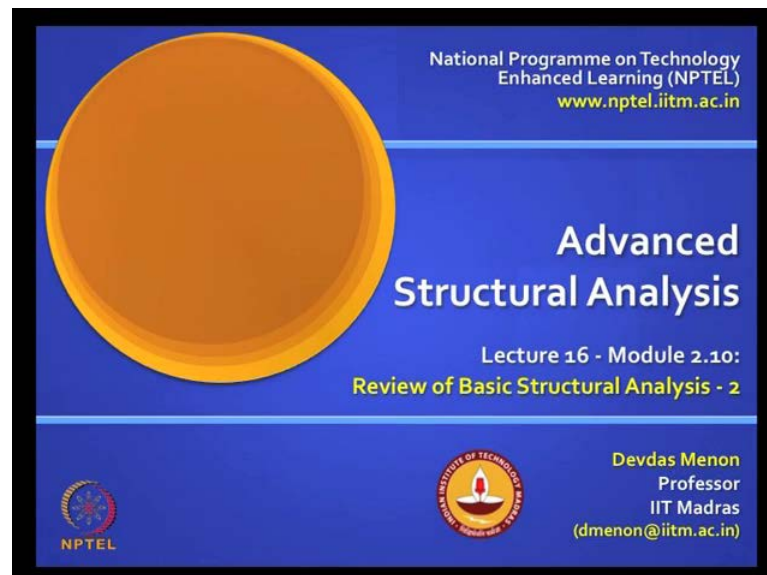


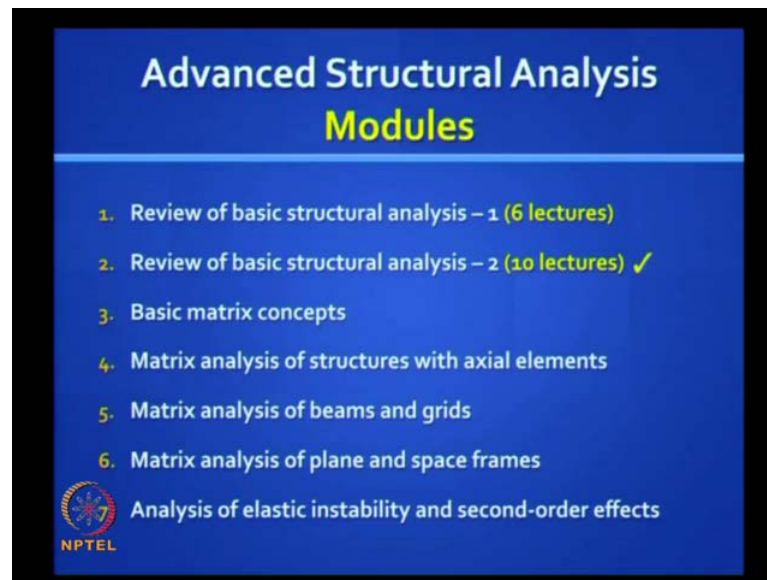
Advanced Structural Analysis
Prof. Devdas Menon
Department of Civil Engineering
Indian Institute of Technology, Madras
Module No. # 2.10
Lecture No. # 16
Review of Basic Structural Analysis – 2

(Refer Slide Time: 00:01)



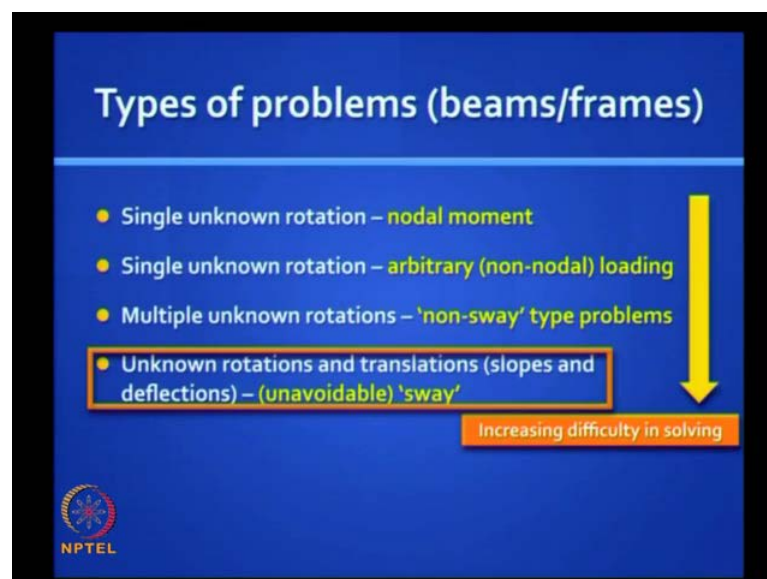
Good morning, this is lecture 16. With this lecture, we will be concluding the second module, which deal with the review of basic structural analysis.

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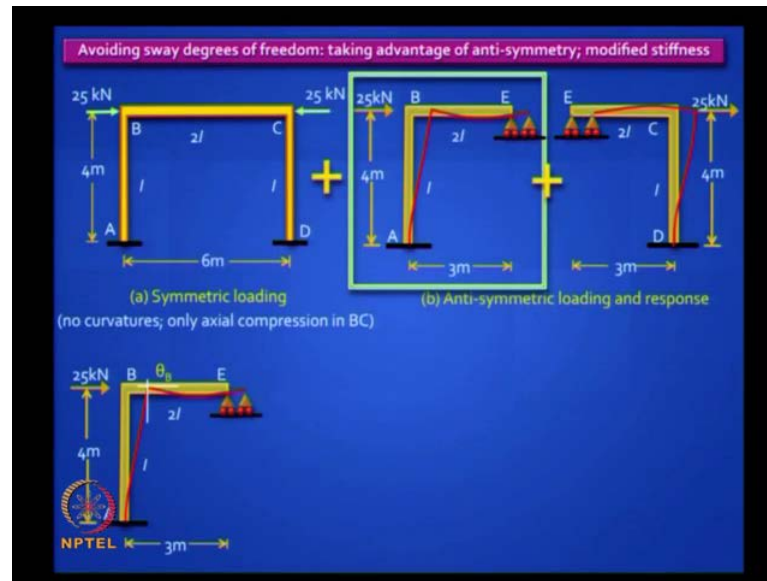
As you can see, we did 6 lectures covering the first half that dealt with statically determinate structures. With this tenth lecture, we will be completing the analysis of indeterminate structures.

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If you recall, we stopped here. You are dealing with sway type problems. We observe that it is possible in some cases to take advantage of the fact that you can avoid including the sway degree of freedom. This is especially true in portal frames which are symmetric and subject to lateral loads. So, let us look at that.

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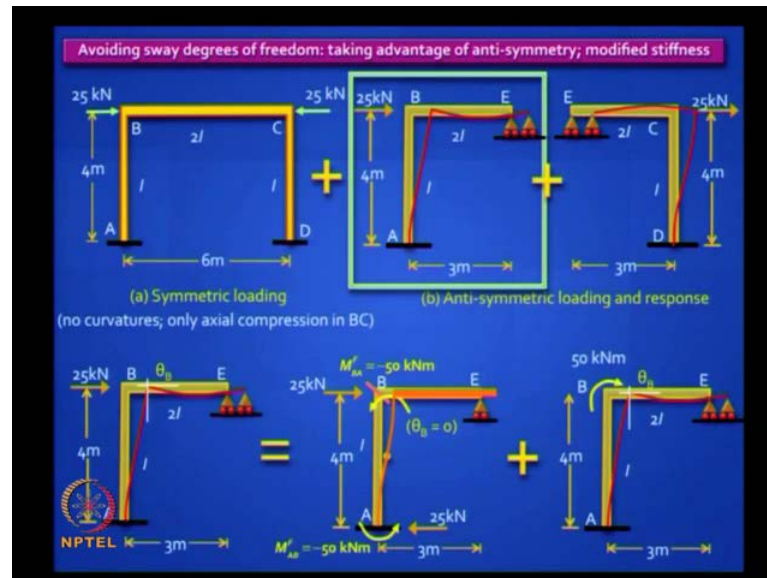
If you recall, we stopped here in the last session. So, the total lateral load of 50 kiloNewton can be divided into two parts: One, which has the symmetric loading and the other which has an anti symmetric loading. It is only the anti symmetric loading that causes any bending in that frame because a symmetric loading simply produces an axial compression in the beam BC. If you look carefully, you will find that there is a definite point of contra flexure exactly in the middle of the beam. We have explored this in detail when we covered approximate methods of lateral load analysis, but we are not sure where the point of contra flexure will be in the column. If you assume it to be in the middle, there will be some error and that is the error we have made till now.

From now on, we do it exactly. All we need to do is to analyze this frame now-one half. (Refer Slide Time: 02:10) If you look carefully at that frame, what is the degree of kinematic indeterminacy? Well, you have θ_B for sure. You have θ_E , which you can ignore because you can modify the stiffness of B because the moment at E is 0. You have a sway Δ_{BE} which also you can ignore, if you realize that AB essentially behaves like a cantilever. Let us look at it more carefully.

We are going to analyze this frame. (Refer Slide Time: 02:26) We are going to say that θ_B is the only unknown, which means, in the displacement method, when we consider the primary structure, we arrest only θ_B ; we do not arrest the sway. So, you have to be careful if you are taking advantage of the shortcut. Remember that you should

arrest, to find your fixed end moments, if any, only those degrees of freedom that you identified as related to your minimum degree of kinematic indeterminacy - in this case, θ_B .

(Refer Slide Time: 03:29)



So, the first step would be, in the broad displacement method concept, to arrest θ_B .

When you do that if you apply the load, you will get fixed end moments because that column is going to move, but it moves in a manner which is very familiar to us because the beam B is going to behave like a rigid beam. It will just slide horizontally and you will have moments in that column which you can easily calculate. Can you calculate the fixed end moments there?

It will be 25 into 2. It is very simple because the entire 25 kilonewton will go to the fixed end A; nothing will go to the roller support E. So, the point of contra flexure is exactly at mid height and you know that the moments developed at the two ends will be 25 into 2 which is 50 kilonewton metre. Will the moments be clockwise or anticlockwise? Well, there are many ways of figuring it out. You can look at the figure and decide.

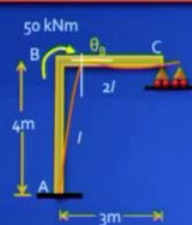
It will be anticlockwise. Another thumb rule you can take is - whenever I have a clockwise chord rotation, I will end up with anticlockwise end moments. We have proved this earlier. So, you know everything. You have got the fixed end moments. Now what should you do in the displacement method? You have to release that moment which

accumulated at the location where you artificially fixed it. So, the correct answer is this plus that (Refer Slide Time: 05:04).

Now, that second figure is something we have analyzed earlier. Remember, it is a single unknown problem concentrated moment; nothing to worry, this is a broad concept. You can use slope deflection method, you can use moment distribution method and you can use stiffness method. Follow this concept and take advantage of the anti symmetry in the response.

(Refer Slide Time: 05:32)

Example 4a: Ignoring sway in frame - Slope deflection method



We can reduce the kinematic indeterminacy (ignore the sway degree of freedom) by considering the cantilever behaviour of AB (with modified stiffness).

Fixed End Moments (with $\theta_B = 0$)

$$M_{AB}^f = M_{BA}^f = M_{BC}^f = 0 \text{ kNm}$$

Slope-Deflection Equations :

$$M_{AB} = M_{AB}^f - \frac{(EI)}{4} \theta_B = -0.25EI\theta_B$$

$$M_{BA} = M_{BA}^f + \frac{(EI)}{4} \theta_B = +0.25EI\theta_B$$

$$M_{BC} = M_{BC}^f + \frac{3(2EI)}{3} (\theta_B) = +2EI\theta_B$$

Equilibrium Equation :

$$M_{BA} + M_{BC} = M_B = +50 \text{ kNm}$$

$$\Rightarrow 2.25EI\theta_B = +50$$

$$\Rightarrow EI\theta_B = +22.222$$

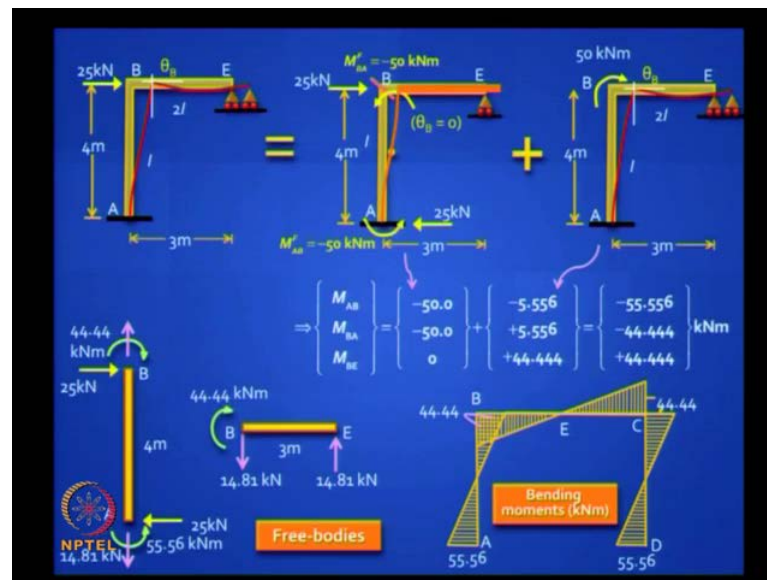
Final Moments:

$$\Rightarrow \begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -0.25 \\ +0.25 \\ +2.0 \end{Bmatrix} EI\theta_B = \begin{Bmatrix} -5.556 \\ +5.556 \\ +44.444 \end{Bmatrix} \text{ kNm}$$

NPTEL

If you do it by the slope deflection method, first you calculate the fixed end moments. You will have fixed end moments only in the column and not in the beam. Of course, here I am not including the fixed end moments because if you go back to the previous figure (Refer slide Time: 05:51) I am analyzing, this is already known. So, I am analyzing this part. I can actually include this in the final answer, but for the time being, I am only analyzing this, and this is a straightforward thing we have done earlier.

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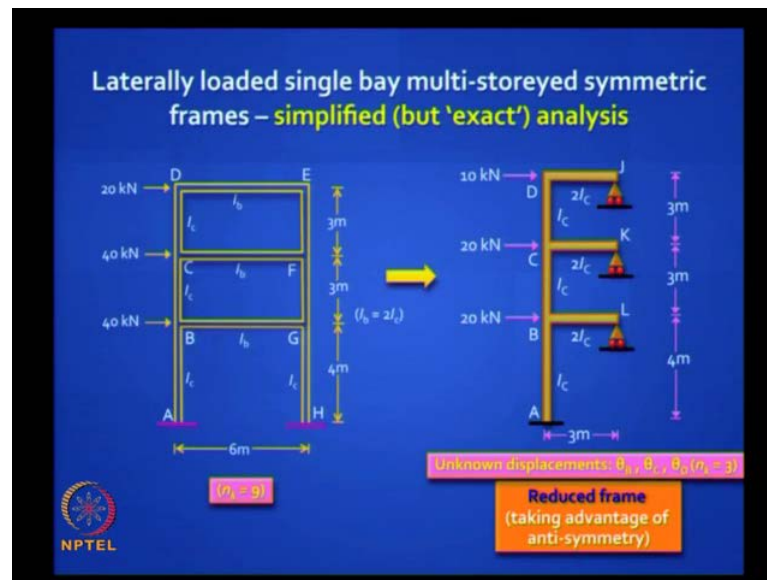


So, you can analyze this and find the bending moment and you simply need to add the two bending moments. From the fixed end moments, these are the moments you get - minus 50; from the constant moment, these are the moments you get. You add them up and that is your final answer.

You can also use moment distribution to distribute this moment in proportion to the two relative stiffnesses. What is the stiffness of AB? It is a cantilever. If $4EI$, only B is prevented from translating; it is EI by L . What is the stiffness for BE? $3EI$ by L , of course, the corresponding EI and the corresponding L . You can work it out either way and you will get the answer.

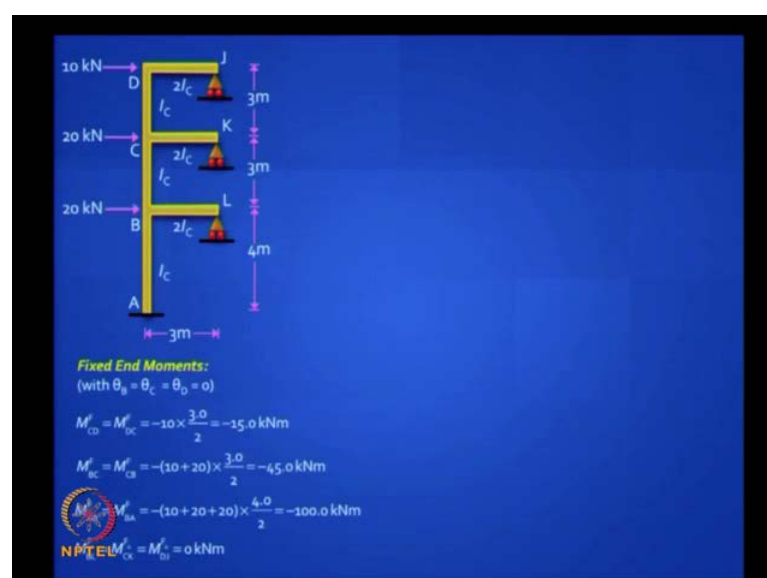
Draw the free bodies; you got the final moment. This is easy to do. Now, let us apply this to a really difficult problem.

(Refer Slide Time: 06:56)



Let us apply to a three storied symmetric frame. We have done the same problem, if you recall by the approximate method of analysis. Let us do it exactly, now using either slope deflection method or the moment distribution method. Let us simplify this problem by recognizing that there is symmetry in the frame by cutting it in the middle and putting roller supports wherever you cut them. So, we just need to analyze this and we have to divide the applied lateral loads by 2 because one half goes to the windward half and the other half goes to the leeward half; you can choose either half to analyze. Now, how do we go about doing this?

(Refer Slide Time: 07:45)



Well, first write down the fixed end moments. Now, what do we mean by fixed end moments? We are talking about a structure which is kinematically determinate and we had identified only three rotational unknowns - θ_B , θ_C and θ_D . We are not taking the sways as unknowns. So, when you arrest θ_B , θ_C , θ_D and you apply the lateral loads, you will have three independent movements, each of those columns in each of the stories will just move like a beam undergoing settlement of supports. Everything is known. Can you tell me what the fixed end moments will be? There we have done. You see the calculation here. (Refer Slide Time: 08:39)

The fixed end moments in the beams are all zero because there is no loading in the beam. So, the beams are all zero. You take AB or let us start at the top, take DC. Now, the storey shear is 10 kilonewton. If you take this column, you fix it at the bottom and you fix it at the top because, θ_B , θ_C and θ_D are arrested and you allow a relative moment. What will be the shear in that column DC? The shear will be 10 because entire 10 comes here, nothing goes to roller support. So, the fixed end moments will be 10 into 3 by 2. Will it be clockwise or anti clockwise? It is anticlockwise because the chord rotation is clockwise. So, that is a simple calculation minus 10 into 3 by 2 is minus 50. It does not take any time.

Take the next flow. In the next flow, what is the shear in that column? It will be 10 plus 20. It will not just be 20, agreed because that is the storey shear. What is the moment you get? 10 plus 20 into 3 by 2 with the minus sign. Lastly, what do you do for AB? It is the total base shear, which is 10 plus 20 plus 20 into 4 by 2, with the minus sign. Do not go wrong here because the shortcut requires you to look at the fixed end moments also and not just at the stiffness. Then only you can take full advantage.

Now, you could have done the same problem the right royal way including sway, but then you have six degrees of freedom: you have three sways and three rotations; You have to write down in addition to three moment equilibrium equations, **you have to write down** three shear equilibrium equations - the way we did in the earlier class, but that is lot of extra work. Solving six unknowns is going to take you a long time, but three unknowns is easy. You have calculated and it gives you the answers straight away. So, it is a powerful technique to use and you get exact results. The only error you make is in ignoring the axial deformations. Now, axial deformations in practice can be ignored except in very tall frames because in very tall frames, the moments are quite significant;

the windward column will actually extend significantly and the leeward column will get contracted significantly. In those instances, you would need to do a second order analysis or you can do something that we are going to learn when we study matrix method.

(Refer Slide Time: 11:32)

Slope-deflection Equations:
 Element AB (EI_c ; $L = 4\text{m}$; $\theta_A = 0$; $\theta_B = ?$; ϕ_{AB} can be ignored);

$$M_{AB} = M_{AB}^F - \frac{(EI_c)}{4} \theta_B = (-100.0 - 0.25EI_B \theta_B) \text{ kNm}$$

$$M_{BA} = M_{BA}^F + \frac{(EI_c)}{4} \theta_B = (-100.0 + 0.25EI_B \theta_B) \text{ kNm}$$

Fixed End Moments:
 (with $\theta_B = \theta_C = \theta_D = 0$)

$$M_{CD}^F = M_{DC}^F = -10 \times \frac{3.0}{2} = -15.0 \text{ kNm}$$

$$M_{BC}^F = M_{CB}^F = -(10 + 20) \times \frac{3.0}{2} = -45.0 \text{ kNm}$$

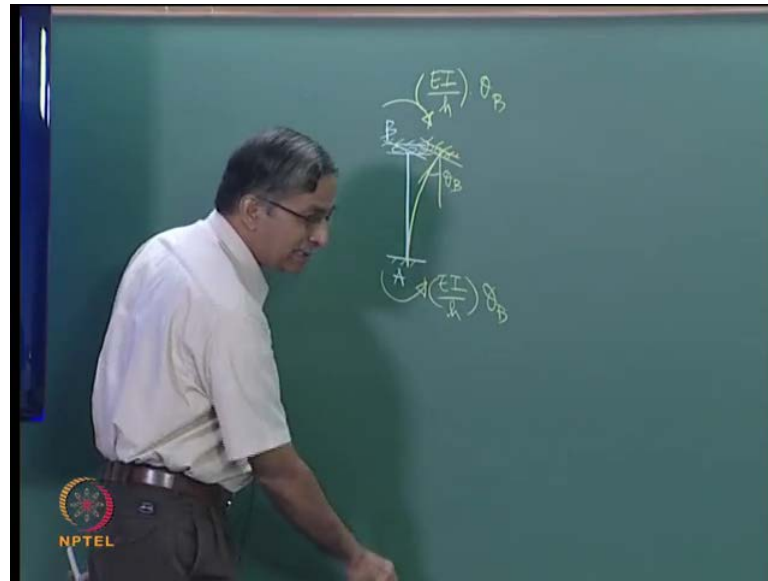
$$M_{BA}^F = M_{AB}^F = -(10 + 20 + 20) \times \frac{4.0}{2} = -100.0 \text{ kNm}$$

NOTE: $M_{CB}^F = M_{DC}^F = 0 \text{ kNm}$

Slope deflection equations.

Let us take AB. M_{AB} is MF_{AB} . Why do we write minus EI_c by 4? Why is it minus? I think let us start with BA. This will be easier for you to understand. M_{BA} is MF_{BA} plus EI_c by 4. It is a cantilever.

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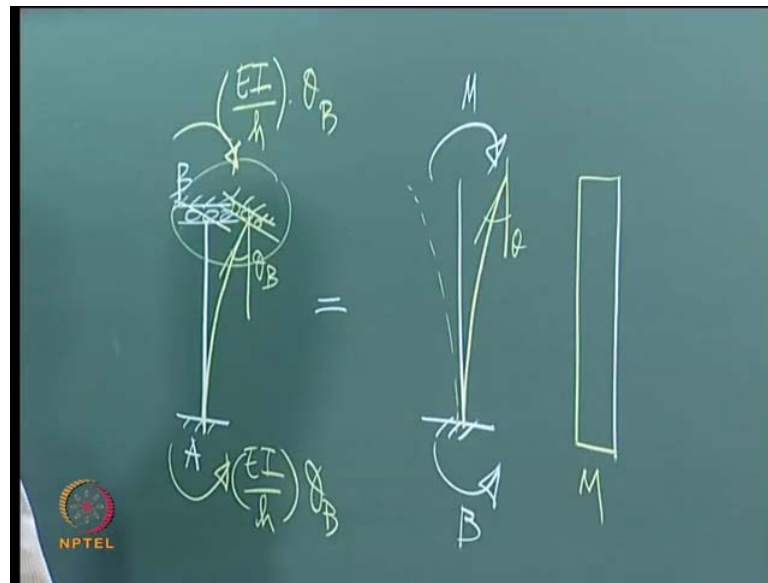


This is A, this is B and we are arresting this with the guided roller support. Now we are allowing θ_B to take place. So, when you allow θ_B to take place, you are letting this undergo a rotational slip which is as good as applying the moment. You will get the moment as a reaction. So, if you let this rotate, it is like releasing the rotational fixity. How is it going to deflect? It is going to deflect like this, like a cantilever. It is going to deflect like a cantilever and you will have a guided roller support; this is the rotational slip.

Now, if this angle is θ_B , what will be the moment that you get? The stiffness over a cantilever is well known. It will be EI into the height of this column into θ_B . It will be clockwise. What is the moment that you get here? It will be in the other way, other direction and **it will also be equal to...** You have done this with the element in the horizontal position in beams. The behavior is exactly the same even if it is vertical, have you understood?

So, that is the reason why the formula (Refer slide Time: 11:32) is MF , the fixed end moments plus or minus $EI \theta$ divided by L . Where is it plus? At the unknown rotation location. Where is it minus? At the remote end. I think that figure (Refer Slide Time: 12:09) explains everything. Is there any doubt on this? Why is the moment clockwise at B? This is the same as this by the way.

(Refer Slide Time: 14:32)



There is no difference. Which way do you think this is going to sway? To the left or to the right? Let us say, this is a common problem of intuitive understanding many students have. What is a bending moment diagram? Just look at this. You will get the same moment here. What is the bending moment diagram going to look like drawn on the tension side? So, the tension is on this side **tension is on that side** and the bending moment diagram will have a constant moment here (Refer Slide Time: 15:13). If it were to move this way, you are going to have tension on this side; so it does not work. So, that is one clue that it has to rotate like this (Refer Slide Time: 15:24). That is why, this has to be clockwise and this is a cantilever; you can use conjugate beam method and show the relation. So, you should not get disturbed by the fact that you can assume it to be rotationally fixed and then release it.

So, the deflected shape gives you the clue. The behavior is exactly the same. That is the modification you need to do.

(Refer Slide Time: 15:58)

Slope-deflection Equations:

Element AB (EI_C ; $L = 4\text{m}$; $\theta_A = 0$; $\theta_B = ?$; ϕ_{AB} can be ignored);

$$M_{AB} = M_{AB}^f - \frac{(EI_C)}{4} \theta_B = (-100.0 - 0.25EI_B) \text{ kNm}$$

$$M_{BA} = M_{BA}^f + \frac{(EI_C)}{4} \theta_B = (-100.0 + 0.25EI_B) \text{ kNm}$$

Element BC (EI_C ; $L = 3\text{m}$; $\theta_B = ?$; $\theta_C = ?$; ϕ_{BC} can be ignored);

$$M_{BC} = M_{BC}^f + \frac{(EI_C)}{3} \theta_B - \frac{(EI_C)}{3} \theta_C = (-45.0 + 0.3333EI_B - 0.3333EI_C) \text{ kNm}$$

$$M_{CB} = M_{CB}^f - \frac{(EI_C)}{3} \theta_B + \frac{(EI_C)}{3} \theta_C = (-45.0 - 0.3333EI_B + 0.3333EI_C) \text{ kNm}$$

Fixed End Moments:
(with $\theta_B = \theta_C = \theta_D = 0$)

$$M_{CB}^f = M_{BC}^f = -10 \times \frac{3.0}{2} = -15.0 \text{ kNm}$$

$$M_{BC}^f = M_{CB}^f = -(10 + 20) \times \frac{3.0}{2} = -45.0 \text{ kNm}$$

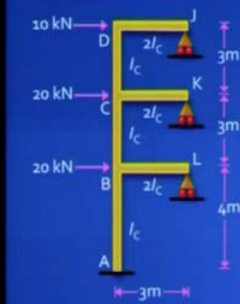
$$M_{AB}^f = M_{BA}^f = -(10 + 20 + 20) \times \frac{4.0}{2} = -100.0 \text{ kNm}$$

NPTEL

When you take the next element BC, will you write down the slope deflection equation for BC? You will get contributions from θ_B and from θ_C . So, that is all. Once you get the hang of it, you will find it is a very powerful shortcut method of solving such problems.

For the element BC - For BC, you will get a positive contribution from θ_B and a negative contribution from θ_C , and for CB you will get a positive contribution from θ_C and a negative contribution from θ_B for the reasons explained there; positive meaning clockwise and negative meaning anticlockwise or counter clockwise. Now it is easy. So, you can finish it by writing it for CD as well.

(Refer Slide Time: 17:14)



Element CD (EI_c ; $L = 3\text{ m}$; $\theta_c = ?$; $\theta_D = ?$; ϕ_{CD} can be ignored);

$$M_{CD} = M'_{CD} + \frac{(EI_c)}{3} \theta_c - \frac{(EI_c)}{3} \theta_D = (-15.0 + 0.3333EI_c \theta_c - 0.3333EI_c \theta_D) \text{ kNm}$$

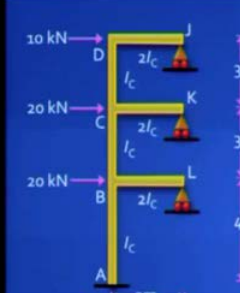
$$M_{DC} = M'_{DC} - \frac{(EI_c)}{3} \theta_c + \frac{(EI_c)}{3} \theta_D = (-15.0 - 0.3333EI_c \theta_c + 0.3333EI_c \theta_D) \text{ kNm}$$

NPTEL

For CD, what about the beam elements? You got three beam elements BL, CK and DJ. What is it going to be? There is no fixed end moment, so the moment will be $3EI$ by 3, the EI here should be $2I_c$. So, it is simple.

So, please understand. **You have to write** Find the fixed end moments correctly, which was actually very easy to do. You have to write the slope deflection equations also correctly and then when we put it all together and it looks so nice, simple matrix form.

(Refer Slide Time: 17:59)



$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BL} \\ M_{BC} \\ M_{CB} \\ M_{CL} \\ M_{CD} \\ M_{DC} \\ M_{DL} \end{bmatrix} = \begin{bmatrix} -100.0 \\ -100.0 \\ 0 \\ -45.0 \\ -45.0 \\ 0 \\ -15.0 \\ -15.0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/4 & 0 & 0 \\ +1/4 & 0 & 0 \\ +2 & 0 & 0 \\ +1/3 & -1/3 & 0 \\ -1/3 & +1/3 & 0 \\ 0 & +2 & 0 \\ 0 & +1/3 & -1/3 \\ 0 & -1/3 & +1/3 \\ 0 & 0 & +2 \end{bmatrix} \begin{bmatrix} EI_c \theta_B \\ EI_c \theta_C \\ EI_c \theta_D \end{bmatrix} \text{ kNm}$$

Equilibrium Equations:

$$M_{BA} + M_{BL} + M_{BC} = M_B = 0$$

$$M_{CB} + M_{CL} + M_{CD} = M_C = 0$$

$$M_{DC} + M_{DL} = M_D = 0$$

$$\begin{bmatrix} -15.0 \\ -15.0 \\ 0 \end{bmatrix} + \begin{bmatrix} 31/12 & -1/3 & 0 \\ -1/3 & 8/3 & -1/3 \\ 0 & -1/3 & 7/3 \end{bmatrix} \begin{bmatrix} EI_c \theta_B \\ EI_c \theta_C \\ EI_c \theta_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} EI_c \theta_B \\ EI_c \theta_C \\ EI_c \theta_D \end{bmatrix} = \begin{bmatrix} +60.17889 \\ +31.38640 \\ +10.91234 \end{bmatrix} \text{ kNm}$$

NPTEL

You have got all the end moments in all the elements. You are not writing down the end moments at J, K and L because you know them to be 0. Now, what do we do? You have to write down three equations and they are easy equations because there is no shear equilibrium here. It is all moment equilibrium. What are the three equations? The net moment at B should be 0, which means?

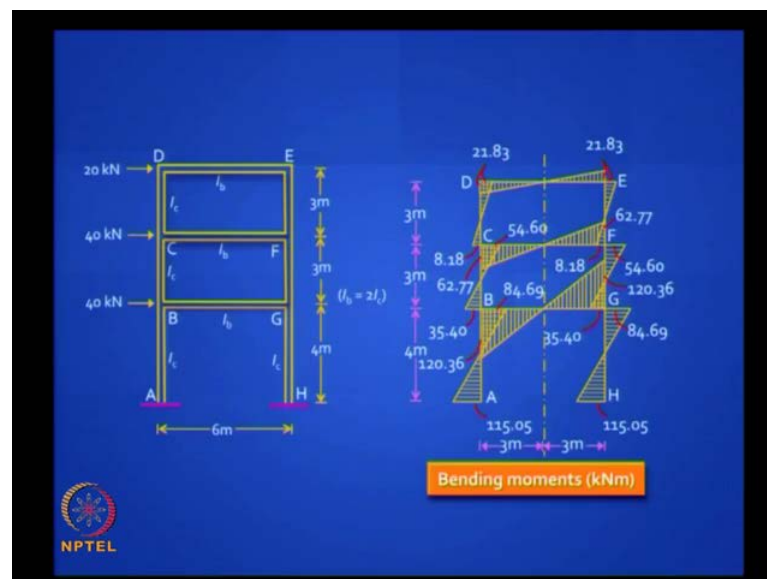
M_{BA} plus M_{BC} plus M_{BL} is equal to zero.

So, you take the second equation, the third equation and the fourth equation or the second, third and fourth rows in that equation; add it all up and put it equal to 0. Likewise, you do for the others.

It is easy to do, and here, again you get a stiffness matrix like we did last time and you will find that you may change these loads, but that only affects the constant vector; nothing happens to this part (Refer Slide Time: 19:05). That only affects this column, the fixed end forces; nothing happens to this. That is the property of a structure and that is what we are going to study in detail, when we take up matrix method.

You can get the answers. Then what you do you do? Plug them back into the slope deflection equation. You got the answers.

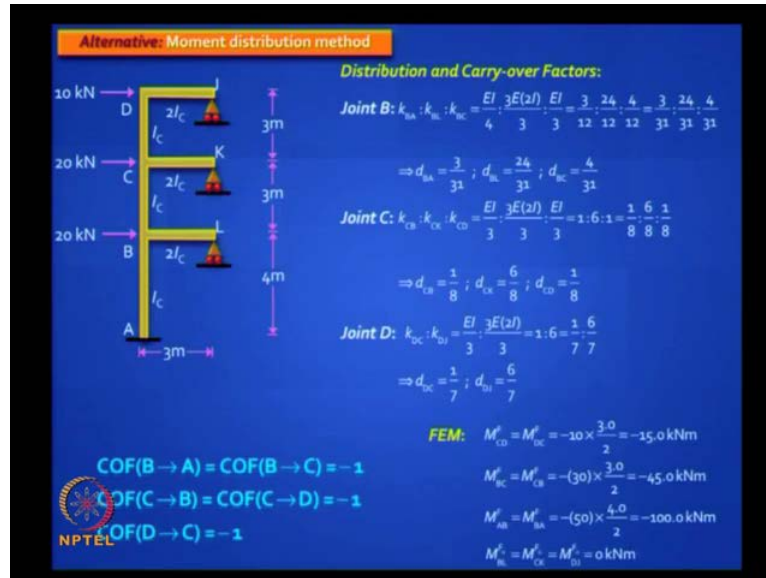
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You can draw the free body diagram; you can draw the bending moment diagram. If you want to do this by moment distribution method, go ahead because you do not have sway and it is not difficult.

(Refer Slide Time: 19:39)

Alternative: Moment distribution method



Distribution and Carry-over Factors:

Joint B: $k_{Bk}, k_{Bl}, k_{Bc} = \frac{EI}{4}, \frac{3EI(2I)}{3}, \frac{EI}{3} = \frac{3}{12}, \frac{24}{12}, \frac{4}{12} = \frac{3}{31}, \frac{24}{31}, \frac{4}{31}$
 $\Rightarrow d_{Bk} = \frac{3}{31}; d_{Bl} = \frac{24}{31}; d_{Bc} = \frac{4}{31}$

Joint C: $k_{Ck}, k_{Cl}, k_{Cc} = \frac{EI}{3}, \frac{3EI(2I)}{3}, \frac{EI}{3} = 1:6:1 = \frac{1}{8}, \frac{6}{8}, \frac{1}{8}$
 $\Rightarrow d_{Ck} = \frac{1}{8}; d_{Cl} = \frac{6}{8}; d_{Cc} = \frac{1}{8}$

Joint D: $k_{Dk}, k_{Dl} = \frac{EI}{3}, \frac{3EI(2I)}{3} = 1:6 = \frac{1}{7}, \frac{6}{7}$
 $\Rightarrow d_{Dk} = \frac{1}{7}; d_{Dl} = \frac{6}{7}$

COF(B → A) = COF(B → C) = -1
COF(C → B) = COF(C → D) = -1
COF(D → C) = -1

FEM: $M_{cd}^f = M_{dc}^f = -10 \times \frac{3.0}{2} = -15.0 \text{ kNm}$
 $M_{bc}^f = M_{cb}^f = -(30) \times \frac{3.0}{2} = -45.0 \text{ kNm}$
 $M_{ab}^f = M_{ba}^f = -(50) \times \frac{4.0}{2} = -100.0 \text{ kNm}$
 $M_{kl}^f = M_{lk}^f = M_{ll}^f = 0 \text{ kNm}$

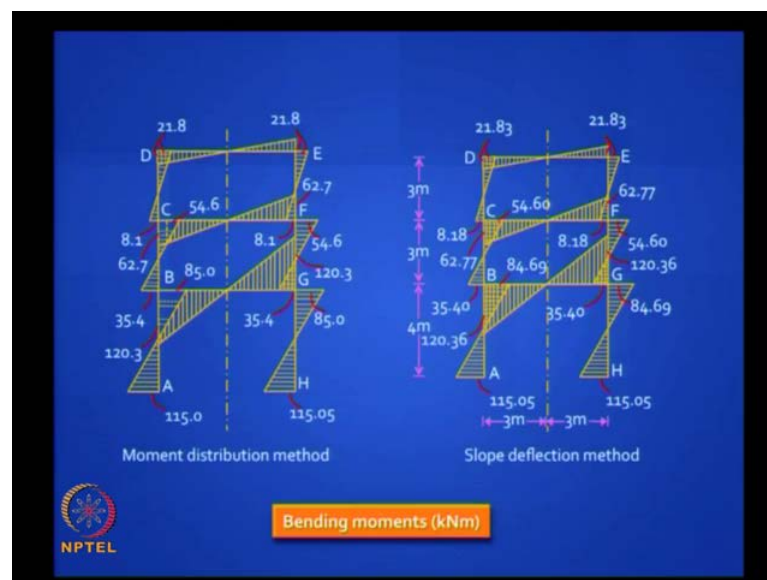
Let us just take a look; I leave the choice to you. Here, the stiffness at B are in the ratio of EI by L, 3EI by L and EI by L. Two columns and one beam, for those columns it is EI by L and for the beam it is 3EI by L. That is it. So, you can do the same thing for the three joints. Write down your carry-over factors. You know that for a cantilever portion only you have a carryover factor of minus 1. You do not have carry-over anywhere else. Calculate the same fixed end moments which you have to calculate for the slope deflection method.

(Refer Slide Time: 20:20)

Distribution table									
	AB	BA	BL	BC	CB	CK	CD	DC	DJ
D.F		3/31	24/31	4/31	1/8	6/8	1/8	1/7	6/7
C.O.F	-1			-1	-1		-1	-1	
F.E.M	-100.0	-100.0		-45.00	-45.00		-15.00	-15.00	
Bal.1		+14.03	+112.26	-18.71	+7.50	+45.00	+7.50	+2.14	+12.86
CoM 1	-14.03			-7.50	-18.71		-2.14	-7.50	
Bal.2		+0.73	+5.81	+0.97	+2.61	+15.64	+2.61	+1.07	+6.43
CoM 2	-0.73			-2.61	-0.97		-1.07	-2.61	
Bal.3		+0.25	+2.02	+0.34	+0.26	+1.53	+0.26	+0.37	+2.24
CoM 3	-0.25			-0.26	-0.34		-0.37	-0.26	
Bal.4		+0.03	+0.20	+0.03	+0.09	+0.53	+0.09	+0.04	+0.22
	-115.04	-84.96	+120.29	-35.42	-54.56	+62.70	-8.12	-21.75	+21.75

Make your table. You will get some answers, the accuracy of which improves as you increase the number of iterations.

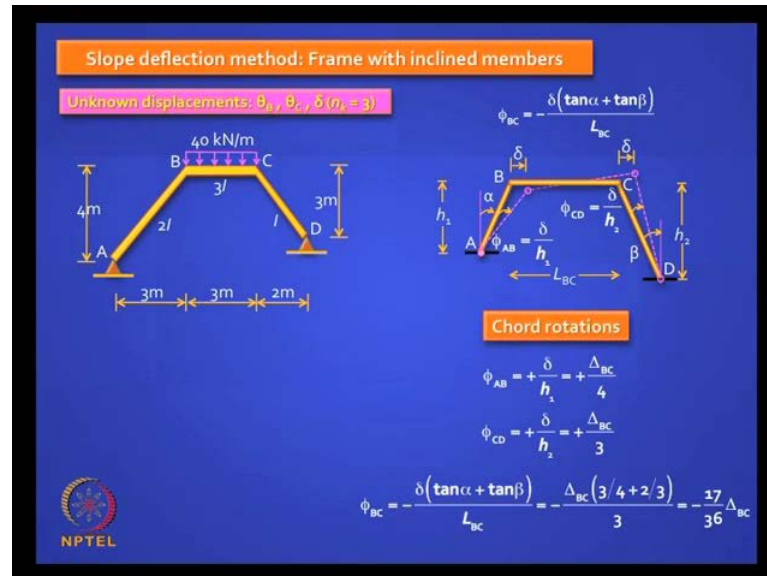
(Refer Slide Time: 20:34)



So, I stop with three and this is the answer you get from moment distribution method. This is the more accurate answer you get from slope deflection method. The moment distribution is good enough. Why is it good enough? Because we are civil engineers. We know that there is lot of uncertainty. Wind loads are very difficult to predict in any case. So, your high refinement to that third decimal place is absolutely meaningless from a

practical design point of view which is why this is good enough; you could have stopped at two cycles, if you wish.

(Refer Slide Time: 21:14)



One last problem - a tricky one. You know I do not want you to solve it, but I want you to tell me how to solve it.

Here, this is a frame with inclined columns which is little tricky. Can you tell me what is so tricky about this, compared to a frame with vertical columns? BC as rotation is not alone the problem. In your shear equilibrium equation, do you see a problem?

[Noise] (Refer Slide Time: 21:51)

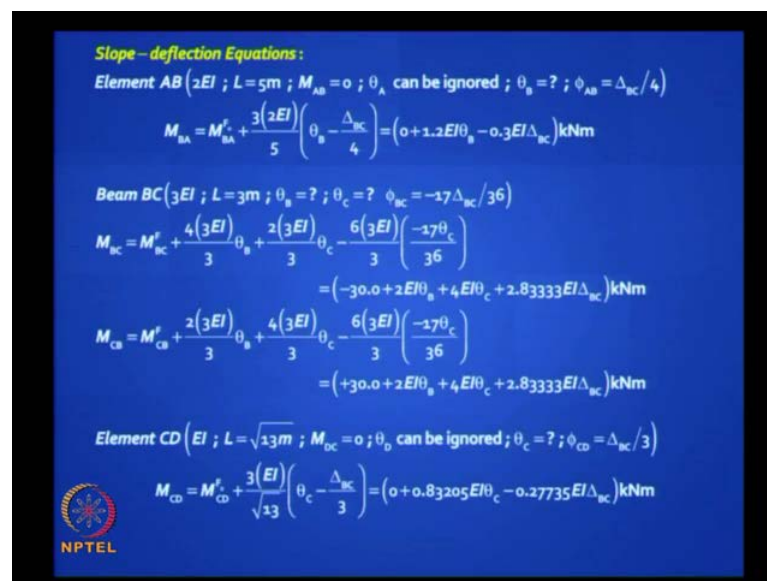
Let us do it; you will get the feel when we try this.

What is the degree of indeterminacy here, kinematic indeterminacy? θ_B , θ_C and sway, fine. θ_B , θ_C , θ_D and sorry, that should not be θ_D , θ_B , θ_C and the sway δ . Now, if it sways δ , the chord AB will rotate clockwise as we discussed in the last class, by δ by h_1 . Chord CD will also rotate clockwise by δ by h_2 and you can prove using trigonometry. I will skip that part, that you can check it out. Compatibility requires that BC will rotate anti-clockwise because the three members are interconnected by that quantity. So, there is only one unknown, δ and you have successfully written all the chord rotation in terms of δ .

You have to make in the mechanism and you will get this proof. What do we do next? What is it? You have to write down the slope deflection equation, but before that you need to calculate the fixed end moment which are very easy to calculate.

These are your chord rotations in terms of delta (Refer Slide Time: 23:26) and your fixed end moments are straightforward - minus $W l$ squared by 12 plus $W l$ squared by 12. You have fixed end moments only in BC. Now you are ready to write the slope deflection equations and they are also very easy to write; you know it by memory now M_{AB} is M_{FAB} plus $4EI$ by L theta_A plus $2EI$ by L theta_B minus $6EI$ by L into phi_{AB}, except when you have a hinge. When you have a hinge, then, $3EI$ by L for rotational stiffness and you have minus $3EI$ by L into phi for chord rotation. That is the only change you need to make.

(Refer Slide Time: 24:20)



Slope-deflection Equations:

Element AB ($2EI$; $L = 5m$; $M_{AB} = 0$; θ_A can be ignored; $\theta_B = ?$; $\phi_{AB} = \Delta_{BC}/4$)

$$M_{BA} = M_{FBA} + \frac{3(2EI)}{5} \left(\theta_B - \frac{\Delta_{BC}}{4} \right) = (0 + 1.2EI\theta_B - 0.3EI\Delta_{BC}) \text{ kNm}$$

Beam BC ($3EI$; $L = 3m$; $\theta_B = ?$; $\theta_C = ?$; $\phi_{BC} = -17\Delta_{BC}/36$)

$$M_{BC} = M_{FBC} + \frac{4(3EI)}{3} \theta_B + \frac{2(3EI)}{3} \theta_C - \frac{6(3EI)}{3} \left(\frac{-17\theta_C}{36} \right)$$

$$= (-30.0 + 2EI\theta_B + 4EI\theta_C + 2.83333EI\Delta_{BC}) \text{ kNm}$$

$$M_{CB} = M_{FCB} + \frac{2(3EI)}{3} \theta_B + \frac{4(3EI)}{3} \theta_C - \frac{6(3EI)}{3} \left(\frac{-17\theta_C}{36} \right)$$

$$= (+30.0 + 2EI\theta_B + 4EI\theta_C + 2.83333EI\Delta_{BC}) \text{ kNm}$$

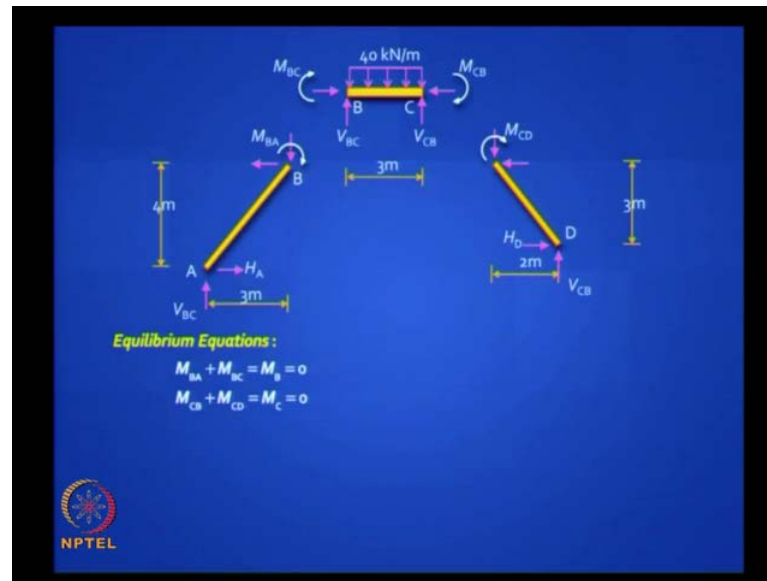
Element CD (EI ; $L = \sqrt{13}m$; $M_{DC} = 0$; θ_D can be ignored; $\theta_C = ?$; $\phi_{CD} = \Delta_{BC}/3$)

$$M_{CD} = M_{FCD} + \frac{3(EI)}{\sqrt{13}} \left(\theta_C - \frac{\Delta_{BC}}{3} \right) = (0 + 0.83205EI\theta_C - 0.27735EI\Delta_{BC}) \text{ kNm}$$

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So, let us say, you correctly write down the slope deflection equations. I am just going through the method and there is nothing special in this. You write it in terms of three unknowns - theta_B, theta_C and delta right. The question I am going to raise is - how do you solve for these three unknowns? You need three equations. Two equations are very easy to get. What are they?

(Refer Slide Time: 24:47)



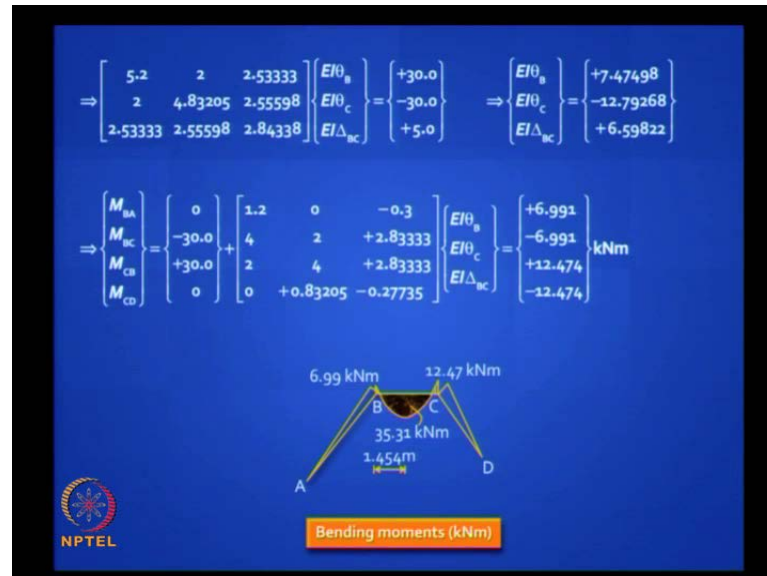
There you are. This we can all write down; third equation, how will you write? Be careful. So, the first thing you should do is draw free bodies of all the three elements with the end moments. Draw the three free bodies. They will look like this. (Refer Slide Time: 25:11) You have M_{BA} , M_{BC} , M_{CB} , M_{CD} and your four slope deflection equations for them. Now earlier in the portal frames, when you wrote H_A plus H_D equal to 0, the H_A involved only end moments in the column.

Now, do you see a complication? What is the complication? You have this guy here. (Refer Slide Time: 25:47) This V_{BC} is going to act down on this column and because it is inclined, it is also going to introduce something that will affect the horizontal reaction.

So, can you write down the shear equilibrium equation? I can ask you such a question in the examination. No calculations, write down the shear equilibrium equation in terms of M_{BA} , M_{BC} , M_{CB} and M_{CD} . That is all you need to do. Calculations are easy to do, concepts are not that easy. So, normally you need to be on the lookout when your columns are inclined. So, it is easy from equilibrium, you can write down an expression for H_A . H_A will be M_{BA} plus V_{BC} into 3 divided by 4 and you agree, this is what you write for H_D . H_D will be M_{CD} minus, this V_{CB} goes here (Refer Slide Time: 27:06) and gives an anti clockwise moment minus V_{CB} into 2 divided by 3. Then, the V itself you have to now express in terms of M_{BC} , M_{CB} and the loading. That is the catch. Does it

make sense to you? It is all equilibrium which you were thorough with in the first course in structural analysis. So, this is discussed at length in the text book.

(Refer Slide Time: 27:48)



Let us say, you can do it because once a concept is there, somebody can work out the calculations and you can solve the equations, substitute and get. I am not asking you to do these problems in detail because they take time. But, I would like you to get the concept behind this.

So, with this, we are done with basic structural analysis and now we will quickly review the problems that you did recently in the 15 minute quiz that you had.

(Refer Slide Time: 28:04)

Quiz 1 questions and solutions

1. Find the force, in terms of the load W , transmitted at the centre E to beam CD in the symmetrical grid arrangement shown in Fig. 1. The two horizontal beams AB and CD , both having the flexural rigidity EI , are simply supported and interconnected at E .

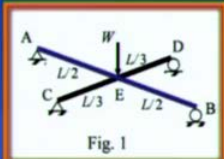
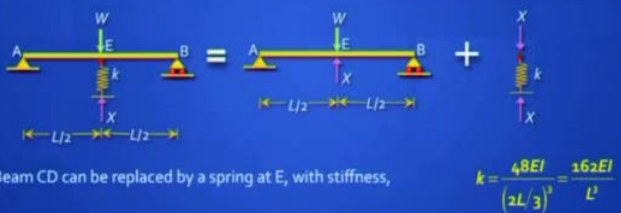


Fig. 1



Beam CD can be replaced by a spring at E , with stiffness,

$$k = \frac{48EI}{(2L/3)^3} = \frac{162EI}{L^3}$$

Compatibility: Deflection at E in beam AB = Deflection in spring

$$\Delta = \frac{(W-X)L^3}{48EI} = \frac{X}{k} = \frac{XL^3}{162EI} \Rightarrow W-X = \frac{X}{3.375} \Rightarrow X = \frac{3.375}{4.375}W = 0.7714W$$

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You have four problems and remember it is a quiz. I am not going to tax you. So, there must be simple ways of doing it, but some of you choose the most difficult ways to do it. You ran out of time and you could not do it. If you go ahead to do it all over again, let us see, how you would do it. So, this was the first problem (Refer Slide Time: 28:39). You have two beams criss-crossing; you have a concentrated load W acting in the middle. That W is going to be shared between AB and CD in some proportion.

The question is what is the fraction of the W that goes to CD ? Very simple and clean question. Both the beams have identical cross section and flexibility. It is a straight forward question. Supposing both the beams have the same span, what is the answer? W by 2, W by 2. Some of you did lot of calculations and ended up proving that the load transfer to CD is 0. That you know by looking at the answer that something is wrong.

Let me ask you another question. Will CD take more of the load W than AB or vice versa? It will take more because the stiffness is more. Let us see how to do this. **Well, the way I have taught, you would have done this, I thought.**

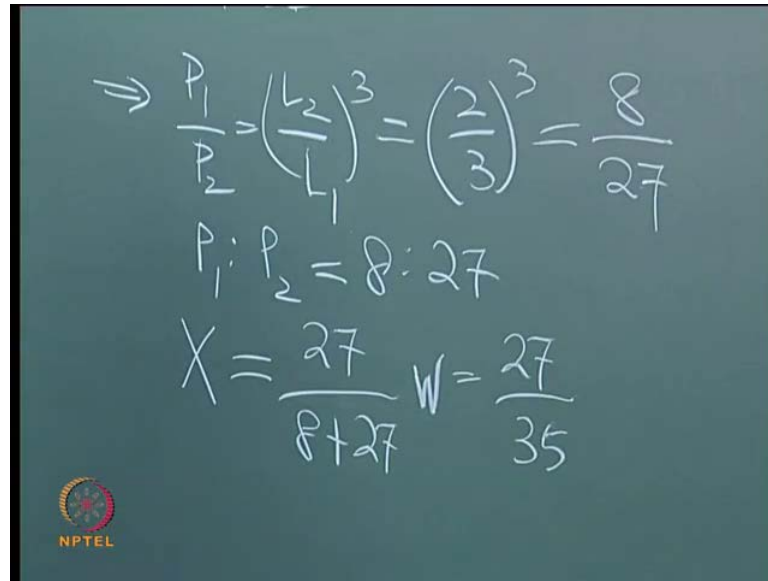
Take AB and replace CD with a spring. The force in the spring is what we are looking for. So, that is the force which is transmitted to CD . Some of you did it the other way. Nothing wrong with that, but then you have to do W minus that to get it. I just want the final answer. This is what you would have done.

Now I did not want any calculation because these are straight forward problems. You can get the answer in one or two steps. So, how do you solve for X ? You need a compatibility equation. Some of you did lot of integration and all that. Not required. Let us play it simple. Theorem of least work will take time. There is a much easier way to do it. First of all what is the stiffness of that spring, $48EI$ by L of CD cubed. Many of you forgot that. So, that L is only $2L$ by 3 . That is the first thing you have to do $48EI$ by $2L$ by 3 the whole cubed, turns out to be $162EI$ by L cubed.

Then what do we do. Then we realize that you can separate out the spring. I am just showing you one clean way of doing it and what is common to the spring and that beam? They both deflect the same. That is compatibility. You can write compatibility in many ways but, that is an easy way to write compatibility. So, the deflection at E in the beam AB must be exactly equal to the deflection in the spring which is the deflection in the other beam. So, the net force acting in AB is W minus X downward and the formula is very clear. W minus X , the whole cubed by $48EI$. What is a deflection in the spring? It is X by k . You have already got a formula for k . Now it is quite easy to solve for X . It does not take you any time.

I would have thought you will crack this problem in 5, 10 minutes. Some of you took much more time and some got the answer. But, I especially liked one solution given by one of the students, John Mathew here. This was the last paper I corrected. He gave a brilliant solution and I want to tell you what it is.

(Refer Slide Time: 32:27)


$$\Rightarrow \frac{P_1}{P_2} = \left(\frac{L_2}{L_1}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$
$$P_1 : P_2 = 8 : 27$$
$$X = \frac{27}{8+27} \quad W = \frac{27}{35}$$

The image shows a chalkboard with handwritten mathematical work. At the bottom left, there is a small circular logo with the text 'NPTEL' underneath it.

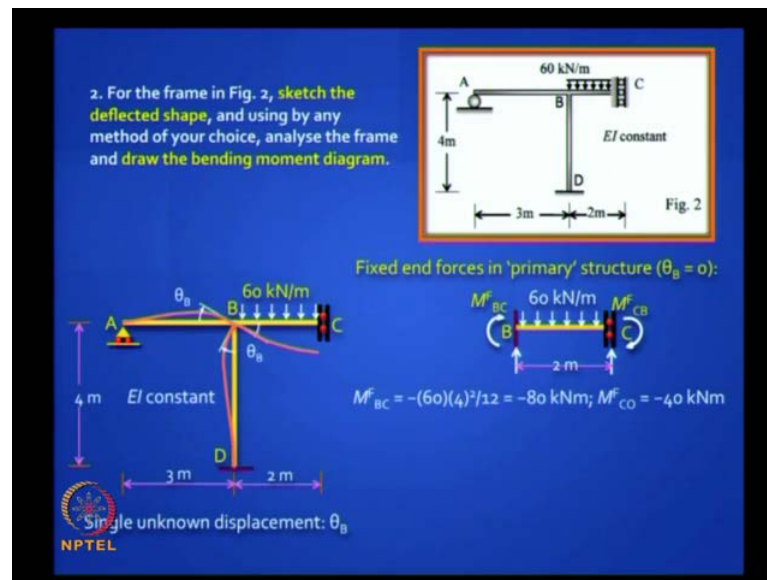
What he said was, that load W the deflection is the same in both the beams. In one beam the constant load, let it be P_1 . $P_1 L_1$ cubed by $48EI$, in the other beam with $P_2 L_2$ cubed by $48EI$, right beautiful answer.

So, he said what is so great about this. Then P_1 by P_2 will be L_2 by L_1 the whole cubed. So, one span is L , the other span is 2 divided by 3, whichever way you can. Let us say 2 by 3 the whole cubed. It works out to 8 by 27. So, the stiffer one must be the one which is taking more loads. So, the P_1 is to P_2 , clearly is the ratio 8 is to 27.

So, CD is stiffer, so the force going to CD must be 27 divided by 8 plus 27 into...

That is a brilliant solution. So, let us give him a hand. John Mathew, get up. Well done, brilliant. How much time does it take? So, this is good thinking.

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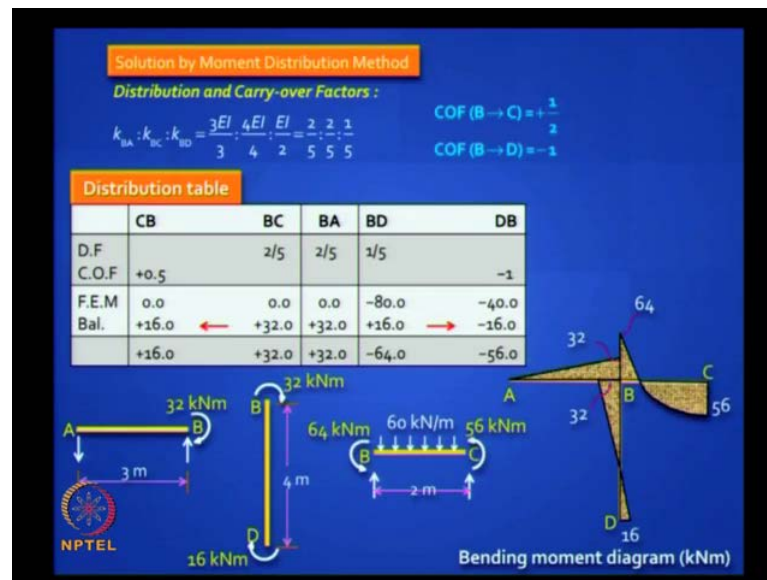
Let us move to this problem which I asked you. Some of you did it right. Some of you messed up. You were asked to draw the deflected shape, many of few lifted up the beam from A. There is a roller support there. How can it fly up? So, this was the problem given to you. Will theta, there is only one unknown rotation here. That is why you should do it by displacement method because you know that this is a power of the displacement method. Force method is a crazy thing to do here.

So, the only unknown is θ_B ; θ_C and δ , we do not want to worry about and what is a deflected shape? Will θ_B be clockwise or anticlockwise? Clockwise. Has to be clockwise, do that first, then it should be easy to draw this. That is all.

You do this, you get some marks. Even before you started batting, you know this is the first thing you should do. Now what do you do.

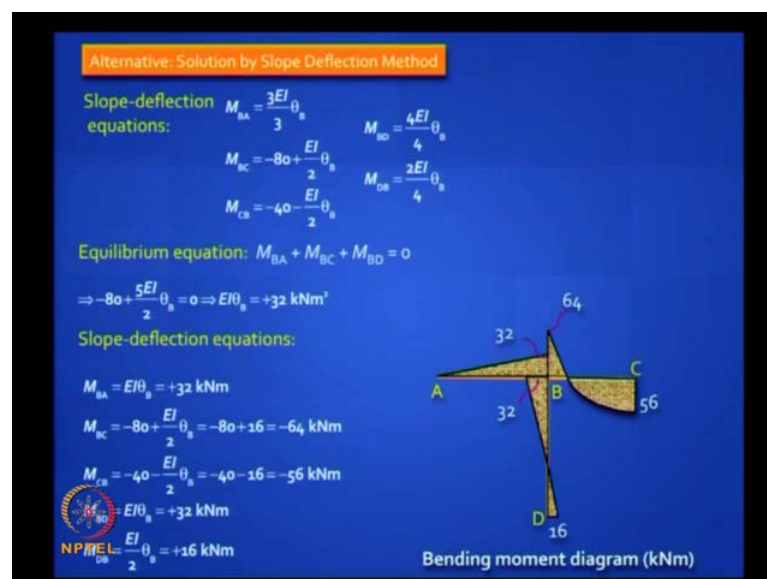
Find out the fixed end moments. Many of you made mistakes in this calculation. **I am not spending very easy**. You double the span and the fixed end moments for the double span. Double span means 4 meter span $Q_0 L^2$ by 12 and in the mid span, it is half that. Both are the other moment minus 40, if this is minus 80. Got it? If you went wrong here, you will get the answer wrong.

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Now here is where, if you do it by moment distribution, here is where many of you mess up. k_{BA} to k_{BC} to k_{BD} you remember, it is $3EI$ by L , $4EI$ by L and EI by L because we had done similar problems in the class, but you forgot that in this problem the L 's were different. So, you are operating on memory which is the worst way engineers should operate and so your relative stiffness is changed here and it is a straight forward calculation. Within 5 minutes you should crack this problem. Draw the bending moment diagram, it will look like that.

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If you want to do this by slope deflection method, you are welcome to do it. So, it is similar, the steps are similar. So, you get the same answer. Some of you did it this way. So, that is the second problem, should not take you too much time.

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3. Analyse the continuous beam shown in Fig. 3 by any method of your choice, considering the effects of the applied load and support settlement shown, and find the bending moments at A and B.

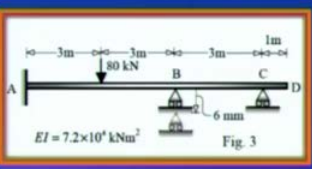


Fig. 3

$EI = 7.2 \times 10^4 \text{ kNm}^2$

$n_k = 1$: Unknown displacement: θ_B

Chord rotations: $\phi_{AB} = +\frac{0.006}{6} = +0.001 \text{ rad}$
 $\phi_{BC} = -\frac{0.006}{3} = -0.002 \text{ rad}$

Fixed end moments:
 due to external load $M_{AB}^f = -\frac{80 \times 6}{8} = -60 \text{ kNm}$; $M_{BA}^f = +60 \text{ kNm}$
 $M_{BC}^f = 0 \text{ kNm}$

Slope-deflection equations:

$$M_{AB} = -60 + \frac{2EI}{6}\theta_B - \frac{6(72000)}{6}(0.001) = -132 + \left(\frac{1}{3}\right)EI\theta_B$$

$$M_{BA} = +60 + \frac{4EI}{6}\theta_B - \frac{6(72000)}{6}(0.001) = -12 + \left(\frac{2}{3}\right)EI\theta_B$$

$$M_{BC} = 0 + \frac{3EI}{3}\theta_B - \frac{3(72000)}{3}(-0.002) = +144 + (1)EI\theta_B$$

Third problem, you had a two span continuous beam with an unnecessary overhang which is just meant to fool you. It is not you can throw it off in your analysis, it does not have any.

It is a simple standard problem. Which method will you choose? You are told to do any method. Half of you did theorem of least work, method of consistent deformation. Nothing wrong, but you will have less time for other problems.

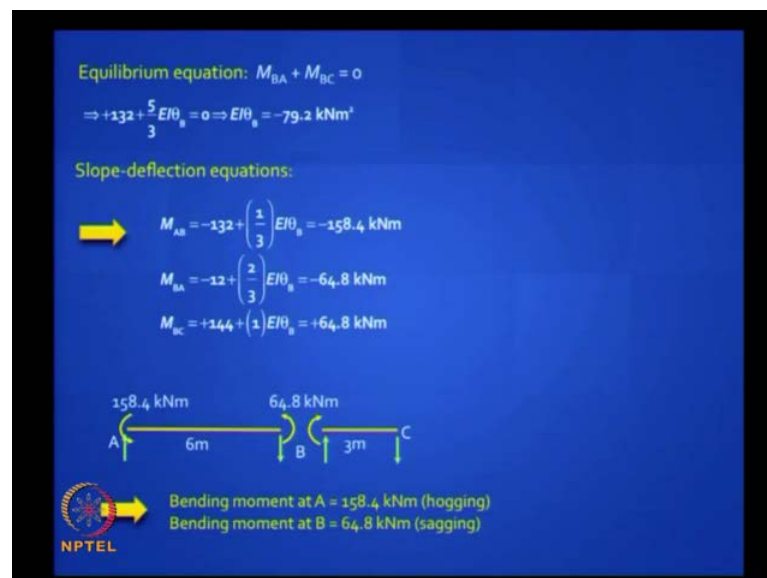
So, here the degree of kinematic indeterminacy is how much minimum? One, that is why I give you quiz. In the end semester examination, it may not be one. You should think. I am not going to tax you. One means 1 minute. If it is slow, what is the other method? method of consistent deformation or theorem of least work that is your two. So, one takes less than half the time it takes for two. Because it is a non-linear process, it does not take time.

So, let us do it by slope deflection method. Very easy, unknown rotation is θ_B . Chord rotation very easy to calculate for AB and some of you messed up in the chord rotation itself.

You wrote ϕ_{AB} is 6 divided by 6 equal to 1. You have 6 apples and 6 oranges, you cannot match or mix them up - actually not; in terms of size, you got 6 lemons and 6 watermelons, you cannot write 6 by 6 equal to 1, 6 millimeters and 6 meters, this is where engineers are supposed to be good. But, if you are like mathematicians, you do not care what is there. That is not right. We are not playing with numbers and they all have meaning.

So, for 0.001 radian, what are the movements likely to be? They are barely visible. If you have 1 radian, it is massive moment. So, please be careful in the units. If you get the units wrong, everything goes wrong. It is a straight forward fixed end moment calculation.

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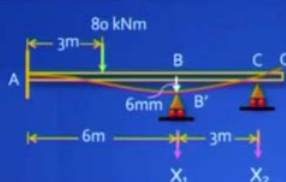
Slope deflection equation - by now, I think you will find they are not difficult to write. Equilibrium equation is straight forward; solve it and you get the answer.

What I wanted was not clockwise or anticlockwise. I wanted sagging or hogging because that is the interpretation. What is clockwise for one beam is anticlockwise for the other. So, should not talk in terms of clockwise or anticlockwise, talk in terms of sagging or hogging and I did not want you to spend time on drawing the bending moment diagram, which is easy to do.

Some of you said moment at B is 0 because M_B is 0. Now, when we said M_B is 0, we are saying the net external moment is 0. The question asks you, what is the bending moment at B, which is an internal moment. So, you must be careful in understanding the terms, bending moments are internal forces. Net moment at B is 0, obviously.

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Alternative: Solution by Force Method (takes more time!)



Compatibility equations:

$$\begin{Bmatrix} D_{10} \\ D_{11} \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} D_1 = 0.006 \\ D_2 = 0 \end{Bmatrix} \text{ m}$$

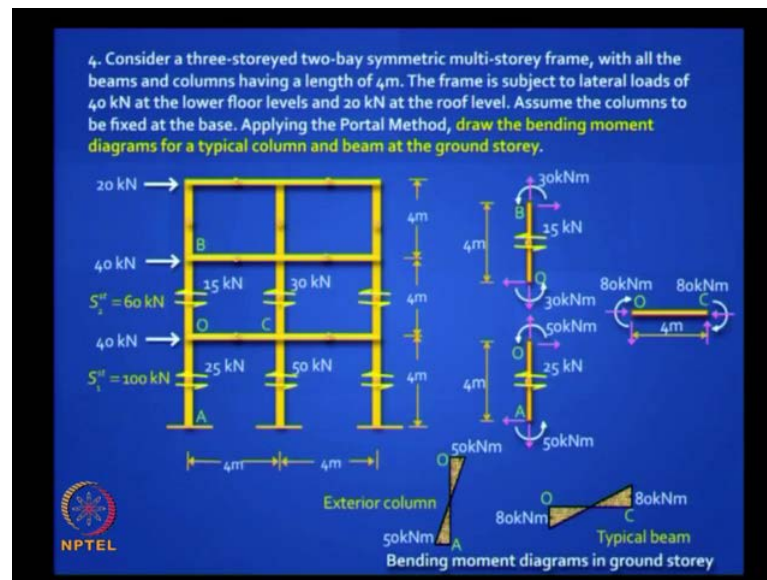
$$\Rightarrow \begin{Bmatrix} \frac{1800}{EI} \\ \frac{2880}{EI} \end{Bmatrix} + \begin{bmatrix} \frac{72}{EI} & \frac{126}{EI} \\ \frac{126}{EI} & \frac{243}{EI} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} D_1 = 0.006 \\ D_2 = 0 \end{Bmatrix} \text{ m} \Rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} +18.78 \\ -21.59 \end{Bmatrix} \text{ kN}$$

$M_B = +(21.59 \times 3.0) = +64.8 \text{ kNm (sagging)}$
 $M_C = +(21.59 \times 9.0) - (18.78 \times 6.0) - (80 \times 3.0) = -158.4 \text{ kNm (hogging)}$

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If you want to do it by this method, I discourage the use of this method for this problem when you have a powerful shortcut method. If you insist on doing it, these are the equations to write and these are the answers you will get. You will get the same answers, but it takes more time much more time.

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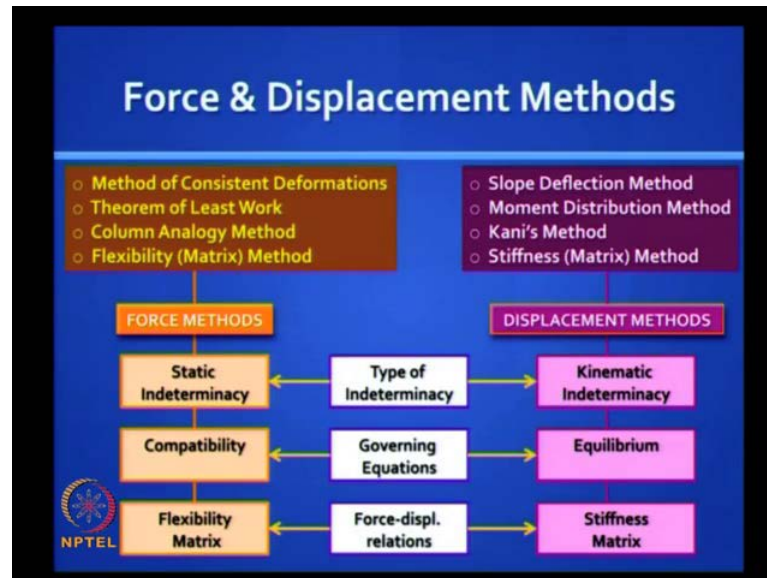
Last problem - this was the easiest problem. 3 marks. You were given three storeyed, two bay symmetric frames. This was given to you and you were asked to draw the bending moment diagram for any column in the ground floor, the grounds storey and a typical beam in the grounds storey. It should not take you more than couple of minutes, but some of you somehow got everything wrong. First of all, you have to get the storey shears. We are doing portal method; so, these are the storey shears. You do not worry about what is happening on top.

Some of you insisted on solving the entire problem and finding the actual forces in the beams. Do not waste your time and mind too. Your job is just to find two moments: one in the ground floor column and one in the ground floor beam. This is all that you need to do - find either for the exterior column or the interior column. Do you agree? The ratio of the shears resisted by exterior to interior is 1 is to 2. Some of you divided everything by 3 which is not the portal method that is your method. Some of you did this ratio, but you forgot to add up and you should add up 200 kilonewton at the base

So, you made all kinds of crazy mistakes. This is all that you have to do. So, 60 kilonewton is shared in the ratio of 15 is to 30 is to 15, it should add up to 60, 100 kilonewton, in the ground storey in the ratio 25, 50, 25. After this, it is child's play, that is it. The moment in the exterior column is 25 into 2 metres, that is 100. In the beam, you have to also look at the second shoddy column. Some of you forgot that that you need to

add it, not the entire 50 does not go here: you get something more from this 30. So, 50 plus 30 adds up to 80 but some of you wrote 50. So, that is the mistake. That is it. These are your answers; you do this and you get full marks.

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So, my friends, we have completed structural analysis - basic structural analysis. In this second part of basic structural analysis, we looked at force methods and displacement methods. We realize that in real problems you have many unknowns and you need to solve many equations. Probably there is a much better way of doing it. Why not make a computer do everything for you? That is what we are going to look at, in the subsequent classes. All I wish to point out here is we have done theorem of least work and method of consistent deformation and the traditional methods for force methods.

If you have the time you can look up the column analogy method. It is there in the book which is a very elegant way of solving some problems, especially problems with non prismatic numbers; it is a beautiful method.

The flexibility matrix method is what we are going to study in future, but most of the softwares that you get to buy in the market are based on the stiffness method and not on the flexibility method. Why is that? They must have a good reason. Flexibility method is too flexible, in the sense, if you have many redundants, you have to choose redundants and there are many ways you can choose. There is no unique choice. When you write a

program that is the problem because the computer does not want to think. It believes only in algorithms, turn left, turn right, walk forward and so on; we have a problem.

Stiffness method - everybody does it the same way because it is a stiff method. You know everybody has to do one way only and the computers love such method and that is why we do it, but for a manual operation, flexibility method really tests your thinking.

As far as the displacement methods are concerned, you now have a very good idea, hopefully of the slope deflection method and the moment distribution method. There is a third method, Kani's method which sadly did not receive the popularity it really deserves. For some reason, obscure reason shall we say, Gasper Kani, he actually found that the moment distribution method was not suitable when you have multi storey building subject to sway. So, he gave an elegant method where you have to do only one distribution including sway, whereas you have to do many distributions in moment distribution method. His name is Gasper Kani and his original paper talked about the method being applied to multi storey frame, but actually it can be applied to any frame. I do not know why this method was not popular, one of the reasons given is that he came from East Germany which was not much favored, despite the belief that all science should be neutral, should not be left or right or center, but he did not get the popularity he needed expect in Russia and somehow it has spilled over to India also.

We did not have time to cover that. It is covered in the book and we will look very closely at stiffness matrix method in the next session.

Thank you.