must be equal to k, and if you follow the right hand , you will find that the z will be coming outside towards me from that plane of x, y. So, if you try to these arrow marks hold not only for forces, and translations where the double arrow in the same direction holds good for moments, and rotations. So, you will discover the z positive direction will demand that I will I should consider clockwise to anticlockwise rotations to be positive, anticlockwise So, this is the first change we are going to make from what we have studied till now. It is a big change, it is an abrupt change, because till now in displacement method we assumed, clockwise - clockwise rotational moments should be positive, because that is the way traditionally. It was done you know slope deflection method revolve like that.

But today we have a big picture proper grasp perfect we want to apply a matrix method, it should be consistent with our idea what is positive. So, we make this change, and we should do it little effortlessly. And it is like getting a getting an international driving license, you know that in some countries including India you have to stay to the left side of the lane, but when you cross over to another country, there you have to be on the right side and you are ok with it. So, then only you get the license to drive, then only you get the license to apply matrix methods after having studied basic structural analysis.

So, having done that you should first write down the coordinates for A, B, C, and D which is easy to do, if someone has given you the geometry of the structure, you can do that x, y, z of A, B, C well you you some people do not like to use any letters at all. So, they will call those joints 1, 2, 3, 4. It is, but you have too many 1's and 2 so, I thought in the early stage it is better to do this, and then you have 3 elements. So, you should identify the 3 elements, the 3 states.

This is a country with 3 states: element 1, element 2, and element 3; it is clear; and I put a circle around it. So, that I do not mix up with other numbers which I am going to show up soon. What is a next thing I need to do by the way there is something more to it to define the element you also to define the orientation of the element. What is a start of the element? What is a start node, and what is the end node?

So, that is you can you have a choice there, but if you move consistently from left to right, and from below to above then A to B is it is called the incidence of my defining defining element A - element 1, B to C is element 2, and C to D is element 3; where I need to do all these - I need to first locate A, B, C, and D. I need to identify elements, and their start node and finish node, and then I I have got that. What node I need to do. So, those are called nodes; nodes are - nodes are locations where you need to locate either a displacement or a force. So, you can have nodes inside an element the for example, we have a constant load and you do not want to convert an equivalent joint load, where you want to find the deflection in the middle of a member. You can a create a node there, nothing prevents you, but the moment you create node you got two elements joining that node.

But we should not make life more complicated than is necessary. So, we normally limit the nodes to the joints, but nothing prevents you from bringing additional nodes. So, you define elements and nodes, and next you need to define degrees of freedom right. We have primarily doing a stiffness formulation later we look at the flexibility formulation. So, the degrees of freedom here are in this plane frame you know B, and C, you have 3 degrees of freedom. And let us align those along are chosen Cartesian coordinate system. So, along the x direction, we say the translation is one; along the y direction positive upwards is 2, and anticlockwise rotation at the joint B is 3, does it make sense.

So, we know at every joint, you potentially at 3 degrees of freedom - 2 translations, 1 rotation D 1, D 2, D 3 would mean that, but then move you move on to the next joint from B to C, because you have only active degrees of freedom at B and C, at C what are your coordinate numbers you if you finish 1, 2, and 3; you need to do 4, 5, and 6; you need to do 4, 5, and 6 right, and that is all you have.

What you have more? You have more degrees of freedom yes at D also you have some more that is a roller. So, it can move, you have an active degree of freedom there. So, it can move to the right, and it can also rotate right. Now, you can choose 7 and 8 there, is it clear; then what else do you have. Then you have, if you want to take care of support reactions you need to have or you have support moments. So, to complete the picture you should also look at what is called as restrained degrees of freedom.

So, you do that. So, you you can bring in 10, 11, 12, and 9, 10, 11, 12 got it; these are the restrained degrees of freedom 9, 10, 11, 12 right. You can do something I have change the color here, but when do it by handle probably using the same black or blue pen. So, there is something nice, you can do to identify restrained coordinates.

You can just introduce a slash, introduce a slash in the arrow marks; the slash tells you that that degree of freedom is not active it is restrained, when you use software packages, you have to feed it is a binary code. You have to show whether, the it is restrained or it is free to move. So, normally the numbers are 0 and 1, 0 and 1.

So, here we are we are doing this. So, with this we have captured all the displacements at the different joints D 1 to D 12, some of them are active, some are restrained; is it clear. And by the way restrained does not preventive from including support settlements, but then you you say that it is still a known quantity; usually it is 0, but it can be non 0. So, you you can define a displacement vector D A and D R which I can partition like that, I am putting all the restrained at the end of the list, is it clear.

The D vector - the displacement vector have displacements at active degrees of freedom, active coordinate locations, and R stands for restrained coordinate locations. Now, luckily A and R also other other words to describe it from a force point of view. So, you can also have F A which is conjugate with D A, and F R which is conjugate with D R.

It is exactly, where you have active degrees of freedom that you can apply loads. is it not? So, A stands for actions which means loads. So, you have F A corresponding to D A, and it is exactly where you restrained displacement you get reactions, and luckily for us R also stands for reactions

So, F A, F R are constitute our first vector in the global picture, and D A, D R constitute our displacement vector in the global coordinate system, does this makes sense. Now, in the word coordinate has two meanings here, we are referring to 1, 2, 3, 4 all the way to 12 also it is coordinates, and the Cartesian coordinate system that we use are also coordinates. So, you have to be careful in in interpreting the word , does it make sense to.

Sir, in the joint you have to provide the like both can…

Rotate both has to rotate at same theta no, what is your if you do not say anything we assume we are dealing with the rigid joint. For example, member 1, and member 2 are rigidly connected at B if you do not make any statement. If you want to and if you dealing with the ends boundaries, there you to have to define the supports is it free or is it restrained, but you can have more complications which will introduce later not in the beginning, supposing I have an internal hinge, then it is little difficult we will study that later.

We should begin simple then we will get it into complication, but it is a point how you can make end releases, but at the as beginners let us not introduce of complications. Let us take simple problems - straightforward problems do you understand this, is it clear.

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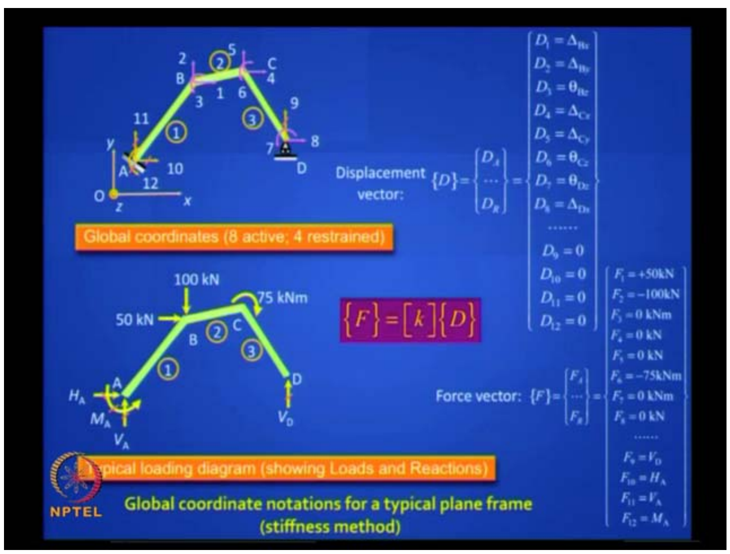

Let us demonstrate how it is done, let us take the same frame, and the physical meaning of D A and D R is shown in this vector, you will find that the first element in this vector D 1 actually points to delta B x. That means, at this joint B, if this moves to the right by some quantity that component of the deflection to the right is called delta B x. D 2 will be the amount it moves upward right at B, D 3 would be the anticlockwise rotation here; if you end up with a clockwise rotation you should put a negative sign. Likewise you can cover the all the way from D 1 to D 8, then you are left with D 9, D 10, D 11, D 12. Those are restrained degrees of freedom, and unless you include support movements they are tacitly taken to be 0, is it clear; no doubt about that.

Now, in this problem you have 8 active degrees of freedom, and 4 restrained degrees of freedom totally 12, 12 degrees of freedom fine, very clear no problem. Let us look at a problem where you have some loads applied, how will you write down, the force vector. So, I have in this problem 3 loads, so what would you call 50 kilo Newton. What is plus 50 kilo Newton? It is yeah you should talk in the language of the coordinates F 1 is 50 kilo Newton, what is 100 kilo Newton

Minus

Yeah. So, F 2 is minus 100, and what corresponds to 75 kilo Newton meter

Minus.

Yeah F 6 is.

Minus 75.

And what about the others.

0.

They are all 0 on the action side F A. So, F A is exactly what we said, you correctly said plus 50 minus 100, and minus 75 the rest are all 0 because though those joints have the potential to take loads for this particular load case those values are 0. And the reactions are not 0, but they weight belong to the unknown category; that is part of the response we need to find about right. Thus it make sense very simple we are going in baby steps, this is very straightforward.

Now, we also recognize there is something called as stiffness matrix for the entire structure, which means if you tell me that I have, you know on account of that loading I have the displacements vector D, then the force vector that cause that displacement vector must be F, and they are related by this equation F equal to k D.

If the displacements are unknown then I can easily get the displacements by…

Inverting the force, but it is not so simple, because you will find that in the vector F, we have F A and F R. F A is known easily, and F R is not known. Similarly, in the displacement vector D, you have D A and D R; D A is not known D R is known.

So, you have to you cannot apply this directly, you have to play with it and we will see how to play with it. Now we will switch from central government to state government. So, how do we do that, what do you suggest?

Things are this.

No, no no they are totally a coordinate system.

See, capital of India is New Delhi. Do you want the capital of Tamilnadu also to the new Delhi.

.

Where do you want it to be.

You want to be here at Chennai right, you want to be in the bus, this is the origin this is a . So, this becomes your reference point, New Delhi talks to Chennai and the whole of Tamilnadu is governed by Chennai. So, your coordinates is your origin should be. So, you have the choice to choose where I mean, it is a you need to put in the middle of the state, but we do not usually put the origin in the middle of the member right. Where would you like to put the origin.

Which one you prefer.

A .

Which. So, that is what that is what, you do you start the origin at A, then you do not align your x, and y in the why should you follow what Delhi says. You choose your own local system right; so, but if you I write same x and y will be confusion. So, when I am dealing over local coordinate system I put a star, I put an asterisk. So, whenever I put a star what does it tell me, I am dealing with local coordinates not global coordinate, is it clear.

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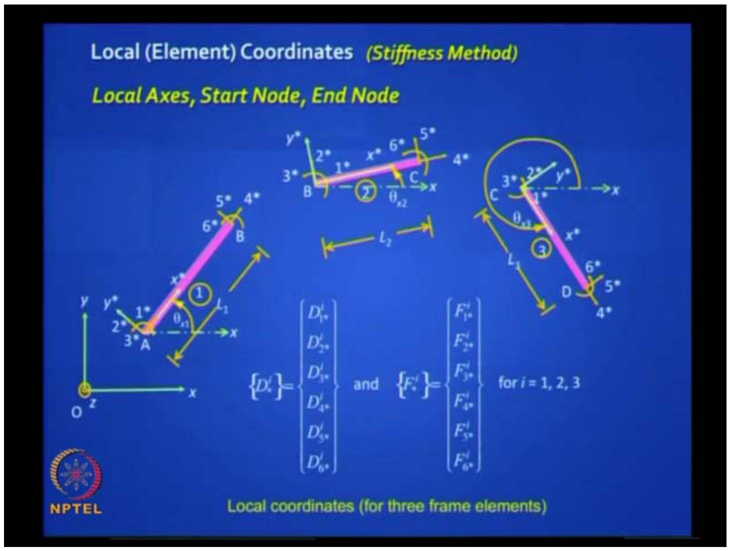

So, I have X star Y star for element 1; and I have to finally, link it to the global, because the X and Y global I shown below here. This X and that X have to be linked; how do I link it? By the relative rotation. If they are parallel theta is 0 right, and I have to choose a direction, so I consistently choose my anticlockwise direction. So, this is my one side define theta X 1, everything is defined; I know, how Y is oriented with Y star? Is it clear? Now, having done that, I can do it for other states right. So, I go number 2, I do the same thing, but now I say theta x 2 and theta x 3. So, the elements are L 1, L 2, L 3 as their links. Assuming they are prismatic, if they are not prismatic; let us see you have a stepped to element, then it is worth creating a node wherever the step is taking place right and treat that as a separate state.

So you have; is this clear to you? Now, what is a next step, next thing we need to do? We need to again bring degrees of freedom. How many degrees of freedom do you have at any joint, any node?

3

3

3. So let us, can we choose it this way? 1 star, 2 star, 3 star, 4 star, 5 star, 6 star, you agree that these are independent. So, I have 6 degrees of freedom for a plane, frame element, what do I label for the element 2? Should I do 7, 8 and 9? No. yeah I will stick to 1, 2, 3. I am only saying state number 2.

1

Does it make sense to you? Clear. Central government, state government, global, local; good; now what? Now for each element, for the i eth element let us say; let us see this is simple structure with 3 elements. I can have a structure with 500 and 25 elements. So, i is a generic index that I will use. So, I defined, now this is a definition I use. If I say D without any star, it goes to the central government automatically. So, I have to put the star, I can put above or below, I am putting it below, because I need to have other indices. And I am putting my i here; i can be 1, 2, 3 to distinguish between the three states. So, does it make sense to you?

So, I have a displacement vector 6 by 1; and I have a corresponding conjugate force vector. Does it make sense? So now, I have creating a framework with which to play, start playing my game, but without creating a framework which all of us share and understand and agree to we will have problems. So, is this clear? Is the notation clear? By the way, different books give different notations; soon get confused that is the concepts are the same. So, you have to define the local axis, you have to define the start node, you have to define the end node, and everything falls into place. The theta is crucial here.

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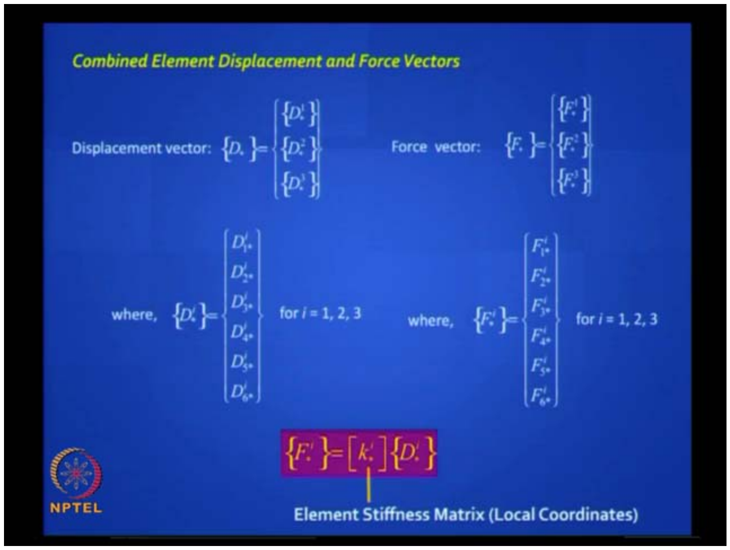

Now, you can also combine all the elements together, and put them all in one group; for example, I call a combined element displacement vector D star without the i, which means it includes D 1 star, D 2 star, D 3 star each of these is 6 by 1. So, what is the size of this one element?

18.

18 by 1, if I want to. If I have 525, I would not waste time doing this, because it is too complicated. The three elements maybe it is nice to put them all together, put all the guys in one line. Similarly so, that is similarly I can do for F star. Is it clear? So, this we will called the combined element displacement and force vectors, it is optional, you may never need to do this, but sometimes for small structures, it is good to put them in one line.

Now, the element has its own governance right. So, at the element level, you have a relationship between the element force vector and the element displacement vector, and you can talk of an element stiffness matrix in the local coordinate frame work. Does it make sense? So, you have an element stiffness matrix; you need for each element in that structure, and you have a common structure stiffness matrix. And obviously, the element stiffness matrices contribute to the structure of structure stiffness. Does it make sense? How how do you putting all together, is what we are going to study.

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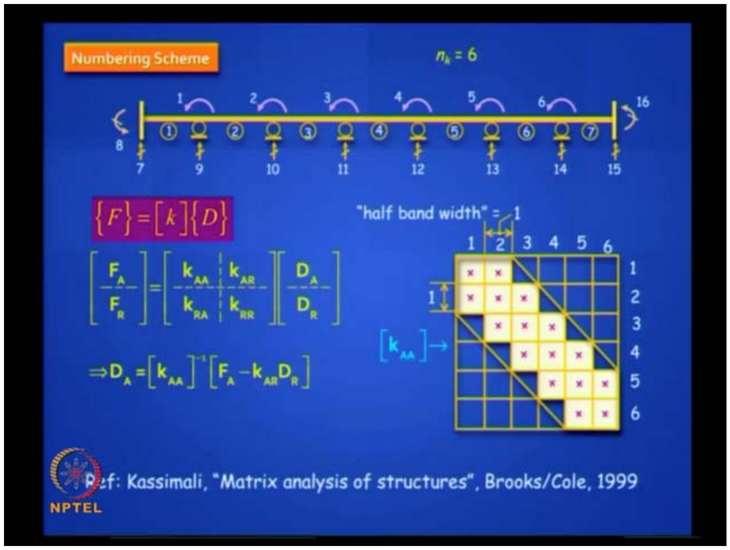

Take a look at this. I am showing you a three examples from a very good book by Kassim Ali, where he shows how we can do the numbering in a in an optimal way; numbering of numbering of a global coordinate system. Here, I shown you a continuous beam, a 7 span continuous beam right 1, 2, 3, 4, 5, 6, 7. How many degrees of freedom do we have? What is the degree of kinematic indeterminacy?

Here 6 rotations I did not write A, B, C, D, but they are the… how will you number them? Well, it is straight forward here; you number them sequentially like this. Remember, anticlockwise positive; we, earlier we did clockwise positive; now we are doing anticlockwise; 1, 2, 3, this is fairly obvious. Suppose, when I do in a funny way, I put 1 here, then I choose to put 2 here, then I put 3 here. What do you think will happen to the stiffness matrix? It is a good question.

It will just shift a column.

Let me ask, you are you are temporarily not in the class. What do you think will happen to the stiffness? Just to wake you up. It is an 8 o'clock class and half of you are half asleep. Tell me.

Any one.

It will just shift the rows or a column.

What will happen to the type of matrix you get?

Same matrix, but same numbers, but elements will…

No, some some beauty in the matrix gets corrupted. What gets…

Symmetry.

Symmetry will… the stiffness matrix has to be to symmetric we proved it; it would not be banded. The all of the place, it does not looks nice; you cannot put it nicely. So, you want to make A banded, it may still be banded, but the bands are you mean, you are pulling in bands stretch to the extremes, you do not want that. So, it is important to keep your band small; and there is a measure for it is called bandwidth or half bandwidth. And these are your restrained degrees of freedom. So, let us just look at the active degrees of freedom; you forget about the restrained degrees of freedom.

Incidentally, you can you can partition your K matrix into active and restrain components using these notations. Does it make sense to you? If I have F as F A and F R, and D as D A and D R, I can write k as k A A k A R k R A k R R, which means F A is K A A into D A plus K A R into D R right. Now, if D R is 0, then the equation is simply F A equal to K A A into D A which is, what is you...So, which is the most common way of dealing with the stiffness matrix. Is it clear?

So, let us look at K A A, and you can find D A from that. If you look at K A A, if you do this numbering system, you get a beautiful band like that. And the half bandwidth thus it is called, turns out to be one that is, that means, you you have the principal diagonal, you have to take only one more cell to the right and to the left of that principle diagonal; that is fantastic; it cuts down your computational work, and your storage to high degree of efficiency. I am not saying, you need to do all this in your class assignments or when you actually doing structures, when you are doing programming, it is important to figure out, how best to do this?

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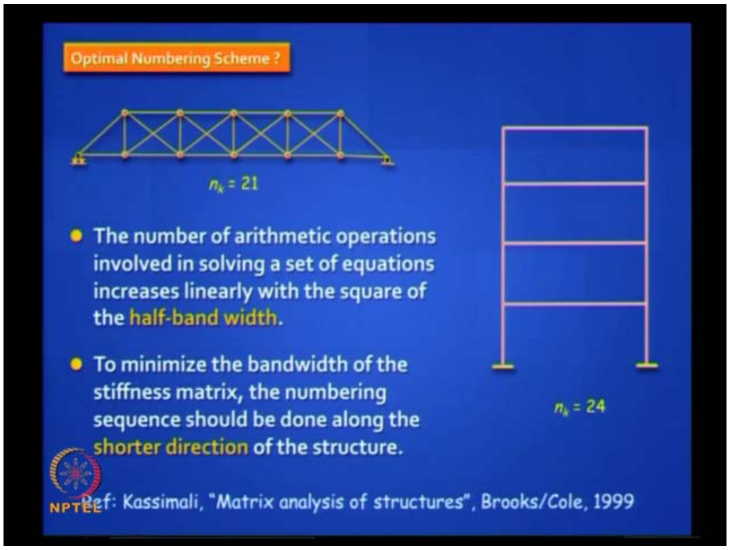

So, let us take a mod difficult structure like a truss or a frame. Now, you will find your 21 degrees of free active degrees of freedom in the truss, and 24 here; we will figure out it. Now, let us take the frame; every joint has three; let us include actual deformation; how many free joints you have? 8; 8 into 3 - 24. Clear, similarly for the truss. So, is there an optimal numbering scheme? What do you think? Would be the best way to do the numbering, so that you still get you you minimize your half bandwidth. So, that helps a lot.

Nearby places.

Nearby places. So, the number of arithmetic operation involved in solving a set of equations increases the linearly with the square of the half of bandwidth into minimize the bandwidth of the stiffness matrix, the numbering, sequence should be done along the shorted direction of the structure. This is what we did in the previous example.

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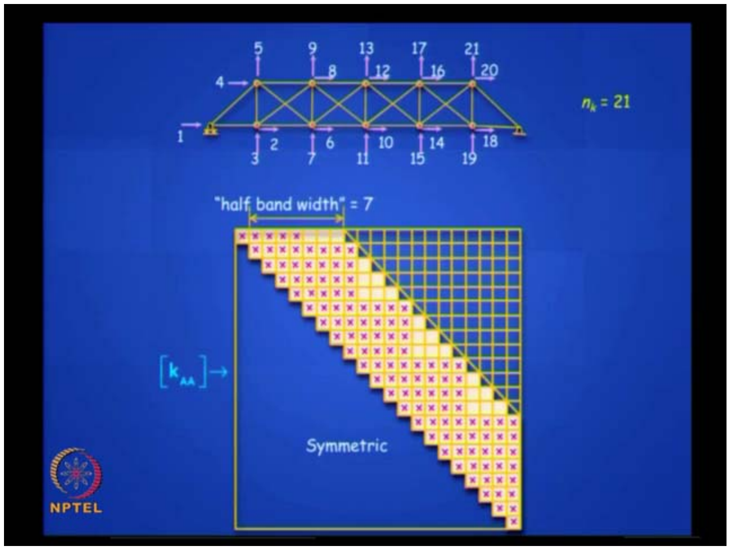

So, this really means if these are active degrees of freedom number them one you got 1 here 2 3 then do not go here do not go along the longer direction go along the shorter direction 4 5 then you can either go here 6 7 8 9 and. So, on does it make sense if you do it this way you will get A Banded matrix which which looks like something happen which looks like this it is a symmetric matrix the half bandwidth is roughly one third the degree of indeterminacy that is very good it is a neat arrangement

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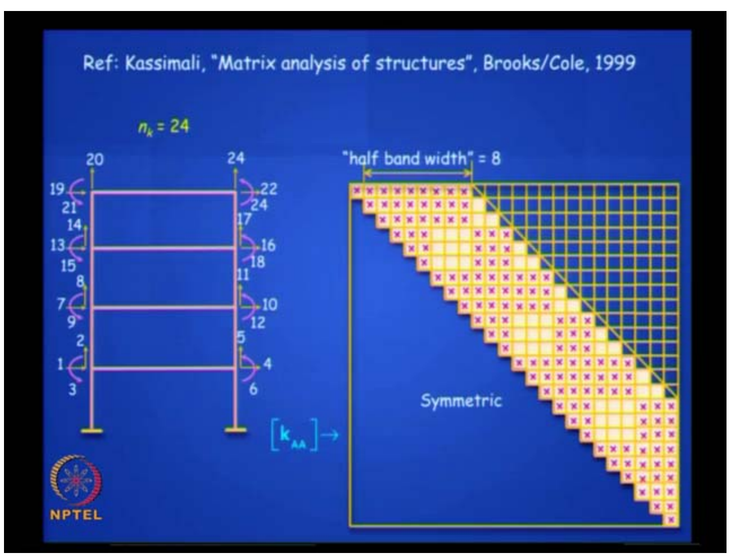

And if you take the other example, you have, you can, you should number them in this way along the shorter direction 1, 2, 3, 4, 5, 6, 7, 8, 9, all the way here you got 24, when you look at the matrix it looks like that. You may still get 0 cells in between, but you cannot get it less than this. So, half bandwidth is one-third and k turns out to be 8. Any questions?

After 1, 2, 3, we are not going up, but we are going

You should go along the shorter direction also

Column is shorter than width.

Not that way. We look at a bay, overall size of the structure. It is not height, the links of the members do not matter at all; links do not matter, because there is just properties, it is… So, this is the important lesson to learn. We will stop here. Thank you.