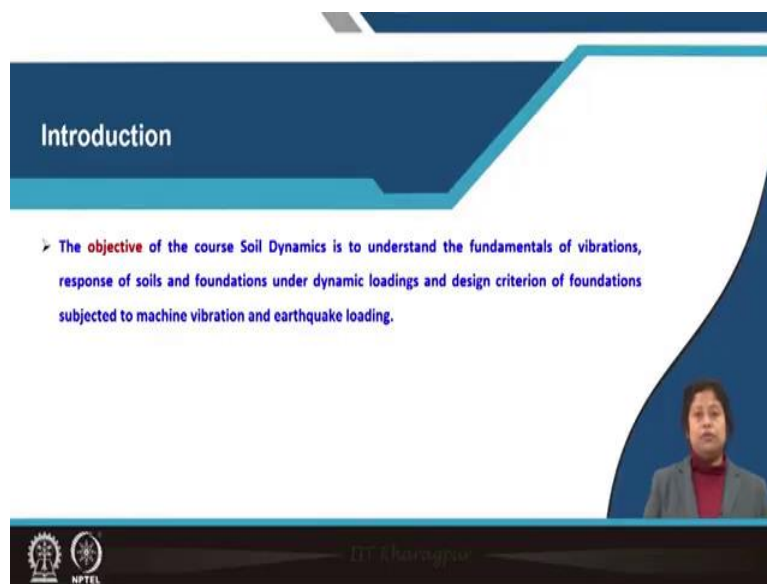


Solid Dynamics
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Lecture - 60
Summary

Hello friends. Welcome to the last class of Soil Dynamics. So, today we will discuss the course which we have studied in this subject Soil Dynamics and I will try to summarize the content, which I have taught during last 12 weeks.

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So, first come to the objective of this course. The objective of the course is to create an understanding on the fundamentals of vibrations. Then to study the response of soils and foundations under dynamic loading and also discuss the design criteria of foundations, which are subjected to machine vibration and earthquake loading. For this the course is divided into 59 lectures, excluding today's one, and there we have studied different topics.

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Summary

➤ **Week 1: Introduction to Theory of Vibrations**

- ✓ Different types of dynamic loading (e.g. earthquake load, functioning of reciprocating and rotary types machines and hammer, blast load, wind load, wave action of water etc.)
- ✓ The definitions of the fundamental elements of the vibrations (such as DOF, time period and frequency, free and forced vibrations, natural frequency, modes of vibration etc.)
- ✓ Theory of vibrations (simple harmonic motions, superimposition of two simple harmonic motions etc.)
- ✓ Single degree of freedom systems (Part 1# equation of motion for freely vibrating undamped system, relationship between static deflection due to body weight and natural frequency)

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So, first we are introduced to the theory of vibrations, where we have studied different types of dynamic loading. So, what are those different types of dynamic loading? You can see here, those are earthquake load, functioning of reciprocating and rotary types machines, that means functioning of reciprocating and rotary type machine can also produce dynamic loading, also hammer can produce impact load, then blast float, wind load, wave action of water, etc.

However, our main focus was on load or dynamic load, which is generated by functioning of reciprocating and rotary type machines and hammer and also on earthquake loading. Then what we have studied? We have studied the different fundamental elements of vibrations, for an example, degrees of freedom, time period, natural frequency, etcetera.

So, after knowing, after discussing these elementary things we have discussed what are free vibrations and force vibration. Free vibration means when the system vibrated due to its inherent forces or due to inertia. For an example, suppose I have switched on one machine and after some time I have switched off, so when I have switched off the machine that immediately after switching off, what is happen?

The machine can still vibrate, but that time there is no external force acting, only because of the inertia it vibrates. So, that kind of vibration is called as free vibration. Forced vibration means when external force is present and because of that vibration is occurring, we call it as forced vibration. Then we have studied the definition of natural frequency and how to calculate it. So, natural frequency of the system, a freely vibrating system.

And because of, in absence of any kind of damping that means undamped system. Then we have studied what is the definition of modes of vibration. After going through the fundamental elements of vibration, next what we have discussed, we have discussed the simple harmonic motions, what is simple harmonic motion, then what is happened when two simple harmonic motions are superimposed.

In this case if two simple harmonic motions of same frequency are superimposed, then it produces, then they produce another simple harmonic motion. However, if the frequency of the two simple harmonic motion is slightly different, then what will be happen, it produce, then these produce beat. Then we have discussed single degree of freedom system under free vibration that means when a single degree of freedom system vibrating freely in absence of any kind of external loading.

And for that type of system, first we have drawn the free body diagram and then we have derived the equation of motion. And from that we have determined the natural frequency. Also, we have the established relationship between natural frequency and the static deflection due to body weight of the mass.

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Summary

Week 1: Introduction to Theory of Vibrations (continued)

- ✓ Single degree of freedom systems (Part 2# different types of damping, equation of motion and the response of freely vibrating damped system considering viscous damping, over damped, critically damped and under damped systems, mathematical formulation of damped natural frequency etc.)
- ✓ Single degree of freedom systems (Part 3# response of under-damped system, definition of logarithmic decrement etc.)

Handwritten formulas:

$$C_c = 2\sqrt{mK}$$

$$\omega_d = \omega_n \sqrt{1-D^2}$$

$$D = \text{damping ratio} = \frac{c}{c_c}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\delta = \frac{x_0 D}{\sqrt{1-D^2}} = \frac{1}{n} \ln\left(\frac{x_1}{x_2}\right)$$

Continued.....

Then we have discussed, we have discussed different types of damping, for an example, the viscous damping, coulomb damping, etc. First, we focused on viscous damping and considering viscous damping we have derived the equation of motion and then found out response for freely vibrating damped system. Here we have seen that three types of damp system considering viscous damping.

One is over damped system, second one is critically damped system and third one is under damped system. For soil we are mainly interested for under damped system. So, how we have the defined over damped, critically damped and under damped systems? For that first we have introduced critical damping. So, if the damping of the system is more than this critical value, then the system is called over damped system.

If the damping constant is equal to the critical value, then it is called as critically damped system and if the damping constant of the system or damping coefficient is less than the critical value then it is called as under damped system. Mathematically we have seen how to calculate this critical value, so the critical value if I write it as C_c , critical value of damping coefficient, then that is equal to 2 times of square root of m times k , where m is mass of the system and k is the stiffness.

Then from that we have discussed how to find out the damped natural frequency. Damped natural frequency we have expressed ω_D in terms of the un-damped natural frequency which is ω_n as ω_n is, sorry, ω_D is equal to ω_n times square root of 1 minus ζ^2 . What is ζ here? ζ is damping ratio. So, damping ratio, how do we define?

We define it as the damping coefficient of the system which is C to the critical value of damping, which is C_c . Now, here ω_n as I said it is the natural frequency of the undamped system and how did we find out ω_n in first week of our lecture, if you remember ω_n can be determined using this expression, ω_n is equal to square root of k by m .

And if we are interested to write in terms of displacement static displacement due to body weight that also we can do. Then what we have studied? Then we have discussed single degree of freedom system considering under damped condition, so in under damped condition what we have seen, we have seen that the amplitude of displacement decreases exponentially over the time.

And because of this nature of response we can find out logarithmic decrement. So, logarithmic decrement we expressed it as δ and that can be expressed mathematically in terms of damping ratio, as I am writing here δ is equal to $2\pi\zeta$ divided by $1 - \zeta^2$. So, in this way or I can write it also as 1 divided by n times log of Z_1 divided by $Z_1 + n$. So, these things we have studied.

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Summary

$M_m = \frac{C_m}{\sqrt{1-2D^2}}$ (Const)

$M_m = \frac{C_m}{\sqrt{1-2D^2} \cdot \omega}$ (Rotating mass type)

- Week 2: Theory of Vibration
- ✓ Single degree of freedom systems (Part 4 to Part - derivation of equation of motion for forced vibrating system with viscous damping considering constant and rotating mass type excitations, derivation of the response of forced vibrating system with viscous damping, dynamic magnification factors, dynamic force transmissivity, Rayleigh's method etc.)

Dr. Khurshid

Continued.....

Then second week we focused on theory of vibration where we have studied how to derive the equation of motion for forced vibrating system. So, first we basically focused on freely vibrating system and second week, we discussed forced vibrating system. However, both the weeks we focused on viscous damping only. And when we have considered the force vibration that means external force is present in the system.

So, how vibration is created considering constant force type excitation, that means the amplitude of force does not depend upon the operating frequency and rotating mass type excitation where amplitude of force is a function of operating frequency. Then we derived the response that means how to find out the displacement for forced vibrating system considering viscous damping.

Thereafter, we have studied the dynamic magnification factor and what we have seen there, the expression for dynamic, the expression of dynamic magnification factor for constant force type excitation differs from the expression of dynamic magnification factor for rotating mass type excitation. Then what we have studied? Then we have studied how to get the maximum value of the magnification factor or dynamic magnification factor.

And eventually, here we have seen that for constant mass type excitation and rotating mass type excitation these two dynamic magnification factors, maximum value of these two the dynamic magnification factors are same. Then from these we have calculated the resonant frequency for two different types of excitation. So, for constant force type excitation resonance frequency.

If I write it as ω_n maximum value of ω that is expressed in one way, which is $\omega_n \sqrt{1 - 2D^2}$ and for, this is for constant force type excitation and for rotating mass type excitation we expressed it as ω_n divided by $\sqrt{1 - 2D^2}$. So, this is for rotating mass type. So, in this case the amplitude of frequency, sorry, amplitude of vibration or force depends upon the operating frequency for the second case.

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Summary

- Week 3 (Lecture 11): Coulomb Damping
- Week 3 (Lectures 12-15): Multiple Degree(s) of Freedom System
- ✓ Two degree of freedom systems (derivation of equations of motion, determination of natural frequencies at different modes of vibrations and mode shapes etc.)
- ✓ Semi-definite system
- ✓ Three degree of freedom system (under free and forced vibrations)

Handwritten equations and diagrams on the slide include:

$$x_n - x_{n+1} = \frac{4\mu N}{K}$$

Diagram 1: A mass m on a surface with a spring K and a damper μN . Displacement is x .

Diagram 2: A two-degree-of-freedom system with masses m_1 and m_2 , springs k , and a force $F(t)$.

Then in third week what we have studied is coulomb damping, which is the another type of damping. So, in coulomb damping what we have seen, we have seen from one cycle to next cycle the amplitude of displacement will reduce and how much the amount of reduction or I can write it in this way, if for n th cycle the amplitude is x_n and for $n + 1$ th cycle it is x_{n+1} , then $x_n - x_{n+1}$ is equal to 4 times μn divided by K . So, what is μ here?

μ is the coefficient of friction, n is the normal force. Normal force means suppose there is a mass which is connected by a spring like this and this mass is resting on the surface. Now, when we will draw the free body diagram what we will see, if this mass is subjected, is trying to move in positive X direction, then the surface on which it is resting that offers resistance force and that also offers one normal force.

So, this normal force is n and the resistance force which opposing the motion acts like this, and that is μn . Other than that, you can write rest of the forces like when it is moving in this direction that time what is happen to the spring, spring is stretching, so consequently spring will try to pull this mass and the magnitude of pulling is K times x . So, in this way basically we have drawn the free body diagram.

Then we have written the equation of motion, and finally, we determine the amplitude for n th cycle and n plus 1th cycle and then we have seen the difference in the amplitude of displacement that is x_n minus x_{n+1} is $4 \mu n$ divided by K . And that is true for all n values. Then next class onwards we discussed multiple degrees of freedom system, where first we have discussed two degrees of freedom system.

And for that we have derived the equations of motion for two different masses, then we have determined that natural frequencies for the different modes of vibrations and also, we have found out the mode shapes. Then we have discussed semi-definite system. So, what was these semi-definite system? Semi-definite system means the system moves as a rigid body and when it is moving as a rigid body that time it can vibrate also.

But it is moving and vibrating as a rigid body, so there is one natural frequency, which is 0 for this type of system, that we have seen. I can show one figure, so here you can see there are two masses m_1 m_2 connected by K , it is rolling on, it is moving on this surface. So, this is the force, so this is one example of your semi-definite system. So, one of the natural frequencies for this system you will see is 0.

Then next lecture we have discussed three degrees of freedom system under free and then forced vibration conditions are considered. So, after finding out natural frequency in this case we get three natural frequencies, so after finding out that what we have done, we have discussed, we have determined the response for this type of system.

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The slide is titled "Summary" and contains the following content:

- Week 4: Wave propagation $v_p = \sqrt{\frac{E}{\rho}}$ $v_s = \sqrt{\frac{G}{\rho}}$ $v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ $v_s = \sqrt{\frac{G}{\rho}}$
- Wave propagation (longitudinal & torsional waves in an elastic rod, longitudinal & torsional waves in an elastic infinite medium, a semi-infinite elastic half space).

The slide also features a video inset of a woman speaking in the bottom right corner. At the bottom, there are logos for IIT Kharagpur and NPTEL.

Now, fourth week we have discussed the wave propagation theory. So, first we have discussed wave of longitudinal and torsional waves propagating through an elastic rod. So, here we have seen that we have determined actually the velocity of the longitudinal wave, so I can write it as V_{P1} , which is square root of E divided by ρ .

And the velocity of the torsional wave or we can call it as shear wave, which is equal to square root of G by ρ . So, what is E and G here? E and G are dynamic elastic modulus or Young's modulus and the shear modulus respectively, whereas ρ is the density of the soil, not soil, in this case it is the material of the elastic rod.

Then we have discussed the propagation of the longitudinal and torsional waves through an elastic infinite medium. So, here we have seen the velocity of the torsional waves, wave propagating through an elastic infinite medium is v_s which is equal to square root of G by ρ that means the velocity of the torsional wave remains same for both the cases.

For longitudinal wave we have seen the velocity v_p is equal to square root of $\lambda + 2G$ divided by ρ , where λ and G are called as Lamé's constant. Then we have discussed the wave propagating through a semi-infinite elastic half space. And here what we have seen we have seen Rayleigh wave which is noted at the ground surface or very close to the ground surface.

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Summary

- **Week 5: Determination of Dynamics Properties of Soils**
 - ✓ Introduced to different low strain tests (e.g. resonant column test, ultrasonic pulse test etc.) and high strain tests (e.g. cyclic triaxial test, cyclic torsional shear test, cyclic direct simple shear test etc.) methodology of resonant column test, estimation of dynamic properties etc., experimental procedure of cyclic triaxial test and determination of secant Young's modulus, damping ratio etc.)
- **Week 6: Determination of Dynamics Properties of Soils**
 - ✓ Different field test like seismic reflection test, seismic refraction test, block vibration test etc.

So, with this background of elastic waves or I can say the with the background of longitudinal and torsional wave propagation we have forecast our discussion on

determination of dynamic properties of soils and that we have done on fourth week, sorry, fifth week and sixth week.

So, fifth week basically we have started how to determine the dynamic properties of soils like dynamic Young's modulus, dynamic shear modulus, damping ratio, etc. by conducting different types of laboratory tests. For an example, resonant column test, cyclic triaxial test, etcetera. So, first what we have studied we have studied different types of tests or their names and we classified these tests into two groups low strain test and high strain level test.

So, under low strain test, resonant column test, ultrasonic pulse test come, whereas under high strain tests cyclic triaxial test, cyclic torsional shear test, then cyclic direct simple shear test are considered. So, then what we have done? We have chosen one, one test from each class that means from low strain test we have selected resonant column test and from high strain test we selected cyclic triaxial test for the discussion.

And we have studied the methodology or testing procedure for resonant column test and cyclic triaxial test, then how to calculate the dynamic properties of the soil from these two tests that we have discussed. Here I should mention one thing in resonance column test what we have seen, we can find out both dynamic Young's modulus and dynamic shear modulus, whereas from cyclic triaxial test we can only find out dynamic Young's modulus.

Because in this case we apply the axial load and for that there are two way, either we can keep the amplitude of the applied load constant over the time or we can keep the amplitude of the axial deformation constant over the time. During sixth week what we have done, we have studied how to determine the dynamic properties of soils by conducting field tests. So, what are those field tests? First one is the seismic reflection test, second one is seismic refraction test and third one which we have discussed was block vibration test.

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Summary

- **Week 7: Liquefaction of Soils**
 - ✓ Introduction to soil liquefaction, mechanism of soil liquefaction, initial liquefaction, final failure condition for soil in liquefaction, soil composition criteria and critical state criteria for soil liquefaction.
 - ✓ Simulation of field condition of soil liquefaction in laboratory tests,
 - ✓ Different factors affecting soil liquefaction
 - ✓ Susceptibility of soil liquefaction
- **Week 8: Design of Machine Foundation**
 - ✓ Design criteria
 - ✓ Analysis of machine foundation using elastic half space theory

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Then we have studied liquefaction of soil where we have discussed mechanism of soil liquefaction, then we are introduced to initial liquefaction, final liquefaction, now what is, what was initial liquefaction, when excess pore water pressure reaches to the applied confining pressure. Now, final failure condition means 20 percent double amplitude of string. So, that we have discussed when studied liquefaction of soils.

Thereafter, we have discussed soil composition criteria and critical state criteria for soil liquefaction. Then we have the studied how to simulate the field condition of soil liquefaction in laboratory test. Here we focused on cyclic triaxial test mainly. Then we have discussed different factors which affect soil liquefaction. Finally, we discussed how to check the susceptibility of soil liquefaction using the SPT data, using the CPT data.

Eight week we have started new topic which was design of machine foundation. So, there we have first studied what are the design, different design criteria, guidelines, then we have studied how to analyze the machine foundation using elastic half pace theory. There what we have seen, we have used analytical method to find out the stiffness of the soil and the damping constant.

And in these, that discussion different modes of vibrations are considered like vertical vibration, foundations subjected to vertical vibration, foundation subjected to sliding vibration, foundation subjected to rocking vibration, coupled sliding and rocking vibrations

and torsional vibrations. And for each case we have the studied how to find out the natural frequency and the amplitude of vibration or response.

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Summary

- **Week 9: Design of Machine Foundation**
 - ✓ Analysis of machine foundation for reciprocating machine (lumped parameter solution).
 - ✓ Analysis of machine foundation for impact type machine/ hammer.
- **Week 10: Design of Machine Foundation**
 - ✓ Analysis of machine foundation for rotary machine (frame type foundation for turbogenerator)-
Resonance Method and Amplitude Method

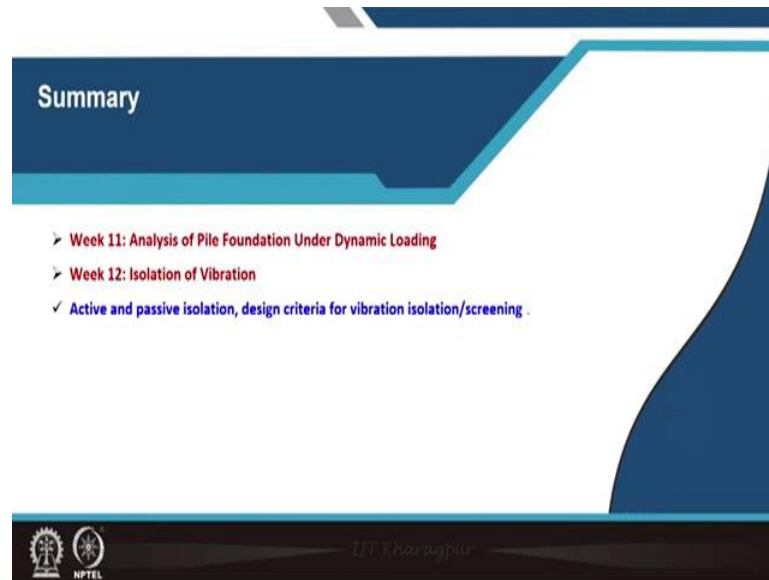
Then we discussed design foundation or we discussed how to analyze the machine foundation for the, for its design for reciprocating machine using lumped parameter solution, so there we did not consider the damping effect, we only considered the spring constant and the mass of the foundation and the machine. So, first we discussed reciprocating machine, thereafter, we discussed impact type machine or we can call it as hammer.

An example of impact type machine is hammer, so that we have discussed. So, if you recall in the, for hammer foundation what we have done, we have seen for low capacity hammer, we can represent the problem as a single degree of freedom system, whereas for medium capacity or high capacity hammer we represent the problem as 2 degrees of freedom problem and then we solved the problem by calculating natural frequencies.

And then the amplitudes of displacement, whereas for reciprocating machine we have used block foundation and determine the natural frequency and the amplitude of response. 10th week we studied the, we studied how to analyze the machine foundation for rotary machine. So, for rotary machine frame type foundation was considered and we have studied how to do the analysis by following resonance method and amplitude method.

These are the two methods which we have discussed and we have solved the numerical problems to understand how to take the loads, how to calculate the design load and then how to do the analysis of this type of, that means frame type of foundation for rotary machine.

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Then we have discussed, we have studied analysis of pile foundation under dynamic loading, here also we focused on the machine foundation loading, so pile, single pile group and pile group with pile cap were considered in the analysis and the different types of vibrations like vertical vibrations, then the horizontal sliding vibration, then coupled sliding and rocking vibration and torsional vibrations are considered in this analysis.

And during this discussion that means from week 1 to week 11, what we have seen, these sometimes-excessive vibration is produced and as a consequence the level of vibration sometimes exceeds the permissible limits for which we need to isolate the source of vibration or the important machine from the source of vibration. So, for that how do we do the isolation that we have studied on 12th week.

So, there we have studied active and passive isolation. What was the active isolation? Active isolation was the isolation of the source itself. Whereas passive isolation basically isolated the important machines from the source. So, passive isolation is generally is created away from the source, whereas active isolation is provided surrounding the source of vibration, so this is the main difference between active and passive isolations.

Then what we have done? We have studied different design criteria for this active and passive isolations and we solved numerical problems considering active isolation and passive isolation. So, with this knowledge of soil dynamics, I hope in future you will be able to, you will be able to take the challenges of design of machine foundation.

You will be able to take the challenges of this, you will be able to take the challenges coming due to earthquake loading, etc. So, I think this course and its contents are useful for you when you will do the design of machine foundation, when you will do design of foundation subjected to earthquake loading that means if you are doing foundation design in earthquake prone zone for that type of cases this knowledge at least will be helpful.

Other than that, we have discussed different codes of practice, what are those, those are different ASTM codes and Indian Standard codes. So, now you are familiar with those names also, so when you will encounter any problem which are related to the machine foundation or which are related to the design of foundation subjected to earthquake loading, you can refer those codes also depending upon your problem.

And from these course other than design theme you have understood what are the parameters, the soil parameters, which are important to consider when you are finding out the dynamic properties of the soil, when you are characterized, characterizing the soil, all these things, so I hope these discussions and this course will be useful for all of you, who are attending this subject or who are attending this course. With this I think I can end the lecture and this is the end of this course also.

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Finally, I would just like to show you the references, here only I have presented the textbook reference. I have not shown the reference of journal papers or conference papers or IS code, you know what are the different IS codes or ASTM codes we have used, so that you can refer. So, I hope this reference lists and whatever we have used earlier that references also be useful to all of you. So, with this I am stopping this lecture and if you have any doubt you can contact me, we can interact online also to clarify the queries and your doubts. Thank you.