Soil Dynamics
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Lecture 55
Analysis of Pile Foundation Under Dynamic Loading (Part – V)

Hello friends, welcome to the course Soil Dynamics. So, today is the fifth class on analysis of pile foundation under dynamic loading. So, far in this analysis of Pile foundation, we have discussed how to find out the stiffness and damping constants for the single pile or pile any group and pile in a group with pile cap total stiffness and total damping constant when pile group is subjected to vertical vibrations subjected to torsional vibration subjected to horizontal sliding vibration and rocking vibrations.

And also, we have discussed a few numerical problems related to determination of stiffness and damping constant for that single pile when it is subjected to only vertical vibration and when it is subjected to only torsional vibration.

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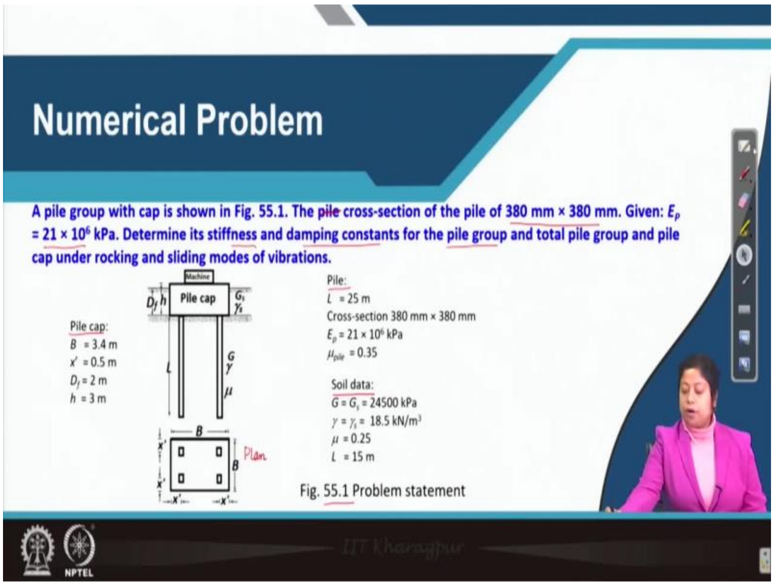

So, today's class we will discuss a numerical problem related to the pile group subjected to sliding and rocking vibrations. So, here you can see the problem statement, it is said that a pile group we kept is given or shown in figure 55.1. So, what we can see in this figure, this is the plan view. So, from the plan view, we can see that there are 4 piles in this pile group. The pile cross section or you can I think it is the cross section of the pile that means single pile is 380 millimeter by 380 millimeter, what is given E P that is 21 times 10 to the power 6 kPa that means, modulus of elasticity for the pile material is provided.

You are asked to find out the stiffness damping constant for the pile group and the total pile group and pile cap under rocking and sliding modes of vibrations. And you can see a lot of informations related to the pile, related to the pile cap and soil data are provided here. So, we need to find out the stiffness and damping constant for this problem considering the pile group, then we will find out these two parameters for the pile cap and then we will find out the total stiffness and damping constant for the pile group and pile cap together alright. So, let us solve this numerical problem now.

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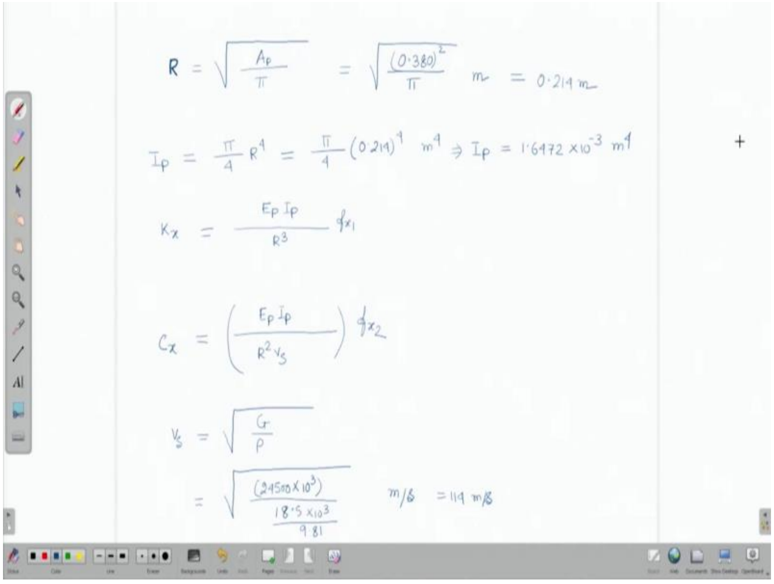

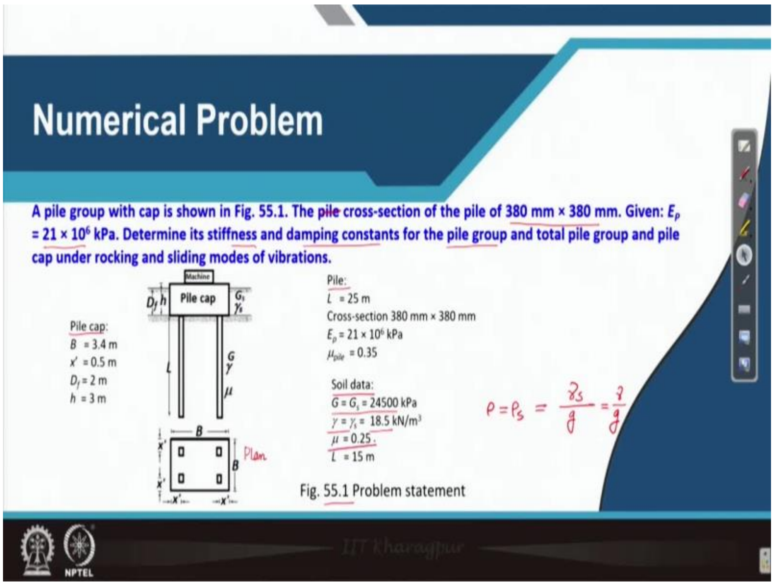

So, the data feature is given first time trying to write that cross sectional area is provided. If I go back to the figure, you can see the individual pile is its cross section is 380 millimeter by

380 millimeter that means, it is cross section is square in shape. So, we need to find out the equivalent radius for this problem.

So, R is that equivalent radius which is equal to cross sectional area of the pile. So, I can write it Ap divided by pi. So, that means, it is equal to 0.380 square because the cross sectional area of the pile is the square here divided by pi it is in meters. So, finally, we will get R is equal to 0.214 meter. So, this is the equivalent radius for the single pile.

Now, we can find out Ip which is the moment of inertia for the circular cross section. So, pi by 4 times R to the power 4 is the cross equation to find out Ip. So, this is equal to pi by 4 R is 0.214 to the power 4 and it is unit is meter to the power 4. So, here we will get Ip is equal to then from these Ip is equal to 1.6472 into 10 to the power minus 3 in meter to the power 4. Alright?

Next is to find out the K theta sorry Kx Cx that means, now, we will consider that the pile is subject or pile group is subjected to sliding vibrations. So, for that Kx is equal to E P times Ip divided by R cube times if x1. Similarly, we can find out Cx which is this (()) (06:59) I can write this little bit below. So, Cx is equal to E P times Ip divided by R square times Vs whole this thing multiplied with a fx2.

So, what is Vs here? Here Vs is the velocity of the shear wave in the soil. So, we have to calculate the Vs so, Vs is equal to square root of G by rho, rho is the density of the soil and G is the shear modulus of soil. So, let us see what is shear modulus of the soil here it is given 24500 in kPa and rho is and rho sorry gamma and gamma s both are equal to 18.5 kilo Newton per meter cube.

So, from this we can calculate the value for rho or n rho is that if gamma s divided by g or we can also write gamma divided by g. So, let us find out then what is the value for Vs. So, here V is is equal to G is 24,500 kPa so, 24,500 kPa times 10 to the power 3. So, now it is Newton per square meter divided by 18.5 into 10 to the power 3 now, it is Newton per meter cube divided by 9.81 so, then the unit for Vs is meter per second.

So, with these we can get Vs is equal to 114 meter per second. So, Vs is known R is known. Now, what we need to find out is fx1 and fx2 if you recall in last class, we have seen how to calculate fx1 and fx2 our table was shown and in that table fx1 and fx2 depend upon Ep divided by g and it also depends upon mu value right.

So, in this problem already mu is mention that is 0.25. Now we need to calculate Ep divided by G.

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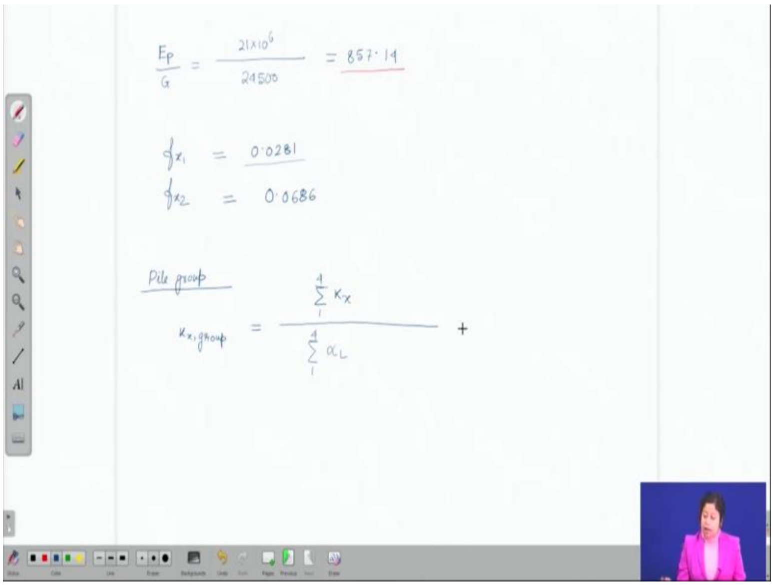

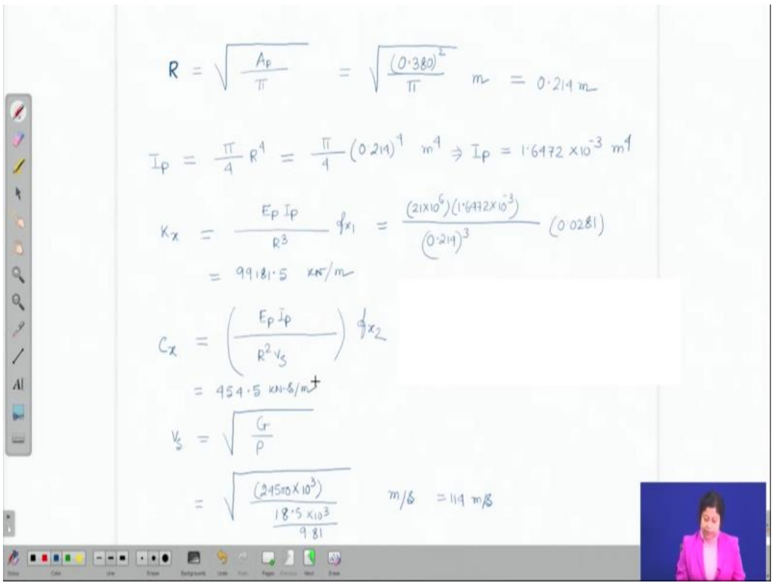

So, let us do this, here Ep divided by G names 21 into 10 to the power 6 divided by 24,500. So, it is coming approximately 857.14. So, now, if we will find out fx1 and fx2 I have done it by linear interpolation the good idea is to plot the fx1 and a fx2 for different values of Ep by G ratio and from that you can find out for the value of this one 857.14 what is the value of fx1 and fx2.

So, approximately fx1 and fx2 were which I am getting R 0.0281 and fx2 0.0686. So, with these values of fx1 and fx2 we can now find out Kx and Cx. So, in this equation, Ep is 21 into

10 to the power 6, Ip is 1.6472 into 10 to the power minus 3, R means 0.214 cube and fx1 that we have already calculated here point 0.0281.

So, this will give us the value of Kx so, let me get it so, it is coming approximately equal Kx if you see it is coming approximately equal to 9, I am writing here 99181.5 and unit is kilo Newton per meter. So, here if you find out a fx1 plotting the curve as I said then probably the value of a fx1 may change and this value of Kx may slightly change. Now, we can find out Cx also. So, we are getting here for 54.5 in kilo Newton second per meter. So, now, we have calculated Kx and Cx.

Next, this is for single pile. Now, we need to find out it for the pile group. If I go back to the figure here you can see there are 4 piles and that equation which we need to use for pile group that I am now writing we are considering pile group for that we can find out Kx for the group which is equal to summation of Kx for there are 4 piles so, 1 to 4 Kx divided by summation of alpha L 1 to 4. So, what is alpha L here interaction factor. So, now, we need to find out actually alpha L.

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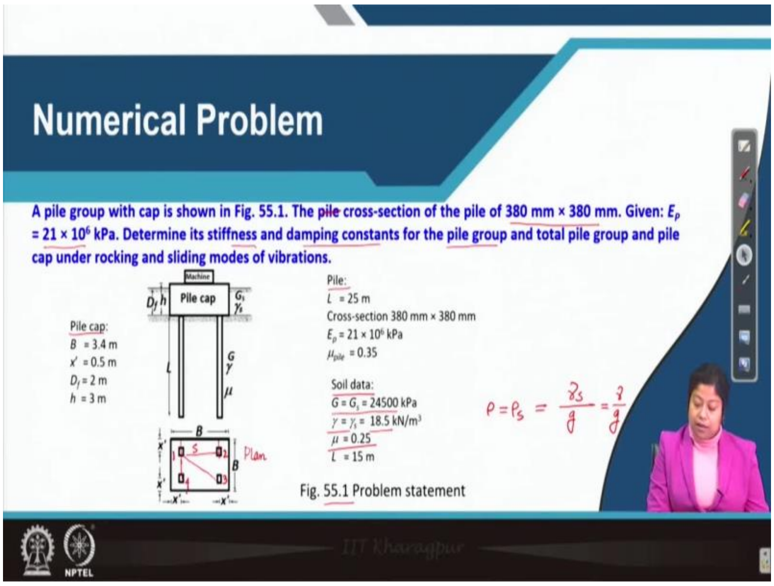

For that we need to also find out S divided by 2R this ratio from the given problem, so, here what we can see you can see here, what is the spacing between two piles, since it is a square shape pile cap. So, these things center to center distance is so, these we will measure the distance and that is S and already the value for R is known. So, from that we can calculate the S divided by 2R value. So, this is for let us give some number for the 44 different piles.

So, this is let us take pile 1, this is 2, this is 3 and this is 4. So, S for 1 to S in between 1 and 2 is marked in this figure what about these distance here from these you can find out this distance likewise from this you can find out the other distance. Now, the we need to calculate the S divided by 2R.

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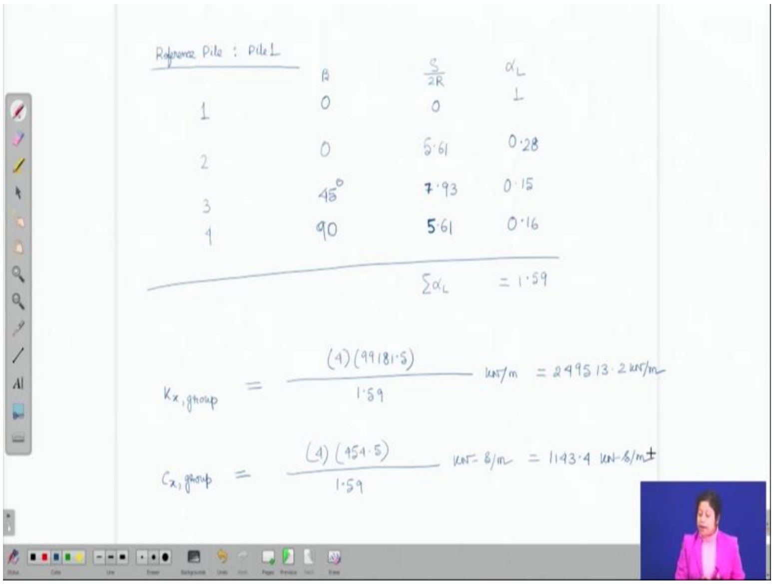

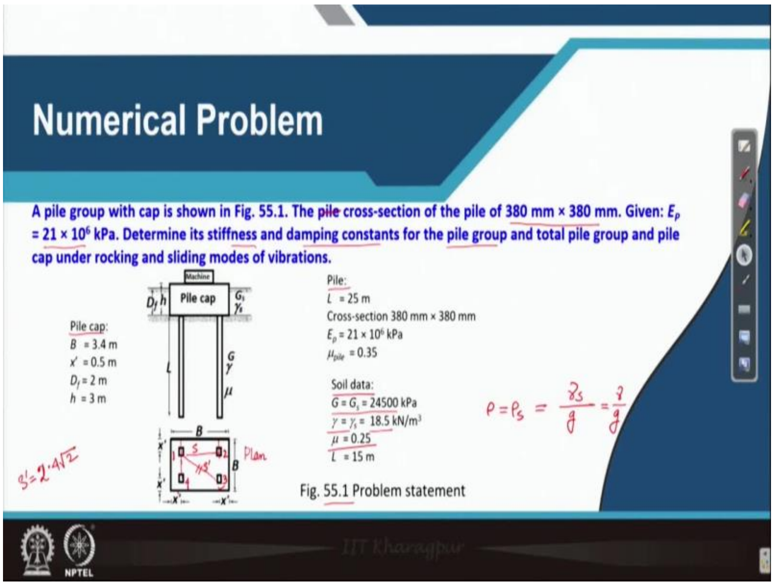

So, I am going to the new page here reference pile let us take a reference pile is pile 1 pile number 1. So, whatever we will calculate that is to find out alpha L that is with reference to pile number 1. Now, for what are the different points for which we need to find out alpha L for pile 1, 2, 3 and 4 and it depends upon beta it depends upon SS divided by 2R it means the value of alpha L.

So, first we will write the beta and S divided by 2R for different pile and from that we will find out alpha L. So, this is the figure which we can now use. So, just let me calculate all right, so. So, now, I am telling you how I am calculating as I shown S for pile 1 with reference to pile 1 is 0. So, this is 0 beta, which is the angle this is also 0 in this case.

So, if you refer the figure, which I have already shown in the last class, then you will find out alpha L is equal to 1 when beta and S divided by 2R both are 0. Now, for the second pile, what is the angle beta it is again 0 and what is the value of S divided by 2R, in this case, S is 2.4 meter center to center distance and then divided by 2R R we have already calculated. So, I am getting these approximately is equal to 5.61 and angle here also it is 0 degrees. So, we this beta value and S divided by 2R we can find out the value of alpha L which is equal to approximately 0.28.

Now, for third pile that means we prefer this one with reference to pile Number 1, what is the value for S is or you can take a S prime. So, basically these distance if you see it is 2.4 times root 2 that is S dash, then you can find out these divided by 2 divided by R, R is 0.214 so, you will get 7.93 in this case. And what is the angle beta here? Here beta is equal to 45 degree. So, now, for this beta value and S by 2R we can find out if alpha L value which is close to 0.15.

Now, the fourth file in this case you can see beta is equal to 90 degree and S divided by 2R is equal to S divided by 2R for pile 2 also. So, this is 5.61, this is 90. Now, we have this value of beta and S divided by 2R we can get alpha L which is approximately 0.16. Now, what we can do we can find out summation of alpha L.

So, it is coming 1 plus 0.28 plus 0.15 plus 0.16. So, we are getting it is equal to 1.59. So, in this way we can find out summation of alpha L considering the considering all the piles in the pile group. Now, after finding out summation of alpha L you can see here already we have calculated Kx for individual pile. So, we can find out now Kx for the ile group and that is equal to now, this is 4 times of 99181.5 divided by 1.59. So, it is coming approximately 2249513.2 unit is kilo Newton per meter.

The same way we can find out Cx group as well we have already seen the equation, I have not written. So, the equation for Cx group also equal to summation of Cx for all 4 piles divided by summation of alpha L. So, then it is 4 times summation of Cx and let us see 454.5 divided by 1.5 time and unit is kilo Newton’s second per meter. So, the value is approximately I can find out 4 to 454.5 divided by 1.59. So, the value is approximately equal to 1143.4 in kilo Newton second per meter. So, in this way we can find out Kx group and Cx group. Next is to find out Kx for the pile cap and Cx for the pile cap.

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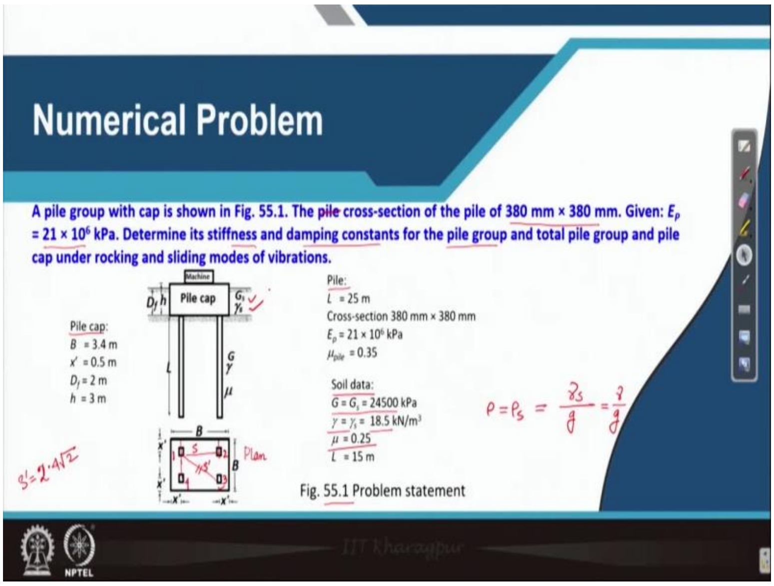

So, for that once again I am going back to this figure. So, here what we can see pile cap is embedded in the soil or I can say after placing pile cap, backfilling is done by the soil and you can see the gamma S and Gs value for the backfill soil.

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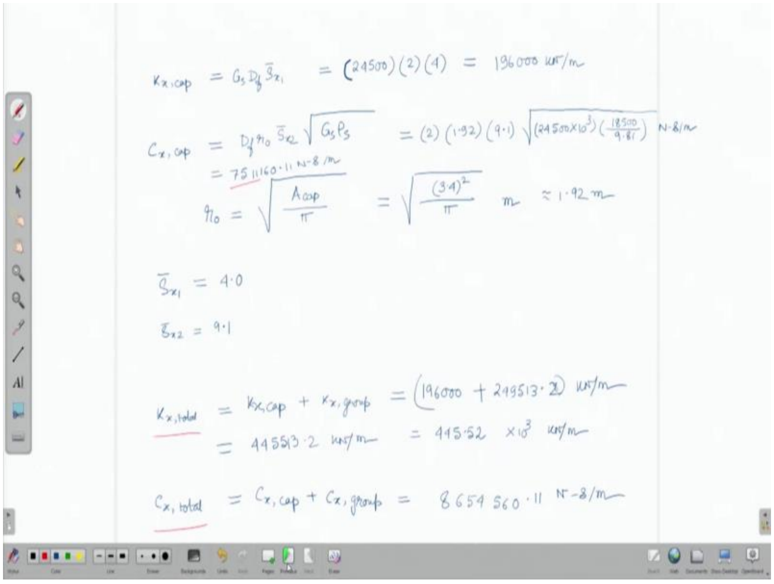

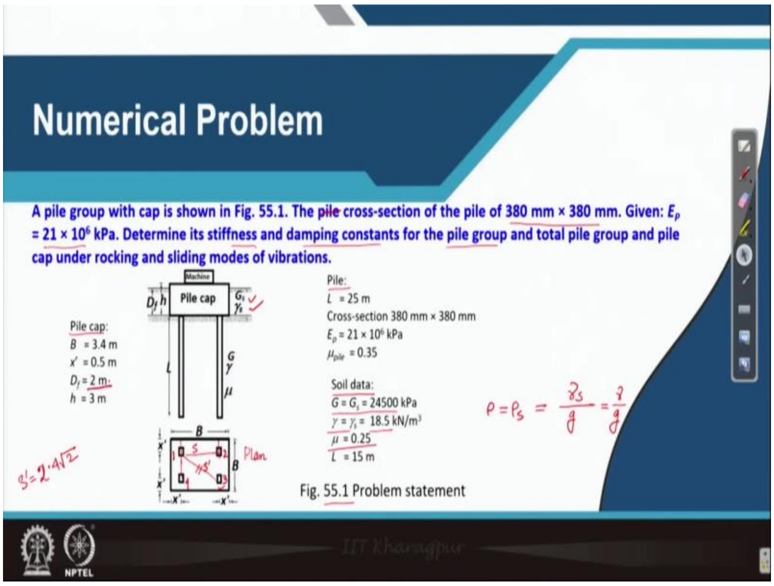

So, from these now, from with these information now, we will find out Kx for pile cap and for that we can use the equation Gs times Df times S x1 or S bar x1 and for Cx cap this is equal to Df times r0 times S bar x2 times square root of Gs times rho s, so, Gs and rho s are shear modulus dynamic shear modulus and rho s is density of the backfill soil respectively. So, and Df refer the figure here Df is said 2 meter.

So, and what is r0 now, this is important. So, r0 is that equivalent radius of the pile cap. So, once again we have a pile cap of square cross sectional area, what is the cross sectional area for this pile cap 3.4 meter by 3.4 meter if you see this data, so, r0 is equal to cross sectional area of the pile cap divided by pi which is equal to 3.4 square divided by pi and this is in meter.

So, we are getting 3.4 square divided by pi it is approximately 1.92 meter alright. So, in this way we can find out the equivalent radius r0. Now, after knowing r0 we can also find out a fx1 and fx2 from the table that that is already shown when we discuss the theory. So, what is the value of fx1 and fx2 that we can find out from that table. So, I am just writing here S bar x1, not S x1 S x2, but it is S bar x1 S bar x2.

So, S bar x1 is equal to 4 whereas, S bar x2 is equal to 9.1. So, with this information now, we can find out Kx and Gs is already given. So, 24,500 times 2 times r0 means, sorry S bar x1 means 4. So, we are getting 196,000 kilo Newton per meter, now, we can find out also Cx cap, so, Df is 2 meter, r0 1.92 meter, S bar x2 is 9.1 and that multiplied with Gs which is 24500 times rho s. Now, Gs value of what I have written here that is in, if you take that is in kilo Newton per square meter so, I am converting at Newton per square meter being rho s rho s is this is in Newton per meter cube divided by 9.81. So, now it is a Newton's second per meter.

So, how much you are getting for Cx cap? For Cx cap we are getting 7511160.11 Newton’s second per meter. Alright, so, now we know the values for Kx cap and Cx cap also we know the values for Kx in Group Cx for the pile group. So, the Kx total is equal to Kx cap plus Kx group and how much then it is coming that we need to check now, it is coming Kx cap if you see the value is already given.

So, I can write the value for Kx cap that is 196000 plus here you can see the group 249513 this is in kilo Newton per meter point 2. So, this is how much it is coming approximately it is coming approximately, 44555513.2 in kilo Newton per meter or I can write it as 445.513 or 5 approximately 52 also I can write in kilo Newton per meter into 10 to the power 3 I have to write into the into the 3 kilo Newton per meter.

Similarly, we can find out Cx total is equal to Cx for the cap plus Cx for the group Cx for the gap already shown here you can see you can take this value and the Cx for group is this one. So, I am trying to write it directly this is in kilo newton second plus this is a Newton’s second 7511160.11. So, total we are getting 8654560.11 in Newton second per meter. So, in this way we can calculate Kx total and CX total. Now, this is for the case when we are considering pile group is subjected to sliding vibration.

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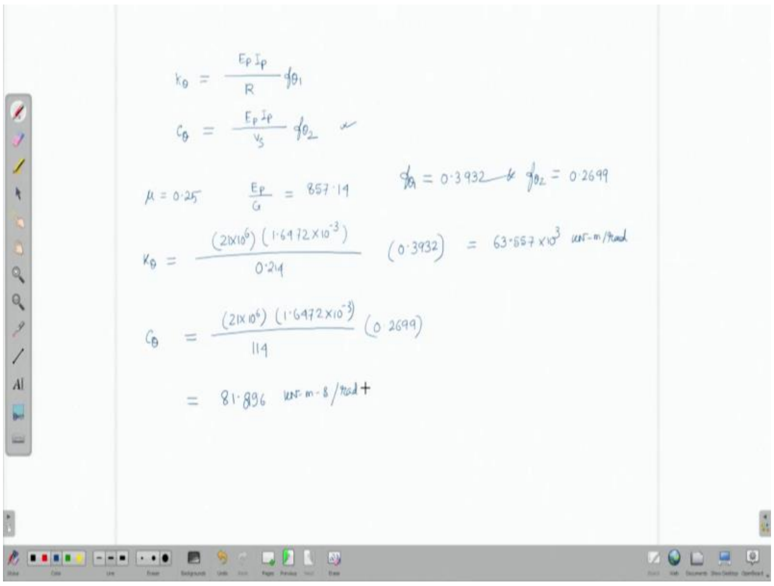

Now, we need to consider also the pile group subjected to rocking vibration for that, how we will find out the K theta total and C theta total. So, for K theta total C theta total, first we need to find out K theta can be calculated by using the equation which I am writing here and C theta will be calculated by using the equation which I am writing now, that means, C theta is equal to Ep times Ip divided by Vs times f theta 2.

Now, f theta 1 and f theta 2 depends upon 2 factors, one is Poison's ratio of the soil which is 0.25 here and the ratio Ep divided by G which is in this case 857.14, then, in this combination, what is the value of f theta 1 and f theta 2 that we need to first write down. I have calculated f theta 1 and f theta 2 approximately these 2 are equal to 0.3932 and 0.2699 respectively.

Now, with this we can calculate K theta. So, K theta is equal to first I am writing the value for Ep, which is 21 into 10 to the power 6 in kilo Newton per square meter times Ip which is 1.6472 into 10 to the power minus 3, unit is meter to the power 4 divided by r which is 0.214 in this case times f theta 1 which is 0.3932. So, if you will try to calculate K theta what will be the final answer that I am now writing it is coming 63.557 into 10 to the power 3 unit is kilo Newton meter per radian.

Now, C theta so, for C theta we will use this equation which is saying Ep times Ip divided by Vs which is 114 in meter per second in this case, this entire thing will be multiplied with f theta 2 which is 0.2699. So, from this we are getting the value of C theta which is coming at 81.9 or you can write it 896 also in kilo newton meters per second per radian. So, now we have the information of K theta and C theta alright.

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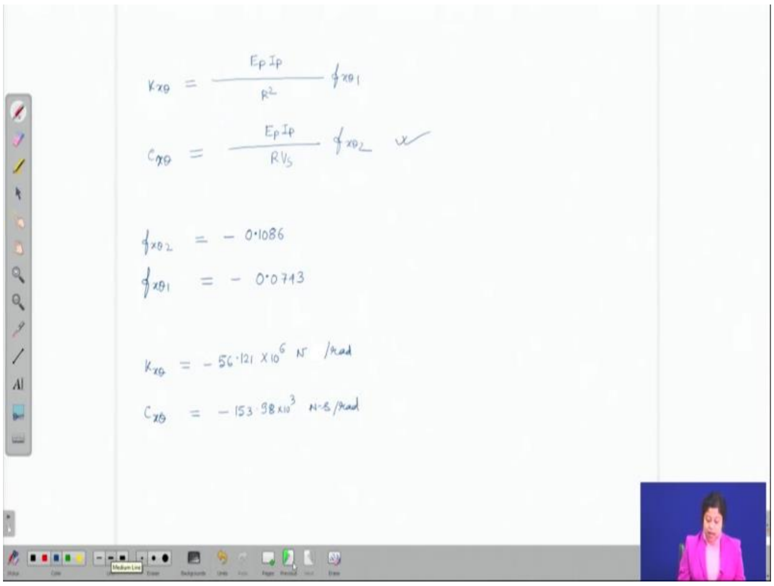

Now, we need to find out Kx theta K for Kx theta that equation is Ep times Ip divided by R square multiplying with f x theta 1 likewise you can find out Cx theta which is equal to Ep times Ip times R sorry Ep times Ip divided by R times Vs together this is multiplying with a fx theta 2. Now, using the tables, you can find out the values for a fx theta 2 and all. So, I am just writing this value directly I have brought this value. So, a fx theta 2 in this case whatever values we know for Ep by G and mu for that we will get a fx theta 2 is equal to minus 0.1086.

Likewise, we can find out a fx theta 1 which is equal to minus 0.0743. Now, using these two values of fx1 and fx sorry fx theta 1 and fx theta 2 we can calculate Kx theta and Cx theta rest of the parameters we have already calculated. So, I am directly writing the value for Kx theta which is equal to minus 56.121 into 10 to the power 6 in Newton per radian.

Similarly, we can calculate Cx theta using this equation and the value of fx theta 2 which is equal to minus 0.1086 and the value of Cx theta is coming 153.98 in kilo Newton second per radian or you can write 10 to the power 3 here, then it is Newton's second per radian. So, in this way, we can find out Kx theta and Cx theta,

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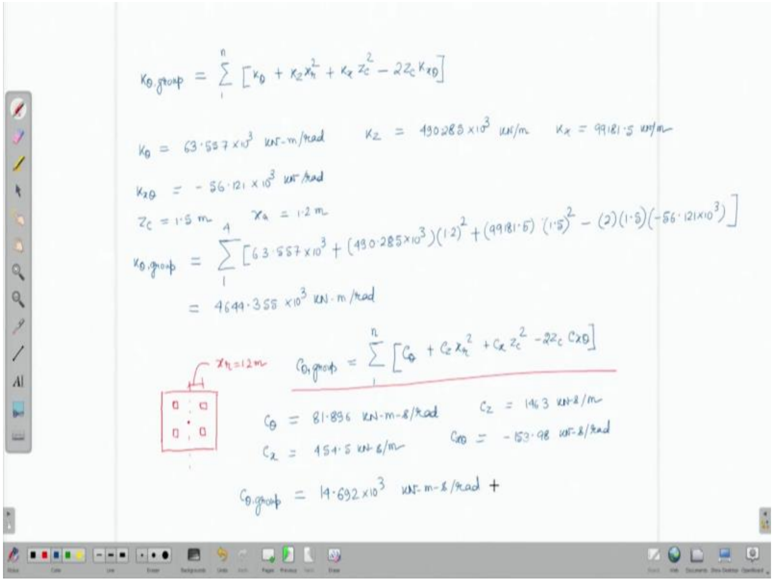

Next, we will calculate K theta group and C theta group alright. So, in pile group, there are 4 piles present. So, how we will calculate K theta group for that, we will use the equation which we have already studied, I am writing the same equation once again here also. So, summation of K theta times Kz times, summation of K theta plus Kz times Xr square plus Kx times zc square minus 2 times Zc times Kx theta.

Now, here we have already calculated the values of K theta Kz, Kx and Kx theta which I am writing here. So, K theta is 63.557 into 10 to the power 3 in kilo newton meter per radian likewise Kz is 490.285 into 10 to the power 3 in kilo Newton per meter Kx is 99181.5 in kilo Newton per meter and what is Kx theta Kx theta is minus 56.1 to 1 into 10 to the power 3 unit is kilo Newton per radian.

Now, we these we can calculate K theta for pile group. So, in this equation I can use the value of Xr and Zc. So, if you see this figure what is Zc? Zc is the distance this distance you can see here so, Zc means how much h divided by 2 which is 1.5 So, I can write here Zc also, Zc is 1.5 meter.

Similarly, if you see this figure what is Xr? Xr is the if I will draw the pile cap with the piles. So, this is the pile cap and there are 4 piles just I need to correct this. So, these are the 4 piles. Now, the distance from the cg of the pile cap to the center of the pile, this is Xr. So, what is given here if we calculate it is from that we can calculate Xr also. So, from this figure we can see Xr s is divided by 2 that means 1.2 meter.

So, Xr is 1.2 meter, I can write it here also Xr is equal to 1.2 meter. Now, we can calculate the value of K theta group. So, there are total 5 piles in these pile group this is K theta then, Kz times Xr square. So, this is Kz times Xr square next term is Kx times Zc square this is what I am writing is Kx times Zc square sorry here it is not square on Kx, but Zc squared.

So, Kx times Zc square alright minus 2 times Zc times Kx theta, which is minus 56.121 into 10 to the power 3 in kilo Newton per radian. So, now, if you will do the calculation, what you will get that time writing So, from these finally, we are getting K theta group is equal to 464 4.355 into 10 to the power 3 kilo newton meter per radian alright.

Now, next term is C theta for the pile group. So, I can write here itself C theta for the pile group for which of each equation we will use first time writing that if there are n number of piles present in the group then summation for n piles and then the term within this is C theta plus C z times Xr square plus Cx times Zc square minus 2 times Zc times Cx theta.

Now, we have already calculated the value of C theta which is 81.896 in kilo newton meters second per radian. Then, we have also calculated Cz which is 1463 in kilo Newton second per meter. We have also calculated Cx which is 454.5 in kilo Newton second per meter and Cx theta which is minus 153.98 unit is kilo newton second per radian. So, with this information we can calculate C theta group using this equation so, I am directly writing the value for C theta group now.

Please remember in the pile group there are 4 piles so, in this case n is equal to 4, so, C theta group which I am getting that I am writing here the value of C theta group is equal to 14.692 into 10 to the power 3 unit is kilo Newton meter second per radian. Now, we need to find out K theta and C theta for the pile cap. So, go to the next page.

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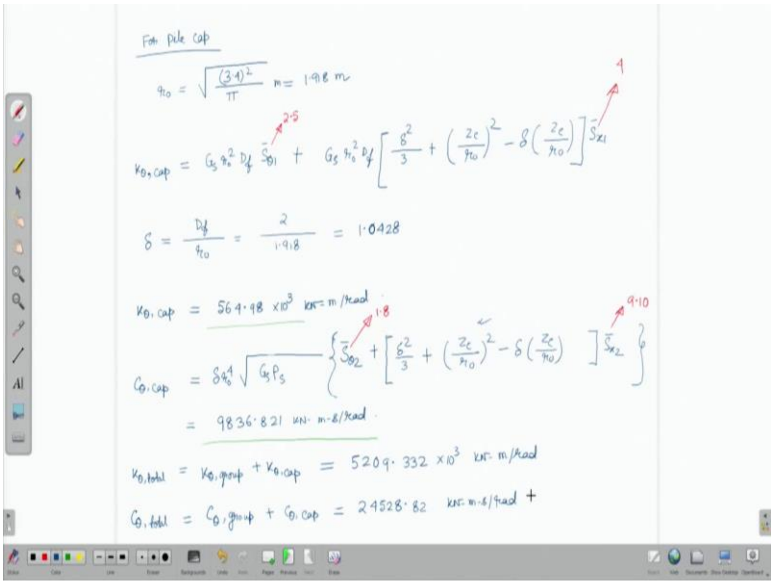

So, for pile cap first we need to find out the equivalent radius r0. So, r0 for pile cap is let us see what is the area for pile cap area is 3.4 meter by 3.4 meter. So, equivalent radius is equal to square root of 3.4 square divided by pi and unit is in meters so, we are getting r0 for pile cap is equal to 1.918 in meter.

Next, we need to find out now, K theta for pile cap. For that, we need to use another equation, which is I am writing K theta cap is equal to Gs times r0 square times Df times S theta 1 or S bar theta 1 plus Gs times r0 square times Df, this thing will be multiplied with the term delta squared divided by 3 plus Zc divided by r0 square minus delta times Zc divided by r0 and these will be multiplied with S bar x1.

Now, how to find out S bar x1 that we have already seen. So, in this case, if we will write the value of x bar x1 and if we will write the value of other parameters, so, actually S bar x1 in these cases 4 what is the value of S theta, S bar theta 1, S bar theta 1 for all combinations of mu and Ep by G, you can get 2.5. So, with these 2 parameters, now, we can calculate the value of K theta cap, I think there is one more term delta which you need to know delta actually is equal to Df divided by r0. So, here Df is 2 meter, r0 is 1.91 meter. So, we are getting the value which is equal to 1.0428. So, delta is a dimensionless number in this case.

Now, using the values of delta r0, Gs, Df and the 2 parameters S bar theta 1 and S bar x1 what we can calculate we can calculate the value of K theta cap and it is coming I am directly writing the value which I have already calculated. So, it is coming approximately 564.98 into 10 to the power 3 unit is kilo Newton meter per radian.

Next step is to find out C theta cap. So, for C theta cap, we will use another equation which I am writing delta times r0 to the power 4 times square root of Gs times rho s this thing will be multiplied by a term which I am writing within the bracket. So, these will be multiplied with S bar theta 2 plus another term multiplying with S bar x2. So, what is the term in this bracket that I am now writing? It is delta square divided by 3 plus Zc divided by r0 square minus delta time z C divided by r0.

So, with this I am getting the value of C theta cap. Here we need to know 2 things what is the value for S bar theta 2 and S bar x2. So, it is bar theta 2 is for all combinations of Poisson's ratio and Ep by G you can write 1.8 whereas depending upon the values of mu and Ep by G you can calculate S bar x2 and that we have already learned and what is that value that is 9.10 for our case.

So, with this what we are getting we are getting C theta cap value which is coming. I am writing the value directly 9836.821, unit is kilo newton meters per second per radian. I hope all the terms are mentioned here. So, now, we know the value of K theta for pile group we know the K theta value for pile cap, from this we can calculate K theta total which is sum of sorry which is sum of K theta group plus K theta cap.

Already if you see the previous page, we have calculated the value of K theta group and here we have calculated the value of K theta cap. So, from these 2 values finally, we can get K theta total which is equal to 5209.332 into 10 to the power 3 in kilo Newton meter per radian. Same way you can find out C theta total also, better here I will write full form total. So, C theta total is equal to C theta for pile group plus C theta for pile cap.

We have already calculated C theta for pile cap you can see here and also, we have calculated C theta for the pile group in the previous page yes here. So, from this we can find out the value of C theta total. So, that is coming 24528.82 unit is kilo Newton meters second per radian alright. So, in this way we can find out the values of K theta total and K theta sorry C theta total.

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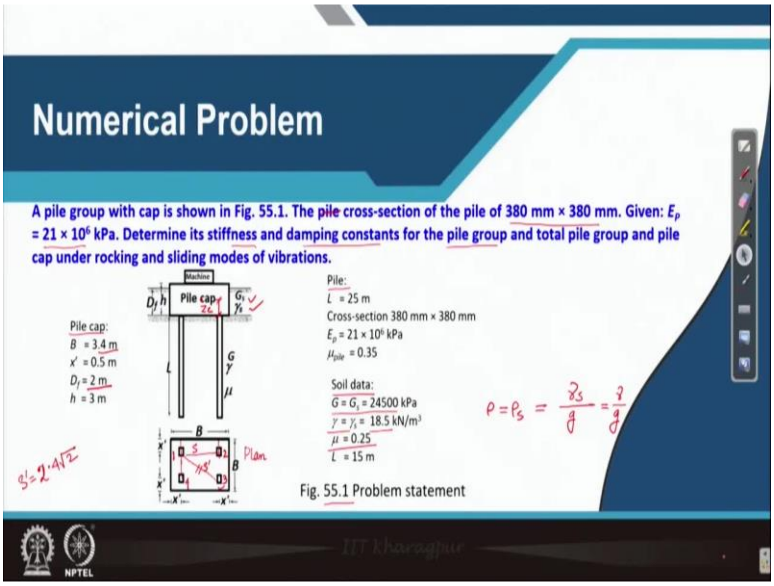

So, in this way we can do that analysis of pile foundation subjected to dynamic loading which comes from the machine when it is operating alright.

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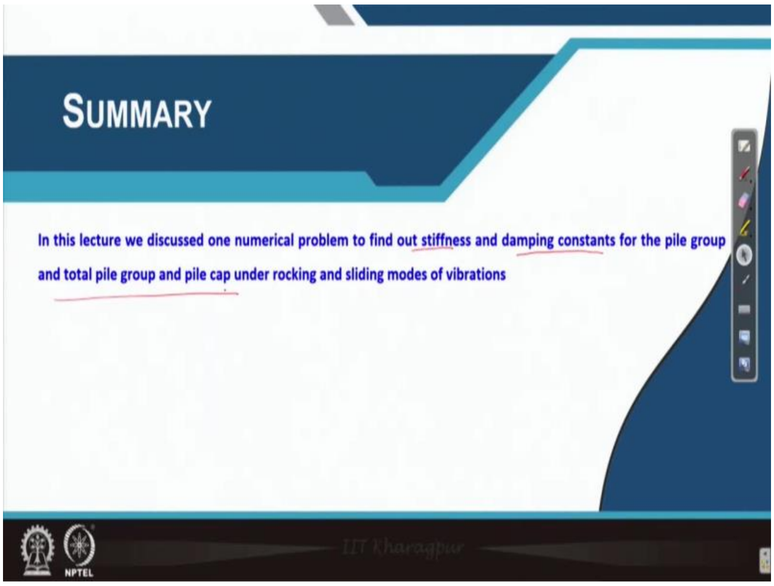

So, then we can come to the summary of today’s class as you know, we have discussed the numerical problem to find out the stiffness and damping constants for the pile group and total pile group and pile cap under rocking and sliding modes of vibrations.

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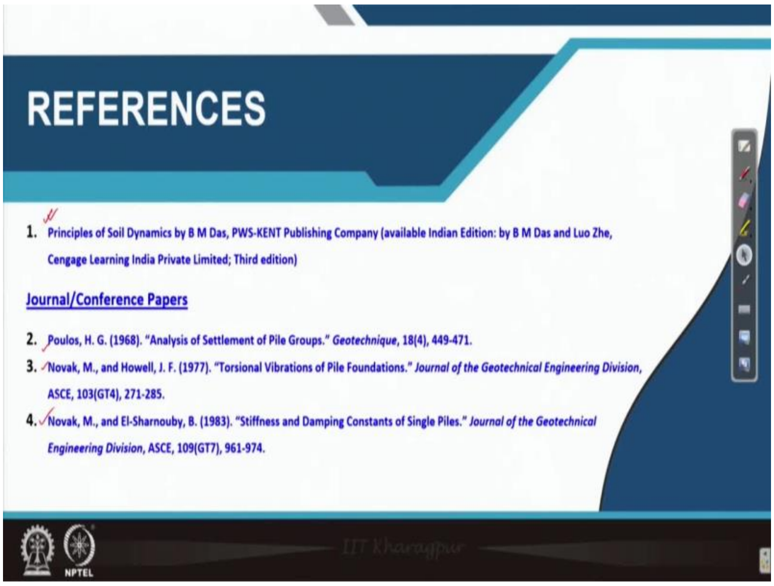

So, this is the main reference textbook from which I have taken the numerical problem and to solve the problem, we use the different charts and tables which are taken from these three references. With these I am stopping today's class. Thank you.