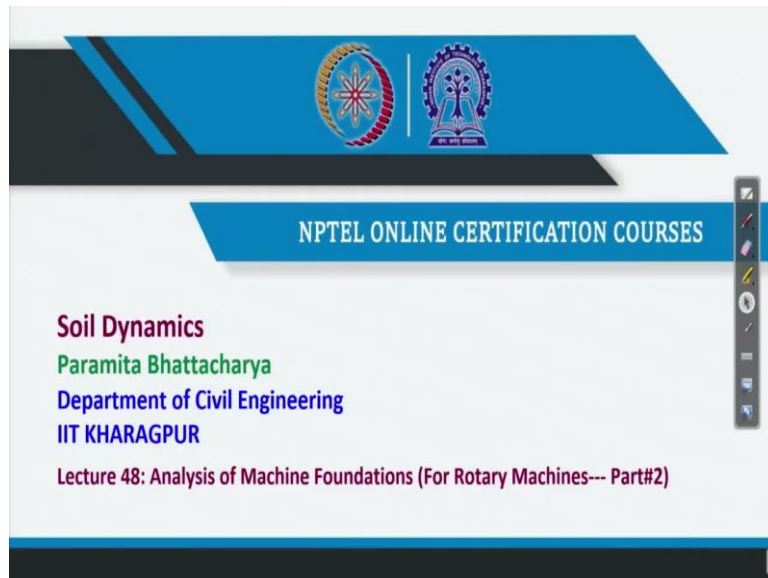


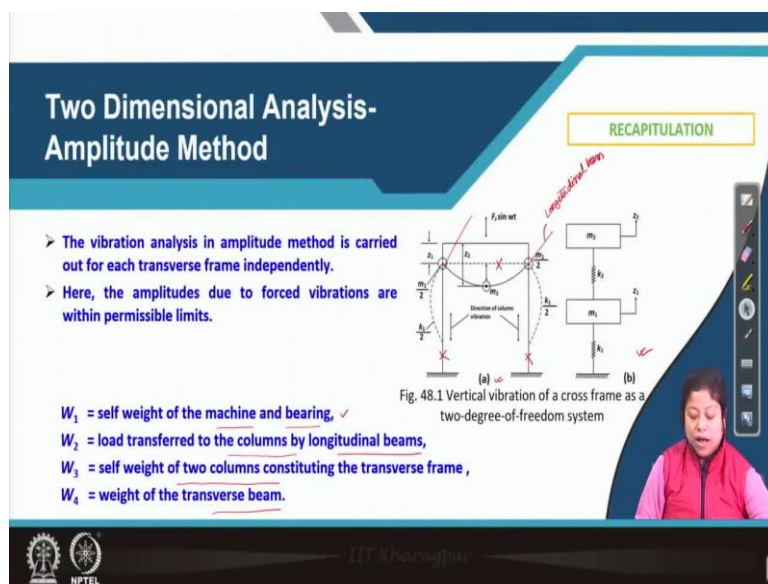
**Soil Dynamics**  
**Professor Paramita Bhattacharya**  
**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 48**  
**Analysis of Machine Foundations**  
**(For Rotary Machines – Part II)**

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Welcome friends to the course soil dynamics. Today, we will continue our discussion on analysis of machine foundations for rotary machines today is the second class on this topic.

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So, last class we have discussed a several things, we have discussed the 2 methods one is resonance method to find out the natural frequency and the amplitude often the frame

foundation is subjected to vertical vibration and horizontal vibration. Then we have started discussion on amplitude method to find out the natural frequency of the frame foundation under vertical vibration. So for that we have considered one independent transverse frame as shown in figure a, and that can be represented by our 2 degrees of freedom system, which you can see in figure b.

Also, in these case the amplitudes due to forced vibrations are considered within the permissible limit. So, before analyzing the problem 2 degrees of freedom problem first what we need to consider we need to find out or we need to know the value of W 1 which includes the self-weight of the machine and the bearing together. Then we need to know the W 2 which is the load transferred to the columns by longitudinal beams.

Then next step is to find out or calculate or determine the self-weight of 2 columns constituting the transverse frame. So, these 2 columns are these 2 which I have marked by the cross sign, also we need to know the weight of the transverse beam that means, this beam. So, one thing when we calculate the load transferred to the columns by the longitudinal beam, longitudinal beam means a beam in this direction.

So, this is the longitudinal beam you can take one likewise the side also the second one. So, these beam is basically simply supported beam rest in each beam each transfers sorry longitudinal beam is simply supported by the 2 columns. So, whatever load you will estimate for the longitudinal beam that will be divided into 2 columns.

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RECAPITULATION

## Two Dimensional Analysis- Amplitude Method

➤ The natural frequencies can be determined by solving the following Equation:

$$\omega_n^4 - (1 + \eta_m)(\omega_{n11}^2 + \omega_{n12}^2)\omega_n^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2) = 0$$

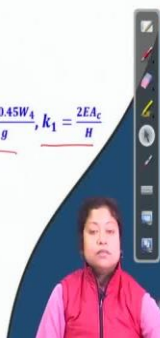
where,  $\omega_{n11}^2 = \frac{k_1}{m_1 + m_2}$  and  $\omega_{n12}^2 = \frac{k_2}{m_2}$  and  $\eta_m = \frac{m_2}{m_1}$ ,  $m_1 = \frac{W_1 + W_2 + 0.33W_3 + 0.25W_4}{g}$ ,  $m_2 = \frac{W_1 + 0.45W_4}{g}$ ,  $k_1 = \frac{2EA_c}{H}$


and  $k_2 = \frac{1}{\delta_{st}}$  where  $\delta_{st} = \frac{L(2K+1)}{96EI_b(K+2)} + \frac{3L}{8GJ_b}$

➤ Thus amplitudes of vibration are: (Vertical vibration)

$$A_{x1} = \frac{\omega_{n11}^2 F_z}{m_1 [\omega^4 - (1 + \eta_m)\omega^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2)]}$$

$$A_{x2} = \frac{[(1 + \eta_m)\omega_{n11}^2 + \eta_m \omega_{n12}^2 - \omega^2] F_z}{m_2 [\omega^4 - (1 + \eta_m)\omega^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2)]}$$





- $H$  is the effective height of the column
- $L$  is the effective span of the column
- Here,  $L = L_0 - 2\alpha b$  and  $H = H_0 - 2\alpha a$

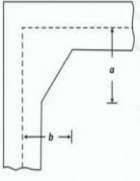


Fig. 48.2 A typical frame with haunches

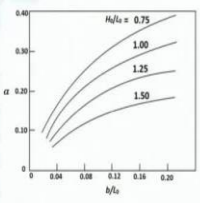


Fig. 48.3  $\alpha$  vs  $b/L_0$

- $\alpha$  can be calculated from the following Fig. 48.3
- $H_0$  is Height of the column from the top of the base slab to the centre of the frame beam
- $L_0$  is the centre to centre distance between columns.

## Two Dimensional Analysis- Amplitude Method

RECAPITULATION

- The vibration analysis in amplitude method is carried out for each transverse frame independently.
- Here, the amplitudes due to forced vibrations are within permissible limits.

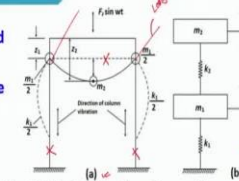


Fig. 48.1 Vertical vibration of a cross frame as a two-degree-of-freedom system

- $W_1$  = self weight of the machine and bearing, ✓
- $W_2$  = load transferred to the columns by longitudinal beams,
- $W_3$  = self weight of two columns constituting the transverse frame,
- $W_4$  = weight of the transverse beam.

Now, after knowing the  $W_1, W_2, W_3$  and  $W_4$  what we can do? We can calculate  $m_1$  and  $m_2$ . After  $m_1$  and  $m_2$  we can calculate the stiffness coefficient  $k_1$  and  $k_2$ . So, here you can see  $k_1$  is equal to 2 times of  $E$  times  $A_c$  divided by  $H$ . So, what is  $H$  what is  $A_c$  here?  $A_c$  is the cross sectional area of the column whereas,  $H$  if you see is that effective height of the column. Now  $k_2$ ,  $k_2$  is equal to  $1$  divided by  $\Delta_{st}$ , where  $\Delta_{st}$  is calculated from this expression.

So, in this expression  $m$  is the effective span length, whereas,  $I_b$  is the moment of inertia of beam,  $A_b$  is the cross sectional area of the beam  $G$  and  $E$  are shear modulus and elastic modulus respectively. So, after knowing  $m_1, m_2, k_1, k_2$  we can calculate also  $\eta_m$  which is  $m_2$  divided by  $m_1$  also  $\omega_n^2$  please write it  $\omega_n^2$  and  $\omega_n^2$ , because here I have used the symbol  $\omega_n^2$  and  $\omega_n^2$ .

So,  $\omega_n^2$  can be calculated from this expression which is equal to  $k_1$  divided by  $m_1 + m_2$ . Likewise  $\omega_n^2$  is equal to  $k_2$  divided by  $m_2$ . Now, after knowing  $\eta$  which is the ratio of the mass  $m_2$  to mass  $m_1$  also knowing  $\omega_n^2$  you can form this equation and from this equation you can calculate the natural frequency of the 2 degrees of freedom system.

So, here if you see, since the degrees of freedom is 2 for this system, you will get 2 natural frequencies  $\omega_n^2$  by solving this equation, after that you can calculate also the amplitudes of vibration this is for vertical vibration. So, this part we have discussed in last class.

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- $H$  is the effective height of the column
- $L$  is the effective span of the column
- Here,  $L = L_0 - 2ab$  and  $H = H_0 - 2aa$

- $\alpha$  can be calculated from the following Fig. 48.3

Fig. 48.2 A typical frame with haunches

Fig. 48.3  $\alpha$  vs  $b/L_0$

- $H_0$  is Height of the column from the top of the base slab to the centre of the frame beam
- $L_0$  is the centre to centre distance between columns.

I just like to mention here what is the meaning of the term used in previous equation. So,  $H$  is that effective height of the column whereas,  $L$  is the effective span of that sorry it is not effective span of the column it is effective span of the frame I can it is better to use the word effective frame. So, what exactly it is if I will draw the frame like these, so basically we have a frame like this.

So, in this frame, we can find out the effective height of the frame and effective span. So, that so, this is your  $L_0$  likewise you can calculate  $H_0$  this one and what is then. So, in this expression now, you know  $L_0$ ,  $H_0$ ,  $H_0$  is the height of the column from the top of the base slab to the centre of the frame beam. So, this is the centerline of the transverse beam. Likewise  $L_0$  is the centre to centre distance between columns.

Now, when you know  $H_0$  and  $L_0$  from if instead of this kind of shape, please note that these lines are not very straight. So, I can give I can write yes, so, now, if hunches are present as you can see here, now, if you can see the hunches are present then what you need to do, you need to find out the values for a and b that you can see here. So, basically b is width of the half of the width of the column, whereas a is the half of the depth of the beam. So, if I will refer to the adjacent frame, then I can write what is 2 a and what is 2 b.

So, this is basically 2 a likewise this is basically 2 b. I hope these diagrams are clear to all of you. So, anyway now, when these hunches are present, then what we need to do, we need to calculate first the values of  $H_0$  by  $L_0$ , you know what is  $H_0$  and  $L_0$ . So, you will find out  $H_0$  by  $L_0$  also you will find out b divided by  $L_0$ . Now, depending upon these 2 parameters, you can find out the value for alpha from this figure 48.3.

Now, after knowing alpha you can calculate a, which is the effective span of the frame, likewise, you can calculate each which is the effective height of the column. Now, this part we have discussed in last class. Now, today what we will do, we will continue our discussion for horizontal vibration of the system.

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• The columns are taken to act as leaf springs and its stiffness is equal to the lateral stiffness of the individual transverse frame.

Fig. 48.4 Spring-mass model for combined horizontal and rotational vibrations of the deck slab

- It is assumed that upper and lower foundation slabs are assumed to be infinitely rigid.
- The equivalent mass  $m_i$  lumped over the spring  $i$  (representing  $i^{\text{th}}$  frame) is:
 
$$m_i = m_{mi} + m_{bi} + 0.33m_{ci} + m_{gi} \quad \dots (1)$$

$m_{mi}$  = Mass of machine resting on cross beam of  $i^{\text{th}}$  frame ✓  
 $m_{bi}$  = Mass of cross beam of  $i^{\text{th}}$  frame  
 $m_{ci}$  = Mass of columns of  $i^{\text{th}}$  frame  
 $m_{gi}$  = Mass transferred from longitudinal girders on either side

So, here you can see the, for horizontal vibration the mathematical model in figure 48.4. So, originally the slab deck was A 1, B 1. Now, after vibration its position is A 3 this is B 3, what is G 1 and G 2? So, G 1 is centre of masses whereas, G 2 is centre of stiffnesses. So, you can see there are 3 masses  $m_1$ ,  $m_2$ ,  $m_3$ , so when we will consider these 3 masses we will get

the centre of masses at G 1, likewise centre of stiffnesses for you can see k a x 3, k a x 2, k a x 1, it is G 2.

Now, the horizontal distance between G 1 and G 2 in this figure is small e. Also, you can find out the distance of the mass m 1 from the from G 2 is d k 1, for mass m 2 it is d k 2, and for mass m 3 it is d k 3. Likewise, these tails of the distance between mass m 1 and G 1 is d m 1, for mass m 2 it is d m 2, and for mass m 3 it is d m 3 and Psi is the rotation with respect to the horizontal you can see in this figure.

Now in this case it is assumed that the upper and lower foundation slabs are assumed to be infinitely rigid. Second thing the equivalent masses the equivalent mass m i lumped over the spring i which representing the i-eth frame and that lumped equivalent mass m i lumped over the spring i is equal to m m i plus m b i plus 0.33 times m c i plus m g i. So, what is m m i? You can see here in m m i is the mass of machine, which is resting on the cross beam of i-eth frame.

Likewise m b i is the mass of cross beam for i-eth frame, m c i is the mass of column for i-eth frame and m g i is the mass which is transferred from the longitudinal girders or beam on either side. So, if we know the values for m m i, m b i, m c i, and m g i, we can calculate the equivalent mass m i using equation 1. Also in this method, the columns are taking to act as leaf springs and its stiffness can be calculated which is equal to the lateral stiffness of the individual transverse frame.

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The equations of motion for the system shown here is:

$$\left( \sum m_i \right) \ddot{x} + \sum [k_{xi}(x + d_{mi}\psi)] = F_x \sin \omega t \quad \dots (2)$$

$$\left( \sum m_i d_{mi}^2 \right) \ddot{\psi} + \sum [k_{xi}(x + d_{mi}\psi)d_{mi}] = M_z \sin \omega t \quad \dots (3)$$

where,  $M_z$  = unbalanced moment  
 $F_x$  = unbalanced horizontal force  
 $m = \sum m_i$  = total mass  
 $M_{mz} = \sum m_i d_{mi}^2$  = Polar mass moment of inertia of all the masses about the vertical axis through G,

$$\sum [k_{xi}(x + d_{mi}\psi)] = x \sum k_{xi} + \left( \sum k_{xi}d_{mi} \right) \psi$$

$$= xk_x + k_x e \psi = k_x(x + e \psi)$$

$k_x$  representing the total lateral stiffness is:

$$k_{xi} = \frac{1}{\delta_{hi}} \quad \dots (4)$$

where,  $\delta_{hi} = \frac{H^3}{12EI_c} \frac{3K+2}{6K+1} + \frac{6H}{5FA_c} \left[ 1 + \frac{A_c H}{A_d l} \frac{18K^2}{(6K+1)^2} \right] + \frac{H^3}{EA_c l^2} \frac{18K^2}{(6K+1)^2}$

Now, from the mathematical model of the 2 degrees of freedom system, which I have already shown, from that, you can calculate the you can derive the equations of motion. So, first one for the horizon is subjected to horizontal force and the second one subjected to. So, these are the 2 equations of motion for the system which already shown to you now, from these if we know that data for F x and M z, which are unbalanced F x is unbalanced horizontal force whereas, M z is unbalanced movement and if you see summation of m i is the total mass that will be already given. So, from that you can first write the equations of motions.

Now, in this equation you can see a term which is summation of m i times d m i square. So, what is it exactly? It is nothing but the polar mass moment of inertia of all the masses about the vertical axis passing through G 1. I hope you have remember G 1 is the centre of masses for this model which is shown in the previous figure. So, these summation of m i times d m i square can be written as M m z. Likewise, the summation of a m i may be written as small m.

Also the second term of equation 2 can be written as k x times x plus e times Psi, where e is the horizontal distance between G 1 and G 2 and Psi is the angular rotation for the slab A 3, B 3. Now, here we need to know k x. So, k x represents the total stiffness and that can be calculated by the expression 1 divided by delta h i. So, what is delta h i then? Here in equation 4 what is delta h i? Delta h i is shown here so, we can calculate delta h i using this expression. So, here we know what is A c which is the cross sectional area the column, A b is cross sectional area of the beam. Likewise, I c is also known the capital H is also known.

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• In Equation (3) the second term on the L.H.S. can be written as:

$$\sum [k_{xi}(x + d_{mi}\psi)d_{mi}] = x \left( \sum k_{xi}d_{mi} \right) + \left( \sum k_{xi}d_{mi}^2 \right) \psi = k_x ex + \left[ \sum k_{xi}(e^2 + d_{ki}^2) \right] \psi$$

$$= k_x ex + k_x e^2 \psi + \left( \sum k_{xi}d_{ki}^2 \right) \psi = k_x(x + e\psi)e + k_\psi \psi$$

Here, the equivalent torsional spring stiffness for the frame columns is :

$$k_\psi = \sum k_{xi}d_{ki}^2$$

• Finally, the equations of motion (i.e. Equations [2] and [3]) can be written as:

$$m\ddot{x} + k_x x + k_x e \psi = F_x \sin \omega t \quad \dots (5)$$

$$M_{mz} \ddot{\psi} + k_x ex + (k_x e^2 + k_\psi) \psi = M_z \sin \omega t \quad \dots (6)$$

• The solution is being given by the equation:

$$\omega_n^4 - (\alpha_c \omega_{nx}^2 + \omega_n^2) \omega_n^2 + \omega_{nx}^2 \omega_n^2 = 0 \quad \dots (7)$$

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Now, in equation 3, the second term that means this term can be written in other way. So, let us see, yes, we can simplify this expression also as  $k_x$  times  $e$  times  $x$  plus summation of  $k_x$   $i$  times  $e$  square plus  $d k_x i$  square and this summation thing should be multiplied with  $\Psi$  which is the angular rotation. And finally, you can write it as  $k_x$  plus sorry  $k_x$  times  $x$  plus  $e$  times  $\Psi$  times  $e$  plus  $K \Psi$  times  $\Psi$ . So, what is  $k \Psi$  hear? This expression actually nothing but  $k \Psi$ , sorry not these expression what is  $K \Psi$ ?  $K \Psi$  is this expression. So, it is your  $k \Psi$ .

So, now, you can use the expression for  $k \Psi$ . Now, after this simplification, what we can do? We can rewrite the equations 2 and 3, which we have already seen in the previous slide as this one and this one. So, here we know all the terms what is small  $m$  what is capital  $M$ ,  $M_z$ , likewise we know how to express  $k_x$  and  $k \Psi$  also.

Now this in order to solve these 2 equations, we need what first we will do, we will first solve the free vibrating system considering the right hand side 0 for that, we will get this equation and in this equation, what is what we will get finally, from this equation? We will finally get the 2 natural frequencies for the 2 degrees of freedom system that already shown to you. So, we will find out  $\omega_{n1}$  and  $\omega_{n2}$  from equation 7.

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• Here,  $\omega_{nx} = \sqrt{\frac{k_x}{m}}$ ;  $\omega_n \psi = \sqrt{\frac{k_\psi}{M_{mz}}}$ ;  $\alpha_e = 1 + \frac{e^2}{r^2}$  and  $r = \sqrt{\frac{M_{mz}}{m}}$

• The amplitude of vibration in translation is:

$$A_x = \frac{\left[ \frac{e^2}{r^2} \omega_{nx}^2 + \omega_n^2 \psi - \omega^2 \right] \frac{F_x}{m} - \omega_{nx}^2 \frac{M_x}{M_{mz}}}{\Delta(\omega^2)} \quad \dots (8)$$

• The amplitude of vibration in rotation is:

$$A_\psi = \frac{\frac{e^2}{r^2} \omega_{nx}^2 \frac{F_x}{m} - (\omega_n^2 \psi - \omega^2) \frac{M_x}{M_{mz}}}{\Delta(\omega^2)} \quad \dots (9)$$

where,  $\Delta(\omega^2) = \omega^4 - (\alpha_e \omega_{nx}^2 + \omega_n^2 \psi) \omega^2 + \omega_{nx}^2 \omega_n^2 \psi$

• The net horizontal amplitude is:

$$A_h = A_x + y A_\psi \quad \dots (10)$$

where,  $y$  = Distance of the point at which the amplitude is being calculated from the centre of gravity of the system.



• In Equation (3) the second term on the L.H.S. can be written as:

$$\sum [k_{xi}(x + d_{mi}\psi)d_{mi}] = x \left( \sum k_{xi}d_{mi} \right) + \left( \sum k_{xi}d_{mi}^2 \right) \psi = k_x e x + \left[ \sum k_{xi}(e^2 + d_{ki}^2) \right] \psi$$

$$= k_x e x + k_x e^2 \psi + \left( \sum k_{xi}d_{ki}^2 \right) \psi = k_x(x + e\psi)e + k_\psi \psi$$

Here, the equivalent torsional spring stiffness for the frame columns is :

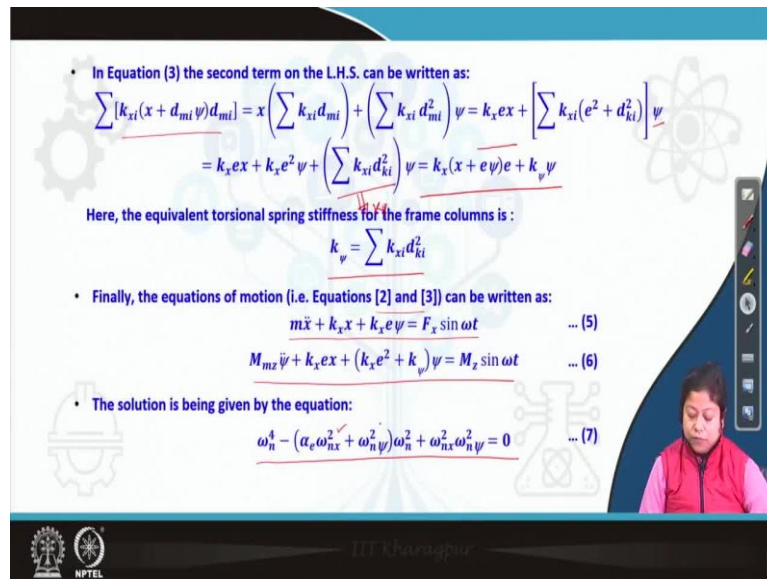
$$k_\psi = \sum k_{xi}d_{ki}^2$$

• Finally, the equations of motion (i.e. Equations [2] and [3]) can be written as:

$$m\ddot{x} + k_x x + k_x e \psi = F_x \sin \omega t \quad \dots (5)$$

$$M_{mz} \ddot{\psi} + k_x e x + (k_x e^2 + k_\psi) \psi = M_z \sin \omega t \quad \dots (6)$$

• The solution is being given by the equation:

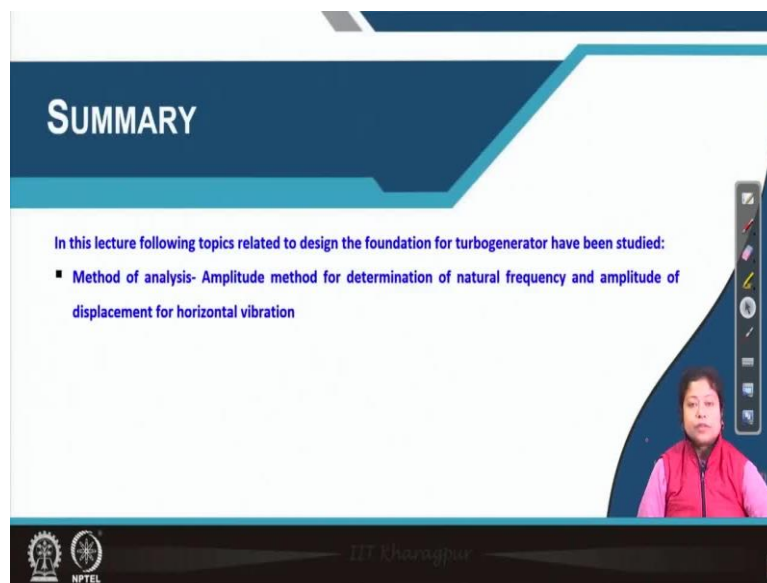
$$\omega_n^4 - (\alpha_e \omega_{nx}^2 + \omega_{n\psi}^2) \omega_n^2 + \omega_{nx}^2 \omega_{n\psi}^2 = 0 \quad \dots (7)$$


Now, here what is  $\omega_n x$  which is used in equation 7, and what is  $\omega_n \psi$  that is mentioned in this. So,  $\omega_n x$  is square root of  $k_x$  by  $m$  likewise  $\omega_n \psi$  is square root of  $k_\psi$  by  $M_{mz}$  and  $\alpha_e$  is equal to  $1 + e^2$  divided by  $r^2$ , where  $r$  is the ratio of square root of  $M_{mz}$  by  $m$ .

Now, after solving the equation 7, what you can get, you can get the amplitude shown in equation 8, you can also find out the amplitude of vibration in rotation which is  $A_\psi$  using equation 9. After knowing  $A_x$  and  $A_\psi$ , you can find out the net result net horizontal amplitude which is  $A_h$  and for that you need to use that expression thing.

Here  $y$  is that distance of the point at which the amplitude is being calculated from the centre of gravity of the system. So, the point under consideration and its distance from the c.g. of the entire system is represented by  $y$  here. And what is  $\delta \omega^2$  here that is this expression, just make it  $\alpha_e$  this is not  $\alpha$ , but  $\alpha_e$  and  $\alpha_e$  you have already seen.

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**SUMMARY**

In this lecture following topics related to design the foundation for turbogenerator have been studied:

- Method of analysis- Amplitude method for determination of natural frequency and amplitude of displacement for horizontal vibration

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**REFERENCES**

1. Dynamics of base and foundations by D. D. Barkan, McGraw-Hill, New York, 1962.
2. Maschinen fundamente und andere dynamisch beanspruchte Baukonstruktionen by E. Rausch, VDI Verlag, Dusseldorf, 1959.
3. IS: 2974 (Part 3) - (1992), "Foundations for rotary-type machines (Medium and high frequency) ".

The slide features a dark blue header with the word 'REFERENCES' in white. Below the header, there is a white area with a blue border containing a numbered list of three references. On the right side, there is a vertical toolbar with various icons. The footer contains the NPTEL logo and the name 'Dr. Khairul Anwar'.

So, come to the summary of today's class. So, today we have discussed the amplitude method for determination of natural frequency and amplitude of displacement for horizontal vibration. So, now, we have learned 2 different methods one is resonance method and the second one is amplitude method using these 2 methods, we will try to solve one numerical problem in the next class. So, these are the references for today's class. Thank you.