Soil Dynamics
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Lecture No. 54
Analysis of Pile Foundations
Under Dynamic Loading (Part – IV)

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Hello friends, welcome to the course Soil Dynamics. So, in this week we are discussing how to do the analysis of pile foundation under dynamic loading. So far, we have considered pile foundation subjected to vertical vibration, then we have studied when it is subjected to horizontal vibrations or sliding vibrations and rocking vibrations individually and when it is subjected to coupled rocking and sliding vibrations. So, today we will discuss the case of pile foundation subjected to torsional vibrations.

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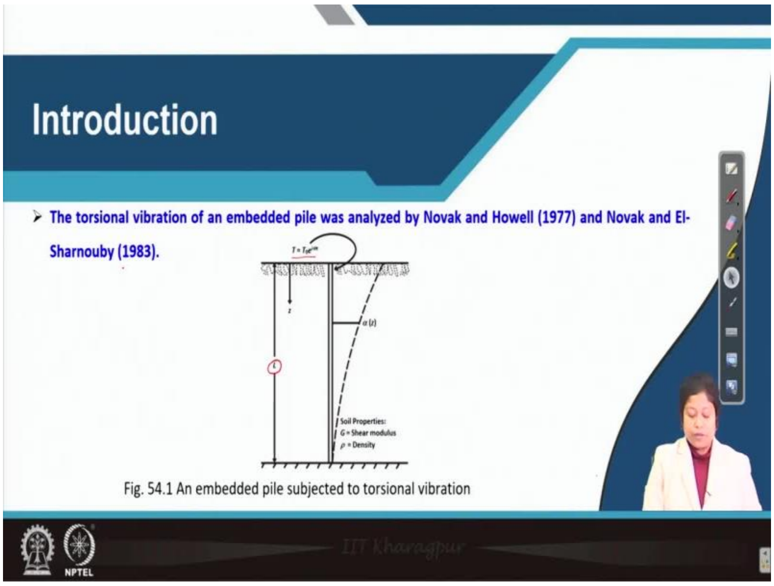

So, here you can see things single pile which is embedded in the soil. So, this is subjected to torsional vibration and the expression for torsional vibration is already mentioned T0 times e to the power i omega t, so T0 is the amplitude of torsional vibration. Soil properties are also given G is a shear modulus of the soil and rho is the density. From the geometry point of view of the pile, only length is given, the cross sectional area of the pile either be circular or rectangular. So, the torsional vibration of these embedded pile this type of embedded pile was analysed by Novak and Howell in 1977 and Novak and El-Sharnouby in 1983.

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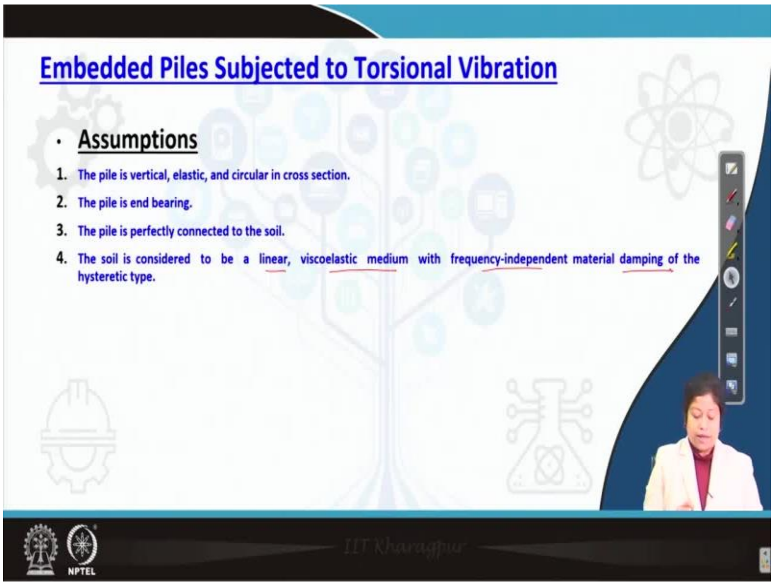

So, what they have proposed that we will now discuss, before that we need to know when embedded piles subjected to torsional vibration that time, to do its analysis, what are the assumptions are involved. So, there are some assumptions. First assumption says the pile is vertical, elastic and circular in cross section. Then the pile is in this case end bearing pile, the pile is perfectly connected to the soil, also the soil is considered to be a linear viscoelastic media with frequency independent material damping of the hysteretic type. So, with these assumptions now, we can analyse the pile foundation subjected to torsional vibration.

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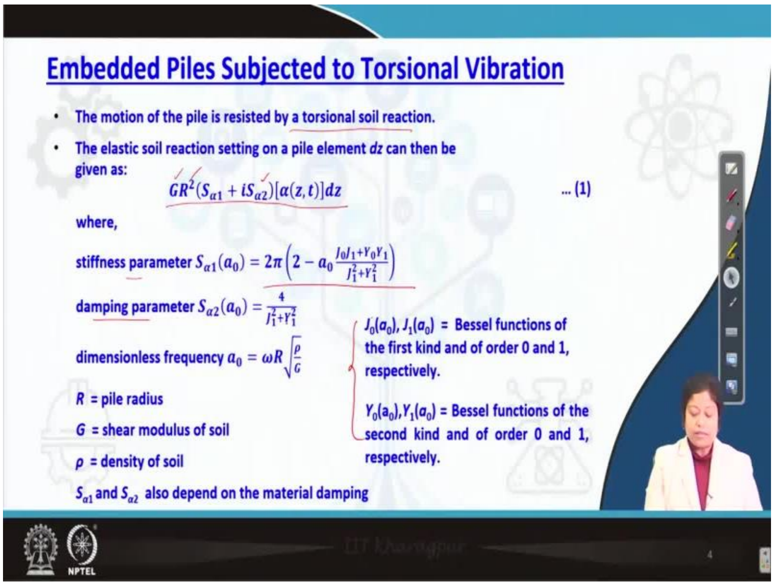

So, here the motion of the pile is resisted we are assuming the motion of the pile is entirely resisted by our torsional soil reaction. Then the elastic soil reaction sitting on a pile element these dz can be given by this equation 1. So, here G is shear modulus, R is the radius of the pile, S alpha 1 and S alpha 2, depends, these two depend upon the material damping. And you can see the parameter S alpha 1 and S alpha 2 how we can calculate.

So, S alpha 1 is related to the stiffness, whereas S alpha 2 related to damping parameter. So, if alpha 1 can be calculated by using this expression, where J0 and J1 are the Bessel functions of the first kind and their order is 0 and 1 respectively. So, for J0 order is 0, J1 order this 1, what are y0 and y1? These are also basal functions of the kind of the second kind and orders are 0 and 1 respectively. So, for y0 order is 0, for y1 order is 1. So, in this way we can define S alpha 1 and S alpha 2.

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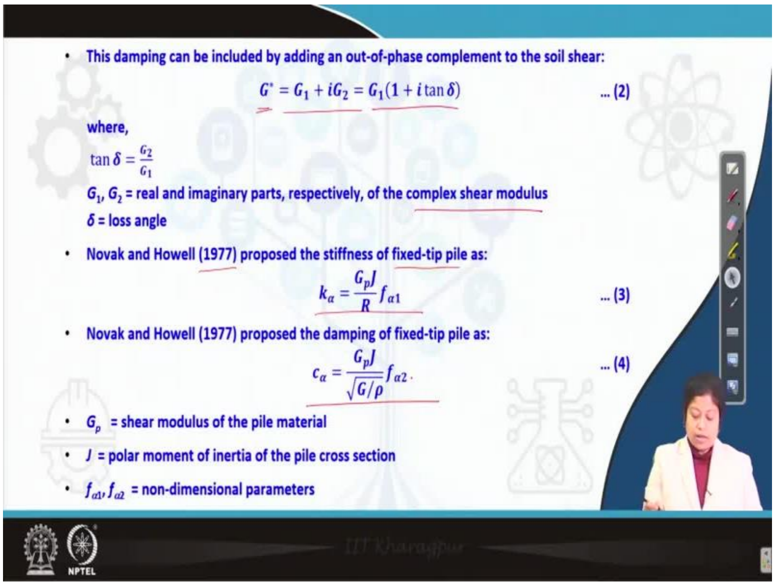

Now, this damping can be included by adding an out of phase complement of the soil shear. Here you can see, we can find out the complex shear modulus G star, we can express basically the complex shear modulus G star by this expression where G1 is the real part and G2 is an imaginary part of the complex shear modulus. What is then in the next step we can in place of G2 we can write G1 times tangent of delta, why so, because you can see the ratio of the G2 divided by G1 is defined as tangent of delta, where delta is the loss angle. So, that is the reason we can write G star which is the complex shear modulus in this form as well.

Now, Novak and Howell in 1977 proposed how to find out the stiffness for fixed tip pile. So, here you can see that expression k alpha is the stiffness that can be calculated by using the expression Gp times J divided by R, whole thing is multiplied with f alpha 1. So, what is Gp here? Here, Gp is the shear modulus of the pile material, R is the radius of the pile, f alpha 1 is a factor. Same way, they have proposed the how to find out the damping for fixed tip pile, for this we can use equation 4.

So, here also you can see there is this Gp times J is common, but here you can see there is a term square root of G by rho. So, basically square root of G by rho is nothing but the velocity of the shear wave. And what is G and what is rho here? These are shear modulus and density of the soil respectively, J is polar moment of inertia of the pile cross section and f alpha 2 like f alpha 1 it is also a non-dimensional parameter.

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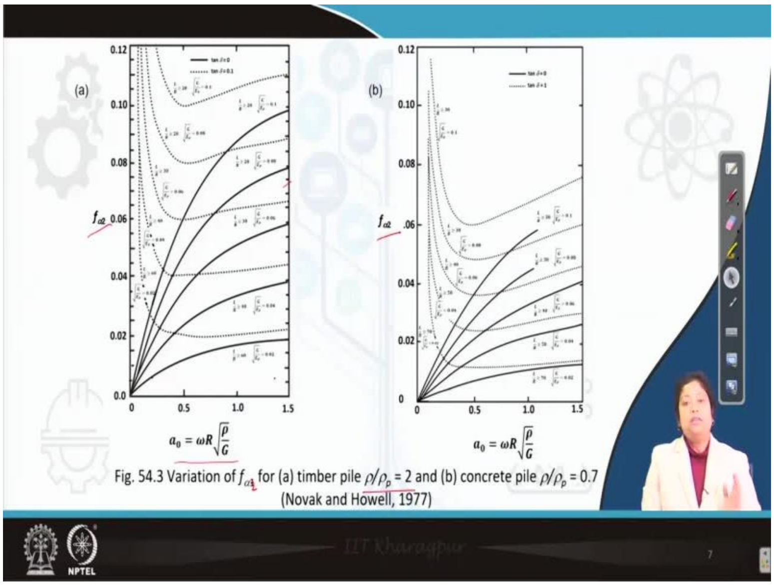

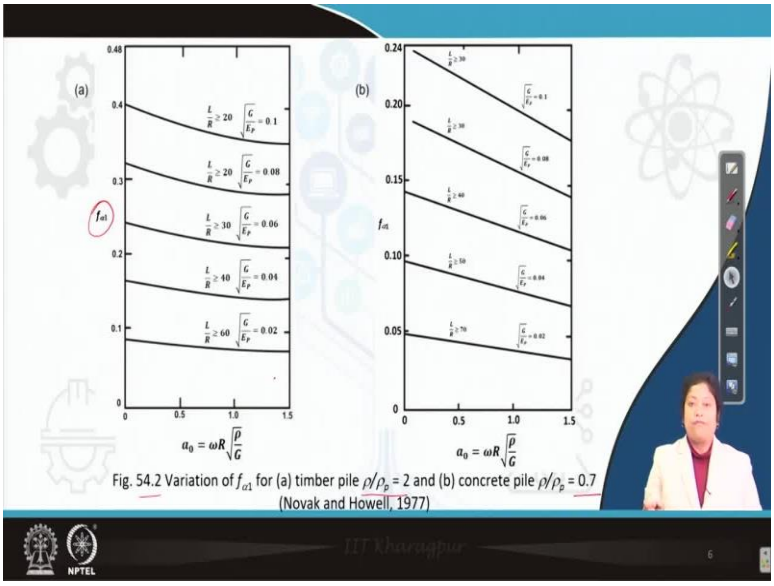

Now, how we can find out f alpha 1 and f alpha 2? So, figure 54.2 shows how to find out f alpha 1, this is for timber pile. For timber pile, you can see rho divided by rho p, rho p is the density of the pile material. So, rho divided by rho p is equal to 2. Now, if it is concrete pile then the ratio of rho divided by rho p is equal to 0.7. So, these charts are proposed by Novak and Howell in 1977. Also, here you can note that the it depends on the ratio L by R, it also depends upon the ratio of square root of the ratio of G by Ep. For this is F alpha 2.

So, figure 54.3 shows the variation of f alpha 2, you can see here this is f alpha 2 for the timber pile, this is for the timber pile, so the rho divided by rho p value is equal to 2 as you can see here. And the right hand side figure that means, figure b for concrete pile, where rho divided by rho p value is equal to 0.7. So, a 0 is non dimensional number, so, how to calculate that is also shown. So, if we know a 0 value, if we know L by R ratio, if we know the square root of the ratio of G divided by Ep, then using either figure a or b we can find out alpha f alpha 2. The same way we can also find out if alpha 1.

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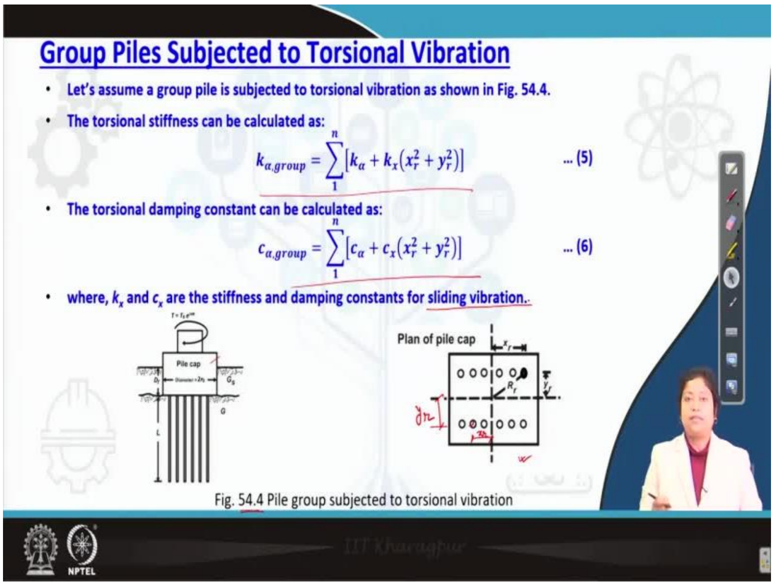

We already know that piles are generally constructed in any group. So, what will be happening between pile groups subjected to torsional vibration, how do we calculate the stiffness and damping constant? So, in figure you can see a pile group with a pile cap, it is subjected to a torsional vibration t. Now, here to calculate the torsional stiffness, we can use this expression.

So, the m is here total number of piles. So, as per this diagram, if we see the plan of the pile cap what we can see there are total 6 plus 6, 12 piles, so m is 12, as per this diagram. So, how we can now calculate damping? Constants for that, we will use this diagram already we have seen how to calculate k alpha and c alpha. So, and also, we know how to find out kx and cx when we studied the pile foundation subjected to sliding vibration.

So, from that we can finally find out k alpha and c alpha for pile group using equation 5 and 6 here how to calculate xr, xr is the distance from the Cg or centerline to the particular pile measure along the x direction. Likewise, yr is the distance from that Cg measured along y direction. So, if this is the pile, sorry, if I am concerned about this one, then how do we do? Then xr is this distance this is xr and yr should be this distance. So, in this way we can calculate xr and yr. And kx and cx are the stiffness and damping constants for sliding vibration that we have already discussed in last class.

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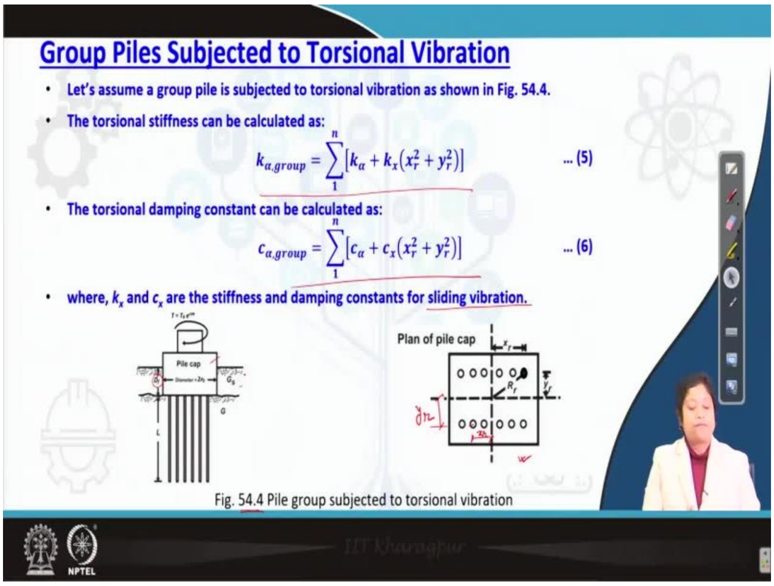

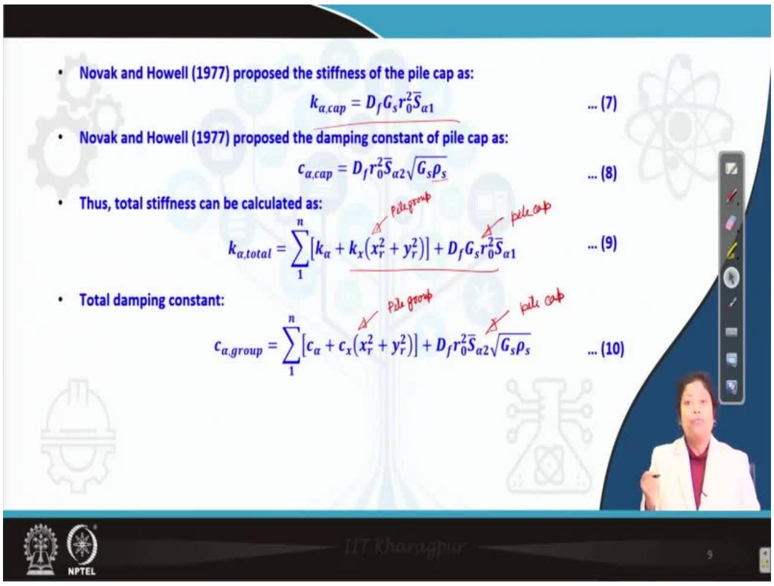

So, next thing to find out the stiffness for the pile cap for that we can use expression 7. So, Df, if I go back to the previous figure here you can see Df is the embedment depth or the depth of the pile which is embedded below the soil or below the ground surface, so this is Df. Gs is the shear modulus of the soil in which this pile cap is embedded we call it as backfill soil. r0 you have already seen it is the diameter sorry radius of the pile cap 2r0 is actually the diameter of the pile cap, if pile cap is rectangular in shape, then we need to find out the equivalent radius.

Now, Novak and Howell in 1977 also proposed the expression for damping constant, which you can see in equation 8. So, here only the thing is rho which is not discussed, so, rho s is the density of the soil which is used for backfilling that means, the soil in which the pile cap is embedded. So, now, we know k alpha for pile group we know k alpha for pile cap. So, if we will add these two expressions, we will get total stiffness.

Likewise, so, this is the total stiffness this component for pile group and this component for pi cap. Similarly, we can also calculate c alpha total which is called as total damping constant. So, the first part coming from pile group and the second part is coming from pi cap. So, this way we can find out the total k alpha and c alpha as shown here.

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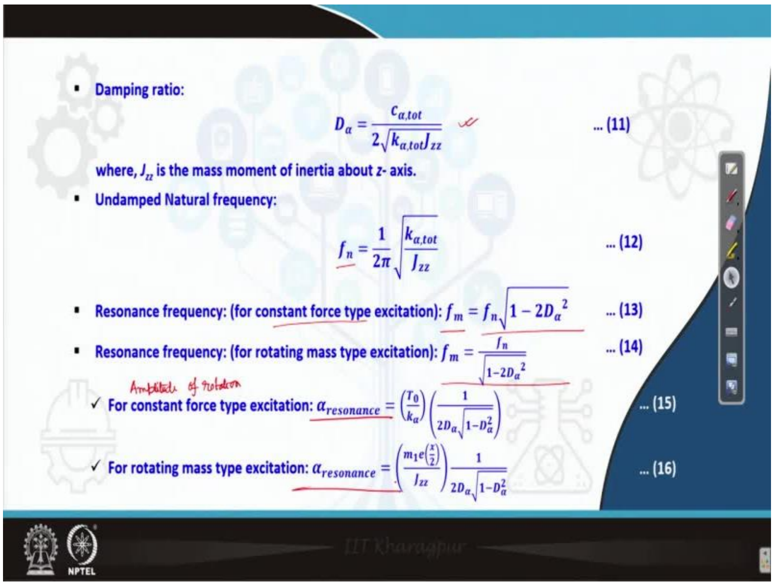

Now, our next task is to find out the damping ratio, which we can do using equation 11. Then what we need to do? We need to find out the natural frequency fn. So, here J z z is the mass moment of inertia about z axis. Next is to find out that that resonance frequency f m using expression or equation 13 and this is for constant force type excitation, if the excitation is caused by rotating mass, then we will use equation 14.

And then we can find out the amplitude also amplitude of rotation. So, basically this is amplitude of rotation using equation 15 for constant force type excitation and using equation 16 for rotating mass type excitation. So, depending upon the source of vibration, we will choose which equation we need to use here.

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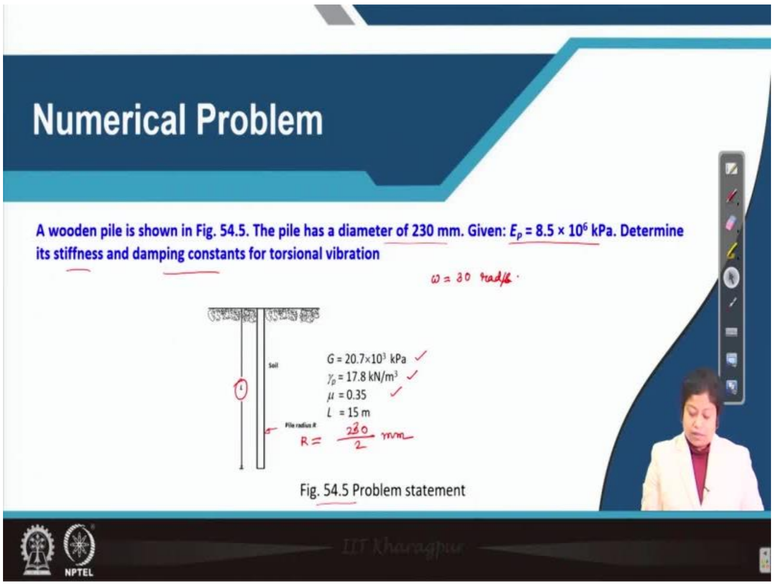

Now, let us come to one numerical problem, a simple problem which is described here we will try to solve. What is said here, this pile is made of wood and the Ep value that means the modulus of elasticity is given which is 8.5 into 10 to the power 6 kilo Pascal, diameter of the pile is mentioned which is 230 millimeter, then radius of this pile should be 230 divided by 2 in millimeter. And what is else l which is the length of the embedded pile is given that is 50 meter.

Related to the soil properties, you can see shear modulus of the soil, then bulk unit weight, and also potions ratio are provided. So, from these we can calculate the we need to calculate the stiffness and damping constant. So, we can take just for completeness of this problem, we can take some omega value also, because that we need to use later. So, I can take it suppose 30 radian per second.

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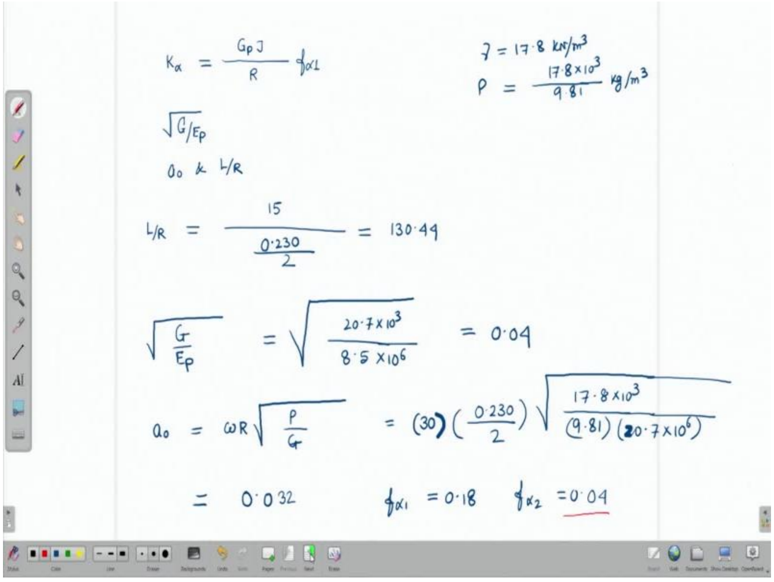


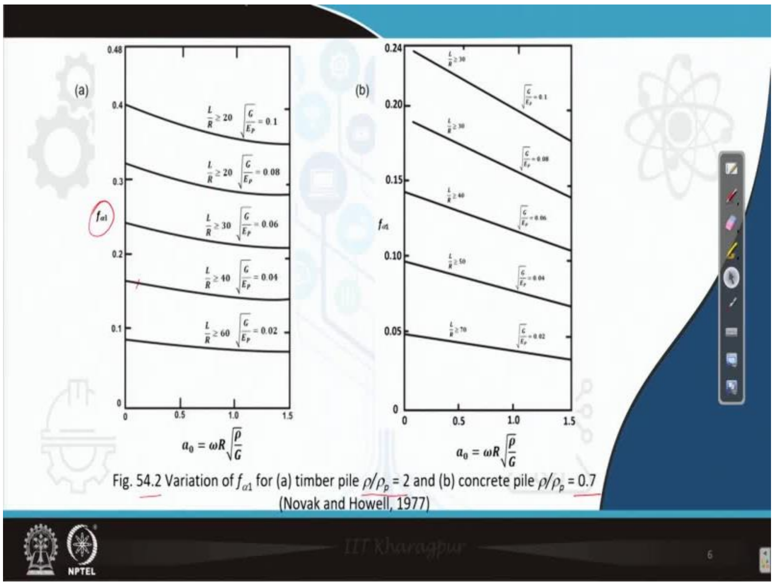


Fig. 54.2 Variation of $f_{\alpha 1}$ for (a) timber pile $\rho/\rho_p = 2$ and (b) concrete pile $\rho/\rho_p = 0.7$ (Novak and Howell, 1977)

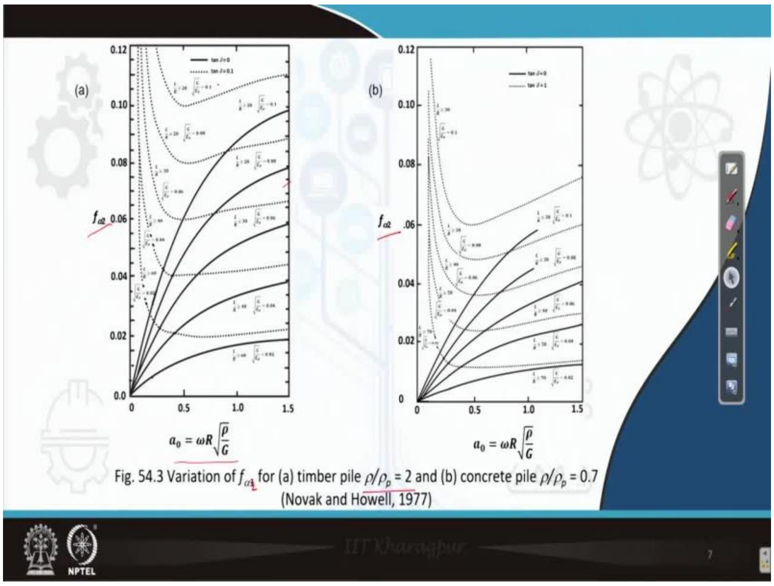
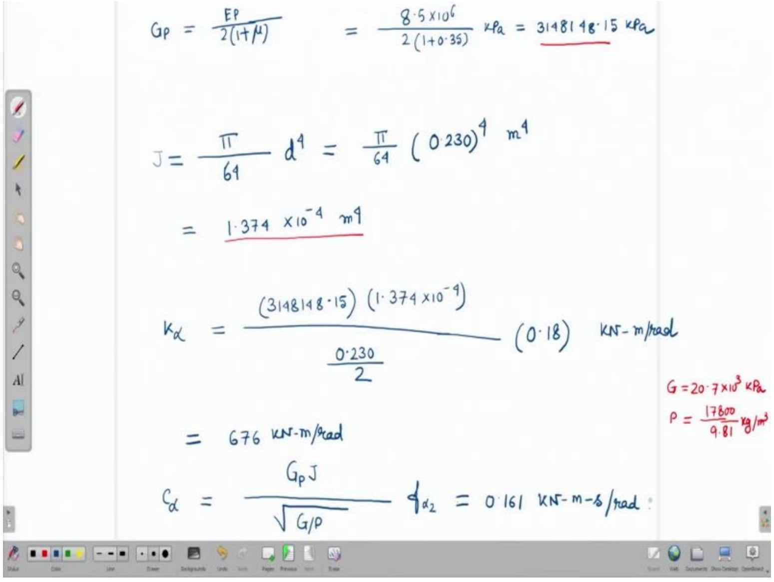
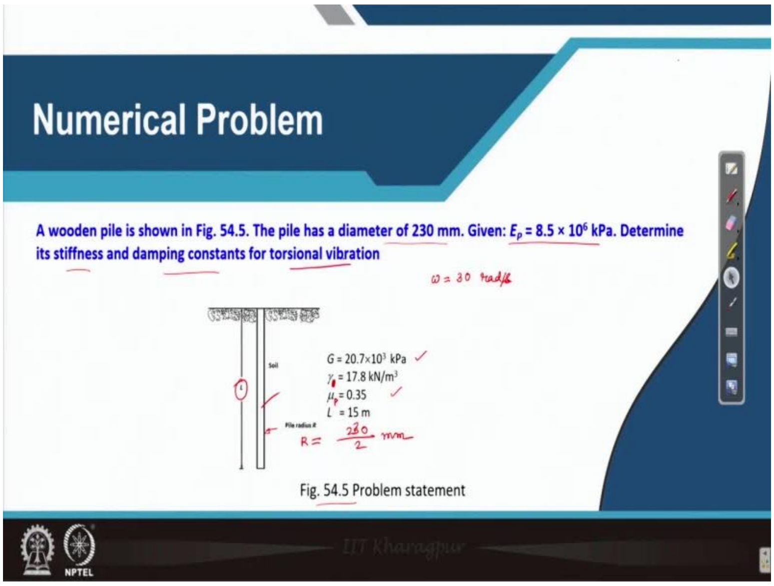


Fig. 54.3 Variation of $f_{\alpha 1}$ for (a) timber pile $\rho/\rho_p = 2$ and (b) concrete pile $\rho/\rho_p = 0.7$ (Novak and Howell, 1977)





Now, let us solve this problem. So, the first thing which we need to do here is we need to find out k alpha. So, what is the equation to find out k alpha? Gp times J divided by R times f alpha 1. Now, f alpha 1 depends upon 2 things, one is square root of G divided by Ep, and it also depends upon a 0 and L by R. So, we need to calculate all these things one by one. First thing, L by R, length is 15 meter and its radius is 230 millimeter by 2, that means 0.230 divided by 2. So, it is coming L by R is equal to 130.44.

Next is to find out square root of G divided by Ep. So, G is 20.7 into 10 to the power 3, let me check yes 20.7 into 10 to the power 3 and Ep is 8.5 into 10 to the power 6, these both are in kilo Pascal. So, divided by 8.5 into 10 to the power 6. So, the value is coming 0.04. For this value of G divided by Ep now, we need to check what is the value for a 0 which is equal to omega R times square root of rho by G.

So, first let us find out the value of we are assuming omega is approximately 30, R is already given that is 0.230 divided by 2 times, what is rho here rho is 17, I think here this is not P disease, this should be rho not for pi, because we do not need the data for rho P we need sorry gamma P we need the data for soil. So, this is gamma and generally 17.8 kilo newton per meter cube is bulk unit weight of soil only. So, this is please make it please correct it this is gamma which is 17.8 kilo newton per meter cube.

So, then if gamma is 17.8 kilo Newton per meter cube, then rho will be how much it is 17.8 into 10 to the power 3 divided by 9.81 in kg per meter cube. So, here I can write it that way 17.8 into 10 to the power 3 divided by 9.81. So, this is all about rho now this divided by G, so G is how much? 20.7 into 10 to the power 6. So, how much then we are getting, let me calculate. So, it is coming 0.032.

So, now, if I will go back to the chart alpha 1 we are using right now, timber wood. So, I can take something and our square root of G by Ep 0.04 that means, we need to use this line. So, you can see here it is coming approximately 0.18. So, I am writing here f alpha 1 is 0.18. The same way now, if I will go to the next slide f alpha 2, I can find out so, for that if I will concede that some value here or I can do averaging also. So, I will get f alpha 2 from these figures 0 point approximately I am talking 0.04.

Now, using these two values, I can find out k alpha. So, k alpha is equal to Gp I think I have already calculated Gp or not? No, I did not calculate, so I have to calculate. So, Gp means Ep divided by 2 times 1 plus mu, this is the value for Gp. So, in the problem given to us actually it is a typing error from my end when I type it, it is this is rho p sorry mu p, mu for the pile that means this material. So, Gp is equal to 8.5 into 10 to the power 6 divided by 2 times 1 plus 0.35 this is in kPa. So, how much it is coming? It is coming 8.5 into 10 to the power 6 divided by 2 divided by 1.35. So, finally, I am getting 3148, finally, I am getting 3148148.15 in kPa.

So, now Gp is known and also I need to find out J. So, J in this case, J is equal to pi divided by 64 times d to the power 4. So, pi divided by 64 into 0.230 to the power 4 unit is meter to the power 4. So, how much we are getting? We are getting approximately we are getting approximately 1.3 sorry 37 not 36, 1.374 into 10 to the power minus 4 in meter to the power 4, this is J. So, now k alpha is equal to Gp, Gp means here 3148148.15 this is Gp times J that means 1.374 into 10 to the power minus 4. Gp times J divided by r which is 0.230 divided by 2.

And what else, multiplied with 0.18 which is f alpha 1 and its unit is kilo newton meter per radian. So, what is the value of k alpha then, let me calculate it is coming approximately 676 in kilo Newton meter per radian. Now, what is the value for c alpha? So, for c alpha equation is Gp times J divided by square root of G divided by rho that means, velocity of the shear wave this entire thing is multiplied with f alpha 2. So, I am just telling you the values for Gp Gp is this one.

Then J we have already calculated J, we know G, G is I am just writing here 20.7 into 10 to the power 3 in kPa that is G and rho also known rho for soil. So, this is 17800 divided by 9.81 in kg per meter cube. So, and f alpha 2 we already calculated that is 0.04 here, so, c alpha is then equal to it is coming sorry it is coming 0.161 in kilo newton meter second per radian. So, in this way we can find out the stiffness and damping constant for a single pile. In this case a single pile is given and for it only we are asked to find out the stiffness and damping constants for torsional vibration, so, that we have calculated.

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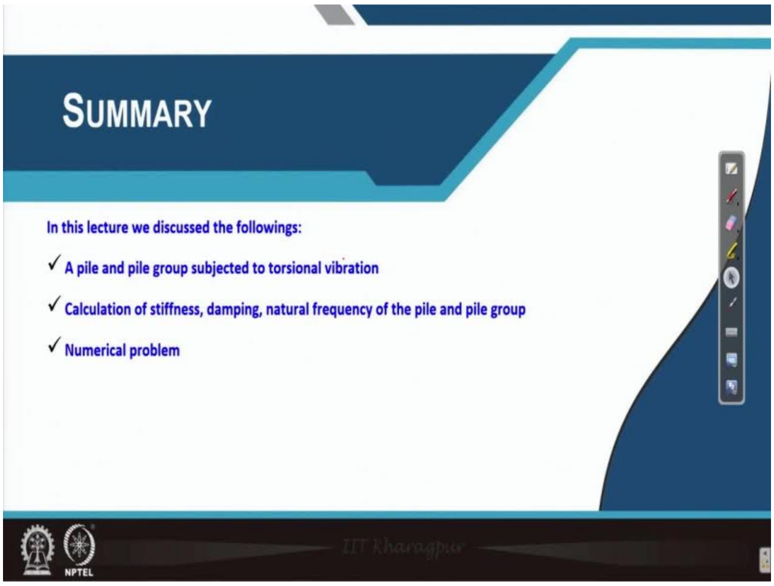

Now, come to the summary of today's class. So, in this lecture, we discussed first a pile and pile group that is subjects that are subjected to torsional vibration and then, how we can find out the stiffness and damping constant. Next, how to calculate natural frequency of the pile and pile group. Then, we have discussed numerical problem to clear our understanding for the single pile.

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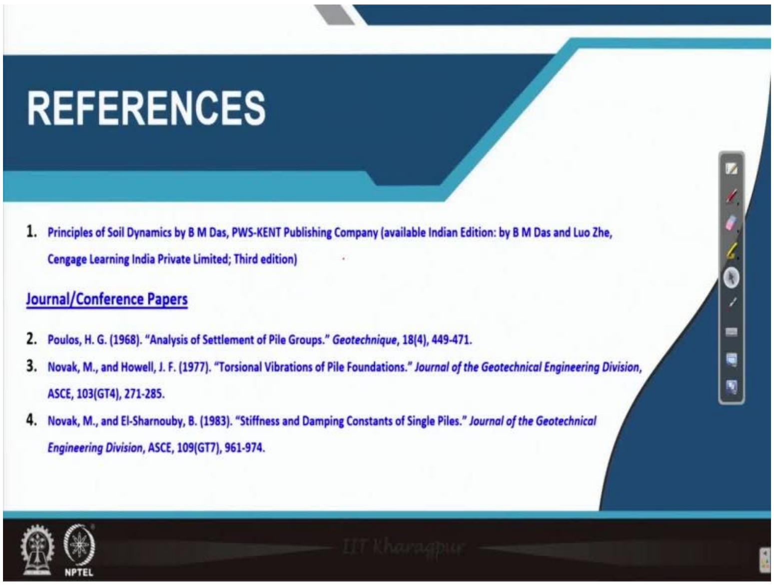

So, here you can see the list of the references for today's class. So, in next class we will discuss one numerical problem so that our understanding related to the pile foundation subjected to horizontal sliding vibration and rocking vibration should be clear. With this I am ending today's class. Thank you.