Soil Dynamics
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Lecture No. 51
Analysis of Pile Foundation Under Dynamic Loading (Part - I)

Hello friends, welcome to the course Soil Dynamics. So, in this lecture, we will discuss how to analyze the pile foundation under dynamic loading. So, dynamic loading means, we will consider only the machine foundation loading in this subject. So, today is the first class on this topic; there will be another four lectures on the same topic.

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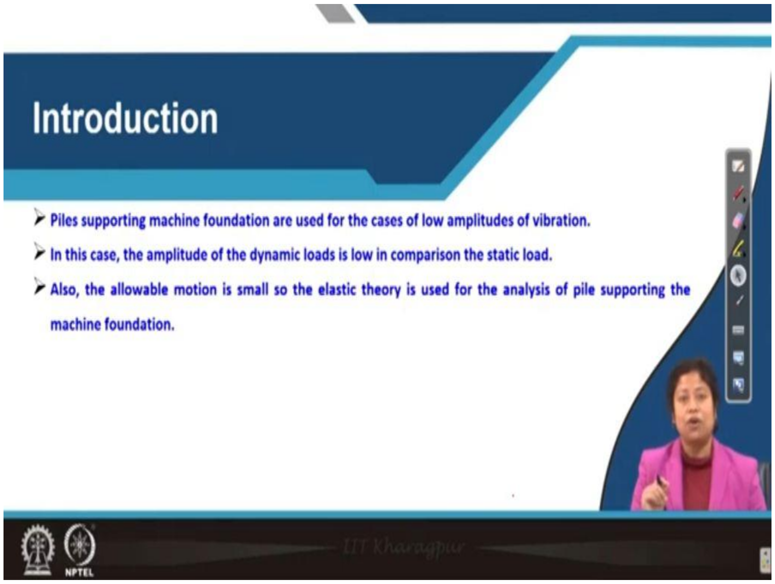

So, the first thing, piles supporting machine foundation are used for the cases of low amplitudes of vibration. Now, in this case the amplitude of the dynamic load is low in comparison to the static load obviously. So, what is happened? The allowable motion is small and thus, we can use the elastic theory to analyze for the analysis of pile.

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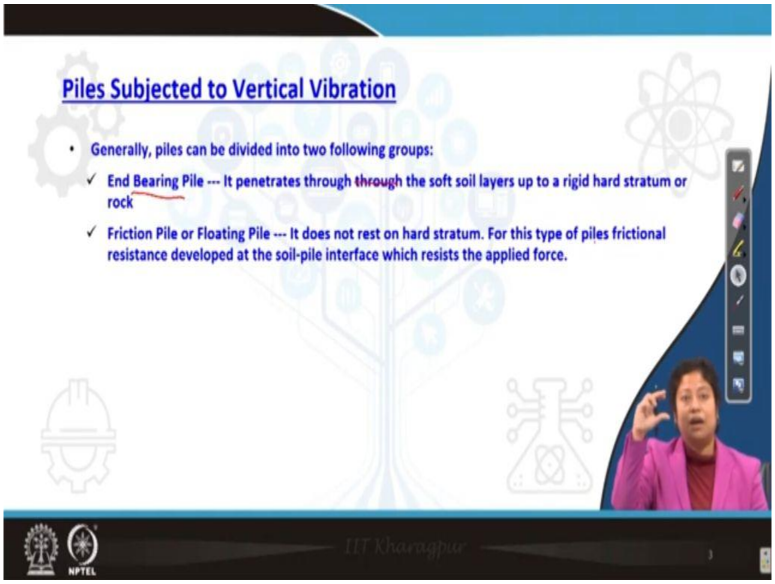

So, we first case which we need to consider is the pile foundation subjected to vertical vibration. So, generally, piles can be divided into two groups. What are those two groups? First one is end bearing pile; it penetrates, there are two through. So, what is end bearing pile here? End bearing pile penetrates through the soft soil up to a rigid hard stratum or rock. So, the rock or hard stratum is considered as rigid in this analysis. The second type of pile foundation is friction pile or floating pile. It does not rest on hard stratum. For this type of piles, frictional resistance developed at the soil pile interface which resists the applied load.

So, basically that means what we have seen here in case of end bearing pile, resistance provided at the tip or at the end of the pile; whereas in case of friction pile or floating pile, resistance is offered by the periphery; or you can consider it as the along the length of the pile.

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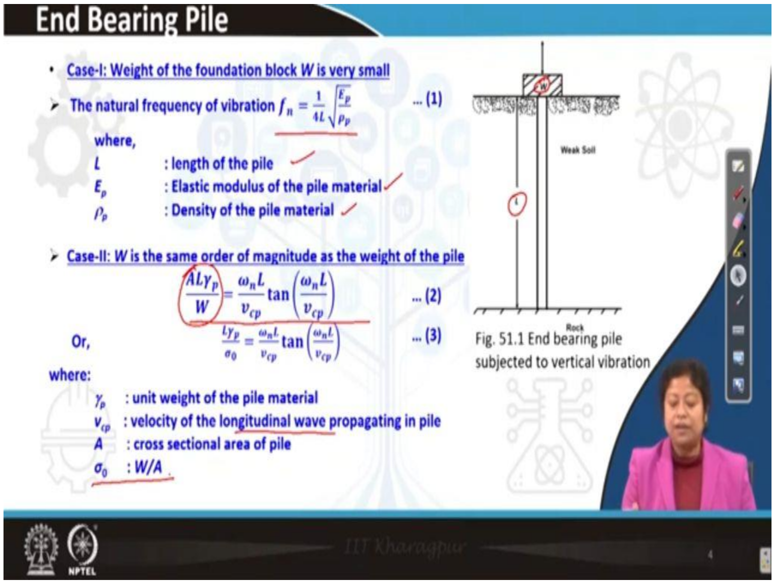

Now, in this figure you can see one end bearing pile which is supporting the machine foundation having weight W; and the entire pile and foundation system is subjected to vertical vibration. So, in this case how do we do the analysis? Here you can see W is the weight of the foundation block and the machine together; it does not include the weight of the pile. So, when weight of the foundation block W along with the weight of machine is very small, then how do we get the natural frequencies?

In this case, we will consider the problem as a; we will consider pile actually as a rigid rod. And we will consider wave is propagating through the rigid road; that analysis we have already done in second or third week of this course. So, the same concept you can use here; and you will get that the natural frequency is equal to 1 divided by 4L times, square root of Ep divided by rho p. What is Ep and rho p here? Ep and rho p are the Young's modulus and unit weight of the pile material; whereas L is the length of the pile.

I made a mistake; rho p is the; can I repeat this part once again (())(05:47). So, in equation 1 what is Ep and rho p? Ep is the Young's modulus; whereas, rho p is the density of the pile material, and L is the length of the pipe; aur ek bar coaching, if we need. So, let us start (()) (06:53). So, W is the total weight of the foundation block and the machine together. So, if W is very small, then how do we do the analysis?

We will consider the problem as a rigid rod through which wave is propagating; and similar kind of analysis we have already studied in second or third week of this course. So, using the same elastic theory of wave propagation we can solve these types of problems. So, what will be then the natural frequency? The natural frequency of vibration under vertical; this is the undamped free system. So, the natural frequency, please cut this portion. The natural frequency of vibration can be calculated by this expression, where capital L is the length of the pile you can see here; Ep is the Young's modulus of the pile material, and rho p is the density of the pile material.

You can see here. Now, the second case is when W is the same order of the magnitude of the weight of pile; so, that time how we will solve the problem. This problem is an is similar to the problem which we have already solved when wave is propagating, P wave is propagating through a rigid rod; and the boundary condition which we impose there at the fixed end. There is no displacement, and at the free end when a block is resting; that time we have considered its weight and solve the problem.

So, same thing if you do, you will get this type of relationship that AL gamma p divided by capital W is equal to omega n L, divided by V cp times, tangent of omega n L divided by V cp; where omega n is the natural frequency, capital L is the total is the length of the pile, and V cp is the velocity of the P wave. You can see velocity of the longitudinal wave propagating in the pile.

Now, just allow me to erase these line, I can. Now, in equation 2, you can see this term on the left hand side. So, what we can write it? We can also write it as L times gamma p, divided by W divided by A. So, W divided by A is sigma 0; and right hand side will remain as it is, that you can see in equation 3.

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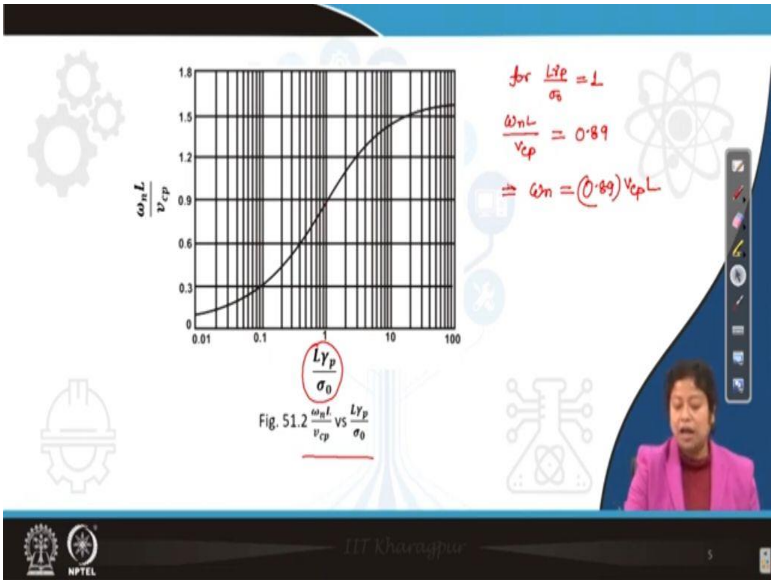

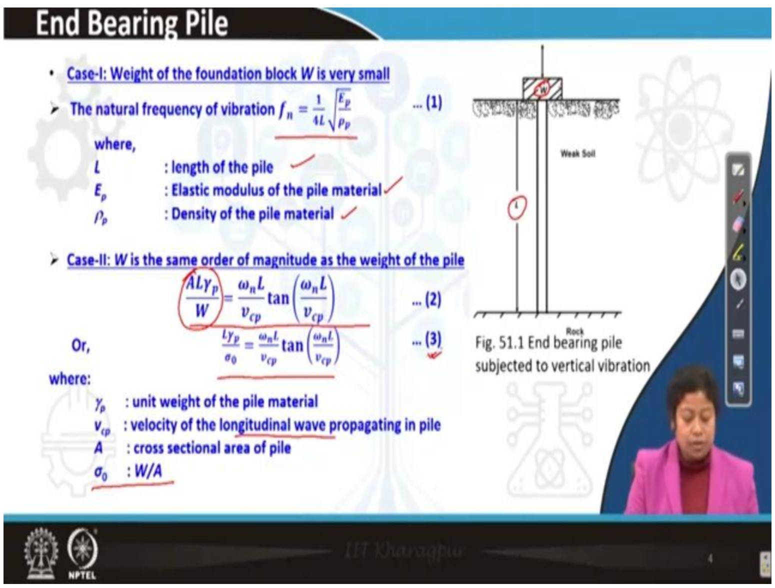

So, now in this figure what we can see? In this figure we see the plot of omega n L divided by V cp versus L gamma p, divided by sigma 0; so, basically the plot for the equation 3. So, if we know here the value of L times gamma p divided by sigma 0, then using this figure what we can get? We can get the values for omega n L divided by V cp. So, after knowing the value of omega n L divided by V cp, suppose, for L gamma p divided by sigma 0 equals to 1; what is the value of omega n L by V cp? You can see here, it is coming approximately 0.89.

Now, from this you can calculate the value of omega n which is 0.89 times V cp times A. So, in this way, you can calculate the value of omega n when the weight of the foundation block is the same order of the magnitude as the weight of the pile.

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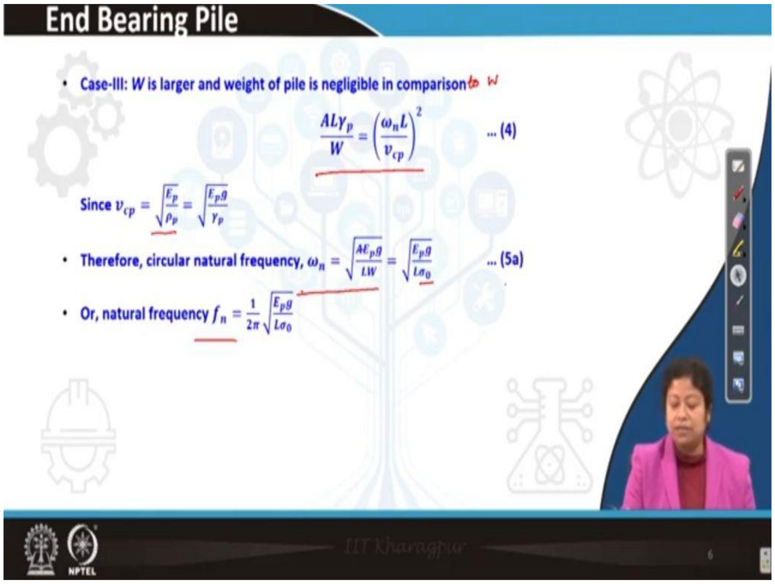

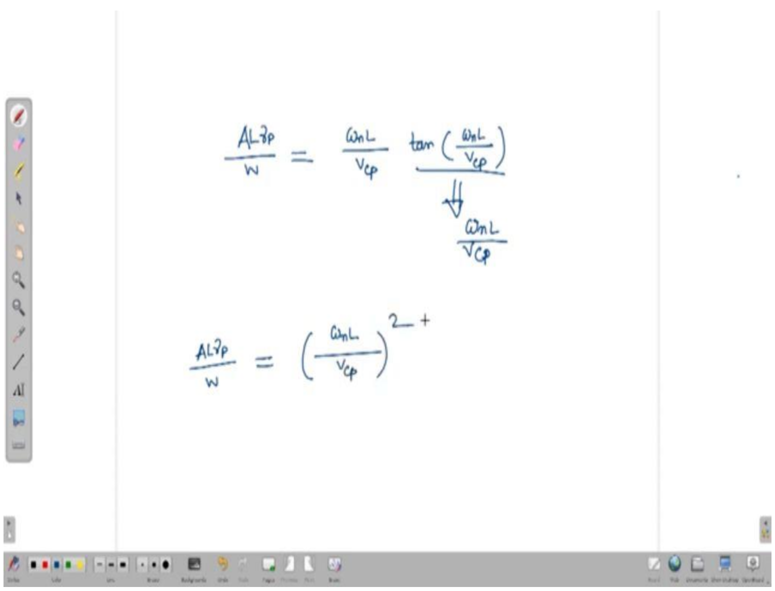

Now, third case. So, in third case, what is the condition that we need to check? Whether W which is the total weight of the foundation block and the machine together is larger, and weight the pile is negligible in comparison to W. In that case, basically, earlier we have seen this equation AL gamma p divided by W; this is equal to omega n Lp, sorry not p. I think better I write it in whiteboard rather than writing in PPT. So, earlier we have seen this equation that AL gamma p divided by W, AL gamma p divided by W is equal to omega n L divided by V cp, times tangent of omega n L divided by V cp.

Now, if the value of omega n L by V cp is. So, in this equation, what we need to consider? This value is almost equal to omega n L by V cp. In that case what we are getting? AL gamma p divided by W is equal to omega n L divided by V cp square. So, we will get then this expression which is shown in equation 4, this one. Now, we know the value for V cp which is square root of Ep divided by rho p, you can see this. So, finally, if we will write it in equation 4 or you can write in place of rho p; you can write gamma p divided by g, where gamma p is the unit weight of the pile material.

Then you can, you will get the natural frequency in this form. Now, here W by A, you can write as sigma 0 and rest of the term as it is; then you finally get equation 5a. So, this is the circular natural frequency omega n. From this you can find out the natural frequency in hertz; that is 1 divided by 2 pi times, square root of Ep times g divided by L times sigma 0; where sigma 0 is already known, it is W divided by A.

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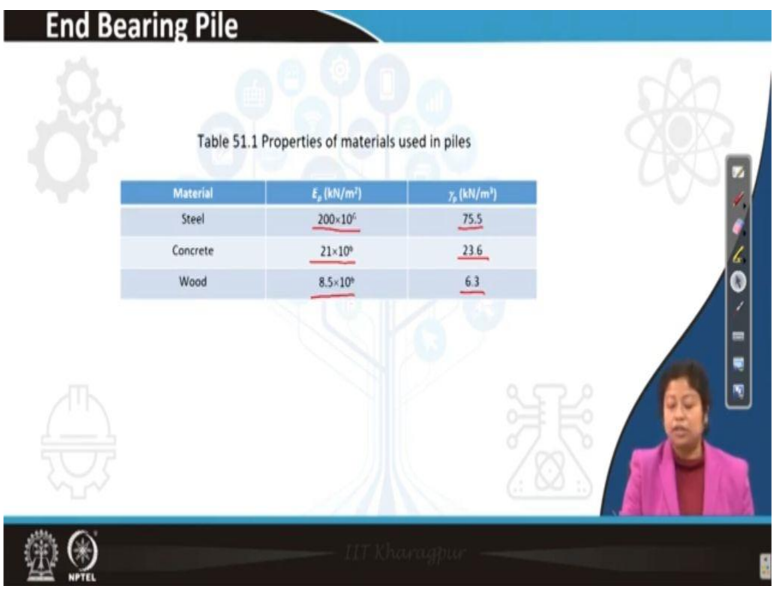

In this table you can see properties of materials that can be used in pile. What are those materials? First one steel. It is Young's modulus value is given in kilo Newton per meter square, 200 into 10 to the power 6 kilo Newton per meter square. Its gamma p value is 75.5 kilo Newton per meter cube. For concrete, Ep is 21 into 10 to the power 6 kilo Newton per meter square; and gamma p is 23.6 in kilo Newton per meter cube. Sometimes we take it also as 24.

For wood, the value is 8.5 into 10 to the power 6, and gamma p is 6.3 into 10 to the power; sorry, 6.3 in kilo Newton per meter cube. I am just repeating for wood, EP is 8.5 into 10 to the power 6 kilo Newton per meter square; and the gamma p is 6.3 in kilo Newton per meter cube.

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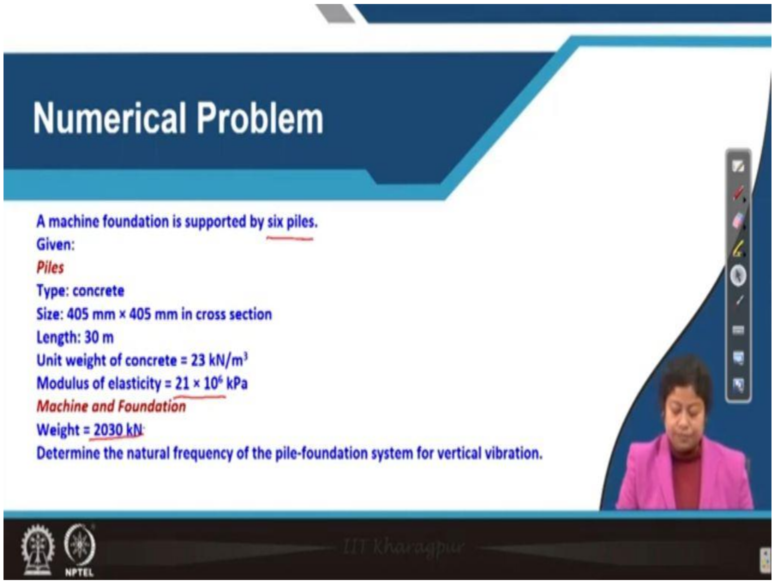

So, with this, now we can solve one numerical problem; so, see here the numerical problem. A machine foundation is supported by six piles; data are given. For piles, it is made of concrete; the size means cross section of the pile is given which is 405 millimeter by 405 millimeter in cross section; that means it is a square cross section. Length is 30 meter, unit weight of concrete 23 kilo Newton per meter cube.

Modulus of elasticity for concrete is reported as 21 into 10 to the power 6 kilo Newton per meter, per square meter. Also the data of machine and foundation provided, weight is given 2030 in kilo Newton. So, you are asked to determine the natural frequency of the pile foundation system for vertical vibration. So, let us solve these problem then.

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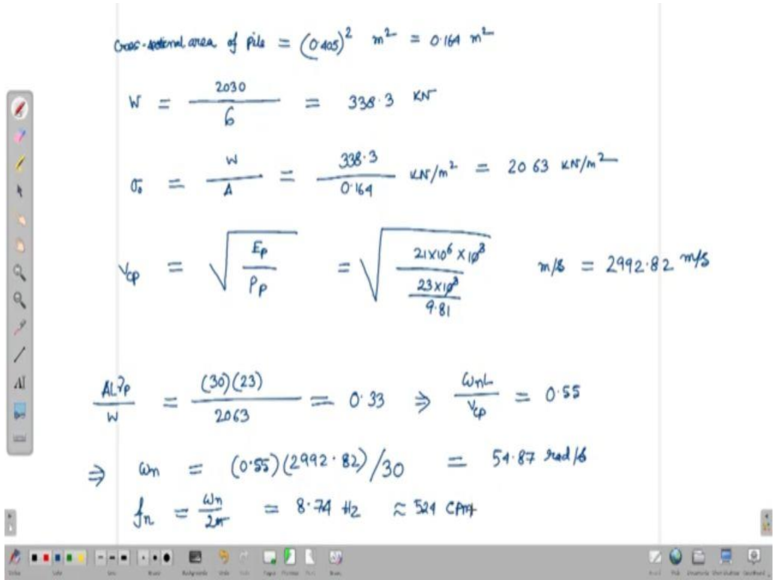


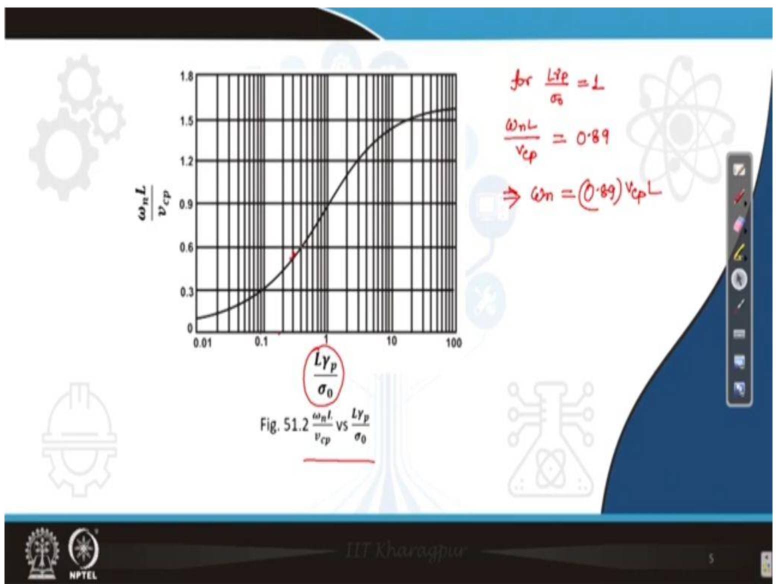
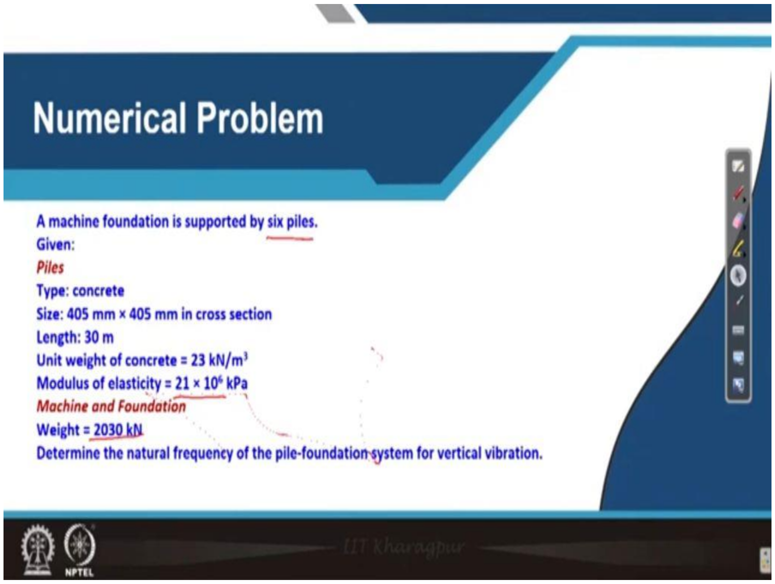


Fig. 51.2 $\frac{\omega_n L}{v_{cp}}$ vs $\frac{L\gamma_p}{\sigma_0}$



I am just writing all the data cross-sectional area pile that is 0.405 square in square meter. So, it is coming approximately, it is coming 0.164 square meter. Now, what is W? W is total load, you can see here 2030 kilo newton is total load; and we have already calculated the cross-sectional area. So, 0.164, this is in kilo Newton per square meter. So, how much we are getting? We are getting 2030 divided by 0.164; we are getting, sorry, we are calculating W. So, W means we have, we are not first calculating stress; first we need to calculate W. You can see here total weight of machine and foundation is given which is 2030 kilo Newton.

So, first we need to find out W which is the load coming to the individual pile; and that is in this case, there are six piles if you see here, so 2030 divided by 6. So, we are getting 2030 divided by 6, it is 338.3 in kilo Newton. Now, sigma 0 is W divided by A, which is 338.3 divided by 0.164 kilo Newton per square meter, and it is. So, you can now calculate the value of sigma 0; I think this is 2063 in kilo Newton per meter square; we can use it or we can indirectly use the expression for W by A.

Next is to find out V cp. What is V cp here? V cp is the velocity of the longitudinal wave through the pile. So, that is equal to square root of Ep divided by rho p. So, Ep is how much? Ep is 21 into 10 to the power 6; this is in kilo Newton per square meter. So, I can write it in Newton per square meter divided by rho p; rho p means 23 into 10 to the power 3, divided by 9.81; and we will get in meter per second. So, this is equal to how much then? I am calculating 21 into 10 to the power 6, divided into 9.81 divided by 23. So, we are getting 2992.82 in meter per second; this is velocity of the longitudinal wave.

Now, we can calculate AL gamma p divided by W, or you can write also AL gamma p divided by sigma 0, both are. So, I think better I should just give me one minute time. I can write it as L means 30; gamma p means how much? It is 23 in kilo Newton per meter cubed divided by 2063. So, you can see here in numerator and denominator both the places, dimension is in kilo Newton per square meter; now, so it is a number. This AL gamma p divided by W is a number; and we are getting it 206, so we are getting the value as 0.33 approximately. Now, already I have shown you one chart; I can just show that here also. Better I can directly show you the chart; so, I am just showing. So, using this chart, now we will find out.

So, we have the value 0.33; 0.33 means this is 01, 0.2, 0.3. So, 0.33 means somewhere here, so the value will be approximately if I will take 0.55 I can take; so, I am taking some approximate value. Here for this value, we are getting omega n L divided by V cp is equal to 0.55. So, from this now, I can calculate omega n L 0.55 times V cp; V cp value is 2992.82, divided by L which is 30. So, what we are getting? I am just calculating and then we will write; we are getting omega n value is equal to 54.87 in radian per second.

Then, from this we can calculate Fn in hertz. So, omega n divided by 2 pi; and I am getting approximately 8.74 in hertz. We can express it also in CPM multiplying it by 60, so, it is coming approximately 524.4; I am writing 524 in cycle spark minute. So, in this way we can find out the natural frequency of the pile.

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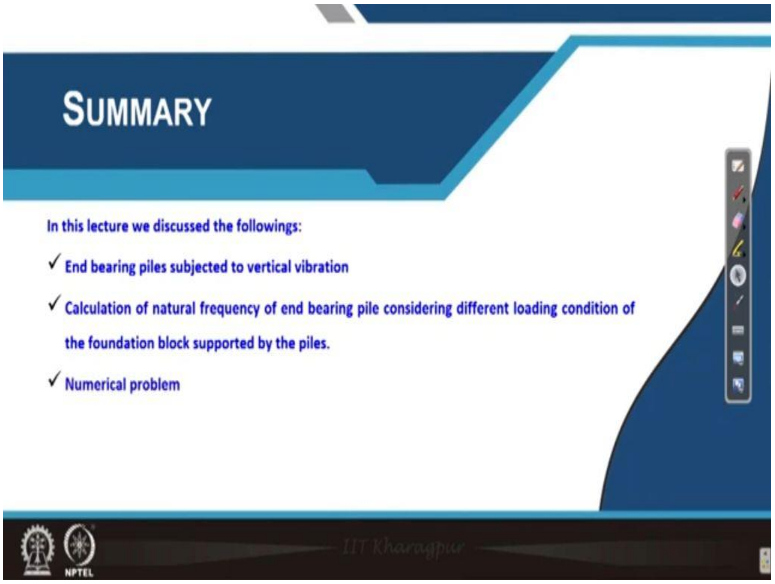

So, now, come to the summary of today's class. So, in this lecture, we discussed the bearing pile, end bearing piles which is subjected to vertical vibration. Then, how to calculate the natural frequency of end bearing piles considering different loading condition, like W is very small, then W is of the same order of magnitude of the weight of the pile. And third case, when W is higher than the weight of the pile. So, three cases we have considered; and for those three cases, how to calculate the natural frequency that we have discussed. Then we have solved one numerical problem.

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So, here is the reference that I have used for this lecture. So, with this I am ending today's class, thank you.