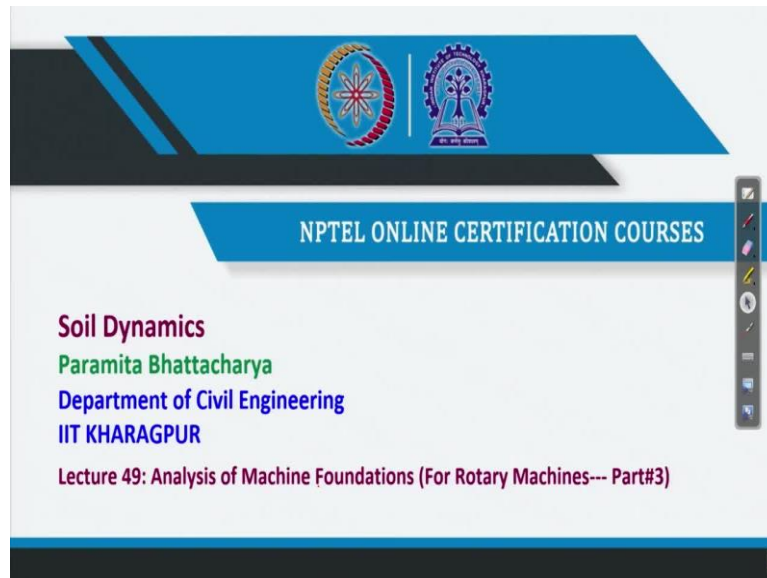


**Soil Dynamics**  
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**Lecture 49**  
**Analysis of Machine Foundations**  
**(For Rotary Machines – Part III)**

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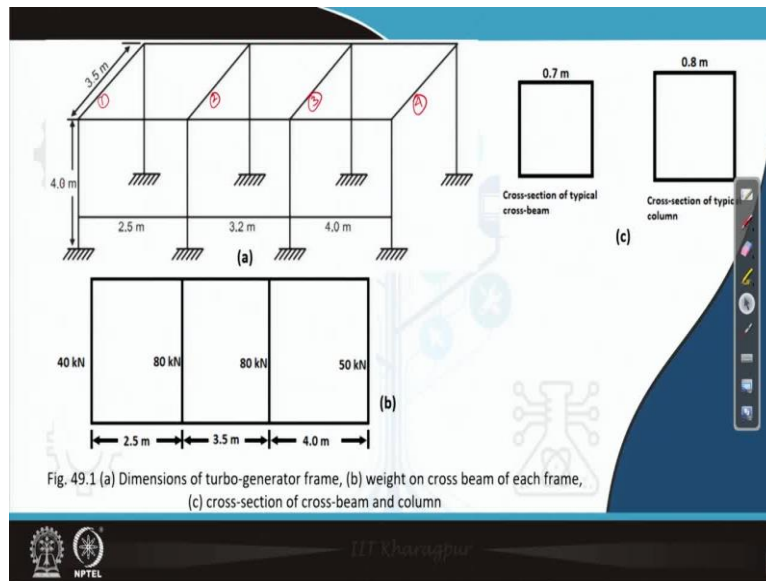
Hello friends. So, today, we will solve the numerical problem related to the machine foundations for rotary machines. Already we have discussed the two methods, one is resonance method and the second one is amplitude method. So, using one of these two methods, today we will solve our first numerical problem.

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**Numerical Problem**

Design a turbo-generator frame foundation shown in Fig. 49.1 with the following data:

- Cross sectional dimensions of each column = 0.8 m x 0.8 m
- Cross sectional dimensions of each cross beam = 0.7 m x 0.7 m
- Cross sectional dimensions of each longitudinal beams = 0.6 m x 0.6 m
- Weight on cross beam of each frame:  $W_{11} = 40$  kN;  $W_{12} = 80$  kN;  $W_{13} = 80$  kN;  $W_{14} = 50$  kN
- Weight of rotating parts acting on each cross frame:  $W_{21} = 10$  kN;  $W_{22} = 15$  kN;  $W_{23} = 15$  kN;  $W_{24} = 20$  kN
- Eccentricity =  $0.05 \times 10^{-3}$  m
- Young's Modulus for concrete =  $3 \times 10^7$  kN/m<sup>2</sup> (E)
- Damping ratio = 0.02;  $\mu = 0.15$
- Operating speed = 4800 rpm =  $\frac{4800}{60}$  cps = 80 cps  $\omega = 2\pi f = 160\pi$  rad/s
- Permissible vertical amplitude = 0.3 mm and Permissible horizontal amplitude = 0.5 mm



Handwritten calculations for the cross-sectional area and moment of inertia of the cross-beam and column:

$$A_b = (0.7)^2 \text{ m}^2 = 0.49 \text{ m}^2$$

$$I_b = \frac{(0.7)(0.7)^3}{12} \text{ m}^4 = 0.02 \text{ m}^4$$

$$A_c = (0.8)^2 \text{ m}^2 = 0.64 \text{ m}^2$$

$$I_c = \frac{(0.8)(0.8)^3}{12} \text{ m}^4 = 0.03413 \text{ m}^4$$

$$\frac{b}{L_0} = \frac{0.4}{3.5} = 0.114 \quad \alpha = 0.19$$

$$\frac{H_0}{L_0} = \frac{4}{3.5} = 1.143$$

So, let us see the problem now. So, here you can see the problem statement and this is the figure which is referred in the problem itself. So, let me read now, so, design our turbo generator frame Foundation which is shown here. Now, that geometry of different components are given what are those things cross sectional area of each column which is 0.8 meter by 0.8 meter, cross sectional dimensions of each cross beam that is 0.7 meter by 0.7 meter, cross sectional dimension of each longitudinal beams that is 0.6 meter by 0.6 meter.

Also the weight on cross beam is given in this diagram also you can see for the weight for acting for different frame first if I write it one it is second frame it is third frame and this is 4th frame then you can see how much weight is acting on that 40 kilo Newton, 80 kilo Newton, 80 kilo Newton, and 50 kilo Newton. So, that is given here also. Also, weight of rotating parts acting on each cross frame that is also given you can see that here.

Now, in addition to these Young's modulus for concrete it should be meters square or square meter not like this. Also, damping ratio 0.02 and  $\mu$  which is the Poisson's ratio is given that is 0.15.

So, we know Young's modulus  $E$ , so if required we can calculate shear modulus  $G$  since, we know the Poisson's ratio. Operating speed for the machine is also given that is 4800 RPM operating speed means this is in  $\omega$  sorry RPM, so this is in  $\omega$  we need to calculate it in  $\Omega$ . So, operating speed is in rotation per minute. So, from these you can calculate cycles per second so, which is a 80 CPS. So, from these you can find out the circular frequency of the machine which is  $2\pi f$  that means  $160\pi$  in radians per second.

And also the permissible vertical amplitude is mentioned which is 0.3 millimeter and the permissible horizontal amplitude is 0.5 millimeter. So, you are asked to design a turbo-generator frame that means, we need to find out the amplitudes of vibrations for horizontal vibration and vertical vibration and we need to find out the natural frequency for this problem. I hope this geometry is clear. Now, let me start there to solve this problem.

So, first thing which we will do here is first we will find out the cross sectional area. So, in the problem cross sectional area or dimension of the beam is already given. So, from that we can calculate  $A_b$  which is the cross I can show you the data here you can see the cross data for cross beam is given cross beam means transverse beam. So, area of the beam then we can calculate this is 0.49 square meter.

Now, we can find out moment of inertia  $I_b$  which is 0.7 times 0.7 cube divided by 12 in meter to the power 4. So, it will be 0.02 meter to the bar 4. Now, for column also we need to do the same exercise. So, here we can see that dimension for the column is also given 0.8 meter by 0.8 meter. So, 0.8 meter depth and 0.8 meter read from these you can calculate  $A_c$  which is cross sectional area.

So, how much it is, 0.64 square meter. Likewise we can calculate  $I_c$  which is the moment of inertia. So, 0.8 times 0.8 cube divided by 12 you we already know. So, the value which we will get here is 0.031 Sorry 0.03413 in meter to the power 4.

Now, next thing which we need to do here is  $b$  divided by  $L_0$ . So,  $b$  is how much? In this case  $b$  is half of the width of the column which is 0.4, what is  $L_0$  here?  $L_0$ , let me go back to the next figure. So, here  $L_0$  is the spacing the sorry span length which is you can see 3.5. So, how much we are getting? We are getting 0.114. Similarly, we can calculate  $H_0$  divided by  $L$

0 which is equal to 480 if you see it is 4 meter and L 0 is already known 3.5 meter, so, we are getting 1.143.

So, now, we know b divided by L 0 and H 0 divided by L 0. So, from this I can calculate alpha. So, if we use these two values we will get alpha we need to do interpolation because there is a range mentioned for in the curve we will see you will get a range. So, from that curve if you need to do some sort of simple interpolation between here H 0 is 1.143 in the curve it is given for 1 and 1.25 that is the reason you need to do interpolation. And finally, you will get alpha which is equal to approximately 0.19.

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Handwritten mathematical derivation showing the calculation of L and H values:

$$L = L_0 - 2\alpha b = [3.5 - 2(0.19)(0.4)] \text{ m} = 3.348 \text{ m}$$

$$H = H_0 - 2\alpha a = [4 - 2(0.19)(0.35)] \text{ m} = 3.867 \text{ m}$$

Relative Stiffness (K) is calculated as:

$$K = \frac{I_b}{I_c} \cdot \frac{H}{L}$$

$$= \frac{0.02}{0.03413} \cdot \frac{3.867}{3.348} = 0.677+$$

Handwritten mathematical derivation showing the calculation of area and moment of inertia for rectangular sections:

For a square section with side length 0.7 m:

$$A_b = (0.7)^2 \text{ m}^2 = 0.49 \text{ m}^2$$

$$I_b = \frac{(0.7)(0.7)^3}{12} \text{ m}^4 = 0.02 \text{ m}^4$$

For a square section with side length 0.8 m:

$$A_c = (0.8)^2 \text{ m}^2 = 0.64 \text{ m}^2$$

$$I_c = \frac{(0.8)(0.8)^3}{12} \text{ m}^4 = 0.03413 \text{ m}^4$$

Calculations for b/L<sub>0</sub> and H<sub>0</sub>/L<sub>0</sub>:

$$\frac{b}{L_0} = \frac{0.4}{3.5} = 0.114$$

$$\alpha = 0.19$$

$$\frac{H_0}{L_0} = \frac{4}{3.5} = 1.143$$

So, now, we will find out capital L which is equal to L 0 minus 2 times alpha times b, L 0 is 3.5 meter minus 2 times alpha which is 0.19 times b, b is here 0.4. So, this is a meter which is

coming equal to 3.348 meter this is L. Likewise, H means H 0 minus 2 times alpha times a. So, H is 4 meter minus 2 times alpha, alpha is 0.19 and a is 0.35 which is the half of the if you see a is the half of the depth of the beam, so it is coming 3.867 meter. Now, from these now we know capital L capital H so using these two parameters, we can find out relative stiffness which is k is equal to I b divided by I c times H divided by L.

So, I can go back to previous page here i b is 0.02 I see is 0.03413 So, I need to use these two value, H already calculated L also calculated, this is H, this is L. So, finally, we are getting the factor is 0.677 this is relative stiffness k. Now, after knowing relative stiffness k what we can do now? Now, we need to do load calculation.

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$$W_1 = (40 \text{ kN}, 80 \text{ kN}, 80 \text{ kN}, 50 \text{ kN})$$

$$W_2 = (10.8 \text{ kN}, 29.624 \text{ kN}, 31.104 \text{ kN}, 17.28 \text{ kN})$$

$$q = (0.7)(0.7)(24) = 11.76 \text{ kN/m}$$

$$\delta_1 = \frac{W_1 L^3}{96 EI_b} \cdot \frac{2K+1}{K+2}$$

$$= (2.292 \times 10^{-5}, 4.583 \times 10^{-5}, 4.583 \times 10^{-5}, 2.865 \times 10^{-5}) \text{ m}$$

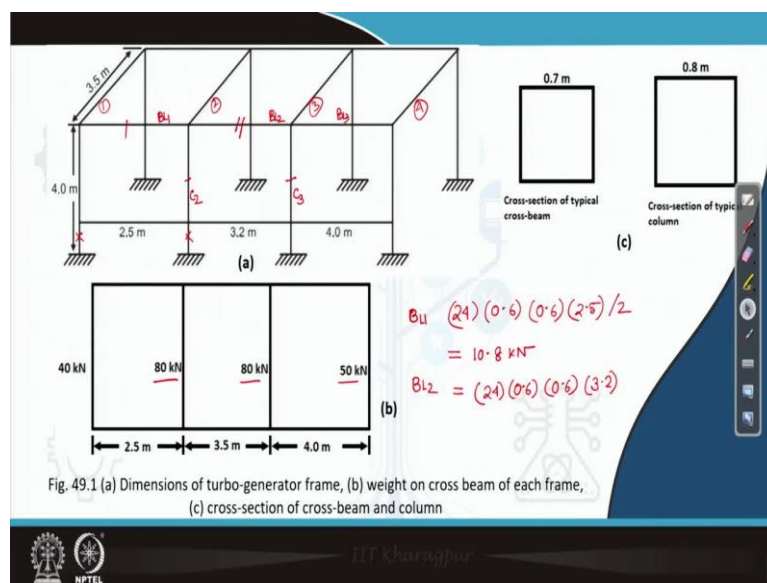


Fig. 49.1 (a) Dimensions of turbo-generator frame, (b) weight on cross beam of each frame, (c) cross-section of cross-beam and column

So, for load calculation, I can go next slide. So, in resonance method load is already given acting on each frame you can see here 80 kilo Newton, 80 kilo Newton for the third frame also 80 kilo Newton, 4th frame it is 50 kilo Newton. So, I am just writing the values of the  $w_1$  40 kilo Newton this is for first frame. I am just writing all the values with separating by comma so, these are the 4 values of the  $w_1$  for the 4 different frames. Likewise, I can write the  $w_2$  also for 4 frames so let us first do it for the first frame.

What is the  $w_2$  the load coming from the trans longitudinal beam that means from this beam. So, what is the self-weight of this beam? Here the self-weight of this beam, if I take unit weight of concrete is 24 kilo Newton per meter cube, then 24 times cross sectional area, which is I think given 0.6, 0.6 times 0.6 and its length is 2.5. So, this is the total weight of this longitudinal beam which is marked in the figure.

Now, this load is divided into two columns, this column and this one. So, when I am analyzing frame number one, that time the first column takes how much load? It takes this divided by 2. So, these divided by 2 is coming 10.8 in kilo Newton. So, the  $w_2$  is 10.8 in kilo Newton. Now, see the second longitudinal beam, so, in this case this is I can write B L 1 now, for B L 2, this is B L 1, this is B L 2. So, for B L 2 total self-weight is how much? Same 24 times cross sectional area times length in this case length is 3.2 meter. Now, this load also be divided in this column and this column.

So, now, if I focus on column C 2 how much load is coming half of the load from B L 1 and half of the load from B L 2. So, this time the load is equal to I am getting it is 24.624 in kilo Newton. Likewise for C 3 we can calculate the load from B L 2 and B L 3 and that will be that is coming 31.104 kilo Newton and for the last frame that column it is 17.28 kilo Newton. So, now the  $w_2$  calculated. Now, small  $q$ , so small  $q$  in this case 0.7 times 0.7 times 24 which is coming 11.76 in kilo Newton per meter.

So, from these values now we can calculate delta 1, delta 2, delta 3, and delta 4, for delta 1 we have this equation  $w_1 \times L^3$  divided by  $96 \times E I b \times 2 k \text{ plus } 1$  divided by  $k \text{ plus } 2$ . So, already I have calculated the value of the  $w_1$ , E is already given which is 3 into 10 to the power 7 kilo newton per square meter, I b we have already calculated. So, from those, you can get the values for delta 1 for 4 different frames.

So, I am just writing the values for you for frame one it is coming 2.292 into 10 to the power minus 5 that is in meter. For frame 2 it is coming 4.583 into 10 to the power minus 5 it is also

in meter for third frame it is 2.865 into 10 to the power minus 5 for this is not for the second frame, but this sorry third frame this is actually for the second last frame. So, third frame same value 4.583 into 10 to the power minus 5. And for the 4th frame, this value is coming 2.865 into 10 to the power minus 5 that is in meter. So, for all the 4 frames, the values of delta 1 are given here in meter.

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The image shows a whiteboard with handwritten mathematical derivations. The first derivation is for  $\delta_2$ :

$$\delta_2 = \frac{qL^4}{384 EI_b} \cdot \frac{5k+2}{k+2}$$

$$= \frac{(11.76)(3.348)^4}{384 (3 \times 10^7)(0.02)} \cdot \frac{5(0.677)+2}{0.677+2}$$

$$= 1.29 \times 10^{-5} \text{ m}$$

The second derivation is for  $\delta_3$ :

$$\delta_3 = \frac{3L}{5(EA_b)} \left( w_1 + \frac{qL}{2} \right)$$

$$= 8.156 \times 10^{-6} \text{ m} = 0.8156 \times 10^{-5} \text{ m (Frame-1)}$$

At the bottom, there is a list of values in red:  $(0.8156 \times 10^{-5}, 1.362 \times 10^{-5}, 1.362 \times 10^{-5}, 0.9523 \times 10^{-5}) \text{ m}$

Likewise, we can calculate delta 2 I think this pen is not working properly today every time I am unable to write too. So,  $qL$  to the power 4 divided by 384 times  $E I_b$  we multiply with 5  $k$  plus 2 divided by  $k$  plus 2. So, delta 2 is coming, how much that I can write here, we I am getting delta 2,  $L$  is 3 point I am just writing the values.

So,  $q$  already we calculated that is 11.76 times  $L$  which is 3.348 to the power 4 divided by 384 times 3 into 10 to the power 7. See here  $q$  is in kilo Newton per meter. So, I am writing  $E$  also in kilo Newton per square meter. So, in numerator and denominator both the place you will get kilo Newton so there will be no problem in the dimension.

So,  $I_b$  is 0.02 here, we already know the values for  $k$  which is 0.677 plus 2 divided by 0.677 plus 2. So, finally I am getting 1.29 into 10 to the power minus 5 in meter. Likewise, and this delta 2 value remains same for all the other 3 frames also. Now delta 4, delta 4 can be calculated by using this expression, I am directly writing the value for delta 4 for frame 1 it is because we already know the values for capital  $L$ ,  $w_1$ , small  $q$ , etc. So, you will get 8.156 into 10 to the power minus 6 meter or you can write it also as 0.8156 into 10 to the power minus 5 meter also this is for the frame 1.

Now what about the other 4 frame, so, I am just writing this is for frame 1. So, for other, if I will include other 3 frames, and I can write these together please correct it here, this is for delta 3, so the values of delta 3 can be calculated using the expression which I have shown here and for first frame, I have already written the values for other frame I am writing 1.362 into 10 to the power minus 5, same value for the third frame also, and for the last frame I am getting 0.9523 into 10 to the power minus 5, these are all in meter.

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$$\delta_4 = \frac{H}{EA_c} \left( H_2 + \frac{w_1 + qL}{2} \right)$$

$$= (1.017 \times 10^{-5}, 1.628 \times 10^{-5}, 1.828 \times 10^{-5}, 1.248 \times 10^{-5}) \text{ m}$$

$$\delta_{st} = \delta_1 + \delta_2 + \delta_3 + \delta_4$$

$$= (5.415 \times 10^{-5}, 8.933 \times 10^{-5}, 9.064 \times 10^{-5}, 6.355 \times 10^{-5}) \text{ m}$$

$$W = w_1 + 2w_2 + qL$$

$$= (100.97, 168.62, 181.58, 123.93) \text{ kN}$$

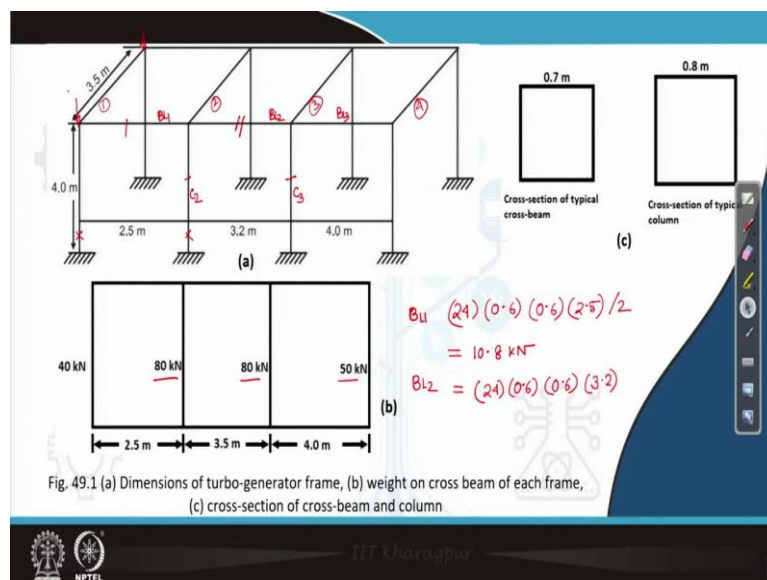


Fig. 49.1 (a) Dimensions of turbo-generator frame, (b) weight on cross beam of each frame, (c) cross-section of cross-beam and column

Now, next is to calculate as I said delta 4. So, for delta 4, we will use this equation the w 2, this is w 2, I do not know why this pen is see, I am getting difficulty to write, w 1 plus q L divided by 2, so using this equation, we can find out delta 4 for the 4 frames. So, I am writing the values of delta 4 now, 1.017 not 7, 017 into 10 to the power minus 5 in meter, then for the



second frame 1.698 into 10 to the power minus 5 in meter, for the third frame this value is 1.828 in 10 to the power minus 5, and for the fourth frame, this value is 1.248 into 10 to the power minus 5 all values are in meter.

So, after calculating delta 4 we need to find out total which is sum of all these 4 delta's, delta 1, delta 2, delta 3, and delta 4. So, now I am writing the delta s t value for all the 4 frames 5.415 into 10 to the power minus 5, this is for frame 1, for frame 2, 8.933 into 10 to the power minus 5, for frame third frame it is 6.355 into 10 to the power minus 5, and for the fourth frame sorry, this is for the fourth frame I am getting. So for the third frame I need to write. So, first I need to write for the third frame which is 9.064 into 10 to the power minus 5 and for the fourth frame it is 6.355 into 10 to the power minus 5 this is in meter.

So, next is total weight, which is the W. Now, if you recall, total, W is the W 1 plus 2 times of w 2 because if I show you there are 2 frames so sorry there are 2 columns, so each column is subjected to the load w 2, so total 2 w 2 that is the reason 2 times w 2 plus q L. So, this is the total load and which is for frame 1 it is 100.97, for frame 2 it is 168.62, for frame 3 it is 181.58, for fourth frame it is 123.93, all these the w's are in kilo Newton.

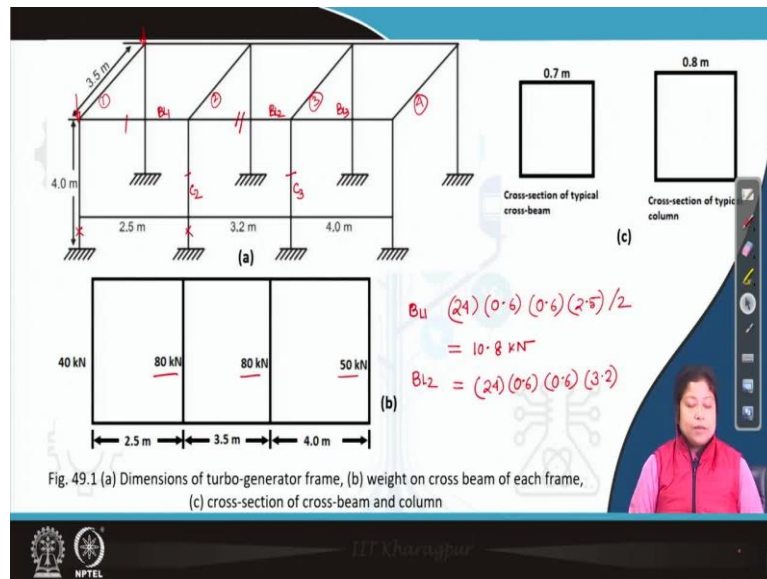
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$$K_z = \frac{W}{\delta_{st}} = \left( 1.865 \times 10^6 \text{ kN/m} + 1.885 \times 10^6 \text{ kN/m} + 2.0034 \times 10^6 \text{ kN/m} + 1.95 \times 10^6 \text{ kN/m} \right)$$

$$\omega_{nz} = \sqrt{\frac{K_z}{m}}$$

$$= (425.65, 331.158, 328.91, 392.88) \text{ rad/s}$$

$$\omega_{nzav} = 369.67 \text{ rad/s}$$



So, now we know  $\Delta s t$ , we know capital the  $W$ , so from this what we can do we can calculate  $K z$  using the expression the  $W$  divided by  $\Delta s t$ . So, for first frame it is coming 1.865 into 10 to the power 6 in kilo Newton per meter, this is for the first frame, for second frame it is 1.885 into 10 to the power 6 kilo newton per meter, for the third frame it is 2.0034 into 10 to the power 6 kilo newton per meter and for the last frame it is 1.95 10 to the power 6 kilo newton per meter.

So these are the  $K z$  values for 4 different frames, from this we can calculate  $\omega n z$  using this expression square root of  $K z$  divided  $m$ . So already you know the value for  $K z$  and  $m$  for all 4 frames and from that you can calculate I am getting for first frame it is 425.65. Likewise for the second frame I am getting 331.158, for the third frame I am getting 328.993 or you can write 329 also. And for the fourth frame it is 392.88 in radian per second. So finally we will take  $n z$  average of these four values so that is coming 369.67 in radian per second.

So with this I am stopping the discussion of the problem which is shown here. In next class what I will do, I will show you how to calculate the vertical how to calculate the vertical amplitude, then how to solve the same problem for horizontal vibration and finally I will show you the answer for amplitude method because there are two methods we have discussed in this content in this topic so that is the reason I will show you what are the difference in the results when using amplitude method to solve this problem. So I am stopping here today.