Mathematical Geophysics

Swarandeep Sahoo

Department of Applied Geophysics

Indian Institutes of Technology (Indian School of Mines), Dhanbad

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Lecture – 32

Hello everyone. Welcome to the SWAYAM NPTEL course on Mathematical Geophysics. We continue with module number 7, Thermofluidic Processes in Geophysics. This is lecture number 2, Geophysical Heat Transfer. In this lecture, the following concepts are covered.

In general, we will be looking into various heat flow processes in the Earth. First, we will look into the concept of heat flow. Second, heat transport mechanisms. Third, conductive heat transfer. Fourth, convective heat transfer.

Finally, we will look into some examples of heat flow in the Earth. So, let us begin. The concept of heat flow. Before understanding the concept of heat flow, it is important to understand why heat flows. Heat flows because of gradients.

Just like we have a diffusion process. Similarly, in the temperature field, if there is a change in temperature with space, then there is a temperature gradient. This temperature gradient is called heat flux. Heat flux depends on the temperature gradient and the conductivity. Now, temperature gradients would lead to heat that flows per unit second, and this heat flow per unit area would be equal to the heat flux.

The unit of heat flux is watt per meter square (W/m^2). Watts is the unit of power, which is energy per unit time. And meter square represents area here. Energy is heat energy or thermal energy per unit time, and per unit area would give heat flux. Now, heat flux occurs when there is a temperature gradient. This can be understood by looking at this adjacent diagram. Consider two parallel planes, 1 and 2. Now, in the left-hand side configuration, the bottom plate is at T_1 while the top plate is at T_2.

The temperature T_1 is less than T_2. Assuming the z-direction is given by the arrow, we can understand that the temperature increases as we increase z. This gives a positive temperature gradient. This means the derivative of temperature with respect to z is positive ($\frac{dT}{dz} > 0$). On the right-hand side configuration, we have the temperature T_3 on the top, which is lower than the bottom temperature T_1. This gives a negative axial temperature gradient ($\frac{dT}{dz} < 0$). Thus, $\frac{dT}{dz}$ is the axial temperature gradient, which is positive or negative as the temperature is increasing or decreasing as the set or height increases. This would lead to the transfer of heat from the bottom or top regions, as the case may be. Now, how do we decide the equation for this heat transfer? We have the Fourier law of heat conduction.

The Fourier law of heat conduction states that the heat flux ($\mathbf{q}$) is proportional to the gradient of temperature (∇T), and k is the thermal conductivity, which is the proportionality constant. The minus sign indicates that the direction of heat flow is opposite to the direction of the temperature gradient:

$$\mathbf{q} = -k\nabla T$$

Note that both heat flux and temperature gradient are vector quantities. The minus sign would mean that they point in opposite directions. Now, considering axially varying temperatures, as given in the diagram, we can have a one-dimensional heat transfer given by the z-component of the heat flux:

$$q_z = -k\frac{dT}{dz}$$

Now, depending upon the direction of $\frac{dT}{dz}$, the heat flux would be determined. This is only a one-dimensional heat flow. There is no heat flux in the x or y direction, as per this equation.

Now, for a positive temperature gradient, that is, $\frac{dT}{dz} > 0$, heat flow or heat flux is negative, which means that For a positive gradient of temperature, heat flows from top to bottom, as indicated by this double red arrow. On the right-hand side configuration, the opposite happens. The heat flows from bottom to top. That is, heat flow is positive.

Note that the temperature gradient is on the opposite side. Now, we look into various heat transport mechanisms. These are the general mechanisms of heat transfer, which can be applicable to any processes, whether in physics, geophysics, or any other physical process. The general concept of heat transfer refers to the movement of thermal energy from one region to another as the temperature diffuses. The temperature difference is the force behind this thermal energy redistribution.

There are three processes through which heat can be transported. These are the well-known conduction, convection, and radiation processes. We all know that conductive and convective heat transfer processes require a material medium for the transfer of energy, while radiation can pass through empty space. Conduction is very important in the crust and lithosphere where solid material exists. In regions of the Earth's interior where liquid is the form of the material, then convection dominates.

These regions are the liquid outer core, where the liquid is molten iron. Also, the mantle behaves as a liquid on long time scales, which means it is moving very, very slowly, just like glass. However, this slow movement is also a convective motion. It is not solid. In the Earth's interior, radiative transfer of heat is the least important process.

Mostly, heat transfer in the Earth's interior occurs through convection and conduction. Only in the regions which are very close to the center of the Earth, where the temperatures are extremely high, does radiation become significant. One can have a look at this diagram for a schematic understanding of the conduction process, convection process, and radiation processes. The conduction process occurs When a person touches a hot kettle, convection transfers heat from the kettle to the person.

Convection occurs within the kettle, where water is moving upside down. This overturning of the material in liquid form is convection. Radiation occurs in high-temperature regions, such as flames. The flames are at a high temperature, leading to radiative heat transfer. Next, we look into the detailed mathematical aspects of conductive heat transfer mechanisms. Now consider a solid bar as shown in this diagram. This solid bar is of length D. The cross-sectional area of the solid bar is A. Its ends are maintained at temperature T_1 and T_2. We assume that $T_2 > T_1$ such that the heat transfer occurs from T_2 towards T_1. If the z-axis is shown as the arrow then the temperature difference across the bar would make the axial temperature gradient less than zero or negative.

This would indicate that the direction of heat flow is positive that is along the z-axis. Now We assume that the side walls are insulating. Thus, the net amount of heat that passes through a small area in a given time is given by ΔQ. ΔQ is the total amount of heat which is moving from the bottom to the top. This is proportional to the temperature difference $T_2 - T_1$, the cross-sectional area A and also the total time under observation Δt. This means the heat flow is enhanced if we have a higher temperature difference, larger area and more time for observation. The heat transfer is inversely proportional to D. This means the longer bars would experience lesser heat flux. This is mathematically represented as:

$$\Delta Q \propto A(T_2 - T_1)\frac{\Delta t}{D}$$

Adding the proportionality constant as heat conductivity k, we have the equation 1 as the heat conduction equation:

$$\Delta Q = kA\frac{T_2 - T_1}{D}\Delta t$$

Now consider the case when the length of the bar is very small or the temperature changes is uniform. Then the ratio $\frac{T_2 - T_1}{D}$ is the temperature gradient. Since we are considering a small elemental length, dz across which the temperature change is also infinitesimal. Thus we can have the temperature gradient given by $\frac{dT}{dz}$.

In the context of earth where vertical flow of heat occurs near the Earth's surface, this $\frac{dT}{dz}$ or the temperature gradient is called geothermal gradient. Substituting the geothermal gradient in the case of Earth's surface, we have the heat conduction equation given by:

$$q_z = -\frac{1}{A}\frac{dQ}{dt} = -k\frac{dT}{dz}$$

q_z is the heat flux across a small thin layer of the Earth's crust.

Measurements have shown that the geothermal gradient is approximately $30°$C/km, which means if we cross one kilometer towards the center of the Earth from the surface we will encounter a change of temperature that is the temperature would rise by $30°$C every kilometer we go towards the center of the earth. However, after certain depth the temperature rise reduces to $10°$C/km. Also, there are location where the active zones of thermal gradients are present. These are hotspot areas in the Earth's crust.

Here, the temperature rise can be as large as $50°$C/km. Now, depending on the location of the place on the Earth's surface, various temperature gradients or geothermal gradients can exist. In geophysics, exploiting the geothermal gradient for energy purposes is of paramount importance, as it is cutting-edge technology for clean energy. Next, we move on to the convective heat transfer mechanism. For convective heat transfer to occur,The temperature gradient would lead to conduction. However, if there is an excess amount of heat that must be transferred, then the convective heat transfer mechanism sets in. For example, if we have $T_2 > T_1$, then heat transfer is supposed to occur in the positive direction. Now, the amount of heat transfer can be enhanced by increasing T_2 while keeping T_1 constant if the medium is liquid. After a certain threshold where T_2 is higher than T_1, the mechanism of convection sets in on top of conduction.

Note that conduction is still prevalent, But due to the amount of heat that must be transferred being in excess of what conduction can handle, heat transfer now takes an additional route: convective heat transfer. So, we have the dual mechanism of heat transfer: both conduction and convection. Now, we will look into the convection process in more detail as it appears in the Earth's interior. We have the adiabatic temperature gradient.

Now, this adiabatic temperature gradient is the gradient where the temperature change occurs in such a manner that the parcel of fluid moving from one height to another does not lose any energy. The adiabatic temperature gradient is prevalent for large temperature differences and also when the material properties change throughout the depth of the Earth's crust and interior. We can have a look at this diagram for more clarity. This direction indicates the depth. The leftmost side represents the surface, and the rightmost end represents the center of the Earth. From the surface to the center, the temperature rises. This is how the temperature rises. We can also look into the solidus line. This is the solidus line. This shows that from region to region, the solidus line becomes higher or lower than the temperature.

When the solidus line is higher, such as in this region, the material is solid, and this is the mantle. When the solidus line is lower than the temperature, the outer core is liquid. Again, when the solidus line is above the temperature, then the inner core is solid. Thus, we have various phases of matter inside the Earth's interior, depending on whether the solidus line is above or below the temperature. The temperature profile, as shown here, represents the adiabatic temperature profile, and the derivative of the temperature with respect to the radius gives the adiabatic temperature gradient:

$$q_{\text{adiabatic}} = -k \frac{\partial T_{\text{adiabatic}}}{\partial r}$$

Now, the difference between the real and adiabatic temperature gradient causes heat transfer to occur. The real temperature gradient is the localized, time-varying temperature at that instant of time. The real temperature gradient can be higher or lower than the adiabatic temperature gradient. When the real temperature gradient is higher, it is called a superadiabatic temperature gradient, and if it is lower, it is called a subadiabatic temperature gradient. Now, a subadiabatic temperature gradient can only lead to a conduction process. Whereas, a superadiabatic temperature gradient can lead to both conduction and convection processes. This is explained further. Suppose the temperature at a certain depth exceeds the adiabatic temperature. The amount of excess temperature is ΔT. This excess temperature causes a fluid parcel to move from one place to another.

Assuming the volume of the fluid parcel as V, the volume expansion coefficient α, the expansion of this fluid parcel due to moving from one region to another is $V\rho\alpha g\Delta T$, where ρ is the density of the material, and g is the gravity. Now, as this volume of the fluid parcel has expanded, it has become buoyant. Now, Archimedes' principle states that due to the expansion in volume, the hot fluid parcel experiences a buoyancy force. This buoyancy force is equal to $V\rho\alpha g\Delta T$. This is F_{buoyancy}. This buoyancy force starts to move the liquid in an overturning manner, leading to convective heat transfer.

This diagram shows the process in more detail. Particles move from a hotter region to a cooler region and then from a cooler to a hotter region because of expansion and contraction, depending on their location. This occurs due to the superadiabatic temperature gradient. Such types of flows occur in the mantle and also the outer core, where the medium is fluid. However, the convective time scales are very different. What is a convective time scale? A convective time scale is the time a fluid parcel takes to complete one cycle. The time taken for a fluid parcel to complete a circulatory trajectory of convective motion is called the convective time scale. In the mantle, it is much higher because the convective process is very slow due to the high viscosity of the mantle.

However, in the core, which has much lower viscosity, the convective time scale is much shorter as the fluid parcels overturn in a much shorter duration. Now we look into the various phases of heat flow that occur in the Earth. Having understood both conduction and convection, we can look into both processes, which occur throughout the Earth's crust and interior. The surface of the Earth. On the surface of the Earth, the heat flow ranges from 0 to $200\,\text{mW/m}^2$. This means in different regions of the Earth's surface, the heat flow can range from $0\,\text{mW/m}^2$ to $200\,\text{mW/m}^2$. In regions where the geothermal gradient is higher, we will have higher heat flow. We also have regions such as volcanic or tectonically active regions. In volcanic regions, a lot of heat flow occurs. In tectonically active regions, a lot of heat flow also occurs from the interior of the Earth toward the surface.

Within the ocean, there are oceanic ridges. In oceanic ridges, magma flows from the ocean bed toward the ocean surface. Here also, the heat flow is much higher. Overall, it can be estimated that $65\,\mathrm{mW/m^2}$ is the average heat flow over continental regions, while $101\,\mathrm{mW/m^2}$ is the average heat flow over oceanic regions. Note that oceanic regions have a net higher heat flow output than continental regions.

This is because the crustal region over oceans is much thinner than that of the continents, which allows heat to pass more easily. Thus, the thinner crust below oceans helps in enhancing heat flow. In the interior of the Earth, we have both conduction and convective heat transfer. Specifically, we look at the core-mantle boundary. The core-mantle boundary is the interface between the mantle and the core.

The mantle is highly viscous while the core is much less viscous. Thus, the heat flux which moves from core towards mantle is accounted for. This heat flux can be estimated from seismic tomography studies which use seismic shear waves as the model for understanding heat flux. Recall that we have looked into the seismic shear wave mechanisms and types of shear waves in previous lectures. The study and detailed timing of these shear waves as they arrive by reflection from the core-mantle boundary helps to understand the heat flux which is occurring across the core to mantle.

From these studies it has been estimated that regions below America, Asia and Australia have higher heat flux. This is shown as yellow patches in this diagram. You can see that this is the map of the world outline. Here we have the Americas and the Asia and Australian regions in yellow color. This indicates higher heat flux at the core mantle boundary.

Note that this is not the surface heat flux. The outline is just shown to indicate the geographical regions. the heat flux is occurring at the core-mantle boundary from core side to mantle side, that is, radially outward. It is also obtained using seismic tomography studies that the heat flux below southern Africa and central Pacific regions, it is low heat flux. These are the Southern African region and the Central Pacific regions, which are shown in blue patches.

The blue patch indicates that the heat flux is lower. Note that the heat flow is neither zero nor negative. It is just higher in yellow regions and lower in blue regions. But the heat flow always occurs from the core toward the mantle. The heat flow map at the Earth's surface is shown in this diagram.

We can see that the heat flow is much higher along the plate boundaries. This is a plate boundary. This is another plate boundary. We can see that these regions have higher heat flux, indicated by red color. The lower heat flux regions, shown in blue, occur within the continents.

Such as these intra-African continental regions, which have very low heat flow at the Earth's surface. This gives us an idea of the various types of heat flow mechanisms that occur in the Earth and the quantitative estimates for the amount of heat flow occurring throughout the Earth's surface and its interior. Now we come to the conclusion of this lecture. Heat flow is the movement of thermal energy from a high-temperature region to a low-temperature region. This movement is driven by temperature gradients.

Heat transfer can occur through conduction, convection, or radiation. Conduction happens through direct contact. It can occur in both solid and fluid mediums. Convection occurs only in a fluid medium. While radiation can occur in matter as well as empty space, and it does not require direct contact.

We also have heat transfer through the advection process, which is the transport of heat through a moving medium, such as the velocity field. On Earth, heat flow arises from primordial heat, which is entrapped in the central regions of the planet during its formation. This heat flow moves outward in the radial direction. From the inner core to the outer core, then to the mantle, and through the surface. Also, we have radiogenic heat transfer and minor contributions from tidal forces in such heat flow measurements.

Overall, it can be said that heat transfer processes drive key geophysical phenomena such as plate tectonics, mantle convection, and volcanism. This is all driven by the heat content that has been trapped in the central regions during the planetary formation period. One can look into the following references for more understanding and details on heat flow mechanisms in geophysics. Thank you.