Advanced Structural Analysis

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Module No. # 2.9

Lecture No. # 15

Review of Basic Structural Analysis-2

Good afternoon this is lecture 15 on course in advanced structural analysis.

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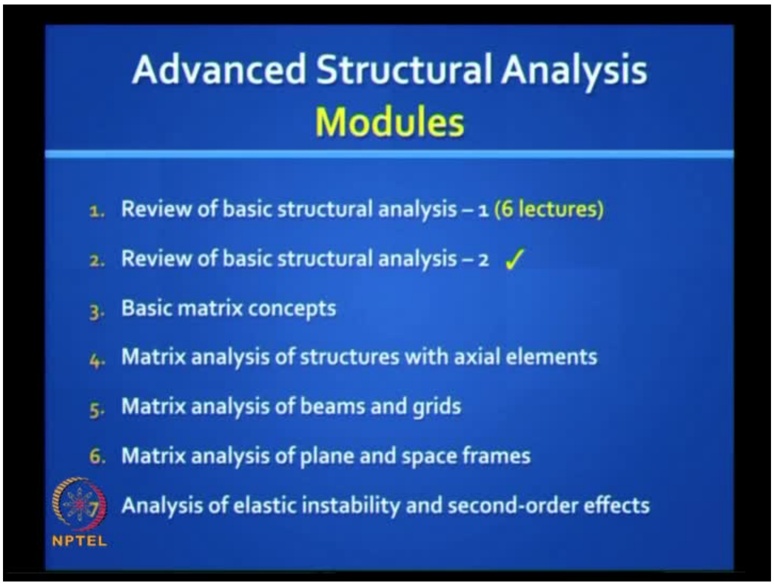

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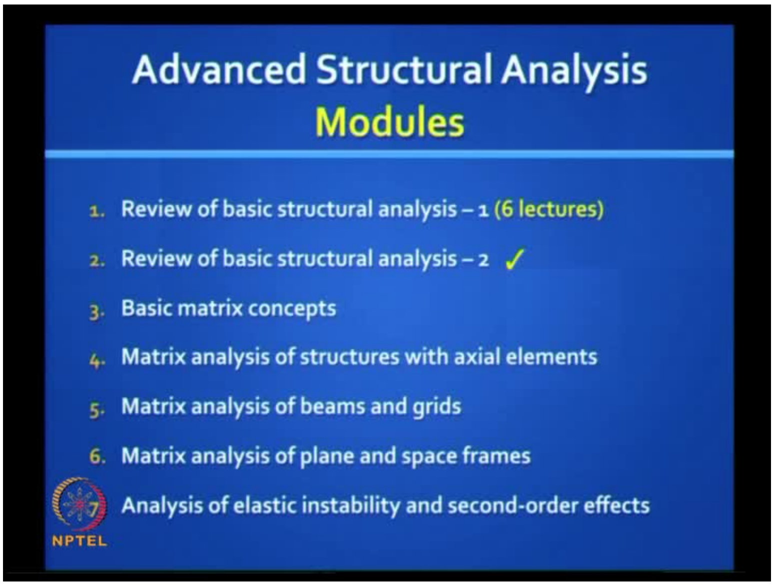

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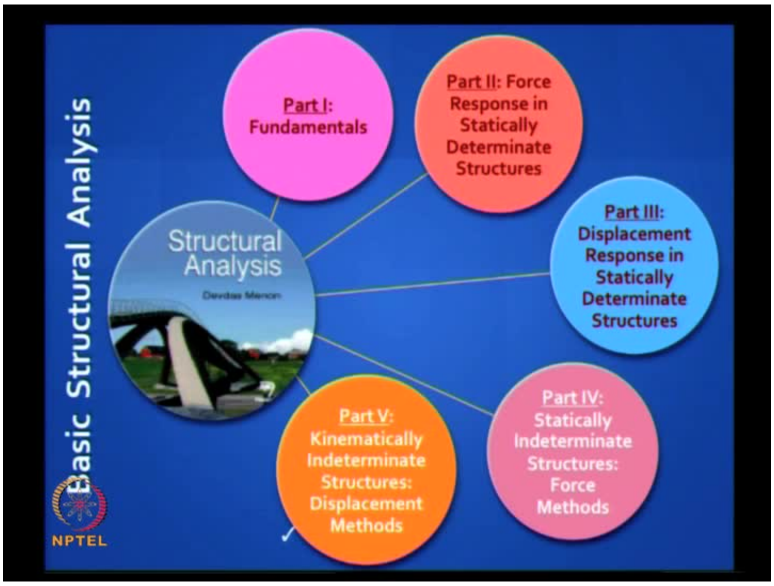

We are about to complete this topic of review of basic structural analysis two. We were looking at displacement methods and this is covered in part five in the book on basic structural analysis.

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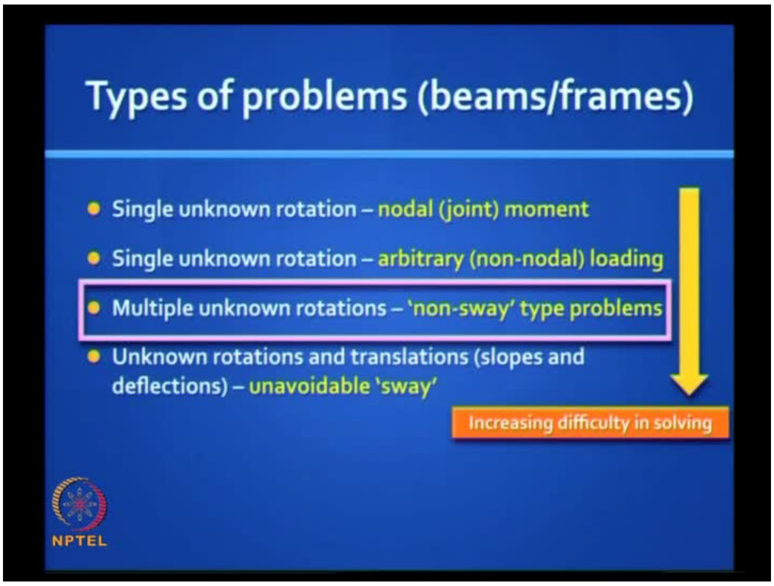

So you will see that the problems that you need to solve fall in four broad categories we finished the first two. In this session we look at how to solve problems by the displacement method when you have multiple unknown rotations but, no unknown translations that is the fourth type, which we will see later.

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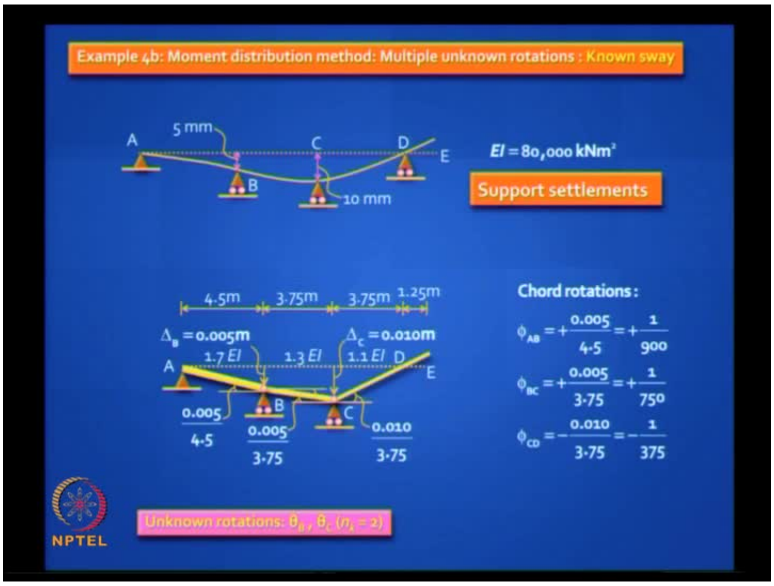

If you recall we had done this problem in the last session. This is a continuous beam subjected to relative settlement of two supports, we had solved this problem by slope deflection method. Let us see how to solve such problems by the moment distribution method.

So, the first step is to convert the relative settlements to chord rotations there are three members you can work out those chord rotations we have done this in the earlier session you will notice that the member C D is the one which undergoes an anticlockwise rotations, so you give a negative sign to that.

What is a degree of kinematic indeterminacy in this problem?

Two, I mean we can reduce it to two and the unknown rotations are at B and C, so at B and C we need to do moment distribution up till now we have done problems which needed moment distribution only at one location and you need to do it in one step and you get an exact solution but, now when you have more than one location where you do distribution it gets to be a little tricky.

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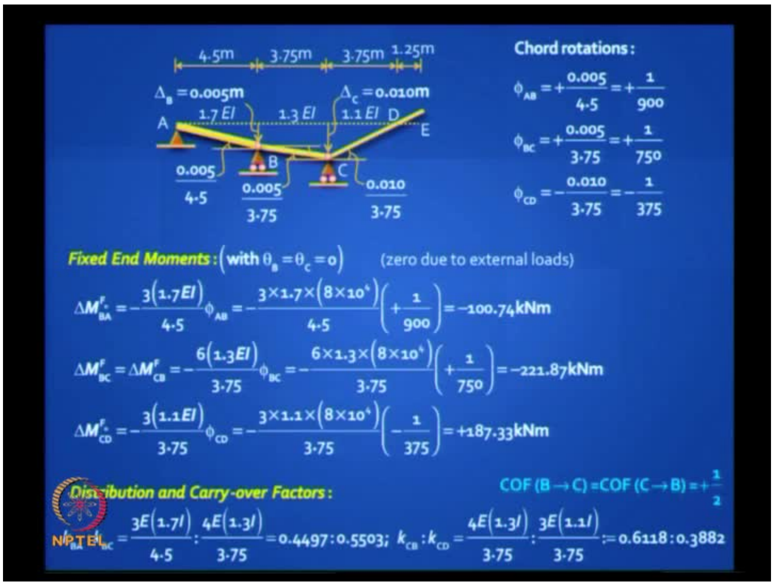

So, let us see how to do that your first job is to find fixed end moments. Are there any fixed end moments in this beam subject to support settlements?, not due to external loads but, you need to calculate the additional fixed end moments you get due to this chord rotations and I hope you remember the formulas these are easy to work out. In the slope deflection method, you do not call them fixed end moments they come in the slope deflection equations under chord rotation but, they are known quantities.

In the moment distribution method, you have to explicitly calculate these quantities and we call them additional fixed end moments right? and what is the next step in moment distribution method after you get the fixed end moments?, up to this step we have done in the last class.

What do we do next? distribution factors carry over factors shall we do that?

You have to find out distribution factors at B and C. Take B it is 3 E i by L and the i's are different for the three elements and 4 E i by L for the middle element right?, so this is straightforward you do not get clean integers here (Refer Slide Time: 02:22), so you can find them to as high and order of accuracy as you wish so here I have given four significant figures, so I hope you know how to calculate the distribution factors at joints B and C, which is the ratio of K B A to K B C and K C B to K C D there is the support at D and as far as carry over factors are concerned you do not carry over to the simple supports, so you have only a carryover from B to C and from C to B and its equal to half.

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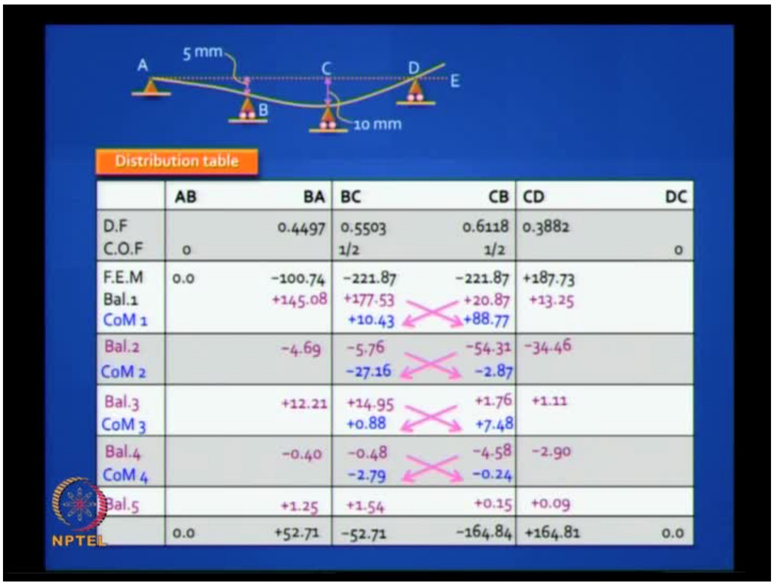

	AB	BA	BC	CB	CD	DC
D.F		0.4497	0.5503	0.6118	0.3882	
C.O.F	0		1/2	1/2		0
F.E.M	0.0	-100.74	-221.87	-221.87	+187.73	
Bal.1		+145.08	+177.53	+20.87	+13.25	
CoM 1			+10.43	+88.77		
Bal.2		-4.69	-5.76	-54.31	-34.46	
CoM 2			-27.16	-2.87		
Bal.3		+12.21	+14.95	+1.76	+1.11	
CoM 3			+0.88	+7.48		
Bal.4		-0.40	-0.48	-4.58	-2.90	
CoM 4			-2.79	-0.24		
Bal.5		+1.25	+1.54	+0.15	+0.09	
	0.0	+52.71	-52.71	-164.84	+164.81	0.0

So, this is the distribution table, as you can see from the number of lines I have drawn you do not get it in one cycle. So, how do we do this? first you write down the element numbers, write down the distribution factors please note at any joint the sum of all the distribution factors should add up to one then you write down the carry over factors and you write down the fixed end moments in this case the fixed end moments are caused by the support settlements right?

Now, let us also give with physical meaning. You have this beam in which you have arrested the joints B and C you got those fixed end moments and at the joint B wherever you arrested it you got a you got a moment, which you need to release and when you release it, it is like balancing that moment which you get by distributing the moments to the two ends. So, now you have take minus 100.74 minus 221.87 add it up to get minus some quantity and that total you have to distribute to the two ends, so how do you do that? and when you do that you also have to do a carryover then you go to the next joint and do the same thing.

Now, there are many ways it can be done and the solution procedure follows pattern technique called relaxation method you know it is a numerical way of solving simultaneous equations. If you notice the superiority of the moment distribution method over the slope deflection method is that we do not calculate theta B and theta C at all, we directly get the moments where as in the slope deflections methods you calculates theta B theta C and plug those values back into the slope deflection methods.

(Refer Slide Time: 04:27) So, here there are many ways to do it, the one that we recommend is you first do at this joint if you have your calculators you can work out it is pretty straight forward you add up the total and distribute 0.4497 to the left end and to the right end, so at this stage if you add up all the moments at that joint it should turn out to be 0 right?

Now, the suggestion is do not do the carry over now, we will do the carry over later for all the joints in one go it really does not matter because the end you have to balance, so this is what we call the first cycle of balancing and then you move over to the next joint and do exactly the same thing you will get something right? It is interesting to note in the second joint you seem to have smaller numbers to distribute in this particular instance it need not to be so for all loading now, is a time to do the carry over.

How do you do the carry over?

You can put some arrow marks to help you 50 percent of the moment gets carried over plus half, so is this self explanatory everything is clear?

Now, after you carried over at the joint B what is the unbalanced moment? It is plus 10.43, so what should you do now? again balance it, so that is what you do and you have to balance 88.77 at joint C, you can do all that in one step, is it clear? it is a simple thing you can do it manually very fast, now what do you do? (Refer Slide Time: 04:27), so this is balance number two carry over again but, as you go down, the quantities which you carry over are becoming smaller and smaller, so when do you stop?

What?

Less than some percent.

Yea, when you done enough number of cycles then you got tired. In practice you see the power of the moment distribution method is lost if you have to put in lot of effort to it, so usually in the olden days they stop to two cycles and they said we live with the error but, today we do not do that we say that stop till you can tolerate you accept as an error that unbalanced moment and that should it is left to you, it is very good if you say if it is less than 1 percent of the maximum moment some people say 5 percent.

So, you can keep on doing this and we have done four cycles here (Refer Slide Time: 04:27) and you can you should end with the balancing because if you leave an unbalanced moment and then you would not satisfy equilibrium, so we leave we stop with the fifth cycle of balancing and then we add up the total and when you add up one way to know whether you got this right to some extent is that when you add up the two moments M B A and M B C they should add up to 0, because there is no net moment there, same at C B and C D, so you got the answers but, there is a little error but, that is a negligible error right? The error comes from also in carry over you will have a little rounding of problem because as you know you have to round off to the decimal places, so but, we will leave with that.

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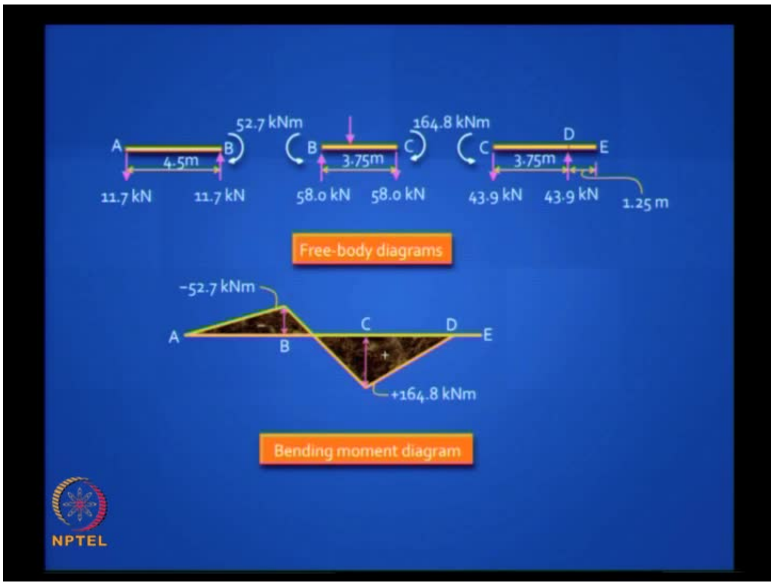

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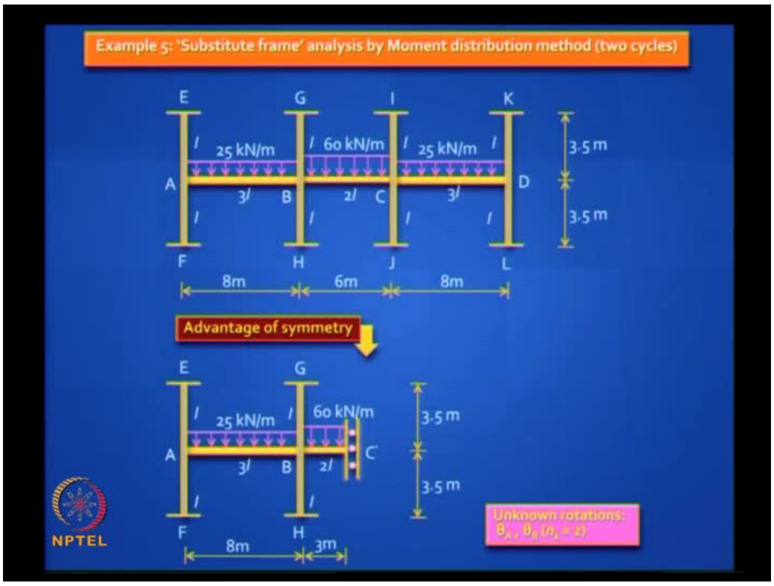

This is how you do moment distribution method when you have more than one joint is it clear? Here I have got an example of two joints, if you have three joints you do parallelly three but, then today we live in the world of computers and we say it is not worth it, so it is really worth it if you have only one member one rotation then you do it very fast and it is an instant solutions 100 percent accurate this is not that accurate compare to slope deflection method and you get exactly the same solution as you get in slope deflection method.

Now, again I want to go back historically, this is how they did multi storied frames in the olden days I mean it is too much to analyze the full frame, so we have this concept of substitute frame you take anyone floor and you assume that there is not much interaction between one floor and the next floor and you can prove this using Mueller bridge Law's principle and as long as there is no sway in the frame you can isolate one frame and assume the columns should be fixed at the top and bottom, so that is what this is for example, what you can do in a three storied a three bay multistoried frame under gravity loads not under lateral load because under lateral loads it will sway.

Now I made our job easier, because I have deliberately chosen a symmetric loading condition in which case you can take further advantage and cut it in the middle and what do you put there?

Guided fixed support

Guided fixed support okay, guided roller support and you know very well how to analyze this. (Refer Slide Time: 10:33) You have two unknown rotations there theta A and theta B and you can you need not worry about that delta C, because we know the modified stiffness how do we proceed?

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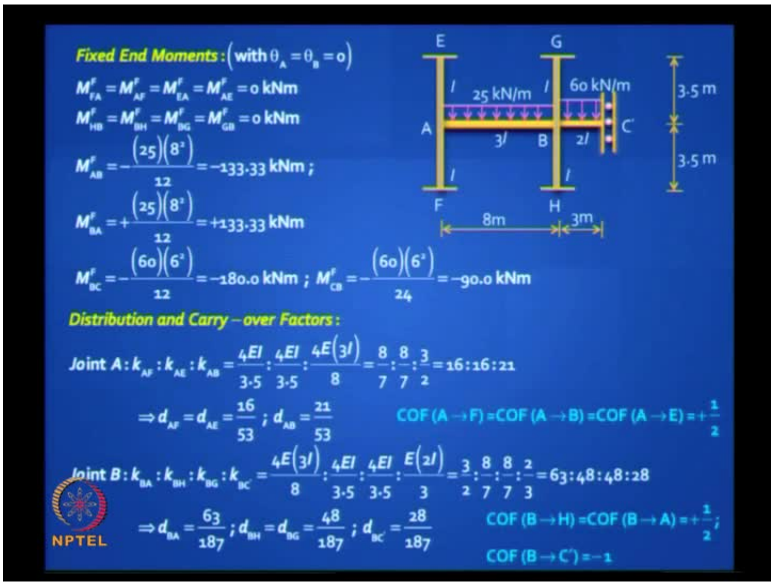

Well, first we calculate the fixed end moments for the column elements they are no fixed end moments because there is no lateral element on those columns. For the beam elements A B and B C dash I hope by know now you know how to do those calculations okay, so let us assume you can calculate these moments correctly then you need to find the rotations of the distribution and carry over factors.

So, let us pause for a while and see how to do this, see at this joint A you have three elements, so you have K A F, K A E and K A B right?, so for K A F it is 4 E i by the height which is 3.5 for K A E likewise it is 4 E i by 3.5 and for K A B,it is 3 E I, 4 E i but, the E i is, 3 E i divided by the span which is 8 are you getting it?

So, you got three members meeting at that joint, so when you make the table you have to arrange it in such a way that you can handle all of them simultaneously and the same thing you do at joint B, at joint B the only point to remember here you have four elements is that the stiffness of B C will be E i and that i is 2 i divided by i, is it clear? So, you can work this out, I want to get the concept clear. So, let us see you got this then it is easy you make your table carry over factors from B to A and A to B is plus half and from B to C dash it is minus 1, clear? Make your table.

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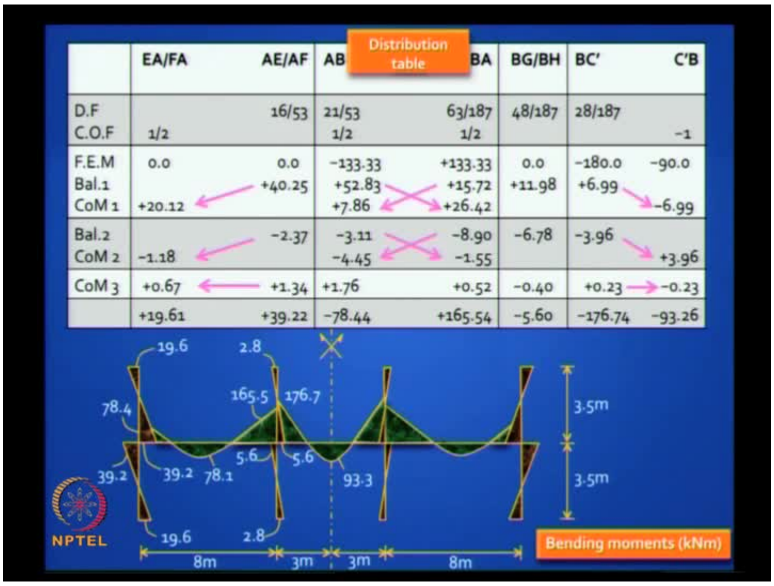

So, let us not do this, let us just see how it is done, it is exactly the same way you have to arrange your table in such a manner that you can fit in most of the quantities nobody does this anymore, so it is a little more of historical importance but, I just wanted you to know that this is how it can be done then you can draw your free bodies and bending moment diagram. This is just a demonstration of how moment distribution method was widely used to design buildings till about the 1980s or 1985 very common.

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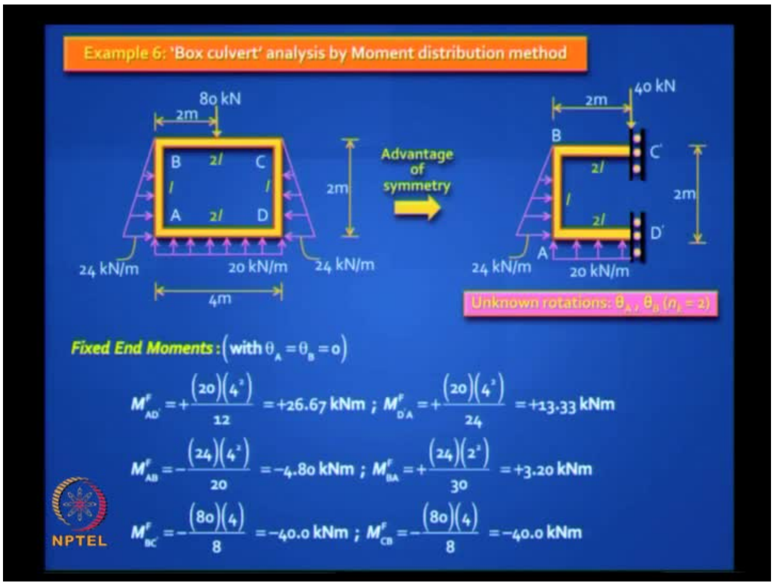

Today we do not do it because we got nice software, which do it automatically for us.

What is this structure that we see here?.

It is called a box culvert, where is it used?

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It has many purposes you can use it for drainage but, let say you got ditch in front of your house and you want to bring your car across instead of putting a slab with a separate foundation and there is a drain going below you can put a neat box like this which is very efficient very stiff and it is called a box culvert right? and if you had a concentrated load in that box culvert say due to your vehicular load coming then it will be balanced by a soil pressure from below which is conveniently assumed here to be uniformly distributed and you could have, what are those triangular pressures you get on this side? Yeah, they come from the soil and may be if there is water around there, so it is a self-equilibrating system. This is a self-equilibrating system you do not talk about supports here okay? It is self equilibrating and you need to analyze this and you can do it beautifully by either moment distribution or slope deflection method but, there is symmetry here, is not it? So, why do not we take advantage of that. When you do that it look like that divide that load 18 by 2 you get 40 right?

Now, how many what is a degree of indeterminacy here? Two, theta A and theta B so can you find out the fixed end moment? yes you know all the formulas, so let us say you know how to do all the fixed end moments of course, you are more used to beams with loads acting downward, so when the loads are acting upward you have to stand on your head in your mind then and get the directions correct it is plus clockwise is always positive and anticlockwise is always negative, so do not that is the only place where you can make a mistake.

Okay, then the distribution factors are easy to calculate I think by now, you are taking advantage of all the modified stiffnesss remember they are only three magic numbers remembers 4 E i by L is very normal 3 E i by L is if you have a hinge support at the extreme end and E i by L is when you have a guided fixed support, is it clear?

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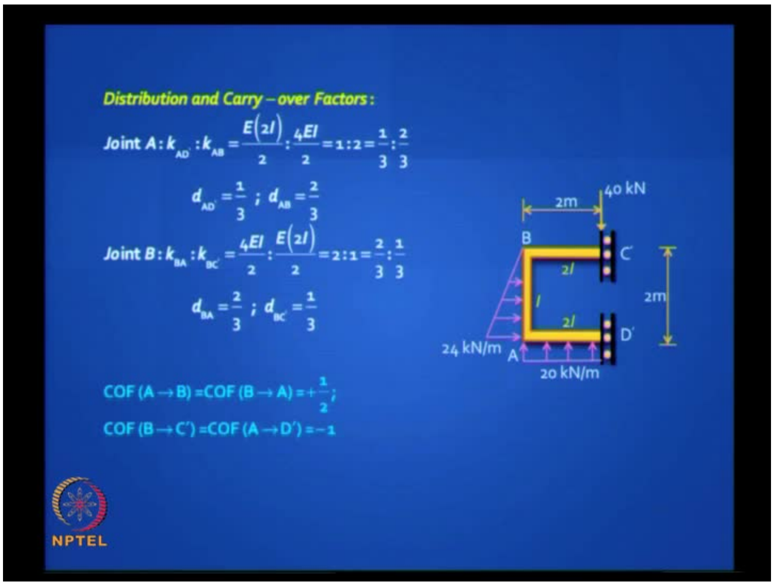

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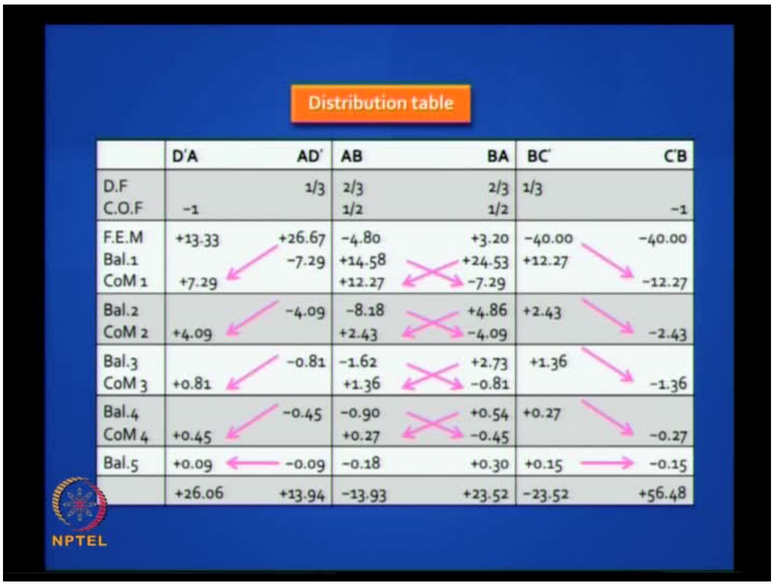

	D'A	AD'	AB	BA	BC'	C'B
D.F		1/3	2/3	2/3	1/3	
C.O.F	−1		1/2	1/2		−1
F.E.M	+13.33	+26.67	−4.80	+3.20	−40.00	−40.00
Bal.1		−7.29	+14.58	+24.53	+12.27	
CoM 1	+7.29		+12.27	−7.29		−12.27
Bal.2		−4.09	−8.18	+4.86	+2.43	
CoM 2	+4.09		+2.43	−4.09		−2.43
Bal.3		−0.81	−1.62	+2.73	+1.36	
CoM 3	+0.81		+1.36	−0.81		−1.36
Bal.4		−0.45	−0.90	+0.54	+0.27	
CoM 4	+0.45		+0.27	−0.45		−0.27
Bal.5	+0.09	−0.09	−0.18	+0.30	+0.15	−0.15
	+26.06	+13.94	−13.93	+23.52	−23.52	+56.48

So, you can work out these and you can work out your carry over factors do your table.

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Sorry.

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Soil pressure when it acts on the retaining wall have you gone through a geotechnical course you are going through, so you will know that. Take water if it is water is hydrostatic if it is water Pascal's law says that the pressure is the same in all directions if it soil it is not the same, so the lateral pressure it is called the active worth pressure is less than the vertical pressure how much less depends on the soil properties, so if you have sandy soil it is roughly around one-third, so whatever vertical load you applying one-third of that pressure goes, so it is a linear variation it is conveniently assume this is ran kinds soil pressure profile but, do not in structural analysis we say somebody is find out those loads and given it to you and you but, in real life you need to work out those loads yourself, so that is why you have to ask those question okay.

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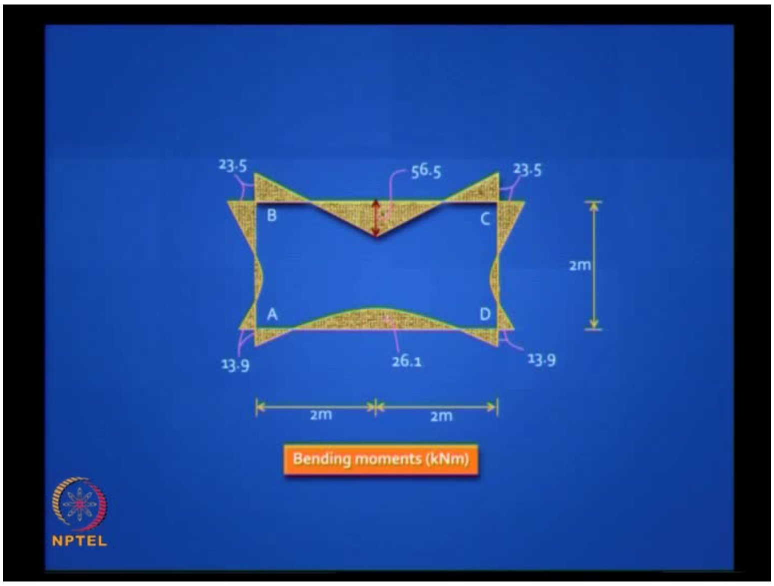

Then you get a beautiful symmetric, symmetric across to planes no across one plane diagram okay. It is very fast so the structure looks little formidable when you saw it in the beginning but, now you see it is quite easy to handle okay.

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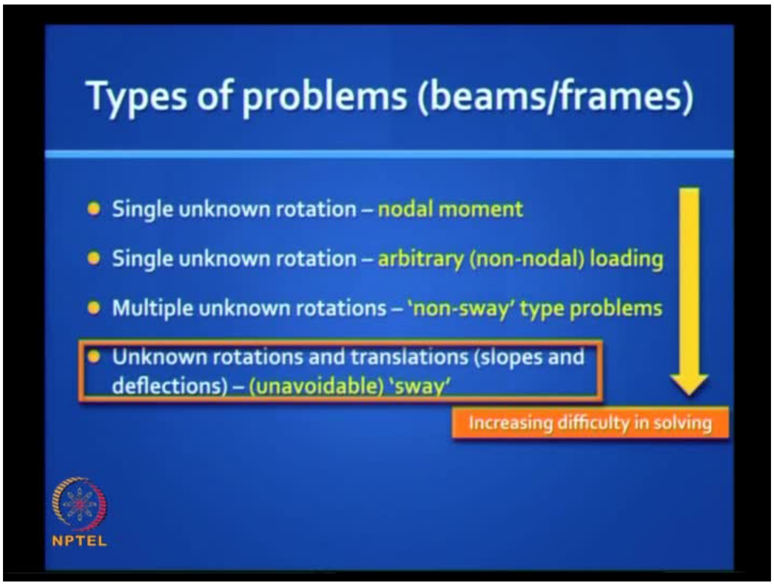

So, the idea is for you to know that such things can be done you done the assignments but, beyond that we are going to do everything automatically using matrix methods, so it is it is important know that this is how many structures were analyzed for many years.

Now, we go to the fourth category, the toughest that is when you have sway which you cannot get rid of it is unknown it is not like a support settlement which is known the chord rotations are unknown in some cases you can take advantage of them. We will look at those cases in tomorrow's class for the time being, let us look at how to deal with this.

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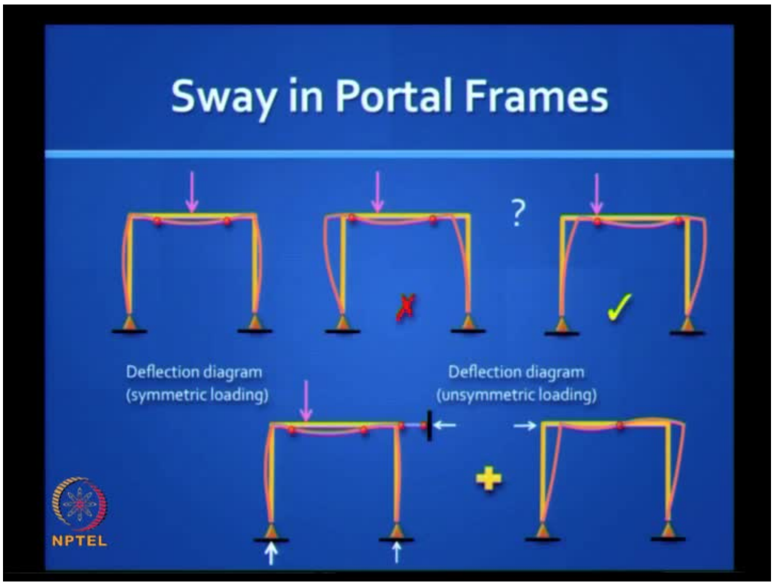

So, if you have a symmetric portal frame with the symmetric loading what is a deflected shape? will it be also symmetric? Yes, it will be symmetric. If the loading is slightly eccentric you have an a symmetric loading then the question I want to ask you is, is it going to sway to the lefts or is it going to sway to the right, what is your hunch?

To the right side.

Did we discussed this earlier?

Okay, now what you need to do to get it intuitively right is keep pushing the load to the extreme length and press it down hard and your mind will tell you which way is going to move, but you can also give a very beautiful analytical way of explaining why it is going sway to the right. Let us look at that, let us say you do moment distribution of this frame okay, you have two joints where you have unknown rotations you can do it right?

Will you get the correct answer? will you get the correct answer you will get some moments but, you will have a problem with the answers you get. What is the problem that you get? that is the problem you get in moment distribution method not with slope deflection method. What is a degree of indeterminacy of such a frame?

Two.

But, there is a sway, so it is 3, so you cannot wish a way that sway degree of freedom, so the moment distribution method the way we have learnt till now where you have unknown rotations is not able to handle sway, so if you do by moment distribution method you are actually doing it for a braced frame which means without your knowledge you are not leading it sway that means there is a restraints there, right? So, you get something from the distribution table but, the frame you are analyzing is not the original frame but, a frame in which someone has arrested the sway degree of freedom right? and then your results will be correct for that problem.

Now, where do you think the moments will be more, in the left column or the right column? (Refer Slide Time: 19:40).

Let us take the beam. If it is a fixed beam where will the fixed end moments be more, left or right? Left, so do not you agree that the it is not fully fixed it is partially fixed, so those moments get affected and whatever moment you get in the beam end will be passed on equal and opposite to the column end, so do not you realize that the column on the left side is likely to have more bending moment at the top than on the right side, right? and the moment at the bottom is 0, because it is a hinge at the bottom.

Now, if you will you have a horizontal force at the bottom? Yes. What will be the value of that force? it will be the moment at the top of the column divided by the height of the column, so you will get two horizontal forces, which one will be more or will they be equal?

Sir they are equal.

Why do they have to be equal?

In opposite

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Yes, right.

So to satisfy equilibrium where will that force have to go?

Hinge that hinge. That artificial.

Artificial support.

That is a key to it you are right. So, you are if you do moment distribution method you would not be able to satisfy equilibrium because your horizontal forces are you will find you need a net horizontal force the way I have shown you and then you have to do another analysis, which is called a pure sway analysis where you have a horizontal load acting, so this is a problem with moment distribution method that is you need to do two distributions, one with the sway arrested and the other you have to figure out how much that sway then do that analysis, so it is not worth it and so, we would not follow moment distribution method beyond this point. We will say if you have such problems you can do it but, we grown up and we do not do it that way okay and here you can clearly see why it is going to sway to the right because the net load is going to act in that direction you have to reverse the load acting on your artificial restrain this is a beautiful powerful left brain understanding of why the sway is always to the right okay.

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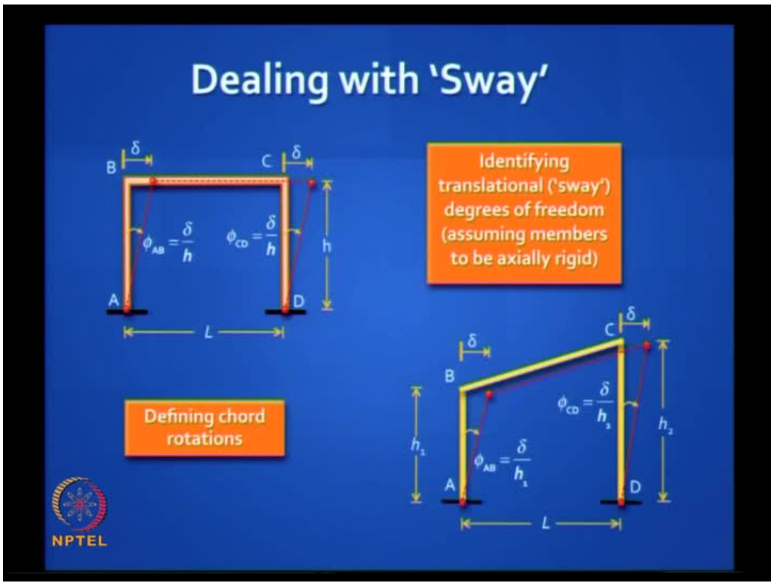

Now, let us say how to deal with sway. I have suggested that do not deal directly with translation always deal with chord rotation, because they are much easier to deal with. So, you will have sway degrees of freedom in a frame like this to figure out how many sway degrees of freedom you need to have you? You conveniently put hinges at all the junctions and let it sway like a mechanism then you will find this there is one unknown degree of freedom here that is delta, is it clear?

So, the other way to do it is at every joint in a plane frame you have two translation degrees of freedom any two orthogonal directions you can take vertical and horizontal, so you have at A and D both are arrested, so you have four potential degrees of freedom at B and C agreed? four translation degrees of freedom but, if you assume members to be axially rigid you are bringing in as many constraints as there are members. So, you have three constraints because three lengths remain unchanged the length of A B, the length of B C and the length of C D, so that is a clever way of figuring out how many independent sway degrees of freedom you have four potential translations degrees of freedom three constraints constraining equations because of the length 4 minus 3 is equal to 1, got it? and you can choose anyone in this case delta. And then you work out your chord rotations if delta is to the right positive then the chord rotation phi A B is delta by H and C D is also delta by H.

What happens if you if your roof is tilted? Okay, it is called a bent portal frame then also you have three members four potential degrees of freedom 4 minus 3 1 then also you have only one unknown and it is delta both will move both will move parallelly both the column will move parallelly as shown here (Refer Slide Time: 24:33) and what is interesting is the beam does not have any chord rotation whether the beam is horizontal or inclined there is no chord rotation, so you will have sway you have chord rotation only in the vertical elements and the chord rotation A B is delta by H 1 the chord rotation in C D is delta by H 4, got it? So, we are going to increase the complexity now.

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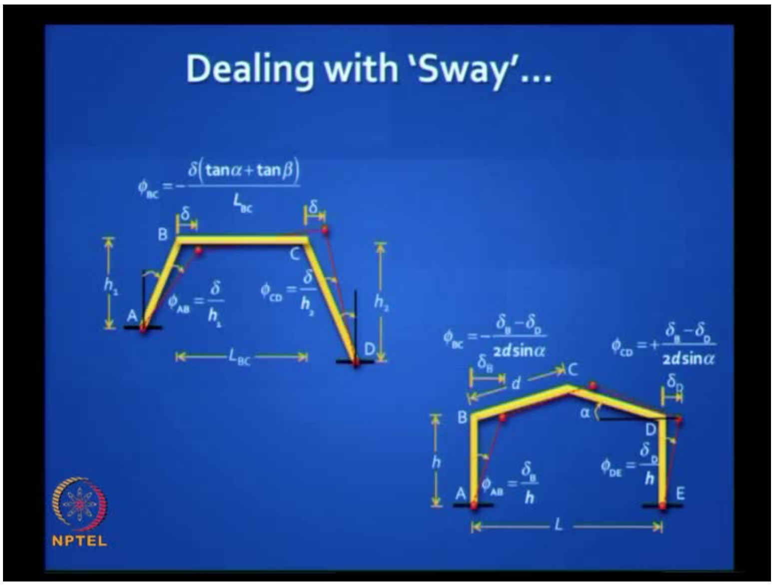

What do you do when you have frame like this? when we will solve this problem? Tomorrow. When do you encounter frames like this? Well you got a house in a hilly sites banjara hills in Hyderabad may be and you are went off from the first floor here and the ground floor there and your architect wants a fancy design with incline walls, so you can have a frame like that, you know your foundation are at two different levels, right? Then also you should remember even though it look complicate and this is a very interesting problem you do not need to solve it but, at least conceptually you should know how to solve it.

You still have only one sway degree of freedom, why? Because four potential translations at B and C, three members, three constraining equations 4 minus 3 1. You can still choose delta but, now your beam is going to have a chord rotation because if you play it with it like as you play with the MeCcano's set it is a mechanism, if you play with it if the if these are rigid members it will take a shape like this, agreed? And chord rotations for A B is simple it is phi A B, phi A B is delta by H 1 for C D also it simple it is delta by H 2 but, it is a little tricky when you calculate you can use trigonometry and calculate and that is the formula that you get.

I would like you to you know this these book are available for you but, that is how you do it, this is how it was done in the olden days in slope deflection method you do not have to worry at all about these things you can do it directly. In the matrix method also its done directly.

If you have a pitch portal frame you can similarly, workout those equations do not worry too much about it.

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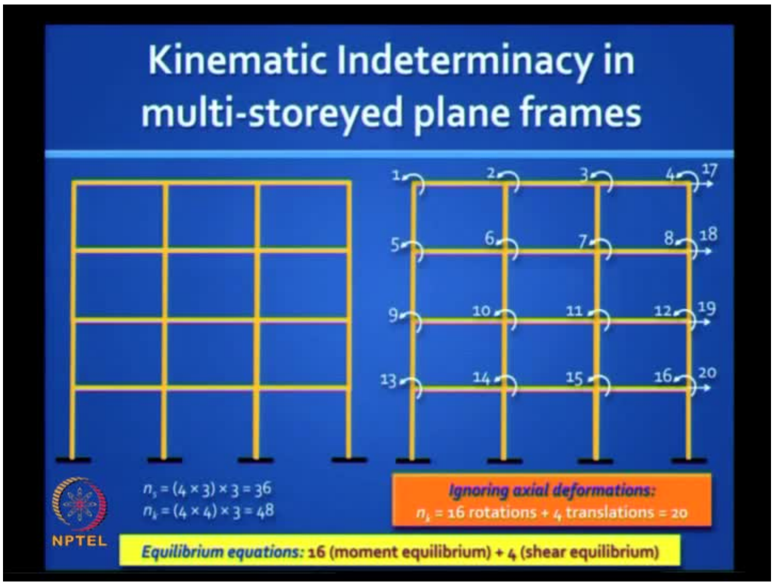

Normally, our buildings do not look that funny, you do not have inclined column you do not have inclined beam you have frames which are called reticulated frames made up of rectangles where your life is made easier and if you say axial deformations are negligible, then you have only one rotational dig unknown at every joint and one sway degree in every floor, right.

Why only one sway degree in every floor? Because all the joints in that floor will move like a rigid body okay, and there are serviceability limitations in a tall building the maximum drift in the building should be limited so usually it is height by 500 under lateral loads, okay?

Can you analyze the frame like this exact solution? (Refer Slide Time: 29:29)

Yes, that is the idea we want to be able to handle any structure any skeletal structure of any shape subject to any loading including indirect loading that is the objective of structural analysis.

Okay now, in slope deflection equations till now we were conveniently having only rotational unknowns and the moment equilibrium equations were M B A plus M B C is equal to 0 kind of equations. Now, you have a translational unknown, so moment equation is not going to work here so you need a force equilibrium equation it is called a shear equilibrium equation. Let us demonstrate that.

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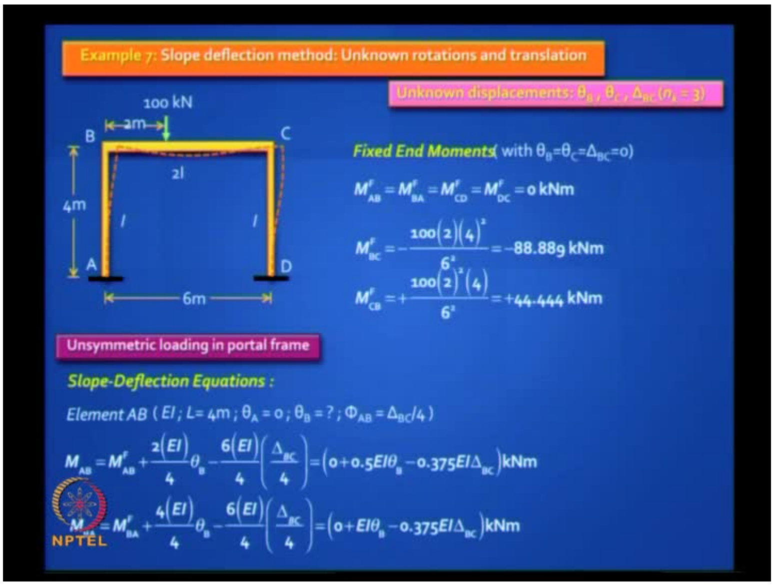

Let us take this example this example okay, where you have a portal frame with an a symmetric load un-symmetric load, right?. First identify what is the what are your unknown displacements, theta B, theta C and you can call it just delta you can call it delta B C because every point in that beam is going to translate by the same delta, so theta B, theta C delta B C fixed end moments well your columns do not have any fixed end moments but, your beam does and you can easily calculate that, right?, you can calculate a fixed end moments at the beam, right? Minus W A B squared by L squared plus W A squared b by L square fine.

Now, what do we do next? Slope deflection method. What do we do next? you have to write down the slope deflection equations chord okay, rotations are clear here, right, so what will they look like? will you try, write down the slope deflection equations for the three members A B, B C and C D, so how many equations to you need to write down six equations six equations and the you can write them mechanically of the six equations, in the six equations you have to put them in terms of three unknown.

What are the three unknowns?

(( ))

Well, instead of phi you say delta, delta by 4 is phi, right?

So, let us demonstrate for A B, do you agree with this? M A B is M F A B plus 4 E i by L of theta A is gone because theta A is, so you are left with 2 E i by L of theta B minus 6 E i by L into phi, phi is delta by 4, is it clear? Likewise you can write for M B A, M B A will be M F B A plus 4 E i by L theta B, minus 6 E i by L into phi, right?

Theta C, where is it? It is outside A B, so do not worry, right? Theta c is not same thing in a continuous beam, right? you had theta B and theta C when you are dealing with this beam, C was far away, so you do not have to worry about it, is it clear?

If you have open them out it will look like continuous beam you are writing down same equations. Is there any doubt on this? In the same manner you can write for B C in B C will there be any sway any chord rotation?, No.

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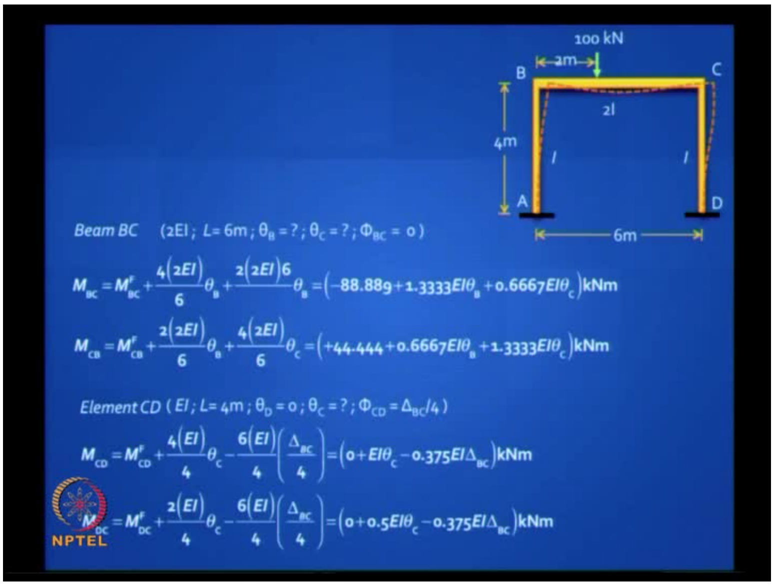

So, for B C it is a straightforward thing fixed end moment you can write down in simple 4 E i by L theta B in 2 E i by l theta C, is it clear? For M B A, for M B C, and for M C B straightforward and for M C D and D C, they will look similar to M A B and M B A, right?

So, actually you have to just copy those equations only thing your theta B gets replaced now by theta C but, you have to put the right theta at the right place. I hope you know now understand that slope deflection equations are a blind method of doing it you do not have to worry about anything you get this.

Now, what is the next step?

(( ))

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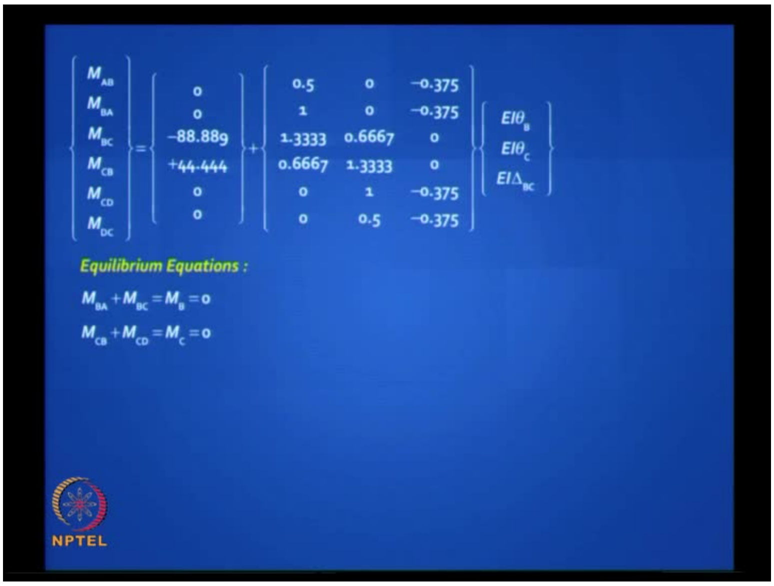

Write down the. So, you need three equations, two of them are very easy, so corresponding to okay, I can summarize a nice rightly in writed nicely in this matrix form, so you look carefully here these are my six moments, right? three pairs three elements these are my fixed end moments only the beam has fixed end moments the columns do not have fixed end moments these are my coefficients related to E i theta B, E i delta C and E i delta B C which comes from either 4 E i L or 2 E i by L or 6 E i by L square, right? And if somehow I get the answers for theta E i theta B E i theta C E i delta then I just plug it in I get the final answers, very simple, is it clear?

Now the big question is how do I get those three equations? Of two of them are straight forward we have done it in continuous beams. There is no net moment at B, no net moment at C you pick up the second and third rows and just add it up you will get two equations.

Where you get third equation?

Please tell me how to write, how to write the third equation?

The clue is you have to write it in terms of these moments. How will you write?

(( ))

No, no.

M A equal to M A B equal to.

(( ))

You tell me yeah this is your structure. Tell me what to do?

(( ))

Vertical reactions.

You will be able to write the reaction in terms of M B C and M C B.

Sir M A B is equal to M D C

How does that help you?

I will give a clue; no I will give a clue.

Sir M A B is equal to M D C, M A B is equal to M D C D C who told?

Horizontal force at B and C those moments.

Horizontal force is different from moment.

That that into this.

I will give you a clue. See you wrote M B A plus M B C equals 0, because there is a B common there, because theta B was the corresponding unknown rotation.

Now, you have delta B C you have a sway degree of freedom, you have to write a force equilibrium equation, right? not a moment equilibrium equation, which force what equilibrium?

The shear force in column A B.

The shear force in column A B, is it a constant shear force? is it a constant shear force is it a constant shear force? Yes.

Yes.

Because you have a column, if you take the free body of the column you have a moment only at the top and the bottom there is no intermediate load and let us say they are clockwise positive you have M B A and you have M A B, so your shear force is?

(( ))

Well, there are both positives, so where is a difference?

2 M B A by the 4.

You add them M A B plus M B A divide by H will give you a shear force well to make your life easier, why do not you look at it as horizontal reaction at A and D? What should those two reactions add up to? see easiest thing you can do. (Refer Slide Time: 34:28)

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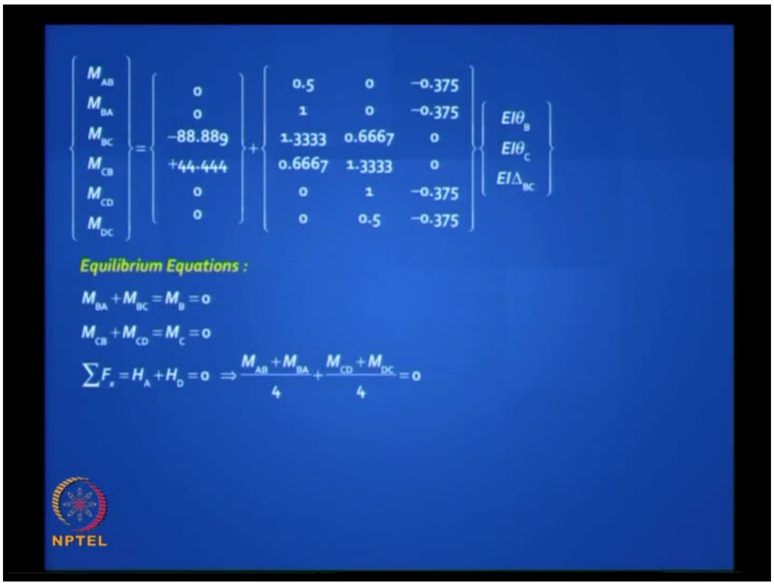

So, you do not break your head too much over it that is all you need to do. Horizontal reaction at A and D should add up to 0 and for the sign convention we have assumed M

A B plus M B A dived by 4 clockwise, where is that reaction pointing from left to right or right to left?

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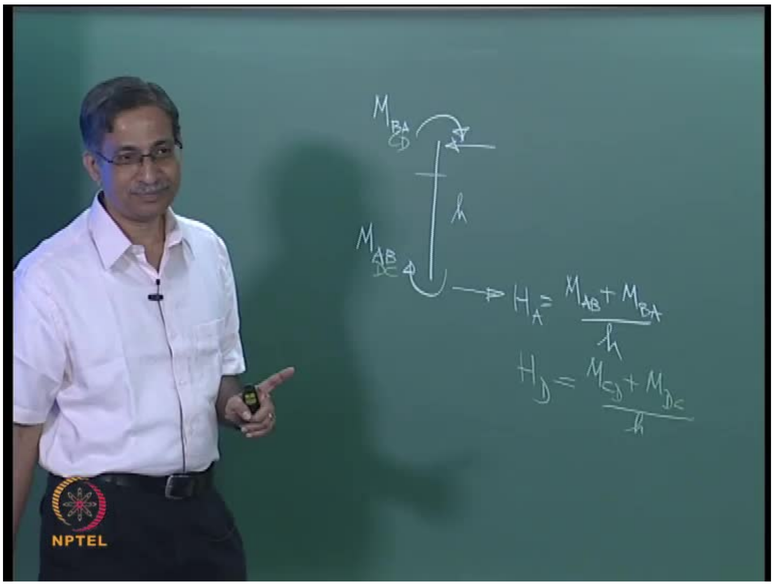

This is, so for equilibrium this will point this way and this will point this way. This is what we called H A and it will be, right? And this will be equal to this end wherever you cut a section the shear force is constant. What happens if you have instead of A B you have D C and C D? well the only change is this A becomes B and you get, so this is the thinking you need to do and now you got expression of M A B, M B A from your slope deflection equation plug it into that equation and you got three equations.

So, George Maney discovered this way back 100 years back powerful method you get actually the last equation you have to play a little bit play bit with it to get the symmetry in that matrix, okay.

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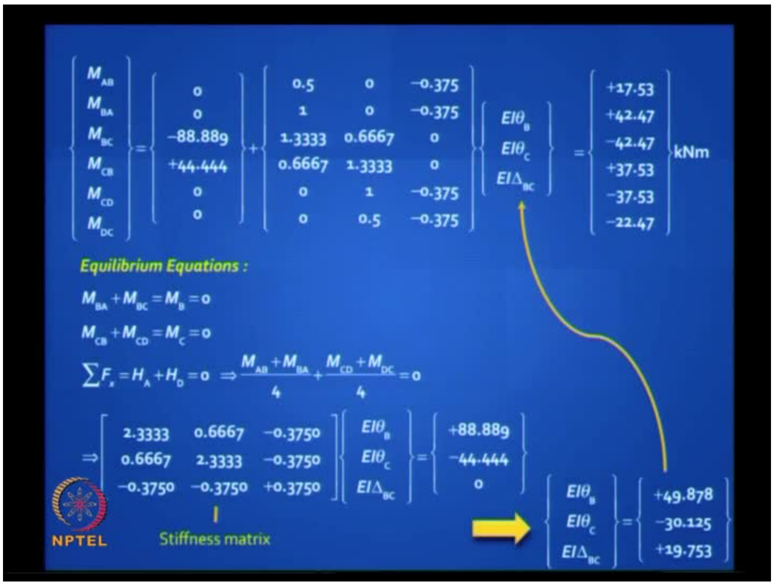

Actually, they do not turned out to be the way it is, I have shown you because there is a minus sign you have to multiply that row with minus 1 you will get the symmetric, okay? Now, what is this matrix look like? what is what is that matrix? Where displacement method or stiffness methods not flexibility method that matrix is a property of the structure.

What is that what is that matrix called?, Well it is related to the stiffness matrix and the beautiful part about this is (Refer Slide Time: 40:41) the vector that you get here is caused by the loads that left side is completely a property of the structure, you change the loads only this will change this will not change and, so we will see how you can generate that stiffness matrix from first principles I am stepping ahead in to into matrix methods but, I want us to go through that exercise.

We got this blindly by just applying equilibrium equation, right? but, with our eyes open can we generate this matrix. Let us do that than you have a real understanding of the physics behind the mathematics okay.

So, you can solve these equations and a now days you calculators which do it you know just by pressing some button so I will not waste time, you will get these answers okay?, and then what should you do? Plug them back into the slope deflection equations you got the answers, so after you got the end moments, what should you do? You draw free body diagrams, I hope you know how to do that.

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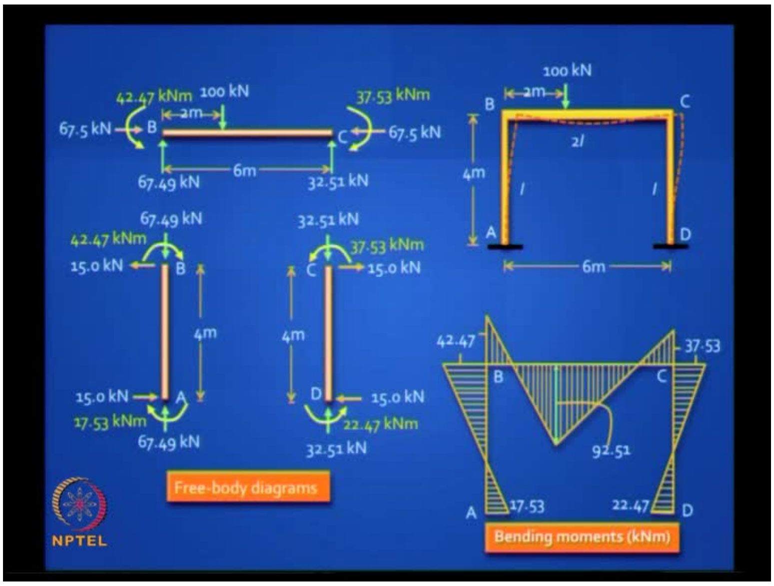

Draw the free body diagrams clockwise moment put clockwise negative sign move put anticlockwise and then everything should match all the equilibrium should be satisfied you can now find the vertical reactions in the beam and the horizontal reactions in the column and you will find that this perfect equilibrium everything is balanced you got the exact solution powerful method, agreed? Then you can draw the bending moment diagram it will look like that. So, you not afraid of sway you know how to deal with sway but, if you have multiple sway degrees of freedom you have to do it little more carefully.

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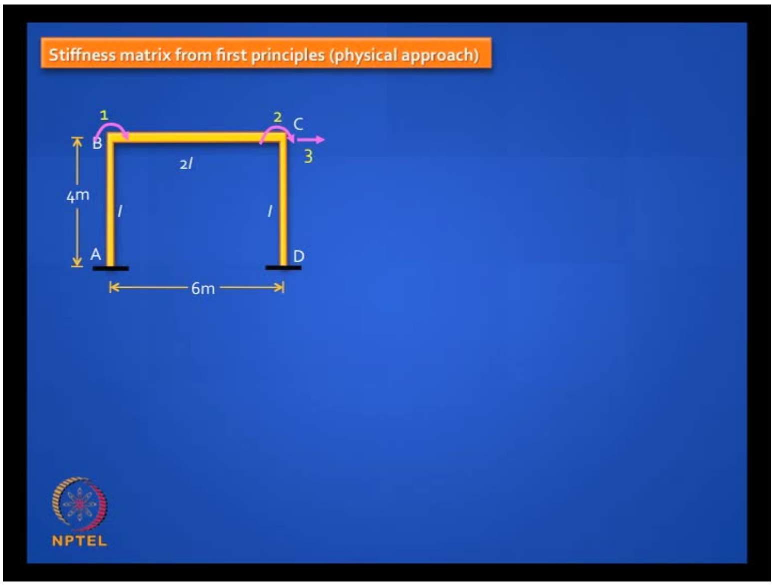

Now, I want ask to go through this small exercise of generating the stiffness matrix of this frame from first principle.

Shall be give it a trail okay?

You will love it, it is simple. How many degrees of freedom are there?

Three, shall we number them 1, 2, 3 Theta B is our first degree of freedom theta C is our second degree of freedom and delta B C is our third degree of freedom. Okay, to write to generate the stiffness matrix of this what should I do, what is the size of the, what is the order of the matrix?

3 by 3

3 by 3, what should I do? From first principle.

Give a unit displacement at that location.

Give a unit displacement; let us say I want to fill the first row. First column of the stiffness matrix what should I do?

First I should arrest everything.

Yes sir.

Rotate (( ))

And then I should allow only.

Unit displacement at.

One displacement at a time, which one?.

First one.

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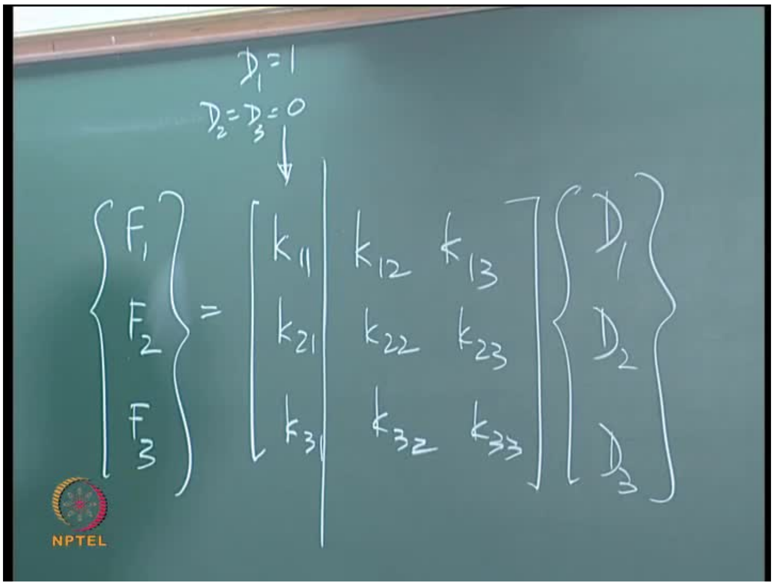

So, shall we call it D 1, D 1 should be 1, D 2 should be 0. Can you sketch the deflected shape of such a frame? Do it. Sketch the deflected shape we are trying to find the first column, so to fill this column we need to put right draw a sketch, what will it look like? it should look like this, right? It should look like this

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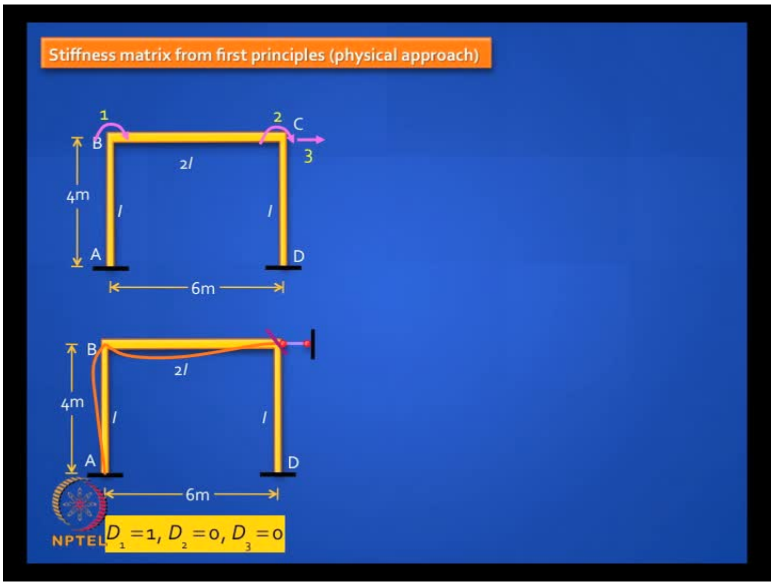

Only a unit rotation at B, no translation, no rotation at C, right? Okay.

Can you draw the next one can you draw the sketch that you need to do for this one? What will it look like?

Okay, let us look at this. (Refer Slide Time: 44:40) What is the physical meaning of K 1, 1.

(( ))

So, when you see when you hear of stiffness you are reminded of a spring, spring constant, right? So, it is a stiffness, so spring stiffness is force per unit deflection here you have two coordinates, so it is a force at the coordinate one caused by unit displacement at the coordinate J with all other coordinates arrested

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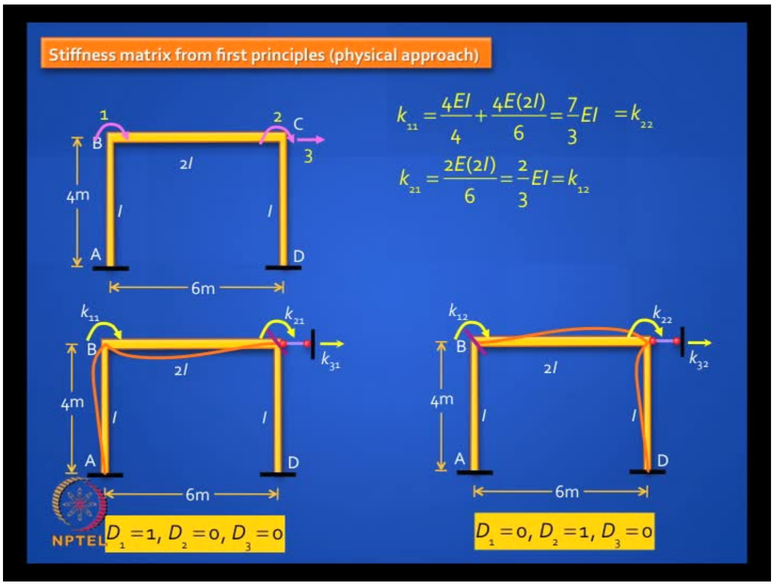

So, here only theta one only D one is there and, so one way to do it is you remove that restraint then you deliberately give an external moment that moment you need to apply to make the joint rotate by a unit angle is K 1 and the reaction that develops at the other artificially fixed end at B is K 2 1 and the reaction that is developed in the direction of the coordinate you identified in the translation degree of freedom is called K 3 1.

Can you calculate these quantities?

Yes sir.

Do it.

Maybe you let us draw this also the next case which is the similar your job now is to calculate K 1 1. What is k 1 1?

Yes.

8 E i by L.

No, wrong.

Let us do it slowly. It is going to be 4 E i by, what is L? 4 E i 4 plus?

4 in to 2 6

(( ))

6 E i by L

These are the other notations, agreed? Right now, we need not do this is equal to what from the other figure? K 1 1 will be equal to K 2 2, is it not? So we need not spend more time on that K 1 1 and K 2 2 will be identical. Can you find out K 1 2 or K 2 1, K 2 1 is how much? it is a carryover moment you get.

2 by 2 E i.

2 by 2 E i.

Which one?

2 by 2 E i.

Wait one minute; do you have a problem with this addition?

(( ))

1 plus 4 E i.

(( ))

7 by 3.

7 by 3 that is right.

7 by 3 you are right well caught, 7 by 3 agreed, 7 by 3 okay.

This one? This is correct?

Yes sir.

Okay, it is 7 by 2 not 8 by 3 you are right, good. This is equal to K 1 2, so there itself you see the symmetry in the matrix. What is left over here? K 3 1, can you find out what k 3 1? That is a clue that is a clue that is a clue,

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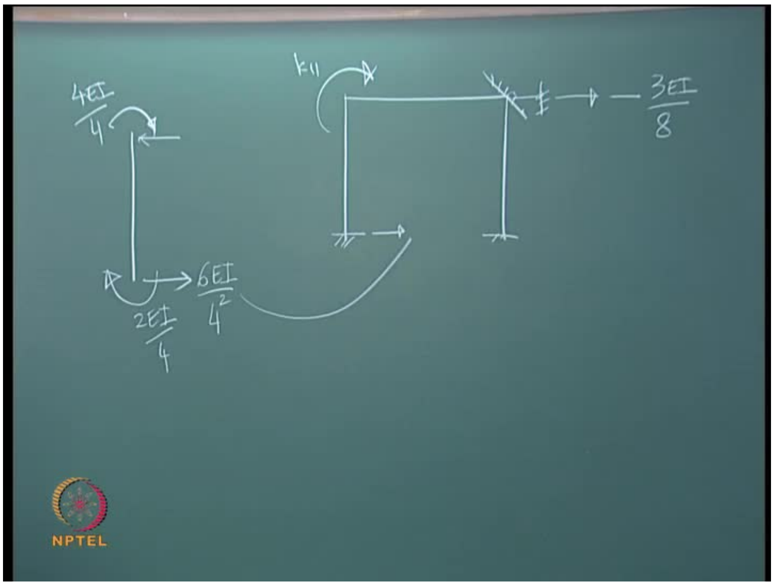

So, you had here if you take that column you had how much? 4 E i 4 and what do you get here? 2 E i by 4 and this will give you shears which are going to be equal to 6 E i by 4 squared and so if you take that frame in which you arrested this degree of freedom and arrested this degree of freedom and you applied K 1 1 equilibrium suggest that you will definitely get horizontal reaction here which is equal to this quantity and you do not get anything here because this does not bend, so the reaction you get here will be equal to how much?

(( ))

No.

Minus.

Minus, because they should add up to 0, so minus how much?

6 E i by 16

6 E i by 16 what (( )) to?

3 E i by (( )).

That is it. This is the physical approach now it is easy now it is easy, right? This will be equal to K 3 2 in the second figure.

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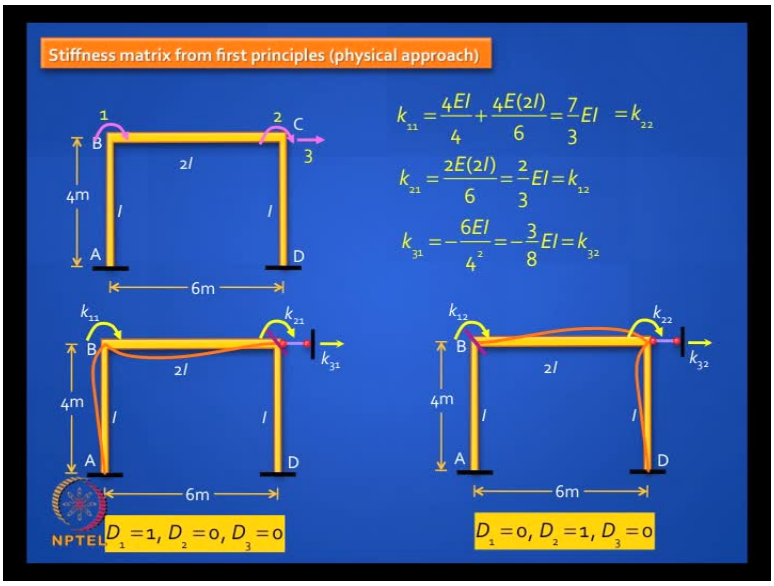

So, we have got the first two columns very easily last column can you draw the sketch.

Infinite rigidity of.

That is right.

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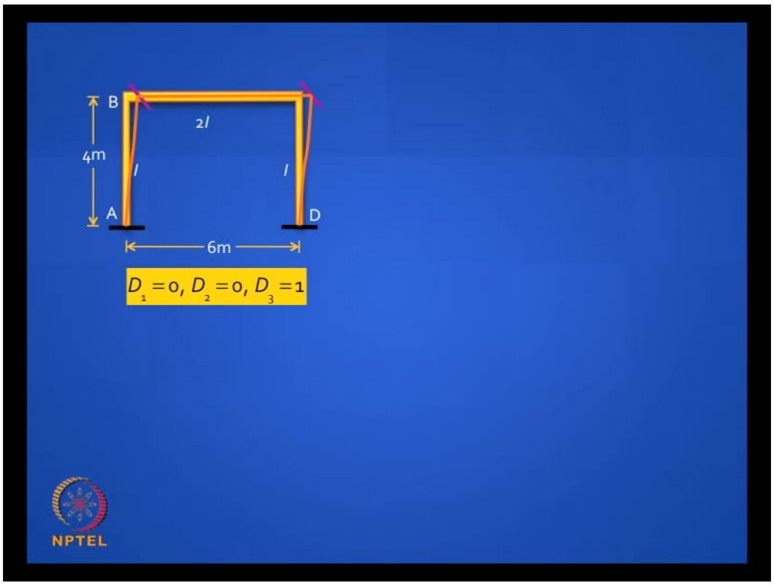

It is like a rigid beam this is how it is going to move? Point of contra flexure will be in the mid height of every column, so what will it look like? Can you calculate? can you calculate K 1 3 and K 2 3, it likes settlement of supports it likes settlement of supports it is a chord rotation, so what is the moment that you get? K 1 2 and K 2 3.

K 3.

6 E i by 4.

Sir K 3 into L by (( )).

Into 1 by 4 because your chord rotation is 1 by 4, so remember this remember this.

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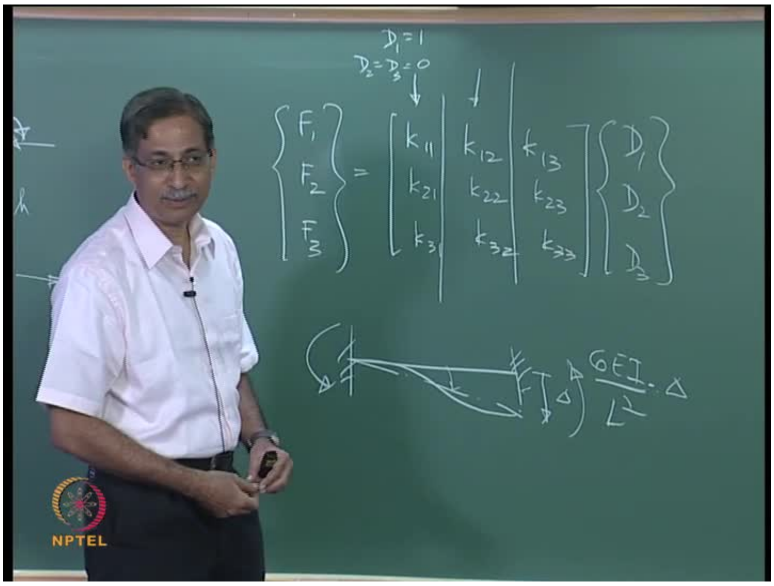

If I have settlement of supports problem and this is delta this is the shape that you get clockwise chord rotation will give you anticlockwise end moments. What are these moments? well 6 now do not talk about 5 it will come to 6 E i l square by delta, because delta by L is 5 do not make mistakes like that you lose everything.

Okay, so that is easy now, you got it? And there is a minus sign clockwise chord rotation will give you anticlockwise end moments, got it?

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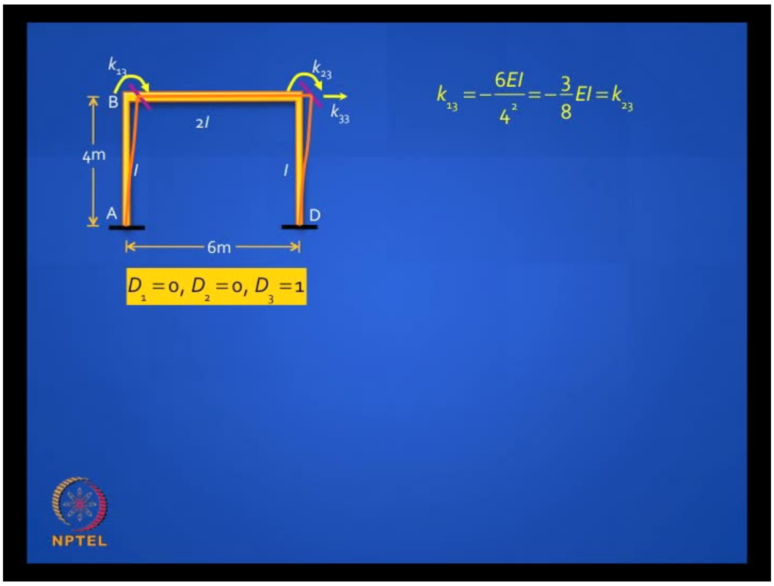

So, you got K 1 3, you K 2 3, you got everything in this matrix except this last fellow. What is k 3 3? will it be positive or negative?

Do you need to think to answer this all the diagonal elements have to be positive why?

Displacement caused it.

You what it mean? I am pushing it is to the right obviously it is going to deflect to the right, if kick a football to the front, it is not going to come back, do you understand?, so that is what all the diagonal elements are doing, I am pushing it in the in the same direction as identified positive degree of freedom, so it has to be positive, do not break your head over it.

Okay, the half diagonal elements can be negative or positive diagonal elements will always be positive. How much will it be? So, what are the shears you get here? This is 6 E i by L square this is 6 E i by L squared the shear will be what? Tell me the value.

3 E i delta by L square L cube.

3 E i delta by L.

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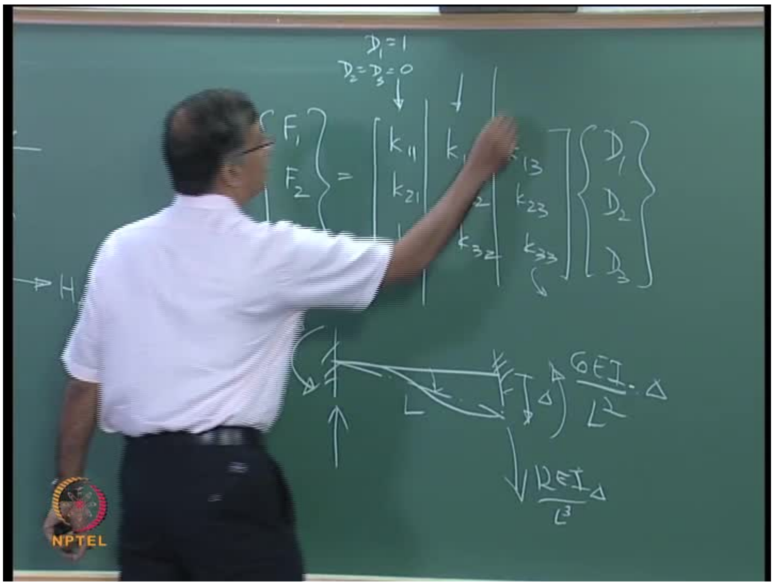

Where did you get three from?

3 E i delta by L.

Come on how where you get 3 from?

By 2 sir.

What by 2?

Moment plus.

Take this free body, what will be this shear? What did we do here? See this place is divided by that.

6 E i delta.

If you are an accountant, your bank balance will have reduced by one-fourth by now, if you play games like this, right? So it is not 3, it is 12, one-fourth the answer you have given is one-fourth the correct answer, is it not?

This moment plus this moment divided by that is span. (Refer Slide Time: 54:13). 12 E i, it is right in front of you by L cube, now that is interesting, so what is the answer? what is K 3 3? What is k 3 3? Give me the answer final answer?

Is it 12 E i by 4 cubed?

Yes sir.

No wrong, answer is wrong.

(( ))

No, no, no what a pity?

Twice.

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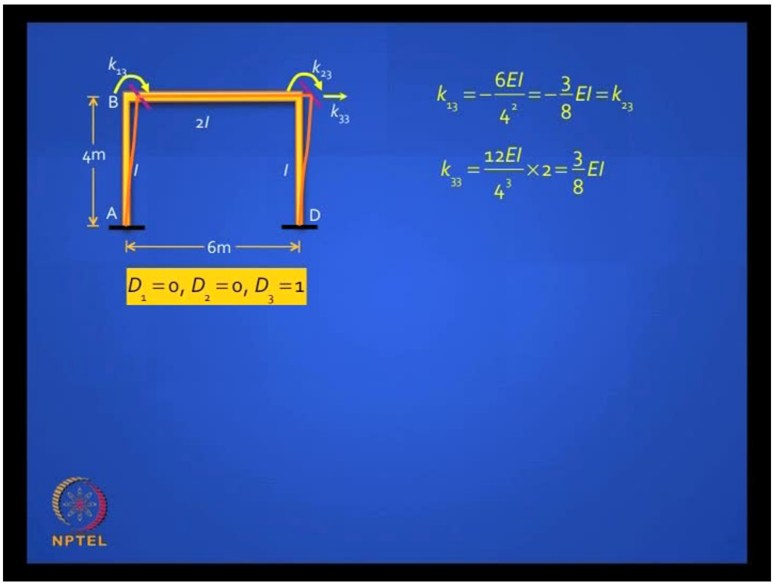

Twice, there are two columns. The two columns at the bottom there you have two reactions they should all add up to that applied force, so in to 2 do not make mistake like this, so physical approach is for bright students who are alert who know how to satisfy equilibrium but, it is wonderful, is it not? So if you do that.

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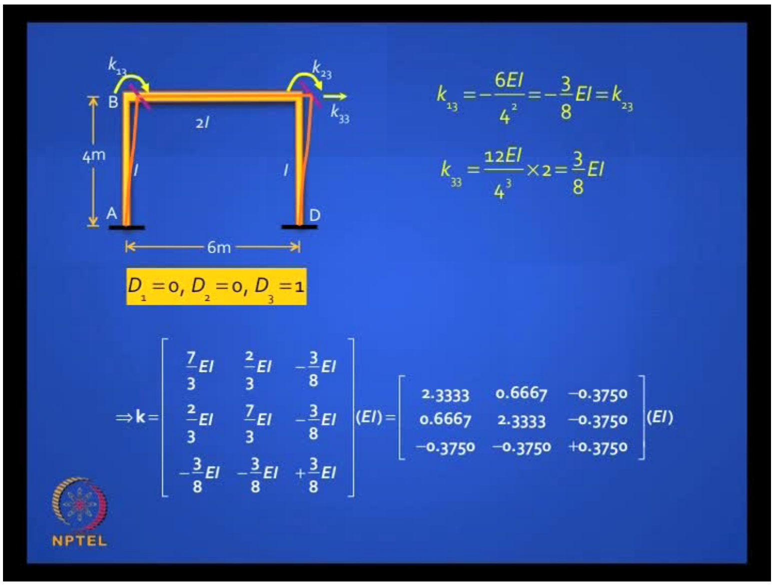

Apart from that small mistake of 8 by 3 which should be 7 by 3 you get the same matrix which we got in the blind manner, right? We did it the blind way and we got the same answer.

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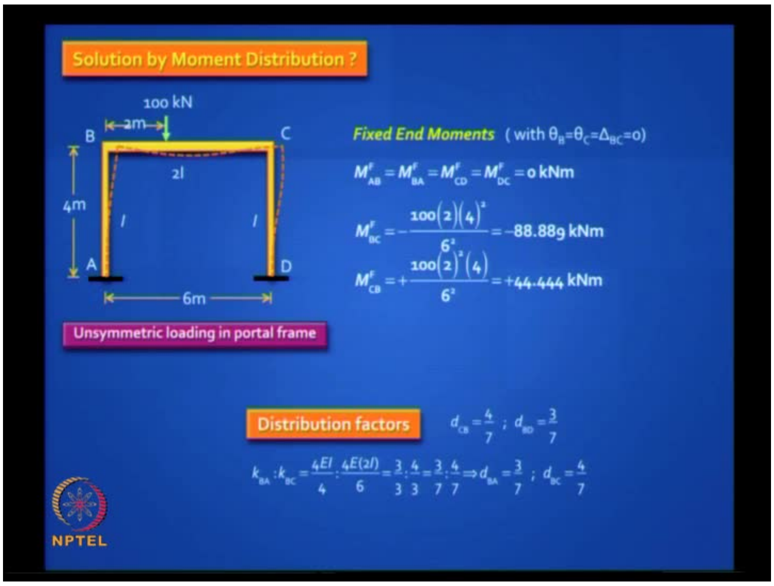

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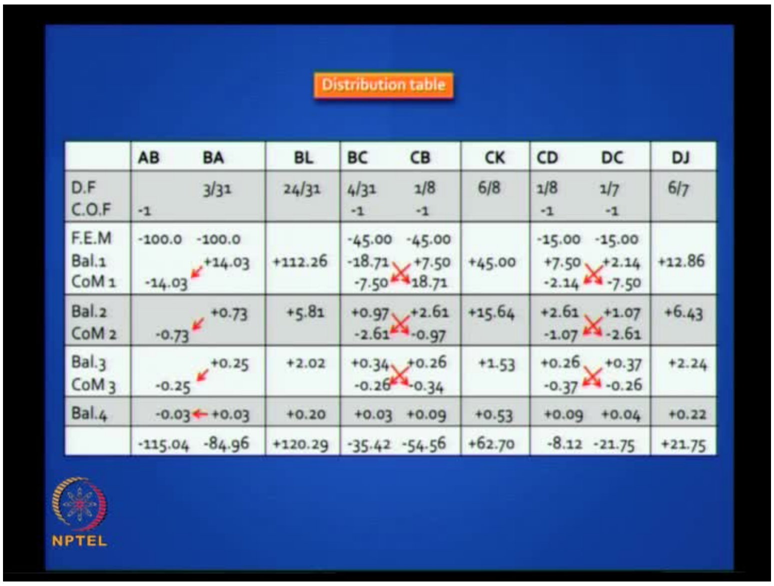

Distribution table

	AB	BA	BL	BC	CB	CK	CD	DC	DJ
D.F		3/31	24/31	4/31	1/8	6/8	1/8	1/7	6/7
C.O.F	-1			-1	-1		-1	-1	
F.E.M	-100.0	-100.0		-45.00	-45.00		-15.00	-15.00	
Bal.1		+14.03	+112.26	-18.71	+7.50	+45.00	+7.50	+2.14	+12.86
CoM 1	-14.03			-7.50	18.71		-2.14	-7.50	
Bal.2		+0.73	+5.81	+0.97	+2.61	+15.64	+2.61	+1.07	+6.43
CoM 2	-0.73			-2.61	-0.97		-1.07	-2.61	
Bal.3		+0.25	+2.02	+0.34	+0.26	+1.53	+0.26	+0.37	+2.24
CoM 3	-0.25			-0.26	-0.34		-0.37	-0.26	
Bal.4	-0.03	+0.03	+0.20	+0.03	+0.09	+0.53	+0.09	+0.04	+0.22
	-115.04	-84.96	+120.29	-35.42	-54.56	+62.70	-8.12	-21.75	+21.75

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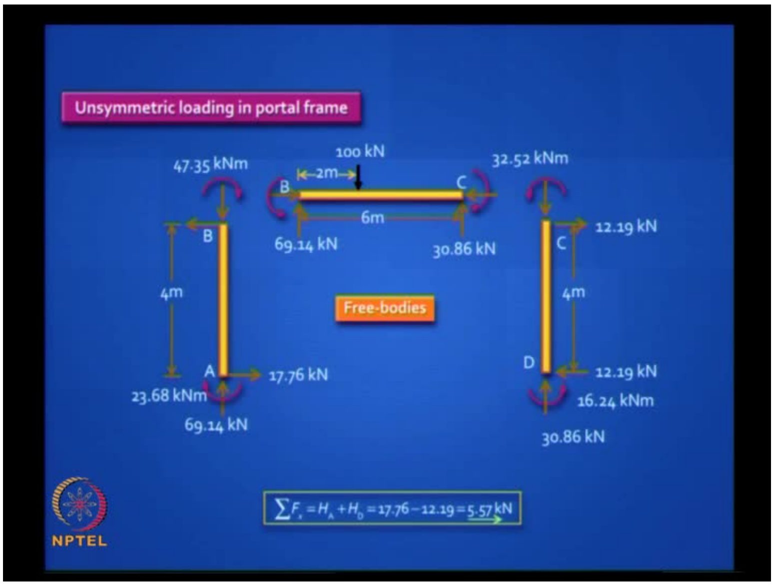

As I said do not attempt this by moment distribution, but if by chance you did it, then you get the same fixed end moments you get the distribution factors you draw the table you get some values but, the problem is you will not satisfy equilibrium and you can check it out you can check it out you will find that the horizontal reactions do not match there is the balance and you need to analyze the frame for that lateral load. So, let us not do moment distribution method when there is sway I will conclude by dealing with one last problem of pure sway.

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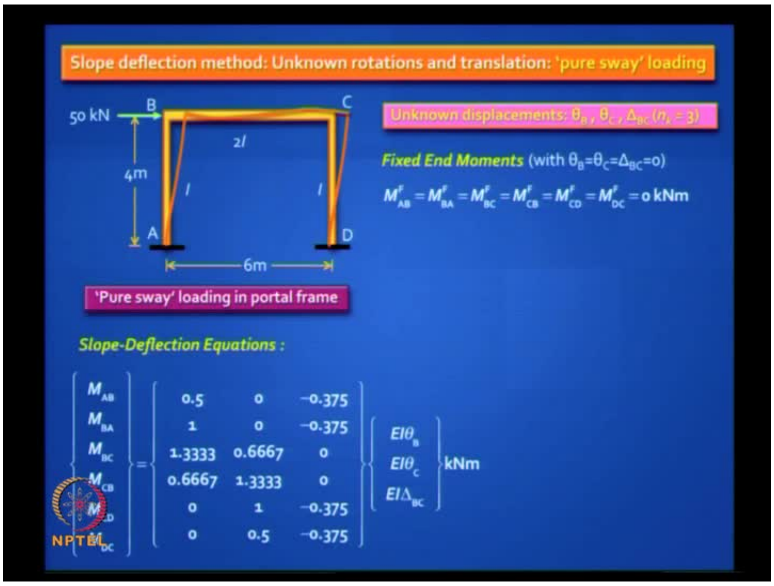

You have this frame subject to pure sway okay. How do you solve this problem, are there any fixed end moments? No fixed end moments deflected shape will look like that. Remember we did an approximate analysis when we did portal method cantilever method we do not do anything approximate now we do exact this is exact solution, so same three unknown displacements I will not waste time write down slope deflection equation because they are the same as in the previous problem with one additional advantage, what is the advantage? No fixed end moments, right? The pure sway that that is going to the node it is not going in between any member then what about your equilibrium equations? Same same except, in that last equation when you put sigma F X equal to 0, you have to add the lateral load which was not there in the earlier problem and do it properly put plus 50 on the left hand side of the equation and not on the right hand side because you will get totally wrong results otherwise, clear?

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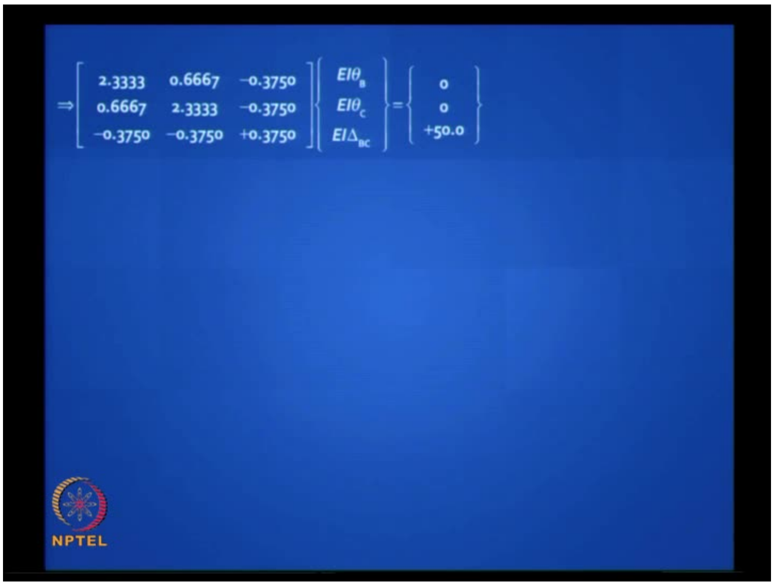

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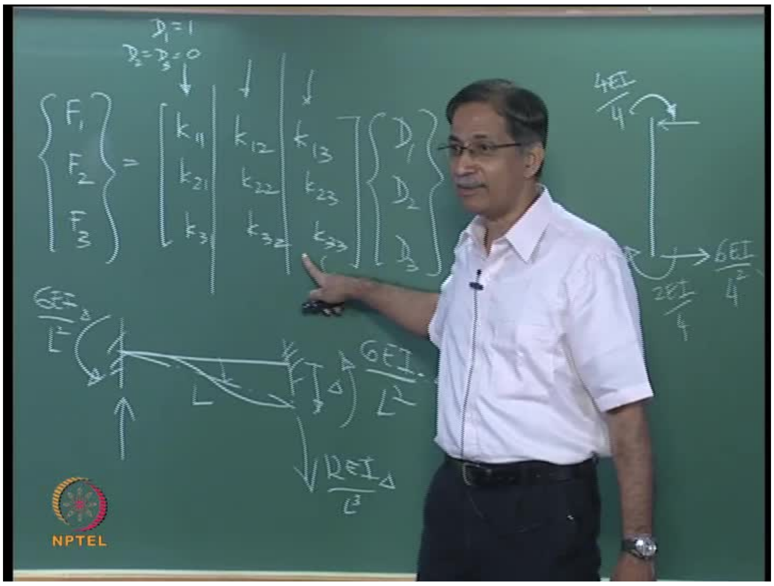

Then you get the same stiffness may in fact if you do stiffness method you can use this you do not have to do slope deflection method F 1 is 0, F 2 is 0, F 3 is 50, right? So you can do it this way also.

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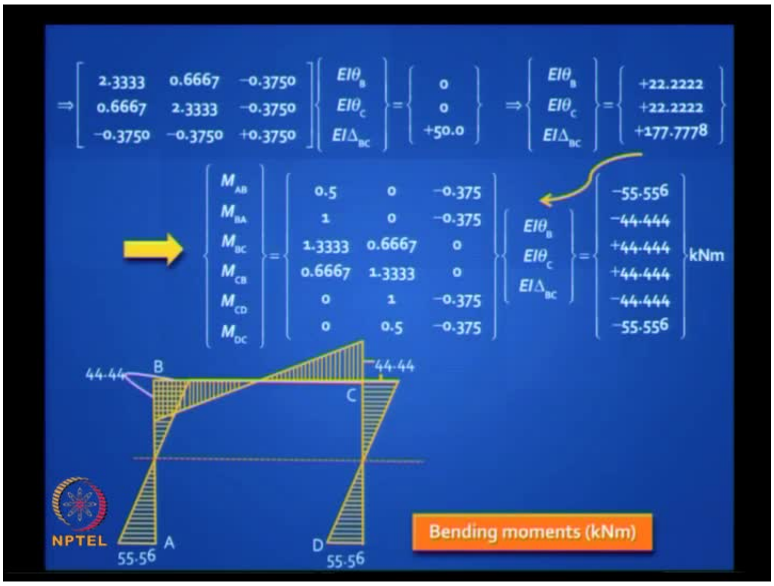

Now you see, this is the stiffness method which we are going to do soon solve it, plug it into the back into the slope deflections equations you get this answer which we.

Now look at this problem I am going to raise a question and will stop this class. There is something nice about this bending moment. What do you call such figure? It is it is not symmetric it is anti-symmetric, so that gives you a clue. It is anti-symmetric and the moment at the middle of the beam is 0, so maybe we could have taken an advantage of this anti-symmetric.

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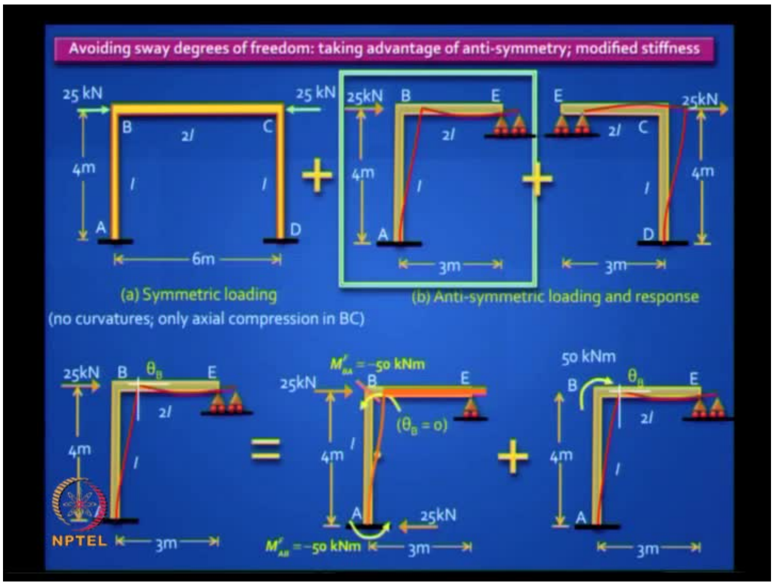
\ 

How do it take advantage of it? you can do this you can do this symmetry plus anti-symmetry and when you cut it in the middle because that is why your moment is 0 you can replace your original structure with this structure. This structure does not have any bending pure axial compression in this member only this will bend and now it is left you can either choose a left one or the right one, your problem has become much less complicated. How many unknowns do you have there? Theta B why? You can ignore sway, because it is a cantilever action we have done a problem earlier like this, so life is easy you have to just take out that theta B is your unknown, right?

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We will look at this in the next class but, this is a beautiful shortcut method this is what we will explore in the next class and this is what you got in your assignment the last problem but, you got two floors.

Thank you.