NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 11: Source parameters, Earthquake statistics.
Lecture 05: Distribution of b-value and Aftershocks

CONCEPTS COVERED

- Distribution of b value
- Aftershocks : Omori’s Law
- Summary

Recap

- **Frequency–magnitude relation** $\log N = a_1 + bM$

  N is the number of earthquakes with magnitude greater than or equal to M occurring in a given time.

- **The data deviate from the b = 1 line for very small (M_s < 3) magnitudes, because the global earthquake catalog is incomplete, with many small earthquakes not detected.**

- **Modified Frequency–magnitude relation**

$$\log N = a_1 - b(\log M_0/1.5 - 10.73) = \alpha - \beta \log M_0$$ **with slope β = b/1.5 ≈ 2/3**

- **For moments above 10^{27} dyn-cm, the data are more consistent with β ≥ 1 than β = 2/3.**

- **Number N of earthquakes with fault area greater than S should obey a frequency–area relation like those for magnitude or moment**

$$\log N = c - \log S$$

Frequency –magnitude relations : Variation of b value

- Although b values approximately equal 1 over long time scales and large spatial scales, significant variations occur on smaller scales.

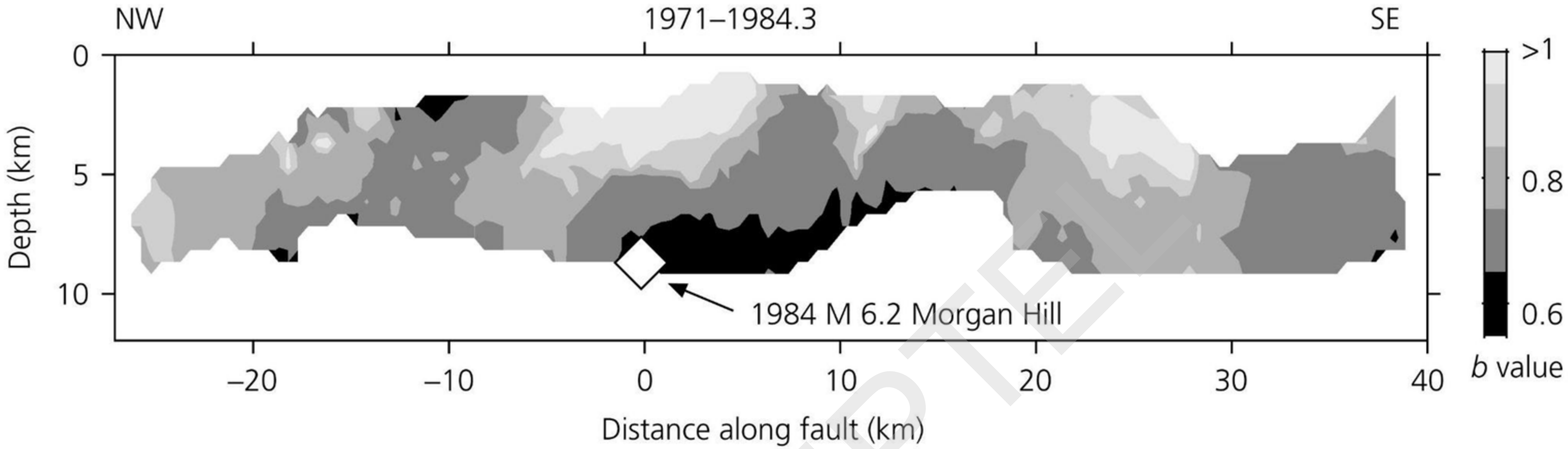
- The b value of earthquake swarms is often much larger than 1, sometimes approaching 2.5.

  Note: An earthquake swarm is a sequence of seismic events occurring in a local area within a relatively short period.

- These swarms, which lack a mainshock, are often associated with volcanic regions, and may result from processes such as the migration of magmatic fluids or caldera development.
- The b value also varies regionally, both spatially and with depth.

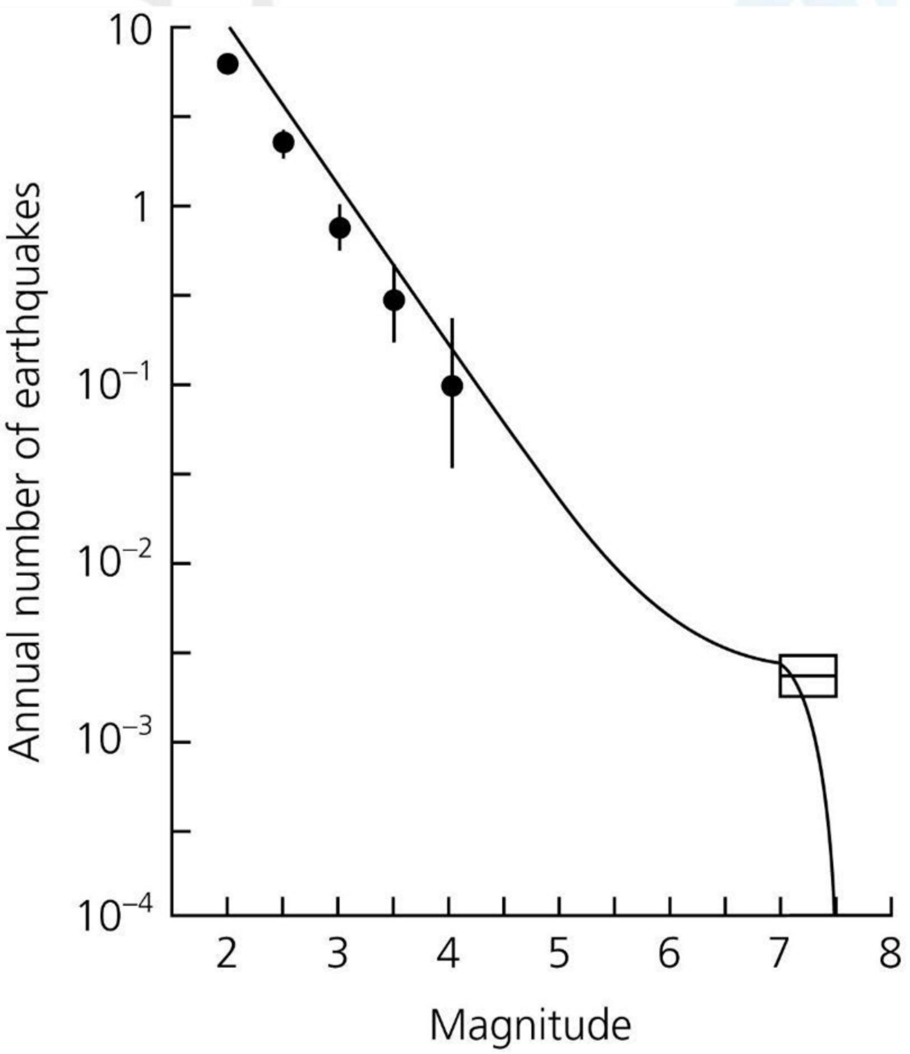
Frequency –magnitude relations : Variation of b value

Figure 4.7-5: Variation in *b* value along the Calavaras fault.



- **In the figure above, b-value variations are shown for Calavaras fault in California.**
- **Some patches have b values much less than 1, implying shorter recurrence time.**
- **These patches have been interpreted as possible asperities or stress concentrations, perhaps reflecting variations in frictional properties along the fault, which may control the recurrence of the next large earthquake and have large moment release during it.**

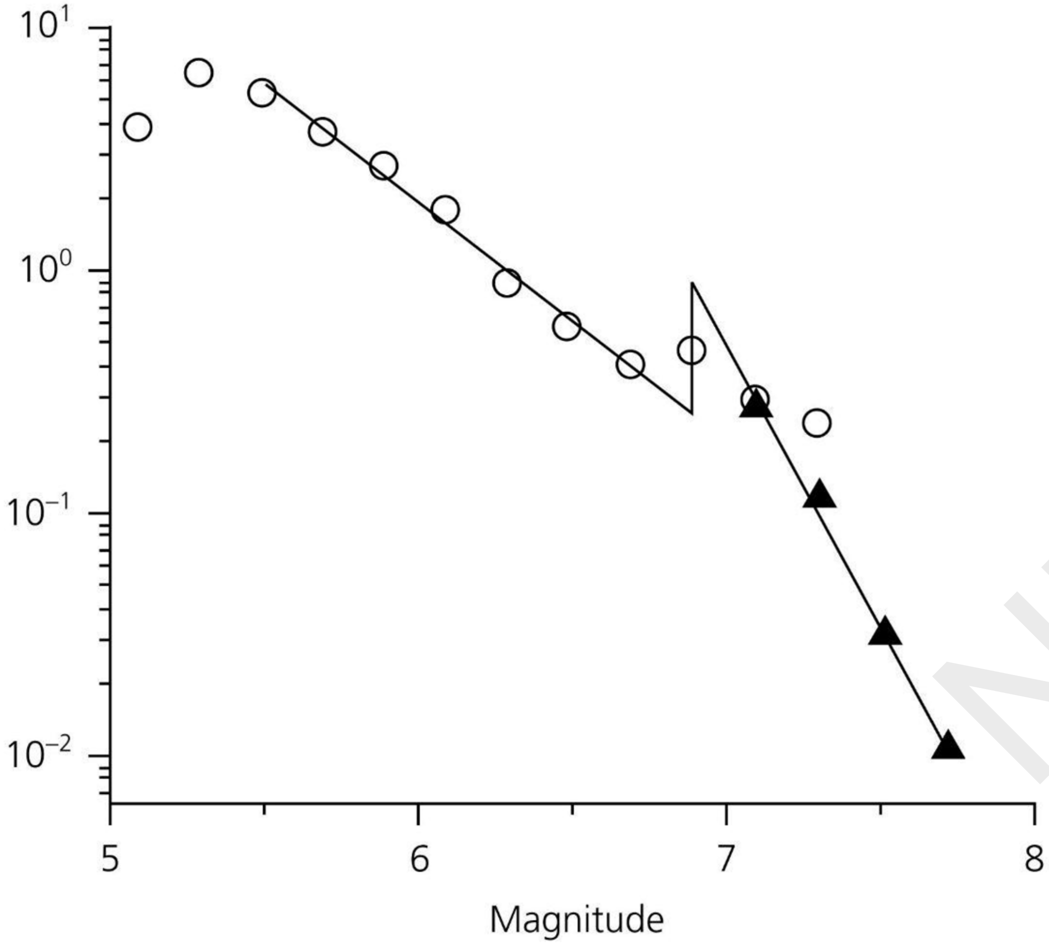
Frequency –magnitude relations : Variation of b value



- Fig shows earthquake magnitudes and frequencies for large earthquakes inferred from geological paleoseismic studies.

- These observations have been interpreted as showing large earthquakes more frequent than would be expected from the linear relation derived from the instrumental data

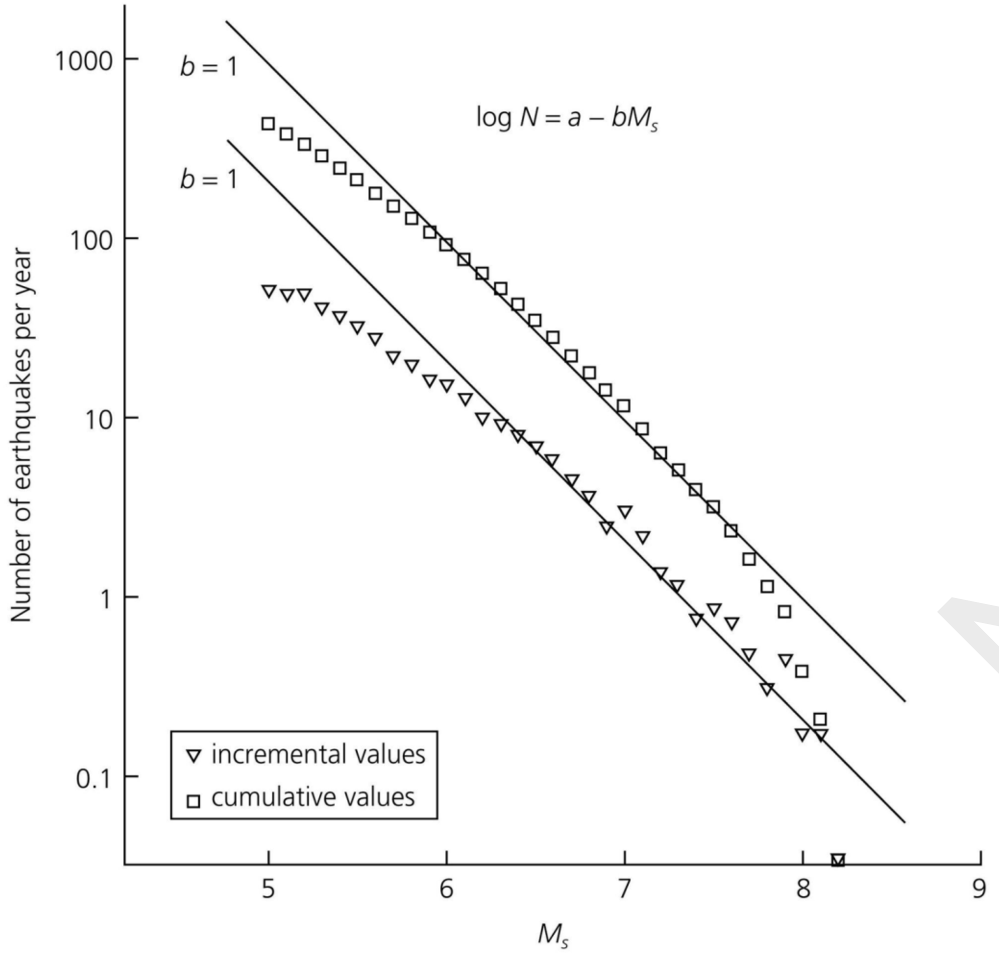
Frequency –magnitude relations : Variation of b value



- A study using seismological data for continental interiors finds that the largest earthquakes are less frequent than expected from the smaller earthquakes

Frequency –magnitude relations : Variation of b value

Figure 4.7-1: Frequency-magnitude plot for earthquakes during 1968-1997.



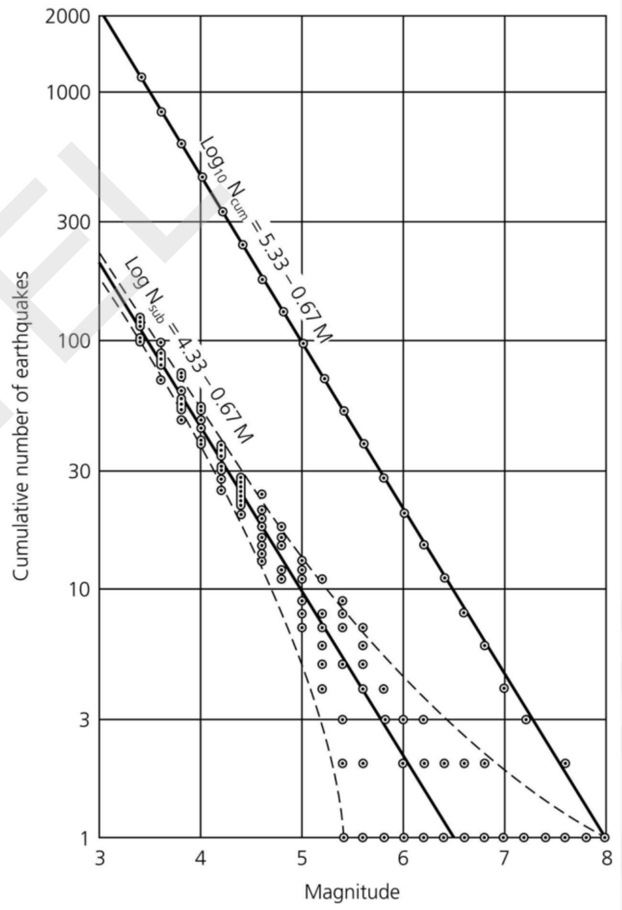
- **These observations are interpreted as showing a possible small deviation toward higher frequency at about Mw 7, followed by a significant decrease as observed in the global data presumably due to finite fault width.**

- **The Gutenberg–Richter relation can be modified to describe the different deviations from linearity.**

Frequency –magnitude relations : Variation of b value

- Some deviations of the largest earthquakes in an area from a linear frequency–magnitude relation may reflect small sampling.
- The frequency–magnitude relations for the subsets have considerable scatter.
- The largest earthquakes appear in some cases more and in other cases less frequent than for the total population.
- Thus the b value is reasonably well estimated from the smaller earthquakes, but not the largest ones.

Figure 4.7-7: Deviations from a linear frequency-magnitude relation due to inadequate sample size.



Aftershocks

Omori’s law

- **Aftershocks occur on or near the mainshock fault plane, so their locations are used to distinguish between the fault and auxiliary planes and to estimate the fault area.**

- **The largest aftershock is usually more than a magnitude unit smaller than the mainshock, and the aftershocks have a size distribution with b near 1 so, the total energy released by the aftershocks is usually less than 10% of that of the mainshock.**

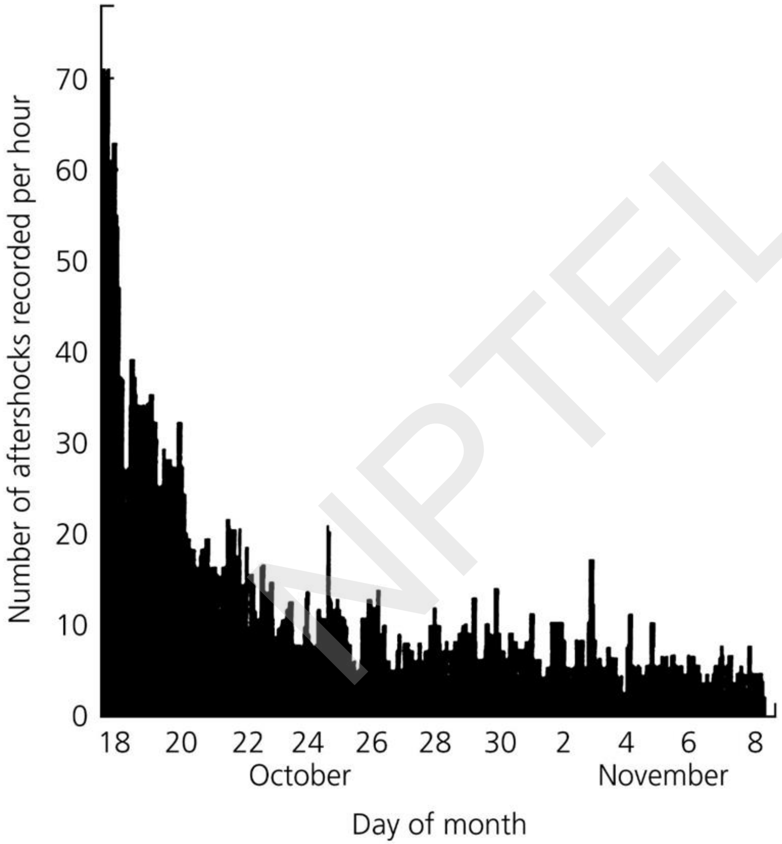
- **The decay of number of aftershocks is represented by the Omori’s law**

$$n = \frac{C}{(K+t)^P}$$

where n is the frequency of aftershocks at a time t after the mainshock, with K, C, and P as fault-dependent constants. P is typically about 1.

Figure 4.7-8: Aftershocks following the 1989 Loma Prieta earthquake.

Magnitude	Number	Effect
5	2	Damaging
4	20	Strong
3	65	Perceptible
2	384	Not felt
1	1855	Not felt
<1	2434	Not felt
Total	4760	



4760 aftershocks of the Loma Prieta earthquake had been recorded by noon on November 7, 1989. The diminishing number of aftershocks with time is typical for large California earthquakes.

Most of the aftershocks occur soon after the mainshock, and the remainder decay with time in a quasi-hyperbolic manner.

Decay of aftershock reflect stress readjustment following the stress changes due to the main shock

An intriguing exception to Omori's law

Most deep earthquakes have many fewer, and often no, detected aftershocks. This difference may reflect deep earthquakes resulting from phase changes in mantle minerals, which could produce slip only once on a fault surface, in contrast to frictional sliding, which can recur.

Summary

- The general form of earthquake magnitude is $M = \log_{10}(A/T) + F(h, \Delta) + C$

  A is the amplitude of the signal,
  T is its dominant period,
  F is a correction for the variation of amplitude with the earthquake's depth h
  Δ is epicentral distance,
  C is a regional scale factor.

- Richter Scale magnitude is: $M_L = \log A + 2.76 \log \Delta - 2.48$

- Body wave magnitude m_b is $m_b = \log_{10}(A/T) + Q(h, \Delta)$

- Surface wave magnitude M_s is: $M_s = \log_{10}(A/T) + 1.66 \log_{10} \Delta + 3.3$

- Body and surface wave magnitudes do not correctly reflect the size of large earthquakes and saturate about 6.2 and 8.3 respectively.

Summary

- There are uncertainties in magnitude estimation due to the earth's variability and deviations from the mathematical simplifications used.

- Different techniques (body waves, surface waves, geodesy, geology) can yield different estimates.

- Moment magnitude is given as: $$M_w = \frac{\log M_o}{1.5} - 10.73$$

- It gives a magnitude directly tied to earthquake source processes that does not saturate.
- Seismic moment is the scale factor for the spectral amplitude at low frequencies ω → 0. This is the reason why it is also called the "static" moment and defined as the

$$M_0 = \mu \bar{D} S = \mu \bar{D} f L^2$$

Here, the fault area is written in terms of a shape factor f and the square of a dimension L

Summary

- The rupture time needed for the rupture to propagate along the fault is approximately

$$T_R = L/v_R$$

- The rise time needed for the dislocation to reach its full value at any point on the fault has been predicted to be about

$$T_D = \mu\bar{D}/(\beta\Delta\sigma) = 16f^{(1/2)}/(7\beta\pi^{1.5})$$

where $\Delta\sigma$ is the stress drop in the earthquake

- Once the moment exceeds about 5×10^{27} dyn-cm, 20 s is to the right of the second corner, on the ω^{-2} portion of the spectrum. Thus M_s saturates at about 8.2, even if the moment increases. M_s saturates even as the moment and fault areas increase.

- m_b saturates at a lower moment (about 10_{25} dyn-cm), and remains about 6 even for much larger earthquakes.

Summary

- The relationship between the slip in an earthquake, its fault dimensions, and its seismic moment is closely tied to the magnitude of the stress released by the earthquake, or stress drop.

- The best-constrained quantity is the seismic moment, so we estimate the average slip, $\bar{D}$, from the seismic moment as

$$\bar{D} \approx cM_0/(\mu L^2),$$

  "c" is a factor depending on the fault's shape.

- Stress drop is given as: $\Delta\sigma = cM_0/L^3 = cM_0/S^{3/2}$

- The stress drop on a circular fault with a radius R is $\Delta\sigma = \frac{7}{16}\frac{M_0}{R^3}$

- Stress drop on strike-slip on a rectangular fault with length L and width w yields

$$\Delta\sigma = \frac{2}{\pi}\frac{M_0}{w^2 L},$$

Summary

- Stress drop on dip-slip on a rectangular fault gives $$\Delta\sigma = \frac{4(\lambda+\mu)}{\pi(\lambda+2\mu)}\frac{M_0}{w^2 L}$$

- Small differences in time function duration correspond to larger differences in stress drop, even for an assumed rupture velocity and fault geometry.

- Stress drop both characterizes earthquake source spectra and gives insight into the physics of faulting.

- The ratio of the slip to fault length is constant indicates that strain release in earthquakes is roughly constant, at about $\epsilon_{xx} \approx \bar{D}/L \approx \Delta\sigma/\mu \approx 10^{-4}$

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THANK YOU !