Advanced Structural Analysis

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Module No. # 1.1

Lecture No. # 01

Review of Basic Structural Analysis-1

Good morning to all of you. Welcome to this course on advanced structure analysis. This is a course that we are offering on video through the auspicious of n p tel. We are starting with the first module, where we will be reviewing basic structural analysis. This is a first lecture in this entire series of about 45 lectures. Welcome once again to advanced structural analysis.

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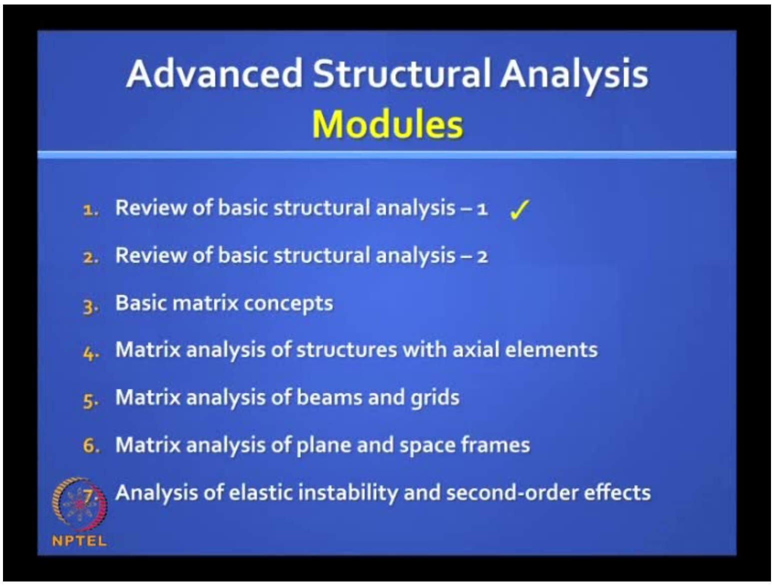

So, these are the modules that we will cover in this course. There are seven modules the first is review of basic structural analysis 1, which is what we will start with today, we will go fast over it because you already studied that. Review of basic structural analysis 2, part of it you studied will be looking at in determinant structures you have learnt forced methods, but you have not learnt displacement methods, so we will study displacement methods in some more detail and then we have basic matrix concepts.

This course is essentially a course which is matrix analysis of structural analysis and then we will do axial elements first, they are truss elements both plane trusses and space trusses. Then we look at beams, we look at grids, look at plane and space frames, and we will also look at second order effects and elastic instability.

So, these are little advanced topics and I hope you will find it interesting, you are free to ask questions at any point.

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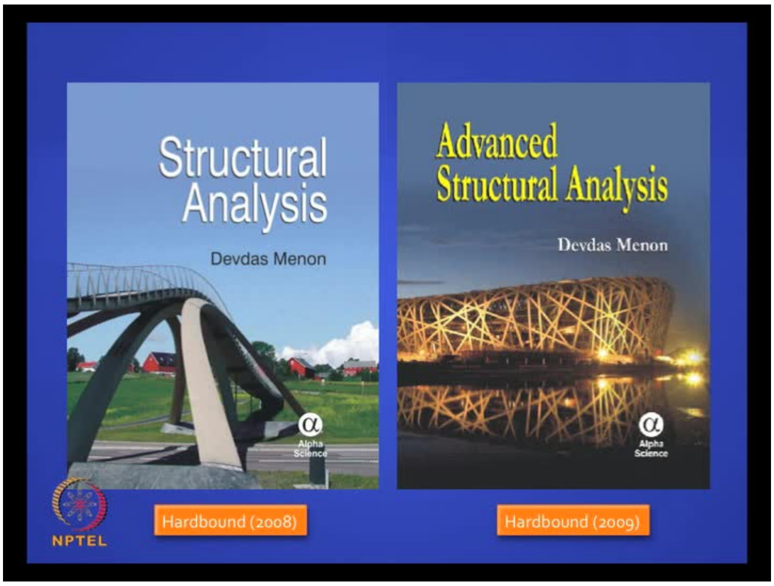

To start with, please note that the primary reference for this course will be the two books which I have authored. Structural Analysis you have already been exposed to except for some chapters at the end which we will cover now and the main text is Advanced Structural Analysis both are published by Narosa in India they paper back and abroad it is Hardbound published by Alpha Science.

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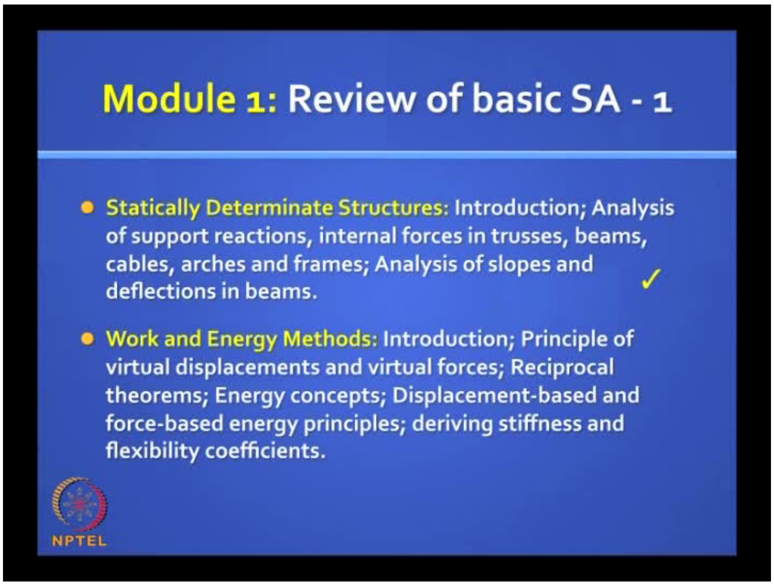

In the first module, we will basically cover introduction to structural analysis in statically determinate structures work and energy methods.

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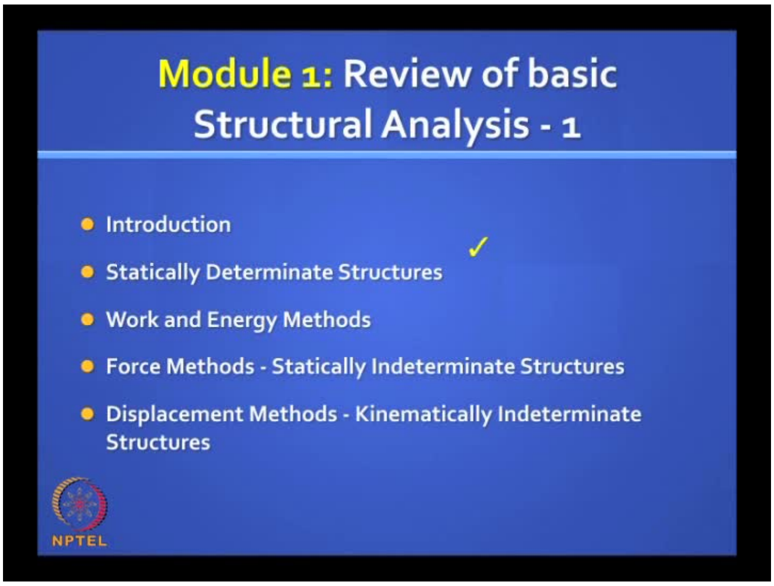

In the next module, in this module itself will be these are the topics, but the last two force methods that displacement methods we will cover in module two.

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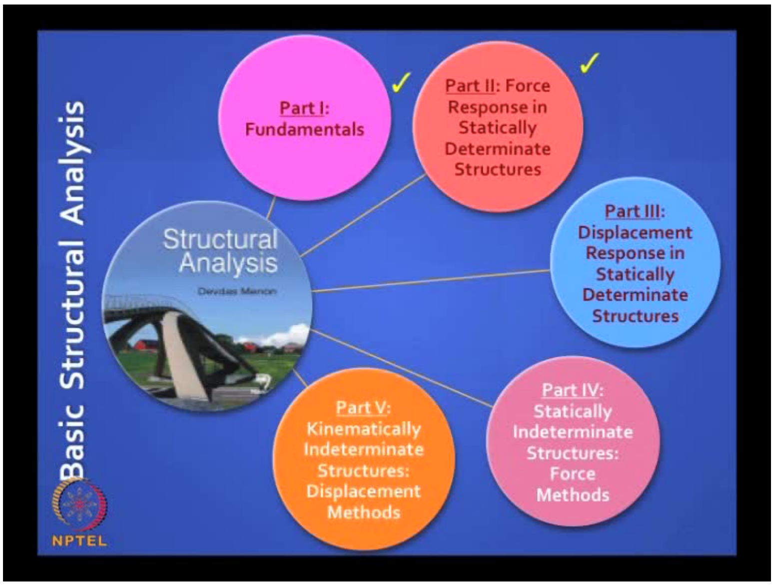

So, if you recall in the book on Structural Analysis, we have five parts and today we will quickly cover parts one and two. To begin with let us refresh our understanding of what Structural Analysis is all about.

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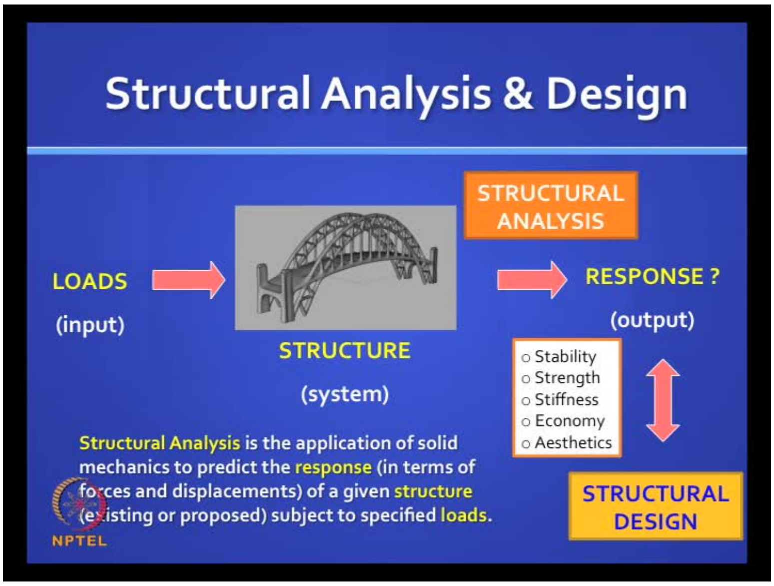

Structural Analysis is the analysis of a given structure subject to some given loads and the idea is to predict the response of the structure, as you may know that this is exactly what is expected of all sciences you can view it as a system.

There is some input to the system, which we call a stimulus and there is an output from the system, which we call a response and Structural Analysis is the application of solid mechanics to predict the response in terms of forces and displacements of a given structure it could be an existing structure or a new structure subject to specified loads.

Along with Structural Analysis, we have structural design your real objective is to do structural design, but in order to do design, you need to do analysis. In design, we proportion the structures we identify the materials and you need to have some initial proportions to do analysis in the first place, so it is an interactive process and there is some requirements of any structure can you name some the main requirements of any structure?

Strength, yes stability, which should come first stability should come first or structure should first be stable strength and?

Durability?!

Durability will come as a party of serviceability okay.

So, basically there are safety related issues and stability strength and stiffness generally cover the safety related issues. Strength and stiffness we have covered stability you all know what it means and then it is not just enough to make a structure strong, stiff, and stable it must be also economical and it must look good it must be aesthetic.

So you will find that economy is there is a tradeoff between economy and safety, because you can you need to invest more to make a structure more safe, but then someone has to pay for it, so the real challenge for a structural engineer is to just about give the right proportions, so that you do not spend too much money in on the structure.

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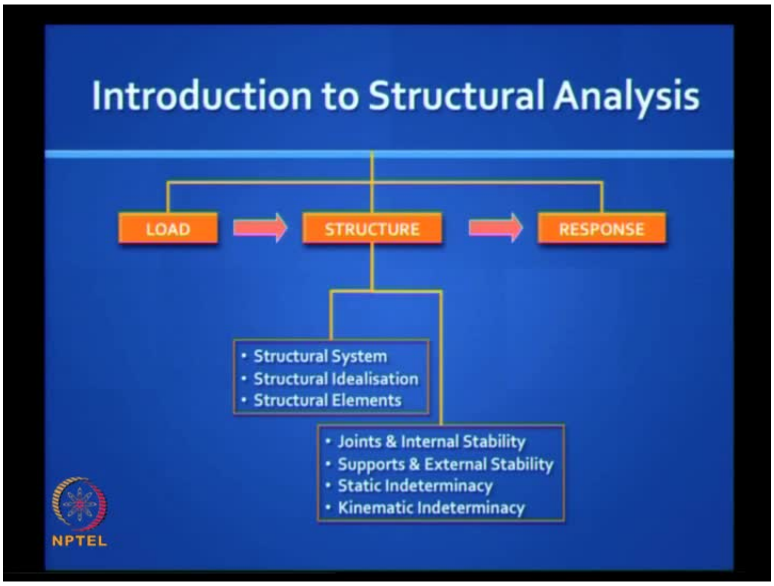

So, we have discuss this structure is really a system and the we actually deal with the mathematical model of the structure, so we reduce the complex three dimensional structure to something that we can handle so, we need to do structural idealization. We have to reduce the structure to elements and the elements are interconnected with joints and there is something called internal stability of the structure.

The structure must also be externally stable especially terrestrial structure on the ground, they should not fly off move away, so we have to have adequate supports and that raises the issue indeterminacy we will look into that static kinematic indeterminacy, so all these put together covers the structure and then loads that act on the structure are of two kinds we will come to that later.

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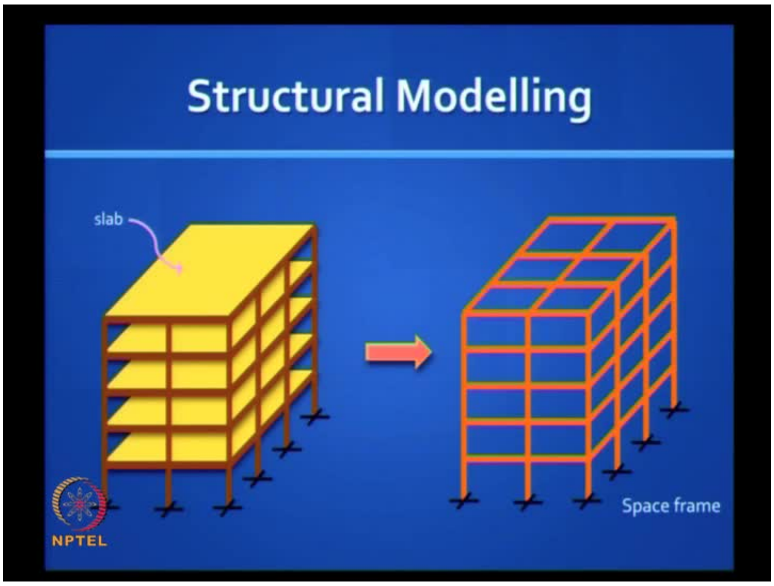

Let us look at the structure properly. So, I have shown here the picture of a building it is a it is pretty complicated you have slabs, you have columns, you have beams, and you have foundations. So for our convenience, we try to separate out the slabs which we deal with independently and separately and we transfer the loads of that slab to the to the frames, so you end-up dealing with the analysis of a space frame.

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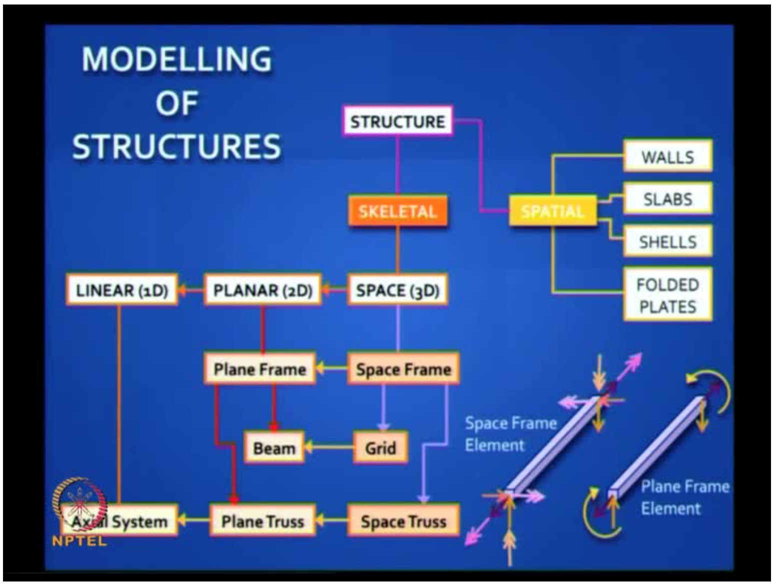

The space frame is made up of elements, which are or the structures are whole is made up of elements which are skeletal and which have also spatial elements. So, spatial elements are those elements whose plan dimensions are large, but the thickness is small for example, a slab or a plate or a shell they constitute spatial elements.

Whereas the skeletal element like the human skeleton is made of line elements whose length dimension is very large in comparison with the cross section dimensions and we all know that the first course in Structural Analysis, we deal with the skeletal elements the spatial elements is little advanced to be taken up later.

So, the most general skeletal structure is a three dimensional space frame, I showed you a picture in the earlier slide and special case of that structure is a planar structure what is how is a planar structure defined?

All those elements and loads are necessary.

All the elements that make up a structure must lie in one plane and the loads that act on the structure must also lie on that same plane, so that is how you get a planar structure and a special case of the planar structure is linear structure for example, the chain where all the elements are connected in one line. So, the space structure is typically made up of space frame elements you can see typical space frame element there, it is cut six degrees of freedom at each of the two ends, you can describe the degrees of freedom in terms of movements translations three translations and three rotations in Cartesian coordinates also, you can think of forces so for example, you have an axial force twisting moment you see the double arrow in the along the longitudinal axis you have shear forces in two planes and bending moments in two planes, so that is pretty complicated,.

So, to simplify we often reduce the complexity and go to a planar structure, which is made up of plane frame elements. In the plane frame element you just have an axial force a shear force and a bending moment at any section and you can see that the planar structures made up of plane frame elements and the grid is the special case of the space frame. In the grid you may have seen networks of beams that especially forming a horizontal planar structure, but the loads are acting out of plane, so it does not qualify to be a planar structure it becomes a space structure a little complicated we will study how to analyze those structures.

In a grid structure you have twisting moments coming into play and that is what makes a different from a beam, because in a beam you have just a shear force and a bending moment, but in a plane frame element you have shear force, bending moment and an axial force and in a grid element we have shear force, bending moment, and a twisting moment, but no axial force and then you have a special case of the space frame called a space truss and you know that a truss member is one which you have only an axial force the shear forces and bending moments are suppose to be negligible and if all the elements in the truss lie in one plane, it is called a plane truss, so plane truss is also a special case of a plane frame and if you have a chain, then you have an axial system this clear, so this is this what we are going to deal with in structural analysis.

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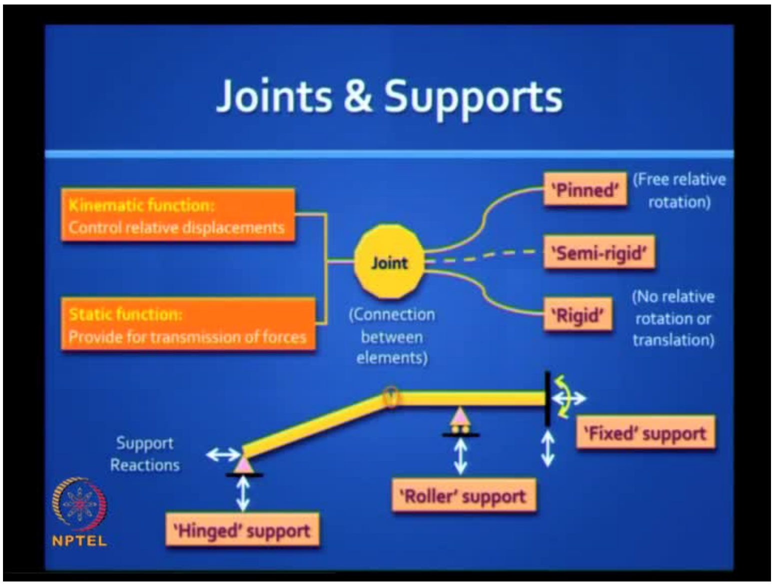

All these elements are connected with joints and what is a function of a joint?

Transfer of loads and moments.

Transferring forces is a static function, but a more important function is.

Constrains the displacements.

Let us take let us take this structure, this is an elbow joint, this is a forearm and this part of the arm are connected with this joint, this joint really holds the elements together, so that if you pull by hand it does not come off you know it is connected to the rest of my body, so one of the main functions of a joint is to ensure that the elements stay together move together and you will allow only those movements that you would like to allow. If you have a rigid connection, you would not allow any relative movement, so in summary you have a kinematic function the purpose is to control relative displacements the displacements could be either translations or could be rotations for example, the joint here the shoulder you have a ball and socket joint which can allow movements in all directions and it is precisely, because you allow those movements at you prevent transmission of those forces corresponding to those movements.

So, no bending moment is transmitted across a hinge joint, but an axial force is transmitted because you do not allow relative axial displacement. So, you have two functions, kinematic function and a static function and ideally these are idealized as either the pinned joint or rigid joint in a pinned joint you allow you do not allow any relative translation, but you allow a relative rotation and it could be in one plane and multiple planes, but if you have a rigid connection, then it is like you have one member one piece, there is no joint in fact, when your bone fractures and it heals again the two elements unite in such a manner you get rigid connection a fully a welded connection for example, is a rigid connection okay.

So, there is something called a semi-rigid connection, which where you have some partial movement we will not look at that in this course and then the supports are joints at the boundaries of the structure here again you can arrest or allow translation or a rotation and you get reactions when you arrest movements okay. So, if you arrest a translation in a particular direction, you provide for reaction it could be vertical or horizontal or incline or you could get a bending moment for example, at the fixed support.

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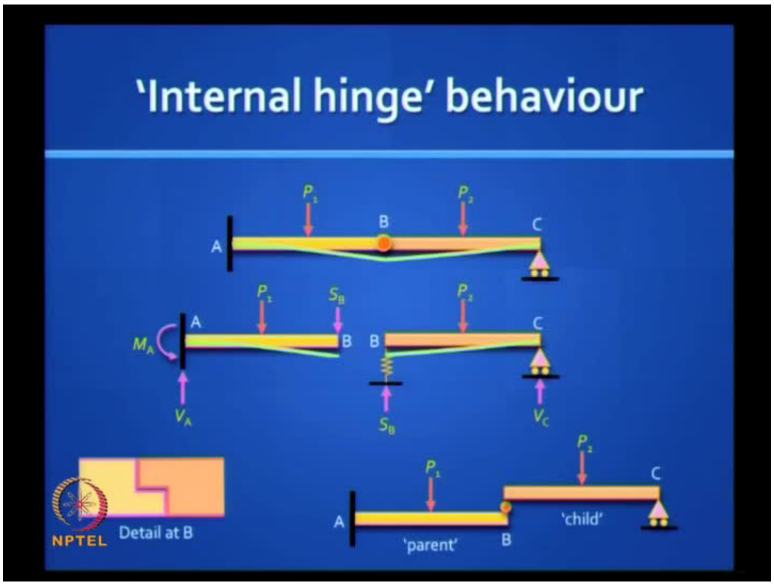

Now, this is a topic which I think many students really have not understood well. What is an internal hinge and how does it work?

So, you have here a propped cantilever A C, which would be statically in determinant, but the provision of a internal hinge in the middle at B makes it just rigid and statically determinant, how does this behave?. Well, take a look at the deflected shape and this is something I have always emphasize a good structural engineer is one who tries to make use of both hemispheres of the brain the logical analytical left brain, which does all the calculations of bending moment shear forces and all that based on equations and the intuitive right side of the brain, which can see directly without doing any calculations and it is necessary to correlate these two and that is why structural analysis is a beautiful subject to develop oneself to develop analytical skills and you also develop intuitive skills.

So, it is a good practice where ever possible to draw deflected shapes for example; the curved shape of the deflected diagram must match with the bending moment diagram. So, here you see that at the joint B, there is a relative change in angle there is no need to satisfy rotational compatibility and you will find that of the two elements A B and B C, one is dependent on the other, which is dependent on which?

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B C is dependent on A B, A B can stand alone on it is on it is own it is a cantilever, but B C is not B C will fall down because there is no support given at B. So, if we have to separate out these two elements, what kind of support would you provide at B?

Spring.

You will provide a spring support, because you can get a vertical force transfer at B, but B can move, so you should ideally module the correct element and here a spring element is appropriate and you can see that the load P 2 will be shared if its right in the middle equally by the forces at B and C and the sheer force develop at B get's transmitted to the cantilever A B and that is how it operates and this is how it becomes statically determinant it is very easy to do that you basically invoked an equation that the bending moment at B 0. No bending moment can be transmitted from one to the other, but even more important you can really appreciate how this works, B C is dependent on A B. So for example, the load P 1 acting on that structure will go entirely to A B because A B can stand on it is own nothing get is transmitted to the support at C, but the load P 2 needs a help of A B, so a part of it reaches A B and if you look at the practical construction these are often used in bridges you find that it is a articulation like this and there is a bearing provided and clearly you can see from the detail at B, B C sitting on A B and B and not vice versa. So, another way to look at it is in this manner here it is very clear which is a child and which is a parent.

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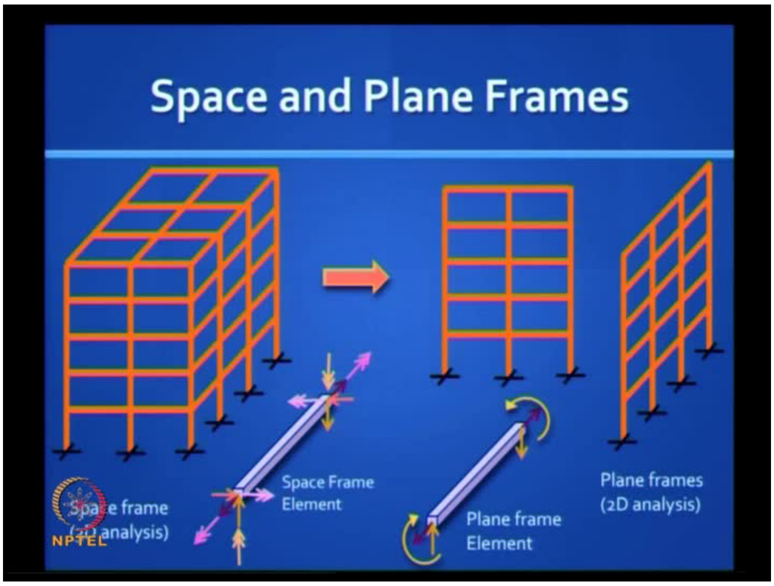

Now, I showed this picture earlier for space frame, a space frame is also very difficult to compute to analyze. Why is it difficult to analyze?

Six degrees there.

Sorry.

Because it has six degrees?

Because it has?

Six degrees of freedom?

Not, that is not the right answer.

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It is highly in determinant okay.

It is highly in determinant, you need to solve many simultaneous equations to crack the problem. So traditionally you try to simplify it so what should you do is? you break it up into plane frames and you make some assumptions the assumption you make is that the frames in the transverse direction and the frames in the longitudinal direction really do not interact, they will interact for example, if the building twists, but if it does not if it is a kind of regular symmetric structure we can make this idealization and if it is a long building most all the plane frames are identical, so you need to analyze only one of them or may be two of them and intermediate frame and end frame.

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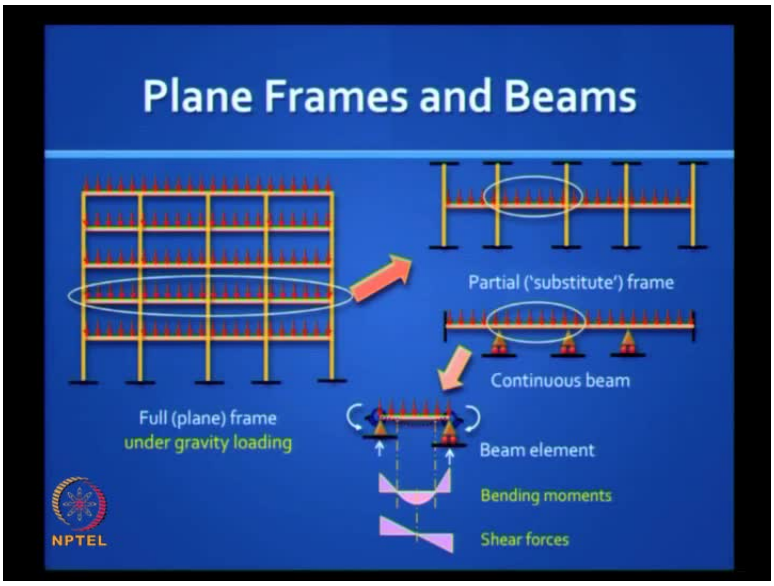

So, it become much simpler this is how it is handled and that is a plane frame element. So, under gravity loads for example, a plane frame will look like this and even this is a little difficult to do in a traditionally the concept of substitute frames have been used to simplify the analysis the argument is the bending moment in any beam will really not be effected by what is happening far away from that end we can prove this through a principle called Muller Breslau's principle, so you could assume you could take out one floor separately and assume the columns to be fixed at top and bottom, so you are actually separating out that frame you substituting the small frame for the big frame there will be errors, but the order of those errors will be not significant you have to be careful when you do this idealization you cannot do it when you when the frame is un-symmetric when the frame is subjected to sway.

Otherwise, you could do this and if you want to take some more shortcuts you could actually take out one beam you could reduce this to a continues beam and say that not much movements get transmitted to the columns, which is true for interior beams because the movement on the left side is more or less equal to moment on the right side you could reduce this to a continues beam and even further, you could reduce take out just one beam. When you take out one beam from the whole big structure, you must recognize that it is not really a simply supported beam because you get some partial fixity at the two ends and so, it is a appropriate put a rotational spring at the two ends and design for some moments.

So, this we know that the bending moment diagram will look like that the sheer force diagram will look like this the exact numbers are something that you need to find out, but a good guess is good to start of when you do modeling of the beam for example, if the total load is W and the span is L it is a general role of some that the bending moments due to gravity load will be in the order of plus or minus W L by 10 and if you design for it, it is good enough, but of course, it is not exactly, so an you can do it more accurately.

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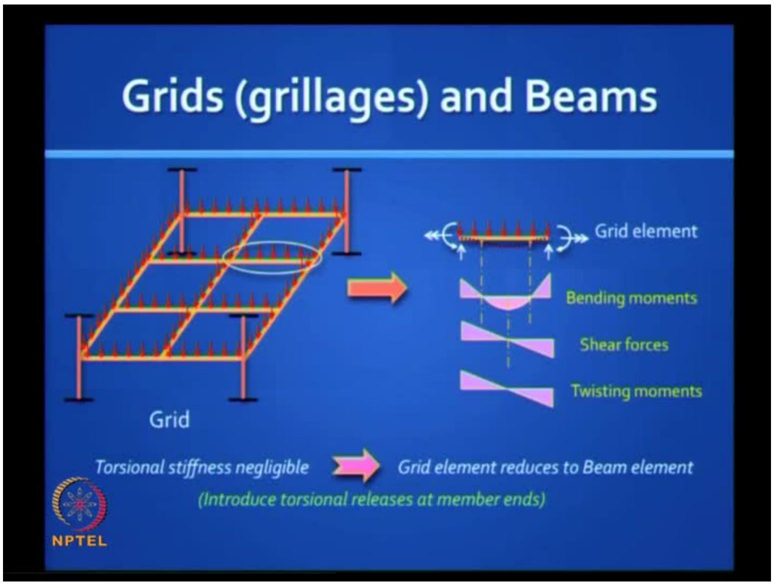

The grillages looks like this, it is all in one floor and as I mentioned the earlier the additional internal force you get is a twisting moment, which is what makes a different from one normal beam.

So, grid l grid element is a beam which also has torsion and if the torsional stiffness is negligible for example, if you are dealing with reinforce concrete and this kind of torsion is called secondary torsion or compatibility torsion. A concrete to concrete connection for torsion is not very strong when this beam bends it is going to have a flexural rotation at this end because it is integrally connected to this beam, that beam is also going to rotate but, what is a flexural rotation for one beam is a angle of twist for the other beam and how much movement is transmitted at that junction depends on the torsional stiffness of this connecting beam in addition to the flexural stiffness of this beam.

Concrete cracks rather easily under torsion and the twisting moment is nothing but, the torsional stiffness multiplied by the angle of twist not to twisting of the torch transmitted at that joint and so, you could have a large angle of twist, but if we have low torsional stiffness and the torsional stiffness degrade can degrade up to 15 percent in normal buildings especially under high loads then you practically get node twisting moment.

Often you can assume that there is no moment transfer and this is allowed by many codes you can treat the beam has been simply supported. So, if you make that assumption the grid element reduces to a beam element.

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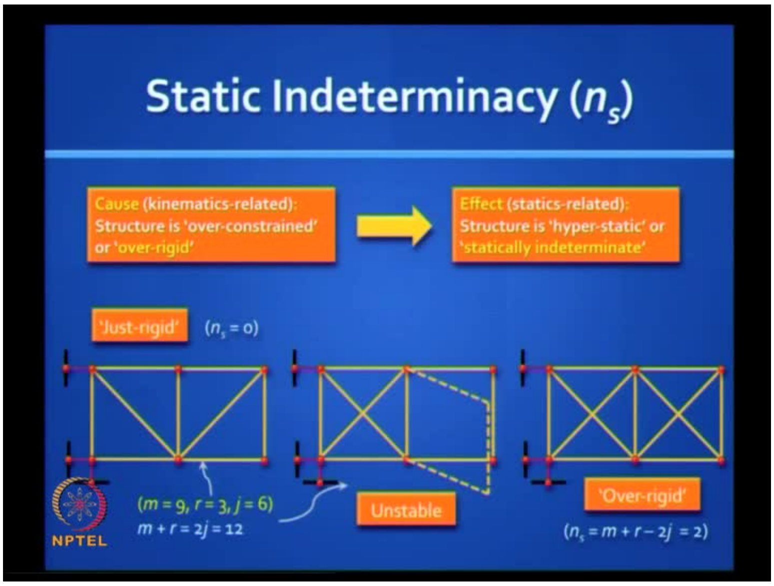

Now, we come to the topic of Static Indeterminacy and it is important to realize that although we use words like statically indeterminate structures. As far the structures is concerned, it is more appropriate to talk in terms of kinematics rather than statics. The indeterminacy is the problem that the analyst faces it is no problem for the structure.

The analyst find the difficult to analyze a structure using simple equations of static equilibrium, so the statics is in determent statics refers to the force filed. Why does it happen? It is happen so because you provided more constrains and the absolute minimum required to keep the structures stable. So, we use a word called over rigid or over constraint to describe a structure which is accessibly constraint and to the extent to which it is constraint accessibly, you have an order of a degree of static indeterminacy. For example, you are familiar with this, this is a simple truss you know that it can be prove that m plus r if it is equal to 2 j, m is a number of bars also signifying the number of unknown internal forces, r is a number of reaction components and j is a number of joints and at every joint in a plane truss you can have two forces that can act you have two equations of equilibrium if we use the method of joints let is how this equation holds good.

Now, here you still satisfy m plus r is equal to 2 j, but the structures are unstable because you have not located the members properly, you do not have triangulation here and that segment on the right can deform and it is important to know how it will move it is unstable those elements cannot taken any shear, but if you provide an extra diagonal element, then you have a degree of static indeterminacy equal to 2. We have studied all this, it is just a simple introduction.

So, here you have an example of an unstable structure in the middle a just rigid structure whose degree of static indeterminacy is 0 and on the right you have an over rigid structure whose degree of static indeterminacy is 2.

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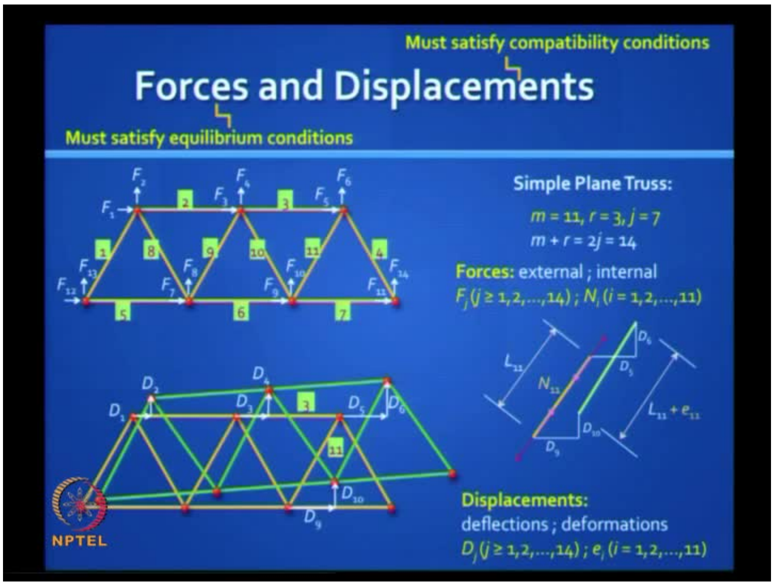

Now, here is something that we will really need to understand it gives us a frame work to do matrix analysis of structures. Take a look at this simple plane truss which satisfies m plus r equal to 2 j. You have 11 members and you have 7 joints, each joint has 2 degrees of freedom so you have 14 degrees of freedom. Let us first see in what all ways we can apply forces, you can apply F 1 and F 2 orthogonally, for convenience we choose x and y directions and we can keep numbering them F 1, F 2, F 3, F 4, F 5, and F 6 all the way to

F 14 and in this numbering you will notice that I have pushed this reactions towards the end of that numbering system.

If you treat this as a simply supported system at the extreme left support, you have a vertical reaction and horizontal reaction those are labeled F 12 and F 13 and at the right side you have F 14 acting upward or downward.

Is it clear.?

So, these arrows show joint forces, some of those joint forces are support reactions and some of those joint forces are potential loads and with this kind of description you have a complete description of the external forces on a structure some of which could be unknown reactions.

Is it clear.?

So, here I have shown a free body of that structure, but the force feel in a structure also includes internal forces and there are 11 bars in that structure each one has an unknown axial force which could be tension or compression. So, if we take out for example, bar number 11, if you take a free body of that bar I have shown in axial tension in that bar the internal forces marked as N 11, N stands for normal force and here is another picture which shows the possible displacements in this structure and we will use the same numbering system if you have F 1 and F 2 at that top corner joint toward the left and right we have the same labeling system for displacements D 1 and D 2

So, 1 and 2 are sometimes refer to this coordinates they kind of unit vectors, because they identify both the location and the direction of either of force or a displacement, but they are all external okay, the forces are external to the structure, so other displacements they act as the joints, so you can have if you have you can have 14 values of displacements.

Now, let us look at that same bar and see it has displaced in a certain manner you will find that the movements at the two extreme joints which we have labeled here D 9, D 10, D5, and D 6 will decide and will dictate how much that bar as D form, so this is simple geometry this is called compatibility.

So, the bar let us say it has elongated and we label the elongation E 11, E standing for 11 clearly E 11 must be a function of the joint displacements vertical and horizontal. So, if you look at the displacement field, the displacement field is made up of joint displacements which I have labeled D j and member elongations which I have E i, j can vary from 1 to 14 in this case and i can vary from 1 to 11 and conjugate with this definition you have F j, the joint forces which can be j can vary from 1 to 14 and N i the internal axial forces i can vary from 1 to 11.

Is this clear?

So, this is a kind of simple frame work which useful for us important thing to notice all the forces must satisfy equilibrium and all the displacements must satisfy compatibility.

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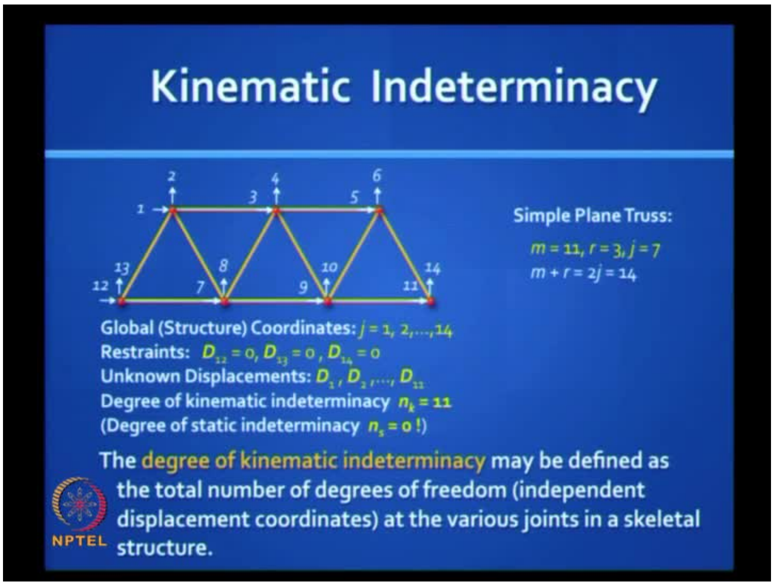

Now, you are familiar with static indeterminacy, it is important to know this new term called Kinematic Indeterminacy. It is very simple our real task in structure analysis to know everything about to force field and the displacement field right?

In the force field, you saw earlier there were some known's and some unknowns. The known's are the loads that are applied and the unknowns are the reactions and the internal forces that is a force field. As far the displacement field is concerned, usually everything is unknown okay the displacement are not known and the bar elongation are not known. If we wish to we should be able to calculate everything and that is a whole idea of doing structure analysis.

Now, kinematics is concerned with movements and statics is concerned with forces. So, in a truss the movements are joints movements which are translations in the x and y directions in this example and the elongations in the different members.

Is it clear?

So, we also found that it is enough to talk about joint displacement because once you know the joint displacements, you also know the bar elongation because there is a compatibility which ties the bar elongations to the joint displacement, so indeterminacy in a kinematic sense in a truss is related to only joint displacements.

Now, in this problem you have 7 joints and in each joint you can have two independent degrees of freedom that is in this case horizontal and vertical translation and so, the degree of Kinematic Indeterminacy you say here is 40. Except at those locations, where you provided supports, because the support ensures that there is no translation, so in this example if it is simply supported one would say that D 12, D 13 and D 14 are known to be 0. So, you indeterminacy now Kinematic Indeterminacy reduces from 14 to 11, 14 minus 3 because you know the displacement there are 0 is 11. So, a truss is a nice example of a structure this particular truss which is statically determinant n s is equal to 0 degree of static indeterminacy if you make it simple supported 0, you could also make it into a cantilever, does not matter. But, the degree of Kinematic Indeterminacy is huge that is why if you were to solve this problem manually, you would not try the displacement method you will try the force method of analysis okay so, this is Kinematic Indeterminacy and you could defined it as a total number of degrees of freedom, which are independent displacements coordinates at the various joints in a skeletal structure.

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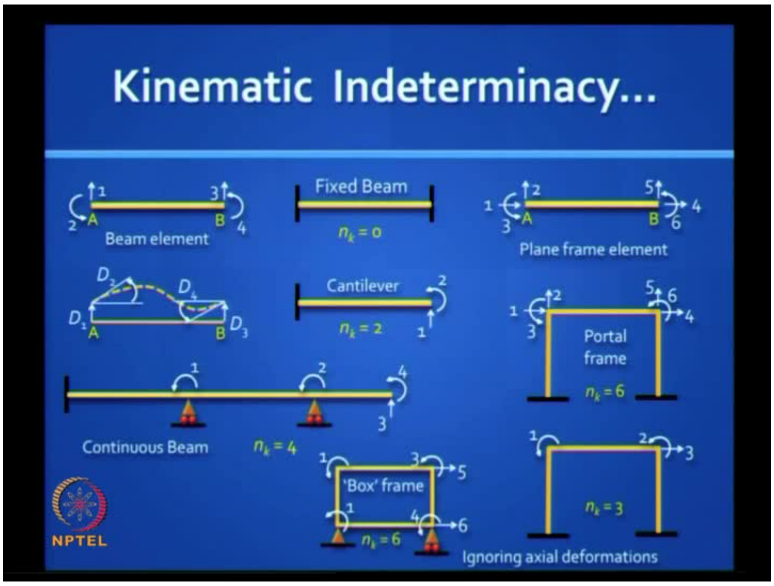

Let us shift from the truss to a beam element. So, I am separate I am isolating a beam element A B and you can see clearly that at the joints there are four movements possible and they are independent D 1 refers to a vertical translation at A, D 2 refers to a slope rotation at the same location A, D 3 refers to a vertical translation at B and D 4 refers to a rotation at B and I have chosen some directions you could change those directions, I have shown upward is positive and anticlockwise is positive okay.

So, here you would say a beam this particular beam element as four degrees of freedom you can erase all the four degrees of freedom as you would in a fixed beam, fixed, fixed beam and that is interesting. The fixed beam is statically is kinematically determinate it is strange that which is kinematically determinate is statically indeterminate okay.

The fixed beam is kinematically determinate. The cantilever has a degree of Kinematic Indeterminacy of 2 because on one end you do not allow movements, on the other end you can have a translation and a rotation.

Does it make sense?

If you have a continues beam, then you can see that there are four movements possible three rotations and one translation. If you have a plain frame element, you have two additional degrees of freedom compare to a beam element because the two ends can move three and four can move and it Is possible for that member to elongate.

One assumption we often make is that axial deformations are negligible, if you make that assumption then one and four are not independent D 4 and D 1 may be inter relative, but, otherwise you can have 6 degrees of freedom.

So, if you take a portal frame with the bottom of the columns as fully fixed, you have 6 degrees of freedom the Kinematic Indeterminacy is 6, I have labeled them is 1, 2, 3, 4, 5, and 6 but, if you ignore the axial deformation that is 6 reduces to 3 because a two column will not change their lengths, so there is no vertical movement possible and the sways is the same for the entire frame that means if that beam element will move horizontally the left end and the right end will move identically.

Does it makes sense?

This is another example of a box frame.

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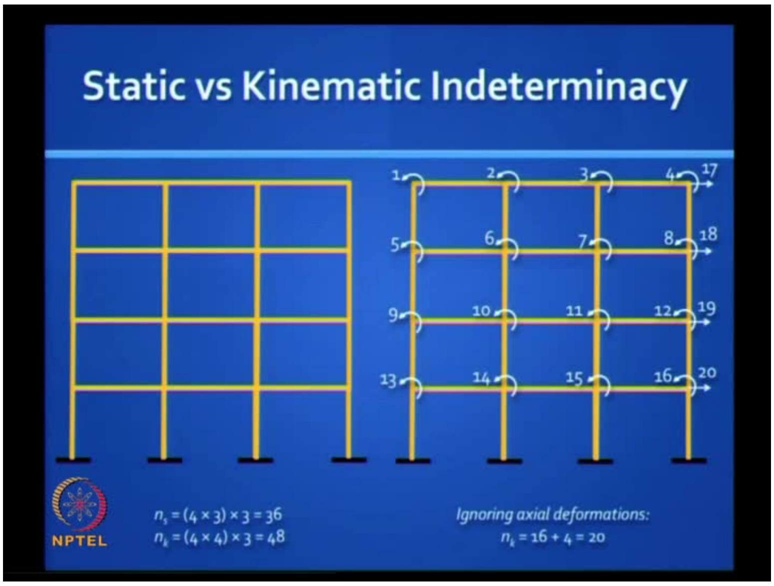

Let us take a proper multistoried building.

Now, what is a degree of static indeterminacy?

The answer is given here each box will have an indeterminacy of 3 and there are many ways of finding this out, you know the tree example you cut it into different trees the trees is just rigid and statically determinate, so you need to make a cut on every beam, so that and every location where you make a cut in a plain frame, you expose how many unknowns.?

Three unknowns.

You will expose a bending moment shear force and axial force, so how many beams are there? There are 4 into 3 beams 12 beams and every cut is 3, so 12 into 3 is 36 okay.

Another way to look at it is, every close box has an indeterminacy of 3 and there are 4 into 3 12 close boxes there the bottom also is like a close box and that is how you can, so you have 36 redundancy in that structure. So, degree of static indeterminacy 36, but the degree of Kinetic Indeterminacy is worse because you see how many joints do you have which can move? you have 4 in each column 4 into 4 is 16 any each joint can move horizontally move, vertically and rotate, so you have except the base so you have 16 into 3 48 okay. The only advantage is if you take advantage of the fact that axial deformations are negligible you can reduce at 48.

Can you tell me how much it will reduce to?

16?

It reduces to 16 rotations but, they are 4 sways possible right? because a whole floor as a whole can move to the left or to the right, so you have 16 rotation degrees of freedom and 4 translation degrees of freedom. So, if you make this assumption then the degree of Kinematic Indeterminacy is much less than that of the static indeterminacy and we will find that the displacement method is a good option.

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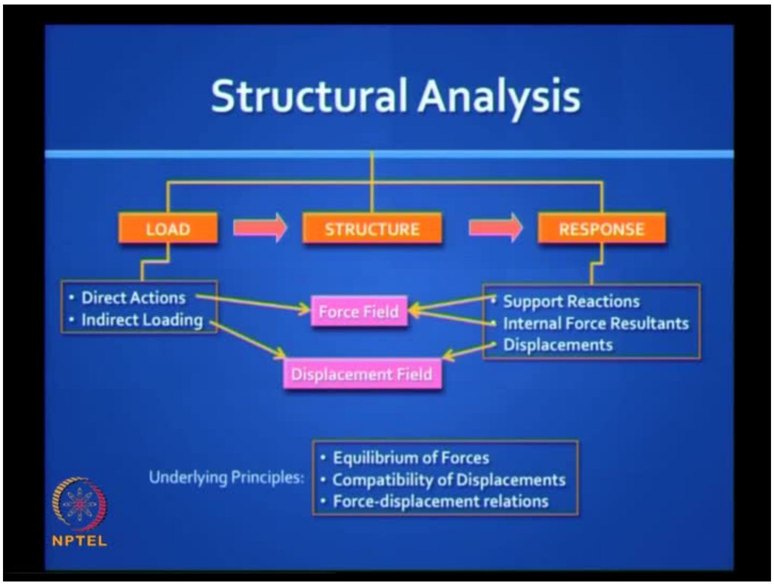

Now, let us look at the loads. There are two types of loads, you can have direct actions and you can have indirect loading I hope you remember this and the response is made up of the internal forces the support reactions and the displacements. Generally, we wish to know everything about the force field but, we may not be interest in knowing all the displacement just a few critical maximum deflections here and there. So, that is a different between the force response and displacement response and the direct actions and the internal forces and support reactions put together make up the force field. The internal forces in a skeleton structure are made up of axial forces, bending moments, shear forces and they could vary from point to point along the length of every element. Whereas the indirectly loading is usually at disturbance in the displacement field.

What is the types of indirect loading that you expect in a structure?

Support settlements, temperature shrinkage environmental related loads which are not very obvious.

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No, no those are all direct actions. There is a third type of loading indirect loading constructional errors constructional errors for example, lack of fitting in trusses.

Well, how do you know that your solution in structural analysis is correct?

You need to have some checks, so if you have a indeterminate structure you can have multiple solutions.

How do you know which solution is correct?

So. the laws of mechanic suggest that you have to ensure that these 3 requirements are satisfied. First all the forces should satisfy equilibrium your equilibrium your force field should be statically admissible.

Number two, your displacement field should be compatible, so these are important words remember equilibrium has to be satisfied compatibility also has to be satisfied and finally, the relationship between forces and displacements, the force displacement relationship should also be satisfied.

Now, the force displacement relationships have two aspects to it, one is the stiffness flexibility aspect that comes from the stress strain relationship of the material and we deal with linear behavior, so you need to have a modulus of velocity well-defined for the material, but there is another aspect which are related to the geometry of the structure. So, stiffness has both the elastic property of the material as well as the cross sectional you know second movement of area cross section area as one.

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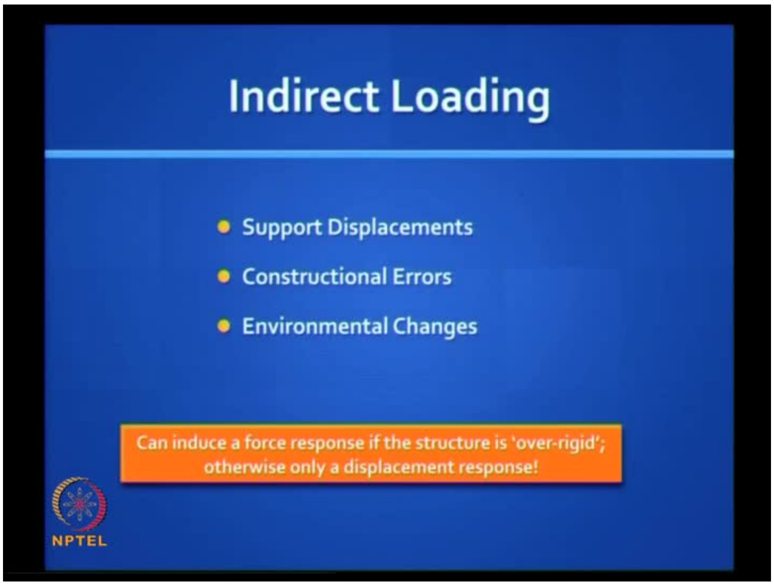

Indirect loading as we mentioned are these three types and it is important note that this can induce a force response if the structure is over rigid or statically indeterminate otherwise you will get only a displacement response.

So, if you have a simple plain truss and all the bars, let us say had a lack of fit or they had different temperature exposures that would cause all the joints to move and the truss will adjust itself but, there would be no internal forces or reactions.

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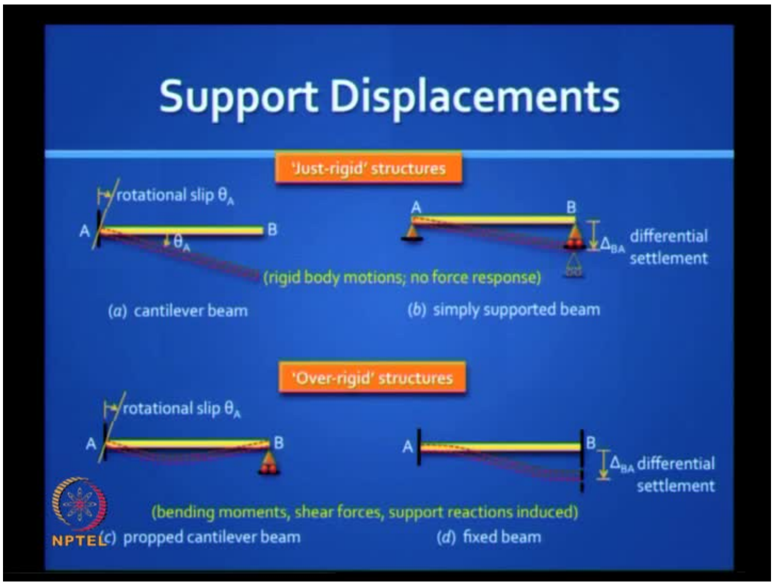

This is an example of a cantilever beam where the support has rotated there are no forces there is a displacement response, but there is no force response similarly, you have a simply supported beam where you have a differential settlement a straight line remains straight, so there is no change in curvature and no bending moment. These are examples of rigid just rigid structures where you have rigid body movements and no force response but, now you take the same cantilever beam with a propped it is called a propped cantilever beam and you do the same rotational slip you will find that kinematically to for the beam to go return to position beam you have to have a change in curvature. The moment in initially straight beam undergoes a change in curvature you know for sure there are bending moments that come into play and the same is true of fixed-fixed beams.

So, here are examples of internal forces and support reactions being generated by differential settlement you do not have a problem of internal forces and reactions if the structure is statically determinate just rigid but, you do if it is not.

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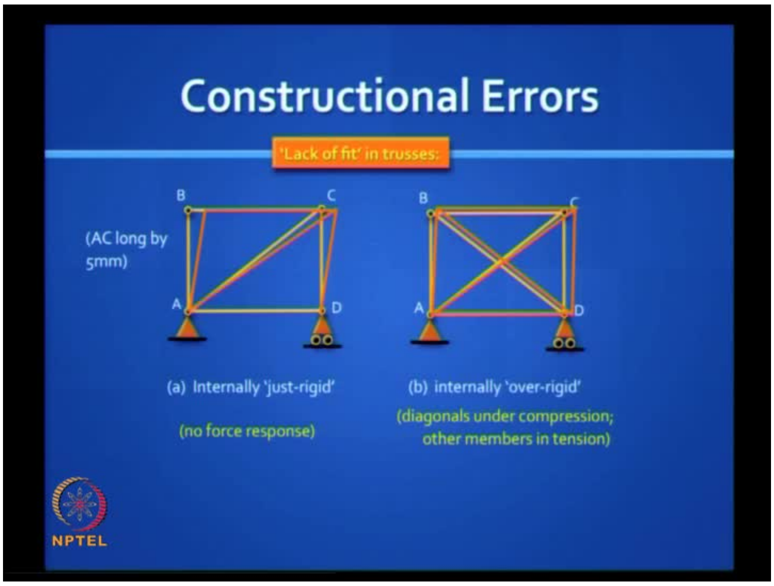

Constructional Errors, lack of fit in trusses you have an internally just rigid plain truss on the left side and you have an extra member redundant member which makes it over rigid. Let us say the bar A C is too long. In the first case, all the bars will just adjust themselves there is every bar gets the length that it is given and the joints will move a little bit.

How to find these movements is?

Something that you can do by principle of virtual work you would have studied that but, if you try to do the same thing in an over rigid structure, you will have problems because some bars will have to be necessarily stretched and some will have to be contracted so, you have an internal force field created which is self equilibrating that is an important point to note.

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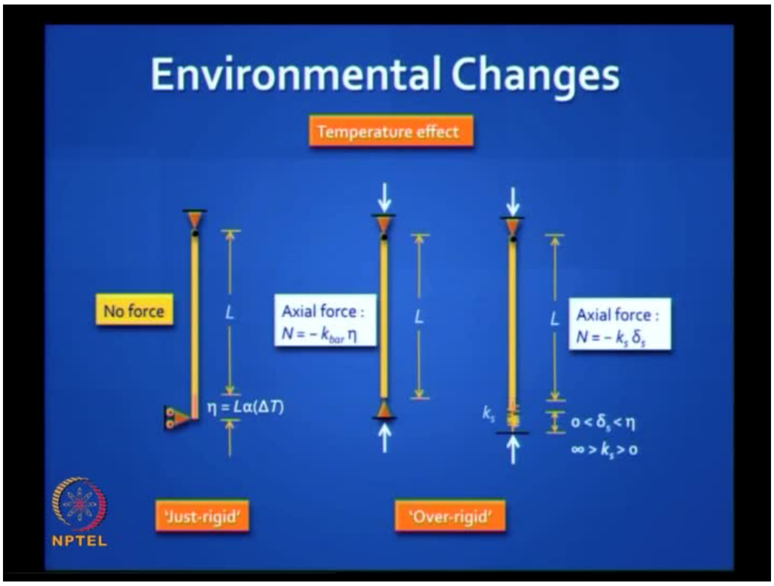

Environmental Changes. Let us take a rod like this, which is free to elongate there is no force but, let us restrain the other end either we prevent the movement fully or we put a spring there and then you have an over rigid structure you have an axial force and you can in the how much it moves depends on the stiffness of the spring.

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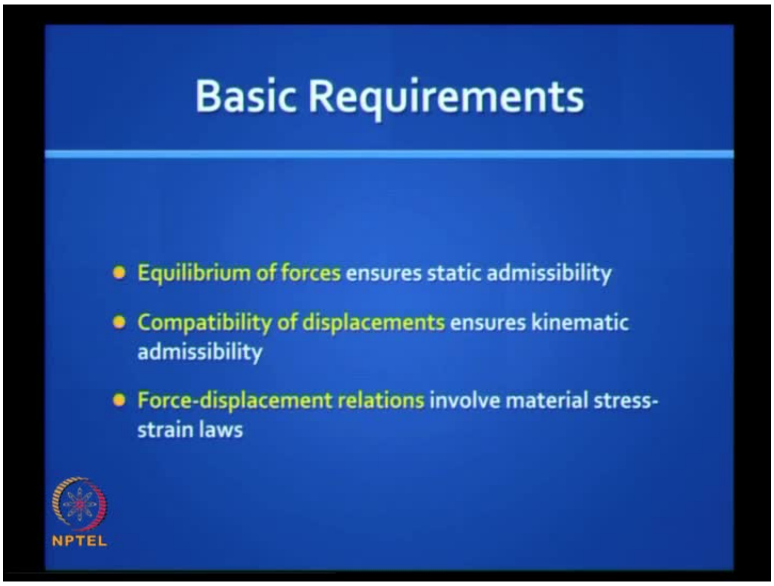

In summary, the structure must satisfy equilibrium and the analysis must satisfy equilibrium so, that you get static admissibility compatibility must be satisfied you get kinematic admissibility and force displacement relations involving material stress strain laws should also be satisfied very quickly this very elementary.

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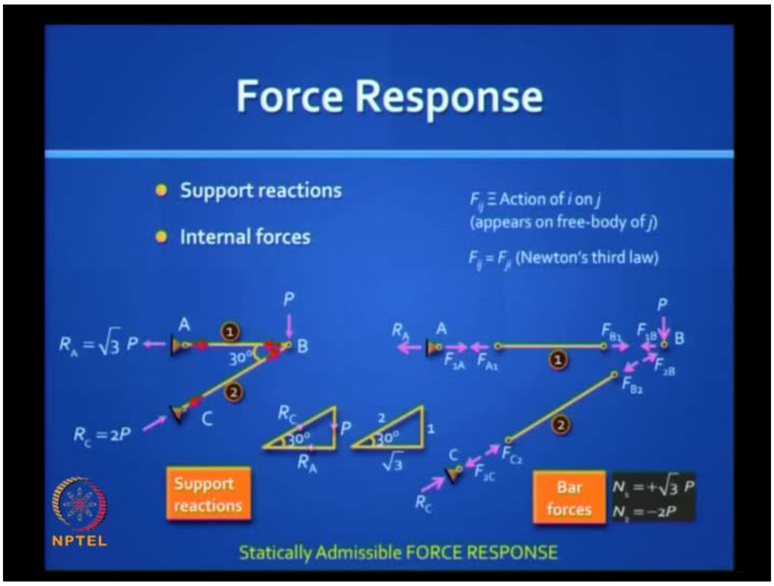

What do you mean by Force Response?

Well in a truss it means you need to know all the support reactions and all the internal forces, so if you take the overall free body of the truss subject to the load P it is easy to calculate the support reactions you can use a triangular forces if you wish or you can use direct equilibrium, but if you examine very closely that is support if you separate out you have an accident reaction coming into play that is Newton's third law. So, you must realize that they are many forces coming into play but, they are really related to one another and you must know how to appreciate the different forces this elementary.

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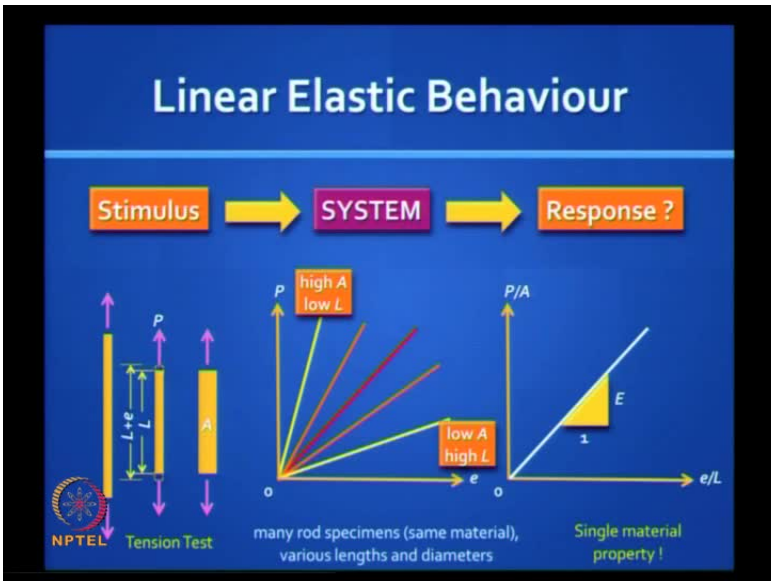

Linear Elastic Behavior and I have asked this question earlier, let us say you do a tension test on a material you can do this in the laboratory you apply a tension P and the bar elongates by E you plot P verses E you get a straight line, if you have linear elastic behavior when you unload it will retrace a path and go back to the origin but, you will find that this line will depend on the type of specimen that you tests.

So, you could play with different diameters of that is specimen you have you know areas of cross section which are larger and which are smaller and you can also change the length of specimen right?.

How will this line change?

Let us say you get so many lines okay.

When will the line be steeper and when it will be flatter intuitively?.

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When the area of cross section is larger, the line will be steeper which means the member is stiffer the axial stiffness larger.

If the length of the member is more what will happen?

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It will become flatter, it is more flexible and it is easier to pull it okay that is true.

So, science is all about trying to reduce all these different lines into a single line.

How will you do that?

(( ))

E by L.

Right, you redraw these graphs dividing the ordinate P by cross sectional area A you gets stress, normal stress and E by L, you get axial strain and you will find that you get a single line which brings out the common material property so, this is a simple understanding of elastic behavior.

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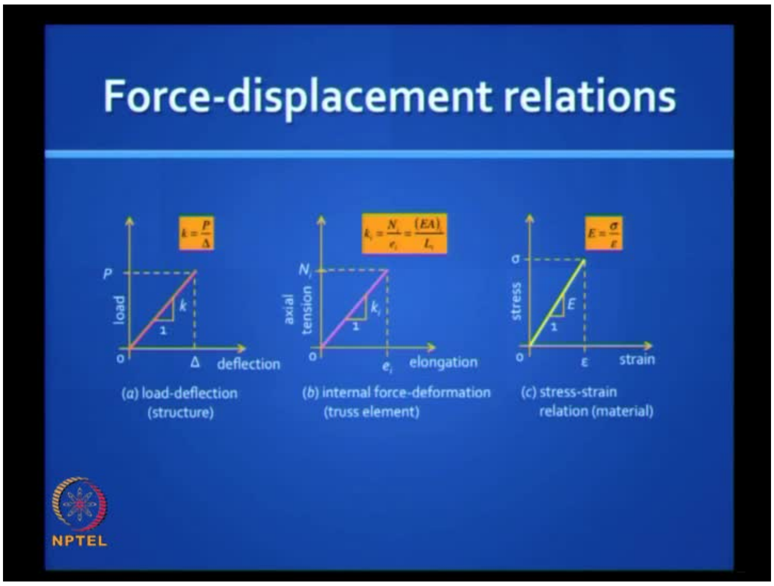

If you look carefully Force-displacement relationships P by delta is if you reduce the element to a spring to an elastic spring you use about spring stiffness right? that is called axial stiffness in this case and you will find that P by delta is the same as N by E for the simple reason that N is equal to P and E is equal to delta for a single element, so if you have different bars, you can define axial stiffness k i for the i th bar s you can prove it to be E A by L from the definition.

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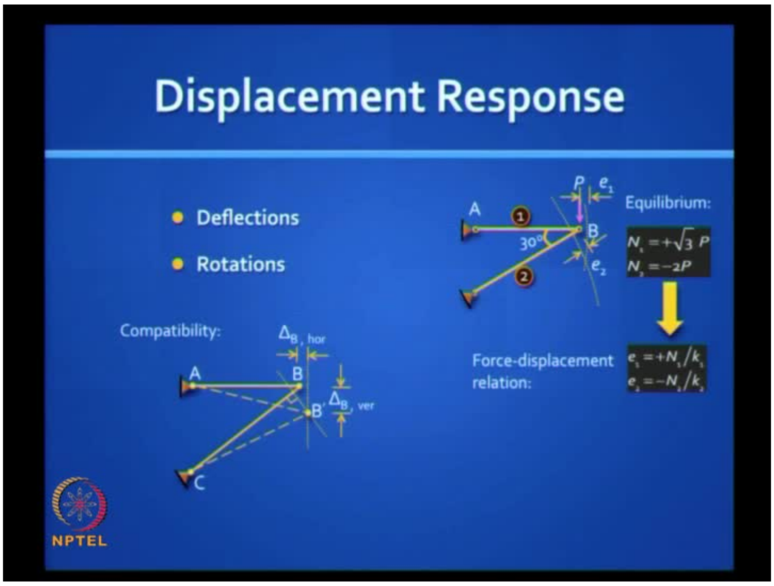

So, we will conclude with this last slide on displacement response. Force response was finding internal forces and support reactions displacement response all about finding essentially deflections and rotations, so if you take that same example once you know the internal forces you know the bar elongations by dividing the axial force in each bar by it's axial stiffness, axial stiffness is E A by L and if you try to plot this, you will find that the bar 1 wants to elongate and the bar 2 wants to reduce in length, so with centre at A and radius A B plus E 1 you try to draw an arc it will come vertically, so this way of construction is possible geometrically, this is called the classic Williot-Mohr diagram, so it is possible to locate locates the fresh location of the joint B and that is how you get the vertical and horizontal displacement.

So, the idea of showing you this slide is to convey to you that there is a strong relationship we will see in matrices, there is a matrix relationship which relate E 1 and E 2 to D 1 and D 2, D 1 being the horizontal deflection and D 2 being the vertical deflection. So, this is a kinematically admissible respond. We will stop here we just doing a review. We will continue in the next class.

Thank you.