Soil Dynamics
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Lecture - 6
Single Degree of Freedom System (SDOF) - Part 4

Hello friends. Today we will continue our discussion on Single Degree of Freedom System.

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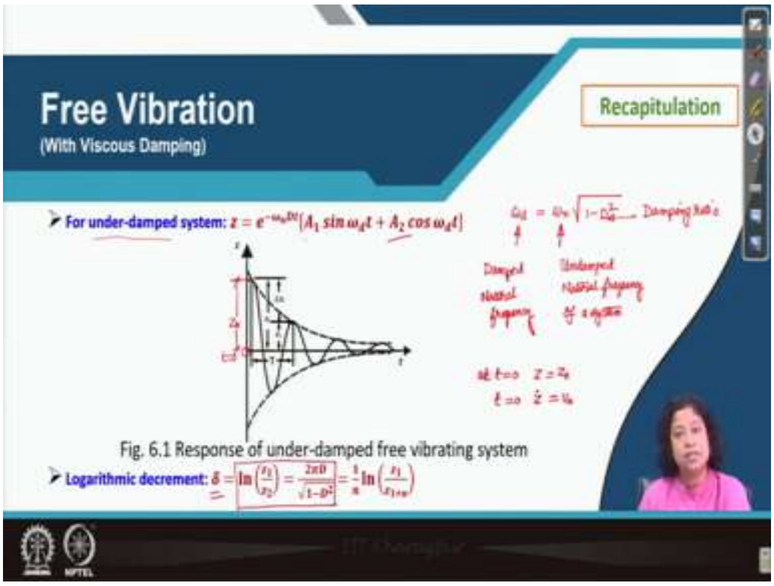

So, last week we have studied free vibration system under different damping condition. First we have studied if the system is undamped and thereafter we have studied if the system is damped with viscous damping and that time we have three different cases of damped system for viscous damping; one is called the over-damp system, the second one is called critically damped system and the third one was the under-damped system.

So, in under-damp system, system we have seen the general equation for vibration is represented by Z is equal to e to the power minus omega n D t times A1 sin omega Dt plus A2 cosine omega Dt. What is omega D here? Omega D was omega n times square root of 1 minus D square that is how we can calculate omega D.

However, this omega D is called damped natural frequency, damped natural frequency. What is omega n? Omega n is called undamped natural frequency of a system and D is the damping ratio. Now, you can see here the response curve for Z versus t for under damped system. So, in these equation there are two unknown terms, one is A1 and the second one is A2. So, how we can calculate or determine A1 and A2?

If we know the boundary conditions, then we can easily calculate A1 and A2. So, how we will set the boundary condition for an example for the vibration which is shown in figure 6.1? If you see at time t equals to 0, we can see what is the mag, what is the value of Z. Here we can take this Z which is probably Z0 may be defined by Z0, so at time t equals to 0, Z is Z0.

Likewise, at time t equals to 0, if we know the velocity which is Z dot we can write Z dot is equal to V0, so the velocity also at this point is V0. If we set these two, if we satisfy these two boundary conditions in this equation, we will get the value of A1 and A2. So, you can give a try to get A1 and A2 in this way.

Then we have studied the logarithmic decrement which is delta here which is the ratio of the two consecutive amplitudes in natural log scale and that can be expressed in terms of damping ratio by this relationship that 2 pi D divided by square root of 1 minus D square.

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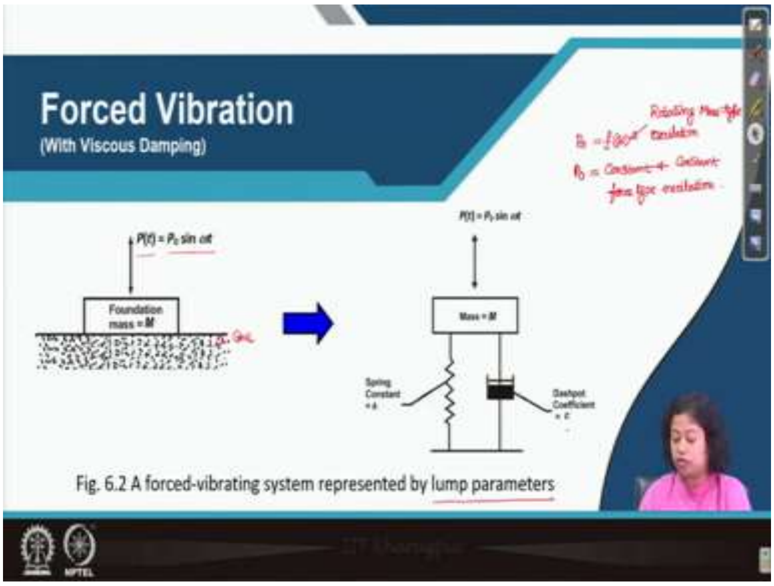

Now today we will study forced vibration. So, under forced vibration system is subjected to external loading. So, if you take an example of this foundation having mass M resting over the soil or ground surface this is our ground surface. So, what we can see here in this case the foundation is subjected to an external force P t which can be expressed by P0 times sin omega t.

So, omega is the operating frequency of the machine for which this kind of force is applied to the foundation. P0 is the amplitude of the force. Now, there are two cases, in one case the amplitude P0 depends upon the frequency or operating frequency of the machine. That means, I can write here as P0 is a function of omega.

In another case, we see that P0 is constant, it does not depend upon omega. So, the second case when P0 is constant is called constant force type excitation. So, this case is called constant force type excitation. Now, if P0 is a function of omega that means the operating frequency, then we call it as generally these can be happened when two masses in machine are allowed to rotate in opposite direction, that time this kind of situation may be arise.

So, we will see this kind of example later. However, this kind of case is called generally as rotating mass type excitation, as excitation is created, occurred by the allowing rotation of the two masses into different direction if one is rotating in clockwise direction the other will rotate in the anti-clockwise direction or counter clockwise direction.

So, first case we will consider the case for constant force type excitation, that means P0 is constant, it does not depend upon the operating frequency omega. So, this kind of system can be represented by lump parameters like k and c, k means spring constant and C is the dashboard coefficient. That means our soil is represented by one spring and one dashboard.

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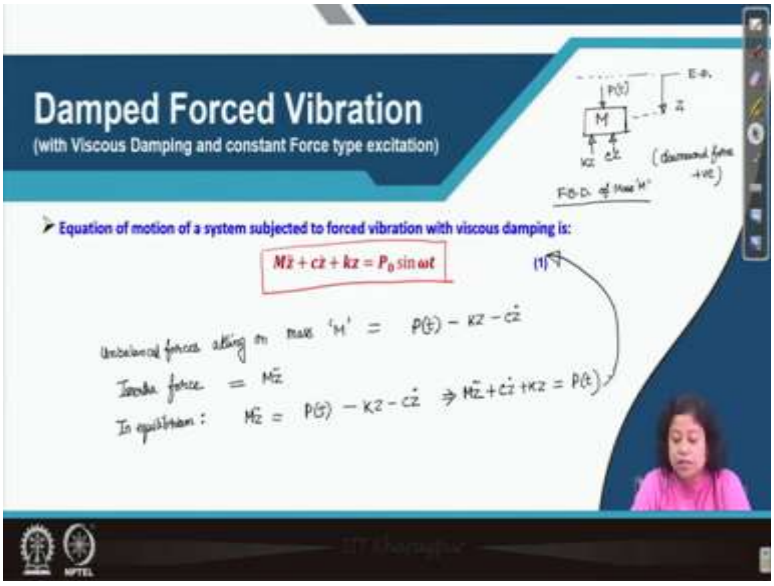

So, in this case first we will see the equation of motion. Here you can see the equation of motion the question you may think or ask that how this equation is coming. So, let us first draw the free body diagram for the mass M, mass M means which represents the foundation so it is resting on the soil.

Now, let us take this is the equilibrium position of the mass, so I am writing equilibrium position by E P. Now, at some time when force is applied to the mass M external force P t at some time t, its position is here, so it is going downward direction this time and sign conventions, as per sign convention which we have followed since beginning downward force means positive, I am writing here if it is upward, then it will be negative.

So, when it is going downward direction because of P t what are the forces or reactions coming from the spring and the dashboard to the mass M? Spring exerts a force of magnitude KZ whereas dashboard exerts a force with magnitude C times Z dot. Now, if we see this free body diagram I am writing here this is the free body diagram of mass M.

Now, if you see what are the unbalanced forces acting on this mass? I am writing here unbalanced forces acting on mass M is or R is unbalanced forces acting on mass in are P0 sin omega t or I can write actually it is better if I write P t, so P t minus KZ minus C Z dot, KZ spring force C Z dot force from the dashboard and P t is the external force acting on the mass M.

Now, what is the inertia force in this case? M Z two dot. So, inertia force should be equal to the unbalanced force, so I can write here, in equilibrium plus or I can I jump to one step better I should write step wise, M Z two dot which is inertia force is equal to the unbalanced force acting in the system.

So, now we can get the general equation of motion which says M Z two dot plus C Z dot plus KZ is equal to P t which is the external force, Z two dot is the acceleration, Z dot is velocity in this case. So, what I have written in equation 1 in this way we can derive. Now, we need to know the general solution for this equation 1.

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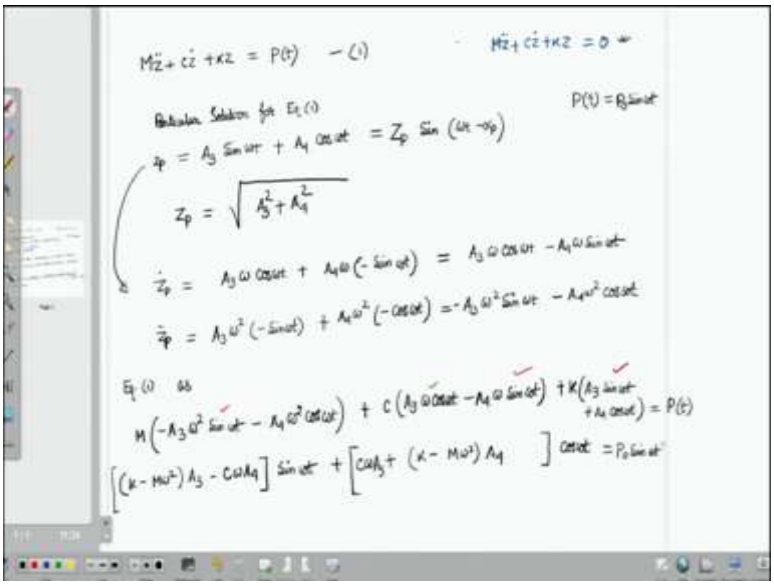

For that let us come to the board. So, first I am writing the equation M Z two dot plus C Z dot plus KZ is equal to P t this is the equation for which we are interested to find out the solution. Earlier we have determine the general solution for free vibrating system with viscous damping and the equation of motion for the free vibrating system was M Z 2 dot plus C Z dot plus KZ is equal to 0.

Now, what is the difference between equation 1 and the equation written on the right hand side? This equation is homogeneous equation whereas; the equation 1 has a particular solution because of the non-zero right hand side. So, in such case what we need to do, we will write first that, we will assume general solution, so in this case it is a particular solution for equation 1, so I am writing it as small zp, let me write it as small z, small zp is the particular solution for equation 1.

And we can write it as A3 sin omega t plus A4 cosine omega t or otherwise we can write it as capital ZP times sin of omega t minus alpha P, Alpha P is phase angle, capital ZP is the amplitude of vibration. So, if I am interested to write in terms of ZP, then I can write ZP also in that means capital ZP also in terms of A3 and A4. What will be that capital ZP is equal to square root of A3 square plus A4 square.

Now, what we need to do? We need to find out the final form. So, here we have assumed ZP which is sum of two periodic motion A3 times sin omega t which is a periodic harmonic motion, A4 cosine omega t is another harmonic motion and both have same frequency also. So, now from this that means this one we can write small zp dashed, small zp dash that means velocity component that is equal to A3 times omega times cosine omega t plus A4 times omega times minus sin omega t.

Then acceleration can be written as A3 times omega square times minus sin omega t plus A4 times omega square times minus cosine omega t. I can rewrite these two once again here, so A3 omega cosine omega t minus A4 omega sin omega t, likewise this one ZP double dot I can write as A3 omega square with a negative sign A3 omega square sin omega t minus A4 omega square cosine omega t.

Now, I will use these Z dot P and Z double dot P in equation 1. So, we can rewrite equation 1 as M times minus A3 omega square sin omega t minus A4 omega square cosine omega t, sorry, plus C times A3 omega cosine omega t minus A4 omega sin omega t plus we have at third term KZ, so for KZ I can write here as K times ZP which is, here I can write it as particular solution for equation 1, it will probably look better.

So, now the third term on the left hand side can be written as K times A3 sin omega t plus A4 cosine omega t and that is equal to your P t. So, now what we will do here? If you see what was actually P t, P t is nothing but P0 times sin omega t. So, we can rewrite the expression which I have just written down once again like P t will be replaced by P0 sin omega t and all the components of sin omega t will be taken together and the, and all the coefficient component means coefficient, all the coefficient of sin omega t will be taken together and all the coefficient of the cosine omega t will be taken together on the left hand side. Whereas, in the right hand side we will write P t as P0 sin omega t.

So, let me do that, first I would like to mark the terms having sin omega t here this one and this one. So, now I can write it as K minus M omega square A3 sin omega t minus, better I write sin omega t at the end. In this case I can use another bracket minus in this case it is C omega times A4, now I can use whole multiplied with sin omega t plus same way I will write the coefficient of cosine omega t here.

So, the coefficient of cosine omega t that is multiplying with A3 that I am writing first so that is C omega this one then I can write the those coefficients which are multiplying with A4, that means now I will write plus K, here I have to write A3, so K minus M omega square A4 and entire thing is multiplying with cosine omega t. So, that is equal to P0 sin omega t.

So, from this we know we can easily write the coefficient of sin omega t on the left hand side is equal to the coefficient of sin omega t on the right hand side, that will give us one equation. And the second equation we will give, we will get by equating the coefficient of cosine omega t on left hand side and the right hand side. In this way, we will get two equations and we have two unknowns which are A3 and A4, so we will get finally A3 and A4 in terms of K capital M and C. So, let us do that.

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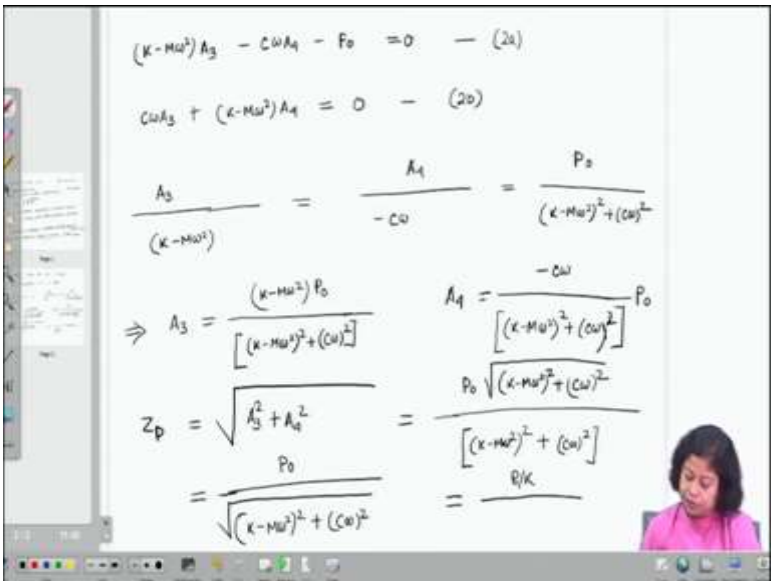

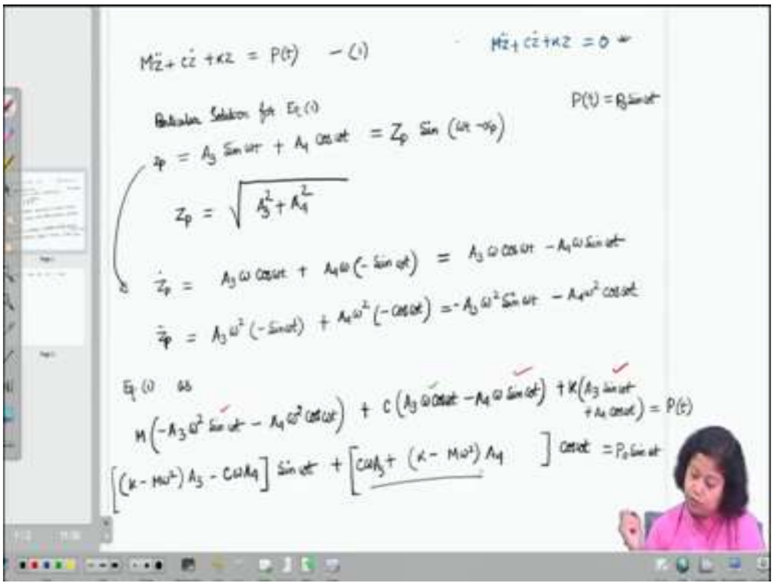

So, first one which we can write is K minus M omega square times A3 minus C omega A4 is equal to P0 or I can take P0 also on the left hand side in such case I will write it as minus P0 is equal to 0, give it a name 2a. Next equation, so first equation is formed, now next equation will be from this one.

So, C omega A3 plus K minus M omega square multiplying with A4, multiplying by A4 that is equal to 0. Let us give it 2b, so there are two equations and we have two unknowns. So, from these we can get A3 and A4 if we do the cross multiplication. So, easily we can do the cross multiplication and get the solution. So, what will be the final product then?

So, in this way we are getting A3 is equal to K minus M omega square times P0 divided by K minus M omega square whole square plus C omega whole square and A4 is equal to K minus, sorry it is not K, it is only minus C. So, minus C omega divided by K minus M omega square whole square plus C omega whole square this whole in multiplying with P0.

So, now we know what is A0, what is A3 and what is A4 in terms of K, M and C and P0. So, from these we can calculate capital ZP which is the amplitude of vibration we already know this is nothing but square root of A3 square plus A4 square. So, it will be your or we can write it as P0 divided by square root of this term. Now, if we take K out of these square root in the denominator, then we can rewrite the same expression that means capital ZP is equal to better I should write it in the next page.

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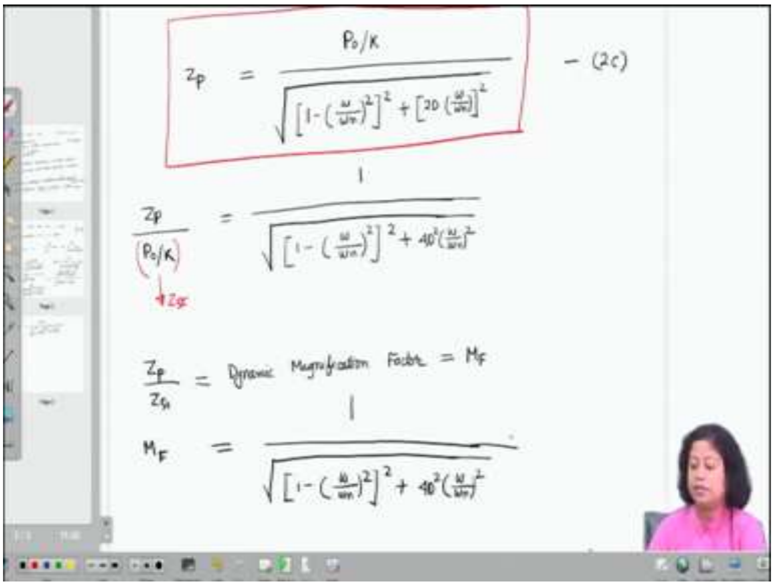

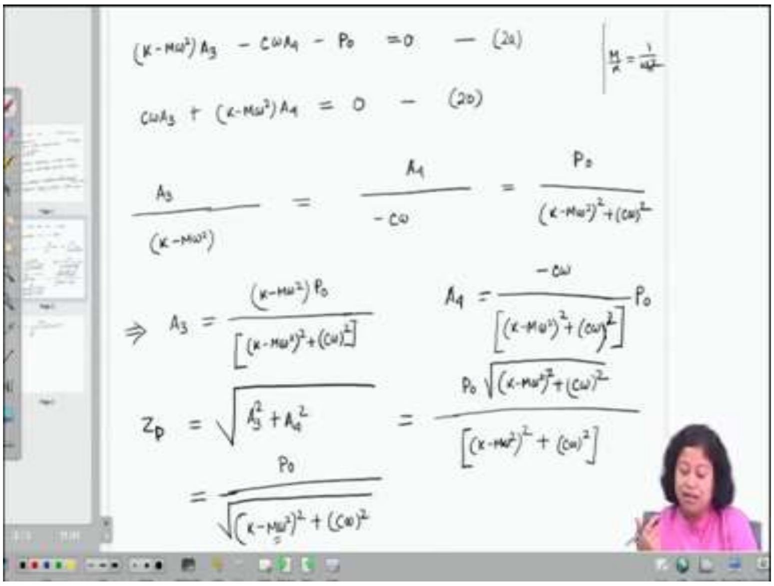

So, ZP, capital ZP can be written here as P0 by K times square root of I am just going back to the previous equation I will take K out of this square root that means the first term under square root will be 1 minus M by K omega square, M by K means 1 by omega n square, omega n is the natural frequency of the undamped system. So, basically what I am saying we will get this term which will be 1 minus omega n square in this place.

So, this is one way and then in the second term under square root will be C by K times omega. Now, C by K means we can use the damping ratio and can rewrite this term once again as C by K means we can write it as 2 D divided by omega n. So, let us write it in the next page.

So, what I said I am just trying to write here. So, finally we are getting, sorry so this is our expression, final expression for ZP. So, what is ZP? This is the amplitude of vibration. Now, P0 if the static load is applied to the system of magnitude P0 then what will be P0 by K? P0 by K will be static deflection of the system. So, now we can write whatever we write, written here if we give a number 2c, then we can write 2c as Z, capital ZP divided by P0 by K which is equal to square root of 1 minus omega by omega n whole square plus 4 D square times omega by omega n whole square.

Now, as I already said P0 by K is static displacement, so I can write this term as Z static, in this way ZP by Z St, ZP is the amplitude of dynamic deflection, whereas Z St is the static deflection so this ratio is called the dynamic magnification factor, the ratio of the dynamic deflection to the static deflection, maximum dynamic deflection in this case that means amplitude of course is called the dynamic magnification factor which can be represented by MF. So, MF is equal to finally 1 divided by square root of 1 minus omega by omega n whole square plus 4 D square divided by omega by omega n whole square.

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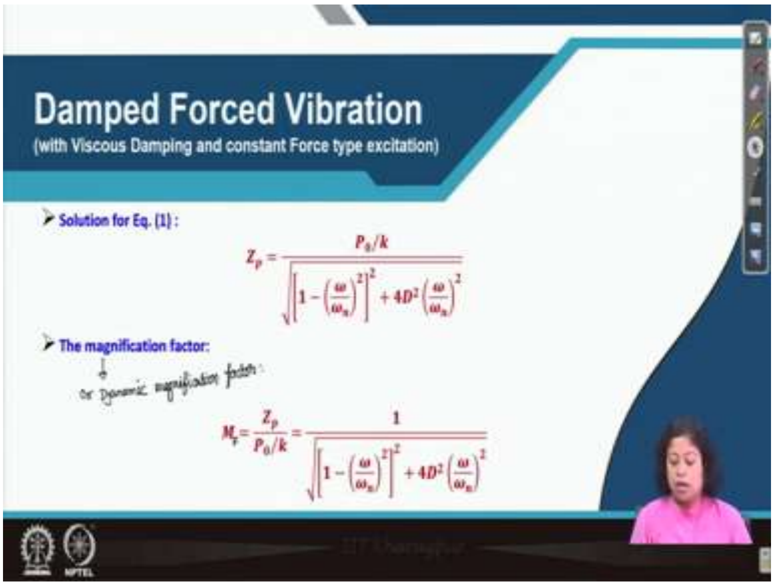

So, you can now see in this slide also we get the same thing, here it will be MF, magnification factor or sometime we call it as or dynamic term may be added, dynamic magnification factor.

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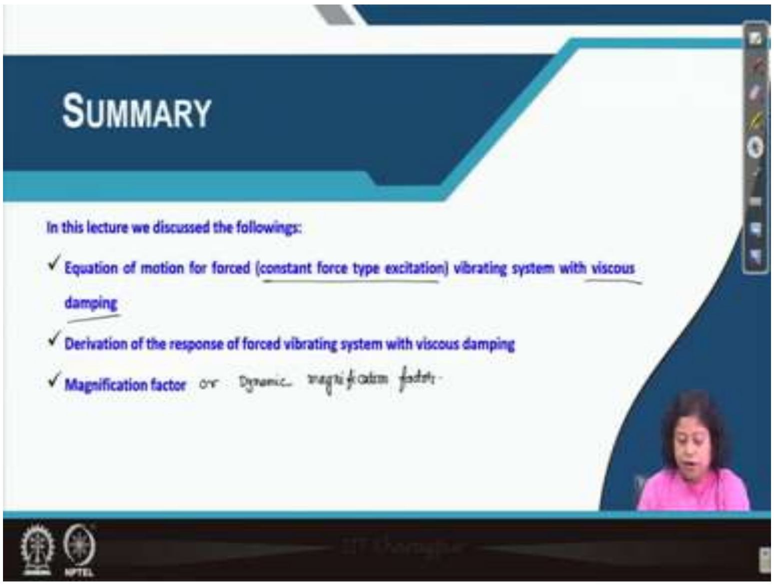

So, finally come to the summary today we discussed the equation of motion for forced vibrating system, forced vibrating system due to constant force type excitation considering viscous damping. Then what we have studied? Then we have studied how to derive the equation of motion for this type of forced vibrating system. Finally, we derived the response and from that the magnification factor or you can call it as dynamic magnification factor.

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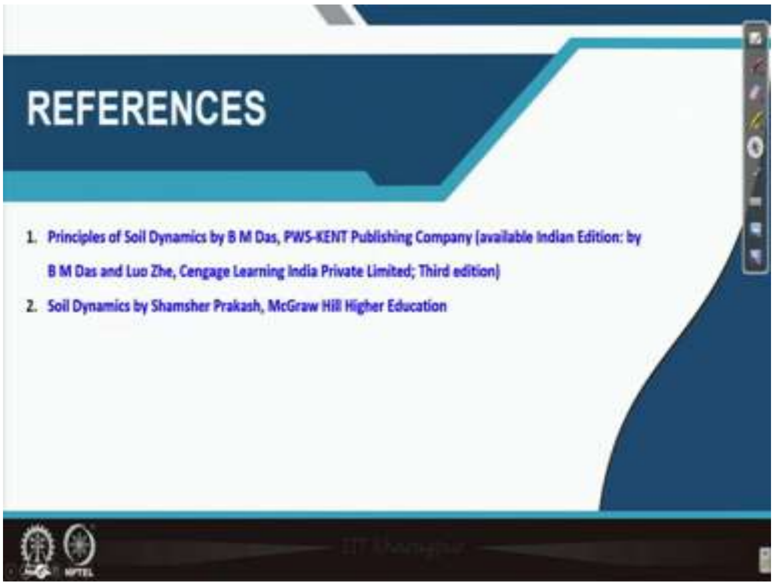

So, these are the references which I have used in this discussion. Thank you.