Mathematical Geophysics

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Hello everyone. Welcome to the SWAYAM NPTEL course on mathematical geophysics. This is continuing module number 6: wave dynamics in geophysics. The present lecture is lecture number 2: seismic waves. In this lecture, we cover the basics of seismic waves and their applications.

The components of this lecture are the concepts of seismic waves, the types of seismic waves, Then we look into the body waves, and finally, we look into surface waves in part 1 and part 2, respectively. We will discuss a few applications in geophysics. So, let us begin. First, let us look into the fundamental concept of seismic waves.

What is a seismic wave? A seismic wave is an elastic wave generated through the sudden release of energy within the Earth. This release of energy can be due to an earthquake or volcanic activity, which are natural causes, or artificially induced explosions or other disturbances of man-made origin. Thus, The important aspect of seismic waves is that they involve the release of energy, the propagation of that energy, and the transfer of this energy within the Earth.

Thus, the seismic wave can have either a natural or artificial origin. Now, for example, look at these diagrams. The study of seismic waves is called seismology. We have the division into two parts: natural seismology and artificial seismology. The natural seismology involves understanding the various processes that occur inside the Earth, triggered by earthquakes, which are natural processes.

Now, artificial seismology involves inducing a source of vibration, such as hitting the ground with a hammer or other kinds of induced explosions, which can generate seismic waves. These seismic waves then propagate into the Earth, as shown by the arrow, and they have different velocities in different materials. On encountering a change in the material, such as shown here, there is a refraction and reflection of the seismic waves. Such changes in the material properties can occur due to water saturation, for example.

Here, sand and gravel are shown as dry materials above, whereas below, the sand and gravel are water-saturated, changing their material properties and hence the speed of the propagation of seismic waves, which induces such reflections. These reflections are then recorded on a seismometer. Seismometers are also called geophones, which are placed on the surface of the Earth to record these seismic waves reflected from the interfaces of various layers with different material properties within the Earth's subsurface.

Now, as The seismic waves have propagated throughout the Earth's interior, they carry information about the Earth's internal structure and the source of the disturbance. Now we look into the various canonical types of seismic waves. These are the overall seismic wave categories. However, any seismic observation made on the surface using a seismometer may be a combination of one or more of these various types. Thus, we look into these fundamentals as unique and separate phenomena.

Two categories of seismic waves are body waves and surface waves. They are divided into these categories based on whether they propagate throughout the volume of the material or along the surface of the material. Using this diagram, it can be better clarified. We can look into the source of the seismic wave located at point P, and we have the surrounding volume into which the seismic wave travels. If the seismic wave travels on the surface, it is called a surface wave. Whereas if it travels throughout the volume, it is known as a body wave. So, seismic body waves travel throughout the interior of the Earth, whereas seismic surface waves travel only along the surface of the Earth.

Now, among body waves, the two categories are P-waves and S-waves. And among surface waves, the categories are Rayleigh waves and Love waves. These illustrations help us understand the characteristics of these waves in a better manner. First, let us consider the body waves. The P-wave is a compression-rarefaction wave.

The material, as shown here, is broken down into grids to depict the compression and dilation. Now, the compression is indicated by thin structures placed close to each other, whereas dilation is shown by thicker structures. Note that the vibration occurs along the direction of propagation, shown by the blue arrow. The materials vibrate along the direction of propagation. This is the P wave.

Now, S waves, on the other hand, have different characteristics. Here, the vibration of the material is in the direction perpendicular to the direction of propagation. One can see the difference in the structural deformations induced by these waves. The P wave induces compression and rarefaction, while the S wave induces shear. This creates crests and troughs in the material, deforming it into a wave-like kind.

Now, coming to surface waves. First, we consider the Rayleigh wave. The Rayleigh wave travels on the surface, as can be seen by the deformation of the material limited to the upper surface. Now, if we consider a particle or a small parcel of the material, its trajectory is circular or elliptical due to the Rayleigh wave.

The particle moves in an elliptical, circular manner as the wave propagates throughout the material. This occurs because the direction of propagation and the direction of the Rayleigh wave displacement are perpendicular. But due to the presence of this wave being limited to the surface, the materials move in a circular direction. On the other hand, we have Love waves, where the materials move perpendicular to the direction of propagation but also in a transverse manner. This is the direction of the displacement for a Love wave.

The Love wave is also restricted to the surface only, whereas in the deep volume of the interior material, Love waves are absent. We can see that the Rayleigh waves induce a circular trajectory, whereas the Love waves induce a shearing motion of the materials. We will look into the details further. First, let us consider the body waves in more detail.

The P wave. The P wave stands for primary wave. It is the fastest seismic wave, meaning its speed is the highest among all four types of seismic waves we have discussed. These are compressional and rarefaction waves, which are also denoted as shown in this diagram. Here, we have a periodic arrangement of compressions and rarefactions, denoted by C and R respectively.

The compressions squeeze the material closer, while rarefaction diverges the material outward. Now, this keeps alternating as the wave propagates, with the material vibrating consecutively along the direction of the wave. Thus, this type of wave is also called a longitudinal wave. The definition follows.

The longitudinal or compressional body wave passes through a medium as a series of dilations and compressions. The equation for the compressional wave in the x-direction is governed by the wave equation, with α^2 replacing c^2:

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

The velocity of the P-wave can be given as:

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

Here, ρ is the density of the material, and K and μ are material properties that vary for different materials.

Next, we consider the S-wave. The S-wave stands for secondary wave. It is a shear wave, which is slower than P-waves. Since it is a shear wave, it depends on the shearing capability of the material. It can only travel in materials that can withstand shearing.

For example, solids. Thus, as fluids such as gases and liquids cannot sustain shear, shear waves cannot propagate in liquid or gaseous mediums.The shearing ability of a material tends to account for the reinforcement or the restoring force for these secondary waves. One can see that shearing displacements can lead to a wavy nature of the material. Such wavy nature cannot be sustained in liquids or gases, and thus the absence of any restoring force in such mediums leads to no propagation of secondary waves.

Thus, transverse waves can be described as the wave in which the particles of the medium move in a transverse direction, that is, a direction perpendicular to the wave propagation direction. This means that the motion of particles is at right angles to the direction in which the energy of the wave is traveling. If β is the S-wave velocity, then:

$$\beta = \sqrt{\frac{\mu}{\rho}}$$

Now, this is the speed of secondary waves. Comparing it to the speed of P waves, which is given by α, we can see that the numerator of α is higher than the numerator of β. Thus, P waves travel faster than S waves.

Next, we look into the surface waves. The first part relates to the Rayleigh waves. Now, as we have discussed earlier, Rayleigh waves are surface waves that cause the particles in the medium to move in an elliptical manner such that the particle motion describes a retrograde ellipse.

A retrograde ellipse means that the motion of the particle is clockwise if we look in a direction such that the direction of propagation of wave is towards the right. Thus, looking from this direction such that the direction of propagation is towards the right, we have anti-clockwise motion that is the retrograde ellipse. The major axis is in the vertical direction while the minor axis is along the direction of wave propagation.

Now for the earth's case, the speed of the Rayleigh wave denoted by v_R is approximately 0.92β. Remember β is the speed of S waves. Thus Rayleigh waves travel approximately at 92% of the magnitude of the speed of S waves. Now it causes the ground motion into an elliptic pattern. Using this diagram we can better understand the Rayleigh wave phenomenon.

The particle motion is given here. Now the direction of propagation is towards the right whereas the depth is towards the bottom. The uniformity of the grids at the depth shows that the Rayleigh wave is only limited to the surface. Now considering the passage of the waves along these two points we can see that this point is going to experience this heightened position after a certain duration but due to the fluctuation along the direction of displacements such as this the particle is going to undergo an elliptic motion.

This would be the location of the particle indicated by red dot in subsequent times as the wave propagates, thus describing elliptical motion. Now, look into the details of surface waves, which are Love waves. The Love wave causes horizontal shearing of the Earth's crust. Now, horizontal shearing means that the direction of the particle motion is perpendicular to both the direction of propagation and to the depth.

If these two are the direction of propagation and depth, the direction which is perpendicular to both these directions is the third axis, which is the direction of the Love wave. Now, looking into this diagram, which is similar to the previous one, we can see that at the bottom, the grids are uniform, indicating that the Love wave does not propagate into the interior.

Now, these materials have been displaced farther away from the screen. While these materials are displaced toward the screen. This describes the Love wave. Now, if we look from the top, The horizontal displacements would look like a wave, which is the Love wave. The particle motion is in the direction as shown here, which is transverse to the direction of propagation.

Now, thus, we can summarize the features of the Love wave as: The particle motion direction being horizontal and perpendicular to the propagation direction. They do not have any vertical or longitudinal displacements. The Love waves typically travel faster than Rayleigh waves but slower than the body waves, such as the P and S waves.

This summarizes the characteristics of Love waves, which are the fourth type of seismic waves. Thus, we come to the conclusion of the various types of seismic waves. Seismic waves are a typical example of wave equation solutions in general, particularly in the context of Earth's interior. These are very vital tools for exploring Earth's interior.

They provide insights into the material properties and geological structures that exist below the Earth's surface. Seismic waves also help us understand dynamical events, such as the slow motion of uniformly distributed materials across the ambient material in the subsurface, by tracking seismic wave properties over time.

Now, seismic waves include both body waves and surface waves. These body waves and surface waves each have unique properties. Their characteristics are completely different from each other. However, a real measurement or a practical seismometer reading would indicate a mixture of all types of these waves contributing to the displacement vector on the seismometer, which records the displacement of ground motion over time.

The speed and behavior of seismic waves depend on the physical properties of the medium, such as density (ρ), elasticity (μ, λ), and temperature as well. Thus, we have a fantastic example of a very widespread application in geophysics of the wave equation in the form of seismic waves, which can be either naturally induced or artificially induced for directed and targeted exploration activities in geophysics.

These are a few references which one can look into for more examples and applications of seismic waves and their various characteristics in geophysics.

Thank you.