Mathematical Geophysics

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Week - 05

Lecture – 22

Hello everyone, welcome to the SWAYAM NPTEL course on mathematical geophysics. We continue with module number 5, Diffusive Processes in Geophysics. This is lecture number 2, Geothermal Gradients. In this lecture, we will cover the general concept of geothermal gradients. In this concept, we have distributed the components of the lecture as heat flow, thermal gradient, and heat sources in the Earth.

Next, we will look into variations in the geothermal gradients, which give rise to various phenomena widely observed on the Earth's surface. Finally, we will look into geophysical applications of the geothermal gradient. So, let us begin. First, we will look into the fundamental aspects of heat flow in a material. Consider this diagram.

This is a solid cylindrical bar of length L. The cross-section of this bar is A, and its ends are maintained at temperatures T_1 and T_2, respectively. We have assumed that $T_2 > T_1$, which means that this end is hotter, and this is the cooler end of this solid bar. Thus, the flow of heat has to commence based on the diffusion process. The flow of heat will occur in such a manner that the heat transfers from the hotter side to the colder side. Now, we will look into the equations that govern this flow of heat along this bar.

The conditions maintained at the cross-sectional ends of this bar are such that heat transfer only occurs along the bar, that is, along the direction shown by the arrow. There is no transfer of heat in the lateral directions or sideways. The net amount of heat, ΔQ, that passes through in a given time from the hot end to the cold end depends on four factors. First, we have the temperature gradient or essentially the temperature difference, $T_2 - T_1$. Next, we have the area of the cross-section, the length of the bar L, and the observation time Δt.

The net heat flow is proportional to the area of the cross-section since a larger area will allow a larger amount of heat to flow from the hotter to the colder end. The heat transfer is also proportional to the temperature difference. The higher the temperature difference, the greater the heat flow. The heat transferred also depends on the total amount of time taken. The longer the observation time, the greater the amount of heat flow.

However, the heat transfer is inversely proportional to the distance L. It means that if the bar is longer, then a lesser amount of heat travels. Essentially, it reduces the amount of heat flow. Now, let us consider the factor $\frac{T_2-T_1}{L}$. This can essentially be written as $\frac{\Delta T}{\Delta x}$, where ΔT is the difference in temperature while Δx is the length. Now, this can be approximated as the gradient of temperature.

Hence, we can say that ΔQ is proportional to the gradient of temperature, like any other diffusive process.

Introducing the proportionality constant K, we have $\Delta Q = KA\left(\frac{\Delta T}{L}\right)\Delta t$. Here, K, the proportionality constant, is the thermal conductivity, which is the property of the material of the bar. Now, let us look into the thermal gradient. We have seen that the temperature difference over the length L can be represented as a thermal gradient. Now, considering the length of the bar being very small or taking a small elemental length of the bar.

This is the ΔL, which is a small elemental length of the bar. It means that the temperature change across it is very small or essentially uniform. However, because the length of the bar is also very small, the temperature gradient is a finite quantity. This becomes $\frac{\Delta T}{\Delta L}$, which tends to $\frac{\partial T}{\partial x}$ as the length of the bar goes to zero. Now, this is the thermal gradient on which the heat transfer Q depends.

The thermal gradients are used to describe the vertical flow of heat out of the Earth. We have the surface of the Earth and the cross-section. There is a constant gradient of temperature from the inner core, 7000 K, to the surface, which is nearly about 30 to 40 degrees centigrade. Now, here we have the geothermal gradient. Thus, the geothermal gradient is the change in temperature across the depth of the Earth.

This geothermal gradient allows vertical flow of heat across the depth. Now, Q_z denotes heat flux, which is defined as the heat flow per unit area per unit time. Thus, we have heat flux equals $-\frac{1}{A}\frac{dQ}{dt} = -K\frac{\partial T}{\partial z}$. This vertical temperature gradient allows the flow of heat from the interior of the Earth to the surface.

Now, let us look into the various heat sources that exist in the Earth. Since we know that the temperature of the Earth at the center is 7000 K, we know that it is very hot. But how does this temperature become so high in the interior of the planet? This is the description of the present slide. From the previous slide, we understood that the interior of the Earth is losing heat due to the geothermal temperature gradient.

Thus, we have the geothermal flux or heat flux due to the geothermal gradient at about 4.4×10^{13} watts. Now, this value of the heat flux is huge, essentially amounting to 1.4×10^{21} joules per year. This indicates that 1.4×10^{21} joules of energy is being lost from the planet due to geothermal gradients. This is essentially an example of a phenomenal diffusive process of heat. Now, coming to the sources of this heat in the Earth's core or the interior.

First is the secular cooling. Now, secular cooling is a general term for the change in temperature across the Earth's interior. This is a general term representing the cooling of the planet due to the loss of heat through the geothermal gradient. This heat was trapped in the Earth's interior due to primordial heat, which developed early in the planet's formation history. Planets are formed through a process known as the accretionary process, which amalgamates various materials and rocks from space due to gravitational differentiation, contributing to 10 to 25 percent of the total surface heat flux.

The accretion process involves bringing materials from the surrounding area through gravitational attraction, and when they merge by colliding, they generate heat. However, as these materials accumulate over time on top of one another, the collision and resulting heat energy released become trapped. That is the primordial heat. Once all this material accumulates, the extreme pressure and temperature cause the materials to melt. This leads to the occurrence of gravitational partitioning.

Gravitational partitioning means the differentiation of heavy elements from light elements. During the planet's formation, which is already in a molten state due to the large amount of stored heat energy, gravitational partitioning facilitates the separation of heavy elements, which sink to the bottom, while light elements rise to the top, forming the Earth's outer layers. Thus, the Earth's inner core consists of solid iron, while the outer core contains traces of lighter elements. The mantle, which lies on top of the core, is mostly composed of silica and other low-density elements. The lightest elements are found near the Earth's surface.

This is gravitational partitioning. Once the planet solidifies and begins to cool, the geothermal gradient develops. Apart from such primordial heat sources of the Earth, we have radiogenic heat production. Radiogenic heat production is related to the radioactive decay of elements such as uranium, thorium, and potassium. The radioactive decay of such elements is also responsible for 75 to 90% of the Earth's heat.

Most of these radioactive elements are present in the continental crust of the Earth. The radioactive elements are also present in the mantle as well as the outer core of the Earth and continuously generate heat due to radioactive decay. This heat also raises the temperature and develops the thermal or geothermal gradient. There are also variations in the geothermal gradient. For that, let us consider the structure of the Earth again.

On the left-hand side, we have a sketch of the Earth's interior. We have the inner core, which is solid; the outer core, which is fluid; and the mantle, which is mostly a highly viscous, semi-fluid rocky composition. We have the hard crust and the atmosphere, which is gaseous. From the interior to the exterior, the temperature falls and forms the geothermal gradient.

You can see that the radius rising from 0 to 7000 kilometers the temperature also decreases from 7000 K to 0 K in outer space. In particular we focus on the small region which belongs to the crustal thermal gradient. Upon magnification, we can understand that the geothermal gradient at the crust is nearly 30°C per kilometer. This is very useful from the perspective of geothermal energy explorations, which is a very widely used application in geophysics.

For that, we also look into the details of the crust. The crust is made up of two distinct parts. One is oceanic crust and the other is continental crust. The oceanic crust forms the seabed while continental crust is the land upon which we stand. Below the crust we have the mantle and crust interface which is essentially the lithosphere and plastic asthenosphere. The plastic asthenosphere is mostly a semi-molten state and the upper mantle is a highly viscous semi-molten state. The temperature change in these areas create a geothermal gradient and drives hot springs and fountains on the surface of the earth. This geothermal gradient is very useful energy resource and is likely to become a major source of energy in near future. The exploitation and exploration of various geothermal gradient and their utilization forms a necessary application in geophysics.

Thus, we conclude with applications of geophysics. Geothermal gradients are the focus of exploration for geothermal energy. To explore or tap into geothermal energy, geophysicists first measure temperature variations at different depths. They identify hotspots or regions with high geothermal gradients. These regions of high geothermal gradients have the potential to be geothermal reservoirs, which are essentially energy reservoirs.

The most prominent examples of geothermal energy sites are volcanic activity regions. Also, tectonic plate boundaries typically exhibit high gradients. The presence of a high geothermal gradient is essential for suitable and economically feasible energy extraction. On a more fundamental level, geothermal gradients are used in crustal and mantle studies. The geothermal gradient provides data for modeling the thermal and mechanical behaviors of various materials in the Earth's crust and mantle.

The study of these materials at high temperature and pressure leads to a better understanding of their material properties and other mineral characteristics. For example, geothermal gradients are also used to understand or estimate the depth of the lithosphere-asthenosphere boundary. This is the lithosphere-asthenosphere boundary, where a sudden drop in temperature occurs when moving from the asthenosphere to the lithosphere. The detection of this boundary is essential to understanding the processes governing geophysics and geodynamics near the Earth's surface. We can also understand the distribution of partial melts and volcanic activity through geothermal gradient studies.

Thus, we come to the conclusion of the present lecture. Thermal gradients are essential to understand the rate of temperature change and the flow of heat transfer. The change in temperature essentially dictates the flow of heat in a material with finite thermal conductivity. The temperature difference with respect to spatial variations gives rise to such thermal gradients. Now, thermal gradients also transfer heat across a material, driving the heat flow from regions of higher temperature to lower temperature.

The thermal gradient is mathematically a vector quantity. It points in the direction of the steepest temperature increase or decrease. We can look at the following references for more details on thermal gradients, geothermal gradients, and their applications in the interior of the Earth. Thank you.