Soil Dynamics
Professor. Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No. 53
Analysis of Pile Foundations
Under Dynamic Loading (Part – III)

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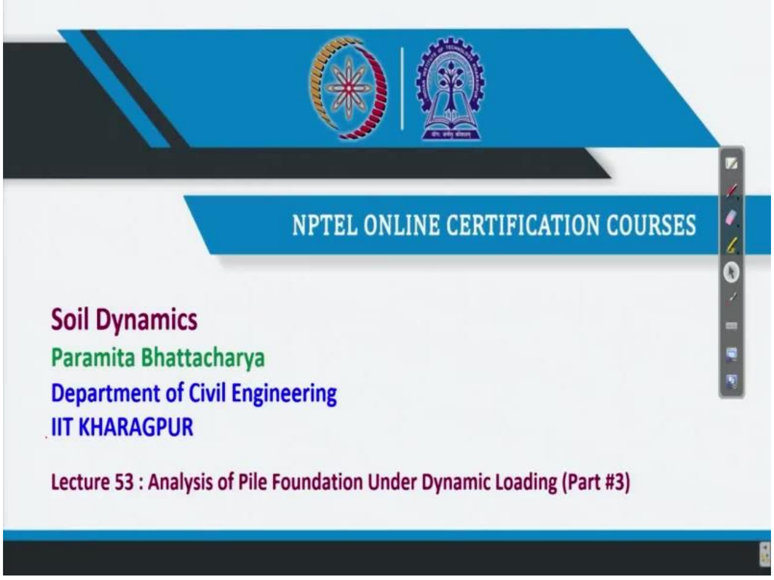

Hello friends, welcome to the core soil dynamics. So, today we will continue our discussion on analysis of pile foundation under dynamic loading. So, today is the third part on this topic.

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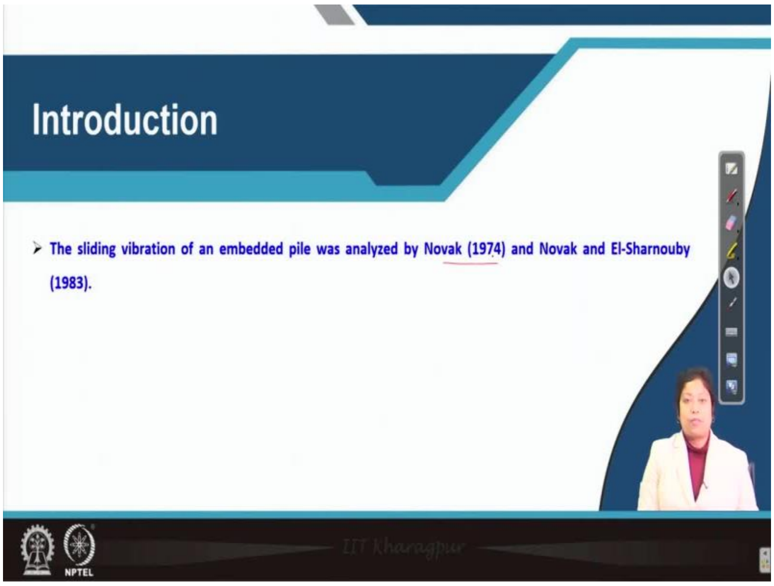

So, what is happening when a single pile or you can take it as an embedded pile is subjected to sliding vibration? The sliding vibration of an embedded pile was analysed by Novak in

1974 and Novak and El-Sharnouby in 1983. So, they have proposed a solution for this type of problem.

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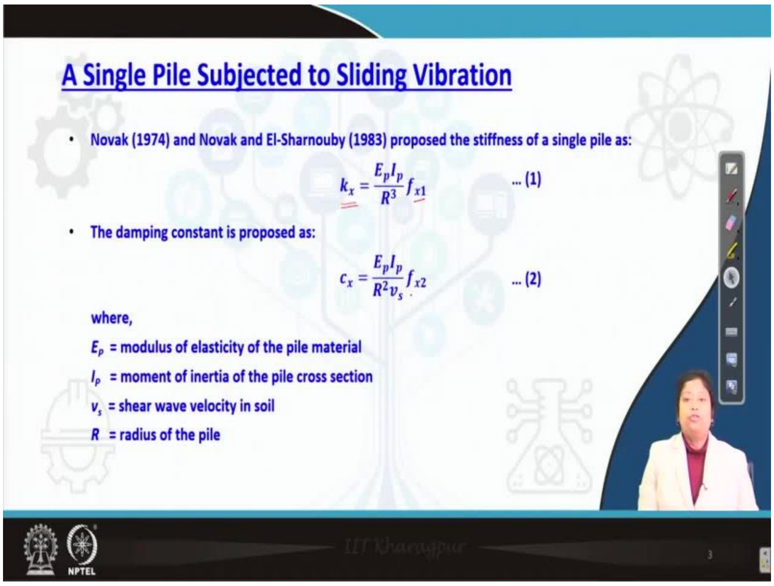

First case which we will study is a single pile subjected to sliding vibration that means, horizontal vibration. So, Novak in 1974 and Novak and El-Sharnouby together in 1983 proposed the stiffness equation for a single pile, which you can see here stiffness when pile is subjected to sliding vibration can be represented by kx. So, kx is equal to Ep times Ip divided by R cube multiplying with a factor fx1.

We have also proposed the equation to calculate the damping constant cx, which you can see in equation 2. So, here what are the meaning of different symbols, Ep is the material modulus of elasticity of the biomaterial. Likewise, R is the radius of the pile, Ip is the moment of inertia of the pile cross section, vs is the velocity of the shear wave and fx2 I have already told fx1 and fx2 these two are non-dimensional parameters.

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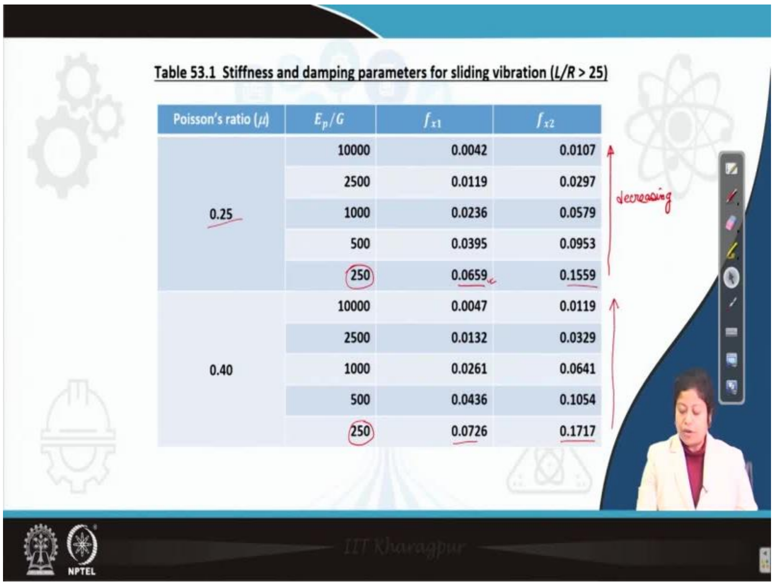

So, now question how we can find out fx1 and fx2? So, in this table you can see the values for fx1 and fx2 given which depends upon the ratio Ep by G, where Ep is the modulus of elasticity for pile material and G is the shear modulus of the soil. And Poisons ratio, so for different Poisons ratio you can see even if the value of Ep by G remain same for an example, when it is 250, these two2 cases depending upon the poisons ratio, the values of fx1 and fx2 can change. So, for higher portion ratio, you can see for higher portions ratio, the value of fx1 is high this is fx1.

So, these value for mu is equal to 0.25 whereas, this one for mu is equal to 0.40. So, 0.40 when mu is equal to 0.40 that and we are getting higher effects one value, same trends you can note for a fx2 as well. Also, what we can see here with the increase in Ep by G ratio the values of fx1 and fx2 decreases. So, here the values decreasing that we can see from this table, the same trends you can see for other new values also.

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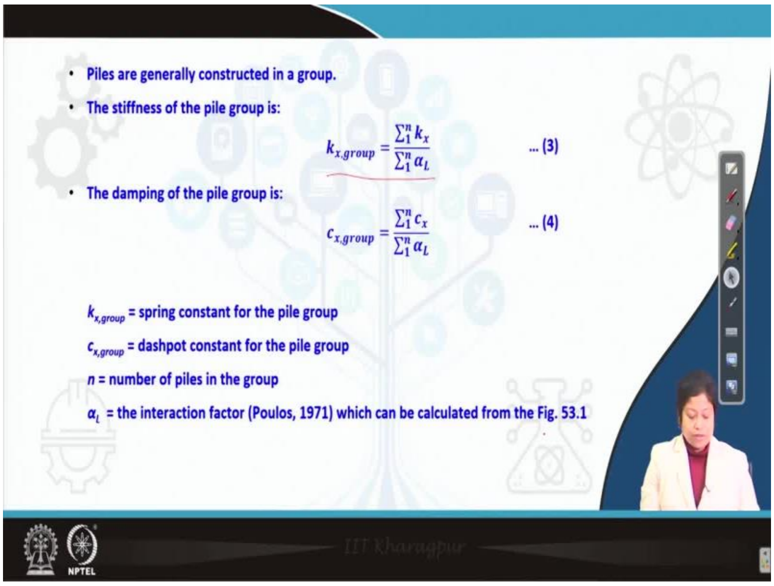

Next thing we need to now know what will be happening when piles are constructed in a group, because this is the common scenario. So, for piles in a group, we can calculate the stiffness using this equation, where alpha l is an interaction factor and n is the total number of piles present in the group. Similarly, we can find out the damping of the pile group by using equation 4. So, here you can see kx group represents the stiffness of the pile group.

Similarly, cx group represents the damping constant for pile group and kx and cx are the stiffness and damping constant for individual pile and alpha l as I said already represents the interaction factor, which we can calculate from the next figure.

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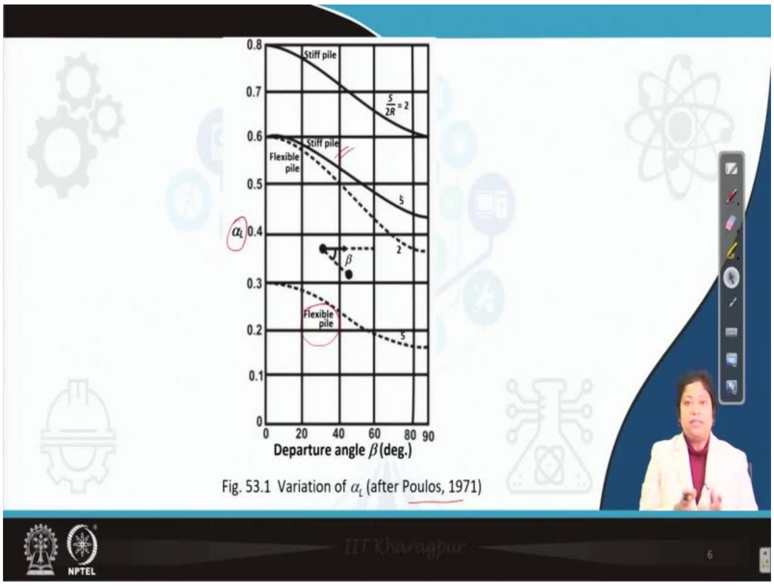

So, these figures actually proposed by Poulos in 1971. Here, if we know the departure angle theta, if we know whether the pile is flexible or it is steep, depending upon the condition and also depending upon the s divided by 2R these ratio we can find out alpha l. So, here, s is the spacing between two piles. So, you can see depending upon this ratio alpha l changes and for higher value of s by 2R the value of alpha l decreases that means, interaction factor decreases when the spacing between two piles is more.

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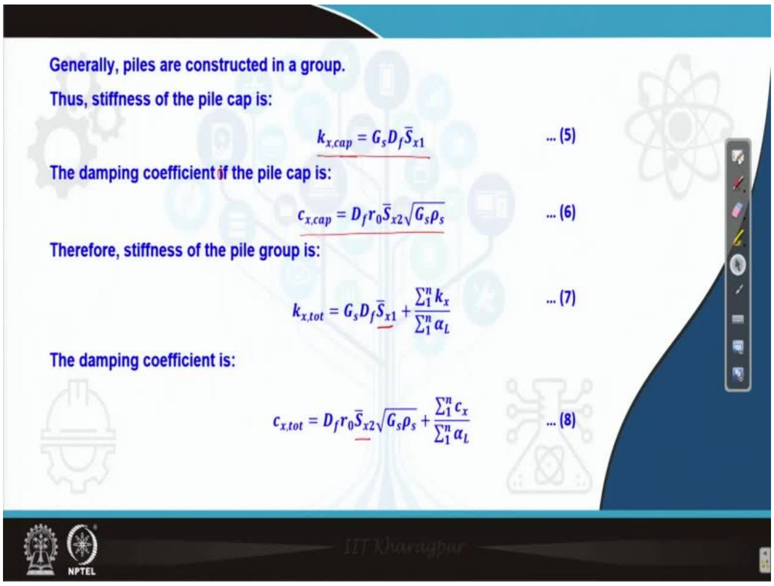

Now, we already know piles constructed in a group, now, the thing is that, what about pile cap because pile at the top of the pile group we need to put up we need to provide a pile cap. So, for pile cap, we can calculate the stiffness using this expression which is shown in equation 5. So, here what is Gs, what is Df and what is S x1 that we need to know S bar x1. Likewise, we can calculate damping coefficient of the pile group this is off of the pile cap is by using this expression 6.

So, here after finding out the stiffness of the pile cap and the damping coefficient of the pile cap, we can find out the total stiffness of the pile cap and the pile group together. Likewise, we can also find out the total damping factor for the pile cap and the pile group together. So, in this expression Gs is the shear modulus of the backfill soil, Df is the depth embedment depth of the pie cap and S x1 and S x2 are the 2 factors.

So, now, we need to know two things; one is what is S bar x1 which is used in equation 7, and what is S bar x2 which is used in equation 8, because the values of these two parameters are required to use in these two equations. So, actually the value of S x1 bar or you can call it S bar x1 and S bar x2 or S x2 bar these two parameters depend upon the Poison’s ratio of the soil. So, if we know that Poison’s ratio, then we can find out the values of S x1 bar or S bar x1 and is x2 bar or we can call it S bar x2. So, I am writing the values for these two parameters for different values of Poison’s ratio.

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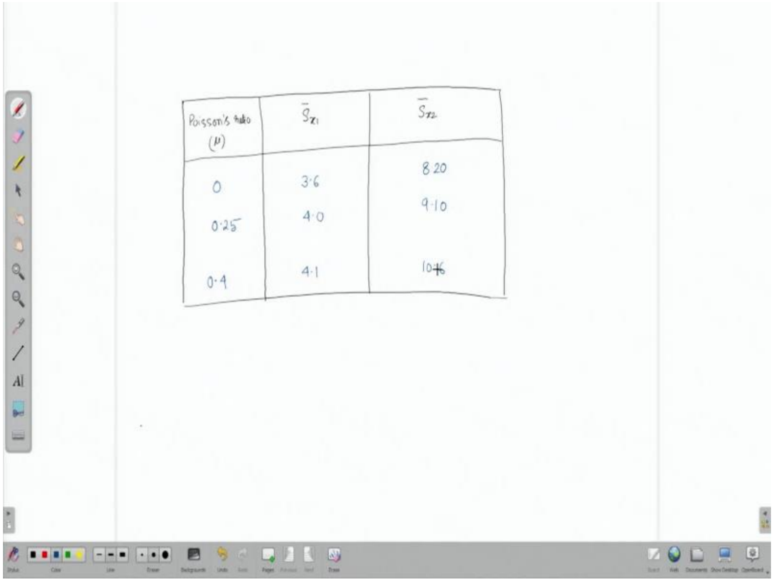

So, that means I can draw a table like this. So, in that table there are three columns so, first column is say, the value for Poison’s ratio of soil mu, second column say the value of S x1 bar, and the third column S x2 bar. So, when Poison’s ratio is 0, that time the value of S x1 bar is 3.6 and the value of S x2 bar is 8.20. When the value of Poison’s ratio is 0.25 that time the value of S x1 bar is 4 and the value of S x2 bar is 9.10. The third case is Poison’s ratio is equal to 0.4 and that time the value of S x1 bar is 4.1 and S x2 bar is 10.6. So, if we know the values of S x1 Poison’s ratio from that knowledge we can calculate we can use the value of S x1 bar and S x2 bar from this table.

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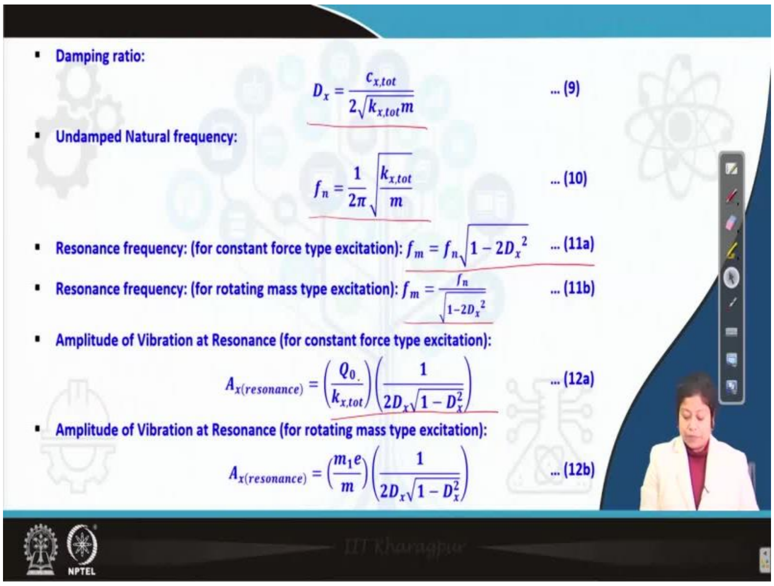

Now, after knowing stiffness and damping factor for a total stiffness and damping factor for the pile cap and the pile group together, next what we need to do we need to find out the natural frequency. So, for undamped system, we can find out natural frequency by using this equation 10, then we can also find out the damping ratio using equation 9. After knowing damping ratio and the natural frequency for the undamped system, we can find out the natural frequency for that damped system, also we can find out the resonance frequency.

So, for resonance frequency, we can use equation 11 a, we are aware that the vibration is produced by constant force type excitation, for rotating mass type excitation, we should use equation 11 b to calculate the resonance frequency. Next is to find out the amplitude of vibration at resonance. So, if it is for constant force type excitation, then we will use equation 12 a that means, this equation if it is rotate for rotating mass type excitation, then we will use equation 12 b. So, here Q0 for in equation 12 a, Q0 is the amplitude of the dynamic force, vertical vibration.

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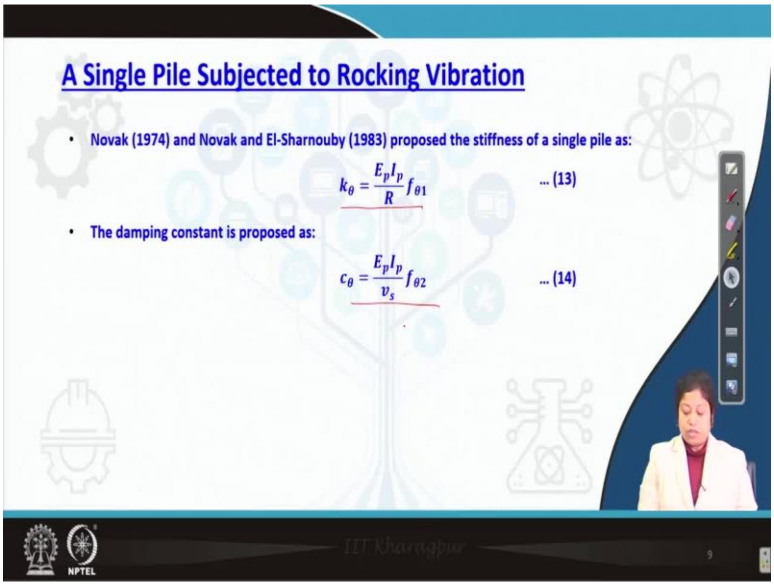

Next thing is a single pile subjected to rocking vibration. So, for that also Novak 1974 and Novak and El-Sharnouby 1983 proposed the solution using which we can find out the stiffness of a single pile. As you can see here in equation 13 also, we can find out the damping factor as shown in equation 14 this one. So, here Ep we already know that is the modulus of elasticity for the pile material Ip is the moment of inertia of the pile cross section, V is the velocity of the shear wave, R is the radius of the pile, f theta 1 and f theta 2 can be calculated using that table in the next slide.

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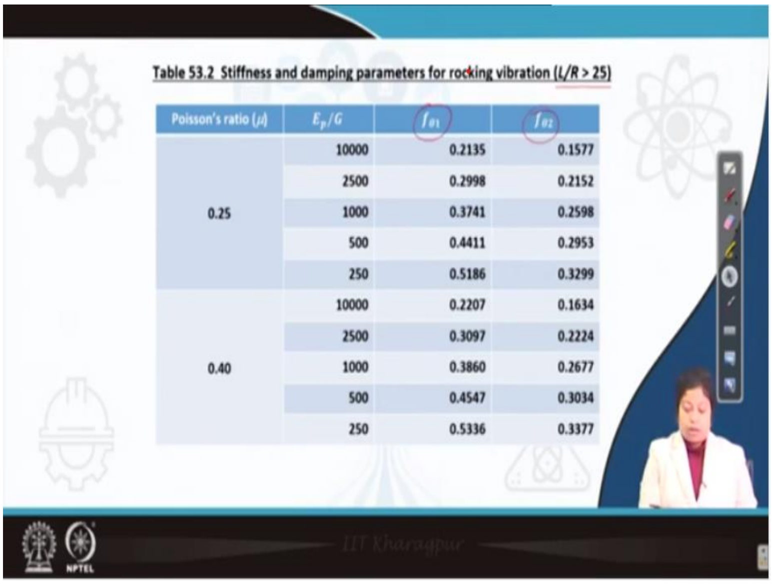
Table 53.2 Stiffness and damping parameters for rocking vibration (L/R > 25)

Poisson's ratio (μ)	E_p/G	$f_{\theta 1}$	$f_{\theta 2}$
0.25	10000	0.2135	0.1577
	2500	0.2998	0.2152
	1000	0.3741	0.2598
	500	0.4411	0.2953
	250	0.5186	0.3299
0.40	10000	0.2207	0.1634
	2500	0.3097	0.2224
	1000	0.3860	0.2677
	500	0.4547	0.3034
	250	0.5336	0.3377

So, here if L by R ratio is greater than 25, then we can use table 53.2 t find out f theta 1 and f theta 2. Already the I have discussed the trends of fx1 and fx2 so, that is the reason I am not discussing it elaborately you can see the trends for f theta 1 and f theta 2 are more or less same to that for fx1 and fx2.

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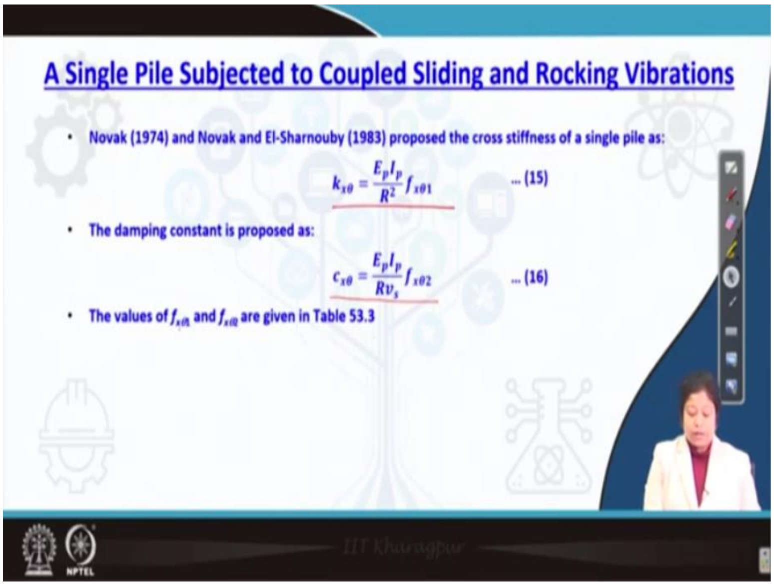

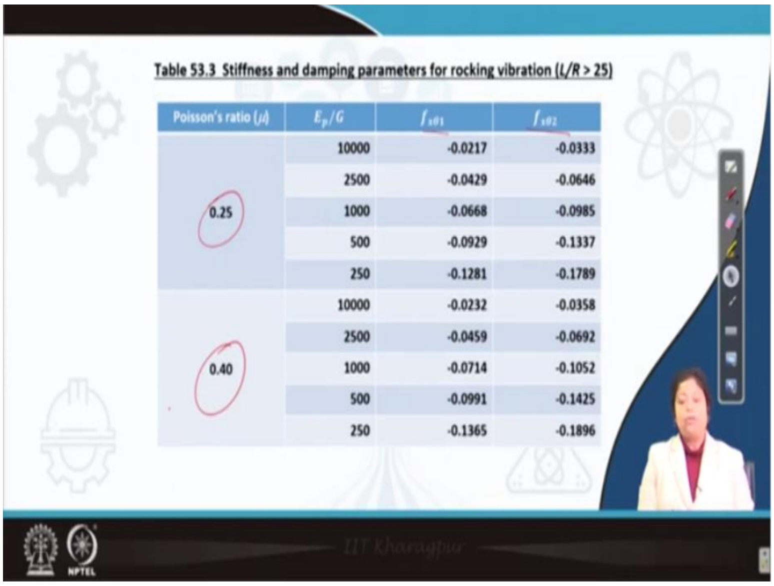

Table 53.3 Stiffness and damping parameters for rocking vibration (L/R > 25)

Poisson's ratio (μ)	E_p/G	$f_{x\theta 1}$	$f_{x\theta 2}$
0.25	10000	-0.0217	-0.0333
	2500	-0.0429	-0.0646
	1000	-0.0668	-0.0985
	500	-0.0929	-0.1337
	250	-0.1281	-0.1789
0.40	10000	-0.0232	-0.0358
	2500	-0.0459	-0.0692
	1000	-0.0714	-0.1052
	500	-0.0991	-0.1425
	250	-0.1365	-0.1896

Next case, what will be happening when a single pile is subjected to coupled sliding and rocking vibrations? So, for that, we can use equation 15 to find out the stiffness for coupled sliding and rocking vibration. So, here you can see the expression which we can use for finding out kx theta which is the stiffness for the single pile subjected to coupled rocking and sliding vibrations. Here fx theta 1 is a non-dimensional parameter.

Similarly, we can find out cx theta using equation 16. Here fx theta 2 is another non dimensional parameter which depends upon the ratio Ep divided by g and the poisons ratio. So, here you can see that values for fx theta 1 and fx theta 2 for different values of Ep divided by g and for different values of portions ratio generally these 2 are required. Now, from these table if required, you can interpolate also the values of fx theta 1 and fx theta 2.

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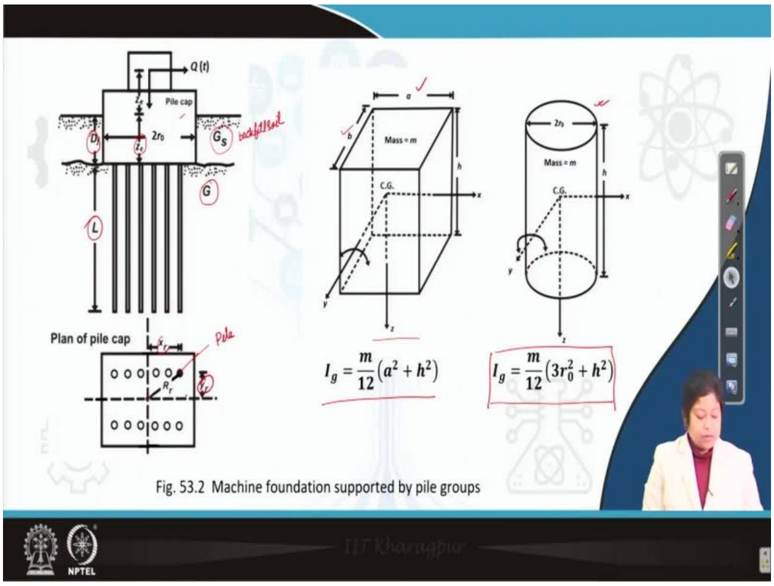

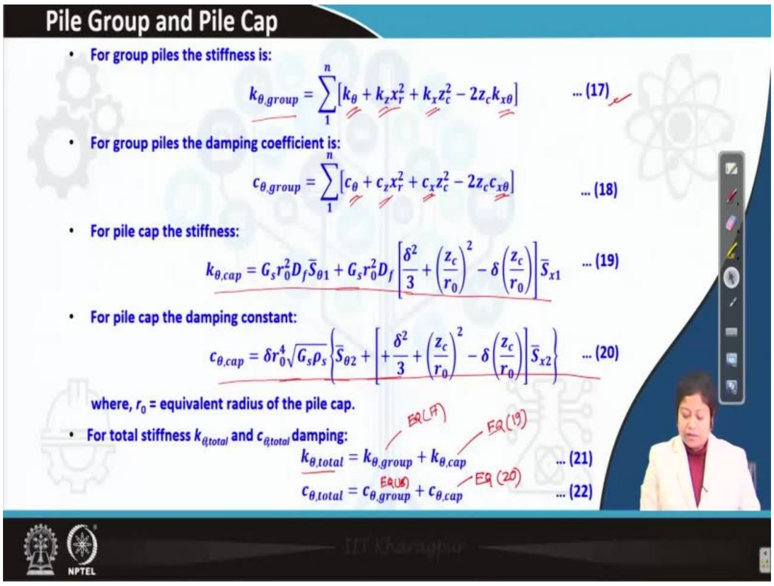

Another thing here what we can see in this figure, you can see a machine foundation. So, this is the machine so, you can see the machine foundation which is supported by pile groups. So, there are 6 piles in one row and there are 2 rows. So, total you can see here 12 piles are used and the length of each pile is L. What about the pile cap? If the pile cap is rectangular, then its width or length is 2r0 or whatever will be given like this case you can see its dimension along the x direction is a and along y direction is b or it may be circular it may be of circular cross sectional area which you can see here.

So, 2r0 is the diameter of the circular cross section. So, this is pile cap, this 12 are the piles of length L, G is the shear modulus of the soil in which piles are embedded, Gs is the shear modulus of the backfill soil, so this soil is called backfill soil. Here you can see Df is the embedment depth of the pile cap. And for rectangular cross sectional area, we can find out I g using this expression, whereas for, so in this case, it is rocking about Y axis that we need to remember and if it is if it is cylindrical, if the pile cap that then we can find out I g using the expression shown here.

So, after knowing all these parameters related to the geometry of the pile cap, what we need to do, we need to find out the stiffness damping factor for pile group which you can see here. So, what is zc? So, here you can see zc is this depth that means, the deep below the which is embedded basically. Now, ze already shown what is xr, you can see suppose I am interested to analyse this pile, so its distance along the x direction from the C G that is xr likewise, its distance along y direction from the C G is yr, as you can see here.

So, for x sorry, so, for k theta for pile group that means, k theta group, we can use equation 17 for group piles, that damping coefficient can be calculated using equation 18. Already we know how to calculate c theta, k theta, also we know how to calculate the cz, kz, last to last class that we have determined today, we also have learned how to calculate kx and cx and kx theta and cx theta.

So, using those expression finally, we can get k theta for pile group and c theta for pile group then, we need to find out stiffness and damping constant for pile cap. So, for pile cap stiffness can be calculated using equation 19. Similarly, for pile cap the damping constant can be calculated using this expression. So, now, if we will add expression, so this is given in equation 17 and this is given in equation 19. So, if we will act the right hand side of equation 17 and equation 19, then we will get k theta total. Likewise, if we will add equation 18 and equation 2,0 then we will get c theta total.

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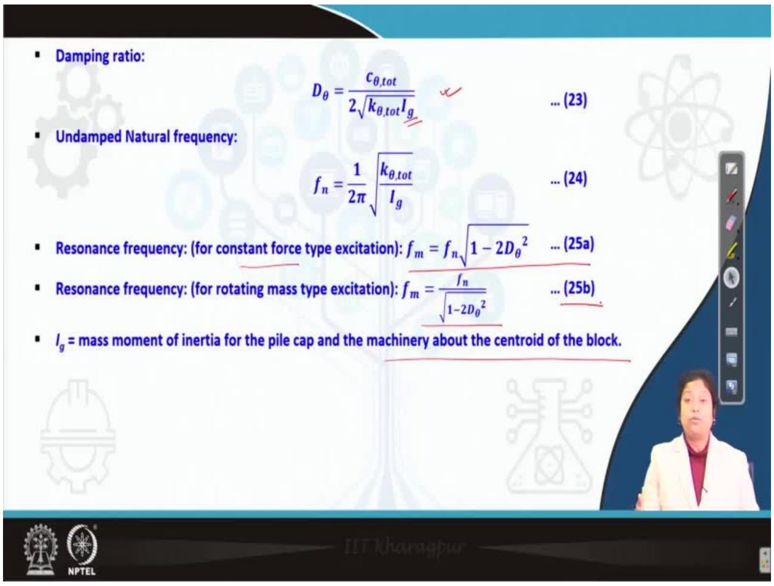

After finding out k theta and c theta, k theta and c theta here the stiffness and damping factor respectively for the pile cap and group together when it is subjected to coupled rocking and sliding vibrations that time the stiffness and the damping constant. So, if we know that then from that we our next target should be to find out the damping ratio which is d theta using equation 23. Already we know how to find out I g which is the mass moment of inertia for the pile cap and machinery about the centroid of the foundation block.

So, d theta we will calculate also we will calculate f n which is the natural frequency of the undamped system, after finding out d theta and f n we can find out the resonance frequency either by using equation 25 a or equation 25 b. What is the criteria to check here what is the source of excitation, if source of excitation is constant force type does not depend upon the frequency operating frequency then we will use equation 25 a, if it is because of the rotating mass type excitation then we will use equation 25 b. So, in this way we can find out the different components like k theta, c theta amplitude at resonance frequency then the resonance frequency itself etcetera.

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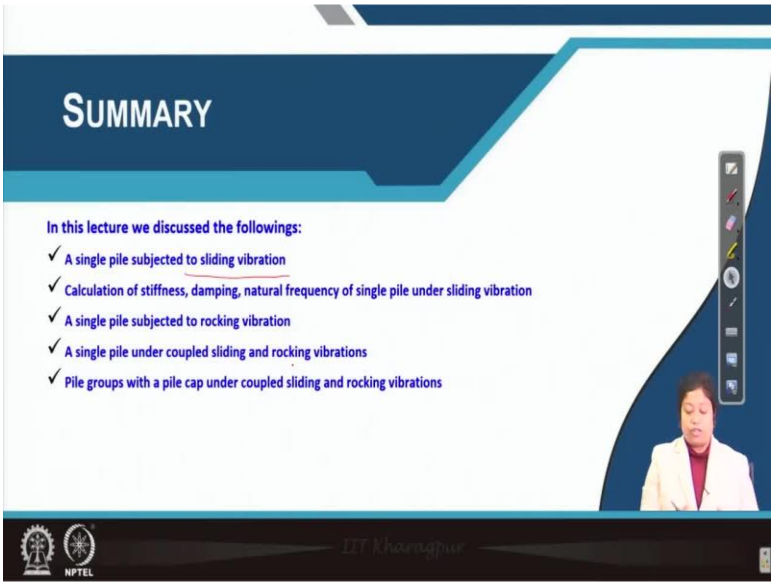

Now, come to the summary of today's class. So, in this lecture first we have studied how to calculate stiffness and damping constant which equation will be used to find out stiffness and to find out the damping constant for a single pile subjected to sliding vibration. Then, we have studied how to calculate stiffness, damping, natural frequency of single pile under sliding vibration.

Then what we have done, we have studied which equations will be used to find out the stiffness and damping constant for a pile subjected to rocking vibration. Then we studied a single pile under sliding and the rocking vibrations together what will be the expression for stiffness and damping constant.

Then what is that expression of stiffness and damping constant for pile cap, then we have studied that for total stiffness and total damping constant, we need to add the stiffness for piles in a group and the stiffness of the pile cap. So, in this way, we have discussed different components related to the calculation of stiffness and damping constant of pile group subjected to coupled rocking and horizontal sorry sliding vibrations subjected to coupled rocking and sliding vibrations.

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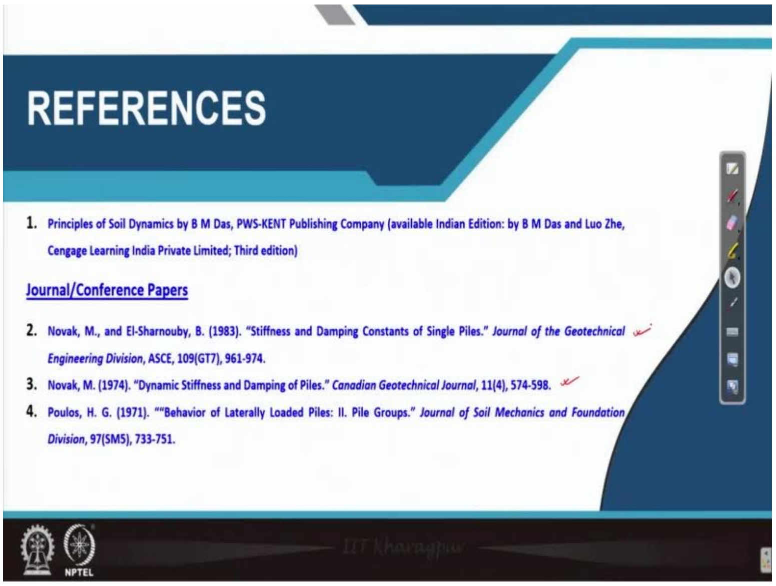

So, these are the references mainly here these two references are used to discuss how to calculate the stiffness and damping constant for different conditions. Then thank you. We will meet the next class to discuss the pile foundation subjected to torsional vibration. Thank you.