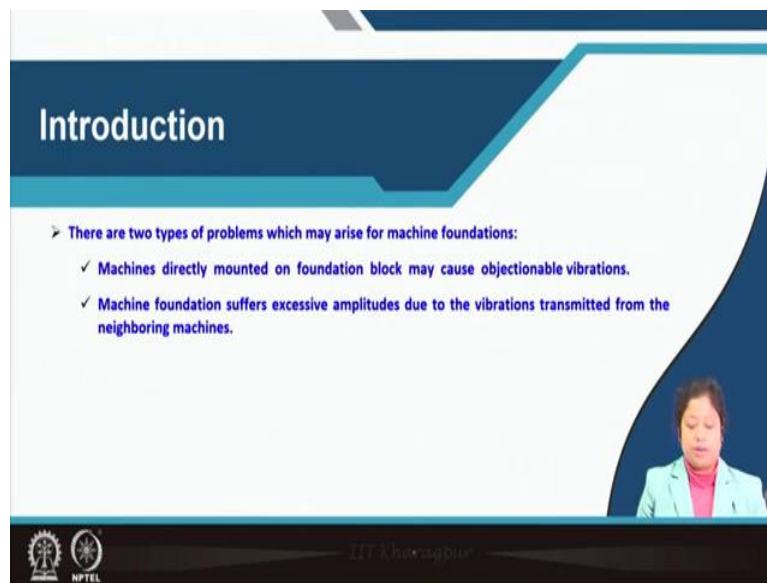


Solid Dynamics
Professor Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture - 56
Isolation of Vibration (Part 1)

Hello friends. Welcome to the course Soil Dynamics. Today is the first class on, isolation of vibration or we can call it as also vibration isolation. So, today, we will see what are the different types of vibration isolation that we can use and the mathematical background of those isolator or vibration screening.

(Refer Slide Time: 1:01)



So, basically, what is happen, there are two types of problems for which we need to provide, we need to provide isolator. So, what are those two types of problems that come from machine foundations? First one, machines directly mounted on foundation block may cause objectionable vibrations. So, that vibration can create trouble to the machine as well as to the foundation itself.

So, the second case of problem or second type of problem, you can see here machine foundation suffers excessive settlement or amplitudes due to the vibrations which is transmitted from the neighboring machines. So, suppose in a lab or in a factory there are two machines, which, one is both are probably a very precise equipment. Now, one machine suppose undergoes excessive amplitudes then what will be happen?

That may cause trouble for the other machines, which are kept in the factory or the laboratory. Probably the machines, the machine which causes excessive amplitude may be designed for that, so that mean that excessive amplitude of vibration may not create trouble to the concerned machine itself, but it can create trouble to the other machines kept in the factory or laboratory. So, we need to know what are the solutions for these two types of problems. Let us see the solutions then.

(Refer Slide Time: 3:06)

The slide is titled "Vibration Isolation" and features a diagram in the top right corner. The diagram shows two scenarios: on the left, a machine (M) is directly mounted on a foundation block (F), which is resting on soil (S); on the right, the machine (M) is mounted on a foundation block (F) that is supported by an isolator (I), which is then resting on the soil (S). Below the diagram, there is a list of bullet points:

- What is the solution of the problem where the machines directly mounted on foundation block may cause objectionable vibrations?
- ✓ Isolating the machine from the foundation through a suitably designed mounting system which reduces the magnitude of the transmitted force.
- ✓ This type of arrangement is also suitable to absorb the vibrations transmitted from adjacent machines.
- ✓ The effectiveness of isolation may be measured in terms of the force (type of isolation is called as **force isolation**) or motion (type of isolation is called as **motion isolation**) transmitted to the foundation.

In the bottom right corner of the slide, there is a small video inset showing a woman in a light blue jacket speaking. At the bottom left, there are logos for IIT Madras and NPTEL.

So, the first case what is the solution of the problem for the machines directly mounted on the foundation and causing objectionable vibration. Machine may be okay for that vibration but foundation may not be, that is also possible. So, for that foundation and soil, so for that case what do we do? Isolating the machine from the foundation through a suitably designed mounting system.

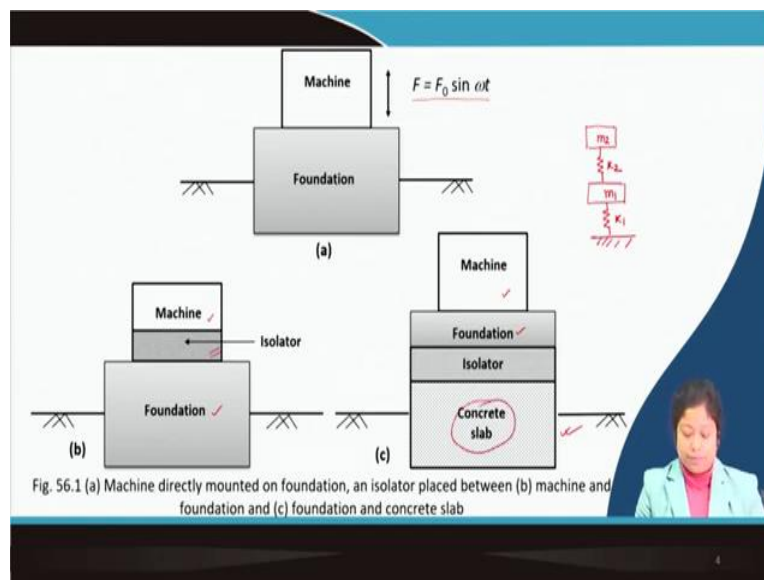
And that can reduce the magnitude of the transmitted force. So, if I will draw here, let us say this is machine, initially it is mounted on this foundation block which is resting on the ground surface. So, this is machine, this is foundation, this is soil. Now, if the machine causes excessive, as I said if the machine causes excessive amplitudes of vibration, then what we can do, we can put one isolator here, on the top of the foundation.

And then, so this is foundation, this is isolator and this is machine. In this way we can reduce the magnitude of the transmitted force to the soil foundation system. This type of arrangement is also suitable to absorb the vibrations, which are transmitted from the adjacent

machines. Now, the effectiveness of isolation can be measured in terms of the force or in terms of the motion of the ground or transmitted to the foundation.

So, in this case the, if we consider the effectiveness of isolation in terms of the force, then the type of isolation which is used is called as force isolation. Similarly, if the effectiveness is measured in terms of the motion, then the type of isolation is called as motion isolation. So, in short we can say that force isolation means it reduces the amplitude of the force or vibration in terms of force which transmitted to the soil foundation system whereas in case of motion isolation, it reduces the amplitudes of displacement of the ground surface.

(Refer Slide Time: 6:22)

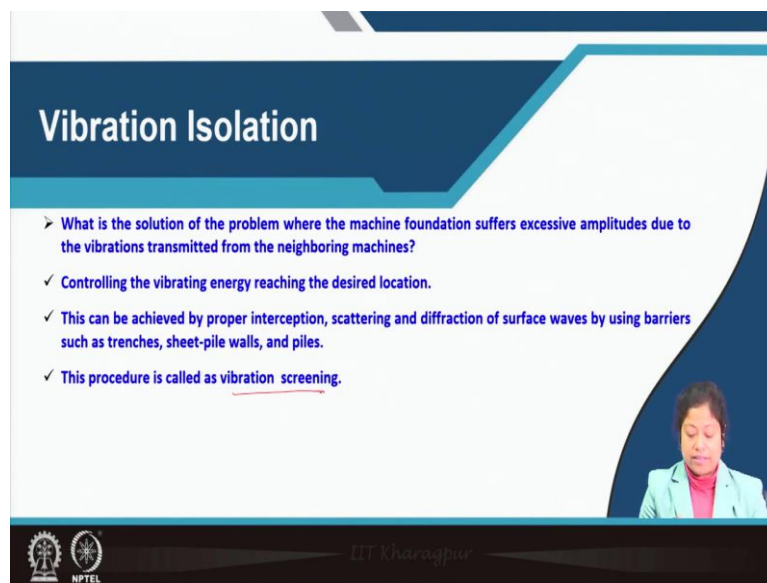


Now, which I have already drawn that you can see here, so machine is resting on the foundation and foundation is in this figure it is partly embedded to the soil, which is more realistic case. So, in this diagram you can see machine is producing dynamic force magnitude or I can say amplitude of dynamic force is F_0 and ω is its operating speed. So, in this case if we encounter excessive vibration then what we can do?

We can put isolator in between machine and foundation which can reduce the amplitude of vibration. Also, what we can do, instead of choosing option b, we can choose the, this option where isolated is provided in between foundation and the concrete slab. So, the difference is that generally we can model this type of problem by two degrees of freedom system where there are two masses and we need to represent the isolator and the soil.

If we are assuming soil is infinitely extended elastic medium, in that case we need to use stiffness for the soil as well, and that time m_1 and m_2 , we need to select carefully. If we are considering b, suppose this is m_2 connected to spring k_2 , which represents the isolator and m_1 , it is connected to spring k_1 represents the soil. Now for case b, m_2 includes the mass of the machine only where and m_1 represents mass of the foundation only. For c, m_2 represents the mass of the machine and the foundation together, whereas m_1 represents the mass of the concrete slab.

(Refer Slide Time: 9:02)



Vibration Isolation

- What is the solution of the problem where the machine foundation suffers excessive amplitudes due to the vibrations transmitted from the neighboring machines?
- ✓ Controlling the vibrating energy reaching the desired location.
- ✓ This can be achieved by proper interception, scattering and diffraction of surface waves by using barriers such as trenches, sheet-pile walls, and piles.
- ✓ This procedure is called as vibration screening.

NPTEL
IIT Kharagpur

Now, in vibration isolation what is the solution of the problem where the machine foundation suffers excessive settlement or amplitudes due to the vibrations transmitted from the neighboring machines. So, in that case solution is to control the vibrating energy reaching the desired location, so before reaching it to the desired location we need to control it.

So, for that what we can do? We can use proper interception scattering and diffraction of surface waves by using barriers such as trenches, sheet-pile walls, and piles. And this procedure is called as vibration screening. So, basically in this case isolator acts as a vibration screening.

(Refer Slide Time: 10:11)

Force Isolation

- Assume soil is infinitely extended elastic media.
- Equations of motion:

$$m_1 \ddot{z}_1 + k_1 z_1 + k_2 (z_1 - z_2) = 0 \quad \dots (1)$$

$$m_2 \ddot{z}_2 + k_2 (z_2 - z_1) = F_0 \sin \omega t \quad \dots (2)$$
- The principal natural frequencies of the system can be obtained by following equation:

$$\omega_n^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \omega_n^2 + \frac{k_1 k_2}{m_1 m_2} = 0 \quad \dots (3)$$
- The solutions of the equations shown above are:

$$z_1 = \frac{\left(\frac{k_2}{m_1 m_2} \right) \sin \omega t}{\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2}} \cdot F_0 \quad \dots (4)$$

$$z_2 = \frac{\left(\frac{k_1 + k_2}{m_1} - \omega^2 m_2 \right) \sin \omega t}{\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2}} \cdot F_0 \quad \dots (5)$$

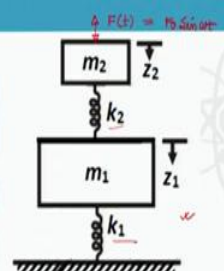


Fig. 56.2 Mathematical model of force isolation system shown in Fig. 56.1 (b)

So, let us consider the case of force isolation, that means in this case excessive force is generated in the machine and consequently, when we will measure the effectiveness of the isolator that should be done in terms of the force transmitted to the foundation. So, here you can see figure 56.2, so m_2 and m_1 , these two masses m_2 represents the mass of the machine, whereas m_1 represents the mass of the foundation. k_2 represents mass, sorry, stiffness of the isolator, whereas k_1 is the stiffness of the soil. Of course, in this case we are considering that the soil domain is infinitely extended and it is of elastic media.

So, if we will draw the free body diagrams for mass m_2 and m_1 , we know we will get these two equations of motion. So, here if you see the, in the equation to right hand side there is a term which is $F_0 \sin \omega t$, so basically this machine is under the vibration $F_0 \sin \omega t$, which is equal to $F_0 \sin \omega t$. Now, we have already seen how to get the solution. So, after solving these two equations what we can get?

We can find out the principle natural frequencies of the system, so for that we need to solve equation 3. After solving equation 3 we need to find out z_1 and z_2 , which are the displacement of the mass m_1 and m_2 respectively. So, z_1 is you can see in equation 4 and the solution for z_2 is shown in equation 5. So, here all the terms are known to us, so I am not explaining it.

(Refer Slide Time: 12:46)

• Force transferred to the foundation block or base slab:

$$F_T = k_1 z_1 = \frac{\left(\frac{k_1 k_2}{m_1 m_2}\right) \sin \omega t}{\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right) \omega^2 + \frac{k_1 k_2}{m_1 m_2}} \cdot F_0 \quad \dots (6)$$

• The transmissibility of the system will be:

$$\hat{T}_T = \frac{F_T}{F_0 \sin \omega t} = \frac{\left(\frac{k_1 k_2}{m_1 m_2}\right)}{\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right) \omega^2 + \frac{k_1 k_2}{m_1 m_2}}$$

The slide also features a small video inset of a woman in the bottom right corner and logos for IIT Kharagpur and NPTEL at the bottom.

Now the, what is, how do we measure the force that is transmitted to the foundation block or base lab? So, the force transmitted to the foundation block or base slab, if we call represented by F_T that is equal to k_1 times z_1 . So, if you put the expression for z_1 , which we have already seen, then F_T can be written like this. So, here ω is the operating speed of the machine, m_1 , m_2 known, k_1 k_2 known.

Now, after calculating f_t that means the force transmitted to the foundation we can measure the transmissibility of the system. So, the transmissibility of the system T_T is equal to F_T divided by F_0 times \sin of ωt . So, F_T is shown in equation 6 and, then if we divide equation 6 by $F_0 \sin \omega t$, what is left that you can see here in equation 7. Now if, so here you can see there are several terms. So, earlier we have seen how to take m_1 m_2 and k_1 k_2 .

(Refer Slide Time: 14:30)

Force Isolation

- Assume: $\frac{k_1}{m_1} = \frac{k_2}{m_2} = p^2$
- Also, mass ratio: $\eta_m = \frac{m_2}{m_1}$
- The natural frequency of the machine-foundation system ignoring the effect of the spring isolator is:

$$\omega_{nz} = \frac{k_1}{m_1 + m_2} = \frac{p^2}{(1 + \eta_m)}$$

Diagram: A mass m_1 is supported by a spring k_1 . A second mass m_2 is shown above it, with a note "Total mass = $m_1 + m_2$ ".

Equation derivation: $\frac{k_1}{m_1 + m_2} = \frac{k_1/m_1}{1 + m_2/m_1}$

Logos: IIT Kharagpur, NPTEL

So, the similar kind of exercise now we will do here. So, first thing we can assume k_1/m_1 is equal to k_2/m_2 and that is equal to p^2 , that means p is equal to square root of k_1/m_1 or square root of k_2/m_2 . Here what is assumed the ratio of the stiffness to mass is same for the, I mean, k_1/m_1 is equal to k_2/m_2 . Also, we need to assume that the ratio of the two masses that means m_2/m_1 is equal to η_m .

Now, with this what we can do? We can, the natural frequency of the machine-foundation system ignoring the effect of the spring isolator can be written. So, that means we are not considering the isolator, we are just considering this is machine, which is resting on the foundation and foundation is resting on the soil. That means for soil we are taking k_1 . So, now in this case total mass is how much?

This is total mass means in this case it is $m_1 + m_2$. So, from this we can find out the natural frequency as shown here. Already we have assumed that k_1/m_1 is equal to p^2 . So, $k_1/m_1 = p^2$ and m_1/m_2 is equal to, sorry, yes, m_1/m_2 is equal to $1/\eta_m$. So, basically if this expression I can write like square root of k_1 divided by $m_1 + m_2$ divided by m_1 . Now, here you can see m_2/m_1 means η_m and k_1/m_1 is equal to p^2 , so that from this we can get this final form. So, let us give it a number 8.

(Refer Slide Time: 17:25)

• For no isolating system, the amplitude of the machine foundation system is:

$$A_z = \frac{F_0}{(m_1 + m_2)(\omega_{nz}^2 - \omega^2)} \quad \dots (9)$$

• If the force F_0 is frequency dependent, i.e. $F_0 = 2m_r e \omega^2$, then

$$A_z = \frac{2m_r e \omega^2}{(m_1 + m_2)(\omega_{nz}^2 - \omega^2)} = \frac{2m_r e \omega^2}{m_1 \left(1 + \frac{m_2}{m_1}\right) \omega^2 \left[\left(\frac{\omega_{nz}}{\omega}\right)^2 - 1\right]} = \frac{2m_r e}{m_1(1 + \eta_m)(\alpha_z^2 - 1)} \quad \dots (10)$$

where, $\eta_m = \frac{m_2}{m_1}$ and $\alpha_z = \frac{\omega_{nz}}{\omega}$

• Assume, ω_{nz} represents the natural frequency of mass m_2 resting on isolating spring.

$$\text{Therefore, } \omega_{nz} = \sqrt{\frac{k_2}{m_2}} \quad \dots (11)$$

Now, for no isolation system that means once again the machine is directly resting on the foundation or placed on the foundation. So, that time what will be the amplitude of machine, amplitude of the displacement for the machine foundation system, A_z better, I should write amplitude of displacement or vibration of the machine foundation system. Then it is already we know A_z equal to F_0 divided by m times ω_{nz} square or...

Sorry, F_0 divided by m times ω_{nz} square minus ω square. So, ω_{nz} we have already calculated which is equal to square root of k_1 divided by m_1 plus m_2 . Now, F_0 is known and m_1 plus m_2 means total mass, so we can find out A_z for the no isolating system. And some cases A_z is, A_z may exceed the permissible value of the settlement. So, in this case if F_0 is frequency dependent then we know how to represent F_0 , so that is done here.

So, finally we can then write A_z is equal to 2 times of m_r times e times ω square divided by mass times ω_{nz} square minus ω square. Now, here we can write this equation, so one step I am writing for you, so here I will take m_1 out, then it is 1 plus m_2 divided by m_1 , likewise I can take ω square out of this bracket, so it is now ω_{nz} by ω square minus 1.

So, if we will consider ω_{nz} by ω is equal to α_z , that means I can write here A_z is equal to ω_{nz} by ω . Actually, I think it is already written in the next line. Yes, it is already written. So, I am, so here you can see, now ω square is present in numerator and

denominator and this is non-zero quantity, so I can write it, both can cancel each other and now for m2 by m1 we can write eta m that is mentioned here.

And for omega nz divided by omega we can write this one. Then the equation 9 can be represented by equation 10, which is shown here. Now, here if we assume omega na represents the natural frequency of the mass m2, which is resting on the isolating spring. Then what we can write for omega na? Omega na then is equal to square root of k2 divided by m2. You can see here in equation 11.

(Refer Slide Time: 21:53)

Then, amplitudes of two-degree-freedom system for frequency dependent force is:

$$A_{z1} = \frac{\omega_{na}^2 (2m_r e \omega^2)}{m_1 [\omega^4 - (1 + \eta_m)(\omega_{na}^2 + \omega_{nz}^2)\omega^2 + (1 + \eta_m)(\omega_{na}^2)(\omega_{nz}^2)]} \quad \dots (12)$$

Or, assuming $\alpha_\omega = \frac{\omega_{na}}{\omega}$ it can be written as:

$$A_{z1} = \frac{a_\omega^2 (2m_r e)}{m_1 [1 - (1 + \eta_m)(\alpha_\omega^2 + \alpha_\omega^2) + (1 + \eta_m)\alpha_\omega^2 \alpha_\omega^2]} \quad \dots (13)$$

$$A_{z2} = \frac{[(1 + \eta_m)\alpha_\omega^2 + \eta_m \alpha_\omega^2 - 1](2m_r e)}{m_2 [1 - (1 + \eta_m)(\alpha_\omega^2 + \alpha_\omega^2) + (1 + \eta_m)\alpha_\omega^2 \alpha_\omega^2]} \quad \dots (14)$$

Finally, the efficiency of isolation system is: ξ

$$\xi = \frac{A_{z1}}{A_z} = \frac{a_\omega^2 (1 + \eta_m)(\alpha_\omega^2 - 1)}{[1 - (1 + \eta_m)(\alpha_\omega^2 + \alpha_\omega^2) + (1 + \eta_m)\alpha_\omega^2 \alpha_\omega^2]} \quad \dots (15)$$

Total force on the isolator, $F_s = k_s \cdot A_{z2}$... (16)

So, then the amplitudes of the two-degree freedom system for frequency dependent force, what we can write its amplitude is equal to this one. Here what is omega na and what is omega nz that we have already seen and what is eta m that is also known now. So, after knowing the expression for Az1, what we can do, we can now assume this omega na divided by omega is equal to alpha omega.

Now, here you can see there is one omega na and also you can see, here also there is another omega na, so this one and this one. So, what we need to do? If we will divide it by omega square then what is the, then we can get this expression. So, basically what we are doing, we can take in, from this expression we can, from the denominator if we can take this omega to the power 4 out of this bracket, then here we can write omega na square divided by omega square and here also we can write omega na square divided by omega square.

And since ω to the power 4 is out of this bracket and if you see the numerator, there is one ω square. So, what we will get in the denominator, they are is a term ω square, so actually what I can do, I can just write one step here itself, so what I am saying here, I am taking ω to the power 4 out of this bracket, then what is left that I am trying to write. So, here I can write it as ω_n divided by ω square times 2 m .

And here I can then write m times $1 - \eta$. Now I have taken ω to the power 4 out of this bracket, that means here there is ω square, so I can write this expression as ω_n square by ω square plus ω_n square divided by ω square plus $1 - \eta$. Here I can write ω_n by ω square times ω_n by ω square.

Now, I think you can, it is easy, now I think it is easy to follow, so here ω_n by ω that is equal to a ω that means this is a ω square whereas ω_n divided by ω is A_z , so this is A_z square. So, finally from this we can get this expression, which is written in equation 13 and for A_{z2} , okay, it is better. I just delete this line. Now for A_{z2} we can, we can express A_{z2} in terms of which is written here in equation 14.

So, after doing A_{z1} and A_{z2} , finally we need to find out the efficiency of isolation system which is ξ here and that is the ratio of A_{z1} to A_z . A_{z1} is the amplitude of displacement for the foundation and A_z is the amplitude of displacement when there is no isolation in between machine and the foundation.

So, A_{z1} when there is an isolation in between machine and foundation and A_z is the case or the amplitude of displacement when there is no such isolation. So, the ratio of these two amplitudes is the efficiency of isolation system and that can be expressed as shown here in equation 15. Now, generally what is happen? Generally, the permissible amplitude is provided, it may be 0.2 millimeter or 0.25 millimeter, that is mentioned.

And for the system of without isolation you can calculate ω_n and from that you can calculate A_z . So, when you know A_{z1} , when you know A_z , you can calculate this efficiency and then you need to write equation 15, where what is the unknown, the unknown quantity is a ω , which is the ratio of ω_n divided by ω . Now, what is ω_n ? ω_n is square root of k by m .

That means the natural frequency if we think it or I should not use the what natural frequency, so a ω is the, ratio of ω_n divided by ω and ω_n is square root of k_2 by m_2 . So, it depends upon the mass of the machine and the stiffness of the isolator. So, if you know this left-hand side, then using equation 15 you can find out a ω because A_z is known. What is A_z here? ω_{nz} divided by ω .

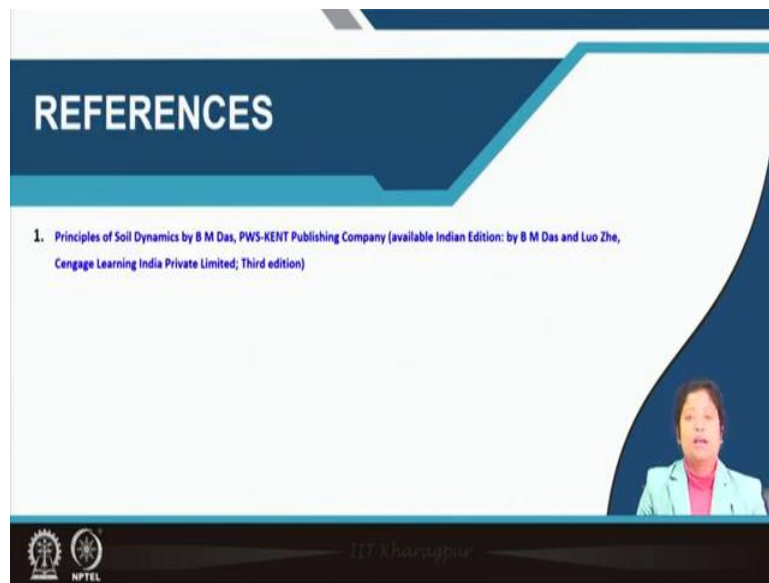
η_m also known, which is m_2 divided by m_1 . Only unknown parameter is A_ω which you can calculate from equation 15. And after calculating a ω from that ω_n and from that k_2 what you can find out, you can find out the total force that the isolator can take, which is this one k_2 times A_z^2 . So, in this way we can, this is the maximum force that the isolator can take. So, in this way we can design the isolator.

(Refer Slide Time: 30:36)



So, come to the summary of today's class. So, in this class we have discussed different types of vibration isolation, what is force isolation, what is motion isolation, that we have discussed. Then we have discussed the mathematical modeling for force isolation and how to express its efficiency and the settlement, etcetera.

(Refer Slide Time: 31:08)



This is the textbook which can be referred to find out the different mathematical form, for an example, Az1, Az2 or how to solve the 2 degrees of freedom system, etcetera. So, with this I am ending today's class, thank you.