Soil Dynamics
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Lecture - 7
Single Degree of Freedom System (SDOF) - Part 5

Hello friends. Today we will discuss the next part of Single Degree of Freedom System. So, let us see what we have discussed in our last class.

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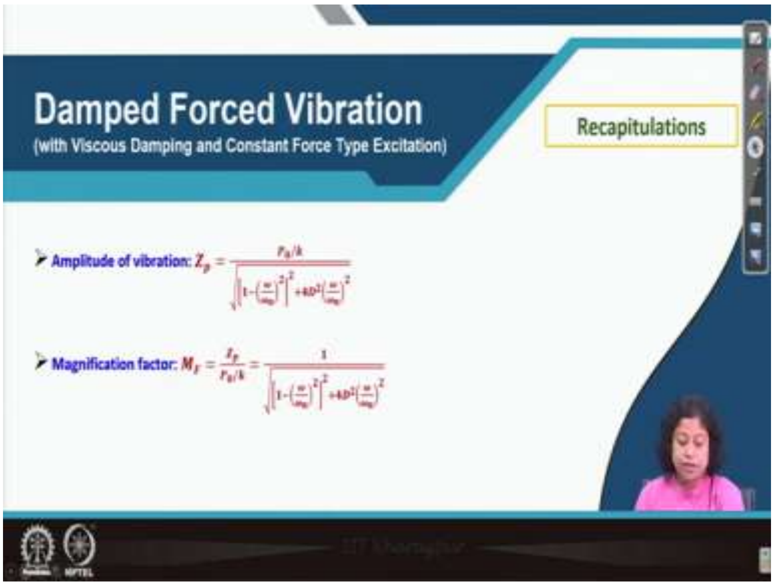

So, last class we have studied the amplitude of vibration for constant force type excitation of forced vibration system with viscous damping. Then we have studied the definition of dynamic magnification factor which is MF, sorry definition of dynamic magnification factor MF. So, sometime we call it also as magnification factor.

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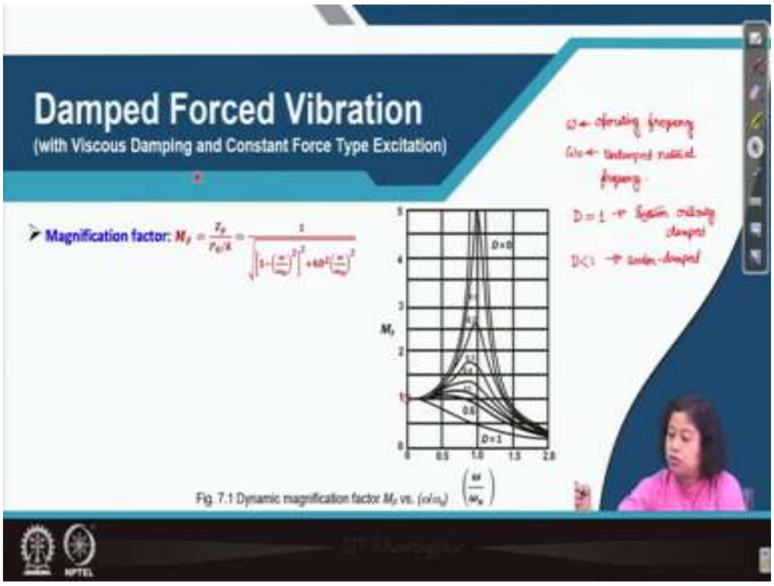

Now, today we will see how this MF varies with the frequency ratio that means omega divided by omega n. What is omega? Once again this is the operating frequency of the machine or you can call it as operating frequency of the external force and omega n is what, it is the undamped natural frequency. So, these are the two terms which we have already studied. So, using these two terms and the damping ratio capital D we can find out the magnification factor.

Now, in this figure you can see if the frequency ratio increases how the magnification factor will change. Let us take the case when D is equal to 1, D is equal to 1 means system is critically damped. That time what we can see the maximum value of the magnification factor is here, that means when the operating when the frequency ratio is 0 and with the increase in the frequency ratio this value continuously decreases like this.

Now, next case is under damped condition when D is less than 1, that means under-damped system. So, in this case you can see the pattern of MF, it starts from one obviously when your frequency ratio omega by omega n is equal to 0 that time magnification factor becomes 1 from this expression. So, if we see the graphical presentation of MF, it starts from 1, from 1 over the increase in frequency ratio the value of MF increases.

Now, at some point if D is equal to 0 that means no damping is present in the system that time when the operating frequency omega is equal to omega n which is the natural frequency of the system, that means the operating system, operating frequency matches to the natural undamped frequency of the system that time what will be happening, resonance will occur.

However, in case of under-damped system what we can see, when operating frequency is equal to the undamped natural frequency of the system, magnification factor MF becomes maximum. And with the increase in the value of D that means with the increase in damping in the system what is happening the peak value of magnification factor reduces, you can see here for D is equal to 0.2, here is the magnification factor.

Whereas when D is equal to 0.5, here is the peak of the magnification factor, that means the peak of the magnification factor or dynamic magnification factor reduces with the increase in damping in the system. That is quite expected. Now, our next interest is to know what will be the maximum magnification factor when system is subjected to forced vibration system by constant force type excitation. So, for that let us go to the board.

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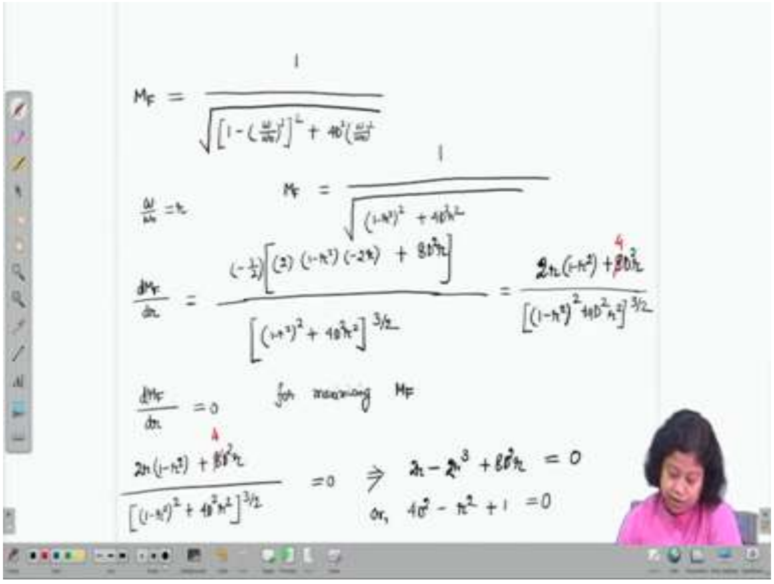

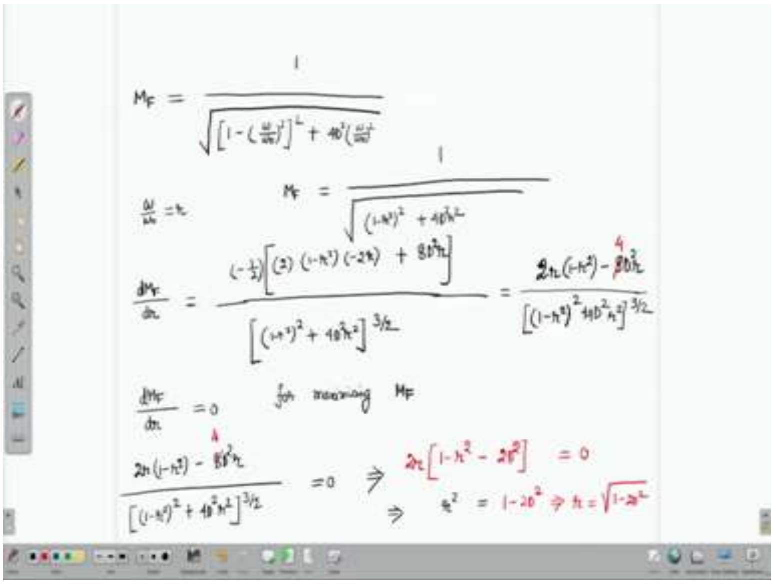

So, in the previous class we have derived the expression for the dynamic magnification factor which is MF here and that is equal to 1 divided by square root of 1 minus omega by omega n whole square entire thing is under whole square plus 4 D square times omega by omega n whole square.

Now, let us take this frequency ratio omega by omega n as r small r, then we can rewrite magnification factor in terms of r which is 1 minus r square whole square plus 4 D square r square. Now, if we are interested to know the magnitude of the maximum dynamic magnification factor, then we need to maximize MF. For that, we need to differentiate MF with respect to r or you can do it with respect to operating frequency omega also.

So, then what will be the expression? I need to erase this. So, this is in the denominator, now let us see what will be in the numerator minus half times, so this is equal to I can write it as 2 times r into 1 minus r square plus 8 D square r divided by, what we have already written in the denominator.

When we are maximizing MF we can write d MF of d r is equal to 0 for maximizing in case of minimizing also we can use the same thing in this case we are maximizing, so for maximizing MF. Therefore, whatever I have written here that should be equal to 0 now, so I am directly writing 2 r minus 1 minus r square plus 8 D square r divided by something in the denominator which is 1 minus r square whole square plus 4 times D square r square times 3 by, to the power 3 by 2 equals to 0 or I can write simply it as.

Now, in this case there are three terms and all the three terms contain r and r is a non-zero value so I can divide both right hand and left hand side by r and or I can divide it by 2r also so then finally what I am getting is. So, let me check if I made mistake anywhere in this expression, yes, I can see one mistake. In this expression here there is one bracket and this minus 1 by 2 is multiplied with the entire term within the bracket, then I need to correct a few things this 8 will be actually 4 because there is one, it is multiplying with minus half here.

So, now you can, here also the same thing it is not 8 but 4. So, I am just erasing one line which is this one and the last one. So, I am writing here it is 2r times 1 minus r square plus 2 D square equals to 0. Then finally I can write from here is that r square is equal to again I think I made again mistake because here since the minus is there so I need to take care of this minus also.

So, please correct here there will be a negative sign, I think now it is written correctly, if so then sorry here also there is one negative sign. So, r square is equal to what we can write 1 minus 2 D square. So, finally I can write then r is equal to square root of 1 minus 2 D square obviously we cannot take the negative sign here, only we will consider the positive sign. So, go to the next page.

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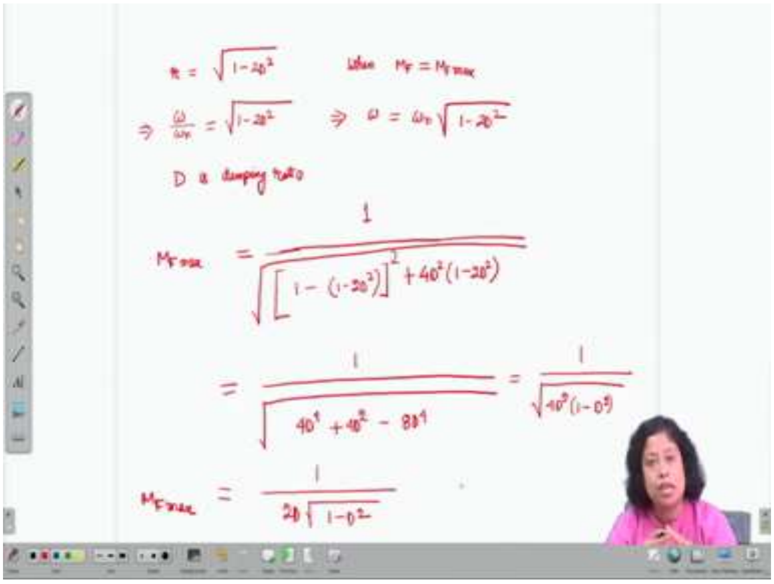

Since r is equal to square root of 1 minus 2 D square when MF is equal to MF max that means omega by omega n is equal to square root of 1 minus 2 D square and then what we can write omega is equal to omega n times 1 minus 2 D square. That means the maximum value of omega, that means when the amplitude of vibration reaches to the maximum value that time operating frequency should be equal to omega n times square root of 1 minus 2 D square.

Or in other words I can see when operating frequency is equal to omega n times square root of 1 minus 2 D square that time the amplitude of vibration will reach to the maximum value. And here D is the damping ratio. Then what will be the value of MF max, I am just going back to the previous page. So, here in place of omega by omega n what we can write, we can write 1 minus 2 D square square root of 1 minus 2 D square then I am writing 1 divided by 1 minus omega by omega n whole square that means 1 square root of 1 minus 2 D square you can see here.

So, here I will write 1 minus 1 minus 2 D square whole square plus 4 D square times 1 minus 2 D square and the entire thing is under square root. So, finally what we are getting from here? We are getting from here is 4 D to the power 4 plus 4 D square minus 8 D to the power

4 or I can write it as square root of 4 D square times 1 minus D square which is your 1 divided by 2 D times 1 minus D square. So, this is our maximum magnification factor.

This is for which case? When the system is subjected to forced vibration with viscous damping and also the type of vibration is, or excitation is constant force type excitation that means the amplitude of the external force does not depend upon the operating frequency.

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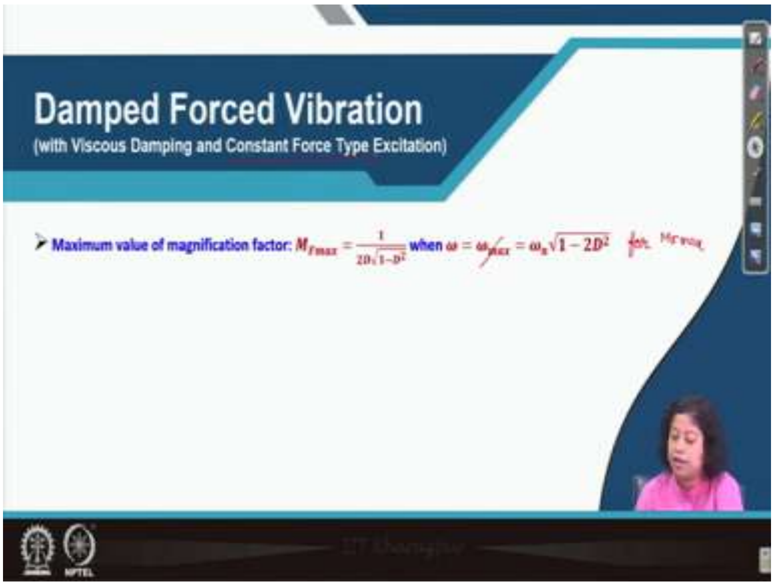

Now, let us see the summary, which we have just now derived. In this case actually what we have seen is that I should not, I may, it will be better if I will not write it as omega max here at least or we can write but the meaning of omega max is the omega value at which ZP becomes maximum or magnification factor becomes maximum. So, this is for F max and condition you can see constant force type excitation.

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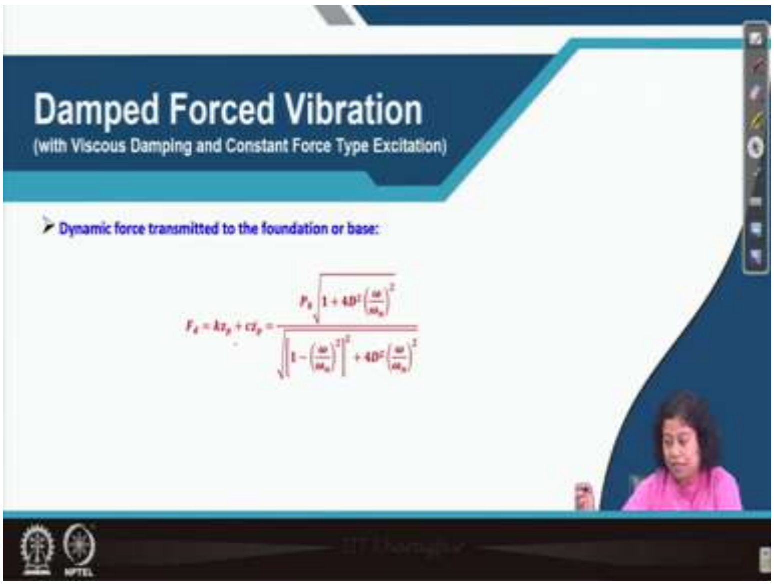

Now, our next topic is dynamic force transmitted to the foundation or base. So, for that what we need to do is something like, basically the dynamic force which is transmitted to the system can be written as sum of K ZP, K ZP is the part of spring force and C ZP dot, we know the general equation for ZP so using that we can calculate K ZP plus C ZP dot let us see how it is coming.

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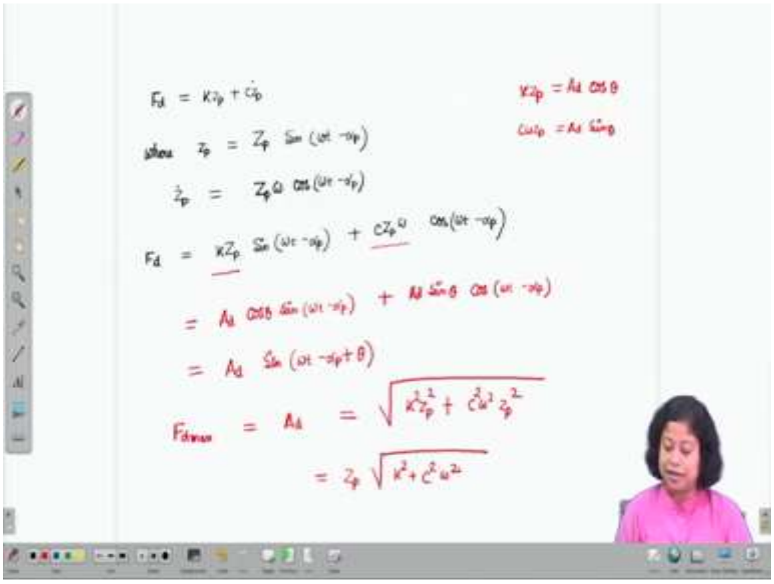

So, right now we are interested to know the maxim, the dynamic force which is transmitted to the foundation or base which is Fd and that is equal to K ZP plus C ZP dot. What is our ZP? Where ZP is equal to capital ZP times sin omega T minus alpha P. Already we know what is capital ZP. So, I am not rewriting here the same thing.

So, now we can calculate Z dot P which will be nothing but ZP times, capital ZP times omega times cosine of omega T minus alpha P. Now, here with this we can write the expression for dynamic force which is transmitted to the foundation that is Fd is equal to K times capital ZP times sin omega T minus alpha P plus C times capital ZP times omega times cosine of omega T minus alpha P.

Now, here you can see, there are two terms first one is k ZP and the second one is C omega capital ZP these two terms, if we assume K ZP here as let us take another variable which may be I can take Ad amplitude of dynamic force transmitted, so A d is into I can take it as cosine of say beta or I can take it theta also, let us take theta so I am taking it cosine theta k ZP and C omega ZP I am taking A d times sin theta.

Then Fd can be rewritten as Ad times cosine theta sin of omega T minus alpha P plus Ad times sin theta times omega T minus sorry cosine of omega T minus cosine of omega T minus alpha P. Now, what we can do here? Finally, with these we can get Ad times sin of omega t minus alpha P minus or plus, plus, plus theta.

Then what is the maximum force or dynamics force that is transmitted to the foundation? Maximum dynamic force is nothing but the amplitude of the dynamic force. So, F d Max is nothing but Ad. What is Ad? From this expression Ad is square root of K square capital ZP square plus C square omega square capital ZP square or I can write it as ZP times K square plus C square omega square. I can take here also K square out of this square root, let us see in the next page.

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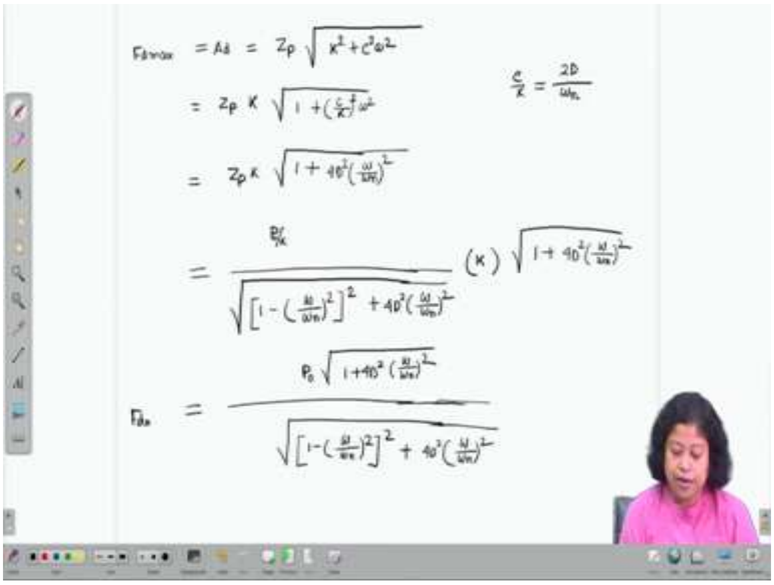

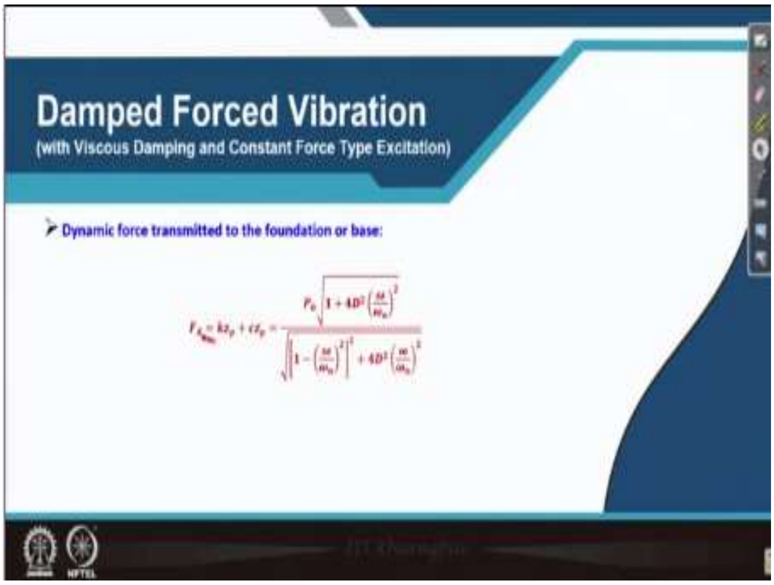

So, for Fd max we get Ad which is I am just going back to the previous page, you can see ZP times K square plus C square omega square and this summation is under square root. So, I can write here as, let us check, yes I have written it correctly here. So, now I am interested to take K square out of this square root.

So, what I am doing here which is nothing but in this case C by K we already know, C by K means 2D divided by omega n so I can, it is square root of sorry it is not square root, I just missed here, C by K is 2D divided by omega n. So, I will take it here. So, in this way we can get the expression for Fd max that means the maximum dynamic force transmitted to the foundation or base or sub grid soil.

Now, whatever we have obtained that we can write in this format also. So, how we can write this way also, we know already if I will once again go back here how to write ZP let us see how we have written ZP earlier, this is our ZP, so if I will use this ZP in this equation then I can get this final form.

So, ZP was actually P0 by K divided by square root of 1 minus omega by omega n whole square entire thing is also whole square under whole square plus 4 D square times omega by omega n whole square, so this is ZP that is multiplying with K and then the term under root which is 1 plus 4 D square this one.

So, what we are getting is P0 times 1 plus 4 D square omega by omega n whole square this thing in numerator and in denominator 1 minus omega by omega n whole square entire thing is under whole square plus 4 D squared times omega by omega n whole square. So, this is the value for Fd max and that you have seen in this expression. It will be better if I will write F d max here rather than just Fd.

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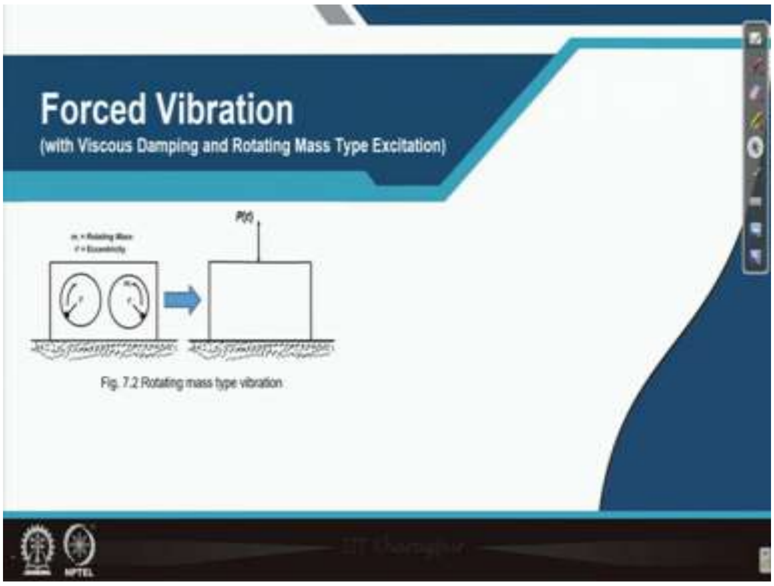

Next class we will discuss forced vibration. So, let me summarize today's class. So, today we have studied the, today we have studied how to get the maximum dynamic magnification factor for the forced vibration with viscous damping when constant force of excitation is applied to the system and then at that time when MF that means the maximum magnification, dynamic magnification factor occur or we reach to MF max that time what will be the value of operating frequency or at what operating frequency we can get maximum dynamic magnification factor.

Now, with these I would like to conclude our today's class. So, next class we will discuss forced vibration, when, where we will apply rotating mass type of excitation which I have shown. Thank you.