Soil Dynamics
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Lecture 08
Single Degree of Freedom System (SDOF) - Part 6

Hello friends. Today we will continue our discussion on Single Degree of Freedom System were we will study today the forced vibration system subjected to rotating mass type of excitation.

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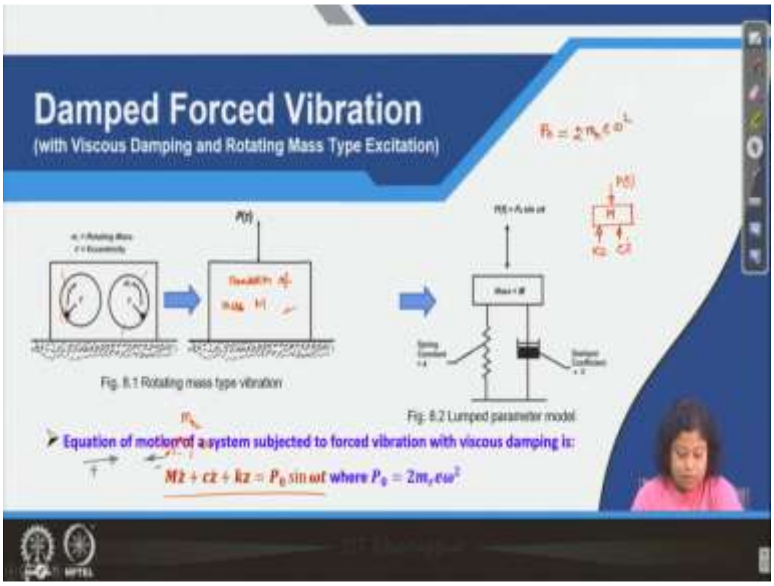

So, let us take this example. What is going on here? Actually, the excitation is created by rotating two masses one mass here, its mass is mr and this one also having the same mass mr. These two are allowed to rotate in two opposite directions. So, one is rotating, you can see this one is rotating counter clockwise direction and this one is rotating clockwise direction with an eccentricity e to the center. So, as a consequence, what is happened?

Vibration is occurred and the vibration can be represented by, I mean force can be represented by P t which is equal to P0 sin omega t. However in this case, P0 is dependent on the omega value. That means, let us take with it is rotating, I am taking this mass suppose, so, this is its path. It is rotating in this direction along this periphery.

So, at some time, let us take its position is mr. So, I am just erasing a little bit once again. So, this is the position of mass m r with respect to horizontal. This angle is omega t. Then mr, I should write in better way. I can write m with subscript r. So, now, what will be and its eccentricity is e. Then what will be the magnitude of P0? You can see when there are two masses rotating in opposite direction to each other, what will be happening with there horizontal component?

If one case, it is directing in, for this it is directing in negative X direction, for the left hand side mass the component, horizontal component is directing towards positive X direction. So, resultant horizontal component is 0 in this case. However, in case of vertical component, what we can see? The total amount of the vertical vibration is P0 which is equal to 2 times mr times e times omega square. We get P0 is equal to 2 times mr times e times omega square. e is the eccentricity, mr is the magnitude of the mass, rotating mass and omega is its angular velocity.

Now, then what will be the equation of motion, in this case? If we consider, for that we need to consider first the free body diagram of the mass m which represents the foundation which is resting on the soil. Here, this is the foundation of mass capital M. So, these arrangement or system can be represented by lambda parameters shown in this figure 2. Then if we draw the free body diagram, we can see on the mass M, forces acting are P t, spring force KZ and resistance force by the dashpot C is Z dot.

So, with this now, if we form the equation of motion, we will get this one. Already, we have actually seen, we have done the similar exercise for constant force type excitation, the same exercise, only thing in this case P0 is equal to 2 times mr times e times omega square. So, P0 actually is a function of omega. Earlier, P0 was constant. Now, what will be the equation after knowing the equation of motion, next thing we need to know what will be the equation of vibration or expression for vibration?

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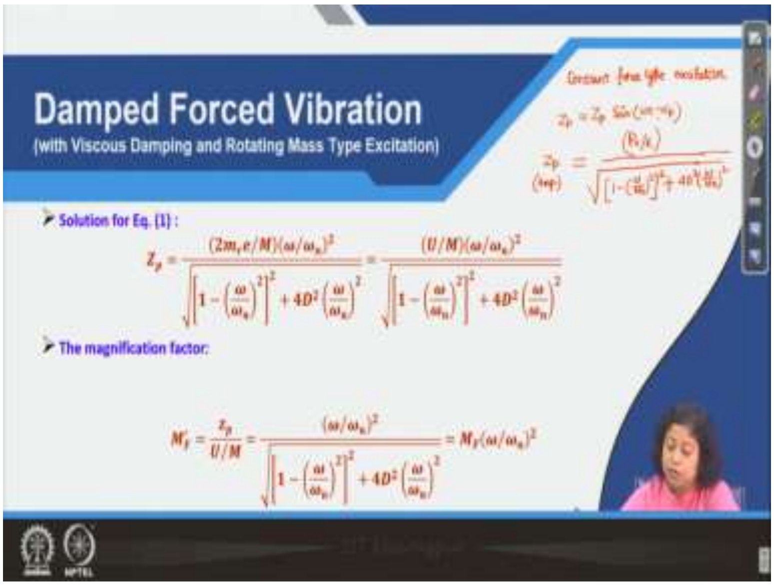

So, in this case, the amplitude of vibration is this one. How it is coming? The same way. Previously, we have seen. Previously means in case of constant, I am just writing, force type excitation. What we have seen? We have seen ZP is equal to capital ZP times some sine omega t minus alpha P and capital ZP, that time it was, this is capital ZP. That means amplitude. I am writing in bracket amplitude.

That was P0 divided by k times 1 minus omega by omega in whole square, entire thing is under square, whole square, under whole square plus 4 D square times omega by omega in whole square. Now, in this case P0 is equal to something else. That is the reason we are getting this expression. So, in this expression, what we have done first? First, we have written in place of P0, let us do this exercise here.

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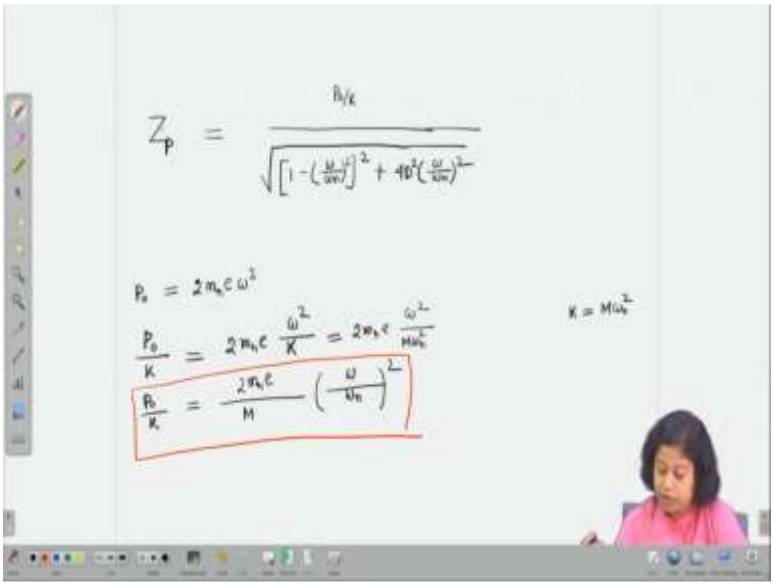

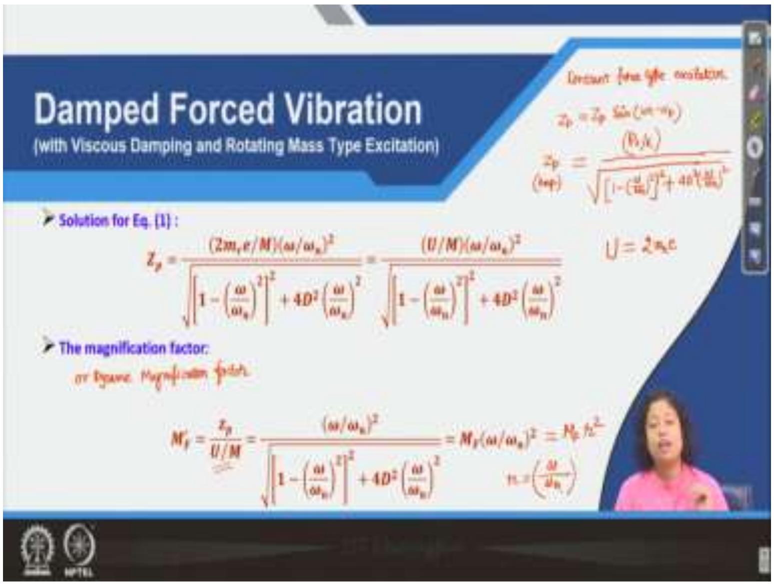

So, as I said previously, we have seen capital ZP is equal to P0 by k times 1 minus omega by omega n whole square, entire thing is under whole square plus 4 D squared times omega by omega in whole square. Now, in case of rotating mass type excitation P0 is equal to 2 times m r times e times omega square. So, what we can write from here?

We have a term here P0 by k. So, P0 by k will be then 2 times mr times e times omega square by k. We all know what is k? k is the spring coefficient and spring coefficient can be expressed in terms of the mass of the foundation and the natural frequency of the system. Natural frequency means in this case, untenthet natural frequency of the system. So, that means k is equal to capital M times omega in whole square. So, in place of that I will now write it as, or I can write it as 2 times mr times e divided by capital M times omega by omega in whole square.

So, if we will write here in place of P0, this term which I have just now written then we get the expression for ZP that means capital ZP which is the amplitude of vibration. And in the next step what I have done? I have introduced a new term capital U. So, what is capital U here? Here, capital U is here, sorry, I think pin is not active. So, in this case capital U is 2 times mr times e. So, capital U is replaced by this 2 mr e and the expression of the amplitude of vibration becomes this one.

Now, what we have done last time? Last time thereafter, we have calculated the dynamic magnification factor. Now, if we see this time, what is the dimension of 2 times m r times e? Mr has the dimension of mass; e has the dimension of length. If it will be divided by capital M, then the dimension of capital U by M has the dimension of length.

Now, then if I will divide ZP capital ZP by capital U by M which I have done here, what will be its dimension? It is dimensionless because capital ZP has also the dimension of length. Then whatever we will get from this ratio is a factor and that factor is called magnification factor or dynamic magnification factor.

So, this time dynamic magnification factor is represented by omega by omega n whole square in the numerator, and in denominator you can see square root of 1 minus omega by omega in whole square, entire thing is under whole square plus 4 D square times omega by omega in whole square. So, if we compare now, the dynamic magnification factor for the case of constant force type excitation, what we see?

This time m Mf dashed is equal to, previously, M got MF times omega by omega n whole square. That means I can rewrite it as MF times r square also. r square is the frequency ratio. I can rewrite here what is r, omega by omega n. Now, next step is to know how MF or MF dashed in this case, how MF dashed varies with r or frequency ratio.

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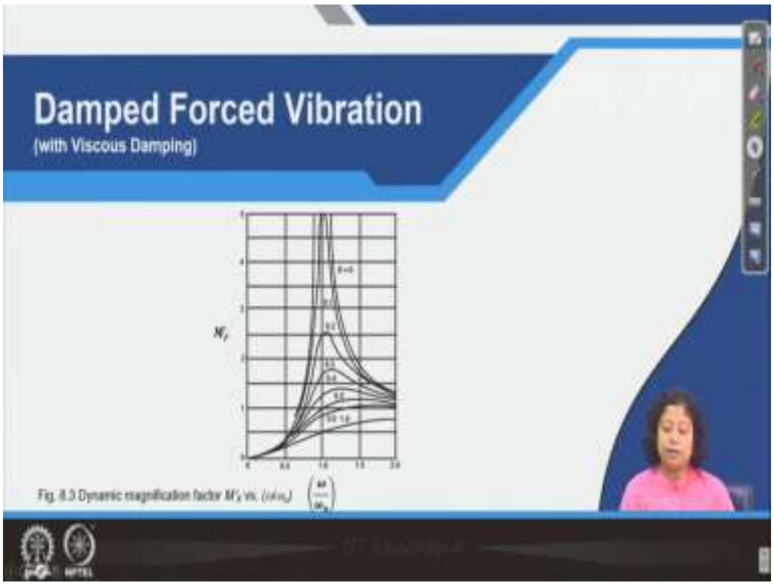

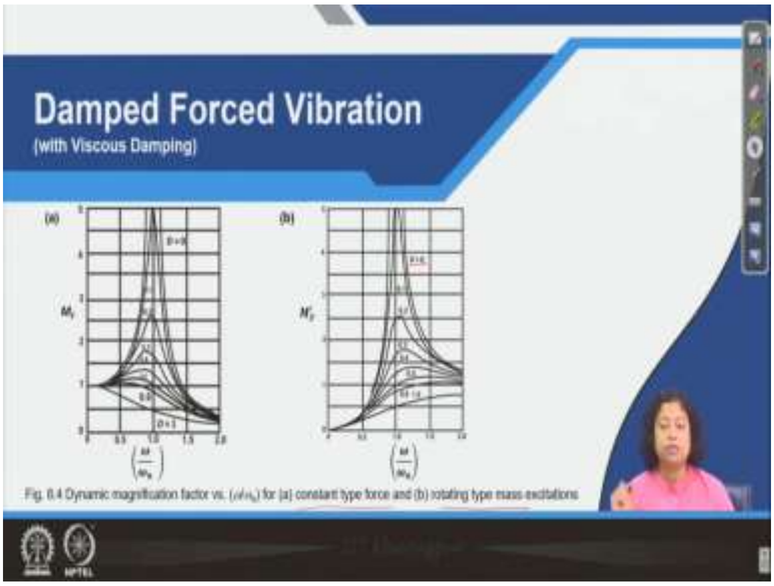

Let us see. In this case, since, in the numerator there is a term r square, so, with the increase in r, what will be happen? When r will exceed 1, then we will get the peak of MF dashed. Earlier, what we were getting? I would just like to show; earlier we were getting peak before r becomes equal to 1. Earlier means this is the case for constant type excitation. Currently, in rotating mass type excitations, what we are seeing?

We are seeing that MF dashed becomes maximum after r becomes equal to 1. Or in case of resonance when r is 1, we can see the maximum value for D equals to 0. Now, our next thing before solving any numerical problem which I would like to do here is to find out what will be the expression for maximum dynamic magnification factor.

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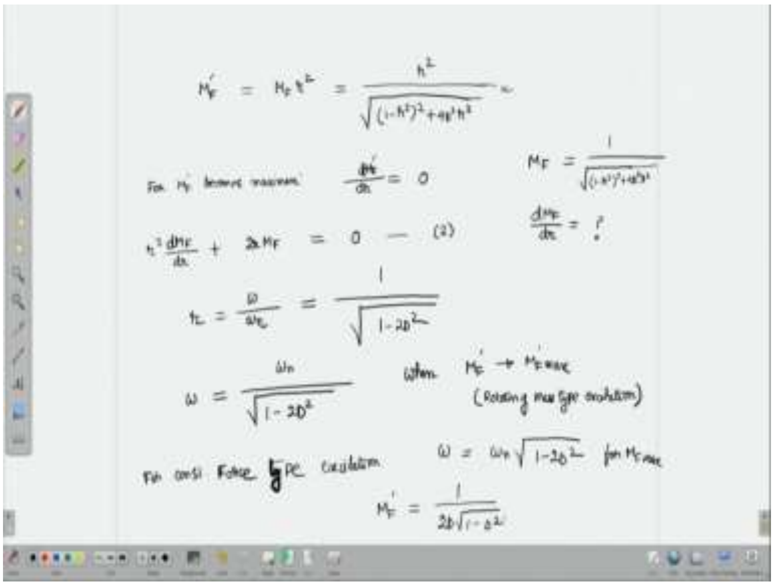

So, I would like to just write the steps here, so that one can follow and derive it. So, in case of rotating mass type excitation, magnification factor can be written as MF times r square. MF is the magnification factor for constant force type excitation. Or I can write it also as r square times 1 minus r square 1 minus r square whole square plus 4 D square r square.

Now, for MF dashed becomes maximum, what we will write? I will write the derivative of MF dashed with respect to r should be 0. What does it mean? It means plus this expression is 0. So, if we will solve this equation, we already know what will be the expression for MF in this case?

MF expression is, once again I am writing, this is the expression for MF. Likewise, we also know what is the expression for d MF of d r. That we have done in last class. Now, if you will use those two expressions in this equation, let us give a name of this equation as equation 2 then you will get the expression for r, r means omega by omega n which is equal to 1 divided by square root of 1 minus 2 D square. That means operating frequency is equal to omega n divided by square root of 1 minus 2 D square when MF dashed reaches to maximum value. So, this is for rotating mass type excitation.

So, earlier what we have seen? Earlier for constant mass type excitation, constant force excitation, I pronounced it wrongly. Earlier we started constant forced type excitation it was this. So, here is the difference. But when we are talking about the maximum magnification factor that means MF dashed, what will be the expression for MF dashed in this case? It will be 1 divided by 2 D times square root of 1 minus D square. It remains same for both the cases.

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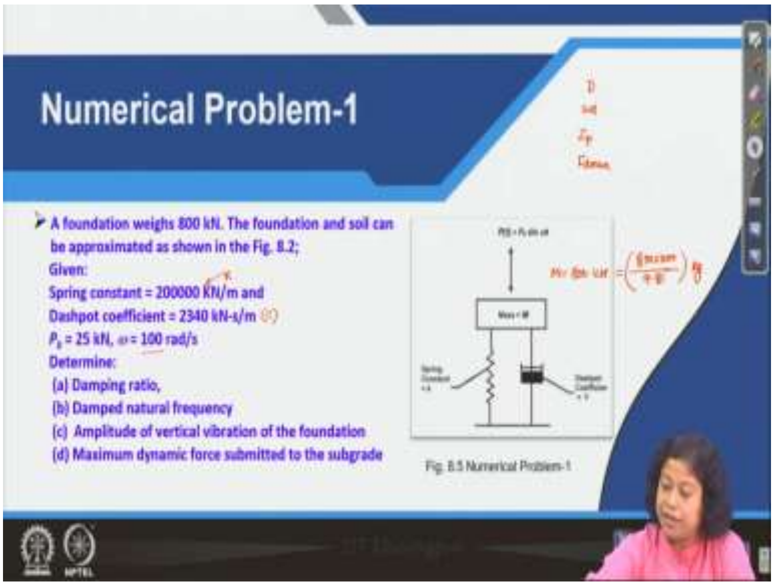

So, with this we can try to solve one numerical problem which is given here. A simple problem. So, a foundation, this one has weight 800 kilo newton that means your capital M is 800 kilo newton which we can write also as 800 into 1000 divided by 9.81 in newton by g. So, basically, it will be in kg and that is the unit for mass. Next, what is stated?

The spring constant, this is k is 200000 kilo newton per meter. Likewise, the coefficient of dashpot is also provided. The amplitude of force P0 is 25 kilo newton. That means it is a kind of constant force type excitation. And the operating frequency is 100 radian per second. Operating frequency means in circular frequency.

So, we are need to calculate the damping ratio, damped natural frequency. So, what we are asked to find out I am writing here. Damping ratio then omega D then we need to find out the amplitude of vibration. Amplitude of vibration means in this case capital ZP. And then the maximum dynamic force Fd max. So, let us solve this.

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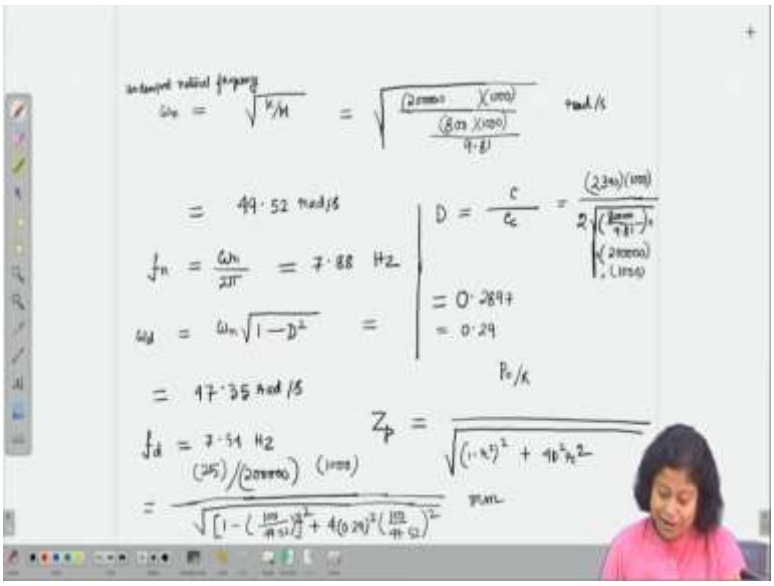

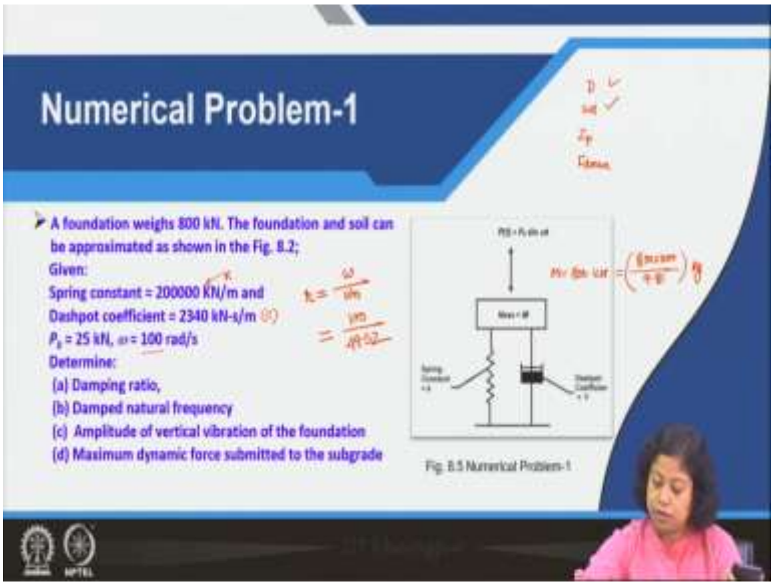

In this case, mass is given, k is given, so, we can find out first the natural frequency, undamped natural frequency. That is square root of k by capital n. Now, square root of capital, so, k is 200000. This is in kilo newton per meter. So, I can multiply it by thousand and change the unit in newton per meter then. So, now it is in newton per meter and M is 800 times thousand divided by 9.81. So, let us see, how much is the natural frequency?

Square root of, it is coming 49.52 in radian per second. Then frequency in hertz is how much? I need to divide it by 2 pi which is coming 7.88 in hertz. Then omega D is, in case of constant force excitation omega D for actually damped natural frequency, that is your omega n minus 1 by minus D square under square root. So, omega n times square root of 1 minus D square is yours, how much?

Let me check. 49.52 here. We need to first calculate D, then only we can get omega D. So, let us do that. D means damping ratio which is C divided by Cc. C is given. I think it is 2340. Yes. 2340. Change the dimension also in newton second per meter. Cc means 2 times square root of Mk. M means, this entire thing is under this square root. So, basically, D will be how much then?

This is multiplying, yeah. Damping ratio D is coming 0.2897. We do not need to go after two decimal place. So, we can do approximation in second decimal place which is giving 0.29 for damping ratio. Then what will be omega D? Omega D is 49.52 times square root of 1 minus point, D square means point, and D square means 0.29 square. So, it is coming, it is coming 47.35 probably, just cheek it. And if I write it in hertz, the damped natural frequency will be 7.54 in hertz. So, we know now the D, we know the omega D.

Now, capital ZP and F d max. So, capital ZP means what? I am just writing capital ZP means P0 divided by k 1 minus r square whole square plus 4 D square r square. Now, in this case, P0 is 25 kilo newton. And if you are interested to find r that is omega by omega n, so, 100 divided by 49.52 probably will be the ratio. So, 25 kilo newton k. So, I am not changing the unit in this case. I am keeping it in kilo newton.

And k you will also be, I will write k also in kilo newton per meter. So, that is 200000. Now, it has unit in meter. So, generally, deflection comes in millimeter for the foundation that is the reason I am multiplying it in 1000 to convert it in millimeter. Now, 1 minus 100 divided by 49.52 whole square and this is in millimeter. In this way, we can calculate Z P which is the amplitude of dynamic vibration. Now, next is asked F d max. That also we know how to find out. So, I am just going back once again to the, sorry, I am going. There is no need to go back. This case.

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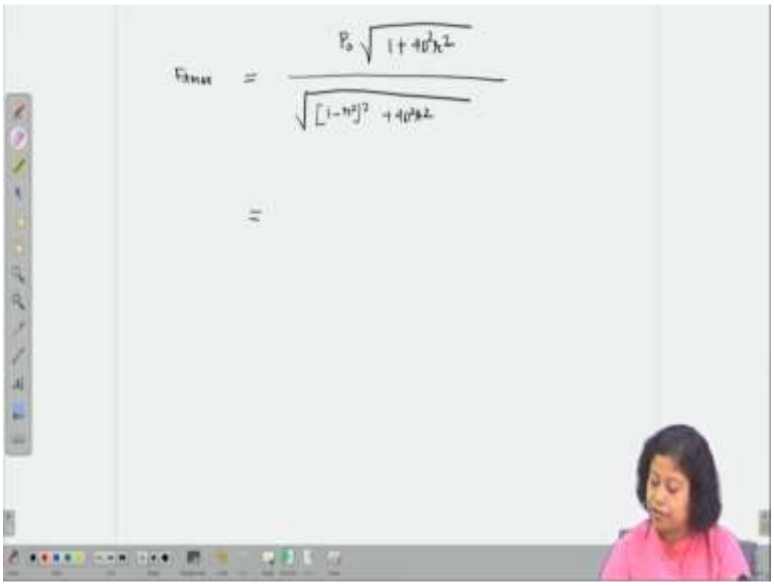

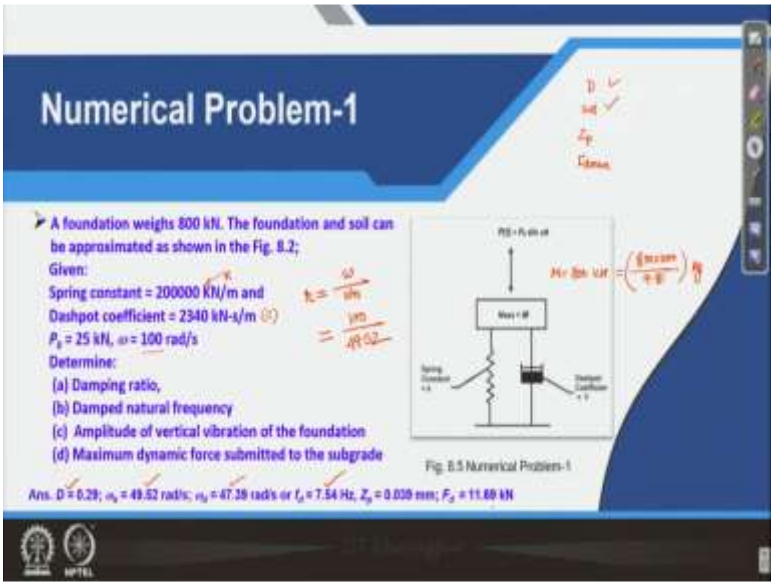

Let us write here itself. So, F d max I need to calculate which also known. What was the expression for F d max1? It was P0 times 1 plus 4 D square r squared square root of, divided by square root of 1 minus r square whole square plus 4 D square r square. Now, we know all the quantity. If we will use this expression, we will get the value. So, finally, what is the final answer that I am showing here.

Already, we get D which is the damping ratio, then we get this natural frequency, we get this damped natural frequency also, in hertz also we get this answer. And the last two cases which I have not completed the calculation, if you will complete the calculation, you will get the same answer. So, with this I would like to conclude this class. What we have studied today?

We have studied the forced vibration system where rotating mass type excitation is applied to the system. And then we studied what will be the magnitude of the, or amplitude of the vibration. Then we have studied what will be the expression for dynamic magnification factor. Then what will be the maximum value of dynamic magnification factor and at which operating frequency we will get maximum dynamic magnification factor that also we have studied today.

So, with this, I conclude our today's class. Next class, we will try to solve, at the beginning of the class we will try to solve another example. But that time, we may take a problem of rotating mass type excitation. So, thank you all.