Mathematical Geophysics

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Lecture – 33

Hello everyone, welcome to the SWAYAM NPTEL course on Mathematical Geophysics. We continue with module number 7, Thermofluidic Processes in Geophysics. This is the third lecture in this module, titled Mantle Convection. In this lecture, the concepts covered are related to convective heat flow in the mantle. In the mantle, we will go through the various types of heat flow, divided into five components which are presented in this lecture. First, the concepts of heat flow. Second, mantle convection models. Third, the mantle convection equation. Fourth, the various non-dimensionalization processes. And fifth, the material properties which affect mantle convection. So let us begin.

First, we will look into the concept of heat flow in the mantle. As we have discussed in the previous lecture, the mantle is highly viscous. So the convective heat flow in the mantle occurs through convection as well as conduction.

So convective heat flow in the mantle occurs predominantly while conduction is also prevalent, although the conductivity is so low that conduction is not as significant as convection. In the mantle, the convective heat transfer process transfers heat from the core-mantle boundary toward the surface, that is, the crust. The convective heat flow in the mantle occurs through the slow, continuous movement of semi-solid rock, which is occurring in the mantle. This behaves plastically over geological timescales. The geological timescale ranges from millions to hundreds of millions of years, which is the time period of overturning of the material in the mantle.

Next, we look into the sources of the heat that is there in the mantle. There are three sources of heat in the mantle. First, the residual heat from the accretion of planetesimals and differentiation of the Earth during its early formation. This is the trapped heat. Next, the heat which is produced by the decay of radioactive isotopes such as uranium, thorium, and potassium-40. These are degenerative heat sources. Finally, we have the heat which is transferred from the Earth's core into the mantle. Thus, we have the trapped heat, the generated heat, and the transferred heat, which add to the sources of heat in the mantle. Thus, if we consider this as the mantle, then we have the entrapped heat shown by dotted regions.

We have generative heat, which occurs due to radioactive decay and the transfer of heat from the core to the mantle. Now, the mechanism of mantle convection. Mantle convection is driven by thermal gradients. The thermal gradients occur due to the temperature difference between deeper layers in the mantle and shallower layers, where the temperature is lower. The hotter and less dense rock in the deeper regions rises toward the surface, while cooler and denser rocks at the surface and shallower regions move toward the deeper regions of the mantle. This movement of denser rock toward the core-mantle boundary and the movement of lighter rocks away from the core-mantle boundary create the convective overturn cycle of the mantle.

Next, we have viscosity. Viscosity is the resistive force of mantle convection. It resists the flow of mantle fluid or highly viscous solids throughout the convective cycle. However, due to such highly viscous flow, the mantle acquires its unique convective signature: a slow, tedious, and continuous movement of rock. High-temperature regions in the mantle semi-solidify the rock, making it partially molten and soft. This allows plastic deformation of the rock, creating a shearing effect and convection currents.

A few features of mantle convection are large-scale convection which are large regions of overturning material cycles. Then there are smaller, localized upwelling zones. These upwelling zones are near the core-mantle boundary, and there are also downwelling zones. These are located at shallower regions of the mantle. Mantle convection is dominated by advection and conduction. Heat is primarily transported through advection, which is the transport of high-temperature materials and low-temperature materials, which carry thermal energy with them. This is material transport. There is also conductive heat transfer. Conductive heat transfer does not require the transport of materials. However, it is only significant in shallower regions near the lithosphere or the upper regions of the mantle, where the material is solid.

Now we will look into the various models of mantle convection. There are three main models of mantle convection. One is whole-mantle convection. This model assumes that the entire mantle convects as a single unit. A single unit or a single-cell convective cycle moves material from the core-mantle boundary up to the lithosphere and back. There are no intermediate cycles. This diagram illustrates whole-mantle convection. We have the core below and the lithosphere at the top. Note that there are only single large-scale cycles of mantle convection. These are single-cell convective models.

A more advanced model of mantle convection is the layered convection model. The layered convection model assumes that mantle convection occurs in different layers. These layers are separated by material properties. For example, this diagram shows the layered convection model in two layers. There is this first layer, which is closer to the surface and smaller than the second layer, which is deeper inside. The first layer has different material properties than the second layer. Hence, the total convection is divided into two layers. This is a dual-cell layered model.

There can be more than two cells or two layers in a layered convection model.The third mechanism of mantle convection is plume-driven convection. Plume-driven convection means that convection is forced from the core-mantle boundary surface. This is illustrated using this figure. Consider this plume.

This plume occurs when a small region near the core-mantle boundary becomes excessively hot, destabilizes, and forms the plume. This plume then rises through the mantle up to the surface or the lithosphere. You can see in this diagram that there are various plumes at certain stages of evolution. This is a new plume. This is an extinct plume, which shows that the plume had evolved over time before being destroyed.

This is an established plume, which has completed its cycle and evolution from its generation at the core-mantle boundary up to reaching the lithosphere. The plume-driven convection model also includes downwellings, or the movement of cold, denser material from top to bottom. This occurs through subduction slabs. These are subduction regions. An evolved plume gives rise to hotspots or volcanic islands.

There are many occurrences of volcanic islands that can be observed on the Earth's surface. For example, the Hawaiian Islands. These islands are evidence that this type of plume-driven convection occurs in the mantle. Other evidence observed on the Earth's surface that justifies plume-driven convection includes the formation of ridges, such as the mid-oceanic ridge, volcanic chain mountains, andesite volcanoes, island arcs, etc.

These are very unique geodynamic signatures that support the plume-driven convection model of the mantle. Now, let us proceed to derive the equations for mantle convection. The equations for mantle convection involve governing equations for fluid dynamics, heat transfer, and thermodynamics. We invoke the unique conditions of the Earth's mantle. They are inherently complex, and the mantle's high viscosity affects the motion of fluid and fluid-like solid rock.

These models are inherently very complex, adapting to the mantle's high viscosity, material properties, and large-scale geometry. So, let us look at the mathematical expressions for the governing equations. First, we have the divergence of $\mathbf{u}$ equals zero, which represents the continuity equation or conservation of mass. Secondly, we have the momentum conservation equation or Newton's law of motion. Here, the high-viscosity fluid requires a low Reynolds number regime, which means there is no inertial term.

That means the $\frac{\partial \mathbf{u}}{\partial t}$ term which is present as acceleration term is negligible. This leads to negligible inertial forces compared to highly viscous and highly buoyant forces. Thus we have the net force equals to zero. There are no inertial term or mass into acceleration term on the right hand side because of the slowly movement of materials going to high viscosity. We have on the left hand side the first term is the pressure gradient, the second term is the viscosity and the third term is the buoyancy term.

Note that the viscosity term includes the viscosity coefficient ν inside the gradient operator. This is because ν is spatially dependent. Finally, we have the conservation of energy. The conservation of energy is also equivalently known as the heat transfer equation. As we know, both conduction and convection heat transfer occurs.

This gives us $\frac{\partial T}{\partial t}$ which is the evolution of temperature through time affected by convection process given by this advective term which is $\mathbf{u} \cdot \nabla T$. The conduction process which is given by $\kappa \nabla^2 T$ and any other heat generation process given by $\frac{H}{\rho C_p}$. These are the advective, conductive and enthalpy related heat generation and heat transfer processes. These three terms can be written as the advective, conductive and the radioactive processes of heat transfer. Here, κ, ρ, and C_p are fluid properties, and H is a heat source or heat sink.

Now, modeling and solving these convective equations for the mantle requires non-dimensionalization. The non-dimensional process makes the equations in a form that is more relevant for understanding the physical mechanisms and parameter regime dependence of the solutions. This non-dimensionalization process involves various scaling factors with which the quantities are scaled. For example, the length scale is given by d, which is the thickness of the mantle. The time scale is given by the thermal diffusion time scale. The temperature is scaled by ΔT, which is the temperature difference across the mantle. The velocity scale is given by $\frac{\kappa}{d}$.

The non-dimensional control parameters are the Rayleigh number and Prandtl number. The Rayleigh number and Prandtl number denote the effect of buoyancy and material properties, respectively. The Rayleigh number is the ratio of buoyancy force to viscous resistance, while the Prandtl number is the ratio of viscous diffusion to thermal diffusion. Particularly for the mantle, since the viscous diffusion is much higher, the Prandtl number tends to infinity for the highly viscous mantle.

Next, we come to the various boundary conditions. The boundary conditions are useful to constrain the solution of the partial differential equations that govern mantle convection. The thermal boundary condition is fixed temperature at both boundaries or fixed heat flux at one of the boundaries. The mechanical boundary condition is a no-slip boundary condition where the velocity is set to zero at the top and bottom boundaries of the mantle. Alternatively, we can have stress-free mechanical boundary conditions where the stress or the normal derivative of the velocity goes to zero at the boundaries.

The heat transfer is estimated using Nusselt's number, which is a non-dimensional number quantifying the dominance of convective processes over conductive processes. It has been obtained through rigorous studies that Nusselt's number varies as the Rayleigh number to the power of one-third, which means as the Rayleigh number increases, Nusselt's number increases. In other words, stronger buoyancy forces would lead to higher convection and convective dominance in the mantle.

For more information, we have provided a table. This table lists the various properties of matter, the units, and the quantities of various physical quantities related to mantle convection. One can refer to them for further details.

Now we move on to material property-dependent mantle convection. The material property of viscosity changes throughout the mantle. The viscosity in the mantle is also temperature-dependent. It is given by $\nu(T) = \nu_0 \exp\left(\frac{E}{RT}\right)$ where E is the activation energy and R is the gas constant.

Now, depending upon high and low temperature regions, mantle convection experiences higher or lower viscous resistances. Let us consider the diagram for a better understanding of this material property-dependent mantle convection. Here, one can see that the temperatures are given by the color. Regions of high temperature are shown in red, while colder regions are shown in blue. Where the temperature is high, in those regions, the viscosity is lower. And in the colder regions, the viscosity is higher. In the adjacent diagram, the temperature and viscosity are shown. With height from the core-mantle boundary, the temperature changes as shown here. This leads to the change in viscosity with height. Higher temperature leads to lower viscosity, and lower temperature leads to higher viscosity.

Thus, mantle convection is to be simulated or solved by taking into account the temperature-dependent viscosity for more accurate solutions. Now, let us consider the various geophysical applications of the mantle convection models. These models explain certain observations on the surface of the Earth. The plume-driven model. It can explain volcanic activity, such as hotspots like the Hawaiian Islands and Yellowstone National Park activities.

If we consider the layered convection model, it predicts slower heat loss from the lower mantle. This explains how Earth has retained the heat that has been trapped for a long duration, nearly 4.5 billion years. This explains the slow loss of heat, without which Earth would have lost all its trapped heat and would have become barren. Lastly, the whole-mantle convection model.

It provides the mechanical link between the surface plate tectonic motion and mantle dynamics. It is only the motion of mantle convection that drives plate motions. The whole-mantle convection model provides good results and estimates for the plate motion with mantle convection dynamics. Also, the subduction slabs act as the primary driver of convection currents in the mantle.

This is supported by the whole-mantle convection model. Thus, we can see that various mantle convection models support various pieces of evidence. Thus, it can be understood that mantle convection includes all these different features from various models to a certain extent. Now, let us discuss the conclusions. Mantle convection models are fundamental to our understanding of Earth's internal processes.

Various manifestations of such convection dynamics occur on the surface, which are observable and provide information about the deep interior. No single model can perfectly explain all the observations of Earth's surface due to mantle convection. Thus, a comprehensive framework must take into account features involving all mantle convection models. The whole mantle convection model explains deep mantle plumes, such as convection slabs extending up to the core-mantle boundary. It also explains the efficient heat transfer from the core to the upper regions of the mantle, which is very effective.

However, the layered convection model reduces the heat flow estimates. It provides geochemical evidence of isolated mantle reservoirs. Additionally, it accounts for the 660 km phase transition in flow patterns. This phase transition involves changes in material properties at a depth of 660 km, which the layered convection model utilizes to refine mantle convection patterns. Thus, the mantle convection model is an excellent example of heat flow processes in Earth.

These are various references that discuss mantle convection in much greater detail. Thank you.