Mathematical Geophysics
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Hello everyone, welcome to the SWAYAM NPTEL course on mathematical geophysics. This is module 4, mathematical modeling part 2. Today we will discuss lecture number 2, time-invariant and time-varying fields. In this lecture, the concepts are related to invariant and varying fields with respect to time. The components of this lecture are the concept of time-invariant field, the concept of time-varying field, time-invariant fields in geophysics, and time-varying fields in geophysics. Finally, we will look into geophysical applications of these types of fields. So let us begin. First, what is a time-invariant field? We have already discussed the concept of a field.

Now, time invariance means the field does not vary with time. It is also known as the steady-state field. Essentially, the field is independent of time. The mathematical representation is as follows. A vector field **F**, which depends in general on space and time, or a corresponding scalar field depending on space and time, can be considered as time-invariant if the value of the physical quantities corresponding to these fields remains constant over time.

That is, the dependency on time is eliminated. We have **f(r)** and little **f** as functions of **r** only. This implies that the partial derivatives of these fields with respect to time are equal to zero. These are the steady-state fields. Some examples of steady-state fields, as they occur in geophysics, are as follows.

The electric field around the charge distribution in a stationary state. These are the field lines of the electric field around the charge distribution. As long as the charge is stationary, these electric field lines do not move. They remain steady; they remain constant over time. Hence, they are steady-state fields or time-invariant fields.

Similarly, the gravitational field of a non-moving mass. The gravitational field can also be considered as a steady-state field. We also have the steady-state field of a magnetic field, which is associated with a steady current flow in a wire. This current-carrying wire will induce a magnetic field. It will remain steady as long as the current flow is steady inside the wire.

These are a few examples of time-invariant fields. Now, we will discuss a few properties of such time-invariant fields. First, no explicit time dependence, as the name suggests. Secondly, the induction effect is absent. This property means that time-invariant fields cannot induce, generate, or amplify any other processes related to other physical quantities.

Had it been time-varying, there would be a possibility of an induction effect. Time-invariant fields cannot represent evolution or transfer effects. For example, in Earth's case, the evolution of various geodynamical processes, such as plate tectonics, etc. and transfer effects, such as heat transfer in the atmosphere or Earth's interior, cannot be represented by time-invariant fields. It is easy to analyze and predict time-invariant fields and the effects of such fields on other processes.

In geophysics and natural processes, time-invariant fields are very rarely found. However, they present a very suitable and fundamental aspect that can be used to understand the underlying physical mechanisms at work in these natural processes. To complement, we have the concept of time-varying fields. In contrast to time-invariant fields, this type of field changes with time. Such fields can either vary spatially or not, but the dependency on time is a must.

In the case of a field varying both spatially and temporally, such fields are known as dynamic fields. As natural processes are dynamically oriented, the concept of time-varying fields is essential for modeling transient and evolving processes in physics, engineering, and geophysics. In particular, geophysical applications such as geodynamical processes—like volcanic eruptions, plate motion, slab subduction, etc. and interior planetary events such as motion in the mantle, outer core, etc.

Oceanic circulations, atmospheric circulations, and their evolution, etc. All present examples of time-varying fields and are necessary for understanding and interpreting these processes. The mathematical representation of time-varying fields. The general representation of vector fields **f(r,t)** or a scalar field **f(r,t)** is the time-invariant field as shown here. The values of the physical quantities change over time, which can be represented as the non-zero partial derivatives with respect to time for these vector and scalar fields.

Some examples of time-varying fields include the time-varying electric and magnetic fields that arise from oscillating current sources in AC circuits. In geophysics, we have time-varying electric and magnetic fields in the upper atmosphere, in the core of the Earth, and various other applications. Large-scale ripples in space-time caused by accelerating masses, such as colliding black holes creating gravitational waves, also present exotic examples of time-varying fields. The properties of time-varying fields can be summarized here.

In time-varying fields, the magnitude and direction can vary as a function of time. This applies to vector fields. This indicates that even if the magnitude of a vector field does not change, the change in direction of the vector field can result in time variance. Time-varying fields can induce other fields due to their change over time. This is the dynamical property.

Time-varying fields are also capable of transferring energy across space. A particular example is wave dynamics, where energy is transferred through space using time-varying processes. Let us look at some examples of time-invariant fields in geophysics. Remember that in geophysics, most phenomena are time-varying. However, for understanding underlying geophysical processes, even slowly changing fields may be considered time-invariant.

Fields that are steady over time and do not exhibit significant temporal changes over geological time scales are also considered time-invariant. Since geological time scales range in millions or billions of years, small and slowly changing fields occur over such large durations. For example, the formation of mountain ranges. These mountain ranges form very slowly. They may be moving, for example, one or a few millimeters per year.

But these changes occur over millions or even millions of years. But these processes can be considered as time invariant because of their slow nature. We also have the motion of tectonic plates, the motion of rocky materials inside the mantle which are so viscous that it may take millions or even hundreds of millions of years for the churning of these materials inside the earth's mantle. Even in the solid iron inner core of the earth materials move at a very very slow pace. Although they are essentially and strictly time varying, but due to their slow nature in geophysical studies, such fields can be considered as steady over time.

Thus, many geophysical fields may still exhibit subtle temporal changes, which remain largely time invariant over the broader aspects. For example, secular variation of the geomagnetic field. The secular variation of geomagnetic field is the changes which is occurring in the magnetic field of the planet over time. There are certain components which vary very fast such as over a day, but then there are other components of the geomagnetic field which only vary over hundreds of millions of years. Thus, the particular components of the geomagnetic field which are very slow in their variation can be considered as steady fields.

As we had discussed earlier, tectonic stress development due to plate motions and their evolution over millions of earth years can be considered as a steady state field. The last example is a very interesting one. We have the glacial rebound theory on gravitational field. The value of gravity over regions where the glaciers have melted also exhibits evolution over time. Due to the removal of large ice masses, the gravitational field changes.

But these changes are very slow and occur over millions of years. Thus, these fields are also considered time invariant in geophysics. The use of time invariance concept in such geophysical processes makes them very suitable for analysis and capturing of the essential physics which describes most of the phenomena which are observable Now let us consider some examples where the main and overall geophysical process is time varying but there are underlying steady components which can be considered as time invariant fields. First is the telluric currents which flow over the earth's surface.

The telluric currents can change over time, but then there are steady components to it which remain invariant over a long duration of time. Similarly, geomagnetic fields. In these examples, the background electric field, the static component, can be treated as time invariant over certain zones. Next, we come to the time varying fields in geophysics. As we had discussed earlier, almost all the geophysical phenomena exhibit time varying properties.

The time varying fields refers to fields that change with small or large geological timescales. So these time varying fields are occurring due to dynamical processes which occur inside the earth or its interaction with external forces such as the atmosphere and oceanic currents. The time varying fields are essential in understanding transient and evolving geophysical processes. The time varying fields are limited by their interpretation capabilities. This is because the time variation makes it very complex and the data which is required for understanding becomes huge in volume.

The complex patterns makes it difficult to interpret the physical processes which is underlying the phenomena. Fundamental insights also become very difficult to disentangle from the observed measurements. Also, there are limitations to getting useful information of geophysical importance from various geophysical measurements. This is because human activities can introduce time variations into the geophysical fields during the measurement. Complicate the measurements and form noisy signals.

For example, electromagnetic fields which are generated by telecommunication networks can obscure actual telluric currents which are useful for geophysical exploration measurements. Thus time varying fields in geophysics although they describe the fields can easily be corrupted through human activities and hence the processes or the methods to disentangle these noisy influence is to be utilized. We look into further examples of time varying fields in geophysics. The secular variation of the geomagnetic field. The geomagnetic field which is encompassing the entire planet changes in its magnitude and direction over various periods.

These periods range from few days to million years. We have the diurnal change in geomagnetic field which is influenced by solar radiations. Now, these solar radiation affect the ionospheric strata in our atmosphere and induce time varying magnetic and electric fields. To summarize, we look into the various applications of geophysics of time varying and time invariant fields. The magnetotelluric surveys.

The magnetotelluric surveys are an important part of geophysical resource exploration. In magnetotelluric surveys, the subsurface resistivity structures are investigated. If $\boldsymbol{\rho}$ denotes the resistivity and $\mathbf{r}$ denotes the radial coordinate, The magnetotelluric surveys aim at providing this information below the Earth's surface. The method involves the measurement of temporal variations in electric and magnetic fields occurring on the surface of the Earth over time.

By further analysis and appropriate interpretation, the magnetotelluric survey data can be used for locating mineral deposits, oil, and gas reservoirs. Next, we look into the induced polarization surveys. The induced polarization surveys are another method of geophysical analysis that monitors subsurface properties. In induced polarization surveys, the temporal variations in electric charge distribution inside the Earth's subsurface are acquired. These surveys are also useful for detecting mineral content and groundwater aquifers.

Next, we have the ground-penetrating radar surveys. Also known as GPR. The GPR surveys utilize time-varying electromagnetic waves to image subsurface structures. It helps in detecting objects and analyzing geological structures. Now, these objects are detected because of their different properties from the surroundings.

For example, different electrical conductivity or different density in gravity surveys and other useful properties of matter. It helps us understand the structure of Earth's subsurface and forms an important part of geophysical applications. Thus, in geophysics, we have various exploratory surveys such as these, which depend on time variation of fields such as electromagnetic and gravitational fields for essential resource exploration. Thus, we arrive at the following conclusions based on the discussions we have conducted in this lecture. First, time-invariant fields are crucial for stable and baseline measurements.

This allows for accurate mapping and resource exploration to a great extent. It also helps us understand Earth's static properties. Overall, time-invariant fields are useful for understanding simple and fundamental physical concepts underlying complex geophysical phenomena. To complement it, we have time-varying fields, which are appropriate for modeling the evolution and transient processes occurring in geophysical phenomena. These dynamic phenomena utilize the time-varying field concept to provide insights into real-time data and monitor geological and environmental phenomena.

Thus, by utilizing both time-varying and time-invariant fields, the modeling of various geophysical phenomena can be done very accurately. One can refer to the following reference for more details. Thank you.