


**Advanced Structural Analysis**  
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**Module No. # 2.4**  
**Lecture No. # 10**  
**Review of Basic Structural Analysis-2**

Good morning this is lecture number 10 on the second module on review of basic structural analysis

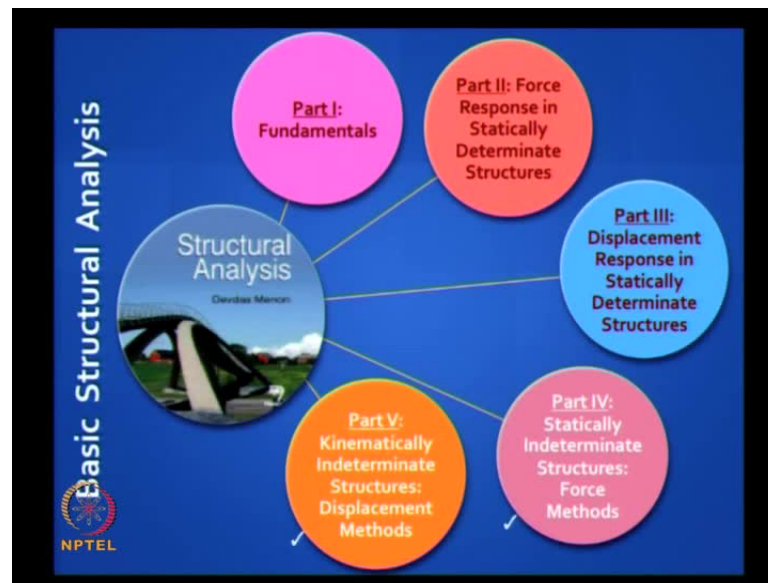
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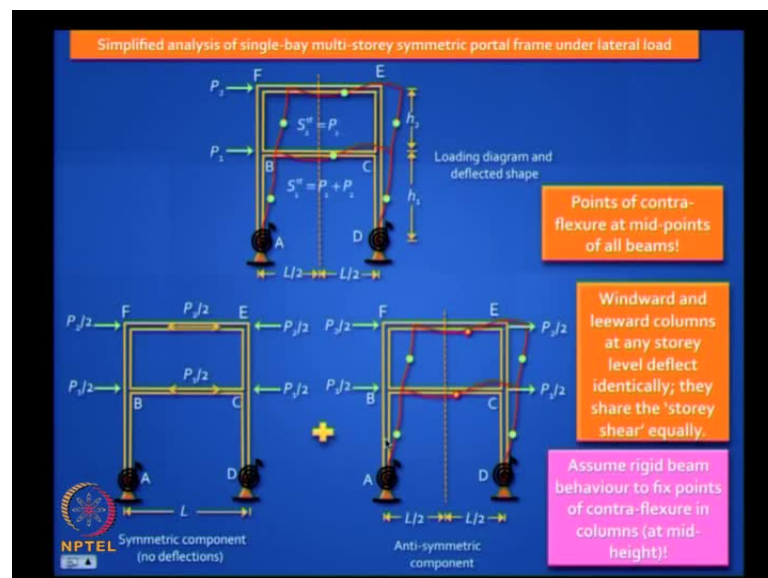
**Module 2: Review of basic SA - 2**

- **Force Methods - Statically Indeterminate Structures:** Introduction; Method of consistent deformations; Theorem of least work; Approx. methods of lateral load analysis. ✓
- **Displacement Methods - Kinematically Indeterminate Structures:** Introduction; Slope-deflection method; Moment distribution method.

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In this session, we will complete approximate methods of lateral load analysis and we will also do an introduction to displacement methods, so we will be finishing part 4 and starting part 5 of this book on structural analysis. You recall this last slide, which we showed at the end of the previous class and you realize that if the frame is symmetric due to the applied loading lateral loading, you are bound to get points of contraflexure at the middle of all the beams, but we are not sure about the points of contraflexure in the columns location in the columns.

This structure statically indeterminate and it would help if we could reduce or entirely remove the indeterminacy, so the idea is do a quick approximate analysis. The other point that is obvious is because **the columns** the two columns at storey deflect identically, they will have the same shear force and bending moment diagrams and **if this**, so the storey shear at any storey level considering cantilever action of that frame must be shared equally by the two column, so these two points are obvious.

We need to make additional assumptions and **we will** this assumption we make we assume that there are points of contraflexure in the middle of all the columns as well, which is an assumption that is valid only if the beams are infinitely stiff as we saw in the earlier session.

So, why would you assume that is it tentatively it is like the inwards sites so different more than the **(( ))**.

Why?

That is where the loading is come indirectly.

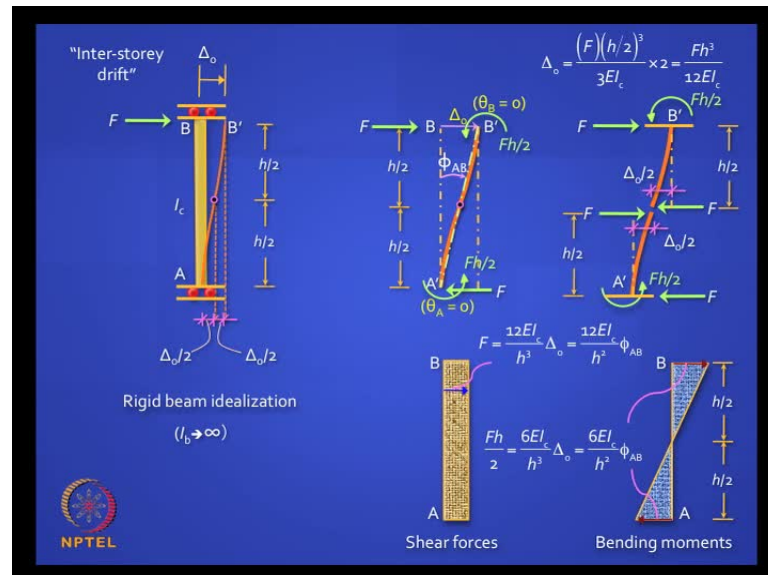
Ok, by the way if you really look at a building that is also a miss conception. Let us see the wind is blowing in this direction and you have building **here**. It is true that there are forces acting the wind pressures are acting directly on the windward phase but, on the leeward side also there is some suction right? and so, really the whole building is effected by the wind field and behave a concept of drag coefficient where you multiply the wind pressure with the drag coefficient and the exposed area and you applied for convenience on the windward phase but, you recognize that its actually including the leeward phase as well number one.

Number two, the floor diaphragms ensure that all the frames participate, so effectively the whole building is moving as a whole, is it clear?

If you look at this superposition that we did that is we divided this loading into two parts a symmetric part and an anti-symmetric part you'll find that from the anti-symmetric point of view, there is a kind of anti-symmetry, it does not matter which is the windward and which is a leeward.

The only difference is the columns on the windward side will be subject to axial tension and the columns on the leeward side will be subject to axial compression and the magnitude will be the same, so that is the overall behavior.

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We also assume that axial deformations are negligible okay, so we make this assumption and let me come back to the point which seem to trouble some of you in the last class, I want you to be totally convinced that that point of contraflexure will be exactly at mid height if we make the assumption of rigid beam and I will go through it once more.

Remember that I have just taken column A B from somewhere in the building and the point A is also not fixed really, A is also moving and B is moving relative to A, so we use a word inter-storey drift to talk about the relative deflection between B and A, **the relative deflection between B and A**, is it clear? A is moving but, B which is located higher will move more than A and we are really interested in the relative moment between B and A, is it clear? That is called inter-storey drift.

Now, really speaking you can put this guided roller support at the bottom also, because it is true it is also going to move, so now this will give you a better idea for symmetry in that frame and of course, it is unstable, so the rest of the building helps in propping in through, so this is a deflected shape and we talk about a cord rotation  $\phi_{AB}$ , which is clockwise and which is nothing but,  $\Delta_o$  divide by  $H$ , is it clear? So, you look at the deflected shape and you can see straight away that the forces would be a pair of

forces, not just 1 F, so it really does not matter where that F is acting and it tells you that the shear forces constant throughout the height of the column and not only that you also get bending moments, what will be the bending moments? What will be the direction of the end moments? clockwise or anticlockwise?.

Anticlockwise.

Anticlockwise, and if you really want to look at it more closely it is (( )) the top half behaves like a vertical cantilever fixed at the top end and the bottom half behaves like another vertical cantilever fixed at the bottom end and there are forces acting at the point of contraflexure, which is equal to F and the end moments are F H by 2, is it clear?

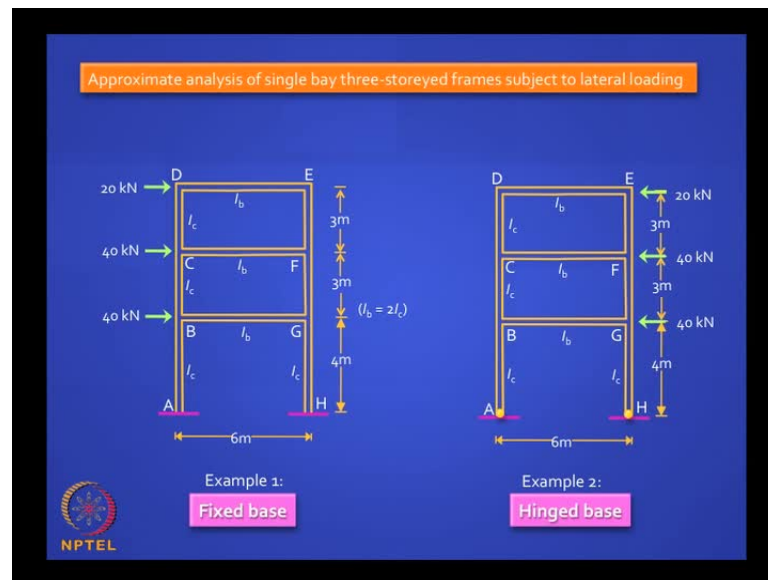
Now, I hope you are totally convince that if you is make the rigid beam assumption, the points of contraflexure going to be in the middle mid height of the column and these are the values, so the total inter-storey drift delta naught is going to be shared by the top half and the lower half, so that value is delta naught by 2 and if you take the simple moment or the deflection equation you get a relationship between delta naught and F and it turns out delta naught is F H cubed by 12 E I c, clear?.

E I b is infinity in this assumption. If you draw the shear force diagram, it will be a constant, it will be equal to f which in turn will be equal to 12 E i naught E i c divided by h cubed into delta naught or 12 E i c by h square into phi, phi is delta naught by L clear?.

If you look at the bending moment diagram, how would it look? well you have two cantilevers there it will look like this drawn on the tension side linearly wearing the point of contraflexure exactly at mid height and the moment will be F h by 2, so it turns out to be 6 E i by h squared into delta naught or 6 E i by h in to 5, is it clear?, so these concepts you need to grasp early.

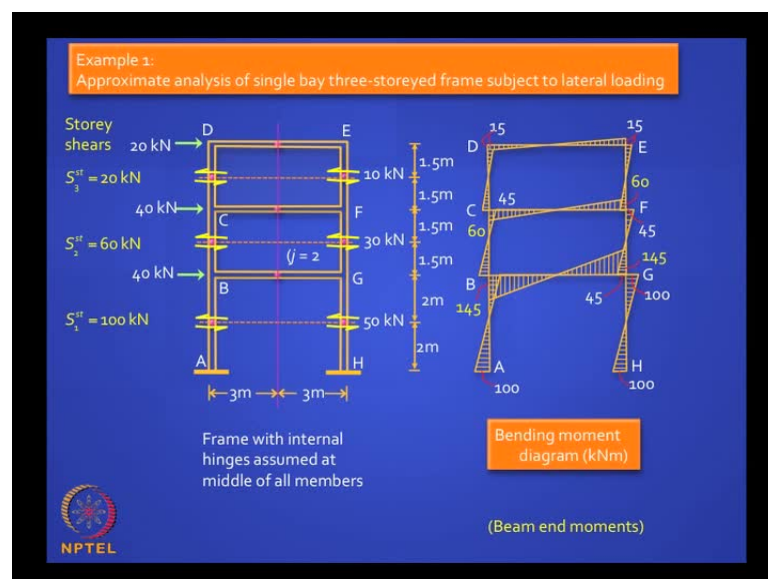
Now, let us try to do a quick approximate analysis of a three storied symmetric frame okay.

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Now, it is give a near that  $i_b$  is equal to  $2 i_c$  but, for the approximate analysis, we are really assuming  $i_b$  is infinite, so that is the error we make in the analysis right? and we will see later how to do a mod accurate analysis of this using a displacement method. Now, we will do two examples, first we will assume the base to be fixed the bottom A and H and the second example we will assume it to be hinged and just to get you of the feeling that loads always act from left to right, we will reverse a loads and see how you draw the bending moment diagram okay.

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Okay, now take a look at this frame. If you assume hinges at the middle of all the beams which is 100 percent accurate and you assume hinges, internal hinges at the mid heights of all the columns which is not accurate which is a big assumption you actually introduce nine moment release locations right?.

If you look at this frame, this frame is statically indeterminate otherwise to the degree nine and you have actually ended-up making it statically determinate, so that is a tremendous simplification but, that makes your life easy you can easily analyze this frame very easily.

Where will you start?

(( ))

Okay, what was a first thing that you will calculate?

Support hinge.

See you need to draw the bending moment diagram. What is a first thing you will calculate?

Support reactions, okay they will be shared equally you get them, so it 40 plus 40 plus 20 that is 100 divided by 3, 33.33 fine but, to draw the bending moment diagram, you look at the storey shear okay.

The storey shear you can do like that, I told you in the last class you cut a section at mid height of the top storey you will find then that 20 kilonewton has to be shared equally by the two columns you get 10, 10 is it clear?

What about the next storey, the second storey?

(( ))

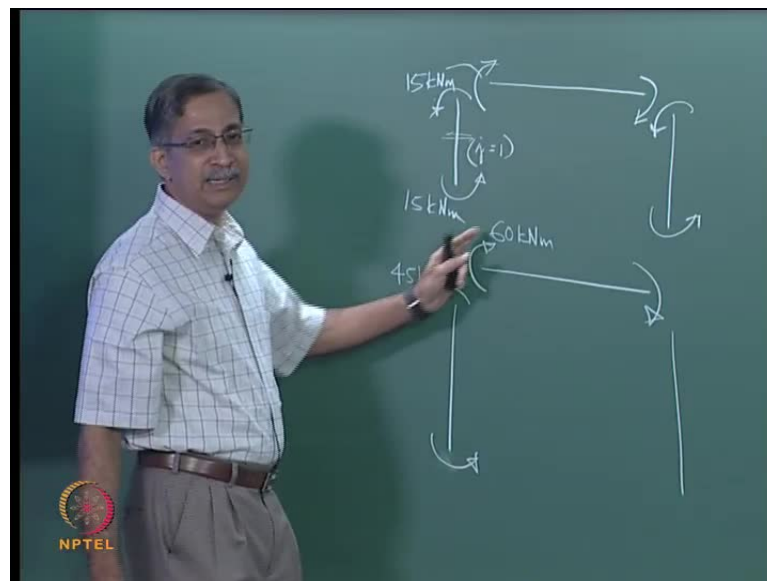
It is 20 plus 40 divided by 2 you get 30, 30 and then the third one will be 50, 50 is it clear?

I have marked those arrows to show you that the direction at the end of that element is it clear?

This is easy thing for you do to.

Now, you have got this can you get the column end moments, you just have to multiply the column shear by  $H$  by 2 that is all you have to do, so it is very easy, so you can draw the shape of the bending moment diagram drawn on the tension side and the lower storey columns will have a moment  $50$  into  $2$ , which is  $100$  the middle storey will have  $30$  into  $1.5$  right? Because a storey as a height of  $3$  meters it turns out to be  $45$  and the top storey column will have end moments equal to  $10$  into  $1.5$  which is  $15$ , this much is clear?

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So, you have got the column end moments. Now, you need the beam end moments how do you get that? **how do you get that?**, so in your mind you must draw the free bodies, let us take the top you got a beam **here**, you have a column **here** remember we drew lines **like this**, so the column moment will be clockwise or anticlockwise it will be anticlockwise right? this is a top storey  $J$  or  $i$ ,  $J$  equal to  $1$ , so how much was this value?

15

So, this was  $15$  kilonewton meter this was  $15$  kilonewton meter.

You had another identical column with the same moments, I am going to write them down. Now, equilibrium demands Newton's third law demands that the beam and moment must be equal and opposite, so it will be clockwise and it will also be equal to



15 kilonewton meter both **here** and **here**, so it is as easy as that, so this is easy **this is** going to be equal to **that**.

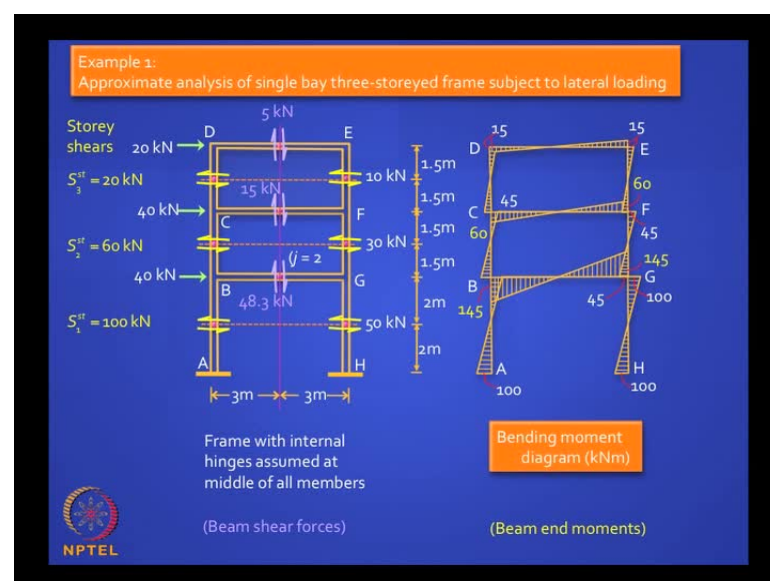
What about the next level?, so you have the second beam, you have another column this moment is, so all the column moments are anticlockwise, what was this value?

45.

This is also 45.

So, what do you think this should be? **this should be clock** it should equilibrate now, you can either subtract or add I know that but, these are both anticlockwise when you join all the elements together you should not have a nodal moment, so you have to add not subtract, do not make that mistake so, it is 45 plus 15, which is 60 is it clear?.

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The same thing holds good here, so you just need to do it at one end and it gets replicated, so it is very easy. What about the last one? **It will be** it will be 100 plus 45, which is 145, so that is it, so it is very easy okay.

Now, you can also draw free bodies and draw the other shear forces axial forces, so it is so easy a problem with the degree of indeterminacy of nine is solved in one minute because we make these assumptions. We will see later how far off we are from the exact solution okay, but this is good enough right?

(( )) backward we know that point of contraflexure for the beam will be at a midpoint that certain, so that can only happen and given that the two beam end moments will be equal and opposite, so.

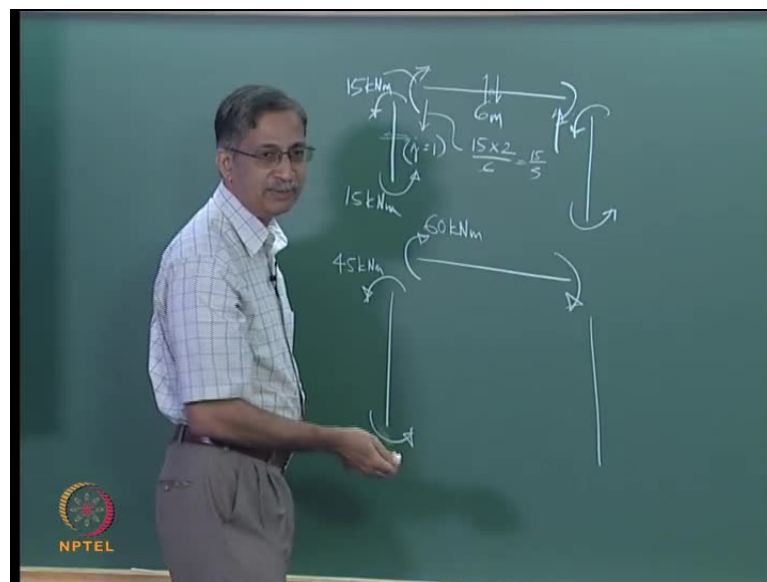
Which is stating the same thing, point of contraflexure at the middle is stating that the two beam end moments will be equal with a same direction.

Then if you draw free body for the column equal and opposite moments you put then you get the point of contraflexure in the middle of column.

You are assuming, we know the beam end moment we do not know the beam end moment look at it very simply, just putting the beam hinges reduces the indeterminacy from 9 to 6 still not enough to get a solution right? it does not work, so this is a beautiful logically meaningful method but, it is approximate,.

Now, let us ask a question how do you calculate the drift in this frame? because people would like to know how much the building will move on the top, you can also get the inter storey drift how do you do that? **So that.**

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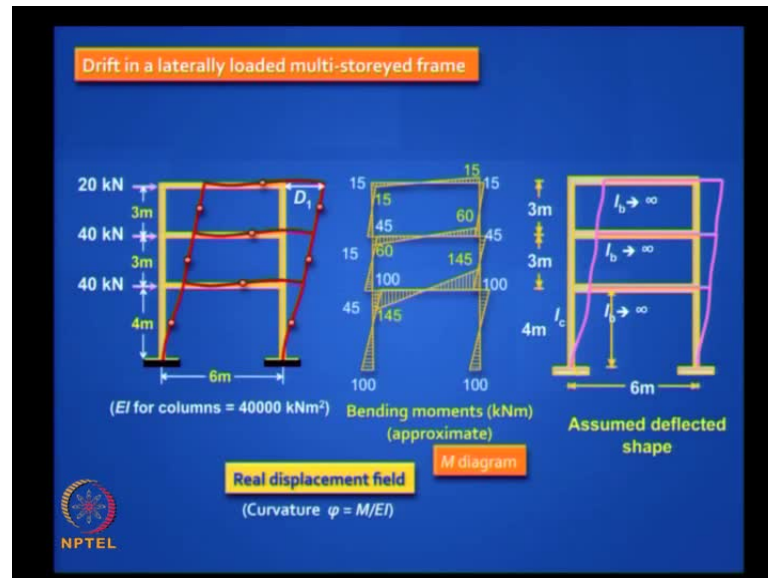


Okay you can also get the beam shear. How do you get the beam shear? well if this is 15 and this is 15 and this pan is how much?

(( ))

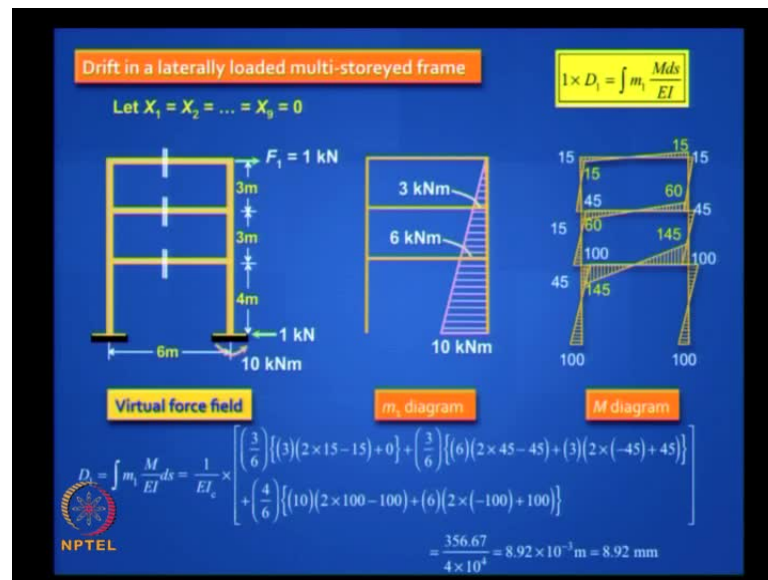
6 meters **six meter**, so you will have a vertical reaction like this right? This is the couple which will, so what will be this value? It will be 15 into 2 by 6 another way to put it is we have a hinge there, so you can draw arrows like that, so you could say this is nothing but, this divide by 3, it is a same thing, so this same as 15 divided by 3, which is 5, so you can work out the beam shears and so on and so forth.

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Now, let us see how to calculate the drift. We have got the bending moment diagram. Can you tell me what  $D_1$  is? Approximately given that the  $EI$  for the columns is some number 40,000 kilonewton meter square, how do we proceed? Remember we also found that deflection in a statically indeterminate truss in your assignment problem, can you do something similar? Incidentally, the deflected shape that you are assuming is not this but, you are assuming this because actually you are assuming the beams to be infinitely rigid is it clear?

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So, you are going to get a solution for this frame where you have tacitly assuming  $i$  is infinite for all these. How do you proceed? You apply a unit load  $F_1$  equal to 1 and do you analyze this frame all over again, assume some of the forces to be 0 so, this is the frame we need to analyze and we do a clever thing this is highly indeterminate the degree of indeterminacy is 9, so let us cut the frame **let us expose some** how many redundant you expose there? 9, 3 at each location a shear force of bending moment in an axial force and conveniently assume all of them to be 0.

So, you get a statically admissible solution. Can you draw the bending moment diagram for this frame? It is very easy, it is just a cantilever, so it will look **like that** so easy right? do you agree, simple 1 into 10 is 10 at the bottom and 1 into 3 is 3 and 1 into 6 is 6, 3 well.

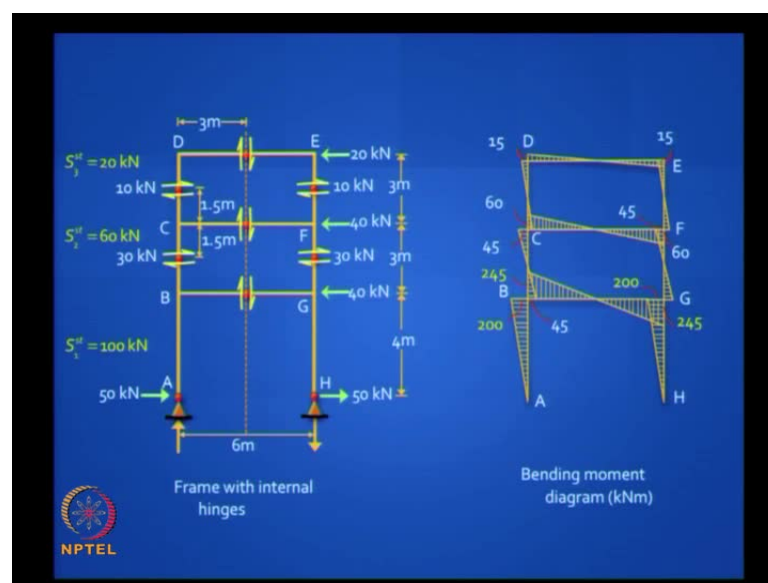
Now, **this is** your  $M_1$  diagram and **this is** the unit load method, you invoke the principle of virtual work, so this was the original bending moment diagram this is a approximate, so the error in your answer is due to the error in this not due to the error in small  $m_1$  diagram but, we live with that error.

You know the axial deflection will be more than the value that you calculate because the beams are not infinite they are flexible, so they will add to the we will get rotations  $\theta$  remember in earlier class, so it will remember this a load bound solution that you get. How do you proceed? now you just multiply, I think you know how to do this, so you

take the live out column you are multiplying one triangle with you could treat that as a trapezium, so remember the formula  $L$  by 6 and so, I hope you know how to do this calculation, I leave it to you to work out the details, is it clear?, So it is a quick elegant fast way of not only in designing a frame but, also estimating how much it will deflect.

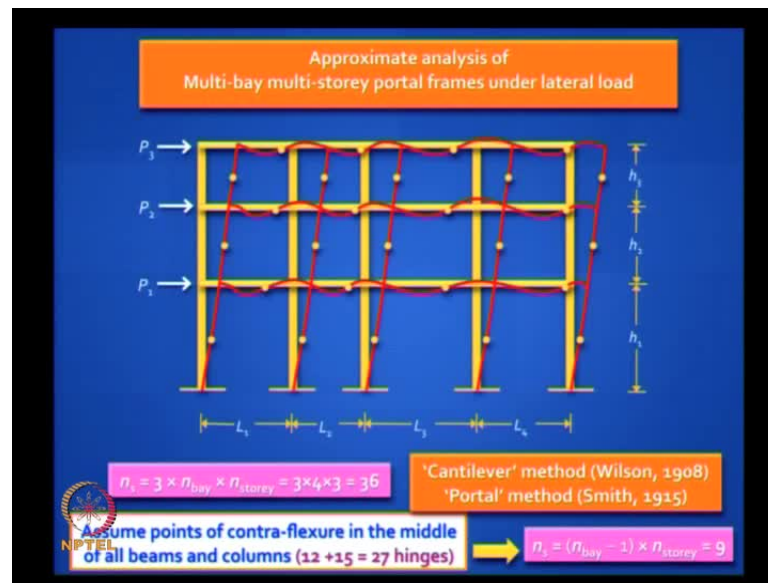
There are practical limits usually for the deflection in design because you do not want the building to move too much roughly around height or inter storied drift is limited to the heights of that storey divided by about 500 or so okay.

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Now, let us look at the second example. In the second example, we assume the base to be hinged at A and H, so we should be showing hinges, so now do not go and put an internal hinge the middle of the lower storey, because that hinge is given at the bottom okay. Hinge is given, so do not put another hinge it will become unstable, so you do the same thing I think it is pretty easy do but, now you should draw the diagram on the tension side it will look this way, is it clear? So, do not be boys that the load should always add from left to right they can act from right to left as well is it clear, so this problem is simple.

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Now, let us take the more general problem of multi-storey multi-bay frames and let us conclude the assumption that you have hinges you have points of contraflexure in the middle of all the beams **which is** which is not 100 percent true also because it is not symmetric anymore and in the mid heights of all the columns if the bottom is fixed, so these are big assumptions we make but, they are still not good enough to make the frame statically determinate.

You can see that the indeterminacy can be generalize, if you have N bays and N storeys, then the total indeterminacy is 3 times N bay 3 times into n storey which in this case is 3 into 4 into 3 36. If you bring in those hinges, the points of contraflexure which are internal hinges you get 12 in the beams and 15 in the columns you get 27, you are still short of 9, 9 additional condition, so how would you solve this kind of frame? this is the problem **which has** which had you know troubled engineers in the early days when tall buildings were made and many suggestions have been given **many suggestions have been given**, can you give some? Let us go back to history and.

(( ))

Sorry.

All the hinges in one floor make it us the hinge support and downward.

I do not understand.

Cut the building sir.

That we can cut at the inter storey in mid height levels that does not solve. What is the additional equation of equilibrium you are going to have?

You need additional equations of condition, is it clear? It still statically indeterminate.

Compatibility equation.

Compatibility is you are just look an equilibrium statically indeterminate you already violated compatibility when you assumed hinges at the mid heights of all the members, so do not talk about compatibility.

We want a quick rough solution. Remember, in the earlier case of a single bay multi-storey frame, which was symmetric we made an assumption that all the columns share the storey shear equally, can we make something like that and do it? Can we make that assumption **here**? No, what do you think is more reasonable to make?

Total.

Proportional.

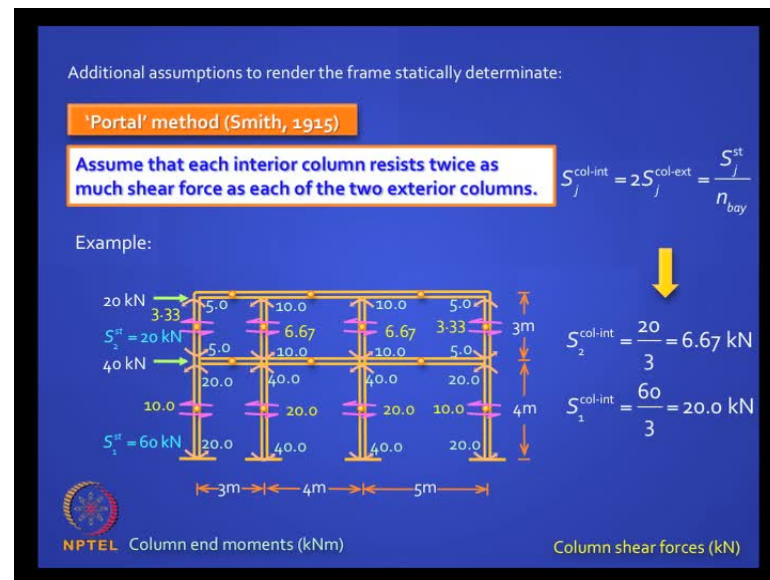
Proportional to.

The distance from.

So **that is** that is your method, you are saying that the total load is shared in proportion to the average bay width, so the exterior columns end up with less column shear than the interior columns right? that was one proposal.

But, the one which gave more acceptance was one with said which with made it simply it is a let the exterior columns take half the shear of the interior columns okay? and that that is one way of doing it, so you will find historically two methods came into being, one is called the cantilever method proposed by Wilson in 1908 the other is a portal method proposed by Smith in 1915 and they actually made buildings with, which were very tall in beam force concrete and in steel.

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Now, let us look at both these methods and see how they simplify. Please note this whole method is approximate, you are ignoring compatibility you are ignoring the relative stiffness of the beams and the columns.

So, let us look at the portal method. The assumption **here** assume that each interior column resists twice as much shear force as each of the two exterior columns, we will find that this one simple assumption cracks the problem beautifully.

Now, let us take this example, I want you to do it, we will do it together and I have given you an assignment of a similar kind, it is a 3 bay 2 storied frame. Usually the ground storey will have a larger height because your foundation is located not at ground level but, at some distance about meter or 1.5 meters below the ground okay?, So take this example and you can work on it, it helps if you have multiple colors in your pens because otherwise just take a look at it.

How do we proceed?

First you mark the hinges okay, put little rounds in the middle of all the beams and middle of all the columns then you do what we did earlier, divide the storey shear okay, so you can do that calculate the storey shear the top storey shear is 20 kilonewton and the lower storey is 20 plus 40, 60 agree?



How is the 20 shared by the 4 columns? If you have to give a formula, you look at that equation you have 2 divide by 3 and you will get the interior column shear right? and you divide, so you right that the interior column shears are 20 by 3, why do we say 3 because there are 2 interior columns and you get half, half contribution from the 2 external 1 so it is just number of bays, is it clear? Do you agree to this? Clean, very easy to calculate, so you plug in those values you got the column shear.

Then for the exterior columns, you put half this value, so 3.33 and 10, is it clear? It is very simple all of you getting it, so you start like this and what to you do next, you can get next the column end moments how do you get them, multiply these shears by half this respective storey heights, so you can mark them they will all be anticlockwise remember our discussions, they will all be anticlockwise which means at the top of the column and the bottom of the column in any storey, you have to put equal numbers and the exterior column will have a moment which will be half that of the interior column, so you to do only one calculation and then multiply by 2, how much does it work out? 3.33 into 3 by 2 is 5, 6.67 into 3 by 2 is 10, is it clear?, so you get 5, 5, 10, 10, 5, 5 in a minute.

Any questions?

Sir point of contraflexure?

Point of contraflexure you made that assumption right in the beginning at mid height.

Sorry.

If you take the extreme side beam.

Yeah.

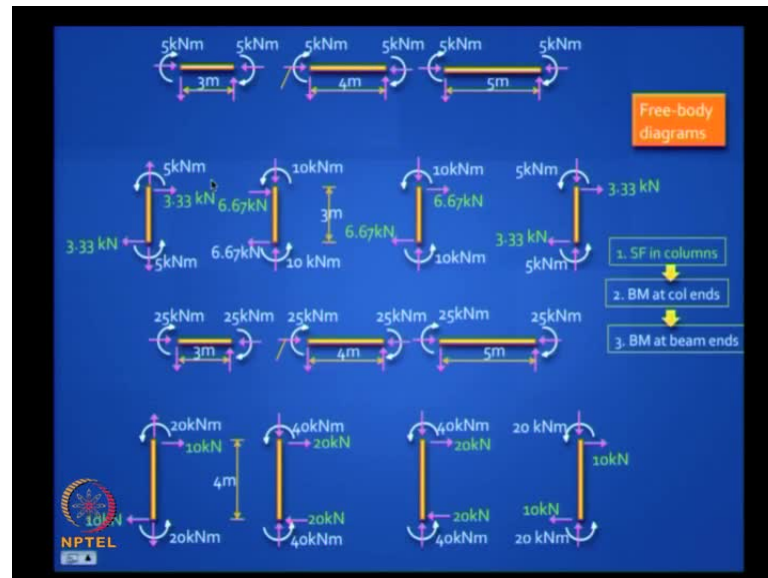
It is 50 and it is 10, it is two end moments.

We are not yet reached the beam end moment we are looking only at the columns.

Yes sir, but actually when you.

You do not go to eventually take one step at a time otherwise, you get confuse. Are you with me till this stage, is there any error in the calculation, is it not straight forward can you not do it in one minute right?.

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Now, let us look at the big picture. So you can fill the bottom storey also 10 into 2 is 20, 20 into 2 is 40 right?, So you got in one shot the column shears and column moments now, we will see what to do it helps to draw a picture **like this**, will you draw this, I have drawn all the free bodies. There are three beams on top, three beams at the first floor level and there are four column segments in the two stories.

We have calculated up to this much, we have calculated the shear force in the columns and the beam bending moments at the column ends yes?.

If you have pens and pencils are different colors it is very easy and very nice to do so that when you look at it, it does not look at crowded, is this clear, all the directions are clear?

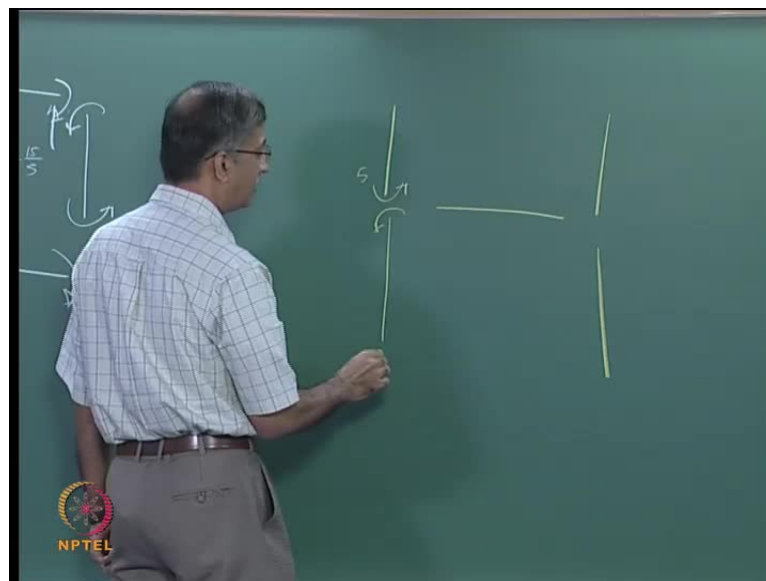
Please note all the column end moments will be anticlockwise and all the beam end moments will be clockwise for reasons which we discussed earlier and so, there are few numbers only that you need to calculate are you with me now up to this stage right?

Shear force in columns bending moment at column ends we have calculated. What is the next step? and we have drawn the free body diagram. Bending moments at the ends of

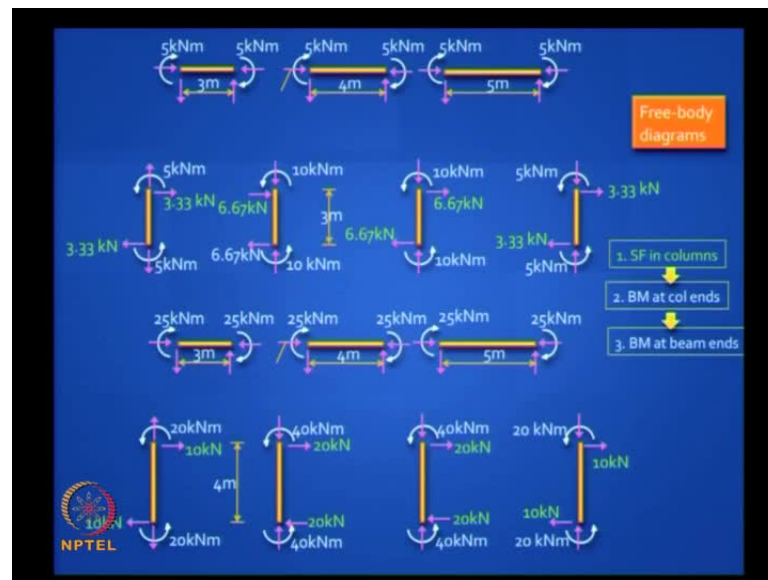
the beams when you say bending moment in the beam, it varies linearly it goes through zero and changes sign, so bending moment at the end of the beam okay, so how do we do that? you start at the top corner may be top left hand corner, you have to do only one calculation why? because the same end moments travels through all the beams in any given flow, let us check it out.

Now take a look **at this** you begin **with this**, this column end moment was 5, so this beam end moment must be 5 because the point of contraflexure in the middle **this must** be 5 and if **this is** 10, **this much** be 10 minus 5 which is 5 and so on and so far, so you will find that that assumption by Smith that exterior column takes half the column shear compare to an interior column greatly simplifies your calculation you have only one beam end to beam end moment to calculate at every storey, so you have done that then you come **here** you got 20 **here**, you got 5 there, so this will turn out to be 25 and therefore, **this must** be 25 but, **this is** 40 right? **This is** 40 but, 40 and 10, please note at that junction in this problem.

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Okay, what was this moment anticlockwise top one was?

10.

10 or 5?

5.

Column 5.

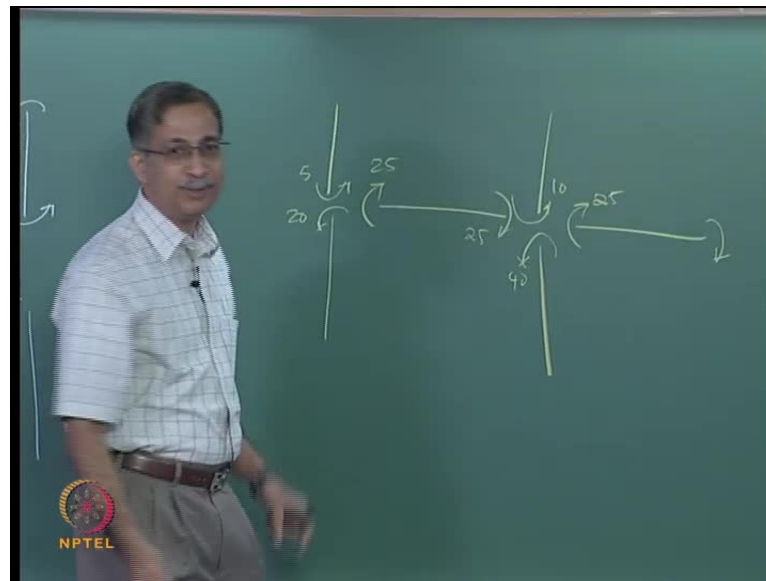
10 sir.

No, no [FT].

Okay what is this?, this is how much?

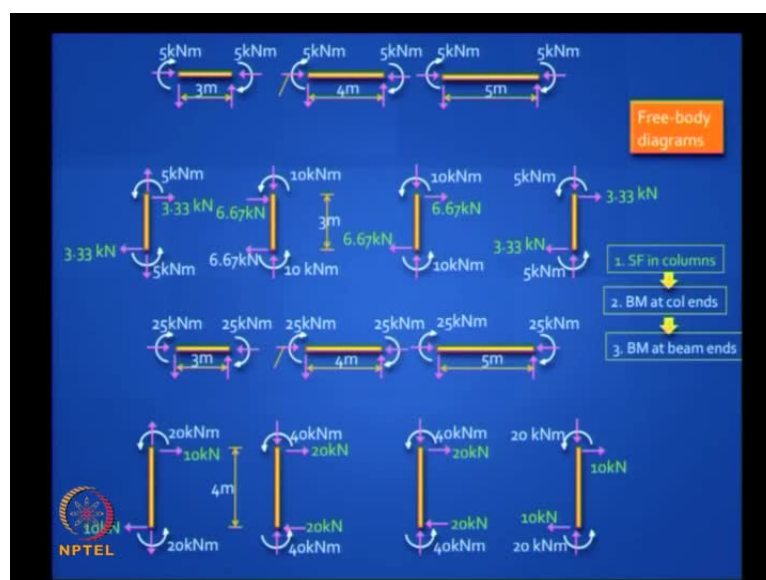
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20 and therefore, this must be 25 clear? So this is 25, this beam what is the moment **here** this is also 25 because all the beams will have the same moment this was 2 times 5, which is 10 and this is 2 times 20 which is 40, 40 plus 20 minus 25 is 25 that is how you work out but, you do not need to do this because this will always work you get only 25 in all the beams is it clear?

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So, we have worked out everything logically does not make sense, so it is very easy to deal, so your beam ends you get effortlessly the first flow that the beams right on top will

have an end moment of 5, the beam at the middle level will have an end moment of 25. What is your next step, if you want to finish this problem? After you find the beam end moments what is deflection is the last you are not ask deflection it just 1.

B M D sir.

No you got you can draw the bending moment diagram, we will do that little later.

Shear.

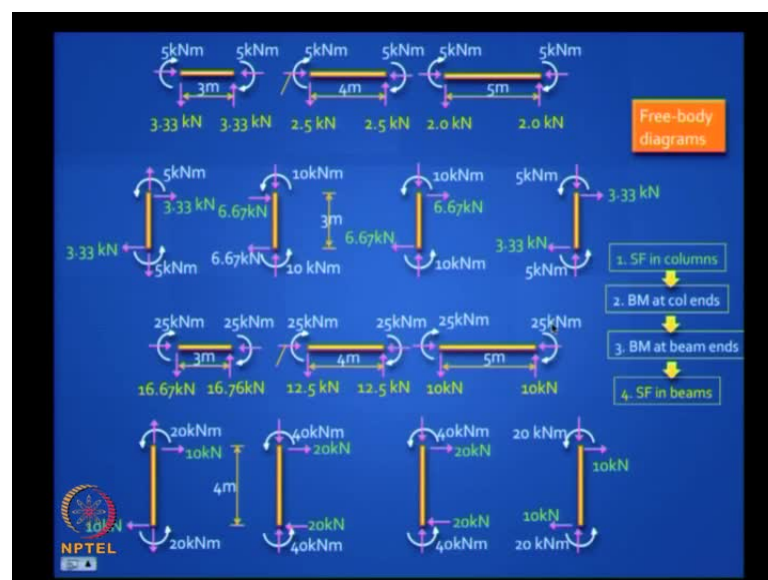
Shear in which?

Beams.

Beams, because you have already got the shear in that column. How do you get the shear force in the beam?

By taking (( ))

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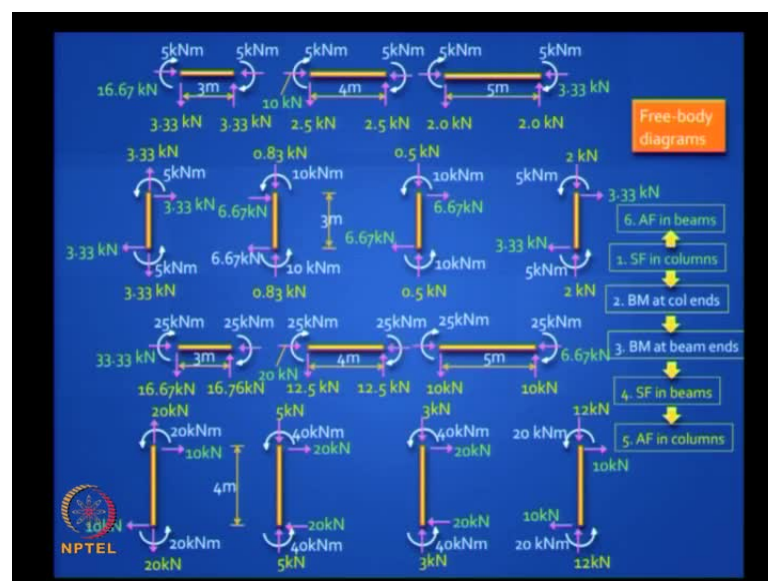
Add the two end moments in the beam divide by the span of the beam, so that is right you guys are good you can do this, so what do you get? 5 into 2 divide by 3 is 3.33 you got all the shear forces in the in the top 25 into 2 divided by 3 you got all those shear force no, no please note the shear forces are going to change because the span of the

beam is different in different bays, so you have to do carefully 5 plus 5 divided by 3 is 3.33, 5 plus 5 divided by 4 is 2.4, 5 plus 5 divided by 5 is 2.0.

Similarly, for this so please note the beam end moments are the same in all the bays in a given storey but, the beam shear forces are different because this bay lengths are different in this particular problem is it clear? Are you with me?

What is a next thing you can do, I mean how we are going to find everything?

Axial force in column.



Axial force in column perfect, how do you do that? you see I am I am showing it step by step. How do you get, it is a same as the shear force in the beam, so start with the left corner if this is 3.33, this will point in the upside direction this is 3.33 and if the top is 3.33 then this must also be 3.33 it is intention. Now you come here, you have to be careful upward 3.33 downward 2.5, so you need another downward 3.33 two 2.5, which is 0.83 and if you got 0.83 then you must be getting 0.83 here also for equilibrium then you move to the next one 2.5 upward, 2 downward, so the difference is 0.5 it must be downward because when you join all three you should get 0, if this is 0.5 this will be 0.5 upward and the last one of course, it should match 2 and 2 clear?

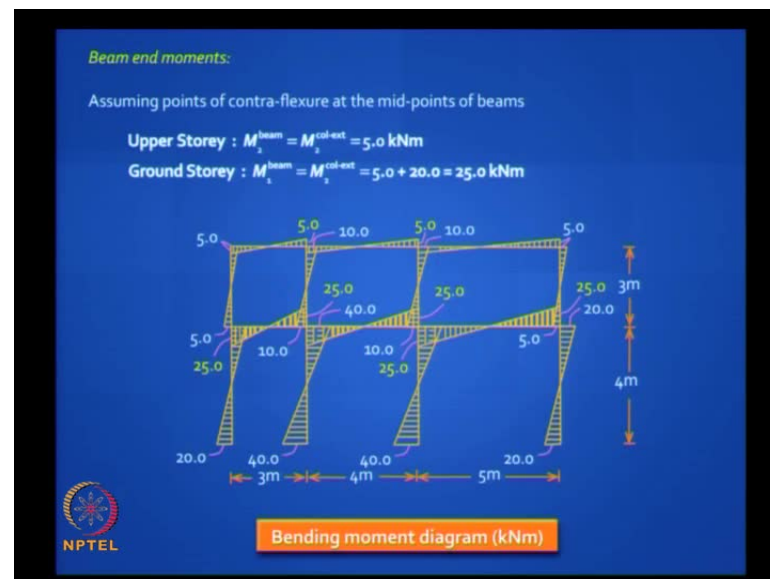
Now, you come below. You want to get this you can you can work it out okay, I am I am not I am not giving you all the answers you can easily work it, what is a message you got the? Column axial force is

What is a next step? What is a next step? Have you got everything?

You can also find beam axial forces, how do you get that? Do you get it from column axial forces?

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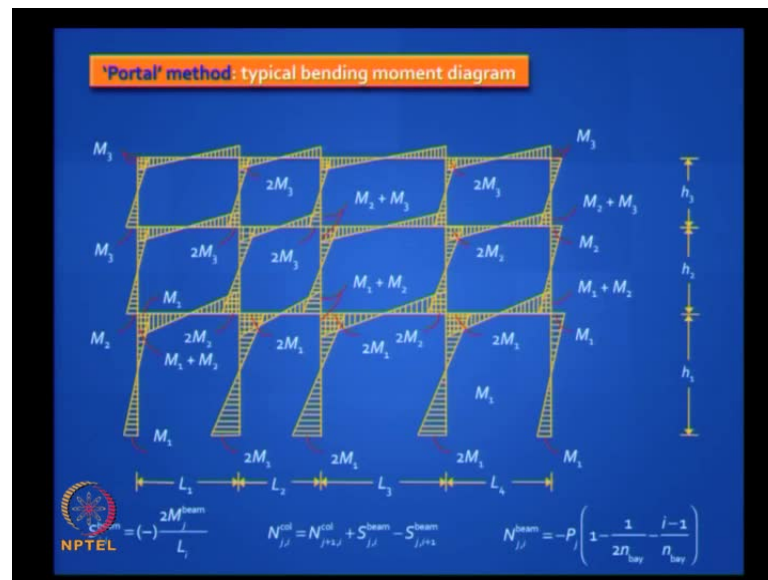
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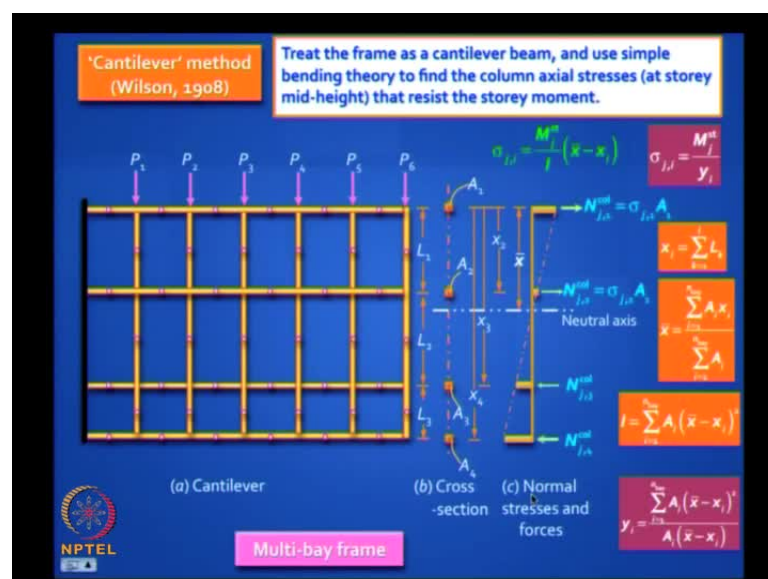
Column shears, so you could have done in right in the beginning if you wanted these are the other value you could have done right in the beginning if you want it but, mind you they are not very important the most important thing is a column shear and the bending moment diagram, so the bending moment diagram is easy to draw points of contraflexure in the middle I hope you understand this you know and remember that the beam end moments are two times of column end moments, is it clear? portal method is easy you can apply it to any problem that you get.



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So, typically if you are given any buildings say three storied four bay building, it will look like this. You can work out formulas but, my suggestion is do not worry about any formula just do it on the on the board on the paper because you will not get many big problems.

In the cantilever method which little more difficult Wilson came up with a suggestion that you treat the frame as a cantilever beam that means the whole building is treated as a

cantilever beam and you use simple bending theory to find the column axial stresses at storey mid height that resists the storey moment.

So for convenience, let me rotate that frame by 90 degrees because we are more comfortable looking at cantilevers which are horizontal right?, So I have rotated the whole building 90 degrees in my mind and fix the left end, so it is got an open web section do you understand that means usual beams are one piece with solid cross section this is an open web section that means I have note the solid areas only located at the column location, if I cut a section anywhere right, is it clear?

Now, If I cut the section, let us say at any line in the middle of a any storey then I do not have any local bending moments in the columns right? But, I have only axial forces and shear forces. Let me plot the axial force variation well according to this theory we treated as a linear variation assuming simple bending theory, it is like a like a truss for example, is it clear? but, you have your beam has solid quantity is **here, here, here, and here** and **these are your** this is your stress distribution is it clear?

Now, it is obvious that you will have compression. Where will you have compression? at the bottom and tension at the top and you have a neutral axis and, so you can use simple bending theory to calculate and do you agree that the actual force in the column axial force will be the stress that you calculate from simple bending theory multiplied by the area of the cross section do you agreed to that?, so that is all.

Now how do you get the stress? go back to Euler Bernoulli theory  $M$  by  $i$  is equal to  $F$  by  $Y$  right? or  $\sigma$  by  $Y$ , so can we right  $\sigma$  like that  $M$ , **here**  $M$  is you are cutting the section at any storey so we need to talk about a storey moment, so do you agree? If I cut a section **here** the moment I need to worry about in the beam, the bending moment the beam is  $P \cdot 6$  into half this distance, do you agree?, So that is my storey moment **here** like we had storey shear, we can talk of storey moment. If I am cutting the section **here** the axial force **here** then I have to multiply  $P \cdot 6$  into **this distance** plus  $P \cdot 5$  into **that distance**.

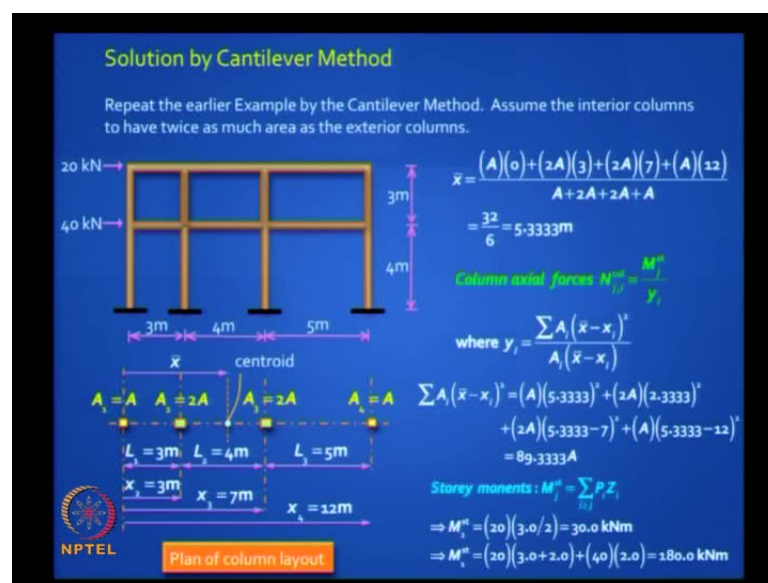
So, the storey moments keep building up as I go to the ground floor ground storey, so you get the largest moments of the lowest end of the building, is it clear?

Now, this formula i o k with  $\sigma = J \cdot i$ ,  $i$  is the storey level and  $J$  is the bay location and  $J$  is a storey level do not worry about the formulas **are you** are you comfortable with this

simple bending theory  $M$  by  $i$  and  $i$  is a property of that section and the  $X$  bar by  $X$ ,  $J$   $x$   $i$  is your  $Y$ .

**41:22 Now, how do you get these properties?** well just areas central of areas and so on so you can usually get the second moment of area and if you substitute this formulas you will get a much of compared formula sigma is  $m$  by  $y$  it is like it is like let us say you replace that complicated frame with just one top chord and one bottom chord and the librium is  $y$  then it is a bending moment divide by  $y$ .

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So this is an equivalent reduced lever on  $y$  which you can prove to be using these formulas you can prove to be this quantity  $i$  i suggest you go through this carefully understand this equations and let us just directly apply to the same problems you'll get a feel for it we are going back to the same problem the portal method would have taken as five to ten minutes to solve cantilever will take you a little more time.

How do you began well first you need to assume something about the cross sectional areas if nothing is given you could not assume all the areas are equal but, here we are told that the interior columns have twice as much area as the exterior columns so you have two  $a$  for the two interior columns and  $a$  for the exterior columns right can you locate this centroid  $x$  bar will you do it quickly what is a  $x$  bar equal to its easy to do take moments of areas about the left edge so it'll be what total area is two plus two plus to six

a six a into x bar is equal to two a into three plus two a into seven plus a n into twelve right will you work it out what do you get.

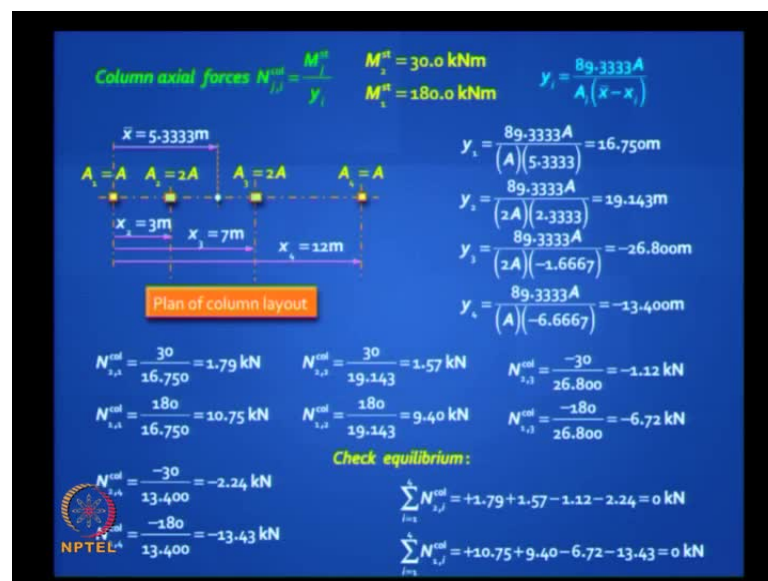
Sixteen by three

Ok sixteen by three or thirty two by six you get five point three three three meters easy to do now can you get the i value second moment of area you you can do that so basically you have to get y i can given you this formula you know the location can i move faster you can do these calculations they are not difficult the numerator is common for all the expressions so you can calculate that and we also need the storey moments tell me what is a storey moment in the top storey.

Twenty into twenty into how much.

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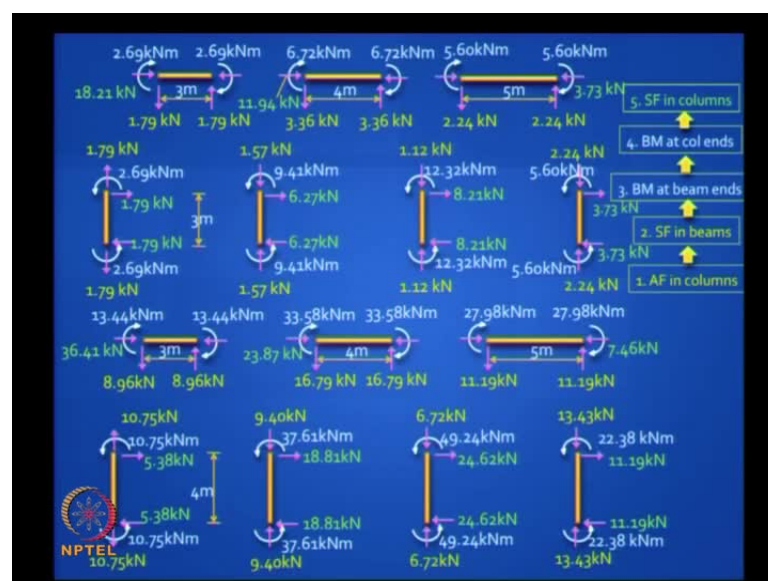
Into one point five is your storey moment it is like cutting a section at the mid height where there are no local moments so the total global moment in that second storey is twenty into one point five what is a storey moment in the lowest storey sixteen no no no you cannot do that twenty into into five plus forty into two good that is it clear you do that and substitute these expressions you can get so every column there are four column locations right every column will have a different equivalent y which is a property of that section ok.

And it turns out to be if  $y$  is negative it shows that it is not going to be tension it is going to be compression the force going to be compression that is it then you substitute there you can get you see we calculate  $y$  one  $y$  two  $y$  three  $y$  four  $y$  one pertains to the column and the windward end  $y$  four pertains to the column at the levered end and  $y$  two is for this and  $y$  three is for this  $m$  storey is known so you divide the  $m$  storey by the appropriate value of  $y$  so you get the force in the top column and the bottom column for the extreme windward column for the next column for the next column and next column

These are simple calculations to but, they they take a little time right so you got the column axial forces if the column axial force is negative what does it mean it means that the column is in compression so that is it once you got and you can check equilibrium if you add all the forces tension and compression at any storey level they should add up to zero because there is no axial force on the on the building due to wind loading so can you all do this step of course, i will give you the choice you can do either portal or cantilever for the assignment we have do the cantilever also but, in an exam it is too much to do.

So you just do the assignment in practice you can take both are approximate people say that the portal method is more accurate when you have a wider frame not very tall and the cantilevers more accurate when you have a very tall frame with narrow width but, it all depends on the relative areas.

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Ok so here i go quickly we are doing the something you've got the column axial forces it is interesting you really have a neutral axis somewhere in the middle on one side you have tension on the other side you have compression so the arrow is get reversed is it clear we calculated these values now what do you do next you got the axial force in the columns.

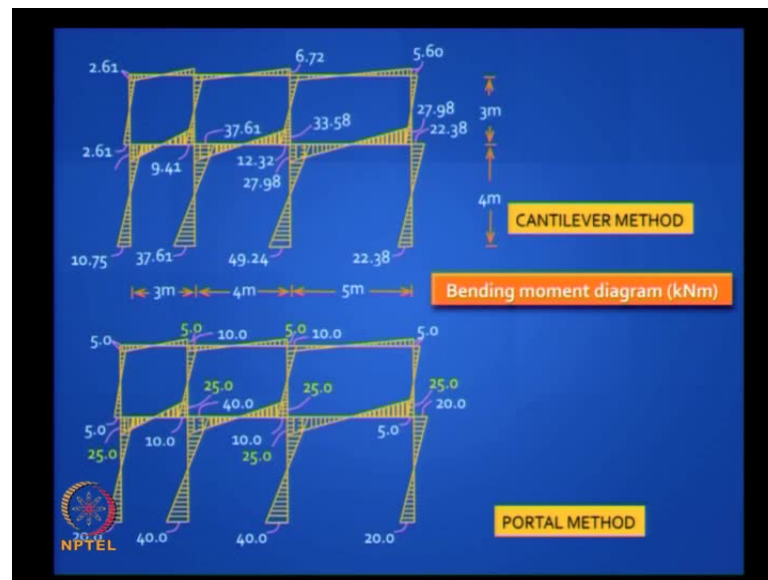
In the portal method we got this last here we getting it first shear force in the beams can you do it just go in the reverse direction you get the shear force in the beams you get the shear force in the beams right what do you do next bending moment the beam if the shear force was the two end moments into span divide by two now you get the bending moment as shear force in the beam into half the span so you can do that so you get you get the bending moment at the beam ends right

please note the big different between in the portal method and the cantilever method is here every beam in every bay will have potentially a different end moment not like a portal method is it clear earlier we you have to calculate only one beam end moment and the same things spilled over everywhere here it does not happen because here share forces are different these spans are different so this needs more work ok

But you know how to do once you have the beam shear forces multiply the shear force by half the span of the beam you get the moment clear have you understood portal method that way is much easier here you do a much more calculation but, you can get the beam end moment after get the beam end moment what do you do next column end moments by satisfying equilibrium and what you will do after that.

Column shear see you are getting the column shear at the fag end and you can check equilibrium and then you can put your shear force you can get the shear force in the columns also is it clear please try this out and if you you can also get the axial force in the beam but, they are not very important.

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So these are the answers we got if you compare these answers with the portal method you'll find that there are lot of variations which is more correct we do not know we do not know seems to be a with your great experience you are saying that the portal method is more accu[rate]- we do not know so we will do this by metrics methods and see which is more accurate.

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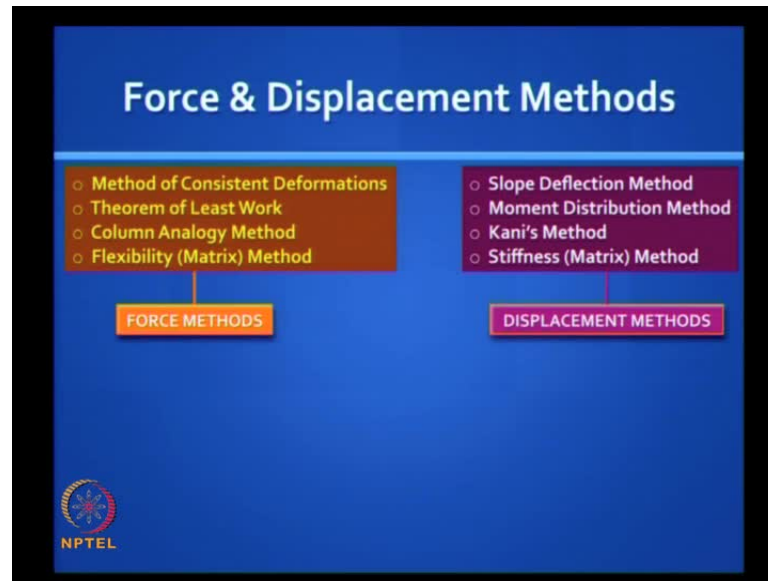


They have an improvements on these methods and people have there is a method called factor method which tries to include the effect of relative stiffness of beam and column



but, it takes away the beauty and simplicity of these methods so in summary the portal method begins with bending moment at the column ends and ends with the axial force in the columns and axial force in the beams the cantilever method begins with axial force in the columns and ends with shear force and columns and axial force in the beams is it clear broadly this is a different.

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So our time is run out we will now look at the next topic in the next class that is we are going to look at displacement methods an introduction in the next class thank you