Soil Dynamics
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Lecture 09
Single Degree of Freedom System (SDOF) - Part 7

Hello friends. Today we will continue our discussion on Single Degree of Freedom System.

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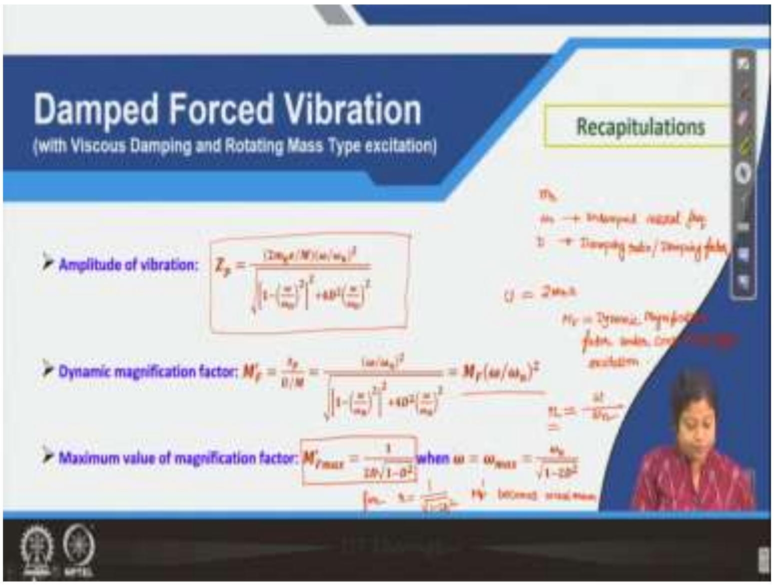

So, here, first you can see what we have discussed in the last class. So, last class we are introduced to the forced vibrating system under rotating mass type excitation. And obviously, so far we have only considered the viscous damping. No other damping is considered in our discussion so far. So, what we have seen in the last class?

We have seen how to calculate the amplitude of vibration for foeced vibration because of rotating mass type excitation. So, we have, so last class, we have derived this expression for the amplitude of vibration. Or we can call it amplitude of vibration under rotating mass type excitation. So, in this expression, what are the different parameters?

This is r. So, Mr is mass of one of the rotating mass which causing the excitation because of its angular motion with a speed omega with an eccentricity e. Now, capital M is the mass of the foundation. What is omega n? Omega n was undamped natural frequency. D was the damping ratio. Sometimes, we call it also as damping factor, damping factor or damping ratio.

Then we have found out the expression for the dynamic magnification factor which is nothing but the capital ZP divided by capital U by M. Where capital U was, what was the capital U in this expression? It was twice Mr times e and capital M already explained. So, MF dashed which was the dynamic magnification factor for the forced vibrating system subjected to rotating mass type excitation, is expressed here in terms of M, capital MF times omega by omega n whole square.

So, what is capital MF here? Capital MF is the dynamic magnification factor under constant force type excitation. So, in this way we have expressed dynamic magnification factor for considering rotating mass type excitation. Then we have studied the definition or derivation of the maximum value of this dynamic magnification factor by maximizing MF dashed or I can say capital MF dashed. This term we have maximized by omega or the frequency ratio omega divided by omega n, which we can symbolically expressed by small r also.

So, basically, we extremised MF dashed by this small r and determine the value of omega or the ratio, frequency ratio omega by omega n for which the dynamic magnification factor becomes maximum. And what we have seen? We have seen for r is equal to 1 divided by, I can write it here for r is equal to 1 divided by square root of 1 minus 2 D square for this value MF dashed becomes maximum.

And what was the maximum value that we have seen in last class. The maximum value can be expressed by this way that 1 divided by 2 D multiplying with square root of 1 minus capital D square. So, here what we have finally seen that the back maximum dynamic magnification factor depends upon only the damping factor or damping ratio.

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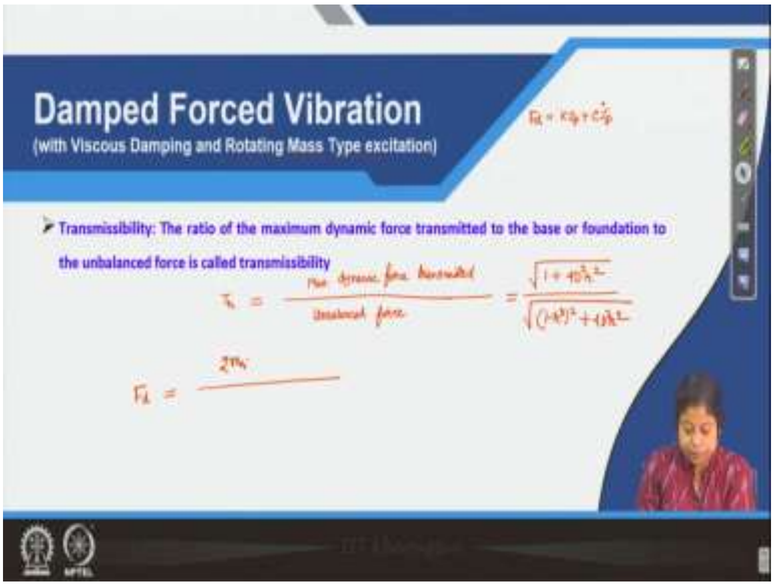

Now, next is transmissibility for rotating mass type excitation. So, what is transmissibility? Transmissibility is the ratio of the dynamic force transmitted to the foundation or base to the unbalanced force in the system is called the transmissibility. So, in case of forced vibration system subjected to rotating mass type excitation, what we can write for transmissibility?

That Tr which is transmissibility is equal to, first I am writing the definition once again here. Maximum dynamic force transmitted divided by the unbalanced force. So, in case of our rotating mass type excitation, the final form of capital T small r is 1 plus 4 D square times r square divided by 1 minus r square whole square plus 4 times D square r square.

Already, if you recall in case of forced vibration subjected to constant force type excitation, we have derived the expression for maximum dynamic force transmitted to the foundation or base. In case of rotating mass type excitation also, we can find out the same way the maximum dynamic force transmitted to the base or foundation by this expression.

And if we will write it, then we will get an expression which is, so using the equation that Fd is equal to k ZP plus c times ZP dot, you can find out the dynamic force transmitted to the base or foundation. In this case, ZP is the particular solution for the equation of motion. Kind of, I am writing just the final expression for all of you that is Fd divided is equal to 2 times mr.

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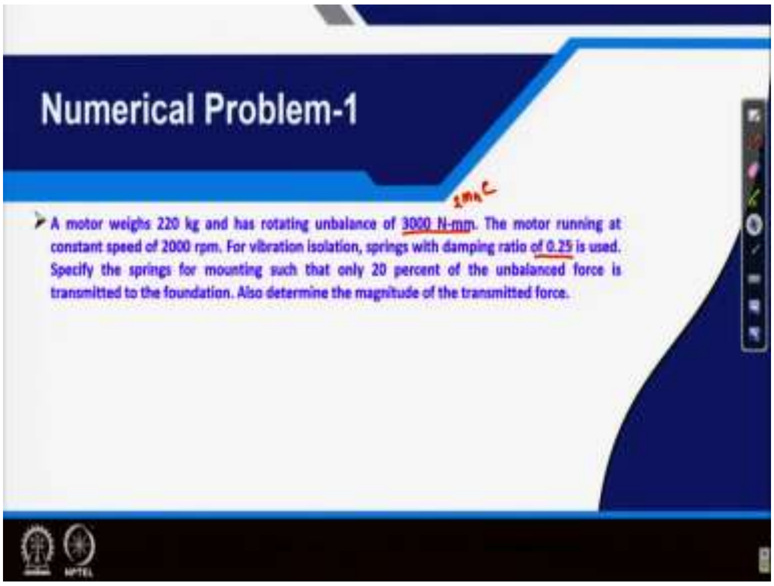

Now, let us see one numerical problem. What is the problem statement here? A motor weighs 220 kg and has rotating unbalance of 3000 newton millimeter. So, we can say this is our 2 mr times e. The motor running at a constant speed of 2000 rpm, rotations per minute. For vibration isolation, springs with damping ratio 0.25 is used. That means the damping ratio of the system is also given for the vibration isolation what is the damping ratio that is provided.

Also, it is specified that the springs for mountings, the spring is mounted in such a way that only 20 percent of the unbalanced force is transmitted to the foundation. We are asked to determine the magnitude of the transmitted force here.

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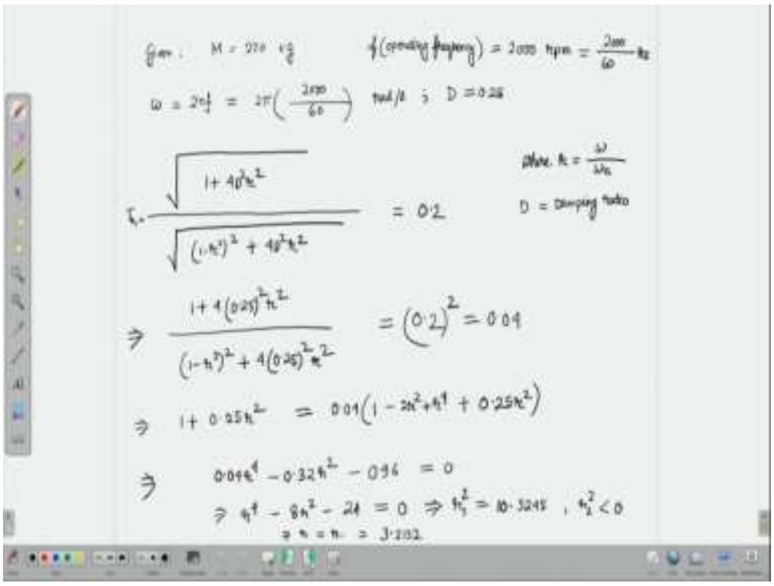

So, given parameter is mass which is 220 kg. This k is small k not capital. Then operating frequency of the machine is mentioned. Then that frequency f which is operating frequency is 2000 rpm. So, we can convert it to cycle per second also which is also called hertz. Then in terms of circular frequency, circular operating frequency omega will be equal to 2 pi f which is nothing but 2 pi times 2000 divided by 60 and unit is now radian per second.

Now, the transmissibility factor is given. So, what is the transmissibility factor? That we have already seen. So, transmissibility factor for rotating mass type excitation is square root of 1 plus 4 D square times r square divided by square root of 1 minus r square whole square plus 4 D square times r square. And it is said that this transmissibility factor or ratio is equal to, let me check, it is equal to 20 percent that means 0.2 where r is frequency ratio which is omega divided by omega n and D is damping factor or damping ratio. So, that is also given which is 0.25.

So, now in this expression, we can write the value of D as well as we can see squaring both the side of this equation. So, 1 plus 4 times D value is 0.25 times r square whole divided by 1 minus r square whole square plus 4 times 0.25 whole square times r square. That is equal to 0.2. So, let me use the calculator to solve this problem. This is not 0.2 but we need to take, we need to square it. So, this is actually 0.04.

So, now we will same simplify this left hand side. Or what we can do we can write it like this way 4 times 0.25 square is coming your 0.25 r square. On the right hand side now we can write it as 1 minus 2 r square plus r to the power 4 plus 4 times 0.25 square which is 0.25 r square. Now, we can take everything on one side and right hand side should be equal to 0. So, let us write it that way. So, 0.04 times r to the power 4. What is the coefficient for r square here? Minus 2 times 0.04 plus 0.25 times 0.04 minus 0.25.

So, what we are getting here is something minus 0.32 r square. I hope I have done the calculation correctly. Yes. And now the coefficient of r to the power 0. That means the term where no r is present. So, that is nothing but 0 4 minus 1. So, 0.96 minus 0.96 is equal to 0. Or I can write it as also r to the power 4 minus 8 r square. This is 24. Yes, it is 24. So, minus 24 is equal to 0.

Now, what we can do here? We can get the two roots. Actually, there are four roots. So, from 4 roots possible root we have to choose, because in our case r is always positive. So, which says that, let me get it. So, that is saying. So, using calculator I am getting r square is or I can write it r1 square which I am interested to consider is this one.

Because the other one is other root that means r2 square is negative. So, from these I can say r or I can write it r1 also, is nothing but 0.3245. So, square root of 10.3245 which is 3.2132. You can take up to two decimal place. So, we get r is equal to 3.2132. From these I can go to the next page.

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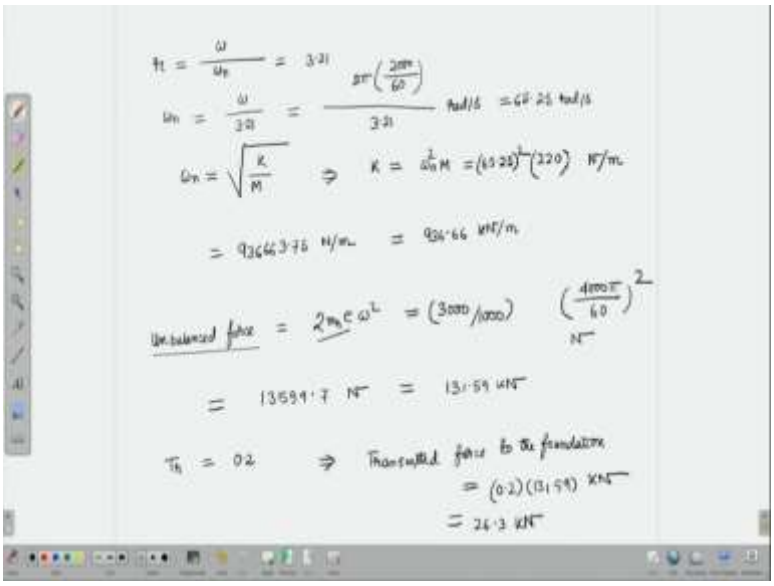

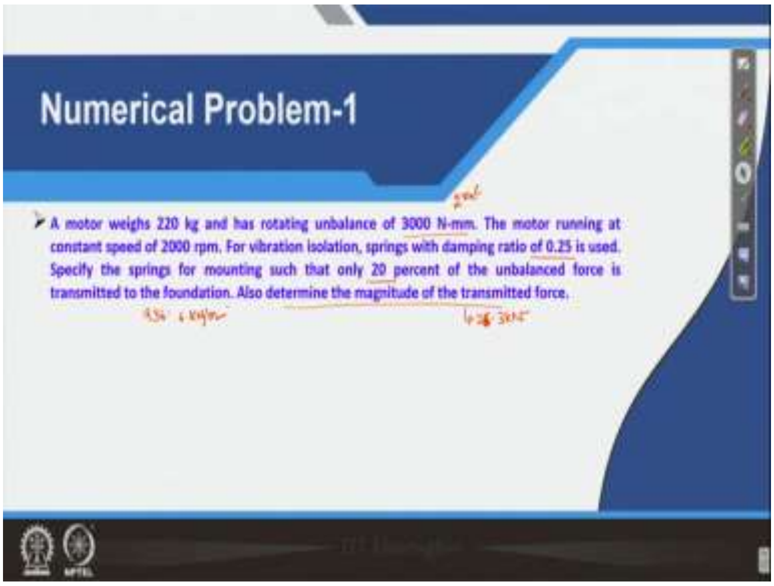

So, r is equal to omega divided by omega n which is equal to 3.21. I am writing only up to two decimal place. Then what will be omega n which is undamped natural frequency? That is equal to omega divided by 3.21. So, omega is how much 2 pi f. f means 20000, 2000 divided by 60 divided by 3.21 in radian per second.

So, let us see, what is the value? 2000 divided by 60 divided by 3.21. So, it is coming 65.25 approximately in radian per second. So, our undamped natural frequency for this problem is

65.25 radian per second. Now, we know the expression for omega n is square root of K by M, where k is the spring coefficient and capital M is the mass in kg.

So, from this we can get spring coefficient or spring constant is equal to omega n square times capital M. So, 65.25 square times M which is 220. So, unit is different. Let me write the correct unit in this case. So, in this case correct unit is newton per meter. So, how much it is coming? Let me check. So, the answer is coming somewhere 936663.75 in newton per meter. Or we can write it as 9.36 or 37 into 10 to the power 5. Or not that way, better I can write it other way. I will write it as 936.66 in kilo newton per meter.

Now, one thing I would like to mention here that some of you may get a slight different value. For an example, I have taken I have approximated r to 3.21. If you are taking 3.213 or one more place this, so, like what we get earlier that is 3.2132 if you take, your answer will differ from the answer which I get here. That is possible. So, omega n is calculated. Now, what is left? What is asked here?

The last part was, also determine the magnitude of the transmitted force. So, if we know the how much force is transmitted that is 20 percent of the unbalanced force. So, here, if we know the unbalanced force, from that we can calculate our transmitted force. So, how much is the unbalanced force in this case? Unbalanced force is 2 times mr e omega square.

So, we have calculated already omega, we know the value 2 times mr e, which is written here already. And the unit I think, newton millimeter. So, I am writing it in newton meter. This is the unit for this part, times omega square. So, omega square means 4000 pi divided by 60 square. I am just erasing this newton meter because I have written it for your understanding only. So, it is in newton. So, how much is the unbalanced force? Let me check.

So, the unbalanced force in this case is 131594.7 in newton. So, we can express it in kilo newton also. So, it will be, just let me check whether I have converted all the parameters correctly or not. So, 3000. Just give me one minute time here. Yes. So, this is that means 131.59 kilo newton. So, this is the unbalanced force.

And we know the transmissibility which is 20 percent or 0.2. From this, we can find out transmitted force to the foundation, is equal to 0.2 times 131.59 in kilo newton which is 26.3 in kilo newton. So, our answer is the, for the second part is 26.3 kilo newton. For the first part, answer is 936.66 in kilo newton per meter.

So, and we can see now what is the answer. Final answer is written here also. For k, it is 936.6 kilo newton per meter and for maximum magnitude of transmitted force or magnitude of the maximum transmitted force is 26.3 kilo newton.

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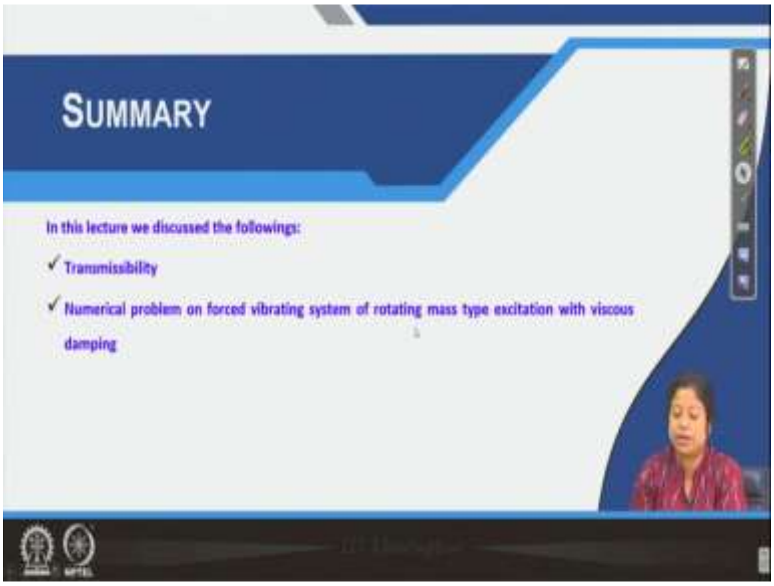

So, come to the summary of today's class. Today, we have seen what is the transpensability. Transmissibility is the ratio of the magnitude of the maximum dynamic force that is transmitted to the foundation to the unbalanced force present in the system. And you see, and then we have seen what is the expression or equation that we can use to find out the transmissibility and using that we have solved one numerical problem today on forced vibrating system considering rotating mass type excitation.

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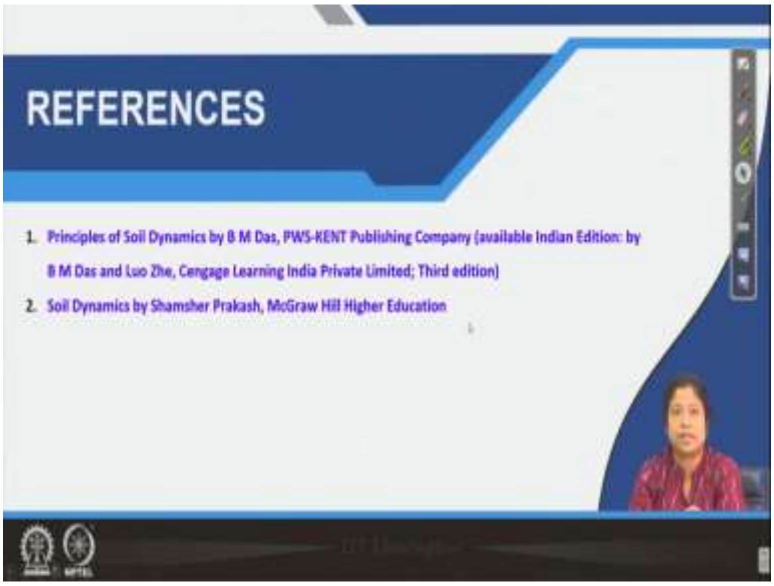

So, these are the references which I have used. So, thank you.