Advanced Structural Analysis

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Module No. # 2.7

Lecture No. # 13

Review of Basic Structural Analysis - 2

With the Review of Basic Structural Analysis part II, in the last session we had covered the introduction to displacement methods. In this session, we will look at slope deflection and moment distribution methods.

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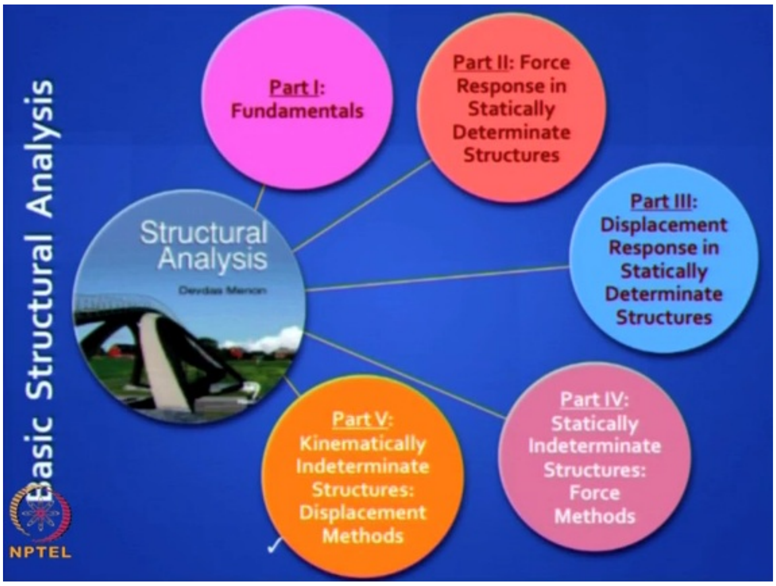

This is covered in part V of the book on Structural Analysis.

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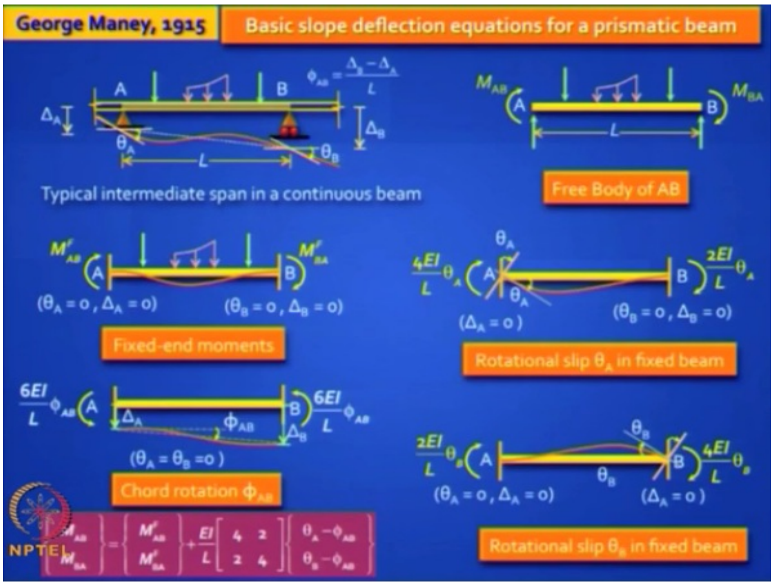

If you recall, we had derived these slope deflection equations, which date back to 1915, which are usually remembered in this manner. If you remember it in this way, it is very useful when you solve these problems.

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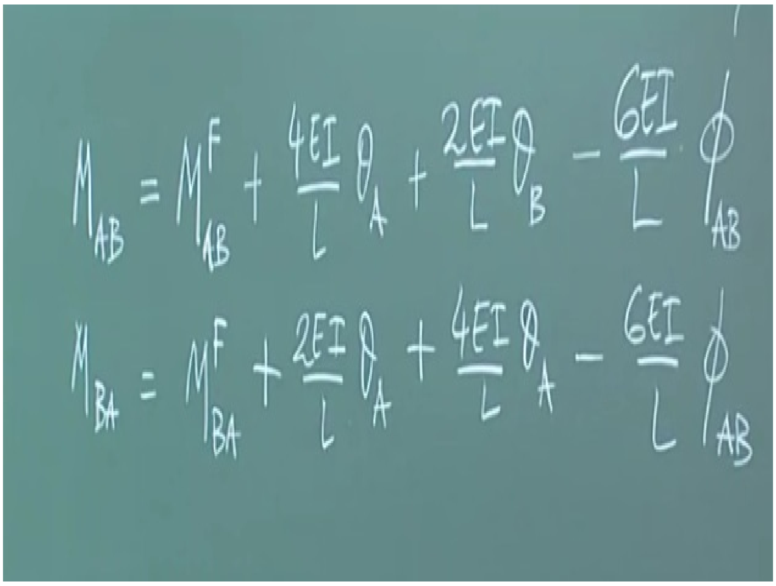

M_{AB} is equal to $M F_{AB}$ plus 4 EI by L $theta_A$. Of course, EI by L belongs to the element AB plus 2 EI by L $theta_B$ minus 6 EI by L into phi, which is phi_{AB}. Incidentally, phi_{AB} is $delta_B$ downward minus $delta_A$ divided by L and M_{BA}.

These are not difficult to remember. If you memorize them, they are very convenient to use, but you should also remember the derivation of it. You can also note that it can be written in a matrix form as shown here.

Now, in this form, we have treated the chord rotation as equivalent to a flexural rotation with a negative sign. You remember, we covered this in the last session - clockwise chord rotations have the same effect as anti clockwise flexural rotation. Are these formulas easy to remember?

[Noise – not audible] (Refer Slide Time: 02:58)

Second part?

It has to be θ_B.

So, where you have a rotation at a near end, it will be 4 EI by L and where you have a rotation at a far end, you have 2 EI by L. So, 4 EI by L is a near end and 2 EI by L is a far end. Is it clear? The derivation is straightforward.

Now, what is the use of these equations? If you are given the slopes, these are called slopes and the deflections, which can be converted to chord rotations. In any beam element, you straight away get the final moments by applying these equations. Mind you, you also need to know the fixed end moments, which is something fundamental in displacement method because they pertain to kinematically determinate structures. So, you should know the fixed moments for any given loading. If you know the slopes and deflections, you can find the equation, but usually in displacement method, the slopes and deflections are unknown. They are known only when you have indirect loading, where you have certain support moments that are known apriori. For example, you have a rotational slip, θ_A and θ_B will be known, or you have δ_B and δ_A because the foundations have settled by known amounts. Then, it is known; otherwise, they are unknown.

Now, you can take advantage in some cases of even unknown rotations and slopes. For example, in this case, where, in a series of continuous beams, the last beam – the exterior beam – has a simple support at the other end. Now, the whole objective of slope deflection method of analysis is to get the end moments. Because once you get the end moments, you can draw the free bodies and the whole beam is statically determinate. This is something like Clapeyron's theorem, where you wanted to know the end moments. Is it clear?

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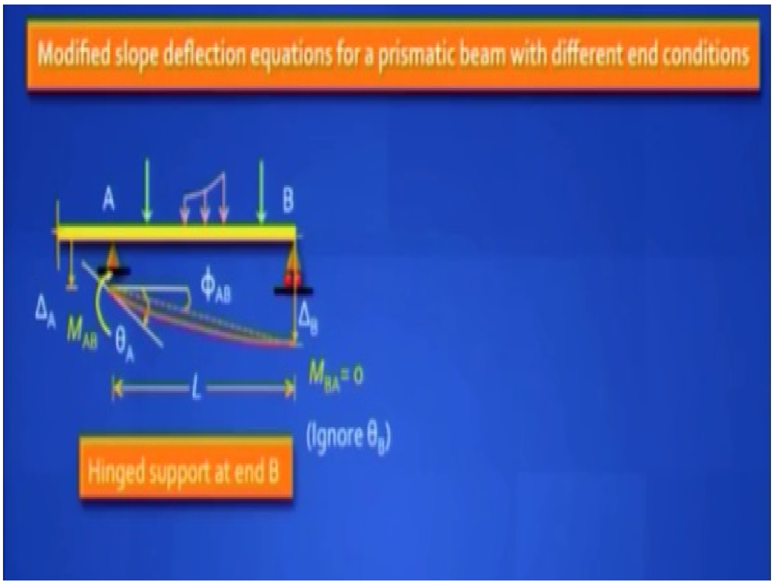

In this particular case, let us say you have a simple support at the extreme end. You know for sure that the end moment here is 0, except when you have a concentrated load; moment acting at B. In which case, you know that concentrated moment. So, it is a known quantity. In such situations, you can ignore theta$_B$ in this instance because the idea of knowing theta$_A$ and theta$_B$ is eventually to get the moment. So, if you know the moment in advance, you do not need to bother about theta$_B$. So, you can take advantage of this situation in which case you need to modify these equations. The first modification you need to do is in the fixed end moments because now, you are not arresting this degree of freedom; you are not treating theta$_B$ as an unknown in this instance.

You are not… You are reducing your degree of kinematic indeterminacy. You are not arresting that degree of freedom in your primary structure, which means your beam AB will now be a propped cantilever and you now know how to get the fixed end moments in propped cantilevers. That is a first correction you need to make. The next correction you need to make is - you can remove theta$_B$ from the picture because you are not treating it as an unknown. This value 4 will change and the second equation is not needed at all because M_{BA} is known.

You have only one equation, one unknown rotation, $theta_A$ and 4 EI by L will become? When the far end is hinged and you apply moment, 4 will reduce to 3, but not 2; 3 EI by L, is it not? We have done this before. What happens to phi? Minus 6 EI by L into phi will reduce to? Can you draw the shape of a propped cantilever? If you are arresting this, you are leaving this as a propped and you are having a chord rotation like this. This comes down. This is phi. Your deflected shape will now resemble what?

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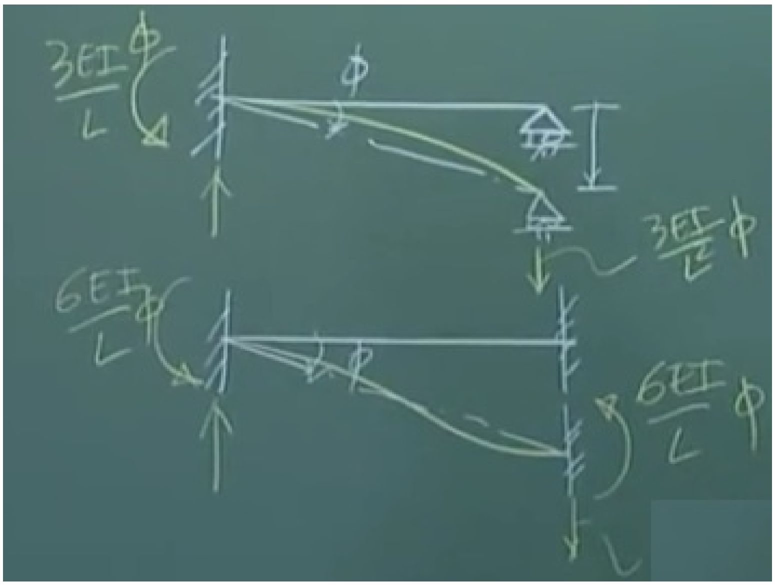

Resemble a cantilever and you have clockwise chord rotations. So, you have an anti clockwise moment. What is this moment in terms of phi?

3 EI by L

On the other hand, if this end is arrested and this goes down so that your chord has rotated by phi, your deflected shape will look like this. You will get anti clockwise moments at both ends and the value will be?

6 EI

6 EI, which is 4 plus 2. Remember we did this earlier. I showed you on the board. Do not you think? into phi. You will get vertical and horizontal reactions. What will be this reaction? This moment divided by the span. This will be always 3 EI by L into L square phi. This reaction will be? This plus this divided by L; (Refer Slide Time: 09:40) It is 12 EI by L square.

Now, you do not worry about these reactions because you can always get them from the free bodies, but you should know the end moments. We have done this exercise earlier. (Refer Slide Time: 09:58)

In case you have forgotten, this was the last picture here. (Refer Slide Time: 10:00)

How did we derive these equations? We took a typical intermediate beam, we identified three possible displacements that affect the end moments θ_A, θ_B and chord rotation phi. Agreed? Then, we arrested all three. When we arrested all three, we got the fixed beam and the first pair of moments you get is the fixed end moments $M F_{AB}$ and $M F_{BA}$. Next, we released only θ_A. Keeping the far end fixed, when you release θ_A, you get moments 4 EI by L θ_A on the left side - clockwise and 2 EI by L θ_A on the right side. Then, we fixed it back.

Now, we release θ_B. Now, you get 4 EI on this side and 2 EI here. Then, you go back, fix everything and you allow the chord rotation only, which means you are allowing this. So, it becomes? They are both going to be anti clockwise. Do you get the background to these equations? Very often students memorize the equations, but they forget the physics behind it. Physics is very simple and straightforward. It can be written in matrix form as indicated.

Now, we are trying to take advantage of the fact that we know the moment at M_{BA}. So, how do we modify these equations?

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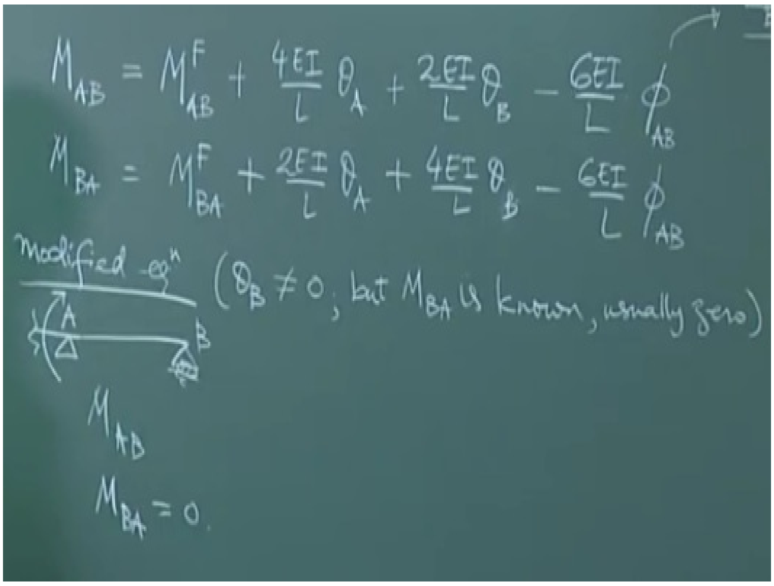

If we ignore theta$_B$, they will take this form. So, the modified equation when you allow theta$_B$ to take whatever value. You are not arresting it, but M_{BA} is known. That is, the far end is simply supported. This is B and this is A. M_{BA} is known. Usually, it is 0 because you do not have any concentrated moments acting there. In such situations, you have only one unknown, M_{AB} because M_{BA} is known. M_{BA} is usually 0 or some given moment.

How will you now modify this first equation? I will write… What should I write for fixed moment?

M F $_{AB}$

I should not write M F $_{AB}$ because now my fixed end moments are what I get when I arrest only A and I am allowing B to rotate. So, it is a… for a propped cantilever. So, how should I write it? No, this term, you remember my notation. I put a not there. This is different from this. In this, B is fixed, A is fixed. Here, B is simply supported. A is fixed. Then, instead of 4 EI by L what do I write? 3 EI by L into theta$_A$. Theta$_B$ does not come in my equation because I am not looking at it at all. I am allowing it to happen. I do not care what it is. I can actually find out, but I do not need to know.

What about the chord rotation? It is minus instead of 6 I get? 3, that is it. 3 EI by L. Now, it strains that many text books take advantage of the far end being hinged in moment distribution method, but they do not take advantage in slope deflection method. We are going to look at all methods from a big picture point of you and we should take advantage of this wherever possible because it reduces your work tremendously. Is this clear?

So, these two equations (Refer Slide Time: 14:28) simplifies to these two when B is hinged at an extreme end. If you have a series of continuous beams the last beam… Or, if you have a portal frame, where the base is a propped, simply supported, you have this.

There is one more simplification possible.

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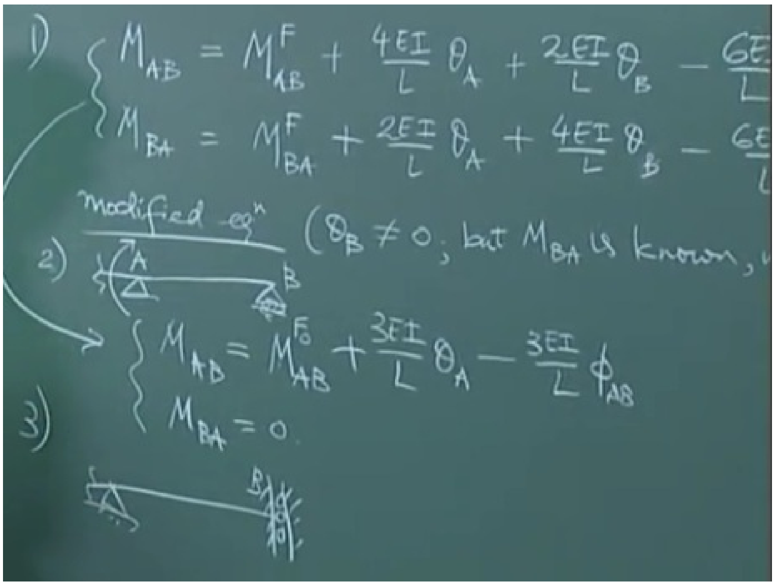

That is, when you have a situation like this and you have a propped cantilever. Here also, you can take advantage. Can you tell me how these equations will change?

This is case one or this is case two. This is the general case one. George Maney only gave these equations. These were subsequent shortcuts discovered and case three.

In case three, how do you think that these equations will change? Here, of course, you will need M_{AB} and M_{BA}. What will be the first equation? That will be F and there is no F_0 because this is fixed against rotation, but you have to take it carefully. You remember? If some loads are acting here, you should take double the span, make it symmetric and do it as we discussed earlier. So, this will remain $M F_{AB}$ this will remain $M F_{BA}$.

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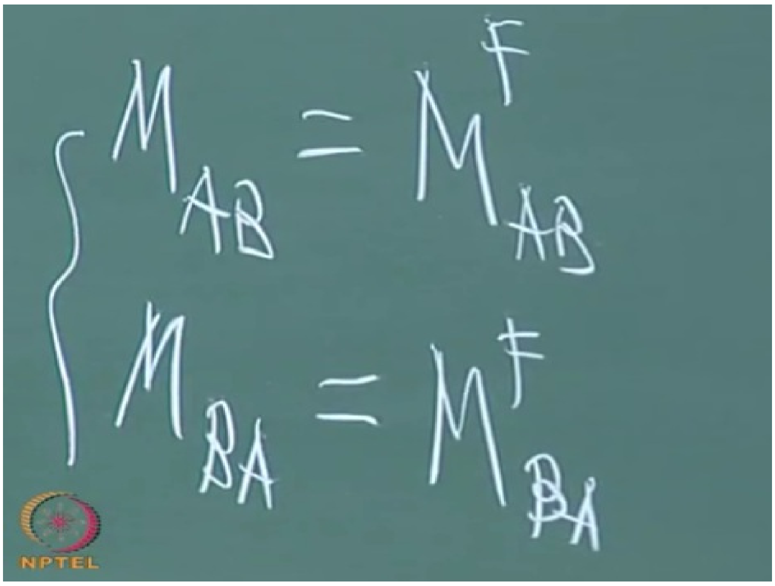

Now, you have $theta_A$. So, due to $theta_A$, what will be the stiffness? It is not going to be 4 EI by L, it is not going to be 3 EI by L, but it is going to be? EI by L. You remember. Is it going to be plus or minus if theta is clockwise? It will be plus. Plus EI by L into $theta_{AB}$. What do you get at the other end? You do not get 0. It is a cantilever. When you apply a moment here, what is the moment you get at the other end?

EI by L

Is it plus or minus?

[Noise]

Let us go back to basics whatever you are trying to do, we are arresting this.

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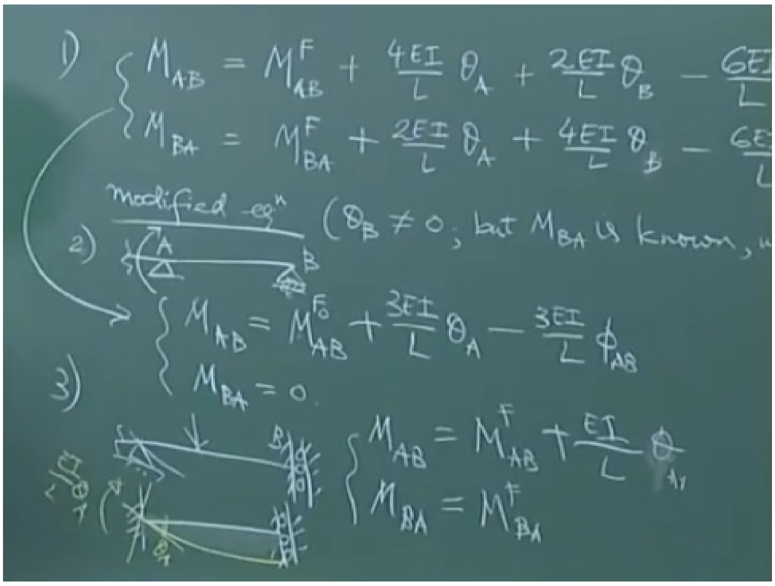

We are leaving this in place. We are not touching it and we are not preventing it from deflecting also. We are saying that this is going to rotate. When it rotates, what is the shape it will take? I am sorry this should look straight. It is going to take a shape like that (Refer Slide Time: 17:37).

Now, if it takes a shape like that and this has rotated by $theta_A$. You say that this moment is EI by L into $theta_A$. What do you think will happen there? Is it not like a cantilever where you applied a moment like that? What do you get here? You will get the same thing, but it will be in the opposite direction. It will be in this direction and it will be anti clockwise. So, what will you write here? That is the only important thing. Minus EI by L into $theta_A$. Got it? That is all you need to know.

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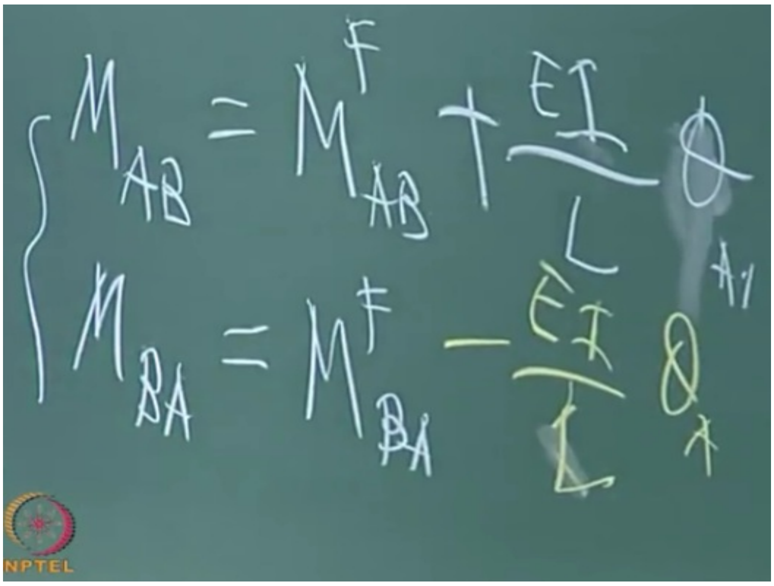

Remember? In the beginning, we drew a graph, where we drew three lines for stiffnesses. I said there are three magic numbers to remember: 4 EI by L, 3 EI by L and EI by L. That is where all of them come into play.

Now, in this case, chord rotation is not an issue at all because chord is always going to rotate as this comes down. You do not worry about chord rotation in such situations. Remember that it is a symmetric thing. So, you do not have a problem. So, these are the simplifications you make. This is the first one.

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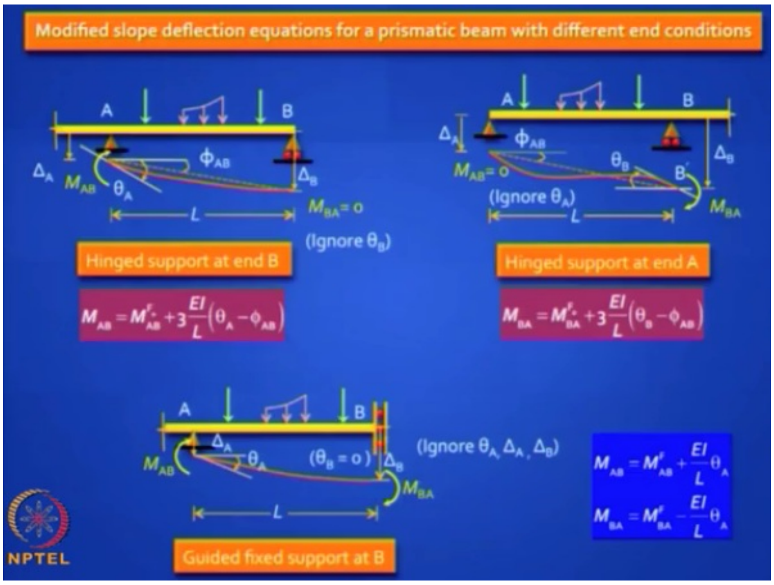

If your hinged support is at the left end, there obviously your equation will get reversed. If you have guided a fixed support, you will get the equations which I showed here. So, these are the equations.

Now, let us look at problems.

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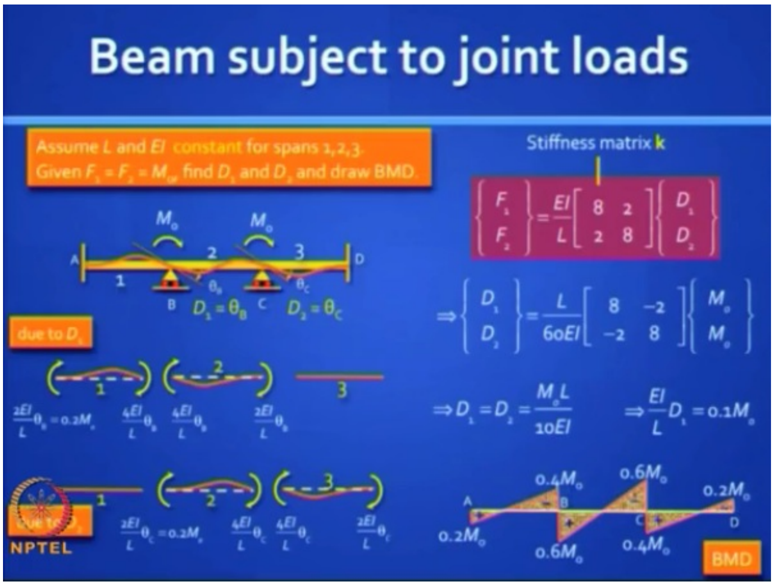

To begin with, let us quickly look at the problem we solved in introduction to displacement method. Remember we did this problem? Introduction to displacement methods. Let us see, if we can solve it in a blind manner using this slope deflection equation. In this problem, if you remember, you had a three span continuous beam and you had two concentrated moments M_0 M_0. We solved this and we did the bending moment diagram, but this needed a little bit of thinking. We had to generate the stiffness matrix from first principles and so on, which was interesting.

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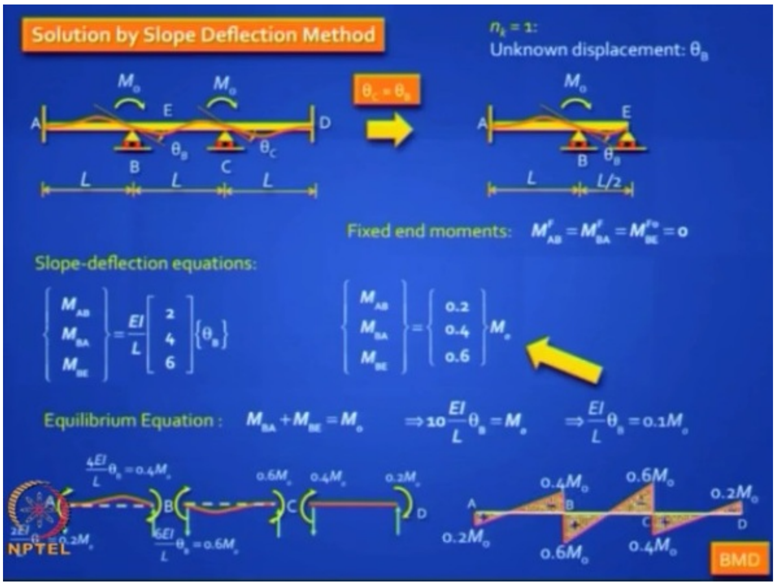

Let us do it in a blind way. Let us solve it by slope deflection method. How will you solve this? You have three beams; so, you should write down three sets of equations. Three sets of equations of this kind, but you unknowns here are theta$_B$ and theta$_{C.}$ So, theta$_A$ is 0 and theta$_D$ is 0. If you were to… You can take one more advantage. What is that? You can see the anti symmetry in the deflected shape and you can see that theta$_B$ and theta$_C$ will be identical. If you take advantage of this, you can cut the beam at the middle E and what will you put there as support? Cut the beam at E.

[Noise – not audible] (Refer Slide Time: 20:55)

What is the bending moment at E? 0. We have seen this deflected shape in portal method, cantilever method. Is it not? So, what you should put there?

[Noise]

Put a roller. So, you see, you know, you have heavily simplified this problem.

Now, you have only one unknown - theta$_{B.}$ Theta$_A$ is 0. Theta$_E$ is something, but you could not care what it is because you can take advantage of this modified (( )). How will you write these equations? Let us see.

Only one unknown displacement. Your fixed end moments are all 0 because there is no intermediate loading. There is no loading on that beam except that concentrated moment.

That moment is going straight to the joint and it is going to be shared by the two beams; it is not being applied in any one beam. Is it clear? That is called a nodal moment. So, I have written the equations in a very simple way.

M_{AB} - you look at this. I have written these two equations for AB. (Refer Slide Time: 22:10) $M F_{AB}$ is 0. $M F_{BA}$ is 0 because there is no intermediate loading on this. $Theta_A$ is 0 in this problem because M_{BA} is fixed. There are no chord rotations so, this is also 0. So, the only term you get is this. 2 EI by L and 4 EI by L into $theta_B$. So, M_{AB} and M_{BA} is known. Now, you have to write down only for M_{BE} because M_{EB} is also known to be 0. What is M_{BE} going to be? It is going to be? (Refer Slide Time: 22:48)

Now, you are going to write this equation because you are going to replace AB with BE. Now, there is no fixed end moment, there is no chord rotation, but there is only this term; with this replaced by $theta_B$. Is it clear? So, it is a simple problem and it is L by 2; mind you. That is span B is L by 2. So, 3 EI divided by L by 2 is 6 EI by L. Do you get it now? So, it is very simple. This is how you write the slope deflection equation. All you need to know is $theta_B$ and you have got the answer. How do you find $theta_B$? Tell me. This is the next step in slope deflection method.

Let me help you. When we did statically in determinate structures, we did something similar, but we solved for the unknown redundants applying what equations? We applied compatibility equation. Now, we are doing displacement method. We solved for the unknown displacement by applying what equation? Equilibrium. What is an equilibrium equation you need to apply here?

[Noise] (Refer Slide Time: 24:00)

Where?

[Noise – not audible] (Refer Slide Time: 24:05)

The clue is wherever you have identified the unknown displacements, in this case, it is $theta_B$. What is a moment corresponding to $theta_B$? It is M_B. So, the net moment at B should be equal to? In this case?

M_0.

M_0 because M_0 is the net moment.

When you join those two elements together, there are the two end moments. M_{BA} and M_{BE} should add up to M_0. That is equilibrium. Is it clear? Have you understood? So, that is what you should do. That is equilibrium. M_{BA} plus M_{BE} should add up to M_0.

Now, you can easily substitute. You have those two equations. So, 4 plus 6 is 10. 10 EI by L theta$_B$ is M_0. You got EI by L. Theta$_B$ is 0.1 M_0. Plug it back into those equations and you get 0.2 M_0, 0.4 M_0, 0.6 M_0. Draw the free bodies. They will look like this. That is your complete bending moment diagram. That is it. Have you got the hang of it? No? Tell me what is difficult in this problem.

[Not audible] (Refer Slide Time: 25:25)

Take the shortcut. Do not worry. Take theta$_B$ and theta$_C$. Solve two equations simultaneously. You will end up with theta$_B$ equal to theta$_C$. You will still get the same answer.

[Noise - not audible] (Refer Slide Time: 25:41)

Why? You tell me if they are not 0 what will they be?

When will you get a fixed end moment in a beam? Let us say I have a beam, when will I get a fixed end moment?

[Noise]

When I apply some load on that beam?

[Noise]

In this case, I do not have any intermediate loads on that beam. I have only a nodal moment. So, I do not have any fixed end moments. These are good questions you are raising, but you will easily get the hang of it as we solve more problems. Suppose we have the load in the middle of ABW, what will be the fixed end moment? Minus WL by 8 plus WL by 8. Remember? If W is 0, what is a fixed end moment? It is 0. So, there is no fixed end moment. Is it clear, any more question?

You have to be alert and see how beautiful and how simple, this method is. You can do this blindly or you can open your eyes and do it.

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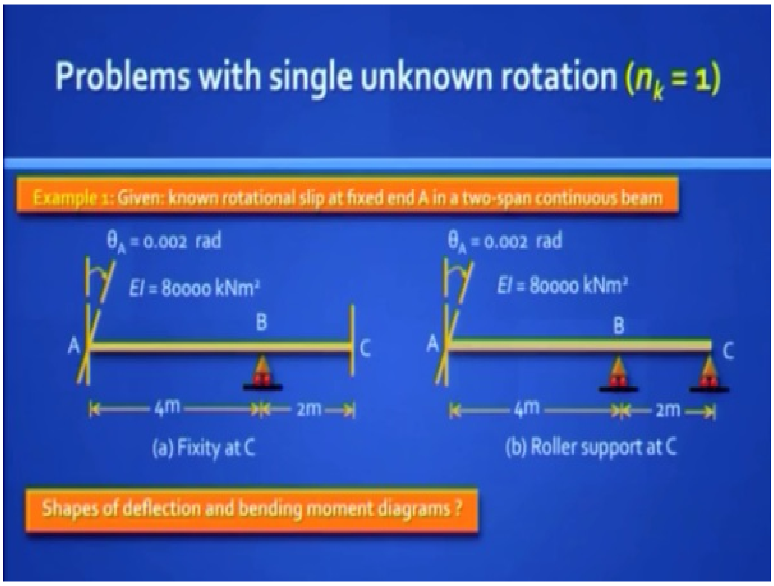

Now, let me wake you up with these problems. I have shown here two pictures of two span continuous beams. The one on the left shows you fixity at C and the one on the right shows you a roller support at C. The question at… In both these beams, you are told that somehow that fixed end at A has rotated clockwise. It is called a rotational slip by a known amount, 0.002 radius and the E I values given.

You can use slope deflection equations here. Now, the question is, do not do any calculations at least, looking at those pictures can you sketch the shape of the deflected diagram? Can you at least try that? What will it look like? What will the deflected shape look like when you have theta$_{A.}$ There is no loading otherwise. It is only an indirect loading in these two beams. What will they look like? Please get used to this. This is prior to do any calculation.

Next, if that is your deflected shape? What do you think the bending moment diagram will look like?

[Noise – not audible] (Refer Slide Time: 28:10)

It is not difficult. Let me explain how this can be done. Look, this has rotated so this tangent must also rotate by the same theta$_A$. Agreed? This has to come back here and naturally this slope must be maintained; $theta_B$ is going to rotate anti clockwise. It must go back, but it has to go back to C and it must have 0 slope here. There is only one way you can draw it and while you are drawing it you get the hang of it. You know that this part is going to be sagging and this has to necessarily hog up to some point, and then again it will sag. There is a point of contra flexure some way out there. You do not know exactly where.

Now, try drawing the bending moment diagram which reflects this deflected shape. Mind you, you will get reactions that is, support reactions. So, the bending moment diagrams will be made up of straight lines. You know that. This is straight line. Can you draw the shape of that?

The difference in this case is, here it hogs all the way because it is not forced to come back to zero slope at C. It is free to rotate at C. Difference is very simple. So, what will be the shape of the bending moment diagram? If you draw the free bodies, you will get a clue. What are the free bodies? At A, you will have a clockwise moment. Do you know the value of that moment? What is it?

We do not know.

You do not know? Had B been fixed against rotation, it would have been 4 EI by L. Had B been an end support, simple support, it would have been 3 EI by L, but this is somewhere in between. You do not know. $Theta_B$ is unknown. Is it clear?

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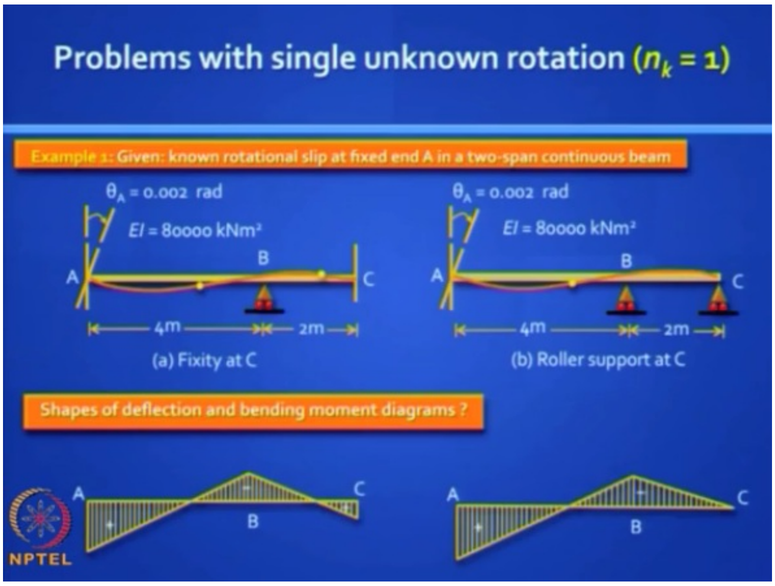

This is what it is going to look like. How many of you got this? Wonderful. This is really, you know, thinking on your feet and doing it and this is what it is going to look like. You have to get used to this. You see, you can always solve problems and get full marks, but being able to intuitively get these shapes qualitatively, is a skill that you need to nurture and quantitatively getting those answers is also important.

In the exam and in real life, you need not draw these in advance. You will get them any way, but it is good to own your understanding. You see how those points of contra flexure are more or less matching deflected shape in your bending moment diagram.

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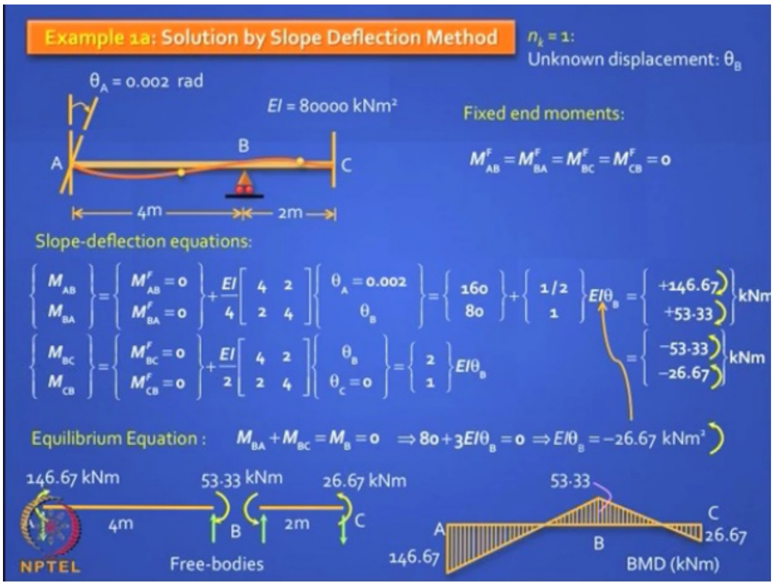


How do we solve this problem?. So, let us solve them by slope deflection method. Let us take the first problem; write down the slope deflection equations for spans AB and BC; Very simple; these equations, the first set of equations and blindly apply them.

Here, again fixed moment is 0 due to applied loading because there is no applied loading other than this known rotational slip; so, MF is off. I have written the equations. Check them out. They are very simple. I have written them in a matrix form because I want you to get used to this formulation. We are going to use it later, but it is basically these same equations (Refer Slide Time: 31:51).

EI by L for the first span is EI by 4. EI by L for the second span, BC is EI by 2. EI is constant. So, it is 4 EI by L into theta$_A$. Theta$_A$ is known. It is 0.002. Theta$_B$ is unknown and theta$_C$ is 0. So, there are only two unknowns, B and C. Is this clear?

So, you have written those equations in terms of EI into theta$_B$. Can you see this? Any doubt? Either you can write them this way or you can write them in the matrix form; I do not mind, but all of you will end up getting these two equations. In other words, this quantity that you calculated is a kind of a fixed end moment that you get because of the known rotational slip; but this 160 and 80 that we got, how do you get this 160 and 80? Now, you have to substitute the value of EI, known that EI is 80,000. 80,000 divided by 4 into theta$_A$ here, will give you these quantities. Have you all got it? Can I proceed?

You tell me what to do next? I have written the slope deflection equations. If someone gives me the value of theta$_B$, I get the answers. How do I get theta$_B$? I need to write down? An equilibrium equation. What will it look like? MB equals 0, which means?

[Noise]

M_{BA} plus $M_{BC,}$ both clockwise should be 0. That is easy to write because you have to expression in terms of EI theta$_B$. You write them down.

Do you get this? Just check it out. That is, I am taking the second equation from the first beam and the first equation from the second beam. I am simply, algebraically adding those equations and I am solving it. I get EI theta$_B$. It turns out to be negative; theta$_B$ as units of radian. By the way, EI theta$_B$ does not have kilonewton meters. It is kilonewton meter per unit into length. It should be kilonewton meter square; only when you divide by…, it becomes kilonewton meters.

Have you all got this? Next step. It is anticlockwise, which you guessed correctly when you drew the shape because clockwise rotations are positive. Now, what do you do once you get this answer? Put this value into… That is, substitute that value into slope deflection equations. Put it there. You can write it there itself and you get the final answer.

Very simple. Please do a few problems and get the hang of it. Very simple. Then, what should you do? What is the next step? You get to… You want to know what this minus plus means. So, draw the free bodies. You know that minus means anticlockwise plus means clockwise.

So, the free bodies will look like this. M_{AB} is plus so, it is clockwise and at B, you will find that for BA, it is clockwise and for BC, it is anticlockwise. They are both equal and opposite, which is why they add up to 0. In fact, that is how they add up to 0. Clear?

Quantity interior. It is not a bending moment reaction.

It is the bending moment at B. See, there are bending moments in both the beams.

[Noise] (Refer Slide Time: 35:45)

Because according to the bending moment diagram and B is not 0; bending moment and B, it is bending.

I do not understand what you are saying. If I cut a section anywhere in the beam, do I get a shear force in bending moment? Now, I am cutting it just at the support, just to the left of the support.

[Noise – not audible] (Refer Slide Time: 36:09)

What is bending moment reaction? It is an internal force. Bending moment is an internal force. So, that is the value. This 53.33 is a bending moment at this junction acting at B in the element BA.

There is no clarity.

[Noise – not audible] (Refer Slide Time: 36:33)

This is that element. This is A and this is B (Refer Slide Time: 36:43). You got some value which was clockwise and you got some value which was? Both were clockwise? Both are clockwise. This is 147 and this is 53(Refer Slide Time: 37:03). Let us round it up. What do these represent? These represent… Now, actually, the real fixity here. You have a support here, but you have a rotational spring here coming from the adjoining beam.

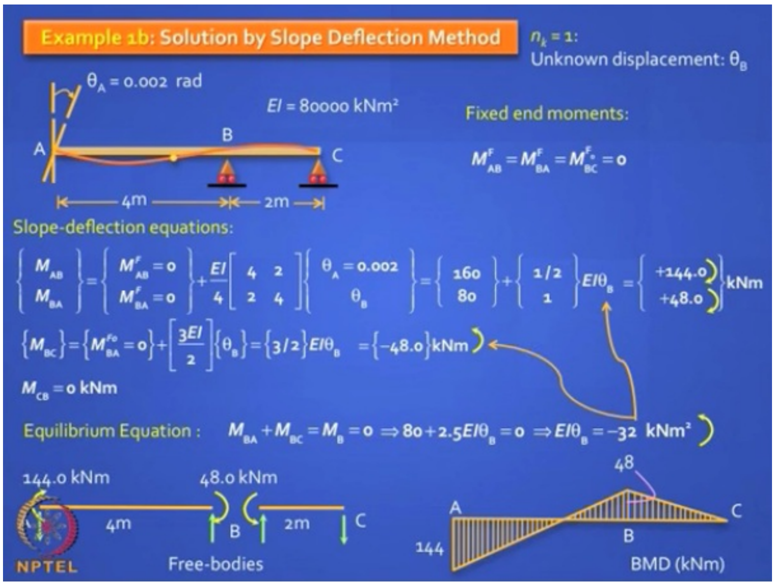
Now, I am just cutting this infinitesimally to the right of A of this beam. I am cutting a section infinitesimally to the left of B. If I cut a section anywhere, I get a bending moment in shear force.

Now, what I am drawing here are… These reactions are nothing, but bending moment of shear force. Now, this reaction is a true reaction because it goes to the support. This you cannot call it really a reaction because the action is taken by the adjoining beam. So, it is an internal moment in the beam, which is the same on the other side; It has to be, because when you join them there is no net moment. Is it clear?

Now, does it make sense? What is the name for this? The name is simply bending moment. If you cut… (Refer Slide Time: 38:22) Let us take a beam like this with the load here. If I cut a section here, if I take this free body, I have a reaction here. I have a moment here which is sagging. Here also, I have a moment, which is sagging. This is called bending moment and this is called shear force (Refer Slide Time: 38:37). So, they are internal. Does it make sense? Any other questions? So, it is easy to do this. The same diagram, but now you got the numbers as well.

Let us take the second problem.

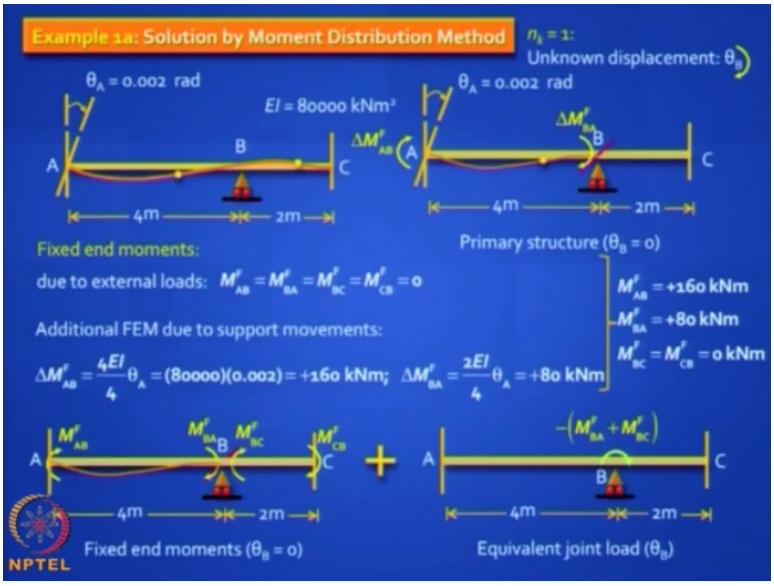
(Refer Slide Time: 38:56)



How does it change from the first problem? The first set of equations do not change. Now that there are no fixed end moment, this set of equations are the same as the previous problem. Only for BC it will change and here deliberately we are ignoring for theta$_C$ because we want to take the advantage of this simplified equation. So, what will that equation look like? It will look like this. 3 EI by L; L is 2 meters in this case and M_{CB} is 0; so, I do not write an equation in terms of theta$_{C.}$ I do not want to find theta$_C$. Incidentally, theta$_c$ will be half of theta$_B$. You can take a look incidentally with the opposite side, but I am not interested in theta$_C$.

Now, what is the equilibrium equation? Same equation. Only your stiffnesses have changed, your answers have changed. Plug it back to get those answers and draw the free bodies. Now, you do not have a moment at C. So, you get a bending moment diagram. Does it make sense? Are you comfortable with this? You have to practice, but it is really simple.

(Refer Slide Time: 40:20)



Now, let us quickly see how you solve these problems by moment distribution method. Let us go back to the first problem. In the moment distribution method, you usually need fixed end moments because you have to distribute some fixed end moments. So, the concept here is, due to external loads there are no fixed end moments, but due to the indirect loading you get fixed end moment because now your arrest theta$_{B;}$ you do not let theta$_{B}$ take place. If you arrest theta$_{B}$ and then, if you allow that rotation theta$_{A}$, will you not get some moments? Do you know those moments? We will call them additional moments. I am in putting delta; delta M F $_{AB}$, delta M F $_{BA}$. What do you think that will be equal to? What is delta M F $_{AB}$ due to that rotational slip? 4 EI by L, which is a known quantity. What is delta M F $_{BA}$?

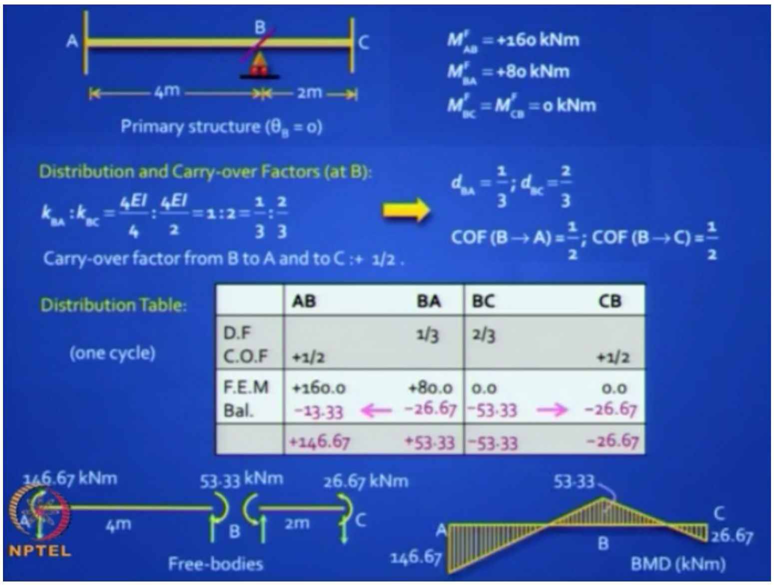
[Noise] (Refer Slide Time: 41:22)

2 EI by L. That is it, which is half the first value. Incidentally, you got the same numbers 160 and 80 in the slope deflection equations from this quantity (Refer Slide Time: 41:37), but now we are treating this as a fixed end moment by arresting theta$_{B}$. Is it clear? So, when you go back to the full beam, this is the picture you have got and you will find that you now need to reverse that moment, which adds up at B. How much adds up at B?

[Noise – not audible] (Refer Slide Time: 42:00)

So, 160 and 80 add up. You need to reverse it. How will you get that moment shared? By? In proportion to their relative stiffnesses.

(Refer Slide Time: 42:16)



That is what we will do. We got these moments. To find the relative stiffnesses, you need to go through the step of finding distribution and carry-over factors.

Now, do you agree to this – K_{BA} to K_{BC} is 4 EI by L is to 4 EI by L by 2. Do you agree to this? The ratio is 1 is to 2 or, 1 by 3 is to 2 by 3 because the distribution factors should all add up to 1. Remember – the end A is fixed so, you have a carryover. Half of what you distribute at B will get carried over to A and it will also get carried over to C. I will make this clear.

All these you can do very nicely in a table called distribution table. So, this is what the table looks like. (Refer Slide Time: 43:12) It is a very simple table; it is a single cycle distribution. This is beam AB; this is beam BC. There are four moments I need to draw: M_{AB}, M_{BA}, M_{BC}, and M_{CB}. I first marked the distribution factors at B when I distributed a moment; One third of that moment will go to BA and two third will go to BC because that is how I worked on my distribution factors. Then, I also have to do a carry-over factor. In case I distribute a moment here, half of it will spill over to here and half will spill over to here because these two ends are fixed.

So, I write them down at the top of the table. Then, I write down the fixed end moments. In this case, the fixed end moments are caused by the rotation and I have calculated those values; I got plus 160 and plus 80 and nothing here. This is caused by the known θ_A and now, I have to balance these moments. Where do I have to do the balancing? At B because I should not get any moments there. There is no net moment there. So, I have to get rid of this 80. How do I get rid of this 80?

[Noise – not audible] (Refer Slide Time: 44:23)

There you are; 80 by 3 here and 280 by 3 there. So, I put a minus sign. I distribute whatever moment adds up here. I apportion it to this and this factor. Is it not easy to do? I show also do carry-over.

Along with this step, I should do a carry-over; Half of 26.67 spills over here, half of 53.33 spills over. That is it. It is all over. Now, I just add up everything. Which method do you prefer, this one or slope deflection? This one?

So, you learn to do both. Very quickly, if you want to do the other one with the hinge, there is only one change everything else is same. There is only one change. What do you think that change is? The carry-over factor is 0, stiffness is also changed; 4 EI by L and 2 EI by L.

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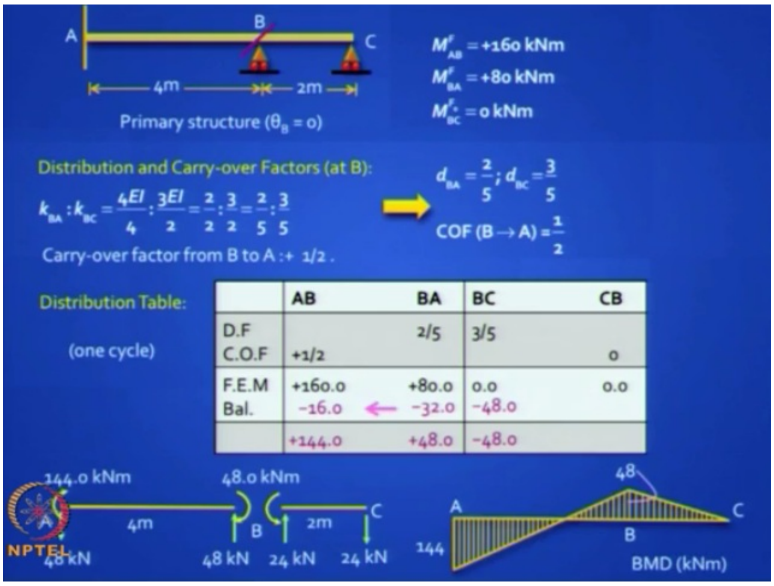

	AB	BA	BC	CB
D.F C.O.F	+1/2	2/5	3/5	0
F.E.M Bal.	+160.0 −16.0	+80.0 −32.0	0.0 −48.0	0.0
	+144.0	+48.0	−48.0	

So, it is now 2 by 5, 3 by 5, carry-over factor is only to one side. There is no carry-over to C. The table looks like this. In this table, only thing that is changed is your distribution factors. When you do the distribution, it is now two-fifths of 80 and three-fifths of 80 and you do the carry-over only to one side. You are very good at addition. You can do this. Draw the free bodies. Draw the…

So, this is powerful. Once you are used to it, within 5 minutes you can crack these problems. Thank you.