Soil Dynamics
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Lecture No. 52
Analysis of Pile Foundation Under Dynamic Loading (Part - II)

Hello friends, today we will continue our discussion on analysis of file foundation under dynamic loading; it will be good to say under machine foundation loading.

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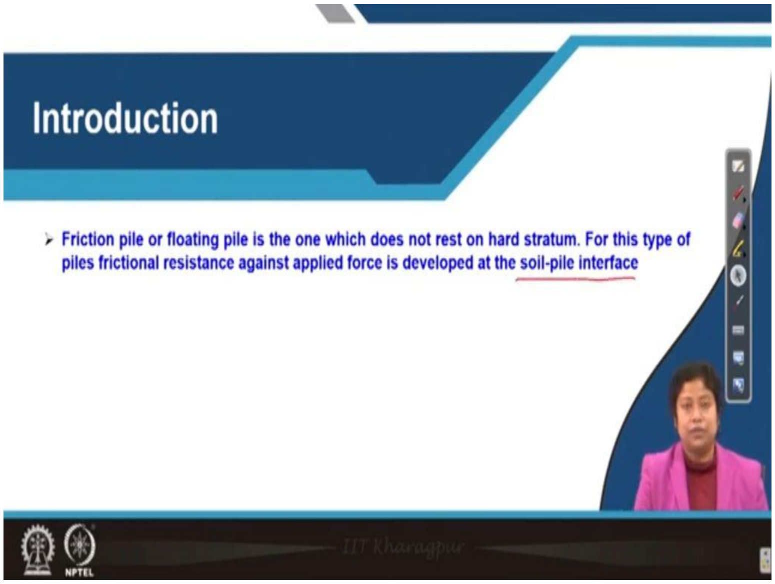

So, in last class we have discussed the end bearing pile subjected to vertical vibration. Now, the second type of pile is friction pile or floating pile. So, that is the one which does not rest on the hard stratum. So, basically for this type of pile it is penetrates; it is, it penetrates through the good quality of soil stratum And for these type of piles, frictional resistance against the applied force is developed at the soil pile interface.

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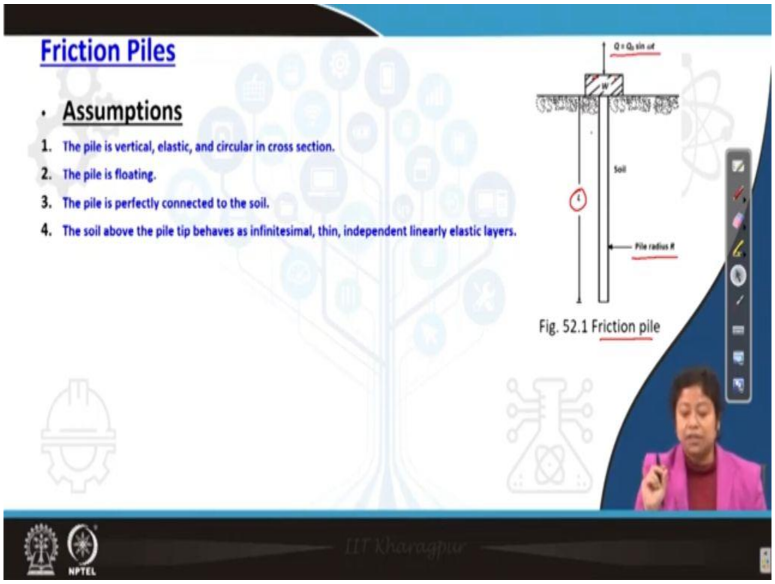

So, here you can see a diagram of friction pile; the length of the pile is capital L, R is the radius of this pile. Now, you can see at the top of this pile, one foundation block is resting; you can think it as pile cap also. And the vertical machine load that means dynamic loading is acting on the because of the machine on this system. So, how we will do the analysis of this type of problem? Before solving the problem, we need to know what are the assumptions that we can use or we can adapt for analysis of friction piles. So, the first assumption is that the pile is vertical, material is elastic, and cross-section is circular; these three things.

Now, if it is, if the cross-section is not circular but rectangular or square, then we need to find out the equivalent radius, that we have already seen how to find out when we have studied the design of block foundation using the elastic half space theory. Second assumption is that the pile is floating type pile. Third assumption: the pile is perfectly connected to the surrounding soil. Fourth assumption says that the soil above the pile tip behaves as infinitesimal, thin, independent linearly elastic layers.

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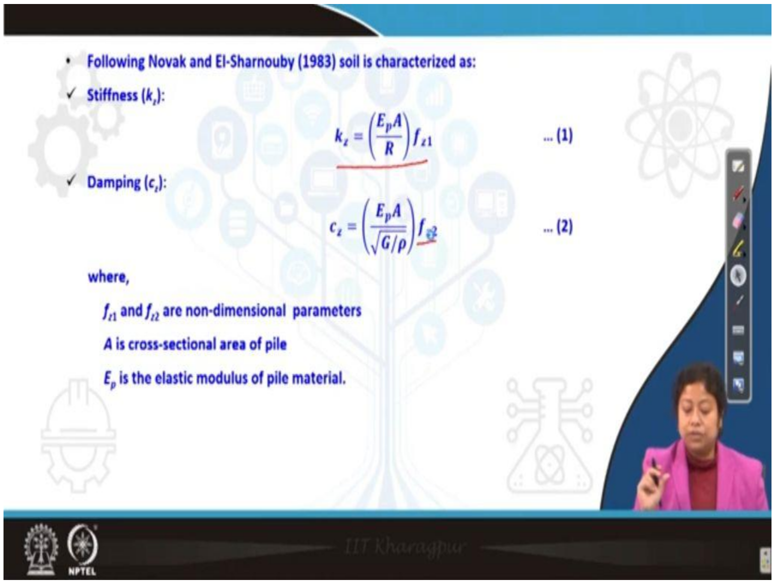

Now, with this assumption, basically we will find out the stiffness and damping using the solution provided by Novak and others. So, what is the solution? Stiffness Kz can this can be calculated by using the equation 1. Here you can see what is Ep? Ep is the Young's modulus of the pile material, A is the cross-sectional area of the pile, R is its equivalent radius. Or if the pile is circular, pile is of circular cross-section, then we can take R is the radius. What is fz1? Fz1 is a factor which is shown in the next slide; I will come to that.

Before that let us see the damping expression which we can use to calculate the damping. So, here is the equation 2 which can be used to determine the coefficient of damping, Cz. So, here G is the shear modulus or dynamic shear modulus of the soil, rho is the density of the soil, and fz2 is a factor.

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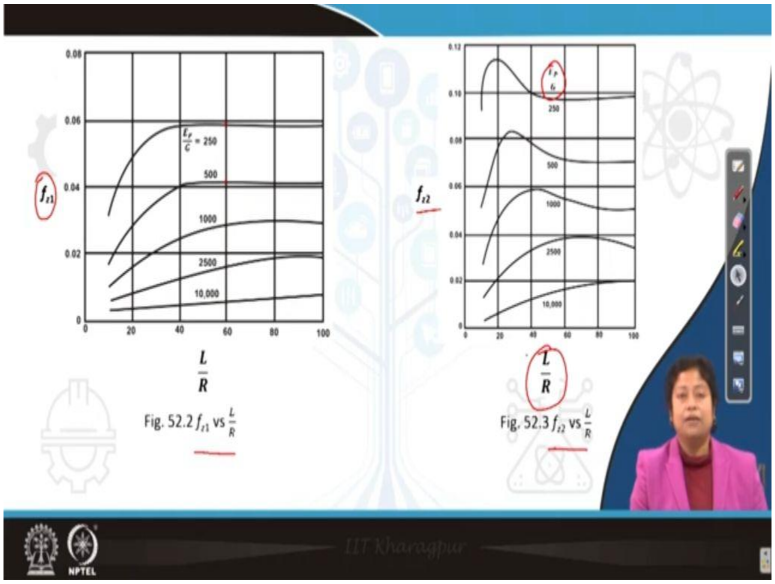

Now, we will see how to find out fz1 and fz2. So, here you can see two different plots; one is fz1 versus L by R, and the second one fz2 versus L by R. L is the total length of the pile, R is its radius. So, you can see this value of fz1 depends upon L by R ratio, and the ratio of Ep divided by G. Ep is that Young's modulus of the pile material and G is the dynamic shear modulus of the soil. So, you can see as the value or the ratio of Ep to G increases, the magnitude of fz1 for a particular value of L by R decreases.

For lower Ep by G for an example, when L by R is 60, you can say, you can see here fz1 is close to 0.59. Whereas, when the same that means L by R is 60, but Ep by G increases to 500; that time the fz1 value decreases to 0.41, you can see it here. Similarly, we can calculate fz2 also, if we know the value of Ep by G and L by R.

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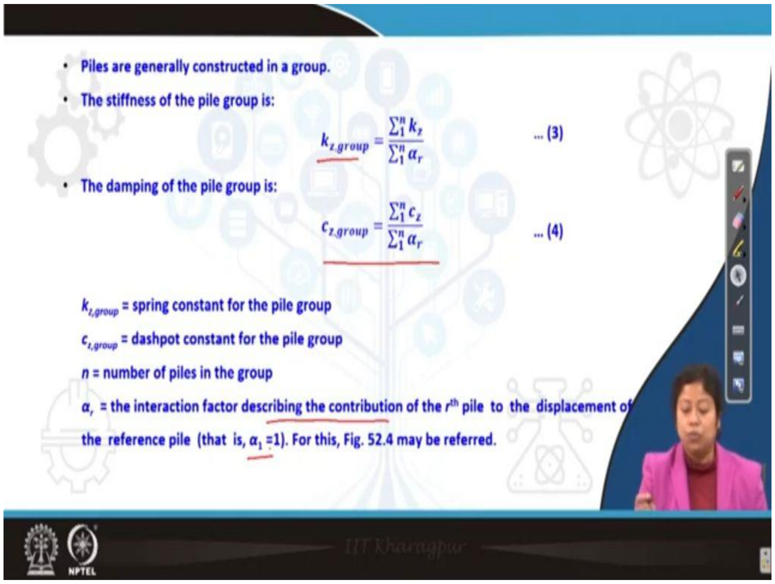

Now, piles are generally constructed in a group. So, when pile is in a group, then we need to find out the stiffness of the pile group. How we will do that? We know the stiffness of individual pile. So, if we will sum it and the sum will be divided by interaction factor alpha r, then summation of alpha r; then we will get the Kz for pile group. Similarly, the damping of the pile group is Cz group and that can be calculated using equation 4 here.

So, what is n here? n is the total number of piles present in the pile group. Alpha r is the interaction factor which describes the contribution of the rth pile to the displacement of the reference pile; that is where alpha1 is equal to 1. Now, how we will calculate alpha r?

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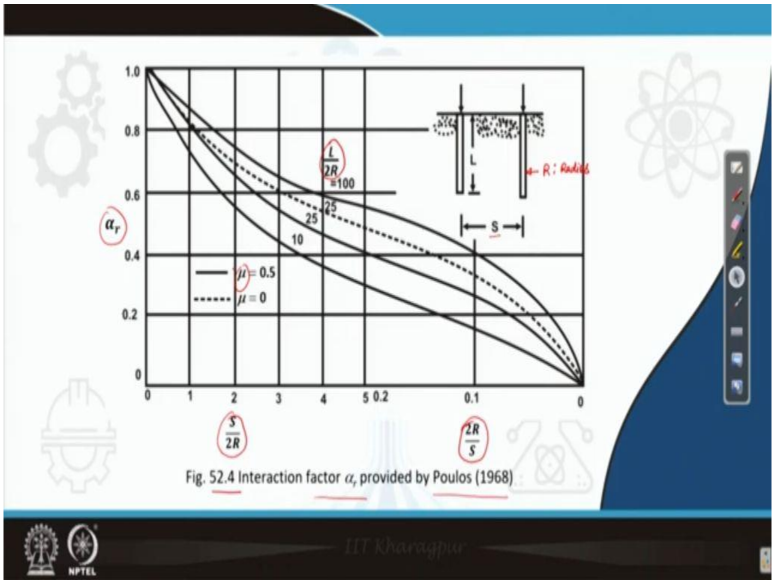

Now, in theory 52.4, we can see how we will get alpha r. Alpha r is interaction factor provided by Poulos in 1968. So, in this figure what we can see? We can see alpha r value depends upon the ratio is the 2R; or you can take it also as 2R divided by S. What is S here? You can see this figure, S is centered to center distance between two piles, and R is radius of this pile; so, R is radius.

Also, you can note here the value of alpha r not only depends upon the ratio of S to 2R; it also depends upon the ratio of L to 2R. 2R means you can call it as diameter also. So, L length by diameter ratio also controls the value of alpha r. Other than spacing by diameter and length by diameter, there is third factor which is Poisson's ratio mu that also can change the alpha r value. So, from this figure we can find out the value of alpha r depending upon the value of S divided by 2R, L divided by 2R, and Poisson's ratio.

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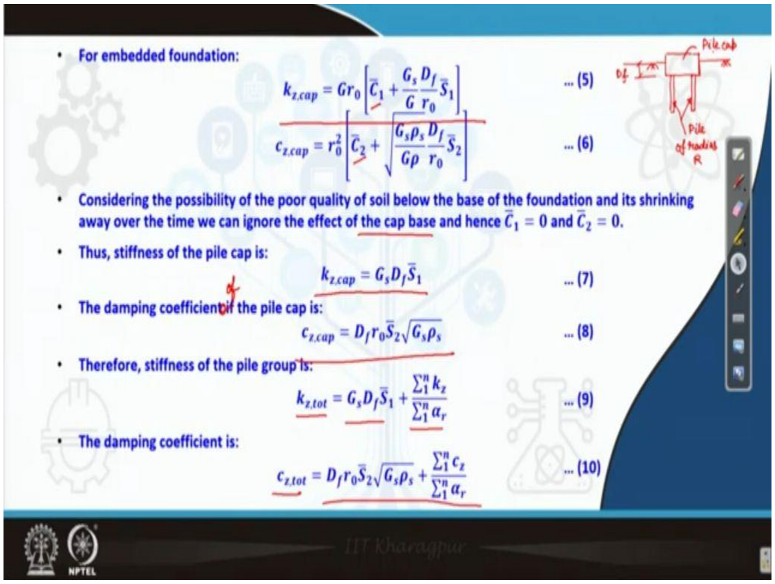

Now, in this case what we have seen? We have seen a pile is supporting a block foundation. So, we need to consider the effect of the foundation block also; and if generally what is happened? This foundation block is also embedded. So, we can use the equation of Kz for the embedded foundation to find out the Kz for this foundation block; or we can call it as pile cap also. So, this is the equation which we can use. Here, say similarly, for Cz cap that means Cz value for the pile cap or foundation block, we can use equation 6.

So, here what is happened actually? In this case when we are using pile cap or when there is a foundation block, always there is a foundation block at the top of the pile. So, there is a possibility of poor quality soil below this, below the base of this foundation block; and also it can shrink away over the time we. So, what we can do? We can ignore the effect of the cap base; and as a result, we need to consider the value of C1 bar and C2 bar in equation 5 and 6 equal to 0. So, the in this way basically what we are doing? We are calculating Kz for pile cap; or you can call it as foundation block in conservative side. And that can be calculated by using equation 7.

Likewise, for the damping coefficient of; this is not if, this is of, of the pile cap. What equation we can use? We can simplify equation 6 to this form. So, in equation 6, if we put C2 bar equals to 0; we will get equation 8. So, now we can find out the total stiffness of the pile group as this form. So, this is for the first part for the pile cap or foundation block, and the second part for the pile groups. Same way, we can find out the damping coefficient also in this way.

In equation 10, what is Df? Df is the depth, embedment depth of the pile cap below the ground surface; so, I can draw a figure for you. This is the ground surface. So, pile cap actually some portion of the pile cap is above the ground level; and most of the part is below the ground surface like this. So, this is a pile cap.

Now, this pile cap is on the top of piles; so, if there are four piles, then in elevation we look we can see the thing like this. So, pile of radius R like this; and this is as I said ground surface. So, Df here is this depth of embedment of the pile cap; that means the portion of the pile cap below the ground surface. r0 is its equivalent radius, and Gs or rho s. Gs is the dynamic shear modulus of the soil surrounding this foundation block, and rho s is the density of the soil surrounding this foundation block.

So, now, how we will find out S2 bar; that is a question. And how we will find out S1 bar also; because these two parameters are required to calculate the Kz total and Cz total. So, let us see.

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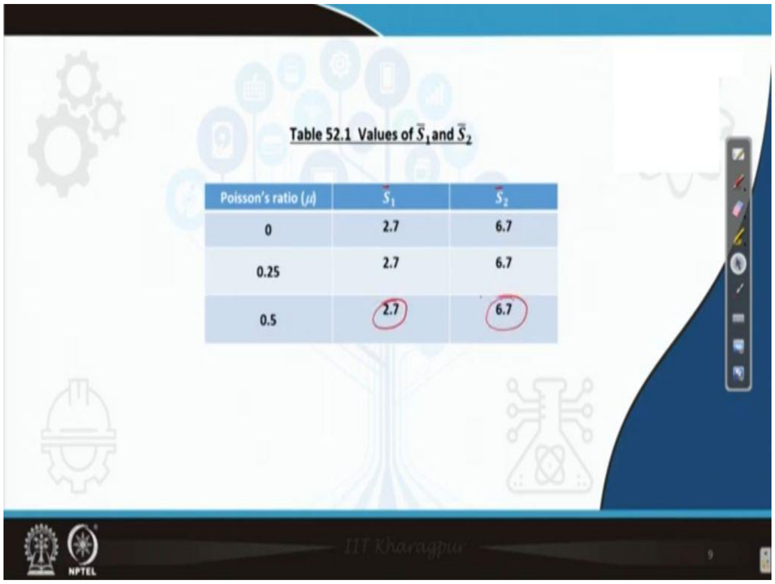

Using this figure 52.1, we can calculate S1 bar and S2 bar; although there is a bar sign, but I think it is not clear that much. So, now in this table what we can see? The value of S1 and S2 remain unchanged for all different values of Poisson’s ratio. So, if Poisson’s ratio is 0, then S1 value is 2.7; and S1 bar value is 2.7, and S2 bar is also 6.7. And this value will remain same for all different values of Poisson’s ratio. That means when Poisson’s ratio is 0.5, that time also this value is S1 bar is 2.7, and S2 bar is 6.7.

Now, after calculating total stiffness and total damping coefficient, that means Kz total and Cz total; what we can.

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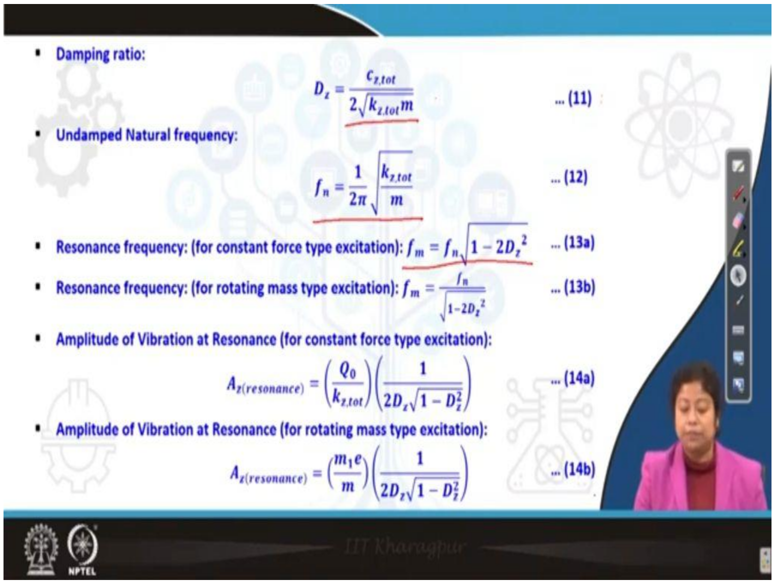

We need to find out the value for Dz which is shown in equation 11. We already know what is Dz; Dz is the damping ratio. And that is the ratio of the damping coefficient to the critical damping. So here you can see, we know how to calculate critical damping; that is equal to 2 times square root of stiffness times mass. So, here stiffness means Kz total. After knowing Kz total, we can also calculate natural frequency for the system. And when we know damping ratio, when we know the natural frequency, from that we can calculate resonance frequency using equation 13a. For constant force type excitation, for rotating mass type excitation, we will use equation 13b to calculate for the calculation of resonance frequency.

Similarly, we can find out the amplitude of vibration at resonance using either equation 14a or 14b. 14a will be used if there is constant force type excitation. If there is rotating mass type excitation, then we will use equation 14b.

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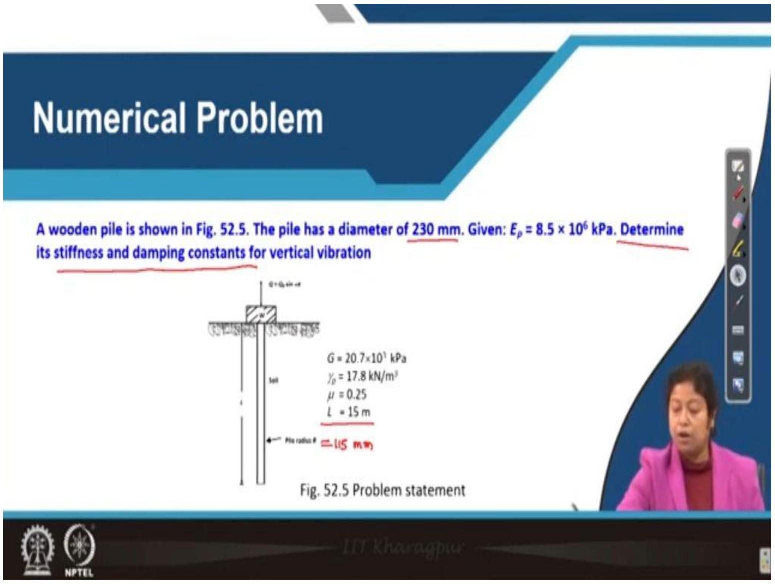

Now, let us solve one simple numerical problem. So, here what we can see? In this problem, one floating pile or you can call it as fiction pile is shown. It is made of wooden wood; so, the diameter of the pile is given, which is 230 millimeter. So, from that we can calculate the radius r, which is 230 divided by 2; it will come 150 millimeter. So, I am just writing here itself. The length of the pile is also given which is 15 meter; the soil property also provided. You can see G is 20.7 into 10 to the power 3 kPa, gamma p is 17.8 kilo Newton per meter cube, and Poisson's ratio is 0.25.

So, what we need to find out? We need to find out the stiffness and damping constant for vertical vibration; so, let us do this exercise quickly.

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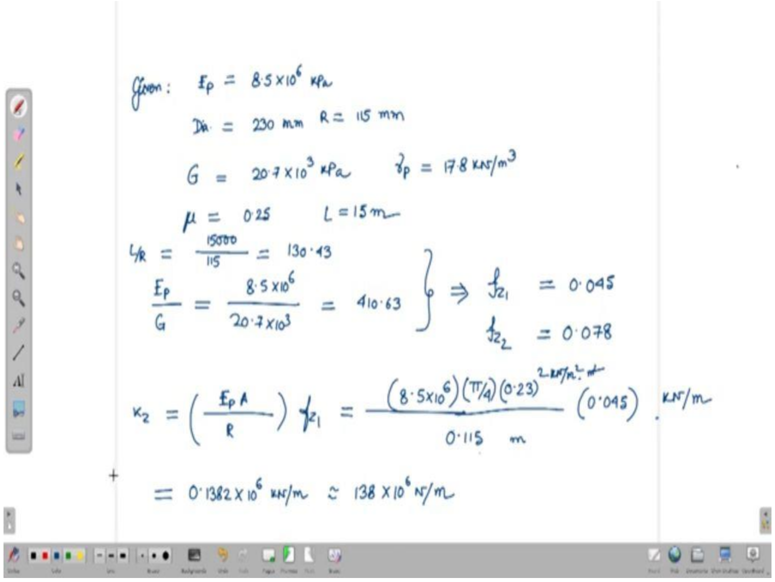

So, given parameter where I can write; let me just take what is given. So, given parameter is Ep which is 8.5 into 10 to the power 6 kPa. Then, diameter is given; I can write it as Dia 230 millimeter, from which I can calculate 115 millimeter. G is equal to 20.7 into 10 to the power 3 kPa, gamma p is equal to 17.8 kilo Newton per meter cube. So, next mu which is Poisson's ratio 0.25, and L is 15 meter. So, from this first thing which we will find out is Ep divided by G. G is the shear modulus of the soil; Ep is Young's modulus of the biomaterial.

So, 8.5 into 10 to the power 6, G is 20.7 into 10 to the power 3; both the same unit kPa. So, we are getting this is equal to 410.63, 410.63. So, from this now if I will use this equation, I need to do interpolation; because you can see Ep by G is equal to 250 and 500 is given, same for the fz2 curve also. So, after interpolating what I am getting? I am just writing directly here with this EP by G value; and I also need to find out L by R. So, L by R in this case is 15000 divided by 115, all are expressed in millimeter. So, what I am getting is 130.43. So, with these two values, I can get fz1 and fz2 using the two figures, which I have just shown you.

And it will be fz1 is 0.045, approximately I am writing; and fz2 is approximately 0.078. Actually, my task is little bit easy; because you can see for L by R 100 or above 100, both the curves become horizontal. So, I directly interpolate from these two values. Likewise, here also from these two values, I can directly interpolate; and find these two values for fz1 and fz2 respectively.

Now, what I need to do? I need to find out Kz, Kz for the pile. So, this is for this, I can use this equation times fz1. So, Ep is given which is 8.5 into 10 to the power 6 in kilo Newton per square meter times A. A means radius is given, diameter is also given. So, using either radius or diameter, you can find out the value; so, it is pi by 4 times 0, sorry 230. So, 0.23 square, this is area divided by R, which is 0.115 times fz1; so, fz1 means 0.045.

So, now, what will be the unit? Unit is kilo Newton per meter. It is meter, it is, this is meter, this is kilo Newton per square meter times meter square; so, you can see the unit is kilo Newton per meter. So how much you are getting from this? So, I am getting 0.1382 into 10 to the power 6 kilo Newton per meter; or I can write it also as 138 into 10 to the power 6 Newton per meter. So, in this way we can calculate Kz.

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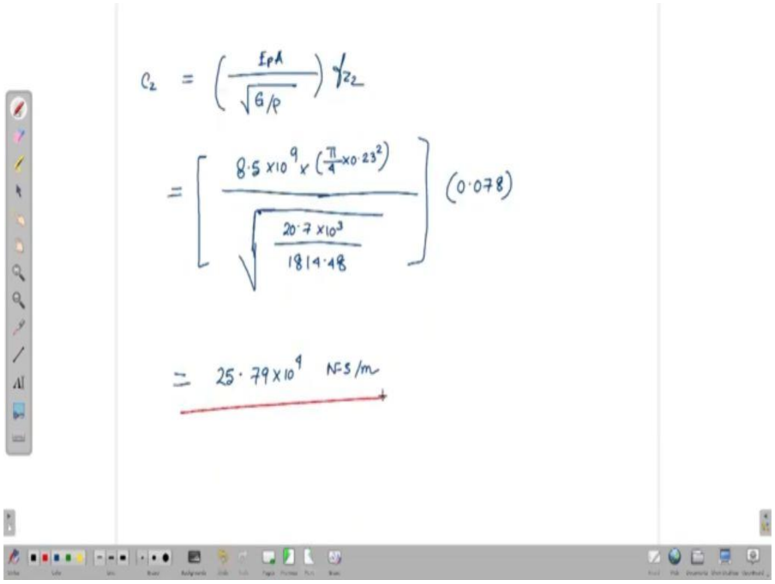

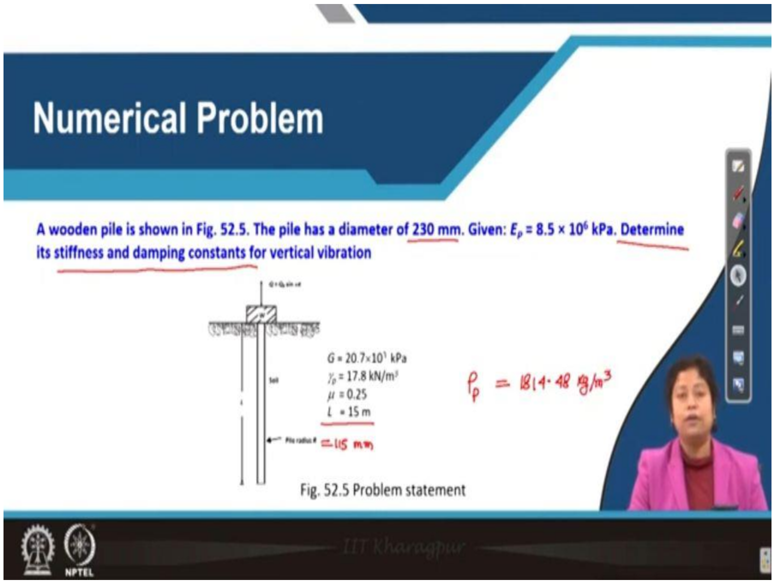

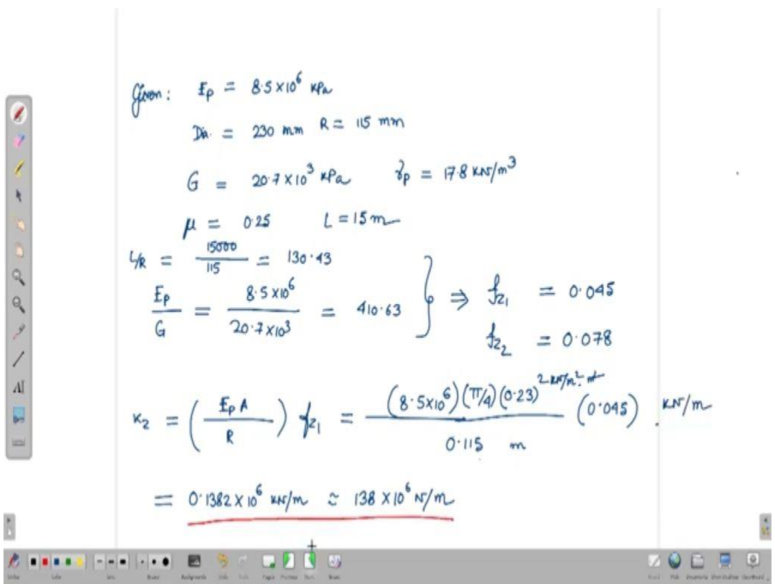

Now, for Cz, we have the equation Ep divided; sorry, Ep times A, divided by square root of G by rho. So, G and rho both are related to the soil only times fz2; already we know the value for Ep. This is 8, what I can do here 8.5; 8.5 into 10 to the power 9; I am expressing it in Newton per square meter times area. So, pi by 4 times 0.23 square, divided by square root of G; that means 20.7 into 10 to the power 3. rho means 17 point, sorry I can; so, this is our numerical problem. So, here you can see rho value is 17.8 in kilo Newton per meter cube; 17.8 into 10 to the power 3, divided by G that will give the value of rho p. So, value of rho p is 1814.48 in kg per meter cube.

So, here I can write this value itself 1814.48. So, I can times fz2; if I will go to previous page, fz2 is 0.078. So, with this now we can calculate the value for Cz. So, it is coming, I am getting 25.79 into 10 to the power 4 in Newton second per meter. So, with this value so the final answer is, this is for coefficient of damping and this is the Kz value. So, this is actually asked in this problem; and in this problem if you see single pile is mentioned, not pile group. That is the reason we have not summed it. So, with this I think now I can summarize today's class.

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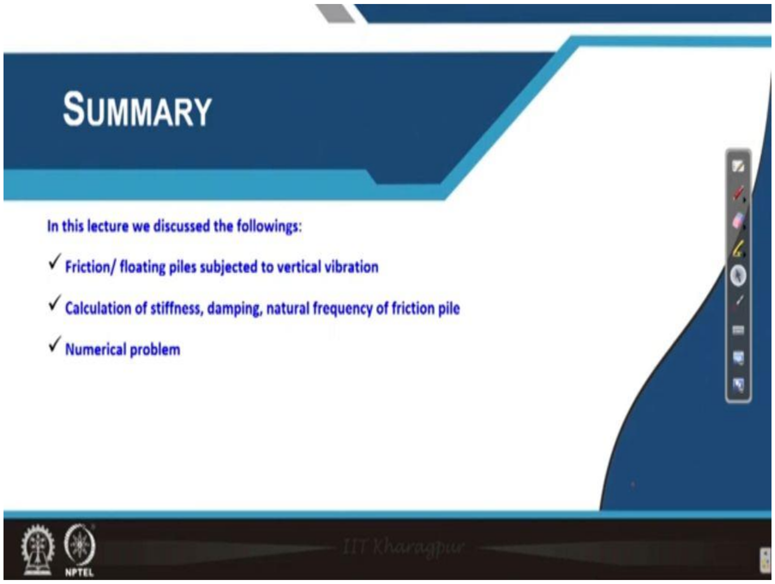

So, today we have discussed first how to solve the problem of the friction pile or floating pile subjected to vertical vibration. How to Calculate stiffness damping natural frequency for friction pile, then how to find out the resonance frequency and resonance amplitude. And finally, we have discussed one numerical problem only considering the pile portion.

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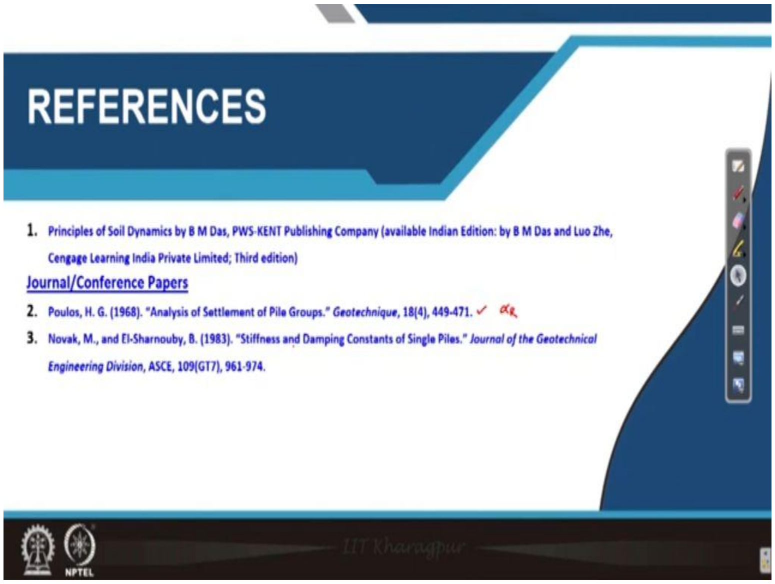

So, these are the references that I have used in today's class. So, this is one curve which this reference for the to get the value of alpha r actually; and this reference for fz1 and fz2 value and the textbook. So, I am concluding today's class now here. Thank you.