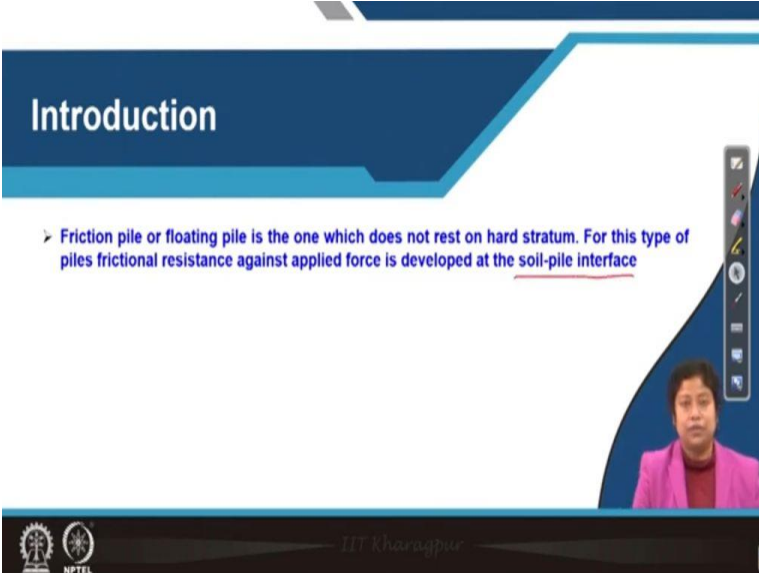


Soil Dynamics
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Lecture No. 52

Analysis of Pile Foundation Under Dynamic Loading (Part - II)

Hello friends, today we will continue our discussion on analysis of pile foundation under dynamic loading; it will be good to say under machine foundation loading.

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The screenshot shows a presentation slide with a blue header containing the word "Introduction". Below the header, a bullet point states: "Friction pile or floating pile is the one which does not rest on hard stratum. For this type of piles frictional resistance against applied force is developed at the soil-pile interface". The text "soil-pile interface" is underlined. In the bottom right corner, there is a small video inset of a woman in a pink jacket. At the bottom of the slide, there are logos for IIT Kharagpur and NPTEL.

So, in last class we have discussed the end bearing pile subjected to vertical vibration. Now, the second type of pile is friction pile or floating pile. So, that is the one which does not rest on the hard stratum. So, basically for this type of pile it is penetrates; it is, it penetrates through the good quality of soil stratum And for these type of piles, frictional resistance against the applied force is developed at the soil pile interface.

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The slide is titled "Friction Piles" and lists four assumptions for analysis. To the right is a diagram of a pile of length L and radius R embedded in soil. A foundation block is at the top, and a vertical load $Q + Q_{ult}$ is applied. The soil is labeled "Soil" and the pile is labeled "Pile radius R ". The diagram is captioned "Fig. 52.1 Friction pile".

Friction Piles

- **Assumptions**
- 1. The pile is vertical, elastic, and circular in cross section.
- 2. The pile is floating.
- 3. The pile is perfectly connected to the soil.
- 4. The soil above the pile tip behaves as infinitesimal, thin, independent linearly elastic layers.

Fig. 52.1 Friction pile

So, here you can see a diagram of friction pile; the length of the pile is capital L , R is the radius of this pile. Now, you can see at the top of this pile, one foundation block is resting; you can think it as pile cap also. And the vertical machine load that means dynamic loading is acting on the because of the machine on this system. So, how we will do the analysis of this type of problem? Before solving the problem, we need to know what are the assumptions that we can use or we can adapt for analysis of friction piles. So, the first assumption is that the pile is vertical, material is elastic, and cross-section is circular; these three things.

Now, if it is, if the cross-section is not circular but rectangular or square, then we need to find out the equivalent radius, that we have already seen how to find out when we have studied the design of block foundation using the elastic half space theory. Second assumption is that the pile is floating type pile. Third assumption: the pile is perfectly connected to the surrounding soil. Fourth assumption says that the soil above the pile tip behaves as infinitesimal, thin, independent linearly elastic layers.

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Following Novak and El-Sharnouby (1983) soil is characterized as:

- ✓ Stiffness (k_z):
$$k_z = \left(\frac{E_p A}{R} \right) f_{z1} \quad \dots (1)$$
- ✓ Damping (c_z):
$$c_z = \left(\frac{E_p A}{\sqrt{G/\rho}} \right) f_{z2} \quad \dots (2)$$

where,

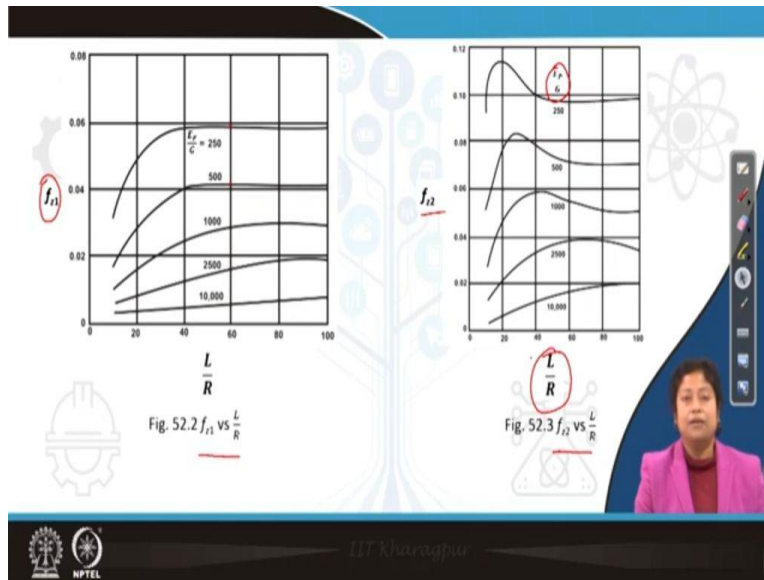
- f_{z1} and f_{z2} are non-dimensional parameters
- A is cross-sectional area of pile
- E_p is the elastic modulus of pile material.

The slide also features decorative icons of gears, a lightbulb, a tree, a hard hat, and a beaker. At the bottom, there are logos for IIT Kharagpur and NPTEL.

Now, with this assumption, basically we will find out the stiffness and damping using the solution provided by Novak and others. So, what is the solution? Stiffness K_z can this can be calculated by using the equation 1. Here you can see what is E_p ? E_p is the Young's modulus of the pile material, A is the cross-sectional area of the pile, R is its equivalent radius. Or if the pile is circular, pile is of circular cross-section, then we can take R is the radius. What is f_{z1} ? f_{z1} is a factor which is shown in the next slide; I will come to that.

Before that let us see the damping expression which we can use to calculate the damping. So, here is the equation 2 which can be used to determine the coefficient of damping, C_z . So, here G is the shear modulus or dynamic shear modulus of the soil, ρ is the density of the soil, and f_{z2} is a factor.

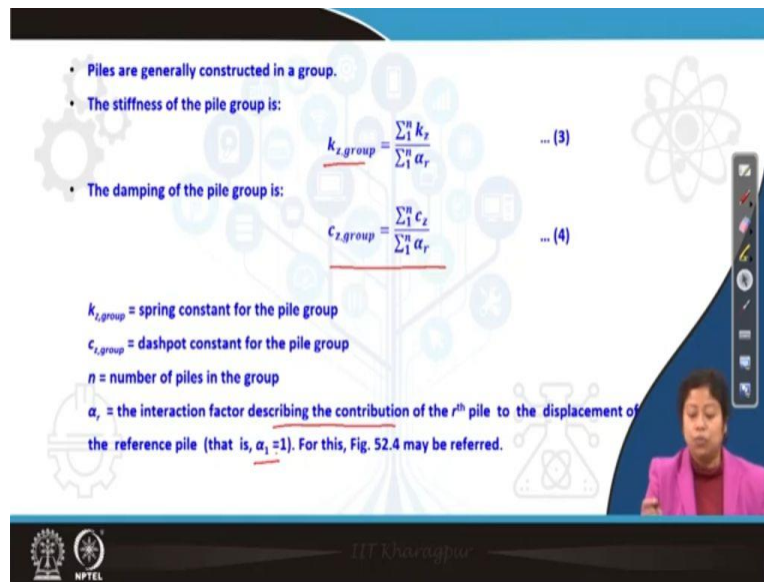
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Now, we will see how to find out f_{z1} and f_{z2} . So, here you can see two different plots; one is f_{z1} versus L by R , and the second one f_{z2} versus L by R . L is the total length of the pile, R is its radius. So, you can see this value of f_{z1} depends upon L by R ratio, and the ratio of E_p divided by G . E_p is that Young's modulus of the pile material and G is the dynamic shear modulus of the soil. So, you can see as the value or the ratio of E_p to G increases, the magnitude of f_{z1} for a particular value of L by R decreases.

For lower E_p by G for an example, when L by R is 60, you can say, you can see here f_{z1} is close to 0.59. Whereas, when the same that means L by R is 60, but E_p by G increases to 500; that time the f_{z1} value decreases to 0.41, you can see it here. Similarly, we can calculate f_{z2} also, if we know the value of E_p by G and L by R .

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• Piles are generally constructed in a group.

• The stiffness of the pile group is:

$$k_{z,group} = \frac{\sum_1^n k_z}{\sum_1^n \alpha_r} \quad \dots (3)$$

• The damping of the pile group is:

$$c_{z,group} = \frac{\sum_1^n c_z}{\sum_1^n \alpha_r} \quad \dots (4)$$

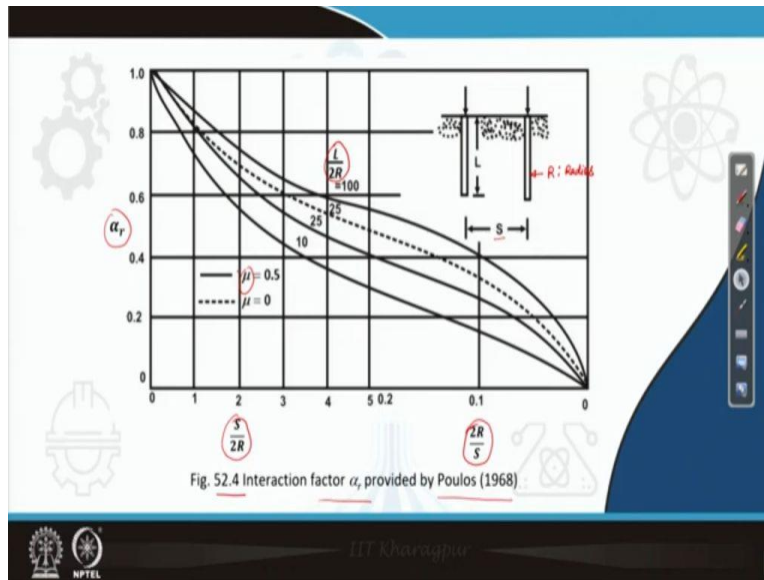
$k_{z,group}$ = spring constant for the pile group
 $c_{z,group}$ = dashpot constant for the pile group
 n = number of piles in the group
 α_r = the interaction factor describing the contribution of the r^{th} pile to the displacement of the reference pile (that is, $\alpha_1 = 1$). For this, Fig. 52.4 may be referred.

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Now, piles are generally constructed in a group. So, when pile is in a group, then we need to find out the stiffness of the pile group. How we will do that? We know the stiffness of individual pile. So, if we will sum it and the sum will be divided by interaction factor alpha r, then summation of alpha r; then we will get the Kz for pile group. Similarly, the damping of the pile group is Cz group and that can be calculated using equation 4 here.

So, what is n here? n is the total number of piles present in the pile group. Alpha r is the interaction factor which describes the contribution of the rth pile to the displacement of the reference pile; that is where alpha1 is equal to 1. Now, how we will calculate alpha r?

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Now, in theory 52.4, we can see how we will get alpha r. Alpha r is interaction factor provided by Poulos in 1968. So, in this figure what we can see? We can see alpha r value depends upon the ratio is the 2R; or you can take it also as 2R divided by S. What is S here? You can see this figure, S is centered to center distance between two piles, and R is radius of this pile; so, R is radius.

Also, you can note here the value of alpha r not only depends upon the ratio of S to 2R; it also depends upon the ratio of L to 2R. 2R means you can call it as diameter also. So, L length by diameter ratio also controls the value of alpha r. Other than spacing by diameter and length by diameter, there is third factor which is Poisson's ratio mu that also can change the alpha r value. So, from this figure we can find out the value of alpha r depending upon the value of S divided by 2R, L divided by 2R, and Poisson's ratio.

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• For embedded foundation:

$$k_{z,cap} = Gr_0 \left[\bar{C}_1 + \frac{G_s D_f \bar{S}_1}{G r_0} \right] \quad \dots (5)$$

$$c_{z,cap} = r_0^2 \left[\bar{C}_2 + \frac{G_s \rho_s D_f \bar{S}_2}{G \rho r_0} \right] \quad \dots (6)$$

• Considering the possibility of the poor quality of soil below the base of the foundation and its shrinking away over the time we can ignore the effect of the cap base and hence $\bar{C}_1 = 0$ and $\bar{C}_2 = 0$.

• Thus, stiffness of the pile cap is:

$$k_{z,cap} = G_s D_f \bar{S}_1 \quad \dots (7)$$

• The damping coefficient of the pile cap is:

$$c_{z,cap} = D_f r_0 \bar{S}_2 \sqrt{G_s \rho_s} \quad \dots (8)$$

• Therefore, stiffness of the pile group is:

$$k_{z,tot} = G_s D_f \bar{S}_1 + \frac{\sum_1^n k_z}{\sum_1^n \alpha_r} \quad \dots (9)$$

• The damping coefficient is:

$$c_{z,tot} = D_f r_0 \bar{S}_2 \sqrt{G_s \rho_s} + \frac{\sum_1^n c_z}{\sum_1^n \alpha_r} \quad \dots (10)$$

The slide also includes a diagram of a pile cap on a pile, with labels for 'Pile cap', 'Pile of radius R', and 'Df'.

Now, in this case what we have seen? We have seen a pile is supporting a block foundation. So, we need to consider the effect of the foundation block also; and if generally what is happened? This foundation block is also embedded. So, we can use the equation of Kz for the embedded foundation to find out the Kz for this foundation block; or we can call it as pile cap also. So, this is the equation which we can use. Here, say similarly, for Cz cap that means Cz value for the pile cap or foundation block, we can use equation 6.

So, here what is happened actually? In this case when we are using pile cap or when there is a foundation block, always there is a foundation block at the top of the pile. So, there is a possibility of poor quality soil below this, below the base of this foundation block; and also it can shrink away over the time we. So, what we can do? We can ignore the effect of the cap base; and as a result, we need to consider the value of C1 bar and C2 bar in equation 5 and 6 equal to 0. So, the in this way basically what we are doing? We are calculating Kz for pile cap; or you can call it as foundation block in conservative side. And that can be calculated by using equation 7.

Likewise, for the damping coefficient of; this is not if, this is of, of the pile cap. What equation we can use? We can simplify equation 6 to this form. So, in equation 6, if we put C2 bar equals to 0; we will get equation 8. So, now we can find out the total stiffness of the pile group as this form. So, this is for the first part for the pile cap or foundation block, and the second part for the pile groups. Same way, we can find out the damping coefficient also in this way.

In equation 10, what is D_f ? D_f is the depth, embedment depth of the pile cap below the ground surface; so, I can draw a figure for you. This is the ground surface. So, pile cap actually some portion of the pile cap is above the ground level; and most of the part is below the ground surface like this. So, this is a pile cap.

Now, this pile cap is on the top of piles; so, if there are four piles, then in elevation we look we can see the thing like this. So, pile of radius R like this; and this is as I said ground surface. So, D_f here is this depth of embedment of the pile cap; that means the portion of the pile cap below the ground surface. r_0 is its equivalent radius, and G_s or ρ_s . G_s is the dynamic shear modulus of the soil surrounding this foundation block, and ρ_s is the density of the soil surrounding this foundation block.

So, now, how we will find out S_2 bar; that is a question. And how we will find out S_1 bar also; because these two parameters are required to calculate the K_z total and C_z total. So, let us see.

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Poisson's ratio (μ)	S_1	S_2
0	2.7	6.7
0.25	2.7	6.7
0.5	2.7	6.7

Using this figure 52.1, we can calculate S_1 bar and S_2 bar; although there is a bar sign, but I think it is not clear that much. So, now in this table what we can see? The value of S_1 and S_2 remain unchanged for all different values of Poisson's ratio. So, if Poisson's ratio is 0, then S_1 value is 2.7; and S_1 bar value is 2.7, and S_2 bar is also 6.7. And this value will remain same for all different values of Poisson's ratio. That means when Poisson's ratio is 0.5, that time also this value is S_1 bar is 2.7, and S_2 bar is 6.7.

Now, after calculating total stiffness and total damping coefficient, that means K_z total and C_z total; what we can.

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▪ Damping ratio:
$$D_z = \frac{C_{z,tot}}{2\sqrt{k_{z,tot}m}} \quad \dots (11)$$

▪ Undamped Natural frequency:
$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_{z,tot}}{m}} \quad \dots (12)$$

▪ Resonance frequency: (for constant force type excitation):
$$f_m = f_n \sqrt{1 - 2D_z^2} \quad \dots (13a)$$

▪ Resonance frequency: (for rotating mass type excitation):
$$f_m = \frac{f_n}{\sqrt{1 - 2D_z^2}} \quad \dots (13b)$$

▪ Amplitude of Vibration at Resonance (for constant force type excitation):
$$A_{z(resonance)} = \left(\frac{Q_0}{k_{z,tot}} \right) \left(\frac{1}{2D_z \sqrt{1 - D_z^2}} \right) \quad \dots (14a)$$

▪ Amplitude of Vibration at Resonance (for rotating mass type excitation):
$$A_{z(resonance)} = \left(\frac{m_1 e}{m} \right) \left(\frac{1}{2D_z \sqrt{1 - D_z^2}} \right) \quad \dots (14b)$$

We need to find out the value for D_z which is shown in equation 11. We already know what is D_z ; D_z is the damping ratio. And that is the ratio of the damping coefficient to the critical damping. So here you can see, we know how to calculate critical damping; that is equal to 2 times square root of stiffness times mass. So, here stiffness means K_z total. After knowing K_z total, we can also calculate natural frequency for the system. And when we know damping ratio, when we know the natural frequency, from that we can calculate resonance frequency using equation 13a. For constant force type excitation, for rotating mass type excitation, we will use equation 13b to calculate for the calculation of resonance frequency.

Similarly, we can find out the amplitude of vibration at resonance using either equation 14a or 14b. 14a will be used if there is constant force type excitation. If there is rotating mass type excitation, then we will use equation 14b.

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Numerical Problem

A wooden pile is shown in Fig. 52.5. The pile has a diameter of 230 mm. Given: $E_p = 8.5 \times 10^6$ kPa. Determine its stiffness and damping constants for vertical vibration

Soil properties:
 $G = 20.7 \times 10^3$ kPa
 $\gamma_p = 17.8$ kN/m³
 $\mu = 0.25$
 $L = 15$ m

Fig. 52.5 Problem statement

The slide features a diagram of a vertical pile of length L = 15 m embedded in soil. The pile has a diameter of 230 mm. The soil properties are listed as G = 20.7 x 10^3 kPa, gamma_p = 17.8 kN/m^3, mu = 0.25, and L = 15 m. The slide also includes the NIPTEL logo and the name 'JIT Khosla' at the bottom.

Now, let us solve one simple numerical problem. So, here what we can see? In this problem, one floating pile or you can call it as fiction pile is shown. It is made of wooden wood; so, the diameter of the pile is given, which is 230 millimeter. So, from that we can calculate the radius r , which is 230 divided by 2; it will come 150 millimeter. So, I am just writing here itself. The length of the pile is also given which is 15 meter; the soil property also provided. You can see G is 20.7 into 10 to the power 3 kPa, γ_p is 17.8 kilo Newton per meter cube, and Poisson's ratio is 0.25.

So, what we need to find out? We need to find out the stiffness and damping constant for vertical vibration; so, let us do this exercise quickly.

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$$\begin{aligned} \text{Given: } E_p &= 8.5 \times 10^6 \text{ kPa} \\ D_a &= 230 \text{ mm} \quad R = 115 \text{ mm} \\ G &= 20.7 \times 10^3 \text{ kPa} \quad \gamma_p = 17.8 \text{ kN/m}^3 \\ \mu &= 0.25 \quad L = 15 \text{ m} \\ \frac{L}{R} &= \frac{15000}{115} = 130.43 \\ \frac{E_p}{G} &= \frac{8.5 \times 10^6}{20.7 \times 10^3} = 410.63 \end{aligned} \quad \Rightarrow \quad \left. \begin{aligned} f_{z1} &= 0.045 \\ f_{z2} &= 0.078 \end{aligned} \right\}$$

$$k_2 = \left(\frac{E_p A}{R} \right) f_{z1} = \frac{(8.5 \times 10^6) \left(\frac{\pi}{4} \right) (0.23)^2}{0.115} (0.045) \text{ kN/m}$$

$$= 0.1382 \times 10^6 \text{ kN/m} \approx 138 \times 10^6 \text{ N/m}$$

So, given parameter where I can write; let me just take what is given. So, given parameter is E_p which is 8.5 into 10 to the power 6 kPa. Then, diameter is given; I can write it as Dia 230 millimeter, from which I can calculate 115 millimeter. G is equal to 20.7 into 10 to the power 3 kPa, γ_p is equal to 17.8 kilo Newton per meter cube. So, next μ which is Poisson's ratio 0.25, and L is 15 meter. So, from this first thing which we will find out is E_p divided by G . G is the shear modulus of the soil; E_p is Young's modulus of the biomaterial.

So, 8.5 into 10 to the power 6, G is 20.7 into 10 to the power 3; both the same unit kPa. So, we are getting this is equal to 410.63, 410.63. So, from this now if I will use this equation, I need to do interpolation; because you can see E_p by G is equal to 250 and 500 is given, same for the f_{z2} curve also. So, after interpolating what I am getting? I am just writing directly here with this E_p by G value; and I also need to find out L by R . So, L by R in this case is 15000 divided by 115, all are expressed in millimeter. So, what I am getting is 130.43. So, with these two values, I can get f_{z1} and f_{z2} using the two figures, which I have just shown you.

And it will be f_{z1} is 0.045, approximately I am writing; and f_{z2} is approximately 0.078. Actually, my task is little bit easy; because you can see for L by R 100 or above 100, both the curves become horizontal. So, I directly interpolate from these two values. Likewise, here also from these two values, I can directly interpolate; and find these two values for f_{z1} and f_{z2} respectively.

Now, what I need to do? I need to find out K_z , K_z for the pile. So, this is for this, I can use this equation times fz_1 . So, E_p is given which is 8.5×10^6 in kilo Newton per square meter times A . A means radius is given, diameter is also given. So, using either radius or diameter, you can find out the value; so, it is π by 4 times 0, sorry 230. So, 0.23 square, this is area divided by R , which is 0.115 times fz_1 ; so, fz_1 means 0.045.

So, now, what will be the unit? Unit is kilo Newton per meter. It is meter, it is, this is meter, this is kilo Newton per square meter times meter square; so, you can see the unit is kilo Newton per meter. So how much you are getting from this? So, I am getting 0.1382×10^6 kilo Newton per meter; or I can write it also as 138×10^6 Newton per meter. So, in this way we can calculate K_z .

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$$Q_z = \left(\frac{E_p A}{\sqrt{G/p}} \right) f_{z_1}$$

$$= \left[\frac{8.5 \times 10^9 \times \left(\frac{\pi}{4} \times 0.23^2 \right)}{\sqrt{\frac{20.7 \times 10^3}{1814.48}}} \right] (0.078)$$

$$= 25.79 \times 10^4 \text{ N/m}$$

Numerical Problem

A wooden pile is shown in Fig. 52.5. The pile has a diameter of 230 mm. Given: $E_p = 8.5 \times 10^6$ kPa. Determine its stiffness and damping constants for vertical vibration



$$G = 20.7 \times 10^3 \text{ kPa}$$

$$\gamma_p = 17.8 \text{ kN/m}^3$$

$$\mu = 0.25$$

$$L = 15 \text{ m}$$

$$\rho_p = 1814.48 \text{ kg/m}^3$$

Fig. 52.5 Problem statement

$$\text{Given: } E_p = 8.5 \times 10^6 \text{ kPa}$$

$$D_p = 230 \text{ mm} \quad R = 115 \text{ mm}$$

$$G = 20.7 \times 10^3 \text{ kPa} \quad \gamma_p = 17.8 \text{ kN/m}^3$$

$$\mu = 0.25 \quad L = 15 \text{ m}$$

$$k_1 = \frac{15000}{115} = 130.43$$

$$\frac{E_p}{G} = \frac{8.5 \times 10^6}{20.7 \times 10^3} = 410.63 \quad \Rightarrow \quad \left. \begin{array}{l} f_{21} = 0.045 \\ f_{22} = 0.078 \end{array} \right\}$$

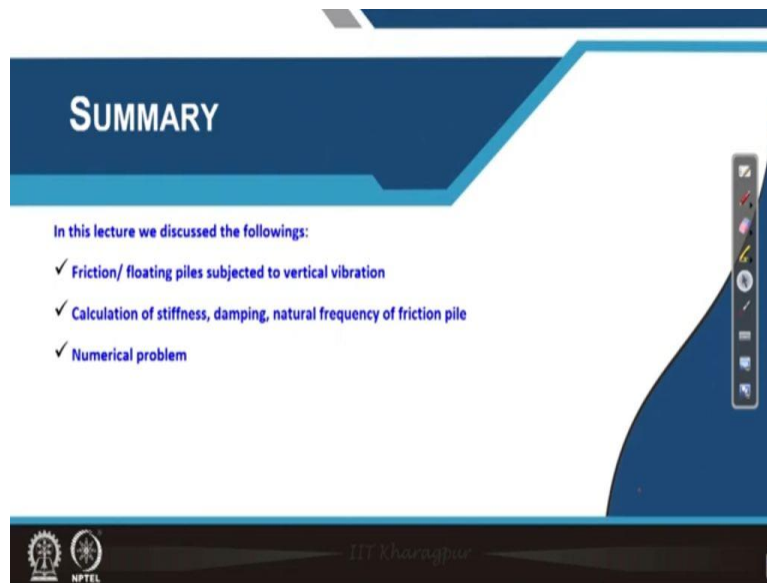
$$k_2 = \left(\frac{E_p A}{R} \right) f_{21} = \frac{(8.5 \times 10^6) (\pi/4) (0.23)^2}{0.115} (0.045) \text{ kN/m}$$

$$= 0.1382 \times 10^6 \text{ kN/m} \approx 138 \times 10^6 \text{ N/m}$$

Now, for C_z , we have the equation E_p divided; sorry, E_p times A , divided by square root of G by ρ . So, G and ρ both are related to the soil only times f_{z2} ; already we know the value for E_p . This is 8, what I can do here 8.5; 8.5 into 10 to the power 9; I am expressing it in Newton per square meter times area. So, π by 4 times 0.23 square, divided by square root of G ; that means 20.7 into 10 to the power 3. ρ means 17 point, sorry I can; so, this is our numerical problem. So, here you can see ρ value is 17.8 in kilo Newton per meter cube; 17.8 into 10 to the power 3, divided by G that will give the value of ρ_p . So, value of ρ_p is 1814.48 in kg per meter cube.

So, here I can write this value itself 1814.48. So, I can times f_z^2 ; if I will go to previous page, f_z^2 is 0.078. So, with this now we can calculate the value for C_z . So, it is coming, I am getting 25.79 into 10 to the power 4 in Newton second per meter. So, with this value so the final answer is, this is for coefficient of damping and this is the K_z value. So, this is actually asked in this problem; and in this problem if you see single pile is mentioned, not pile group. That is the reason we have not summed it. So, with this I think now I can summarize today's class.

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So, today we have discussed first how to solve the problem of the friction pile or floating pile subjected to vertical vibration. How to Calculate stiffness damping natural frequency for friction pile, then how to find out the resonance frequency and resonance amplitude. And finally, we have discussed one numerical problem only considering the pile portion.

(Refer Slide Time: 31:06)

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So, these are the references that I have used in today's class. So, this is one curve which this reference for the to get the value of alpha r actually; and this reference for fz1 and fz2 value and the textbook. So, I am concluding today's class now here. Thank you.