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Week - 01

Lecture – 05

Hello everyone, welcome back to the Swayam NPTEL course on Mathematical Geophysics. This is Module 1, Basic Concepts, Lecture Number 5, Geometric Model of Fields. In this lecture, we are going to cover the following concepts. The geometric model of fields, which is essentially a representation technique of the various types of fields we have considered and discussed in previous lectures. This involves understanding the components of geometric models, the algebraic representation of geometric models, and various types of vector lines that can geometrically represent the field models.

Then we will look into various applications of the fields and the geometric models in geophysics. We will be focusing on the Earth's gravitational and magnetic field vector lines. We will also be looking into the geometrical representation of interacting magnetic and gravitational fields. So, let us begin. First, let us look into the basic components of the geometric model of fields.

First, we will look into the field and its generator. So, a field is a quantity or property of a system at each point in space. This we have already discussed in the previous lecture regarding conservative fields and non-conservative fields. Then comes the concept of the field generator. What is a field generator?

A field generator refers to the source or mechanism that creates or produces a field. For example, an electric charge. An electric charge is the source of the electric field surrounding it. Alternatively, we can also examine the mechanism of a pressure gradient creating a fluid flow velocity field. This would represent an electric field around a charge.

Now, let us look at the vector lines. The vector lines are used to represent the direction and magnitude of a vector field at various points in space. For example, let us look at this diagram. Consider this a two-dimensional domain, the X and Y axes. If we divide this domain into a grid consisting of intersecting points as nodes, we can obtain the value of a vector field at each node as follows.

These are the points where the vector can have its values, also at other intersecting points. So, we have to find a way to represent the vectors at these points. Commonly, vectors are represented using arrows. So, that can also be a way to geometrically represent a field. The arrows shown here represent the geometric model of the field.

Similarly, we can construct vector lines by joining such arrows. Consider these two arrows and the line joining them. So that would be an example of vector lines. So these vector lines are useful for visualizing vector fields. The lines indicate the direction of the field, and the density of the lines provides a sense of the strength of the vector field.

For example, we can see that in one part of this figure, the vector lines are closely spaced. So that would represent a strong vector field. While there is a large gap between these vector lines, indicating a weaker vector field. So that gives us an understanding of a geometrical representation of fields using vector lines. Now let us look at the next concept, which is a normal surface.

A normal surface is a surface that is oriented such that, at every point on the surface, the normal vector is aligned with the direction of the field or the force. So let us consider a pictorial example. Consider this surface. So these are the examples of vectors that are representing the force.

So the surface which we have shown here is perpendicular to the force vectors. Thus, this surface can be called a normal surface. So the normal surface is also a very interesting way to represent the geometrical model of a field. Coming to geophysical applications, gravitational, magnetic, and electric fields are very appropriately represented using vector lines as well as normal surfaces. The temperature field, being a scalar quantity, can also be represented by surfaces which pass through the points where the temperature holds a particular value.

Those surfaces are usually called contours. So contour surfaces are another way to represent the geometric model of a field where the field is a scalar quantity. Now, what are the generators of the fields in geophysics as listed here? We have the mass, which is the field generator for gravitational force. The current is the field generator for magnetic fields.

The electric charges are the field generators for electric fields, as we had discussed earlier. The earthquakes or any vibration in the earth's surface or beneath it are the generators for seismic fields. The seismic field is nothing but the field of displacements inside the earth, generated due to any sources such as an earthquake. Now, in the earth, where the temperature varies along the radius, we have the geothermal gradient, which is the source of temperature fields. In the atmosphere, the same temperature field would be developed as a result of prevailing currents.

In air and the solar radiation. All these fields in geophysics have their physical origins, which are termed as the field generators. In the context of the gravitational field, vector lines are gravity field lines which are tangent to the Earth's surface. In the context of the gravitational field, we will be discussing the vector lines and normal surface. For the gravitational field, the vector lines are the gravitational field lines, and the tangent to the Earth's surface is the normal surface.

So let us So let us consider this as the Earth's surface. So let us consider this as the Earth's surface. The tangent to the Earth's surface is the normal surface. Shortly, we will look at the vector lines. Before that, let us consider the algebraic representation of geometric models of the fields. In other words, the geometrical constructs such as lines and surfaces should be represented in a algebraically concise way so that they can be better understood and analyzed further for proper understanding first.

Let us consider the field map. So M as a function of P is a field map. M stands for map, which is a function of the position vector P. It contains field vector lines denoted by L_N. This can be seen in this adjacent diagram, which is shown here. M1, M2, and M3 are three field maps which correspond to the vector lines L1, L2, and L3.

Along these vector lines, we consider small line elements which are denoted by DL1, DL2, and DL3. Overall, this constitutes the field map. Thus, L1, L2, and L3 are vector lines. dl1, dl2, and dl3 are the elements of these vector lines. Now consider the angle between the field M vector and the field line element dl vector, which is denoted by θ.

The condition for Ln to be a vector line is This box highlights the two conditions: θ equals 0, which means the M vector and the dl vector have to be aligned. It can be seen here. The M1 field is aligned with the dl1 vector of the elemental line. Now this results in the cosine of the angle θ being 1.

In other words, we can say that the field M vector is tangential to the field lines L1 or L2, as the case may be. And the M vector and the DL vector need to be parallel to each other. This leads to the equation for the vector lines, which can be described as follows. Consider this set of equations where we have the ratios dl1 to M1 equal to dl2 to M2, which is also equal to dl3 to M3.

$$\frac{dl_1}{M_1} = \frac{dl_2}{M_2} = \frac{dl_3}{M_3}$$

This equality of the ratios of elemental lines to the fields gives the equation for the vector line.

All it means is that the fields M1, M2, or M3 are parallel to dl1, dl2, and dl3. Their respective magnitudes can be obtained from the closeness of M1, M2, M3 and L1, L2, L3. For particular cases of coordinate systems, such as the Cartesian coordinate system, this equation of vector line reduces to d_x by m_x, d_y by m_y, d_z by m_z,

$$\frac{d_x}{m_x} = \frac{d_y}{m_y} = \frac{d_z}{m_z}$$

where m_x, m_y, and m_z are the fields at particular points along the x, y, and z directions, respectively, while dx, dy, and dz are the line elements along the respective directions. Similarly, in the spherical coordinate system, we have the equation of vector lines as given here.

This can be described in terms of the radius r, the azimuthal angle φ, and the polar angle θ. So this reads

$$\frac{dr}{M_r} = \frac{rd\theta}{M_\theta} = \frac{rsin\theta d\phi}{M_\emptyset}$$

So in spherical coordinates, we can see that the respective elemental lines are taken as ratios with the corresponding directional field vectors. So from this, the vector line equations can be derived for any given force field. And the lines, when represented on a map or a field over a particular domain, would give us the geometric model of the field.

Now let us consider the next topic, which is the types of vector lines. So this is an interesting thing, as there are two types of vector lines. One of the vector lines is an open vector line, while the other is a closed vector line. This can be easily understood by considering the adjacent diagrams.

First, the open vector lines have terminal points such as These points end on the charge itself and spread out to infinity. Reversing the direction of the vector field lines still makes them open vector lines because they have a terminal point. So, open vector lines consist of terminal points, such as the field line of a point charge. On the other hand, closed vector lines are unbounded and form a loop, which makes them bereft of any terminal points.

As the name suggests, closed vector lines can be understood using the following diagram. Here, let us consider the dipolar structure consisting of positive and negative poles. The magnetic field lines diverge from the positive pole and converge into the negative pole. They form a closed loop. Even if it is not able to be shown in this limited space, the lines diverging out would form a closed loop.

So, these are examples of closed vector lines. So, vector lines are very common and appropriate to serve as a geometric model of a field, and terminal points characterize the generators of the field. For example, terminal points exist for single charges, which means that the source is a singular point from where the field is generated and diverges out. In the case of a magnetic dipole, as shown in the second figure,

The source dipole is generating a magnetic field that forms closed loops, and hence this is a different type of generator of a field that is represented using closed vector lines. Vector lines can illustrate not only the direction of the field but also the magnitude of the field. Consider the circle as drawn here, very close to the electric charge. You can see the density of the field lines is higher compared to a bigger circle. The bigger circle has a larger surface area, but the number of field lines is the same.

So the density of the field lines is less, indicating a reduced magnitude of the field. Thus, if the line density is αM_ρ, the number of vector lines piercing any elementary surface ds can be obtained as αM_ρ* ds. This is by definition of the line density. Consider this elementary surface.

If the line density is αM_ρ, then the number of vector lines would be simply a multiplication of the density with the surface area, that is αM_ρ * ds. Now, if we take this elemental surface area to some other location such as this, we can see that the number of lines has drastically reduced, indicating that the field becomes weaker farther away from the generator. So this is a very useful geometrical way to represent vector fields. Now, in geophysics, we have plenty of Conservative fields, as well as non-conservative fields, require to be represented geometrically for better understanding. Shown here are the globes of our planet Earth with its most commonly used and popular fields: the gravitational field and the magnetic fields. You can see that the gravitational field lines are terminating on the planet, while the magnetic field lines form closed loops. The gravitational field is directed along the radial direction, while the magnetic field lines form a complex loop. So, this complex loop is similar to that which is generated by a bar magnet and is popularly known as the dipole field of the Earth.

These are the magnetic field lines which converge on the North Pole. These are the magnetic field lines which converge close to the geographic South Pole of the Earth. And diverge from the geographic North Pole. The blue line indicates the magnetic axis, where the N and S are the magnetic poles. The magnetic South Pole is adjacent to the geographic North Pole and vice versa.

Thus, the magnetic field lines, as shown here, are representative of the magnetic field of the Earth that is generated within its deep interior. Moving on to the geometrical representation of interacting magnetic fields. As we know, Earth is a planet which has its own magnetic field. Also, other planets have their own magnetic fields.

For example, the gas giant Jupiter, which is the fifth planet in our solar system, consists of a gaseous atmosphere and generates its own magnetic field. Now, the Earth's magnetic field and Jupiter's magnetic field interact with each other, and that interaction distorts the individual fields of the planets and can be represented using vector lines. Notice that Jupiter's magnetic field has its poles, the north and the south poles. Adjacent to the geographical north and south poles of the planet, respectively. While for the Earth, the magnetic north and the magnetic south poles are aligned with the opposite poles, which are geographically located.

Now, the interacting magnetic field lines resemble a completely different geometrical structure than the individual field lines. While the individual field lines represent a dipolar structure, the interacting magnetic field lines represent an elliptical structure. Thus, this interacting field lines can be very useful to understand using vector lines or field lines. Similarly, we have the interacting gravitational field lines between the planet Earth and our only natural satellite, the Moon.

Now the alignment of the planet Earth's axis and the Moon's axis is shown here. Regardless of the alignment, the gravitational field lines get distorted due to the influence of the individual gravitational fields. Thus, the interaction distorts the individual gravitational field, which is radial in direction, such as this, and converts it to elliptical field lines.

So this change in the gravitational field can be represented very appropriately using field lines, which otherwise would make it very hard to understand just by the virtue of equations or algebraic expressions. Thus, in this lecture, we come to the conclusion as listed here. First, the geometric modeling of fields is a crucial aspect of geophysics. It enables the representation and analysis of complex field distributions, making it easier for the user to understand. Also, the geometrical modeling

helps to capture the spatial relationships or the distributions and patterns inherent in geophysical fields, such as we have seen in gravitational, magnetic, and electromagnetic fields, which interact among themselves among different planets and other celestial bodies, distorting the fields and making them spatially complex. In such cases, geometrical modeling offers improved interpretation of the geophysical data. However, there are some challenges due to the complexity of the geophysical systems and uncertainty in the obtained data, which are measured using various equipment. In conclusion, we can say that the geometrical modeling of fields is a powerful tool in visualizing geophysical data, and it enables researchers and various other scholars to better understand and interpret complex geophysical phenomena. So this concludes the lecture on geometric representation of fields. And also concludes the first module, which are the basic concepts.

We will start the next module with a new lecture to follow soon. Thank you.