Advanced Structural Analysis

Prof. Devdas Menon

Department of Civil Engineering

Indian Institute of technology, Madras

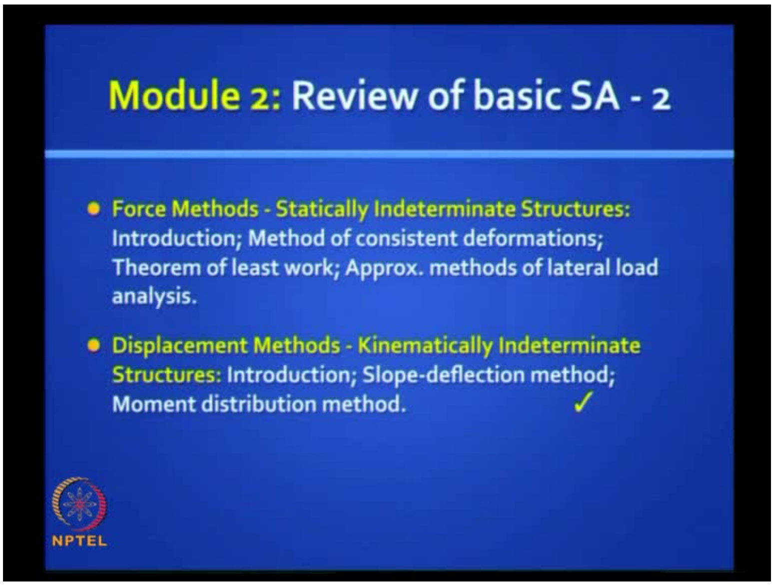
Module No. # 2.6

Lecture No. # 12

Review of Basic Structural Analysis -2

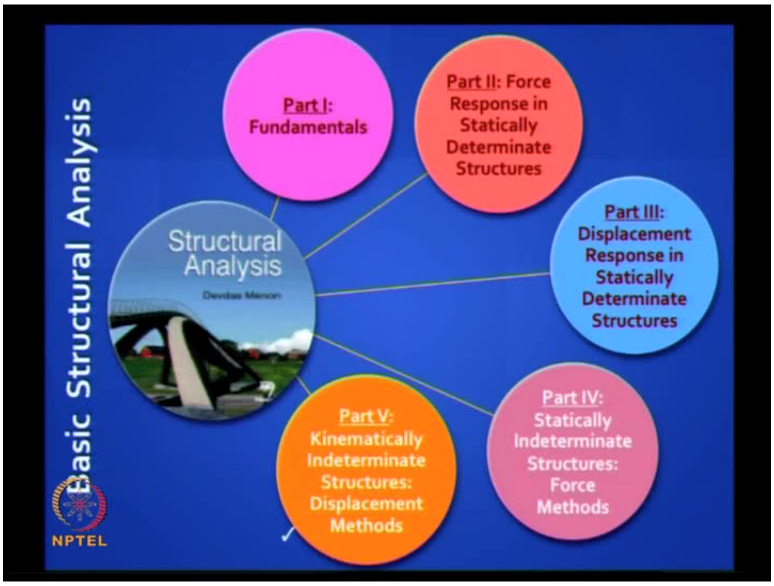
Good morning to you. We are now on to lecture 12 in the second module on the review of basic structural analysis part 2. We will be covering displacement methods which we started in the last session

(Refer Slide Time: 00:21)

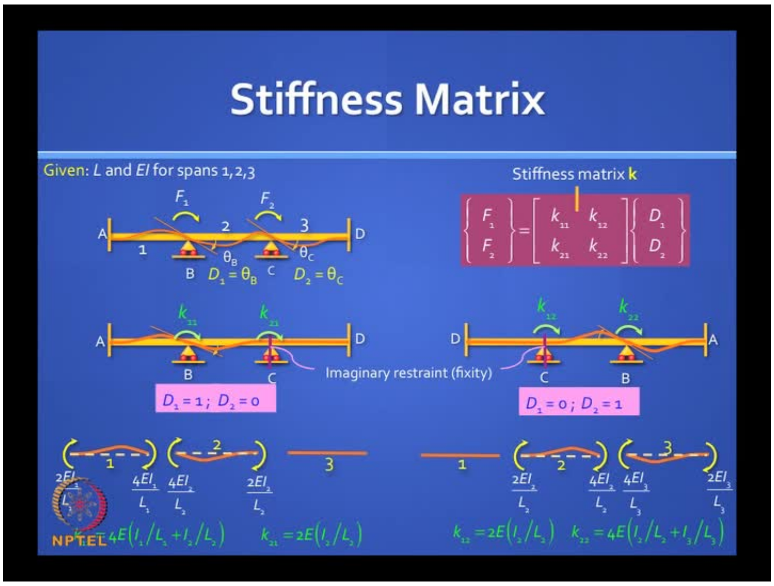


We will continue with the introduction and get into slope deflection and moment distribution methods.

(Refer Slide Time: 00:35)



(Refer Slide Time: 00:42)



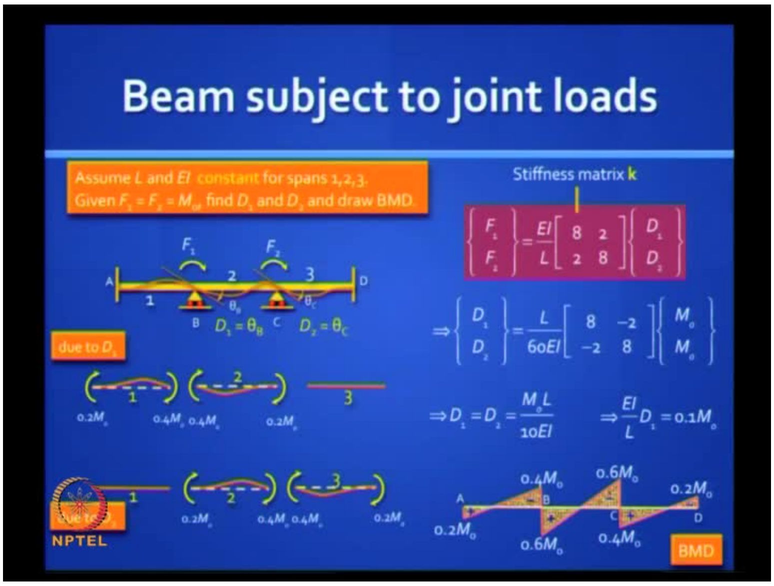
So this is covered part 5 of this book on structural analysis. If you recall the last class, we actually learnt quite a bit about the meaning of stiffness matrix and how to derive the stiffness matrix from first principles you recall that.

Now, can you write down the stiffness matrix for this? given that all the three spans are identical that means they all have the same L and they all have the same E i, what will that matrix look like can we do it?, It will be a symmetric matrix, what will it look like?

(Vocalized Noise)

That is right, that is right, that is right.

(Refer Slide Time: 01:27)



Now let us solve a problem from first principles. Let us take this beam and let us say it is subjected to F 1 equal to F 2 equal to M naught okay, so two concentrated moments applied at the joints B and C and the challenge is to analyze this beam completely, which means find the bending moment diagram after finding the unknown rotations D 1 and D 2 theta B and theta C right?

So, the stiffness matrix for this is this I presume you got this from the last slide right?

How to proceed from here, F 1 is known F 2 is known both are equal to M naught, so just invert the matrix will you find out theta a and theta b quickly. So, inverting a 2 by 2 matrix is simple. The constant E i by L gets inverted to L by E i then the diagonals get then you have to find the determinant and 1 divide by the determinant comes out side as a constant diagonal elements in the main diagonal gets flipped over the cross diagonal elements you will attach a negative sign am I right? will you will you get D 1 and D 2 for this problem. The determinant is 60 is this okay? Simple, so in a jiffy we got the rotations theta A and theta B with the help of the stiffness matrix of the structure.

Now, that you know the displacements it is kinematically determinant can you find the bending moments from first principles? can you draw small sketches showing the kind of end moments you get when you apply only D 1 with D 2 restrain? the answer is both D 1 and D 2 are equal to M naught L by 10 E i, which means E i by L into D 1 is 1 by 10 M naught which is 0.1 M naught, is it clear? this is straightforward.

Now, let us draw the two pictures. The free bodies showing only the moments okay, you will actually have shear forces, let us not draw it right now. Can you fill in the blanks? what are those end moments? remember in the last session we work them out. They are either 4 E i by L or 2 E i by L in this case the carry over moment will be 2 E i by L remember?.

So, this is what it look like in this problem all the E i's are constant and all the L's are constant so do you agree that this will take this form? it is 0.2 M naught 0. 4 M naught that is it we have only two options and in that matrix multiplication you have to add up the contribution of both D one and D two, so you can physically add it up.

Can you draw can you construct the final bending moment diagram with these free bodies? what will be the moment at A? 0.2 M naught will it be sagging or hogging? what is the moment at B? You have to add up.

0.2 M naught hogging, sagging

At B At B.

0.2 M naught hogging, 0.2 M naught hogging.

How can it be? look at this look at this free body, look at the free body due to D 1 you agree?

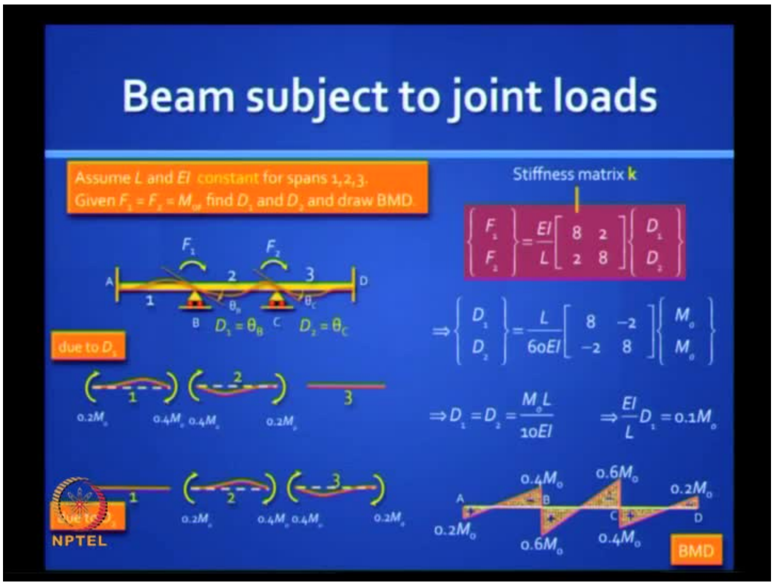
At this junction you got 0.4 m naught right? you got 0.4 M naught this is what you get right? this is what you get. So for, the element 1 that is the bending moment diagram. When you apply D 2 nothing happens to element one, because you have restraint the joint B, nothing spills over to the element one, is it clear? you should have no doubts about this. I am adding those two free body diagrams that is superposition I am saying apply the effect of D 1.

Well, let us look at it from a pure displacement point of view. In the displacement method what is your kinematic kinematically determinate primary structure? it is a structure where you arrest all the rotations. Then you allow one at a time, so we are now allowing only D 1 when you allow D 1 you will have bending moments only in elements one and two, you will not have element moments in element three. When you allow theta C, which is D 2 you will have bending moments only in elements two and three but, nothing in one then you just add up the contributions, is it clear?, So in element one you will get bending moments only due to D 1 when you are doing superposition and not due to D 2 right? in the restrain structure in the primary structure so, what do you get in D 2? yeah what do you get in two, in two you will have to add up both the contributions or in three it will look like this right? it gets flipped over and in element two 0.6 and 0.4 does it make sense?. So we did this from first principles and look at the deflected shape it is more or less showing the right thing you get hogging in some region you get sagging in some other region there is a point of contra flexure does it make sense?

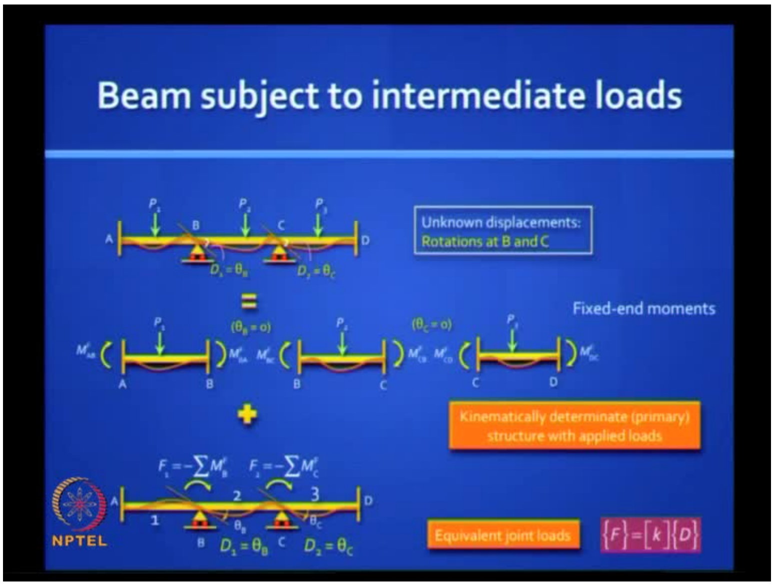
So, we did something from completely different perspective fundamental approach, is it clear?

Now, this is not how the subject of displacement methods evolved. The whole concept of stiffness matrix came later. George Maney discovered invented if you wish the slope deflection method where he made you solve a series of simultaneous equations and get the results very quickly but, this is a much, much better understanding where you really know what is happening step by step.

(Refer Slide Time: 08:36)



(Refer Slide Time: 08:42)



Now, this was an easy problem in the sense the loads were given to you conveniently as concentrated loads F 1 and F 2. In reality, the loads are more likely to look like this right? these are intermediate loads. How will you handle this problem? with the same stiffness matrix right?

So, we are getting into the formulation of the stiffness method of analysis, which is what we need later when we do matrix methods. How will you handle this problem? it was easy enough to analyze the previous problem because the moments were all conveniently nodal moments right? F 1 and F 2 here F 2 and F 2 look like they are 0 but, inside you have intermediate loads arbitrary loads P 1, P 2, and P 3. How will you handle this situation? I want you to.

(( )) be by adding moment and force.

How do you do that?

You are right.

Somehow, you have to convert this problem to the previous problem that means you have to get an equivalent F 1 and F 2 because you can solve by the stiffness method only if you have joint moments. How do you do that? How do you do this conversion? it is called equivalent joint loads. How do you create that situation?

Then it is nice to you know to go through it conceptually. I want you to see how learning takes place. There are two kinds of learning, one in which you see the big picture and you develop the concept that is that is the higher level of development.

The second is somebody does that work for you and you just apply step by step, step one you do this, step two and you solve when you get the answer. We would like you to develop both abilities right? you see the big picture if lay down the map and you also know how to get into the detail and crack the problem and we will do that here.

So I want you to go back to the force method of analysis where. What did you do in the force method of analysis? in the last row.

Yeah, please stand up and tell me please tell me how did you solve the statically indeterminate problem by the force method?

You know it, we have done this before. The problem is statically indeterminate right?, so we know how to solve only statically determinate problem, so what did we do?

Cut in the middle of the

Take any structure, we are talking generic in a truss you do not need to introduce anything so, talk generic somebody else can answer?

Stand up stand up and answer.

Conjugate method beam method we can do.

You know in a truss you cannot use of the conjugate, so talk generic what is a basic principle we follow? one of you one of you girls, stand up and answer.

Choose a primary structure.

Choose a primary structure, so that is the first thing you are comfortable with statically determinate structure so, some how make the structures statically determinate how do you that?

By choosing.

By identifying the redundants and making providing releases which could include providing hinges that is one way of providing a release okay, sit down. (Refer Slide Time: 08:42)

So, we did it there, we can work only with a statically determinate structure then we sequentially apply loads including the redundants on that primary structure which we can handle and we do superposition. So, you can apply the same concept here, so what is your primary structure here? is it a statically determinate structure? No this is the displacement of analysis. What is a type of indeterminacy here? what is a type of indeterminacy

Kinematics.

Kinematics so, what is your primary structure

(( ))

A kinematically determinate structure is one in which you arrest arrest the rotations we arrests rotation B and C you do not get any moment at B and C you will get deflections but, you arrested the two unknown you see there is a similarity in the two approaches, so shall we do that and this is something we can handle, so we arrest it. The moment you have arrested those rotations at B and C there is no interaction between those three beams any more right?

You can actually separate them out and look at them independently and you are suppose to know everything about statically kinematically determinate structures, so you must know the end moments, so fixed end moments and we know how to do that, we can actually show you tables which can handle any type of loading you can get a table of fixed end moments, but we at least would remember what is the fixed end moments when you have a concentrate load at mid span or U D L, which are the most common cases, so we know M F A B, M F B A, M F B C, M F C B, and M F C D anyway M F D C, is it clear?

We apply some formula and get those moments. Now what we will do? follow the same procedure we did in force method but, using displacements. What we will do next logically? there what did we do, we satisfied compatibility here we have to satisfy equilibrium, how do you satisfy equilibrium? right now you do not satisfy equilibrium because you are getting moments at B and C but, in the original structure there are no moments, so what should you do?

We see the moments.

How do you do?

Keep the moment for this and analyze separately without moment.

No.

Keep couple of moments.

No, you have got these values, let us say at B you have M F B A and M F B C clockwise positive what should you do?

To make it 0 we have to introduce a moment and then.

Perfect, you have to make that thing 0 then only you will satisfy equilibrium because you know original structure M B is 0. How do you make it 0?

We will make it 0 by adding moment and also subtracting moment in the separate case and superposition.

No, you need to do only one case so, you can do this. (Refer Slide Time: 08:42) Go back to the old beam apply nodal moments F 1 and F 2, which is the negative of the moments that accumulated there when you artificially arrested those rotations.

Now does this make sense, so the argument is when I add up this picture with this picture I get back the original picture right? I am satisfying equilibrium because there are no moments, I am adding these two and subtracting them here, I am putting it in the opposite directions here, so when I add up everything here I get what I get here in this case there is no nodal moment here there is no nodal moments here you got it? this is a powerful method.

Now, do you know how to handle this problem? we just did it. So, we cleverly converted intermediate loads equivalent joint loads crack the problem solve this and after you get the solution of this you add up the response that you get here and that is the complete solution.

So, you have to add up the bending moment diagram or bending moments that you get from the fixed end condition to what you get from the nodal moment condition, is it clear? do you have a doubt?

Sir basically we using the arresting the rotations at B and C for.

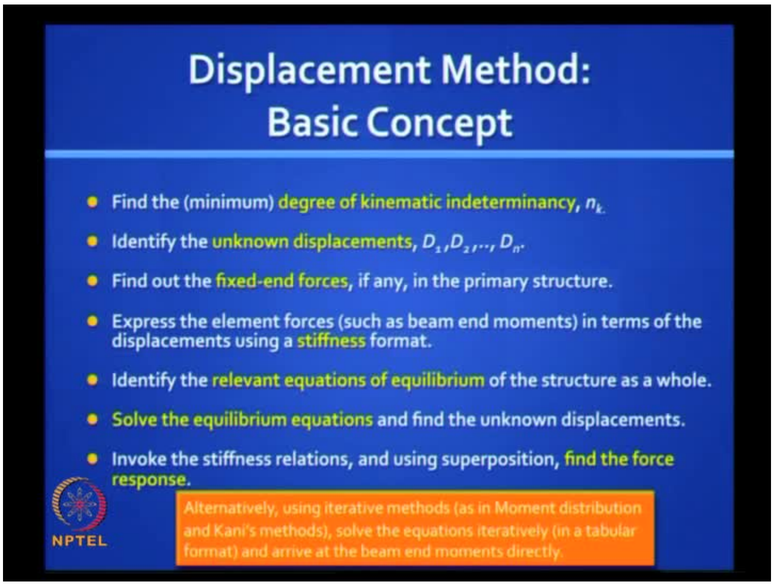
To get the primary structure and then releasing them and you can do it actually one at a time, we saw it. If you do everything together you need a stiffness matrix to help you out is it clear does it make sense.

There is another interesting point I want you to note. These are called equivalent joint loads F 1 and F 2 here are called equivalent joint loads they are joint loads which give you the same rotations as you had in the original structure at the coordinate location identified. The D 1 and D 2 that you get by analyzing this structure which you can get from the stiffness matrix will be exactly equal to the D 1 and D 2 from in the original structure, why? because this is equal to this plus this (Refer Slide Time: 08:42) and D 1 and D 2 in this system is 0.

That is a powerful concept and this is the real foundation the real basis of displacement methods all displacement methods follow this path though sometimes in textbooks they teach them as independent disconnected chapters you study moment distribution method in one chapter you probably learnt that way you study slope deflection method in another chapter then you study stiffness method in a third chapter they like short stories in a in a rather than a novel where the chapters are continued.

So, we are going to look at all together because they are all the same they all the same this is the underlying concept and you can use this beautiful equation which is essential in equilibrium equation which relates forces with displacements.

(Refer Slide Time: 18:43)



So, let us summarize the first step in displacement method is to identify the degree of kinematic indeterminancy take advantages of modified stiffnesses and reduce a number as we did earlier from 5 to 1 we did it. Identify the unknown displacements they will as many unknown displacements as the degree of indeterminacy find out the fixed end forces that is you arrest the structure if any meaning if you did not have any intermediate loads you would do not have to go through this step, in trusses you do not need to go through this step of arresting because all your nodes loads are always nodal there is no intermediate loading there it is only in beams and frames you have this.

Then you express the element forces such as the beam-end moments in terms of the displacements using a stiffness format remember, we wrote that 4 E i by L into D 1 and 2 E i by L into D 1 so, you have to go through this step.

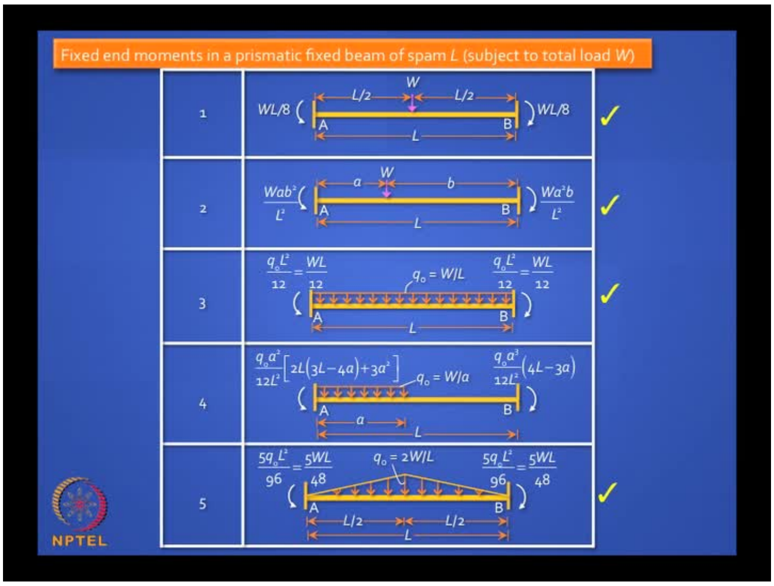
Then put it all together identify the relevant equations of equilibrium of the structure of whole, so you have the F equal to K D equation you have to bring into this format you know K, which is the property of the structure F is given to you or you find it in equivalent terms and then you invert that matrix and or solve it by some system and find the unknown displacements you know the relationship between the unknown displacements and the end forces, so find the force response this way.

There is a 1 to 1 correspondence between the path that the force method takes and the path that the displacement method takes but, they look completely different but, they lead to the same goal and it is you should be comfortable doing both methods okay.

Now alternatively instead of solving simultaneously many equations, there are iterative methods which will give you directly the beam-end moments so the moment distribution method in Kani's method unlike the slope deflection method this we will study shortly give you the moments directly.

In fact, there is a question that is often asked is moment distribution method of force method because you get the moments directly you do not need to find any unknown rotations or displacement and that is because you bypass this step of explicitly finding the rotations and calling this invoking this stiffness relations and just distribute the moment directly to the to the component beam elements but, the path is the same, the overall path is the same as is it is just that in solving the equations you do not solve the unknown displacement the explicitly and then substitute and get the moments you bypass that and you solve it iteratively, so it is a displacement method because your governing equation is an equilibrium equation.

(Refer Slide Time: 21:46)



So, you need to know fixed end moments under various loading conditions they are available in many handbooks in the books that I referred to it is all available in a table, so in practice you can look at these and pick them up but, you should know at the very least some standard load cases like these.

The first load case is a concentrated load on a prismatic beam of span L, load is W so the fixed end moments will be hogging at both sides equal to W L by 8 its symmetric.

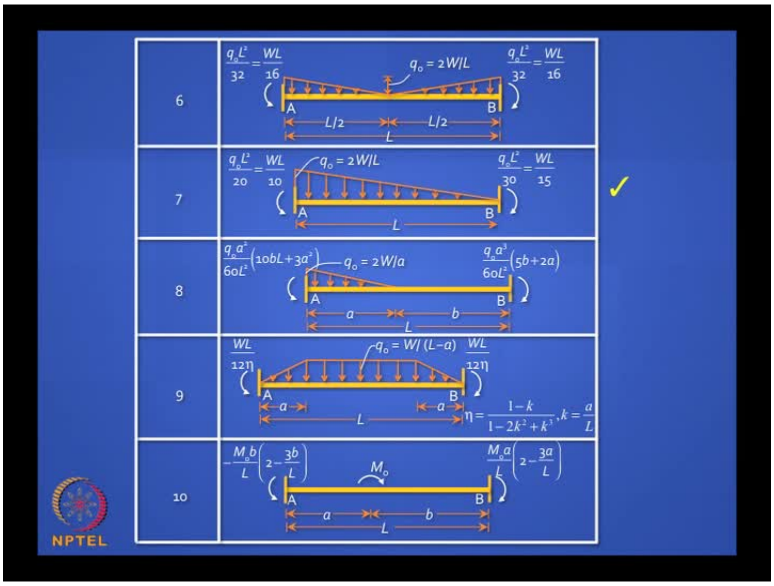
Second load case also you should know if you have an eccentric load with those dimensions A and B it will be W A B square by L squared and W A squared B by L square these two formulas you should know.

The third one is very easy, if your total load is Q the intensity is Q and you have a uniformly distributed Q naught then the fixed end moments are Q naught L square by 12 or W L by 12 where W is the total load which is Q naught which is equal to Q naught into L. The last case if you wish. Where do you get that triangular load?

(( ))

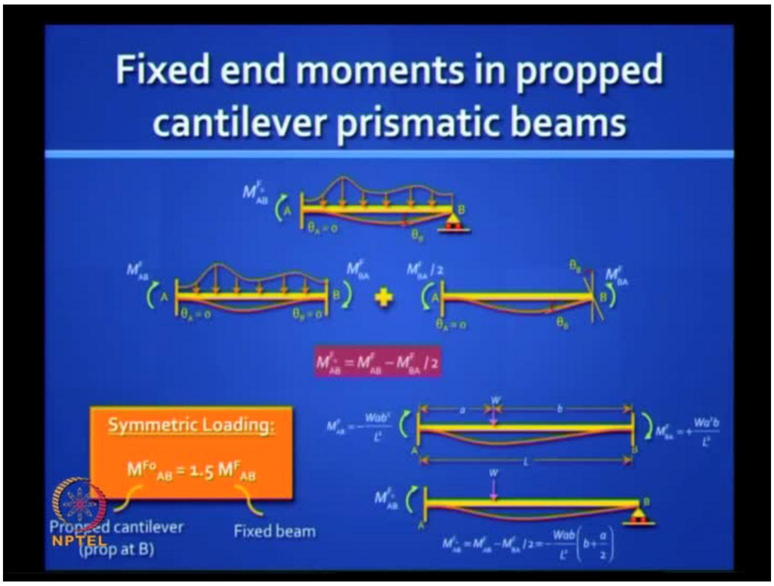
Yeah, for example in two-way slabs the short beams will get triangular load component from the slab, so there the formula is 5 W L by 48 just remember these formulas if you forget them which is the fastest way of recovering those formulas conjugate beam method, which we have studied at length.

(Refer Slide Time: 23:27)



Then there are some more load cases you can skip all of them except the seventh one. Where do you encounter this triangular load distribution? usually earth pressures on retaining walls or hydrostatic pressures okay, if the total load is W remember it is W L by 15 and W L by 10, obviously the higher value will be where the intensity is more. So, remember those formulas in case you forget them, you can derive them the others are not very important.

(Refer Slide Time: 24:04)



Now, you know the fixed end moments in beams which are fixed-fixed but, sometimes you can take advantage where the far end is a hinge so it is a it is a prismatic propped cantilever now and you remember I used a notation earlier I introduced this M f naught A a to remind you at the at the other end you do not have fixity it is a released, right?

Now, do you need a separate set of formulas for these? or you can pick them up from the fixed-fixed beam? You can pick them up from the fixed-fixed beam using a simple logic.

Let us take the same beam and fix the end B, you are supposed to know the fixed end moments M F A B and M F B A how is M F naught A B related to M F A B and M F B A and all you need to do is to add to this another picture where you reverse M F B A, right? When you do that at the end A you will get a moment, what is the moment you get?

M F B A by 2.

M F B A by 2, we have studied that, right? and it will be anticlockwise, so you get M F B A by 2, so when you do superposition obviously, this is equal to this minus that if you say see it as clockwise positive it will be usually add up and if it is a symmetric beam, let us take a case like this you can easily work it out, so it is easy to remember. If it is symmetric it is even easier what is it?

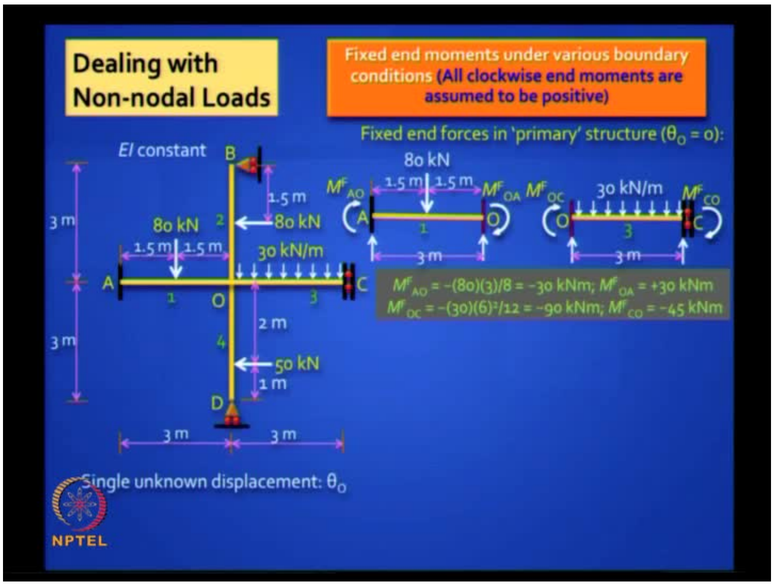
It will be?

Yeah the additional thing is the half so that final thing is?.

Three-by-fourth of it.

Common, this you can work that it will be one and-a-half you got 1 and you got half extra from the carry over is it clear? easy to remember, so you must take advantage of this?

(Refer Slide Time: 26:08)



Now, let us quickly do the same problem we attempted earlier but, with not a concentrated moment at O but, with all kinds of loads all over the place right?

We will do it fast because you are not going to actually do problems of this kind but, I want to show you it is not difficult you still have only one unknown rotation theta naught, so here now what is the first thing you have to do after identifying theta naught?

you must arrests that rotation at O and you will get fixed end moments you have to separate out those four beams.

So, let us take the first one, if you separate it out now we are saying M f a naught will be having a negative value because we are taking a sign conventions that clockwise moments are positive. Can you write down the value of M f a naught for this loading condition? it is equal to minus of 18 into 3 divided by 8.

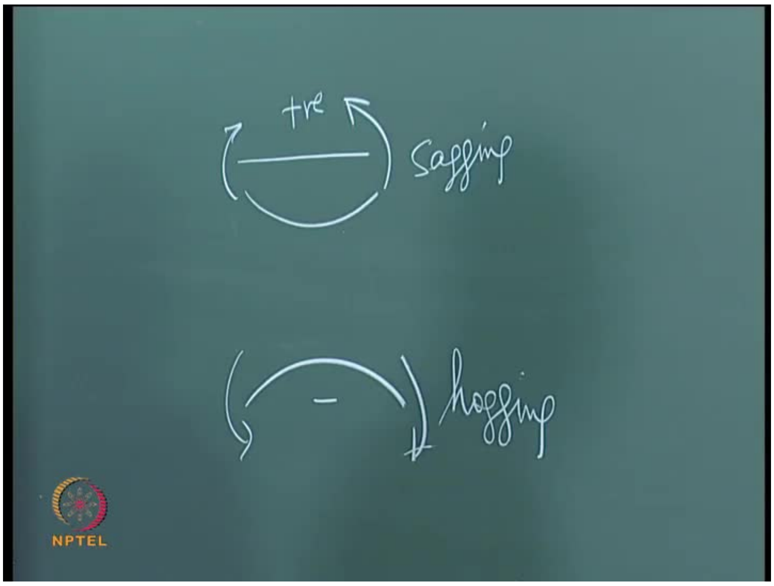
Divided by 8 you are right.

What about the second case?

(( ))

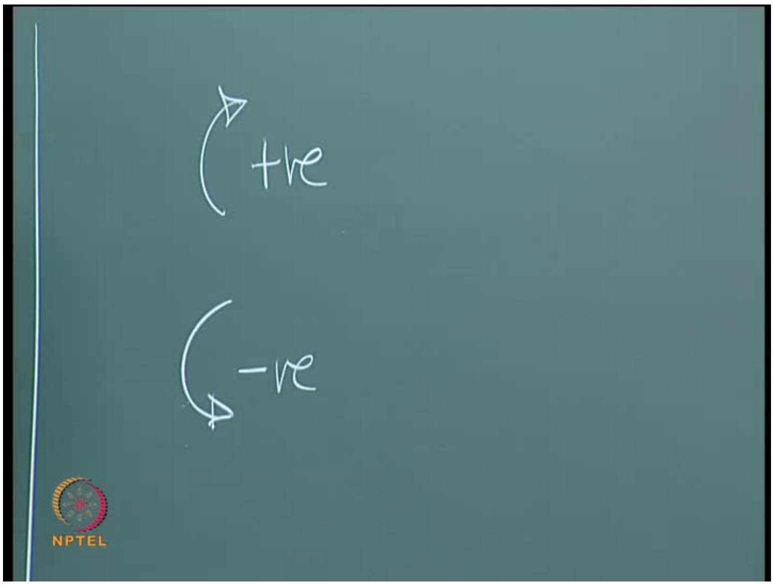
Q naught L squared by 12 right. So, you can write those equations if you get one you can get the other it is symmetric remember the left side will always be negative because it is actually anticlockwise and the right side will be clockwise do not mix up this sign convention with the sign convention of sagging positive and hogging negative which we use when you we draw bending moment diagrams is it clear?, So do not get confuse with that.

(Refer Slide Time: 27:59)



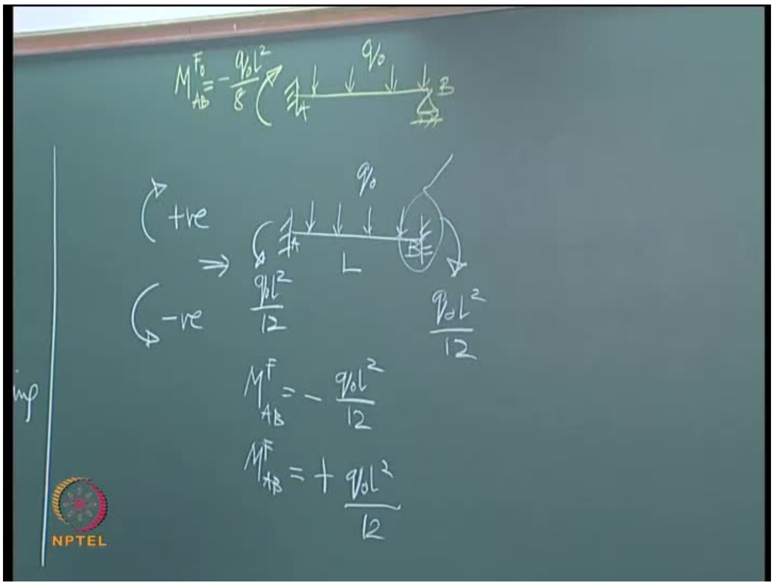
Let me clarify. Our earlier sign convention was if you apply moments like this it will like this, this is positive and this is negative. This is sagging and this is hogging, which we will still use when we draw bending moment diagrams so, let this convention be there.

(Refer Slide Time: 28:32)



Now, for the sake of analysis for the sake of analysis we are using a different convention. The convention we are using is clockwise whether it is on the left or right it is positive and anticlockwise is negative historically this has been the convention so, you will see this is positive as far as the beam-end is concerned and this is negative.

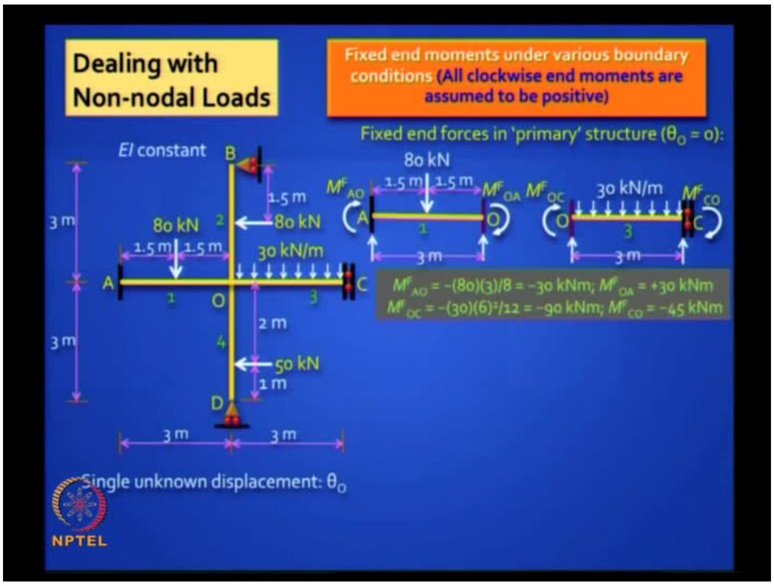
(Refer Slide Time: 28:54)



So, using this convention, let us say you have a U D L, Q naught L you can write it this is the correct answer we all know that but, you will write M F, let us say this is A and this is B you will write M F A B as minus Q naught L square by 12 and M F B A as plus Q naught L square by 12 and if by chance this end were to be a simply supported end with the same loading A B, Q naught then the notation we will use clockwise positive is M F naught A B and this will turn out to be minus one-and-a-half times this, which is Q naught L square by 8 is it clear? These are the sign conventions we will follow.

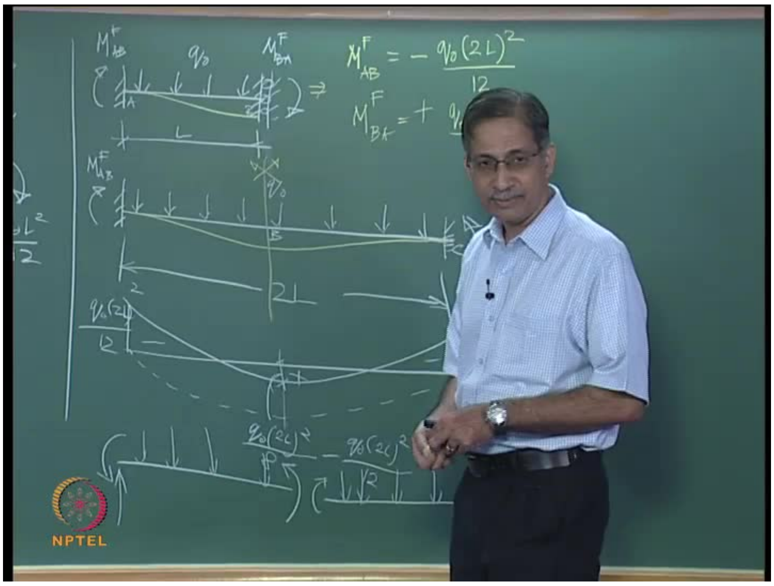
After we draw the final end moments and free bodies then we may draw a bending moment diagram there we put plus and minus in the bending moment diagram but, for analysis by displacement method we will follow this convention. Later we will reverse this convention for a very important reason which has to do with vector algebra okay, but traditionally this is how it is being done, is this clear?. All of you are clear?

(Refer Slide Time: 30:46)



Now, the second case is interesting. We are trying to take advantage of that guided fixed support, so how will you solve that problem?

(Refer Slide Time: 30:53)



So now we are saying, let us say you have a problem like this.

(( ))

Wonderful, you are right.

Let us say this is Q naught and say this is A this is B and this is L. I want to find out these values. What is a M F A B and what is M F B A.

(( ))

Perfect, he is right.

So, remember whenever you see a picture like this it reminds you that there is a mirror image on the other side which will give us the same condition because this deflected shape will be like this and if we had a beam, which was twice this length 2 L with the same loading Q naught that would deflect in much the same way and here right in the middle you have that condition of symmetry this is usually shown like that.

So, you can work backwards this we know these answers do not change, this we know let us say this is B and this is C, so M F A B does not change and if you remember the bending moment diagram here will be, right? hogging hogging sagging this moment is Q naught into not L, 2 L the whole squared by 12 and what is going to be this moment at the middle? well remember if it is simply supported it is a same curve like that, what will be this value? Q naught into 2 L the whole squared by 8 the whole curve gets lifted up by Q naught 2 L by whole, so remember that you might forget but, you remember it is Q naught 2 L the whole squared by 8 minus which turns out to be (( )) that is right that is right by 12 if you remember the final answer go ahead and do it straight away 24 and it is important to note when you break it up into 2 this is going to be hogging, this is going to be sagging remember.

Does it make sense?

And there is no shear here but, there is a shear force here shear force here which will equilibrate with the U D L very interesting not very well-known, is it clear? (Refer Slide Time: 30:53).

So, at the end of the day what are the answers you will write for this problem M F A B? will be minus or plus?

Minus.

Minus, it is hogging right?

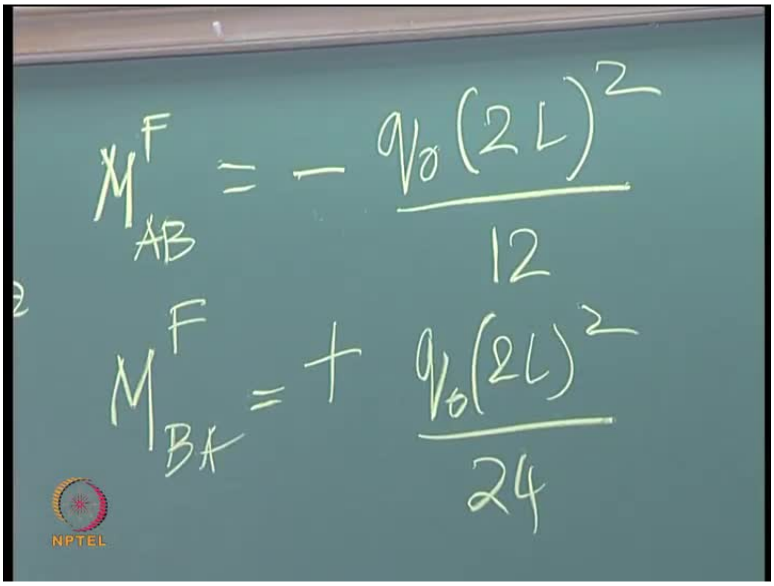
Minus Q naught L square.

Q naught not L square again you are making 2 L squared by 12 and M F B A will be what? well you can write this equation if you wish, is it plus or minus?

Plus.

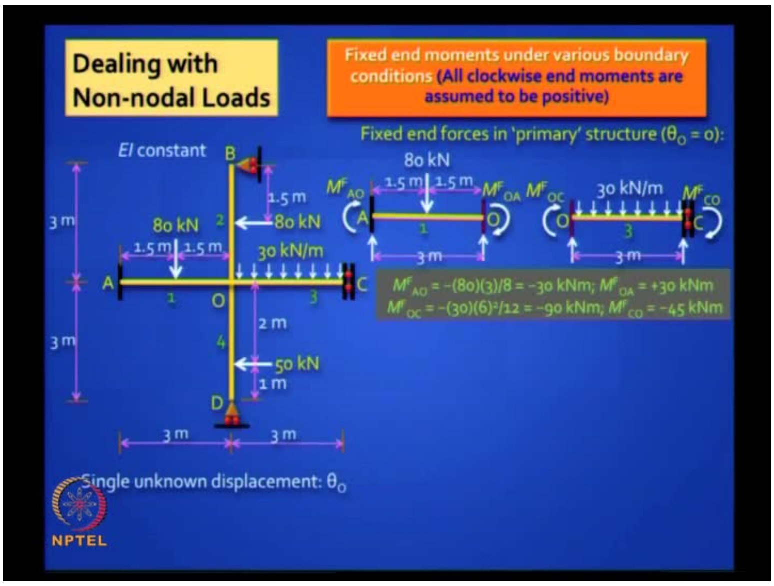
Plus, it is sagging. It is plus Q naught by 24 which is this half that value.

(Refer Slide Time: 35:10)



You have learnt something here so this is very interesting and that is how we get those answers here.

(Refer Slide Time: 35:18)



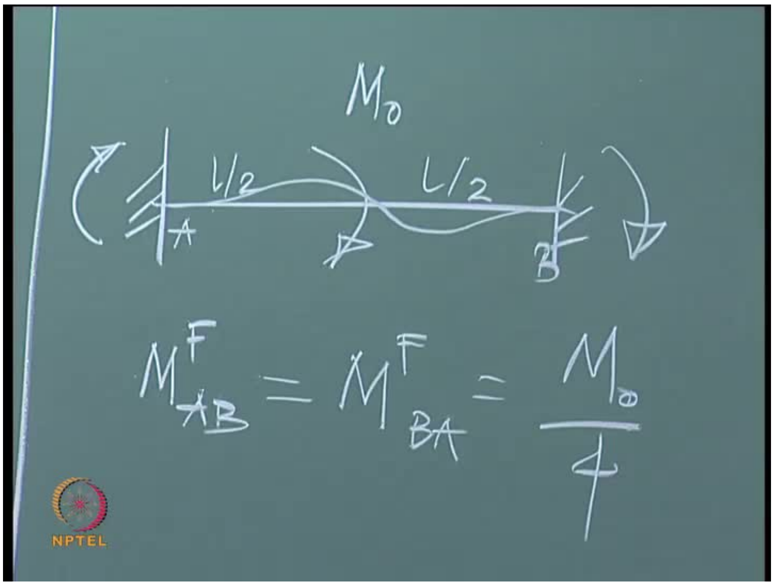
M F O C you can see here M F O C is minus Q naught L squared 2 L the whole square right? L is 3, 2 L is 6 and this is half that and I think there is a mistake here this should be; no?, it is right it is right it is right, this is right please note this is sagging which means it is this way which means it is minus it is right. (Refer Slide Time: 30:53).

(( ))

Yeah.

No, no, no look at this picture (Refer Slide Time: 30:53), this is sagging, so this is anticlockwise which in this sign convention should be minus, so this is a rare case where not only this is minus but, this is minus on the right side usually it is minus and plus left side is minus and right side is plus on the gravity loading this is one rare case where the right side is also of the same sign as a left side which means both are minus, is it clear? Have you understood? (Refer Slide Time: 28:54)

(Refer Slide Time: 36:30)



There is another rare exception I might well point it out. Let us say you have a concentrated load M naught right in the middle of a beam. The deflected shape will be right? deflected shape will look like that and M F A B, what do you think the direction here will be? clockwise or anticlockwise?

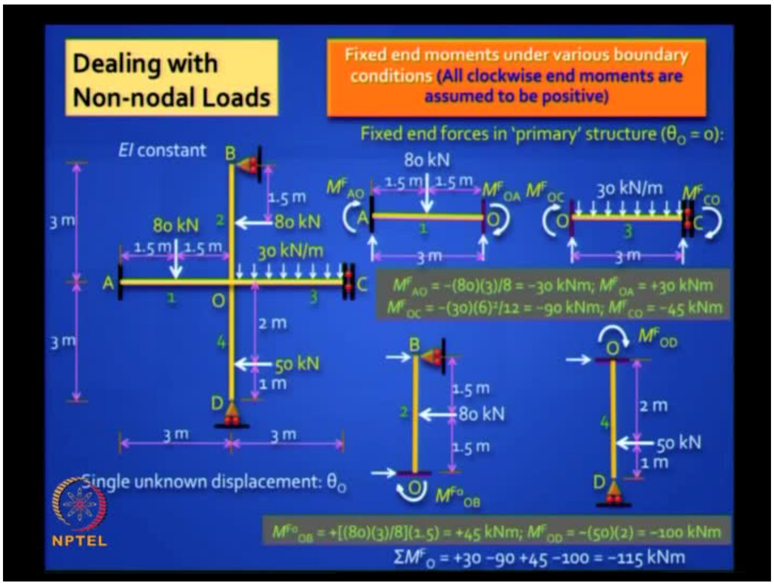
Clockwise.

Where this part looks like is being lifted up, so it will be clockwise and this will be this will be also, no it is being pushed down pushed down it will be clockwise, so this is another rare case where M F A B is equal to M F B A and the answer is the it is worth remember remembering is M naught by 4 some standard cases worth knowing okay.

In the table, I have given you a generalize formula where that concentrated moment is located anywhere in the beam but, in the middle it will be M naught by 4, so far so good.

Can we proceed?

(Refer Slide Time: 37:54)

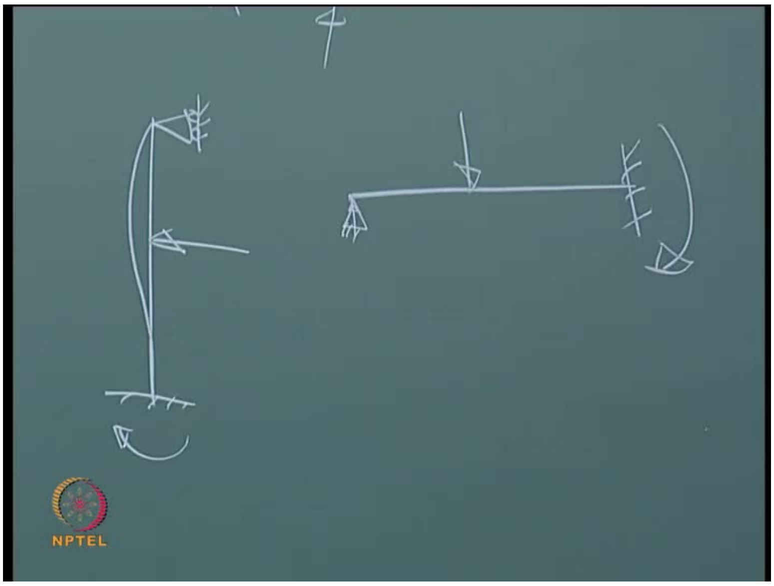


You take these two remaining cases take this case this case is this beam sorry this beam where you have a roller support on top, so this is easy to calculate what is the answer M F naught O B will be one-and-a-half times what?

W L by 8.

W L by 8, so one-and-a-half times W L by 8 and it is positive because I mean you have to rotate it and then you will you will realize why is it positive got it, is it clear? Is not it is positive, you have a doubt on that? not clear to you? No, okay is it crystal clear or not?

(Refer Slide Time: 38:46)

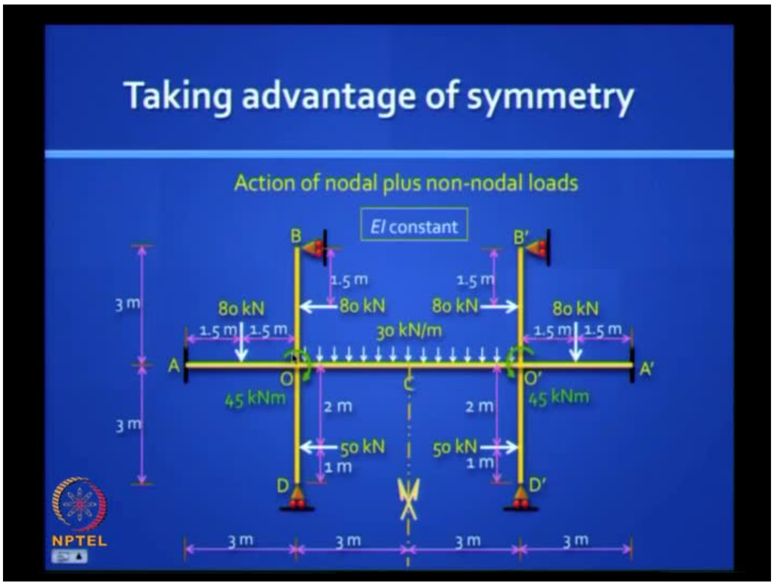


Okay, so here you have a situation where that beam is vertical you have a case like this right? you have arrested this end and this you probe though it is shown like this where is your load acting? the load is acting here what is the deflected shape? all this you should bear in mind it is like this so you can turn it over if you are more comfortable with this situation, now you do not have any doubts this is going to be always clockwise, is it clear?

Are you okay?

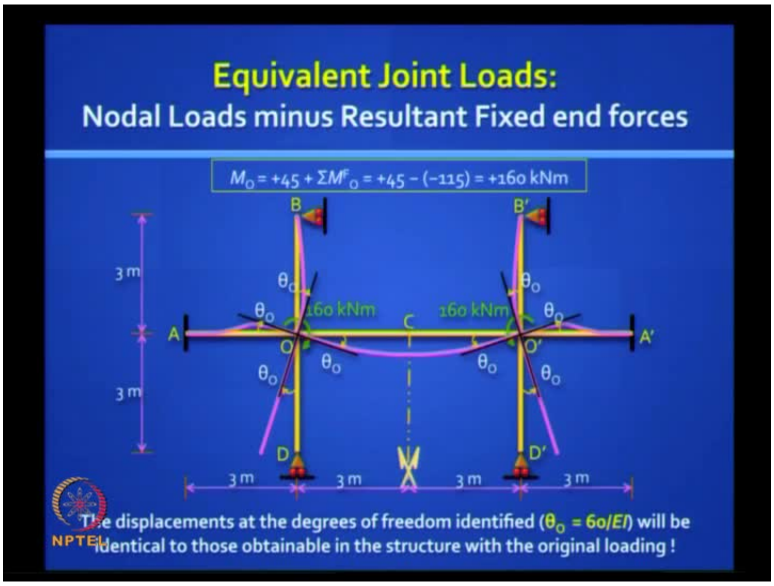
So, do not make mistakes with signs it is very important, got it? So, M F naught, so all these are easy to calculate. Now if you add up all the moments that accumulate at joint O you will get something in this case you get minus 115 kilonewton, so this is the equivalent joint load which you can handle easily.

(Refer Slide Time: 39:54)



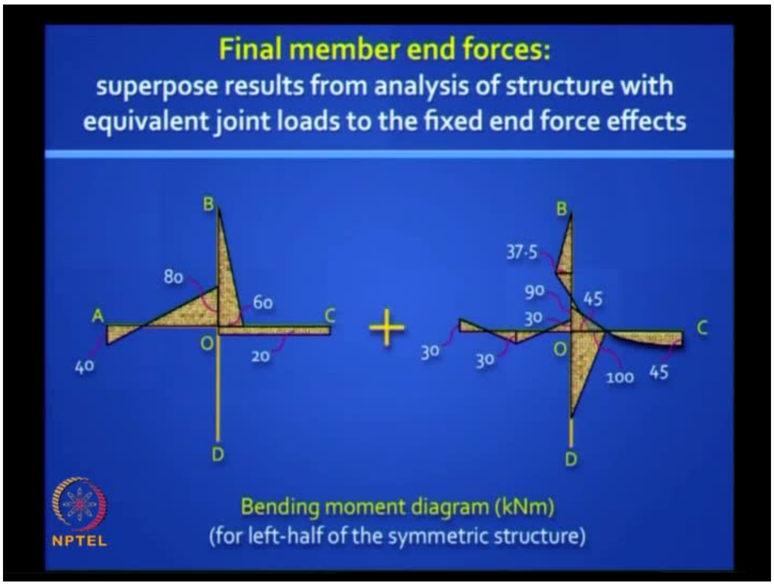
Now, let me take another case where do not worry do not take this down just want to demonstrate where you also have in addition a concentrated moment here and here of 45 kilonewton meter, now this picture looks scary, is it not? but, you are brave warriors who have learnt displacement methods and you will first recognize this is symmetric and you will say if you cut one-half you will do what we did in the previous slide.

(Refer Slide Time: 40:34)



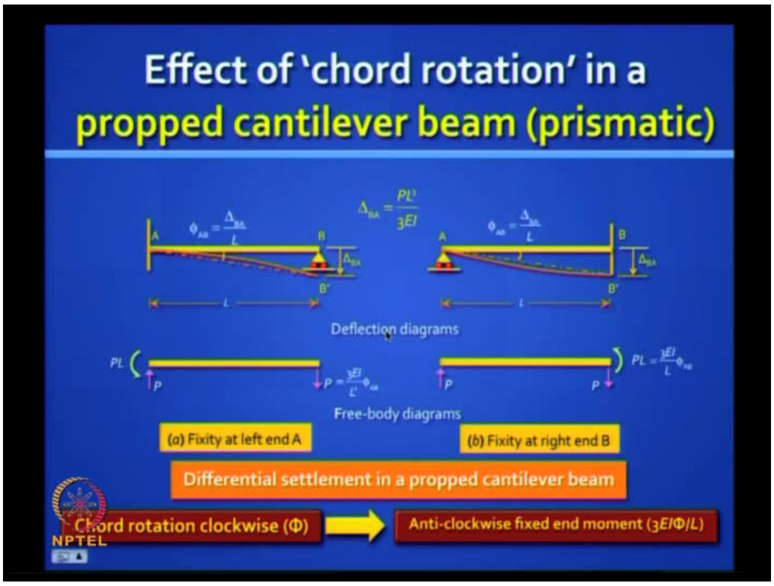
The only change is you have an additional concentrated moment 45, which you have to add which you have to have to add and 45 and 115 when you add because this you have to put negative sign or to add up you get 160 and this we have done in an earlier class and you can solve the problem, is it clear?, so you have to use our brain, you cannot say I am sitting there it is not coming naturally to me until you have learn to master this subject clear?

(Refer Slide Time: 40:58)



So, do not forget there is a fixed end force diagram which is here which you are suppose to add to the diagram that you get from the nodal moment then only you get the complete solution because the actual problem is a distributed load problem and do not forget to add that

(Refer Slide Time: 41:17)



One last point to note before we start the slope deflection method. What happens if you have in addition to rotations, you have translations right? in additions to rotation you have translations, so let us take the same beam element and we have two types.

Let us first say that one end is hinge in the other is fixed, so let us say you have a deflection at B related to A of delta B A which is the same as saying you have a clockwise rotation 5 which is delta B A by L, so here the right support went down here also the right support went down but, we flipped the two ends now this is fixed and this is hinged okay, so here the left end is fixed here the right end is fixed can you draw the free body diagrams and the bending moment diagrams.

What is the answer?

Well you can we did it recently. What is the relationship?

(( ))

Okay, what are the values? what is the fixed end moments you get?

3 E i by L.

That is right, what is it?.

3 E i by L.

Well, this is true you I mean you are going to have a downward reaction at B this is true. What is p, how is it related to delta?

Sir 3 E i is delta.

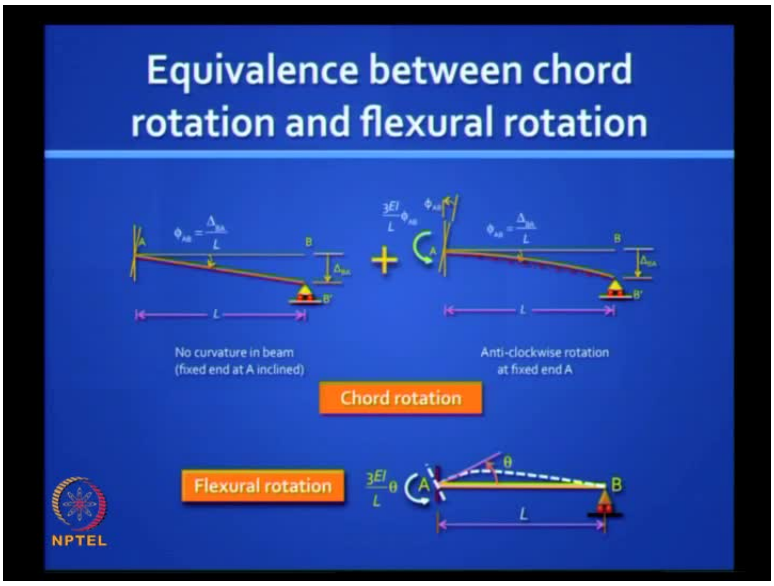
Delta is P L Q by 3 E i you are right.

So, that is how you get this relationship and if you substitute you get 3 E i, L square into five and this is your answer (Refer Slide Time: 41:17) your fixed end moment is 3 E i by L into 5 or 3 E i by L squared into delta, is it clear?

It is very interesting to note that you have a clockwise rotation here but, the moment you get is anticlockwise clockwise chord rotations. Now I want to ask a question, this 3 E i by L is familiar to us you get three E i by L in a propped cantilever when you when you apply a moment unit rotation the relationship between, is it not? there is the 3 E i, is that 3 E i by L and this 3 E i by L related, is it related there you had a flexural rotation here you have a chord rotation, is there a relationship, can you give a physical meaning to it? We will try.

We will try.

(Refer Slide Time: 44:00)

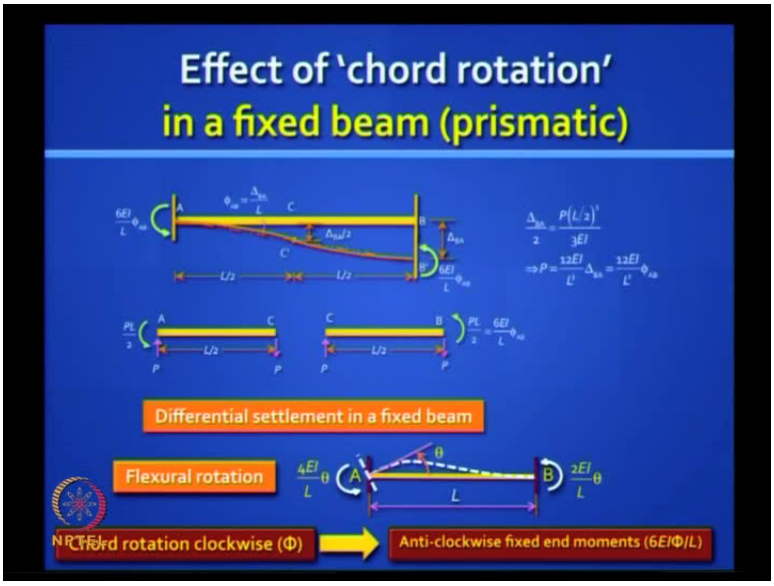


So, we will do in clever way. Let us say originally itself this was like this, this was straight okay. Now what do you need to do to get other picture, you have to rotate A in the anticlockwise direction that means you are now visualizing a rotational slip, if you do that and you get the same shape we got earlier and when you do this what is the moment that you need to what is the reaction moment that you get?

3 E i by L.

3 E i by L, so you know there is a one to one similarity and this is very interesting and you can you can you know use either interpretation and get the same answer.

(Refer Slide Time: 44:47)



Similarly if you have we have done this recently, if you had both ends fixed, what is the fixed end moment that you get? instead of 3 E i by L into 5 you get you do not get 4, you do not get 4, you do not get 3 you do not get E i by L, or you do not get 2 E i by L, you are trying all permutation and combinations. Let us do it this other beautiful intuitive way okay, we inclined both the supports with rotate one, when you rotate one what is a moment you get there? 4 E i by L and what you get at this side?

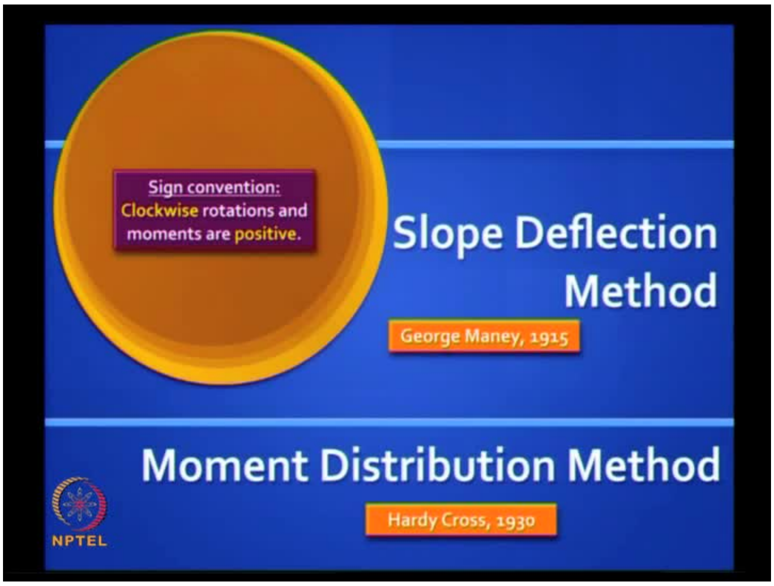
2 E i by L.

2 E i by L, then you rotate this also, what do you get here? you get 4 E i by L and you get 2 what do you get totally?

6 E i by L sir.

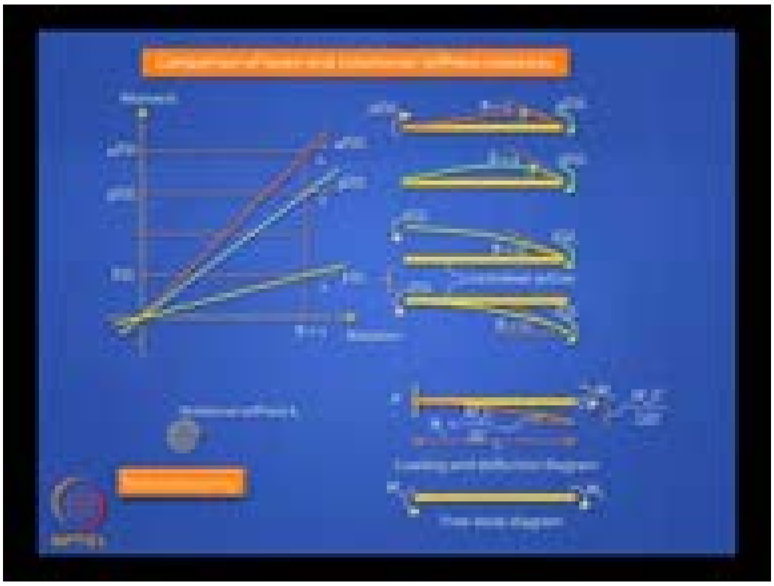
Right, so you forget that number 6 and you can prove it you can prove it and it is very easy and very elegant you can also do it from deflection saying that the deflection in the middle is delta naught by 2 remember. So, you can prove this easily so in general when you see pictures like this you look for chord rotations. If the chord rotation is clockwise you get an anticlockwise fixed end moment it is either 3 E i by L if the far end is hinged or it 6 E i by L at both ends if both ends are fixed right? and vice versa.

(Refer Slide Time: 46:17)



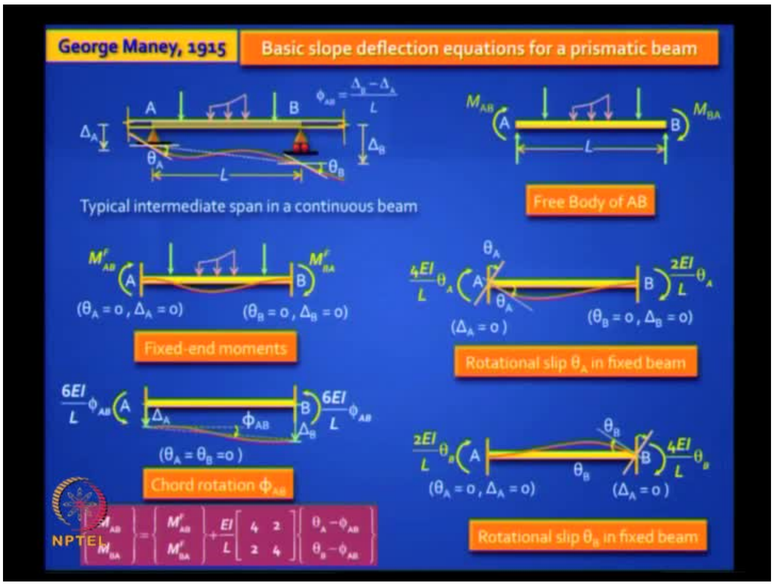
Okay, now we start with the slope deflection method. Let me just introduce it to you it was formulated by George Maney in 1915, it is the foundational method it is where all the displacement methods started and subsequently hardy cross in 1930 he was a professor at Illinois, he suggested a simple method called the moment distribution method and parallelly in east Germany Gaspar Kani proposed his method called Kani's method. Now, throughout this we will assume a signed convention which I have already mentioned that clockwise moments and clockwise rotations are positive.

(Refer Slide Time: 46:58)



You will recall this that if you want to take advantage of reduced indeterminacy then you can play with those stiffness values 4 E i by L and 2 E i by L combination or it is 3 E i by L and 0 or it is E i by L and minus E i by L, is it clear? So, you remember this.

(Refer Slide Time: 47:17)



Now, this is how George Maney proposed this method. He derived some basic slope deflection equations for a prismatic beam. Now let us imagine we have a continuous beam or a beam which is part of a frame it is a same thing it does not matter.

Let us take a continuous beam and let us take some intermediate beam in that system called A B, right? You remember we wrote the three moment equation earlier well it is similar to that and let us imagine the worst kind of loadings possible very messy intermediate loads. Let us also assume that the support deflected this support deflected by delta A and delta B and let us say the deflected shape is this funny shape like this for the beam element A B. We are trying to preserve in this shape some signed conventions three signed conventions we preserved for theta A, theta B and rotation chord rotation phi A B all are clockwise the shape I have shown theta A is clockwise it is a flexural rotation, theta B is clockwise it is a flexural rotation and chord rotation for that entire script phi A B is also clockwise can you see that?, I have assumed delta B to be more than delta A, if delta B is equal to delta A then that chord does not rotate at all you do not have a problem, you do not have differential settlement, is it clear?

Now, how do our very interest is to write expressions for the end moments at A and B using so this is what you want, this a free body A B in the free body of A B you do not show displacement because you free body from the external constraints, so can we right an expression for M A B and M B A from first principles?

We can, how do we do it in displacement formulation? you have to first arrest you are kinematically determinate structure is one in which theta A and theta B are both 0, so you do that you catch hold of theta A theta B artificially pin them arrest them so that they do not rotate then due to the loads you get fixed end moments which you are supposed to know how to calculate right? so no chord rotation and no flexural rotation you know how to get M F A B and M F B A, clear? Next what do you do? release theta a see you are you are rediscovering you are reinventing this beautiful method.

So release theta A let it rotate. When you do that what is the moment that you get far end is fixed in this case? It will be it will be 4 E i by L theta A and 2 E i by l theta A on the other side, agreed? So we have to superpose these then we have to superpose two more pictures.

What is a next thing we will do?

Release theta B.

Release theta B, but arrest theta A because we are going to do superposition we will do that and you will this very straight forward. Finally you have to do one more picture, what is that? take the primary structure and do not forget there is a chord rotation give that also then that completes the picture. Let fix them allow that chord rotation delta A comes down delta B comes down there is a rotation phi A B and the moments are if you have a clockwise chord rotation you have anticlockwise moment 6 E i by L into phi A B, is it clear?

Now can you write down equations for M A B and M B A what will they look like from these four pictures? if you add up these four pictures you get back the original pictures both in terms of the force field and the displacement field. So what they look like this M A B is M F A B plus 4 E i by L theta A plus 2 E i by L theta 3 minus 6 E i by L phi A B which can be written in this format M B A is M F B A plus 2 E i by L theta A plus 4 E i by L theta B minus 6 E i by L phi A B. We will stop here with this introduction.

Thank you.