Advanced Structural Analysis

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Module No. # 3.1

Lecture No. # 17

Basic Matrix Concepts

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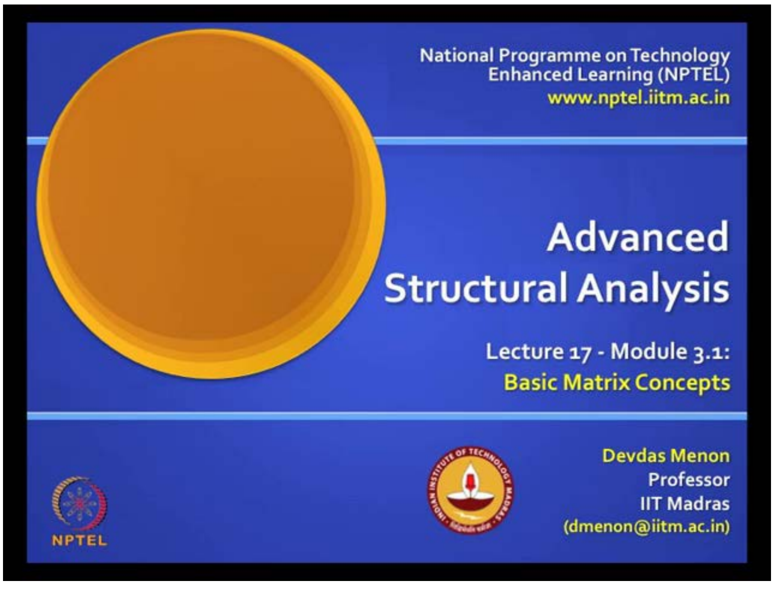

Good morning. We are now starting lecture 17. This is our first session on module 3, basic matrix methods.

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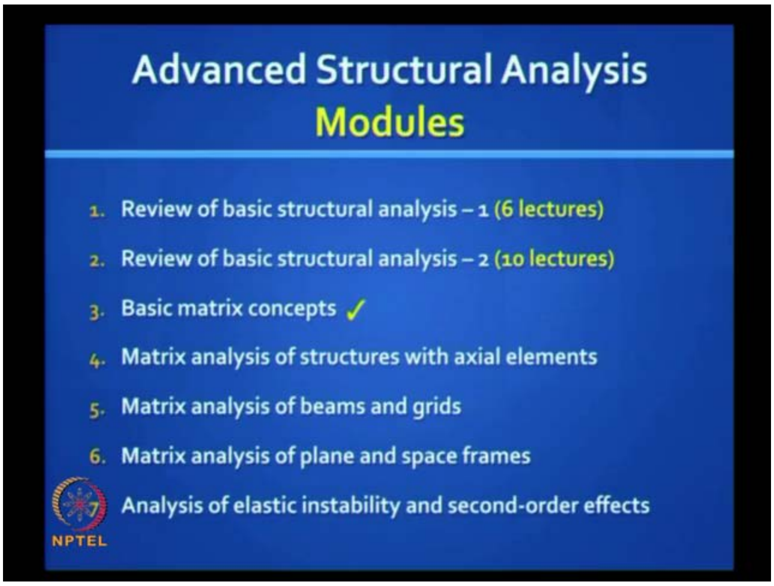

We have completed the review of basic structural analysis. We took 16 hours to finish 16 lectures. The reason why we spent so much time is - you really cannot do anything advanced unless you are strong in the basics; that is the continuous problem we face with structural engineers. I think it is a very good idea to always go back to the basics and strengthen them.

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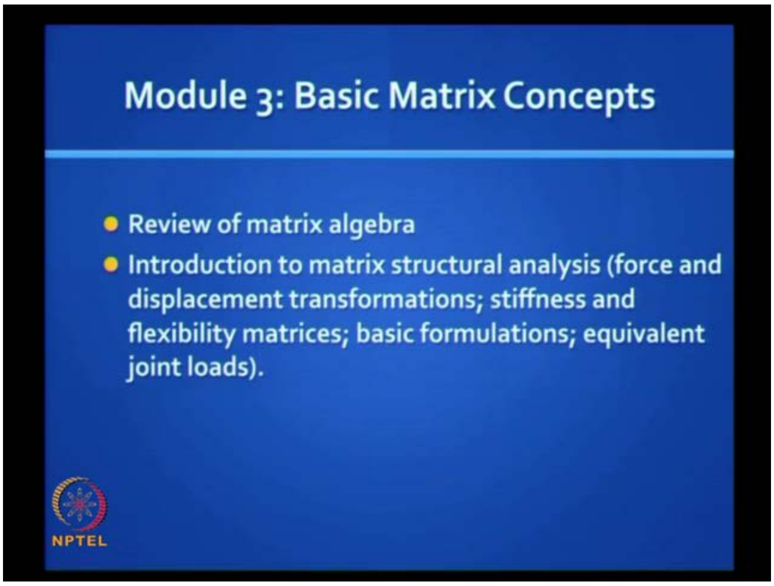

This is module 3, basic matrix concepts. We will first cover simple mathematics, principles of linear algebra which you have probably already studied. So, it is a quick review and then will get into how you can apply concepts of matrices to do structural analysis.

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The book we will be following is the book I have written called Advanced Structural Analysis published by Narosa in India and Alpha Science International; it is a hardbound edition abroad. The topics covered in this course after the review of basic structural concepts, will be matrix concepts and methods. Then, we look at how to apply these concepts to structures with axial elements like trusses; next, how to apply them to beams and grids; next, plane and space frames. Finally, in the seventh module, we will be covering a bit of non-linear analysis as applied to elastic instability and second order analysis.

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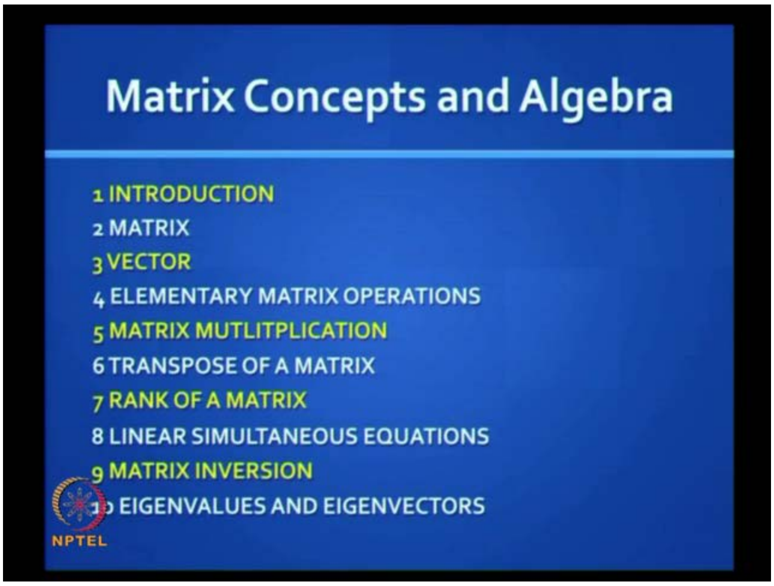

In this particular session where we cover the mathematics underlying matrix concepts, we will review what we mean by a matrix, a vector, the elementary matrix operations including matrix multiplication and transposition. We look at what it means to know the rank of a matrix; how it is useful in solving linear simultaneous equations, which is what we need to do in structure analysis and how it can be done using matrix inversion. Finally, we look at Eigen values and Eigen vectors because they have a role to play - especially when you do elastic instability analysis, and later, in dynamics.

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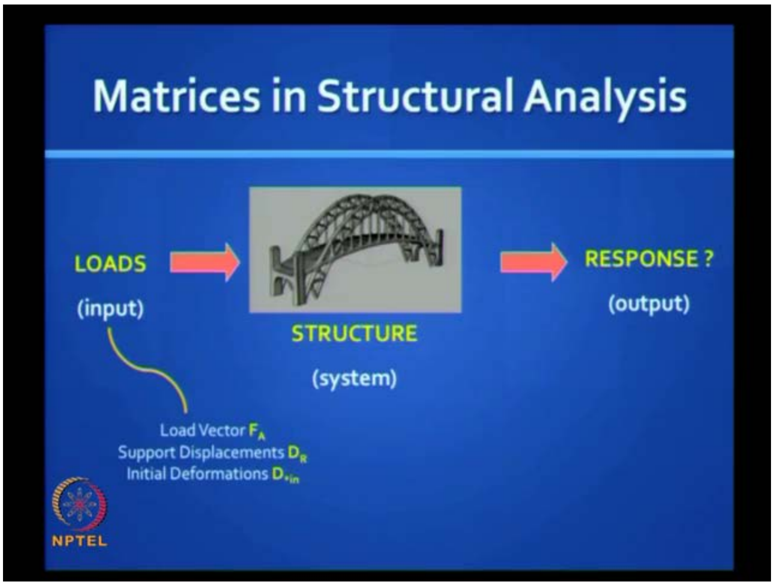

Before we do all that, we should see the role that matrices play in structural analysis. Why do we need to study this topic which you had studied in mathematics? How does it have a role in structural analysis? Could someone give an answer to that question?

Structural analysis is all about finding the response, the force response and displacements response of a given structure when it is subject to given set of loads. The loads could be direct loads, direct actions or indirect loads including support movements, environmental changes and construction errors. So, where do matrices come in for a one load to the other points at the locations and we can build the matrix for different position different forces what would be the reaction for these forces. Now, we are actually stepping back from calculation and trying to look at the big picture because if you want a digital computer to do structure analysis, you have to see the big picture and you realize that basically you are dealing with numbers.

If you want to find unknown forces, internal forces, bending moments, shear forces and actual forces at different locations, you are actually dealing with numbers which can be put nicely in arrays. You are dealing with vectors and you are dealing with properties of the structure like the stiffness matrix and the flexibility matrix. That is how you can deal with it in a matrix configuration. For example, from the input side, you definitely have the load vectors which are loads - error applied at different joint locations. You may have known support displacement and so you have a support displacement vector; you may have some initial deformations or temperature changes.

What you need to find is the displacement at all the degrees of freedom that you identified and the active coordinates; you need to find the support reactions; you need to find the internal forces; you may also want to find the member deformations like the elongations in truss members. In short, all the variables, all the quantities that you need to find in the force field and displacement field can be expressed in a numerical manner, but you need to conveniently do it and you need to do it in a systematic way.

If you want computer to do it, you also need to first model the structure itself, which is not that easy. That means you and I can see a truss and visualize it, but how does a computer do it? Well, the computer does not think. You should be able to even define the geometry of the structure using coordinates, know Cartesian coordinates and you need to also check whether what the computer interprets is the structure that you have. Please note you are dealing with a mathematical model of an idealized structure. In modern software packages you have a tool visual ability, where you get back the picture of the structure that you input. If you find some members are out of place or oriented the wrong way, you can make corrections. You can use visual basic for example, to do such things; that is a separate field.

Basically, you have some unknown quantities and some known quantities in the displacement field and the force field. You need to know certain laws, which are hidden in the structure and which enable transformations to take place.

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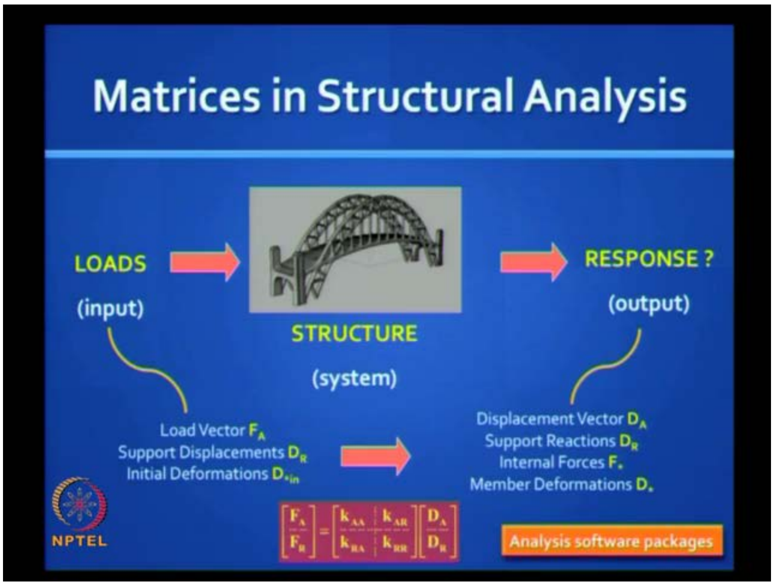

So, basically, we want to transform the loads to the responses and this becomes easy, relatively easy, when you are dealing with a linear analysis because it is a linear transformation. You are actually converting one vector to another vector with the help of a matrix. (Refer Slide Time: 03:35) For example, one of those sets of relationships are, as I have shown there, relating forces, the force field to the displacement field with the help of the stiffness matrix which you can partition.

You have a large number of analysis software packages available today. Unfortunately, today, we are in a stage where structural engineers are no longer doing any manual analysis and they are learning how to use the design or the analysis manual that comes along with the package. You input some numbers and you get some output, but if you really do not know what is going on, it is garbage in and garbage out. In fact, if you read the preface to this book on advance structural analysis, I have raised some of these concerns. In the olden days, when there were no computers, there was no choice and you had to understand structural behavior. You had to know what is you have to know whether the bending moment that you calculated makes sense or not. You have to know whether it should be sagging or hogging, but now you do not know because you are getting some numbers thrown up by the computer; the need to do analysis faster and quicker is so strong now that there is no time to pause and reflect. That old breed of structural engineers who really understood what was going on is a vanishing species. It is important to always step back to have some rough checks, which is where you need to have a good foundation of basic structural analysis.

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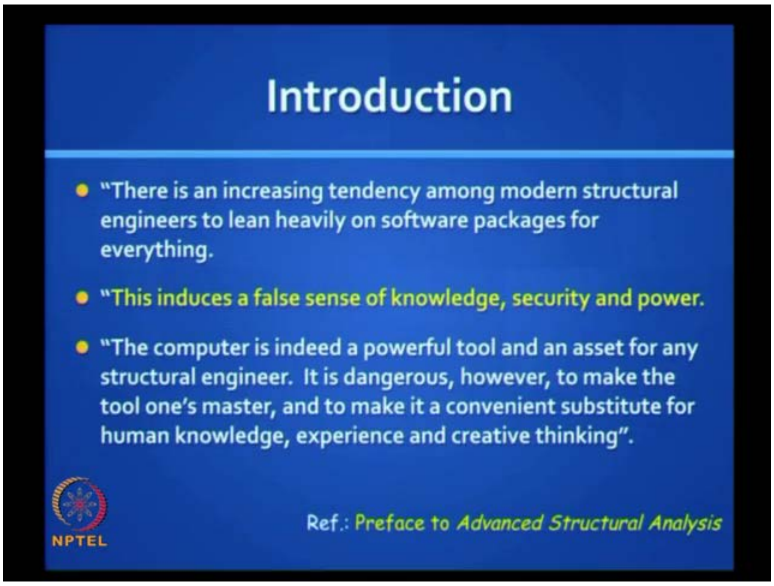

Here is the first sentence in the preface. There is an increasing tendency among modern structural engineers to lean heavily on software packages for everything. In fact, even design is done without thinking that you have computer aided design. At the press of a button you get big reports which the client receives and thinks you have done a phenomenal job. Actually, all you did was to press a button. In fact, today we have reached a stage where quantity surveying, planning everything is programmed. The only thing the computer has not yet been able to do is actually construct buildings at the press of a button; that may also happen and hopefully the building will stand.

This induces a false sense of knowledge, security and power. The computer is indeed a powerful tool and an asset for any structural engineer, but it is dangerous to make the tool one's master and to make it a convenient substitute for human knowledge experience and creative thinking. I hope you appreciate this. I hope you enjoyed what we did in basic structure analysis. You must not lose that touch and always go back to it and have simple ways of checking your answer. So, that is where matrices come in. Let us quickly cover the mathematical concepts.

What is a matrix? Now you should think general because matrix concepts can be applied to a wide variety of not just engineering issues, but even issues in biology, in social sciences, economics and so on.

What is a matrix? Set of things? There are no things. A Computer does not see things, it sees set of numbers.

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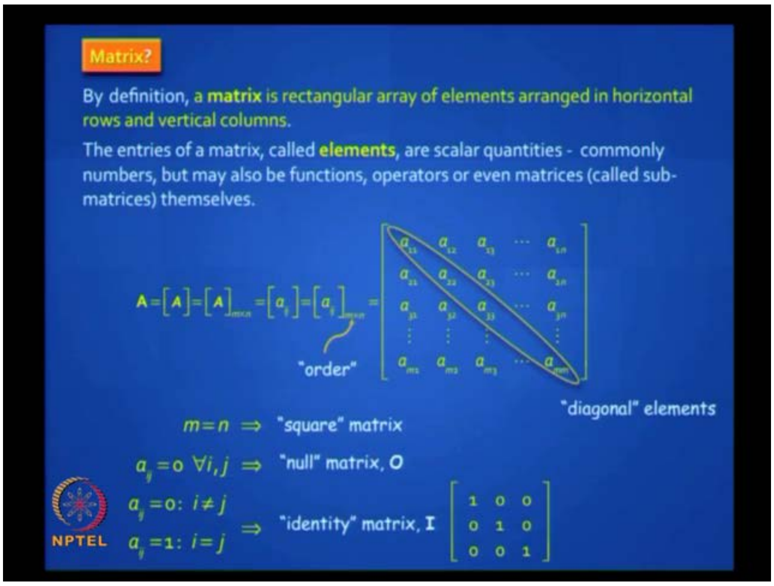

By definition: a matrix is a rectangular array of elements arranged in horizontal rows and vertical columns. So, it should look neat and every number has a place - a_{ij} , i th row, j th column. So, it is a rectangular array. I wish you had said rectangular, you did not say it. It is not a circular array. It is a rectangular array.

The entries of a matrix are called elements. They are scalar quantities. They are not things; they are commonly numbers. They can also be functions; they can be operators;

they can also be matrices which are called sub matrices. They could be algebraic functions, but they should make sense. You have mathematics and you have physics. Our job is to link the physics to the mathematics. Every number has a place and a meaning.

Typically, there are many ways you can write or designate a matrix. In text books, you will find usually with the bold capital letter A, as shown bold, but when you do it on paper you cannot make it bold unless you have the bold ball point pens. What we normally do is we put braces outside box brackets commonly, as shown here.

There are alternative ways you can do it, but actually that one letter stands for a huge set of numbers and variables which can occupy numerical values as shown here. Now, what is m into n, m rows and n columns? That is called the order of the matrix. You know very well, it is called the order of the matrix. Let us see.

When m is equal to n, what do you call that type of matrix?

Square matrix, very good.

When all the elements are 0, it is a null matrix. It has a symbol capital O. When the off diagonal elements are 0, and all the diagonal elements are unity, it is an…

Identity matrix. We can go fast because you seem to know everything, but at least you know that you should know the notations used. For example, a three by three identity matrix looks like that. (Refer Slide Time: 13:50) What are these elements called? You have two diagonals in that matrix a you have a diagonal in the other way. That is the principal diagonal. A matrix which has non-zero elements only in the diagonal is called a diagonal matrix.

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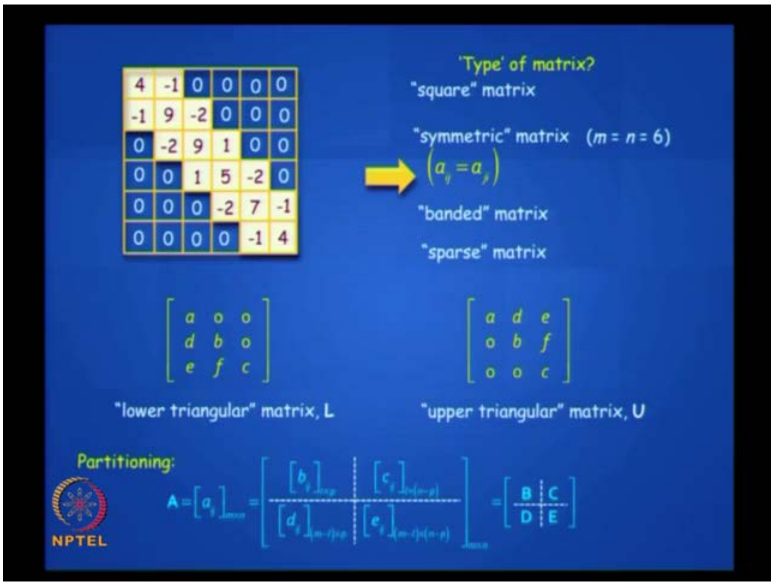

Is this a diagonal matrix? No. How would you.., what are the different adjectives you can give for this matrix? By the way, a stiffness matrix looks like this. Typically, a stiffness matrix looks like this. That is why I chose this. It is symmetric, very good. What else? It is square. Can a non-square matrix be symmetric? No. So, a symmetric necessarily includes... Is it an upper triangle? All the diagonal elements are positive. That is true. What is the name of the matrix? I am not asking you to describe the contents of the matrix. This can be any matrix. In structural engineering, it could be a stiffness matrix. It is a banded matrix.

Why is it a banded matrix? Because you have only the diagonal elements and you have elements next to the diagonal. So, it is forms a band. That is good. It is a banded matrix and many of the elements are zero. There is an advantage in terms of storing the matrix if many are zero, why? You can store only the banded portion. So, storage is easier. That is called as sparse matrix. So, square matrix, symmetric matrix, banded matrix and sparse matrix. Sparse means not filled, empty. This class is relatively full, but on days before you have other exams, you have sparse matrix here. Some of you do not show up. Now, it is not sparse.

What are these matrices called?

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Lower triangle is typically shown with capital L and upper triangle shown with U.

You are familiar with partitioning of matrices. When do you need to partition matrices? When you want to make life simpler and when you deal with a complex system, you want to operate with sub systems. You do not want look at the big picture all the time. You pick out little pieces from your big picture, but you should know where it is, how it is positioned and so on. You can draw these partition lines and sub divide. You have different sub matrices. They will have different orders, but they should all add up to the order of the full matrix. Quickly, you seem to know everything about matrices.

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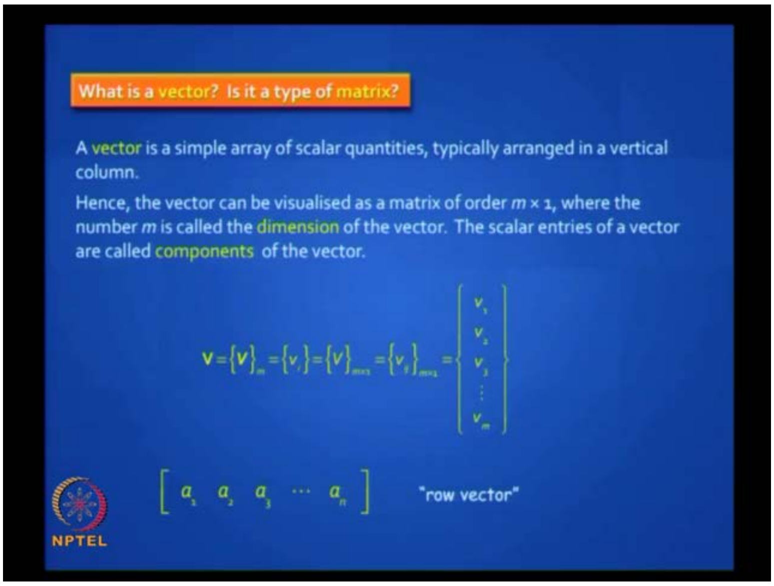

What about a vector? Is it a type of matrix?

Yes sir, it is a matrix.

Which is bigger? Matrix or vector, which is more general?

Matrix.

So, vector is one type of matrix. What is a vector?

It is a matrix which has only one column. A vector is a simple array of scalar quantities, typically arranged in a vertical column. Hence, the vector can be visualized as a matrix of order m into one. You do not use a word order anymore; you use the word dimension. 'm' is the dimension and you can say elements, but the right word to use is components.

You have components in a vector and elements in a matrix. You have dimension in a vector because one of the other order items is equal to unity. You have an order in a matrix.

It is typically arranged like this (Refer Slide Time: 18:10) and again you have a bold letter to describe the vector. Here the brace we use, just for convenience, is a curly brace, but if you put a square brace you are not really making a mistake because it is after all a type of a matrix. Do not put a curly brace on a matrix of size m into n where, n is not equal to 1, but you are allowed to put a square brace even for a vector. When you have something like this, some people call this also a vector, but actually the transpose of this that is the vector. This is sometimes referred to as the row vector, but strictly not a vector.

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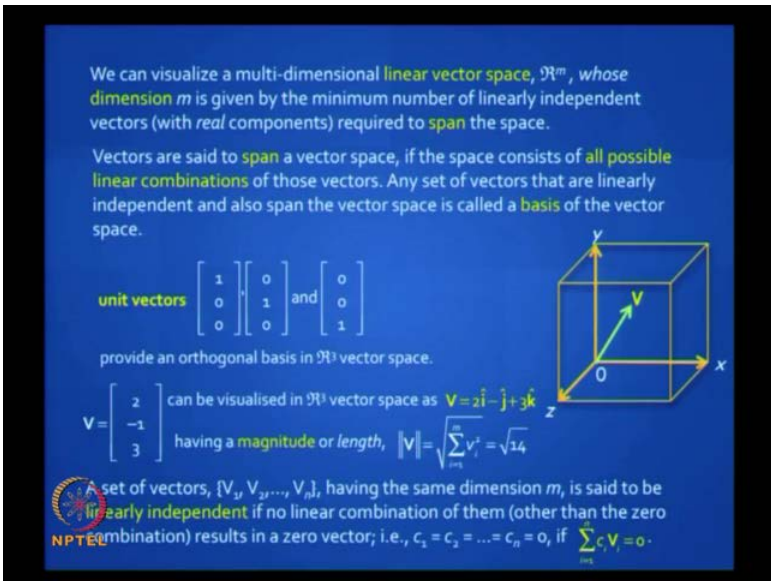

Vectors are important you know we are dealing with displacement vectors and force vectors. So, they have some meaning. It is good to see the background. We can visualize a multi-dimensional linear vector space R m whose dimension m is given by the minimum number of linearly independent vectors with real components required to span the space. These are beautiful words. There is a word called spanning, a word called linearly independent vectors and there is another word called basis.

What is basis? Sorry, one at a time.

Linearly independent vectors, which can span the space. Why do you need to get a basis?

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Do you see it has a role not just in physics, but in other fields of knowledge? Let us take social sciences. In fact, it is more difficult there because if you really want to get complete picture of say human behavior, you need to be able to construct a frame work where you have these linearly independent vectors. It is very difficult because you have to establish that they are independent and that provides a frame work for you to describe behavior. So, these are not just mathematical terms; they have wide applications.

So, vectors are said to span a vector space, if the space consists of all possible linear combinations of those vectors. You should be able to capture any vector in that space with the help of your linearly independent vectors. Any set of vectors that are linearly independent and also span the vector space is called a basis of the vector space. What is a simplest basis that we the use in physics? i, j, k. In a three dimensional space; you are familiar with this. You have i, j, k and you have unit vectors which form a good basis. So, a 1 0 0, 0 1 0 and 0 0 1.

Any vector in that space, for example, 2 minus 1 3 you can write it in a column, but it actually has a sense of direction V equal to 2i minus j plus 3k and it has a length which is obtained by the resultant of the scalar quantities. So, you are familiar with it. It is also called the magnitude. So, vectors are meaningful and a set of vectors $V_{1,}$ V_2 to V_n having the same dimension m is said to be linearly independent, if no linear combination of them other than the zero combination results in a zero vector. That is the proof whether you really landed up with linearly independent vectors. Otherwise, you have a problem because there is a correlation between vectors.

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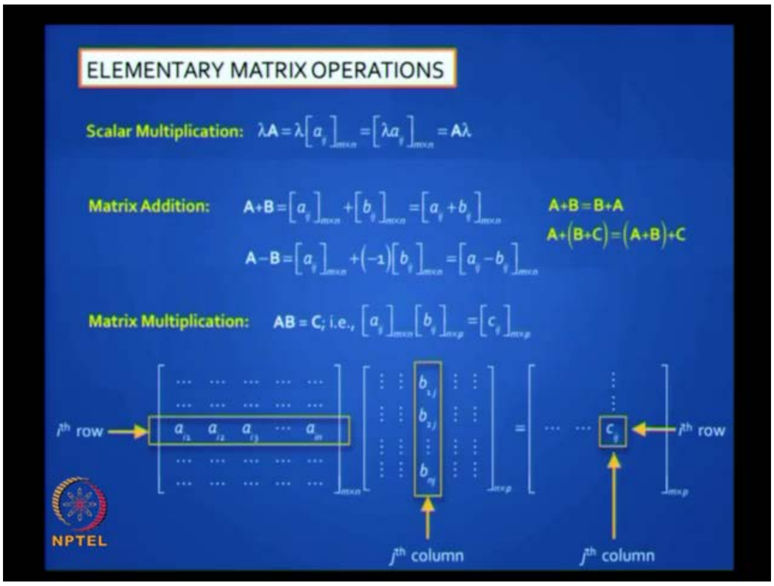

Elementary matrix operations: I will go very fast you are familiar with it. You have scalar multiplication. The scalar is never shown in bold; so, lamda is a scalar. You have matrix addition and A plus B will be equal to B plus A - that is a property. What is the property called? Commutative property, if you put a negative sign. So, we only say addition, but addition is generic. It includes subtraction because you are multiplying with minus 1, which is a scalar.

Matrix Multiplication is, when you studied it first time, it looked very funny because it did not look like the multiplication of scalars, but now you are used to it. You have a distributive property also. A plus B plus C is the same as A plus B which is a matrix plus C. AB equal to C; you cannot just multiply any two matrices there. You have to satisfy some requirement, what are those requirements?

Basically, you know what to do. If you want to find $c_{ij,}$ you have to take the i th row of the A matrix and the j th column of the B matrix and you have to multiply each term with its corresponding term. They must match in terms of dimension. That is why m into n order multiplied by n into p order will give you a m into p order. You are familiar with this.

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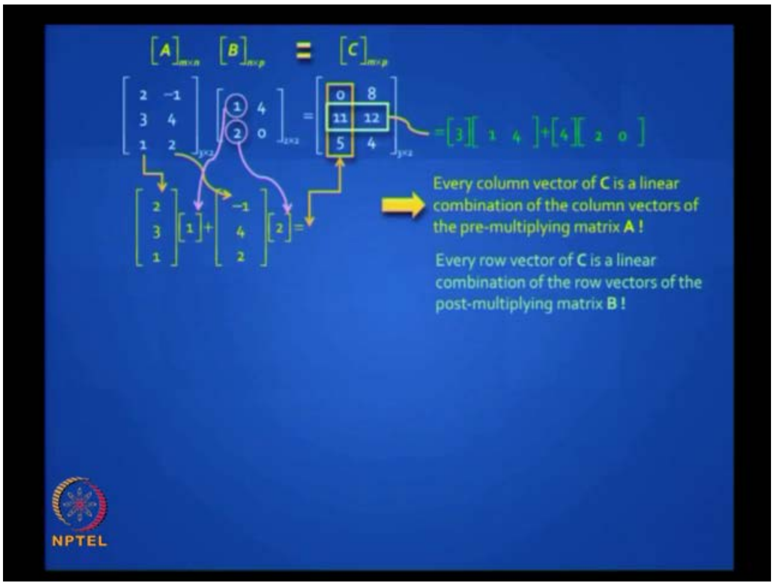

Let us look at it more closely. You know how to do it, but let us see if we can make some physical sense out of it. I will give you a clue. You can do that multiplication in this manner. What does this manner suggest? Now, this is something you may not have studied when you did linear algebra. What does this signify? I can get the same answer by doing the multiplication in this way. What does this suggest? It suggests a certain principle. I took the first column. I took the first element of the first row. I took the second column. I took the second element of the first row and I get the first column in the resulting matrix. What does this principle show? Linear combination of… Every column vector of C is a linear combination of the column vectors of the pre-multiplying matrix A.

Incidentally in terms of direction, if you multiply a vector with a scalar, you are getting the same direction; you are only changing its length. You can do another operation in the other direction and you prove that every row vector of C is a linear combination of the row vectors of the post-multiplying matrix B. That is just a physical meaning. The moral of the story is - you can blindly do matrix operation; you can also do them with your eyes open and figure out everything makes sense. You are playing with vectors.

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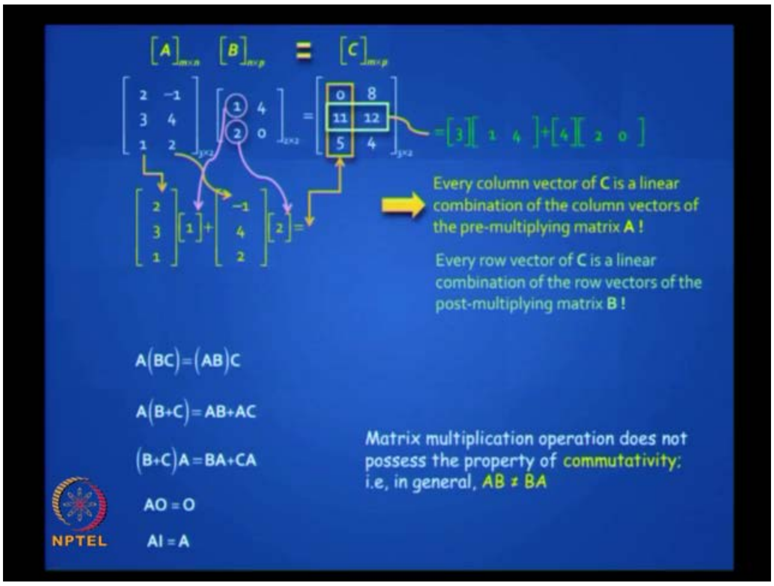

And you have these properties A B What are these properties? You are familiar with all of them, associative and distributive. Then, if you multiply with a null vector, you get anything multiplied with zero is zero. You get identity only replicates. The commutative property is not there; you are familiar with it. AB is not equal to BA. You may not even be able to do the multiplication sometimes.

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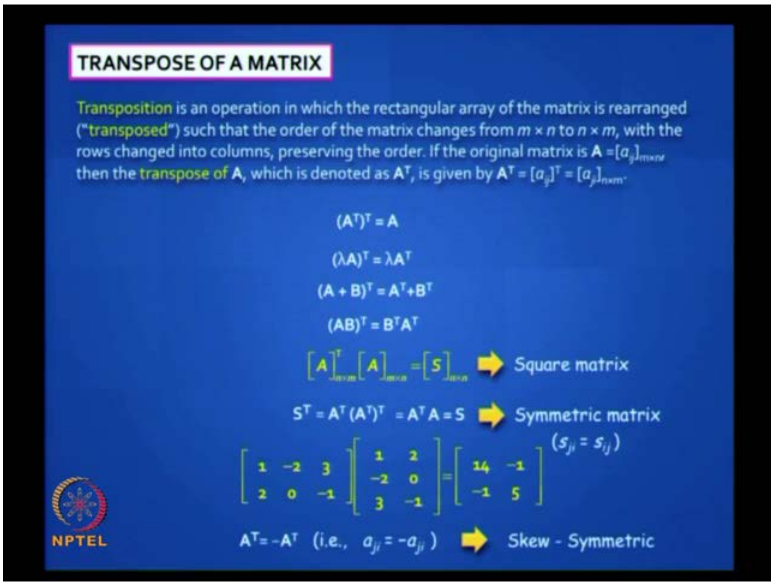

Transpose - you are familiar with it. Transposition is an operation in which the rectangular array of the matrix is rearranged or transposed such that the order of the matrix changes from m into n to n into m with the rows change into columns preserving the order. If the original matrix A is a_{ij} and the transpose will be a_{ij} transpose which is a_{ji} and the orders changes from m into n to n into m. You are familiar with this and you have these properties. That is, if you take the transpose of a transpose, you get back the original matrix. If you pre- multiply it with a scalar, you can take it out side and this is interesting.

The third one is also simple. A plus B the whole transpose is A transpose plus B transpose, but AB the whole transpose is B transpose A transpose. Here, you have to do a switch; then only it works. Supposing B and A are the same, then what is the kind of matrix that you get? If you pre-multiply a matrix with its own transpose, what is the type of matrix you will get? You get a square matrix plus it will be symmetric. How do you know it is symmetric? You take the transpose of this matrix and you will get back the original matrix. So, that is very interesting. You can demonstrate with examples. We will not spend time. Sometimes, you see matrices where the transpose gives you a negative sign. So, this is called a skew symmetric matrix.

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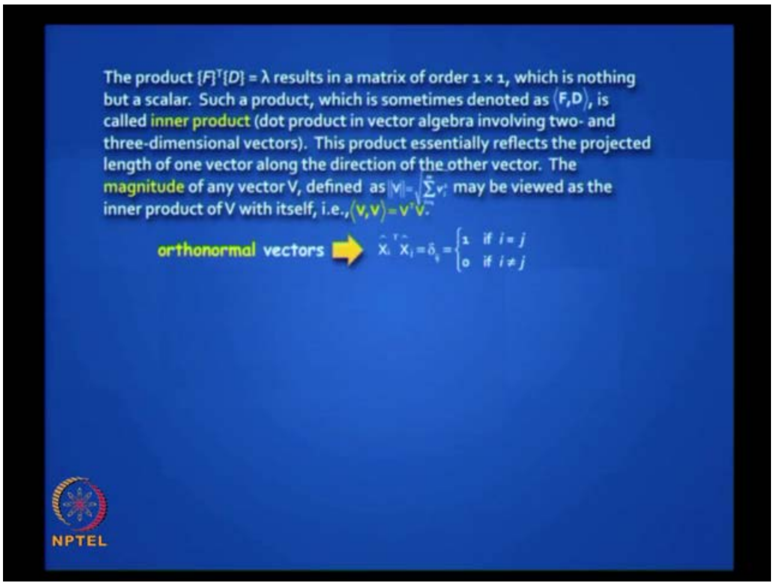

Now, next, we will talk about the products of two vectors. You cannot multiply two vectors; you know that. Why? Because they both of the You have to take the transpose of one of them. You commonly meet this situation, when you want to find work. In structural analysis, you have the force vector and you have the displacement vector. You cannot multiply F into D; either you take the transpose of F and multiply it with D or you take the transpose of D and multiply it with F. Then, you will end up with one by one quantity. What is that? That is a scalar. So, what kind of product is this? Well, we say dot product in physics when you deal with two and three dimensional spaces, but it is called an inner product. In general, it is called an inner product. You can also show it in this fashion. This is symbolic way of showing the inner product.

I will read this: The product, F transpose D equal to lamda results in a matrix of order one by one, which is nothing but a scalar. Such a product which is sometimes denoted as F comma D with those triangular braces is called inner product or dot product in vector algebra involving two and three-dimensional vectors. This product essentially reflects the projected length of one vector along the direction of the other vector; we are familiar with this in physics. The magnitude of any vector V can also be viewed as the inner product of V with itself; you are familiar with that. Vectors are said to be orthonormal, if you do their inner product you either get 0 or 1. What is this symbol called? This delta is called a Kronecker delta.

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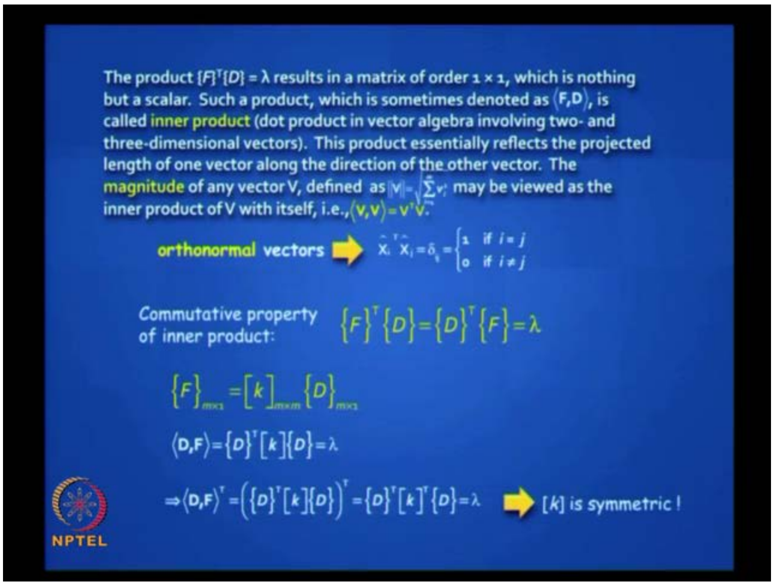

The commutative property of the inner product is what I explained a while back. You can calculate work either by converting the force vector to a row vector or the displacement vector to a row vector, you get the same answer. If you introduce the relation between force and displacement through the stiffness matrix, you can arrive at an interesting conclusion. Can you tell me what that conclusion is?

See, the first is a commutative property, the second is a stiffness relationship, and it is a law. If you plug in the second property to the first property, what can you end up proving?

So, you take the inner product in this manner and you get lamda. Now you substitute and take the transpose of this; you still get lamda. Then, when you compare these two expressions, D transpose k, D is the same in both, but in one you have k transpose. So, you end up proving k is equal to k transpose. It is a very elegant mathematical proof, requirement of why the stiffness matrix must be symmetric.

We will study symmetry from the structural analysis point of view later. Now, let us get to the real need for bringing in matrix. We need to solve equations and we want the computer to do it; we do not want to do it ourselves because it is a pain sometimes.

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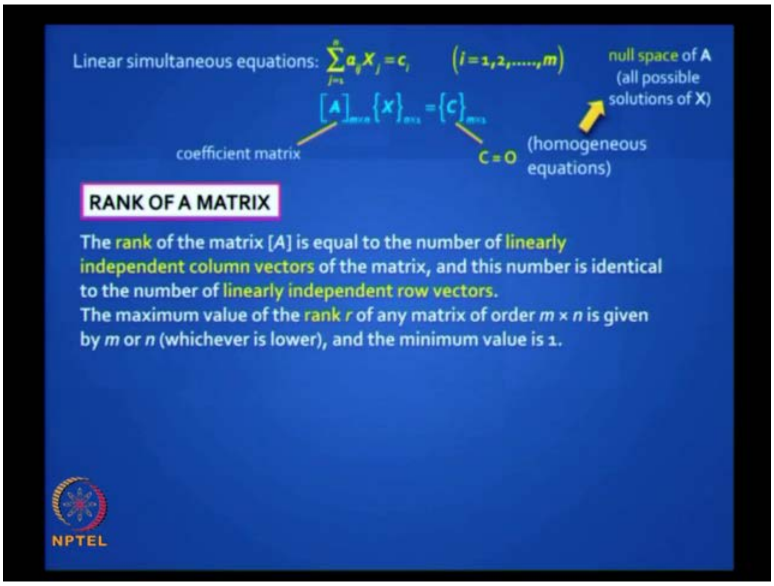

Linear simultaneous equations can be written as AX plus AX equal to C. A is called the coefficient matrix, as we saw in this case of the stiffness matrix, it is a property of the structure. X is a vector like the displacement vector and C could be a load vector C is some set of constant.

If C is 0, well you should not say 0, you should say O because 0 is a scalar, O is a vector or a matrix, then those set of equations are called homogeneous equations. You are familiar with them. When do you need to look at the solution for homogeneous equations? In solving linear simultaneous equations, even if you have a non-zero C, a constant vector you may still need to inspect this solution. That is called the null space of A, all possible solutions. When you need to do this? It is very interesting, though we do not directly need it in structural analysis.

You need to do it because it is related to what is called the rank of the matrix.

What is the rank of the matrix A? It is a beautiful concept. Independent number of independent we can find out.

You kind of vaguely know what it means. Let us explore it - rank of a matrix. The rank of the matrix A is equal to the number of linearly independent column vectors of the matrix and this number is identical to the number of linearly independent row vectors. So, the maximum value of the rank r of any matrix of order m into n is given by either m or n, whichever is lower and the minimum value is 1.

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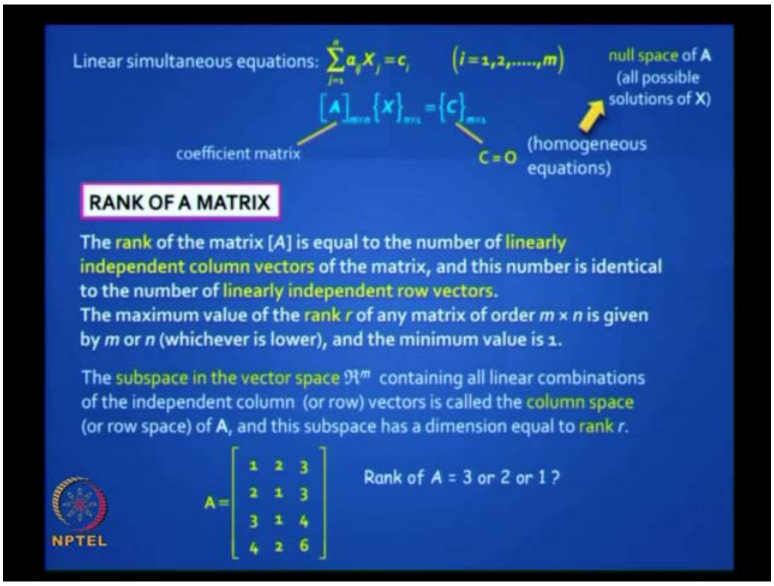

Let us look more closely at this: the subspace in the vector space R m containing all linear combinations of the independent column or row vectors is called the column space or row space of A and this subspace has a dimension equal to the rank r. You have to finally get the number of linearly independent vectors. Take a look at this matrix. It is a four by three matrix. Obviously, the rank cannot exceed 3.

Is the rank 1, 2 or 3? Why do you say 2? That is good, that is quick, smart looking, but in general it is good to keep quiet and do an analysis, before you pop out with the answer. Three, you should never say because you cannot spot it so easily. Sometimes it is cleverly disguised, you cannot catch it.

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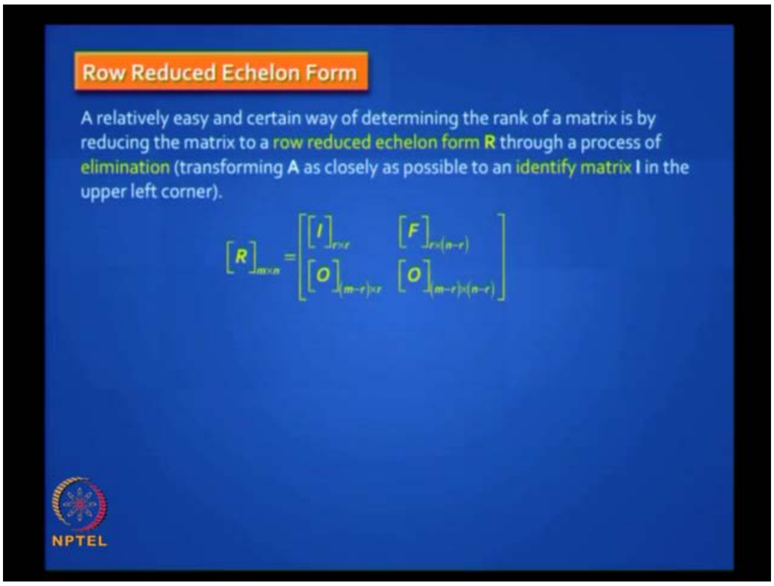

How do you capture the rank of a matrix? There is a beautiful technique. You have studied this. Your math lab will do this easily. It is called the row reduced echelon form. Relatively easy way of determining the rank of a matrix is by reducing the matrix to its form R through a process of elimination you are familiar with this.

Yes or no? Yes, sir.

Now you need not assert yes, you should have the care and concern for other members of your group. This is a matrix. Do not be a dominant diagonal. You should care for those who are saying they do not have an exposure. You cannot insist they have an exposure. All you can insist is, I know, that too we will check shortly whether how well you know it. It is always good to say I do not know. I want to learn or I want to relearn.

So, the idea is, though it is a refresher for many of you, incidentally, we are just covering the theory. I would say it is more for fun because nobody is going to do at this stage in your life. You are going to use press buttons and let the computer do it, but there is a nice story it is telling and I think it is worth understanding this. How do we do that? Any matrix R you can finally, through doing elimination. Many of you, those of you who said no, are you familiar with Gauss elimination? You basically play around with rows. You multiply and add different rows. You are allowed to do that, but you have to do the all the way including the column vector. You can play with them and that is allowed. You do not lose anything in doing so. You can multiply row with 2 and subtract 3 from the next row and so on. Those are row operations that you can do, but the idea is you do them so that you can get an identity matrix in the left hand upper most corner.

You get an I matrix there and the idea is to get 0 below. You can always reduce R to this form where you have null matrices at the bottom, identity matrix at the top left corner and you I will get some matrix F. What is F called? It has a name, those of you who said you know everything, please answer. F is called the free variable coefficient matrix. Why is it called that? Let us see, it is very interesting. Take a look at this matrix.

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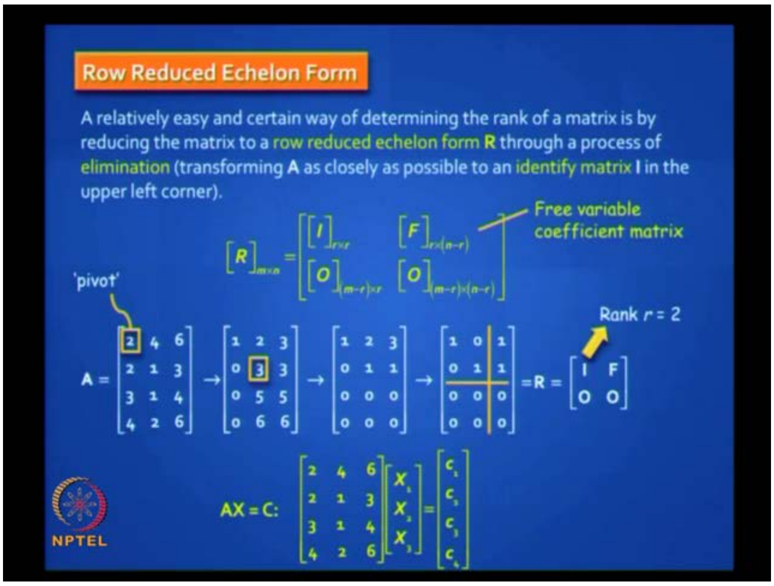

Let us do it is step by step. What is the first thing we should do? Make what, one? Top variable, so that is called the pivot.

You divide that row with 2 and then do an operation and get zeros in the other one. This can be done. We will just go through it fast. You have done it before. In gauss elimination you would have done this. That is our pivot, you will get that. You have got lot of zeroes already. The top one also should be 0. This is ultimate. You got the identity matrix.

Actually the last step is optional when you do simultaneous equation because you can do back substitution and get it. You got this. Then you look at the partition. You captured that I, F, O, O, beautiful. What is a rank of this matrix? It is the same matrix we looked at earlier. Rank is clearly 2. It is the size of the identity matrix. So, it is R into r. That is clear. Now, this is how you can do it. In fact you can write a program and it can do on its own. Rank is 2. Let us look at a set of equations like this.

Can this set of equations have a solution? Yes or no.

Infinite number of solutions.

Infinite solutions? Will they always have infinite solutions? Yes. It may not have because first the constant vector must satisfy conditions of consistency. It is very interesting, but we do not need too much of it. I want you to read on your own. I will just rush through it.

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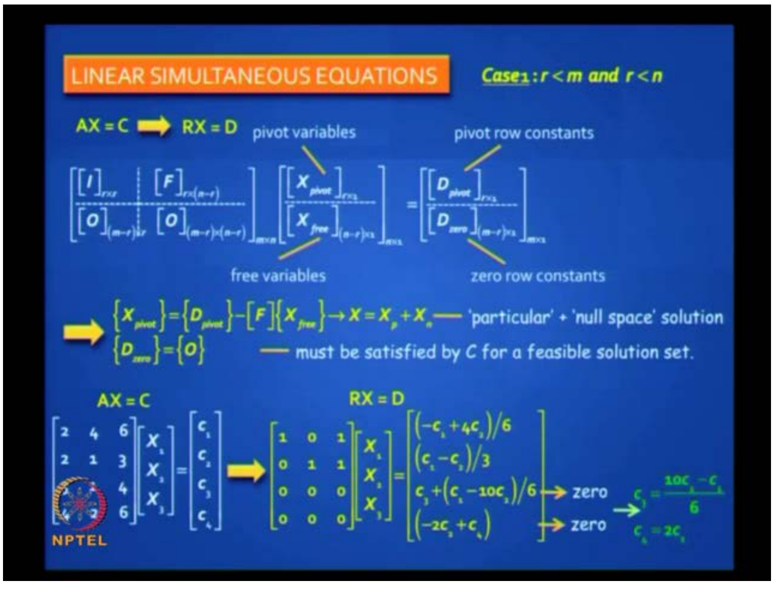

You will find. Let us look at this. Finally, you have to solve equations like this. Do you agree to this? If you take out those two rows, they should look like this. That is $D_{0,}$ should be O and this must be satisfied by C for a feasible solution set. If it is not satisfied, you cannot solve this equation. Still, you might get infinite solution. That is a different matter, but you cannot touch this, if you have a violation of consistency.

And whenever you have a problem like this, when you have free variables, you have to look at the infinite solutions come from the null space. So, you get one particular solution. You must have done this in differential equation. Get one particular solution, then you have to add the null space solution from the homogenous equation. Let us take a look at this. We have X_1, X_2, X_3 and you have this situation. If you take the row reduce echelon form, it looks like that. Well, the last two rows, the constants should be zero. You run the operations right through. They look like that. So, you cannot do this for any arbitrary C. Do you agree? Yes. First that has to be satisfied and then only you talk of a solution. Then also you may not get a unique solution because the rank is only 2, it is not 3. So, first you have to satisfy this. That means you can express c_3 and c_4 in terms of c_1 and c_2.

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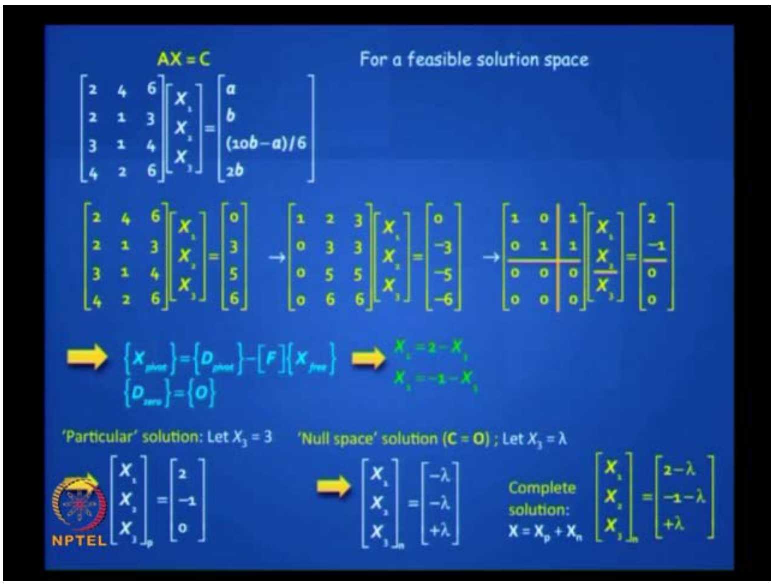

Let us look at a problem like that. Let us say c_1 is A, c_2 is B, and then c_3 and c_4 must necessarily satisfy this requirement. Then you talk of the solution, but even then you would not get a unique solution because the rank is not full. Let us take an example like this and you do the row reduced echelon form, you will get something like that. How do you solve this problem? Well, partition it; you can throw away the third and fourth rows because they already satisfy consistency C. They are not going to help you. You have only two rows, but you have a problem there. You take the first set of equations. You can write X_1 and X_2 in terms of X_3. Clearly, you do not have a unique solution. So, you take a particular solution; you can choose any value of X_3 including minus 342.625, who cares, any particular solution.

For convenience, we took X_3 equal to 3 then you get a unique solution for X_1 and X_2, but this is not a complete solution. You have to take the null space solution, the easiest null space solution; let us take some scalar like lamda. Let X_3 be lamda, plug them into those equations and you will find that if X_3 is plus lamda X_1 and X_2 must turn out to be minus lamda, minus lamda. So, what is your complete solution? That is how. This is how give the (( )). It is for you to learn for a general knowledge. We do not fortunately have such problems except, when your structure is unstable. In which case, you better worry about this structure falling down rather than solving these equations. You have to make the structure stable.

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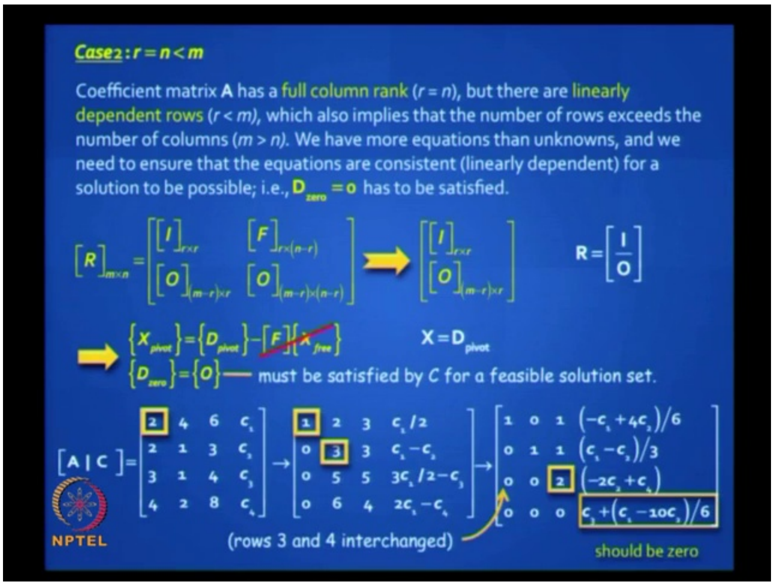

Now the other case we have is when R is equal to n; n is the number of columns which is rare, less than the number of rows. So, coefficient matrix A has the full column rank, but there are linearly independent rows, which also imply that the number of rows exceeds the number of columns. When do you have such a situation? When you have more equations than unknowns, then you need to satisfy consistency. Again this comes in unstable structure; so, we will just rush through this. You do not need it really. Now it takes this form. There are no free variables here and the constant still must satisfy, because it belongs to the same type. You run this exercise.

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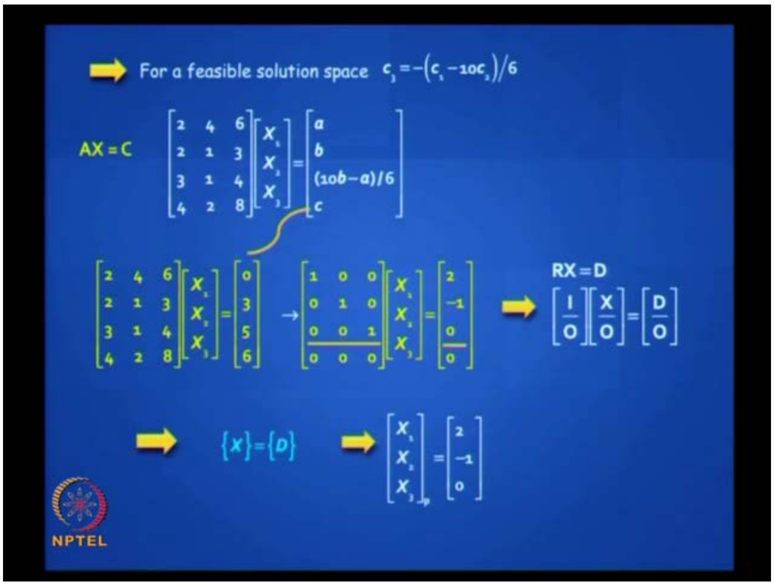

Let us just rush through this. Once you satisfied it, you get the unique solution. You do not need to have the null space here. This is a second type. We are not dealing with it in structural analysis.

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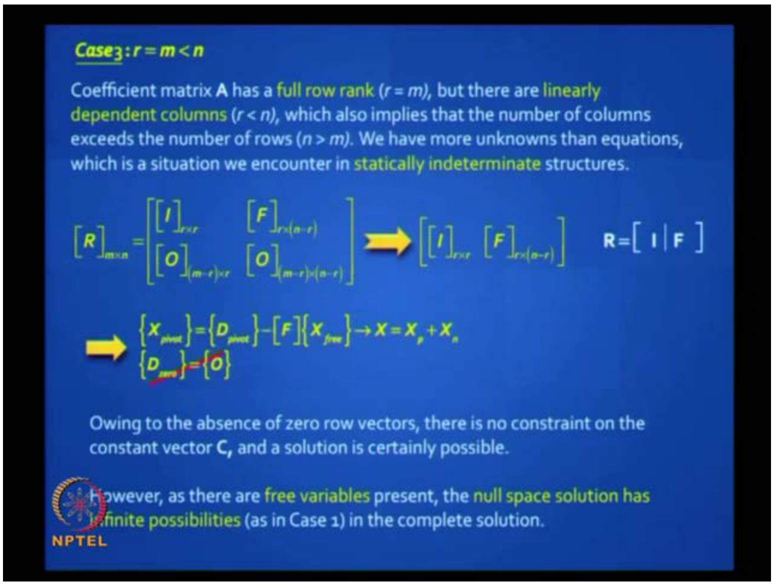

What do we deal with in structure analysis? Case three and case four.

What is case three? R equal to m less than n. When do you get such a situation? When do you get more unknowns and less equations?

Statically indeterminate structure.

So, coefficient matrix A has a full row rank, but there are linearly dependent columns which also imply that the number of columns exceeds the number of rows. We have more unknowns than equations. It is a situation we encounter in statically indeterminate structures. Now, you have only I and F, those O’s vanish. You have a full row rank. Your row reduced echelon form looks like that, I and F. You see, you have infinite solutions, but you do not have to worry about consistency. Those equations are not there. Since there are free variables present, you have infinite possibilities.

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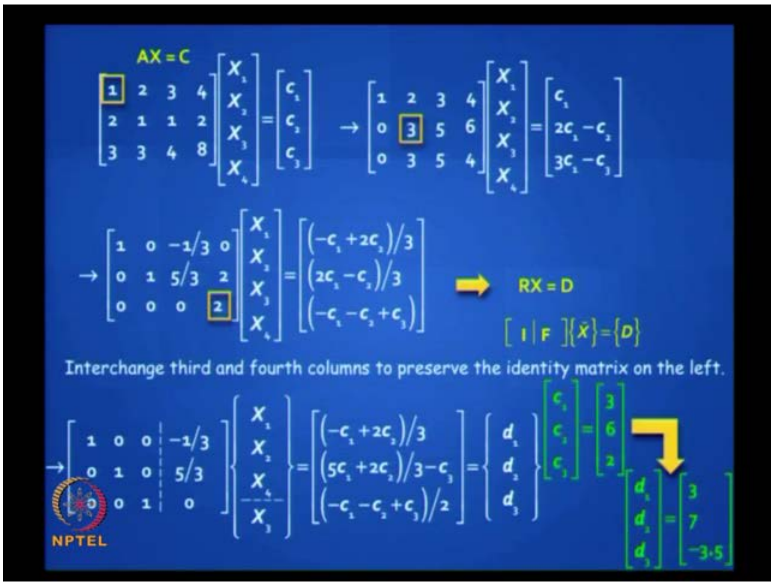

Let us taken an example like this: AX equal to C. You run this operation till you end up with the row reduced echelon form. They are consistent. You do not have to check on, but now, we will have to preserve. To make it look like in identity matrix, sometimes you have to interchange rows. You know that operation. You have to interchange rows. In this case you have to do that. They look like this and if your constants have numbers like 3, 6 and 2, after your elimination, they look like 3, 7 and minus 3.5.

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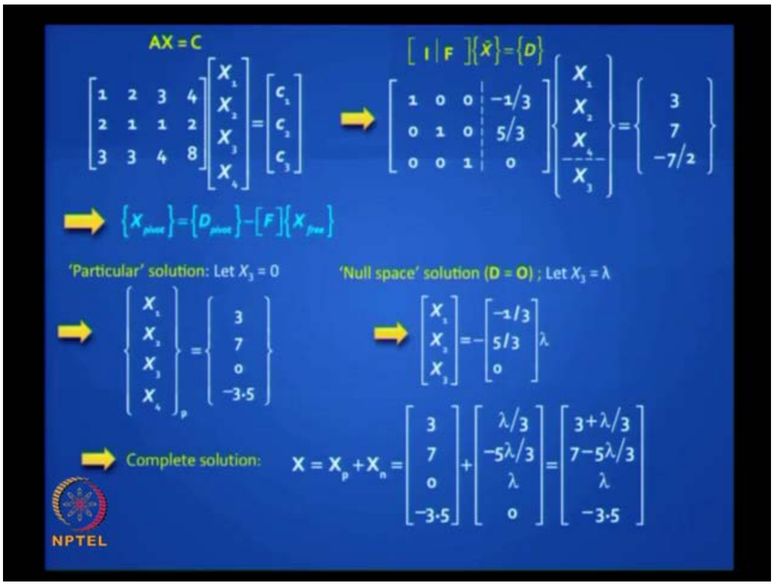

You solve this; find the particular solution; find the null space solution and you get it. It is similar to the fist type of problem except consistency is not there. We do not even do this. We do it. When did we do it? We wanted to find displacements in statically indeterminate structures after having analyzed it, remember. There we chose lamda to be 0. We did not waste time doing all this because you need any statically admissible solutions to apply the principle of virtual work.

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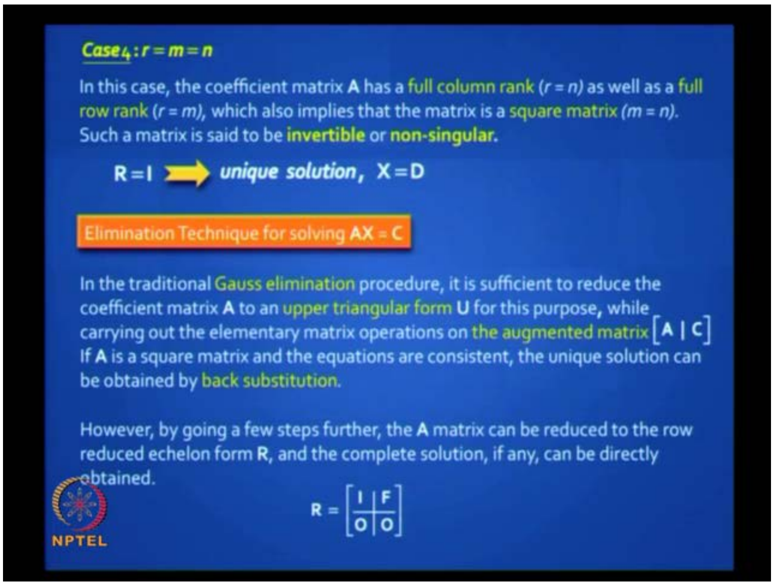

This case is straight forward. Full rank, if you are lucky, but if your structure is stable, you are likely to get this situation. If it is statically determinate, this is straight forward. In this case, the coefficient matrix A has a full column rank as well as full row rank. This also implies, this matrix is the square matrix. Such a matrix is said to be invertible or non-singular. If you have a matrix like this and all our stiffness matrices for stable structure will look like this, then you do not have a problem; you also have a symmetric matrix in our case.

You have a unique solution. You can use the Gauss elimination method or you can use a row reduced echelon form and you can get the answer. In earlier days, we used to make students actually painfully do all this, but today we do not do that. We say, you use the computer; certainly be familiar with software like math lab where you just press a button; enter the matrix. All of you are familiar with math lab. There are some free ware called sky lab and so on, but I think we have many versions of math lab and math cad. So, it is very easy today to find the inverse of a matrix.

What is a problem with some matrices regarding finding inverse?

They should not be singular.

That is true. So, in structural analysis, where do you see a problem? We will see that.

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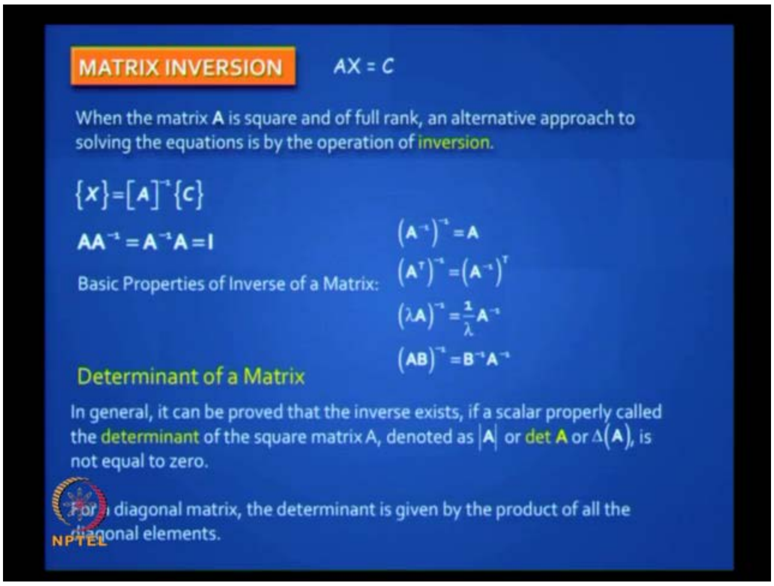

Matrix inversion: When the matrix A is square and of full rank an alternative approach to solving the equations is by the operation of inversion. X is equal to A inverse C. A multiplied by A inverse is A inverse A is identity matrix. You are familiar with these. A inverse, inverse gives you back A and so on. The last property is very interesting. It is like what you got in transpose. A B the whole inverse is B inverse A inverse. Some mathematician gave a beautiful physical feel to this. He gave an analogy. Do you remember? I think I mentioned this earlier. He said how do you put on shoes? You first put on socks that is A matrix, then you put on the shoe. How do you reverse the process? You have to first remove the shoe, and then you remove your socks. That is why A B inverse is B inverse A inverse; very easy to remember, very nice analogy.

What is a determinant of a matrix? Why do you need to know it? Whether it is in so In general, it can be proved that the inverse exists if a scalar property called the determinant of the square matrix denoted A with those vertical lines or det A or delta A is not equal to zero. For diagonal matrix, the determinant is given by the product of all the diagonal elements. You know that. It is an easy way to find out.

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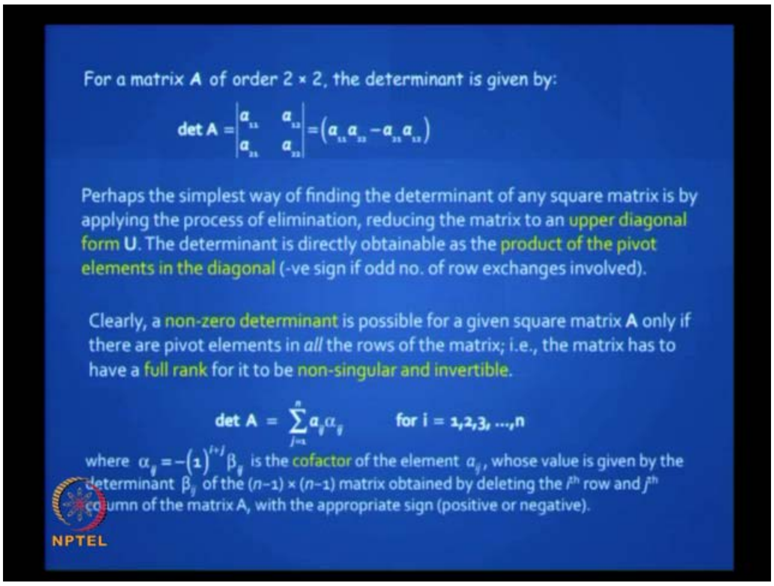

I think you are familiar with finding determinants. We will not spend too much time on this.

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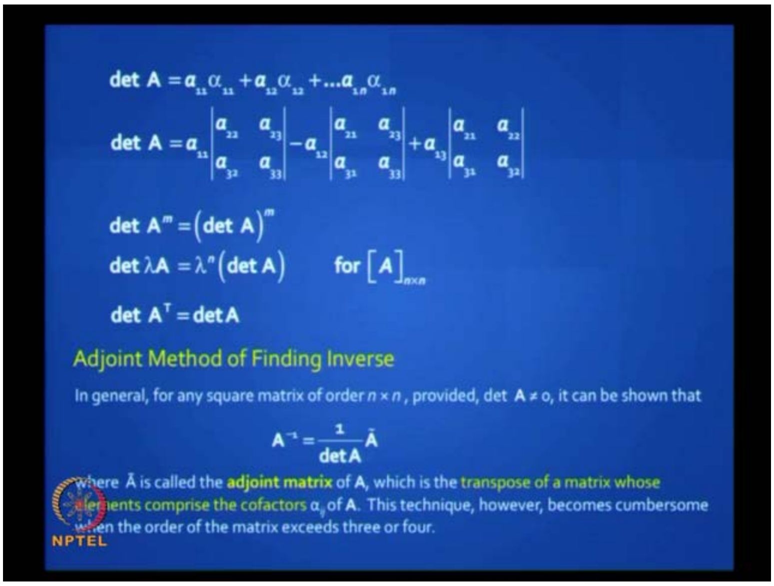

You know about co-factors. You really you do not need to do in this course, matrix methods, but it is good to refresh these concepts. You know about the adjoint method of finding the inverse, yes or no? You know about Cramer's rule?

Yes.

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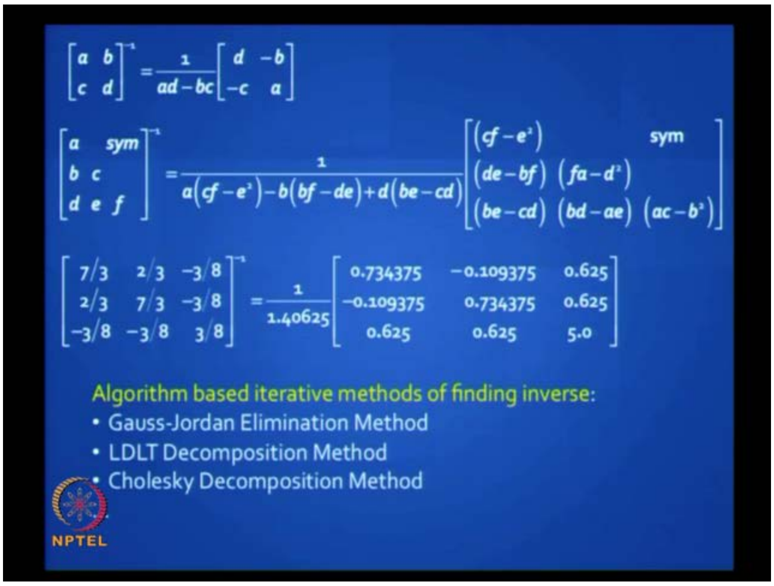

Those who said no and if you are interested please read good book in matrix algebra or just read the second chapter in this book on advance structure analysis. It is good to browse through these and we can spend hours discussing these different methods of finding the inverse like the Gauss-Jordan Elimination method, Cholesky Decomposition method. You know these are very good for stiffness matrices, but we will not do that.

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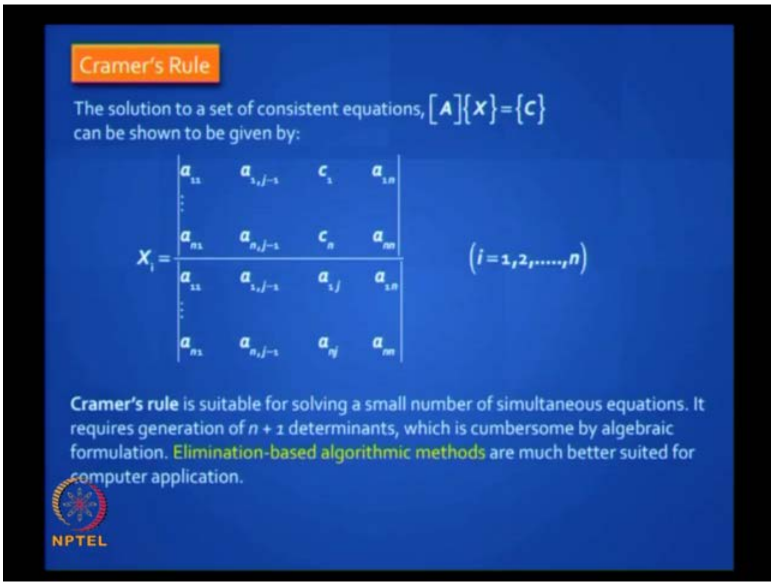

Are you familiar with Cramer's rule? you are familiar with It is very easy to do when your order is up to 3. Beyond 3, manually, it is not worth it and we use eliminations methods to do so. We will stop here.

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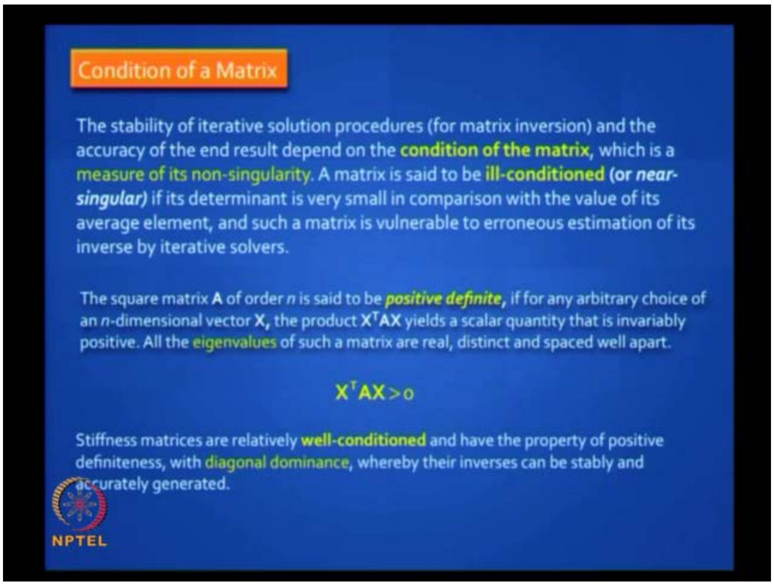

We will discuss that…, may be we will finish this condition of a matrix. A matrix if it is ill-conditioned, you are not sure about the accuracy of the inverse that you get. How do you know whether inverse is right or not? You again multiply with original matrix, you should get the…. You will not get, not necessarily. You have lot of rounding of errors while you do elimination. Unless your matrix is healthy, it is in good condition and there are tests to check the condition of a matrix, you cannot be sure of finding its inverse. Therefore, solving, you are not show about the answers you get.

Now, fortunately for us, our stiffness matrices are all healthy matrices. They are well conditioned. You know about the property of positive definiteness. You should read this. I will just finish this. A square matrix of order n is said to be positive definite, if for any arbitrary choice of n dimensional vector X, the product X transpose AX yields a scalar quantity that is positive. If it is negative, it is ill-conditioned; if it is zero it is called semi definite. You can use Eigen values to decide. Basically if your diagonal elements are heavy and they are positive, you got a well conditioned matrix. That is the thumb rule and stiffness matrix will be like that. Flexibility matrix, there is no guarantee. That is another reason why flexibility matrix is not preferred for. It is too flexible. You can run into problems of finding the inverse.

Thank you. Let us stop.